WHEN CHEAPER IS BETTER: FEE DETERMINATION IN THE MARKET FOR EQUITY MUTUAL FUNDS

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Abstract

In this paper, we develop a model of the market for equity mutual funds that captures three key characteristics of this market. First, there is competition among funds. Second, fund managers' ability is not observed by investors before making their investment decisions. And third, some investors do not make optimal use of all available information. The main results of the paper are that 1) price competition is compatible with positive mark-ups in equilibrium; and 2) worse-performing funds set fees that are greater or equal than those set by better-performing funds. These predictions are supported by available empirical evidence.

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Abstract

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1 Introduction

In 2003, total mutual fund assets in the U.S. were worth 7.4 trillion dollars and constituted an estimated 18.4 percent of the total financial wealth of U.S. households (Investment Company Institute, 2004). The increasing reliance of American investors on mutual funds has raised concerns among industry commentators and regulators alike about the extent of competition in the industry and, in particular, about the level of fees that mutual funds charge investors for their services. These concerns have prompted regulators to commission reports (Securities and Exchange Commission, 2000; General Accounting Office, 2000) to analyze the evolution and determinants of mutual fund fees. More recently, a series of scandals have triggered regulatory initiatives that, among other things, strengthen mutual fund fee disclosure requirements so as to promote price competition in the industry.

Underlying this debate is a long overdue question: How are fees determined in the mutual fund market? To address this question, in this paper, we develop a model of the market for equity mutual funds.

One of the main concerns fueling the debate over mutual fund fees, and a key motivation for the regulatory changes requiring improved fee disclosure, has been the degree to which investors are aware of the fees associated with fund investments and their impact on the return of those investments. A survey by the SEC and the Office of the Comptroller of the Currency (Alexander et al, 1997) reports that fewer than one in six fund investors understand that higher fund expenses—which are deducted from the fund’s assets and mostly consist of management fees—lead to lower returns. The survey also documents that investors are not familiar with the level of fees they are paying for mutual fund services (not even 20% of the respondents was able to give an estimate of the expenses paid for their largest mutual fund) and reveals other gaps in financial literacy. More recently, a study by Barber et al. (2004) has provided further evidence of investors’ difficulties understanding the effects of

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1 For example, in a recent article about the issue in The Economist, one could read: “Retail investors [...] have seen precious little competition on prices. Even for wretched performance, reductions in fees have been all too rare.” (The Economist, 2004).

2 See, for instance, Dwyer, Borrus and Young (2003) for an account of these scandals.

3 The Securities and Exchange Commission has issued a rule establishing stronger disclosure requirements for mutual funds (Securities and Exchange Commission, 2004). The “Mutual Funds Integrity and Fee Transparency Act of 2003”, which imposes strict governance and disclosure requirements passed the the U.S. House overwhelmingly in November, 2003.
mutual fund fees. In the light of these findings, it is our view that a satisfactory model of the mutual fund market should account for the presence of a significant fraction of investors who make less than optimal use of the information available when making their investment decisions.

The model developed in this paper has three main ingredients, which, we believe, characterize the market for equity mutual funds. In the U.S., investors can choose from a large pool of mutual funds, even for relatively narrowly defined investment categories. Therefore, the first ingredient of our model is competition among—possibly many—funds for investors’ money. This ingredient is key, because we would like to know whether existing concerns about the level of fees charged by mutual funds can still be warranted in the presence of mutual fund competition. Since our main focus is the market for equity mutual funds, the second ingredient is quality uncertainty: different fund managers have different abilities to generate returns, but those abilities are not known \textit{ex ante} by investors. Even though in actual markets, funds’ past returns could be used as a signal of performance, this signal is, at best, highly noisy. Therefore, the assumption that quality is not observable appears as a reasonable first approximation. The third ingredient is the presence of a fraction of investors, which we will label \textit{unsophisticated investors}, who do not make optimal use of all available information when making their investment decisions.

The model enables us to address several questions: 1) Should we expect price competition among funds to bring equilibrium profits down to zero? 2) Will high-quality funds drive low-quality funds out of the market? If low-quality funds manage to survive in equilibrium, 3) will high-quality funds charge higher fees, so that, in equilibrium, all investors earn the same expected net returns? If, to the contrary, net returns differ across funds, 4) which funds will be more likely to overcharge investors: high- or low-quality funds? These questions lie at the heart of the debate over the extent of competition in the mutual fund industry and are of key importance to evaluate proposed regulatory changes. The model shows that in equilibrium 1) funds earn positive profits, 2) high- and low-quality funds coexist, 3) high-quality funds never charge higher fees and may charge lower fees, and that, as a consequence 4) low-quality funds greatly overcharge investors.

The intuition behind these, perhaps surprising, results is as follows. In the mutual fund
market, the revenues earned by a mutual fund are the product of its fee and the fund asset value. Therefore, in a setting in which quality is unobservable, high-quality funds—namely, those that deliver a higher value—may be able to differentiate themselves by setting low fees. If those fees are low enough, low-quality funds would not break even by imitating them. Unable to compete for the sophisticated segment of the market, low-quality funds would focus instead on extracting rents from unsophisticated investors. It should be noted that although the presence of unsophisticated investors is necessary to obtain this sort of price differentiation in equilibrium, it is not sufficient. The model’s results follow from the interaction between asymmetric information and the existence of those investors.

The available empirical evidence is fully consistent with our model: funds not only fail to adjust fees so as to offset differences in before-fee returns, but, to the contrary, funds of lower quality charge higher fees. Elton et al. (1993), for instance, document significant differences in after-fee returns across mutual funds and show that funds that charge higher fees deliver significantly lower before-fee returns. Additional evidence provided by Gruber (1996), Carhart (1997), Harless and Peterson (1998), and Chevalier and Ellison (1999) has consistently confirmed the power of high fees to predict underperformance. Surprisingly, this anomalous pattern in the data has not been explained by any previous formal model of the mutual fund industry.

Although there exists a relatively large theoretical literature, initiated by Bhattacharya and Pfleiderer (1985), that aims at characterizing the optimal compensation contract in a delegated portfolio management problem, few studies have analyzed fund fees as the outcome of the strategic interaction of competing mutual funds. Recently, Hortacsu and Syverson (2004) have developed a search model of the market for S&P 500 index funds. In contrast to our paper, however, they analyze a sector in which financial performance differences across funds are relatively small and thus focus on non-portfolio fund differentiation and search frictions as potential sources of fee dispersion. In another recent paper, Berk and Green (2004) have proposed a model of the mutual fund market with no informational asymmetries or search frictions. The goal of their model is to explain why money may rationally follow past good performance even when past performance is not a strong

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4See, e.g. Palomino and Prat (2003) for a recent contribution.
predictor of future performance. The paper is, however, silent about the determinants of the observed distribution of fees. Das and Sundaram (2002) and Metrick and Zeckhauser (1999) have analyzed fee setting in a duopoly context with asymmetric information about fund quality. While Das and Sundaram (2002) compare the performance of two types of incentive schemes for fund managers in such a context, the goal of Metrick and Zeckhauser (1999) is to explain why high- and low-quality producers may charge the same price in certain markets (including the mutual fund market), and is, thus, closely related to ours. We discuss their results in section 2. In another related paper, Nanda et al. (2000) have developed a model where mutual fund managers of observable quality bear the cost of stochastic investment redemptions, and therefore wish to attract investors who are less likely to experience liquidity needs. In equilibrium, more skilled fund managers impose exit fees and cater investors with lower liquidity needs, while less skilled managers become liquidity providers. Finally, Christoffersen and Musto (2002) have recently explored empirically the effect of investors’ performance sensitivity on fund fees.

Our paper is also related to the more general literature on the role of prices as signals of quality. In this literature, high prices generally signal high quality (e.g., Bagwell and Riordan, 1991), although in some contexts involving repeated purchases, it has been shown that low introductory prices can be used as signals of quality (Schmalensee, 1978). This set of work, however, has mostly focused on the case of a single seller of unknown quality. Only recently, Fluet and Garella (2002) and Hertzendorf and Overgaard (2001) have studied price and advertising signaling for the case of a duopoly. In their models, high-quality firms charge higher prices in any separating equilibrium. Of related interest is a recent working paper by Gabaix and Laibson (2003), in which the authors argue that product—or pricing–complexity can allow firms to obtain positive mark-ups in equilibrium and that increases in competition may actually exacerbate the problem.

The rest of the paper is organized as follows: Section 2 presents the benchmark model; Section 3 extends the model to include unsophisticated investors; Section 4 discusses the available evidence on the relationship between performance and fees; and, finally, section 5 concludes.
2 A Model of Fee Determination in the Market for Mutual Funds

Consider a simple setting in which there is a continuum of investors of mass one who have one dollar to invest, and \( N \) mutual fund managers. These managers can be of two types depending on their ability: good \((g)\) and bad \((b)\). G-managers earn gross expected return \( R_g \), and b-managers \( R_b \), where \( R_g > R_b \), and \( R_g > 1 \). Returns are assumed to be independent of fees, so we can abstract from moral hazard problems. The \textit{ex ante} distribution of types is given by the probability \( p \) that a manager is good. Once the types are realized, fund managers observe their quality but not the quality of their rivals and decide what fraction \( e \) of the fund’s final asset value to charge to investors. The assumption that fees are determined as a fraction of asset value is made to reflect actual practice in the mutual fund industry. In section 4, we discuss in greater detail the different fees paid by mutual fund investors.

Investors do not observe quality, so they decide where to invest on the basis of the prior distribution and the fees charged by the different funds. We will assume that a fraction \( \gamma \) of all investors are unsophisticated, in the sense that they do not make optimal use of all available information. Since we will first analyze the case with no unsophisticated investors, we postpone the discussion of the precise form in which these investors behave until they are introduced in section 3.

The costs of managing the fund are \( cw \), where \( w \) is the amount of money managed by the fund manager. It is assumed that costs are low enough to make it profitable for g-funds to operate if their type is known. We assume that all market participants are risk-neutral, and that the only alternative investment is a risk-free asset paying zero interest rate. Therefore, the maximum fee a fund of type \( k \) can charge if its type is known is such that the net return for investors is equal to one: \( R_k(1-e) = 1 \), that is \( e = \frac{R_k-1}{R_k} \). Therefore, the assumption that g-funds may profitably operate if their type is known can be expressed as follows:

**Assumption 1**

\[
\frac{R_g - 1}{R_g} > \frac{c}{R_g},
\]
where the right-hand side of the inequality is the break-even fee for g-funds.

We also assume that, given \( c \), b-funds may find it profitable to operate for some fee less than one hundred percent:

**Assumption 2** \( R_b > c \).

We denote by \( e_k \) the fee charged by a manager of type \( k \) as a proportion of the value of the fund at the end of the period and assume that there are no other fees. Therefore, the amount paid by an investor who invests \( w \) dollars in a fund of type \( k \) is \( we_k R_k \), and payoffs are \( w(e_k R_k - c) \) for the manager and \( w(1 - e_k) R_k \) for the investor.

Finally, the timing of decisions is as follows. First, managers simultaneously set fees. Then investors decide where to invest. We make the assumption that, if several funds have the same net expected returns, investors allocate their wealth among them with equal probability.

### 2.1 Benchmark Case: Complete Information

Before solving the model, it is instructive to investigate the relationship between fund quality and fees when there are no unsophisticated investors and quality is observed both by competing funds and by investors. It is straightforward to show that, in this case, b-funds will be driven out of the market whenever there are g-funds:

**Proposition 1** *With complete information, there do not exist equilibria in which both fund types operate simultaneously.*

**Proof.** First, note that for both types of funds to have a positive market share:

\[
(1 - e_b) R_b = (1 - e_g) R_g, \tag{1}
\]

which implies that \( e_b < e_g \).

With complete information the number of funds of each type is given and commonly known. We study first the case in which there is only one g-fund active, and then the case in which there are at least two g-funds. So, suppose first that there is only one g-fund. For b-funds to operate \( e_b \geq \frac{c}{R_g} > \frac{c}{R_b} \), which implies that \( e_g > \frac{c}{R_g} \). Therefore, if \( e_b \geq \frac{c}{R_b} \), it would be profitable for the g-fund to lower \( e_g \) slightly and attract the whole market. It
follows that, in equilibrium, b-funds cannot operate. The g-fund will set the minimum fee that guarantees that b-funds do not want to enter, which is given by:

\[(1 - e_g)R_g = (1 - \frac{c}{R_b})R_b\]  

(2)

If there are several g-funds, the same argument applies for any \( e_g > \frac{c}{R_g} \). The only possible equilibrium fee is \( e_g = \frac{c}{R_g} < \frac{c}{R_b} \), so b-funds remain inactive. □

Therefore, good and bad funds cannot coexist in equilibrium with complete information: whenever it is profitable for b-funds to operate, it is also profitable for g-funds to lower fees.\(^5\) As a result, b-funds are driven out of the market.

### 2.2 Asymmetric Information

We now investigate what happens when there is asymmetric information regarding funds’ ability to generate returns. To understand the contribution of the model’s different assumptions, we maintain the assumption that there are no unsophisticated investors. We use the Perfect Bayesian Equilibrium as our equilibrium concept and focus only on pure-strategy symmetric equilibria (i.e., equilibria in which all funds of the same type play the same pure strategy). To limit equilibrium multiplicity, we require investors’ out-of-equilibrium beliefs to satisfy the property that they do not assign positive probability to managers setting fees that are certain to yield them a negative profit. That is, investors cannot assign a positive probability to a fund of type \( k \) choosing a fee less than \( \frac{c}{R_k} \). Therefore, throughout the paper, by equilibrium we will refer to a pure-strategy Perfect Bayesian Equilibrium satisfying this restriction on investors’ beliefs.

First, note that in a separating equilibrium in which both types are ever active simultaneously, it has to be the case that net returns for investors are equal across types, since otherwise investors would not invest with the two types when both are available. Equality of net returns, in turn, implies that the expected market shares of g- and b-funds also have to be equal,\(^6\) since investors are indifferent between both types and, therefore, allocate the

\(^5\)In models of vertical differentiation (e.g. Shaked and Sutton, 1982), equilibria in which low- and high-quality producers coexist—with the former charging lower prices—are possible if consumers display differences in their willingness to pay for quality. In the mutual fund industry, however, where the good provided by sellers is end-of-period dollars, one would expect that all consumers have the same willingness to pay for quality: nobody would pay more cents than anybody else for a dollar.

\(^6\)The expectation is taken over the possible realizations of the number of b- and g-funds.
money across funds with equal probability. But this implies that, if \( e_g > e_b \), it would be optimal for b-managers to imitate g-managers. On the other hand, if \( e_g < e_b \), no rational investor that observes both fees would invest with a b-manager. Therefore:

**Proposition 2** If all investors react optimally to differences in expected payoffs, there are no separating equilibria in which both fund-types operate simultaneously.

According to Proposition 2, we should observe no fee dispersion in equilibrium: for any realization of the number of b- and g-funds, all the funds with a positive market share must charge the same fees. Equilibria at which b- and g-funds are active simultaneously and the latter charge higher fees are not possible.

It is straightforward to show that, with asymmetric information, equilibria like the one obtained with complete information, in which only g-funds are active and make zero profits, are not possible. The reason is that any g-fund faces a positive probability of competing only against b-funds. Therefore, there exists a strategy that guarantees positive expected profits to g-funds: setting a fee greater than the break-even fee for g-funds (\( \frac{c}{R_g} \)) but lower than the break-even fee for b-funds (\( \frac{c}{R_b} \)). Such a fee guarantees positive profits if the fund is ever able to attract any money and, as long as it is not too high, ensures that the fund would attract investors’ money at least when there are no competing g-funds. The fact that g-funds have a strategy that guarantees positive expected profits immediately rules out Bertrand-like equilibria, like the one that results from complete information. It is possible to show, though, that, for a broad range of parameter values, there are equilibria in which both types of funds are active, set the same fee and obtain positive profits. With complete information, equilibria with positive profits are not possible. If there is asymmetric information, however, undercutting other funds’ fees can be interpreted by investors as a signal of low quality, so that, if investors are sufficiently pessimistic in their assessment of deviating funds, equilibria with positive profits can arise.\(^7\)

We would like to note here that the existence of an equilibrium at which funds of different qualities set the same fee has already been proposed by Metrick and Zeckhauser (1999), although in a very different context. Metrick and Zeckhauser (1999) study a vertically-
differentiated duopoly characterized by sequential price setting (with good funds setting fees–front-end loads–before bad funds) and by investor heterogeneity along two dimensions: on the one hand, different investors value the “good” provided by mutual funds differently; and, on the other hand, some investors can observe quality, while others cannot. In this context, an equilibrium in which both funds set the same price can arise when the qualities are similar enough. The reason is that competition for the investors who can observe quality is strong in this case. As a result, the good fund may find it optimal to set a fee low enough to force bad funds out of the informed segment of the market. It is important to note that, in their model, good funds attract more money than bad funds in a pooling equilibrium, since the latter do not get any money from informed investors. It is also worth noting that, in their model, there are also separating equilibria in which both funds are active and good funds charge higher fees. In our model, these equilibria are not possible.

3 Unsophisticated Investors

In the introduction, we argued that a satisfactory model of the mutual fund industry should account for the fact that a significant fraction of investors do not make optimal use of all available information when making their investment decisions. Available survey evidence shows that many investors have a limited awareness of the fees charged by the funds they own or others and of the effect that fees have on fund returns. In particular, a survey commissioned by the SEC and the Office of the Comptroller of the Currency (Alexander et al., 1997) shows that 81.2 % of respondents could not give an estimate of the expenses of the largest fund they owned and that, of those investors, only 43% claimed to have known their largest fund’s expenses at the time they first invested in the fund.

The evidence, thus, shows that the behavior of a large fraction of investors is markedly different from that of the sophisticated investors we considered in the previous section, who not only knew the fees charged by all funds, but rightly understood in equilibrium the relation between fees and expected performance. In this section, we do not rule out the existence of these sophisticated investors, but include a fraction of investors with imperfect understanding of the relation between fees and expected net returns. The evidence is, unfortunately, not detailed enough for us to understand the process by which these investors
make their investment choices, so we have opted to model this process in the simplest possible way that satisfies the following three requirements: 1) When purchasing funds, unsophisticated investors do not perform a full search over all possible alternatives; 2) Unsophisticated investors are not completely unresponsive to fees: although they may invest in funds with relatively large fees, they would not invest in funds with outrageously large fees; 3) Unsophisticated investors do not optimally infer fund quality from fees.

To reflect the first requirement, we assume that each unsophisticated investor is paired with a mutual fund at random. This assumption is made for tractability and can be relaxed; what matters is that, of all available alternatives, unsophisticated investors consider only a relatively small set. Once paired with a fund, unsophisticated investors do not invest blindly: they invest only if the fee charged by the fund is below a threshold denoted by $e_U$. This threshold reflects the fact that, even though investors may not be fully aware of the exact value of fees when they lie within a reasonable range, they would detect fees that are conspicuously high. We take a conservative approach and assume that this threshold is relatively low and given by:

$$e_U(1 - \bar{R}) = 1,$$

where $\bar{R} = pR_g + (1 - p)R_b$ is the expected gross return. That is, unsophisticated investors invest only if the expected net return, given the distribution of fund types, is at least as high as that of the alternative asset. Arguably, this requirement demands too much sophistication from unsophisticated investors. As we discuss below, however, our results do not depend on the specific choice of $e_U$ and are consistent with both higher and lower values of this variable. Finally, we assume that unsophisticated investors do not infer any information about a fund’s quality from the fee it charges. Again, we could somewhat relax this assumption. However, as we argue after presenting our results, we do not think that doing this would increase much the realism of our assumptions and would unnecessarily complicate the results.

To derive the market equilibrium when there is a fraction $\gamma$ of unsophisticated investors, note first that a reasoning similar to Proposition 2 still applies in this case: in equilibrium, b-managers and g-managers cannot both serve the sophisticated market segment and charge
different fees. If $e_g > e_b$, b-managers would mimic g-managers’ pricing strategy. If $e_g < e_b$, sophisticated investors would not invest in b-funds. Therefore, if the presence of unsophisticated investors allows for the existence of separating equilibria in which both fund types are active simultaneously, sophisticated investors must prefer one type of fund over the other.

The possibility that b-funds offer a higher net return in equilibrium can be ruled out, since it is straightforward to show that there cannot exist separating equilibria in which g-funds serve only unsophisticated investors and b-funds serve sophisticated investors. Therefore, we investigate next whether there can exist separating equilibria in which both fund types are active simultaneously and in which b-funds serve only unsophisticated investors as long as there are competing g-funds. Obviously, for these equilibria to exist, b-funds must be able to at least break even by charging $e_U$, as, otherwise, they would never be active.

**Assumption 3** $e_U > \frac{e_b}{R_g}$.

The following conditions must hold at this type of equilibrium:

\[
\begin{align*}
    w_g^{U}(R_g e_g - c) &\geq w_b^{U}(R_g e_b - c) \quad \text{(NIg)} \\
    w_b^{U}(R_b e_b - c) &\geq w_g^{U}(R_b e_g - c) \quad \text{(NIb)} \\
    w_b^{U}(R_b e_b - c) &\geq 0 \quad \text{(Pb)}
\end{align*}
\]

where $w_k^U$ is the wealth that a fund setting $e_k$ expects to obtain conditional on all other funds playing the equilibrium strategies. If b-funds serve only unsophisticated investors whenever there are g-funds, $w_g^U > w_b^U$.

The first two conditions above are no-imitation constraints for g- and b-funds, respectively, and the last condition is a participation constraint for b-funds. A participation constraint for g-funds is not necessary, because it is implied by (NIb) and (Pb). Note that, since $w_g^U > w_b^U$, condition (NIb) requires that $e_g < e_b$: in this type of equilibrium, g-funds must set lower fees.

Fees also have to be low enough to convince both sophisticated and unsophisticated to participate:

\[
\begin{align*}
    e_g &\leq \frac{R_g - 1}{R_g} \quad \text{(4)} \\
    e_b &\leq e_U \quad \text{(5)}
\end{align*}
\]
Finally, it cannot be profitable for b- or g-funds to deviate and set an out-of-equilibrium fee. To evaluate these deviations, we need to make assumptions about sophisticated investors’ out-of-equilibrium beliefs. To prove the results below, we assume that sophisticated investors interpret any deviation from equilibrium as coming from a b-fund unless it yields negative profits for such a fund. These extreme beliefs are chosen to simplify the proofs and are not necessary. For our results to hold, all that is required is that sophisticated investors assign a sufficiently high probability to a deviator being of low quality. The next proposition shows that there are parameter values such that all the above conditions hold simultaneously and there are no profitable out-of-equilibrium deviations (all omitted proofs can be found in the appendix):

**Proposition 3** There exist separating equilibria with unsophisticated investors at which:

1. b-funds serve unsophisticated investors only and charge $e^*_b = e_U$.
2. g-funds charge $e^*_g > \frac{c}{R_b}$ and serve both sophisticated and unsophisticated investors.
3. $e^*_b > e^*_g$.

Figures 1-2 show that separating equilibria of this sort can exist for reasonable parameter values. The figures graph the minimum and maximum values of $c$ (plotted along the y-axis) for which these equilibria can exist for each possible value of $\gamma$ (plotted along the x-axis) for the case in which $p = \frac{1}{2}$.

The equilibria described in Proposition 3 are the only possible separating equilibria, but there may also exist pooling equilibria for certain parameter values. At these pooling equilibria, g- and b-funds set the same fee and both types make positive profits.

**Proposition 4** For some parameter values, there exist pooling equilibria in which both types set $e_p > \frac{c}{R_b}$.

Given the complexity of the conditions that define both the pooling and the separating equilibria, we have not attempted to ascertain which type of equilibrium holds for a greater range of parameter values or investors’ beliefs. As we did above for the case of separating
equilibria, we have numerically found the range of values of \( c \) and \( \gamma \) consistent with the existence of a pooling equilibria for the case in which \( p = \frac{1}{2} \). Figures 3-4 shows that pooling equilibria can also exist for reasonable parameter values.

The model in this section departs from the benchmark complete information model in two dimensions, and it is instructive to see how each of these dimensions contributes to the existence of separating equilibria like the ones described in Proposition 3. First, the existence of unsophisticated investors allows b-funds to survive while setting fees that differ from those of g-funds. As we saw in the previous section, this would not be possible if all investors held correct beliefs in equilibrium and could move their money freely. In this respect, our model resembles models of price dispersion based on the presence of search costs (like Salop and Stiglitz, 1977), where impediments to search allow firms charging higher prices to obtain positive market shares. Second, the presence of asymmetric information limits the competitive pressure on g-funds. If sophisticated investors could observe fund quality, competition among g-funds would drive \( e_g \) down to \( \frac{c_R}{R_g} \), but such a situation could not be an equilibrium with unsophisticated investors, because g-funds can set a higher fee, sell to unsophisticated investors and make a positive profit. It should thus be emphasized that the existence of unsophisticated investors alone cannot generate separating equilibria with both fund types active.

In the light of Proposition 3, it is worth reevaluating whether our characterization of unsophisticated investors implies that these investors behave in a way that would be considered unreasonable even for investors as those described in the survey discussed at the beginning of this section. A possible concern is that:

\[
e_U > \frac{R_b - 1}{R_b},
\]

where \( \frac{R_b - 1}{R_b} \) is the fee that guarantees the reservation return when investing with a b-fund. Therefore, at a separating equilibrium, some unsophisticated investors (those paired with b-funds) would do better by investing in the reservation asset. We do not think that this requires too much unsophistication from investors for two reasons. First, unsophisticated investors earn an average return at least as high as that of the alternative asset, since a fraction \( p \) of unsophisticated investors are paired with g-funds. The average return for
unsophisticated investors is:

\[ p(1 - e^*_g)R_g + (1 - p)(1 - e_U)R_b > p(1 - e_U)R_g + (1 - p)(1 - e_U)R_b = \]
\[ = (1 - e_U)(pR_g + (1 - p)R_b) = (1 - e_U)R = 1 \] (6)

The second reason is that most studies coincide in that the average actively managed mutual fund has historically delivered below market returns, at least after expenses and transaction costs are deducted (see, Wermers, 2000, for a recent analysis). In this sense, our model would seem to require too much, rather than too little, sophistication from investors.

Another possible concern relates to the assumption that unsophisticated investors do not update their beliefs about funds’ quality based on the fees charged by those funds. Given that, in a separating equilibrium, good and bad funds set different fees, one may wonder whether separating equilibria like the ones described above could survive. There are again two reasons why we are not especially worried by this concern. The first reason is theoretical. Although we have assumed that \( e_U \) is the maximum fee that an investor would like to pay for a fund of average quality, the proof of Proposition 3 shows that a separating equilibrium can exist as long as \( e_U > 1 - \frac{1}{R_b} \), that is, as long as the maximum fee that unsophisticated investors are willing to pay is greater than the maximum fee a sophisticated investor would be willing to pay for a low-quality fund. Therefore, as long as unsophisticated investors do not fully update their beliefs (i.e., if they do not believe that a fund setting \( e_U \) is bad with probability one), separating equilibria can exist. The second reason is empirical. In the survey discussed above (Alexander et al., 1997), investors were asked about the relation they thought existed between expenses and performance. About 20 percent of the survey respondents believed that mutual funds with higher expenses produced better results, 64.4 percent believed that funds with higher expenses produced average results, and only 15.7 percent of the survey respondents believed that higher expenses led to lower-than-average returns. As we discuss in the following section, at the time the survey was conducted, there already existed several studies that had documented an inverse relation between expenses and performance. It seems that, at the time of the survey, a significant fraction of investors had not optimally updated their beliefs. Further, investors’ money, and not just investors’ opinion, seems to show that a sizeable proportion of them is not
making the right investment choices. As Martin Gruber put it, we see that money remains in funds that can be predicted to do poorly and that in fact do perform poorly (Gruber, 1996, p. 807).

4 Evidence on the Relationship between Performance and Fees

4.1 Mutual Fund Fee Structure

In the market for mutual funds, the fees paid by investors take two forms: periodic fees (operating expenses) and one-time fees (loads).\(^8\) Expenses mostly consist of management fees, but also include 12b-1 (distribution and marketing) fees, custody fees, and administrative fees, as well as operating, legal, and accounting costs. They are computed as a percentage of assets under management—termed the *expense ratio*—and are deducted on a daily basis from the fund’s net assets by the managing company. Fees paid to brokers in the course of the fund’s trading activity are not included in the fund’s expense ratio.

Loads are generally used to pay distributors and they differ from operating expenses in that they are paid by the individual investor as a fraction of the amount invested at the time of purchasing fund shares (*sales charge on purchases*) or redeeming fund shares (*deferred sales charge*). Since fund returns are typically computed from the fund’s net asset value, quoted returns are net-of-expenses, but before loads.

4.2 Empirical Evidence

The quality of an actively managed fund is commonly defined as the manager’s ability to deliver returns above those that any investor could obtain following a passive strategy, such as investing in an index fund. Differences in managerial quality could translate into differences in after-expense returns if quality were not fully priced. If higher quality were partly priced, that is, if better funds charged higher expenses, differences in after-expense returns would be smaller than differences in fees. On the other hand, if high-quality funds happened to charge lower fees, differences in after-expense returns would be greater than

\(^8\)Mahoney (2004) provides a review of mutual fund fee practices and regulation. For a more detailed description, we suggest that the reader visit the Online Publications section of the SEC internet site.
differences in fees.\textsuperscript{9}

Elton et al. (1993) divide a sample of U.S. mutual funds available in the 1965-84 period into quintiles by expense ratios, and measure average after-expense risk-adjusted returns\textsuperscript{10} for funds in each quintile. They find that funds with higher expense ratios perform significantly worse, and that performance differences between funds in the best and the worst quintiles exceed differences in fees, which suggests that funds with higher expenses exhibit lower before-expense returns. Put differently, low-quality funds seem to be more expensive.

Gruber (1996) studies cross-section differences in after-fee performance in the 1985-94 period.\textsuperscript{11} When ranking funds according to performance, Gruber (1996) finds that differences in expenses are negatively and significantly correlated with larger differences in performance.

Harless and Peterson (1998) analyze data employed in early performance studies and find that the predictive power of expense ratios with respect to performance also extends to the 1954-64 period. They conclude that “all the studies show that funds with the lowest expense ratios tend to perform best, and funds with highest expense ratios tend to perform worst.”

Carhart (1997) proposes a different measure of performance\textsuperscript{12}. When regressing this performance measure on expense ratios, he estimates that in the 1962-93 period funds with annual expenses of 100 basis points above the average had on average 154 basis points below mean after-expense performance. Again, the effect of fees is to amplify rather than mitigate differences in before-expense performance.

Finally, Chevalier and Ellison (1999), using a measure of performance similar to Carhart’s

\textsuperscript{9}If we let $r_i$ be the before-expense return of fund $i$ and $e_i$ the expenses charged by this fund, then the after-expense return $r_{\text{a}}$ is given by $(1 + r_{\text{a}}) = (1 - e_i)(1 + r_i) = 1 + r_i - e_i - e_i r_i \approx 1 + r_i - e_i$. Therefore, $r_{\text{a}} - r_{\text{a}} = r_i - e_i - (r_j - e_j) = (e_j - e_i) + (r_i - r_j)$. It follows that if fund $j$ charges a higher fee ($e_j - e_i > 0$) and $r_{\text{a}} > r_{\text{a}} > 0$, then $r_i - r_j > 0$, that is, fund $j$ must have lower before-expense returns.

\textsuperscript{10}In particular, they measure performance as the intercept term from the regression of annual returns on three indexes, tracking the evolution of stocks in the S&P 500 index, non-S&P stocks, and bonds. This measure of performance can be interpreted as the value added by the fund manager with respect to a passive strategy.

\textsuperscript{11}Gruber (1996) analyzes three alternative proxies for performance: (i) the fund’s average return relative to the market; (ii) the fund’s average return in excess of the fund’s expected return according to the Capital Asset Pricing Model; and (iii) the fund’s average excess return according to a four-index model.

\textsuperscript{12}Carhart (1997) employs a four-factor model which captures the fund’s exposure to sources of undiversifiable risk.
(1997), report that manager and fund characteristics—such as the portfolio turnover ratio and log of assets—contribute to explaining differences in performance in the 1988-95 period. When controlling for these variables, they provide estimates of the effect on after-expense performance of a 100 basis point reduction in expense ratios that range from 152 to 225 basis points.

Put together, the empirical evidence implies that superior management is not priced through higher expense ratios. To the contrary, the effect of expenses on after-expense performance (even after controlling for funds’ observable characteristics) is more than one-to-one, implying that low-quality funds charge higher fees. Price and quality thus appear to be inversely related in the market for actively managed mutual funds.

5 Conclusion

In this paper, we have shown that, in the mutual fund industry, better-quality funds should not be expected to charge higher prices. Moreover, investors’ limited ability to evaluate fund quality may lead to equilibria in which worse-performing funds charge higher fees. We thus obtain a form of inverse price differentiation which is consistent with existing evidence on mutual fund performance.

The fundamental role played by mutual funds and the current demands for regulatory action call for further analysis of the mutual fund industry. Our model suggests several directions for future research. First, in this paper, we have considered a single period, so investors cannot base their decisions on past fund performance. An intertemporal extension of the model would make it possible to investigate the relationship between fees and past performance and their relative role as signals of fund quality. Second, while we have taken fund quality as exogenous, mutual fund management companies may, to some extent, set the quality of the funds they offer through their choice of managers or their expenditure in market analysis. Third, in our model, unsophisticated investors are equally likely to buy from good and bad funds. However, it is more realistic to think that funds may differentiate themselves not only through fees but also through their marketing decisions: in a separating equilibrium, lower-quality funds may not only charge higher fees, but also invest more in their distribution networks or advertising to make sure that they attract a
larger proportion of unsophisticated investors.

Although in this paper we have specifically modelled the market for mutual funds, the insights of the model may be generalizable to other markets where quality assessment is costly. The existing empirical evidence on the relationship between quality and price suggests that the correlation between these two variables is typically not strongly positive and that, in a significant number of markets, the correlation is indeed negative (e.g., Caves and Greene, 1996). In our model, the negative correlation between price and quality results from the combination of asymmetric information about product quality and the presence of a subset of unsophisticated investors. Future work may further explore, both theoretically and empirically, how these and other factors affect observed price-quality correlations.

Our results indicate that the complexity associated with the evaluation of fund quality may, on the one hand, weaken competition, leading to high average fees even in the presence of a large pool of competing mutual funds, and, on the other hand, lead to a segmented market in which a fraction of investors pay higher than average fees for underperforming funds. Whether this state of things could be improved by regulation and the optimal form of this regulation are questions that merit further scrutiny. Currently, both the SEC and NASD, the self-regulatory arm of the U.S. securities industry, impose limits on redemption fees and the loads that can be charged to mutual fund investors to pay for brokerage services. Our results open the question as to whether some form of cap on expenses could be beneficial in this context. A less contentious alternative, already having been pursued by the SEC, is to require that funds improve the disclosure of their fees, so as to allow investors to realize the dollar cost of the expenses paid. The SEC, however, has stopped short of requiring funds to periodically disclose to each investor the exact dollar amount of the expenses paid on the grounds that such a requirement would impose large processing costs on mutual fund companies. Given the large potential costs that a poor understanding of the impact of fees on returns has for unsophisticated investors, our model would suggest reevaluating this cost-benefit analysis. Our model also suggests that requiring funds to disclose the level of fees charged by the fund compared to the average or median fees in the corresponding investment category—information that is already voluntarily disclosed by some mutual fund management companies—could greatly contribute to prevent funds from
overcharging unsophisticated investors.
Appendix.

Proof of Proposition 3.

First, notice that $e_b \leq \frac{R_b-1}{R_b}$ cannot be an equilibrium fee, as slightly undercutting such $e_b$ would guarantee the deviating b-fund all the sophisticated market in case there are no g-funds and would only marginally reduce its profits in all other cases. This implies that, in equilibrium $e_b > \frac{R_b-1}{R_b}$, so a necessary condition for the existence of a separating equilibrium is $e_U > \frac{R_b-1}{R_b}$, which is satisfied by $e_U = 1 - \frac{1}{R}$.

Next, notice that, if a separating equilibrium exists, $e_b^* = e_U$. Any $e_b \in \left(\frac{R_b-1}{R_b}, e_U\right)$ cannot be an equilibrium, as such a fee will not convince sophisticated investors to invest with a b-fund even if all funds turn out to be of type b, and $e_U$ yields greater profits from the unsophisticated investors. Since $e_b \leq \frac{R_b-1}{R_b}$ cannot be an equilibrium fee either, the only possible equilibrium fee for b-funds is $e_b^* = e_U > \frac{e}{R_b}$.

Given $e_b = e_U > \frac{R_b-1}{R_b}$, $w_b^U = \gamma \frac{N}{N}$, so the participation constraint for b-funds and the no-imitation constraints read:

\[
\frac{\gamma}{N}(R_b e_U - c) \geq 0 \quad (\text{Pb})
\]
\[
\frac{\gamma}{N}(R_b e_U - c) \geq w_b^U(R_b e_g - c) \quad (\text{NIb})
\]
\[
w_b^U(R_g e_g - c) \geq \frac{\gamma}{N}(R_g e_U - c), \quad (\text{NIg})
\]

where $w_g^U \in \left(\frac{\gamma}{N}, \frac{\gamma}{N} + (1 - \gamma)\right]$. The no-imitation constraints can be rewritten:

\[
e_g \geq \frac{\gamma}{N w_g^U} e_U + (1 - \frac{\gamma}{N w_g^U}) \frac{c}{R_g} = \alpha e_U + (1 - \alpha) \frac{c}{R_g} \quad (\text{NIg}')
\]
\[
e_g \leq \frac{\gamma}{N w_g^U} e_U + (1 - \frac{\gamma}{N w_g^U}) \frac{c}{R_b} = \alpha e_U + (1 - \alpha) \frac{c}{R_b} \quad (\text{NIb}')
\]

where $\alpha \equiv \frac{\gamma}{N w_g^U}$. Since $w_g^U > \frac{\gamma}{N}$, $\alpha < 1$. Therefore, the incentive constraint (NIb') implies that $e_g < e_U$, which proves part 3.

Let us assume that sophisticated investors’ beliefs are such that if a fund sets $e \in \left[\frac{e}{R_g}, \frac{e}{R_b}\right)$, it will be believed to be a g-fund, while for any $e \geq \frac{e}{R_b}$ (other than g-funds’ equilibrium fee if $e_g \geq \frac{e}{R_b}$), it will be believed to be a b-fund. This implies that $e_g \in \left(\frac{e}{R_g}, \frac{e}{R_b}\right]$ cannot be an equilibrium fee. Since $e_g$ needs to be strictly greater than $\frac{e}{R_b}$, the only possible equilibrium fees satisfy $e_g > \frac{e}{R_b}$.
Let \( m_b \) be the minimum fee that would make it profitable for a b-fund to deviate if it captures the whole sophisticated market:

\[
\left( \frac{\gamma}{N} + (1-\gamma) \right) (R_b m_b - c) = \frac{\gamma}{N} (R_b e_U - c), \quad i.e.,
\]

\[
m_b = \frac{\gamma}{N - \gamma(N-1)} e_U + \left( 1 - \frac{\gamma}{N - \gamma(N-1)} \right) \frac{c}{R_b} = \lambda e_U + (1 - \lambda) \frac{c}{R_b}, \tag{7}
\]

where \( \lambda \equiv \frac{\gamma}{N - \gamma(N-1)} < 1 \).

Similarly, let \( m_g \) be the minimum fee that would make it profitable for a g-fund to deviate if it captures the whole sophisticated market:

\[
\left( \frac{\gamma}{N} + (1-\gamma) \right) (R_g m_g - c) = w_g^U (R_g e_g - c) \tag{8}
\]

Rearranging:

\[
m_g = \frac{N w_g^U}{\gamma + (1-\gamma)N} e_g + \left( 1 - \frac{N w_g^U}{\gamma + (1-\gamma)N} \right) \frac{c}{R_g} = \phi e_g + (1 - \phi) \frac{c}{R_g}, \tag{9}
\]

where \( \phi \equiv \frac{N w_g^U}{\gamma + (1-\gamma)N} < 1 \).

Let \( M_b \) (\( M_g \)) be the be the minimum fee that would make it profitable for a b-fund (g-fund) to deviate and capture the whole sophisticated market only when there are no g-funds:

\[
\frac{\gamma}{N} (R_b e_U - c) = (R_b M_b - c) \left( \frac{\gamma}{N} + (1-p)^{N-1}(1-\gamma) \right) \tag{10}
\]

\[
w_g^U (R_g e_g - c) = (R_g M_g - c) \left( \frac{\gamma}{N} + (1-p)^{N-1}(1-\gamma) \right) \tag{11}
\]

Notice that these inequalities imply \( m_g < M_g \) and \( m_b < M_b \).

If \( e_g > \frac{c}{R_b} \), the maximum fee that a deviating fund can charge while guaranteeing the whole sophisticated-investor market with probability one is \( d \equiv \max\{ \hat{e}, \frac{c}{R_b} \} \), where

\[
\hat{e} = \frac{e_g R_g - (R_g - R_b)}{R_b} \tag{12}
\]

is defined by \((1 - e_g)R_g = (1 - \hat{e})R_b\).

Similarly, the maximum fee that a deviating fund can charge while guaranteeing the whole sophisticated-investor market in case all other funds are of type b is

\[
D \equiv \max \left\{ \frac{c}{R_b}, \frac{R_b - 1}{R_b} \right\} \tag{13}
\]
Therefore, the no-deviation conditions for b- and g-funds are, respectively:

\[ m_b \geq d \quad \text{(NDb)} \]
\[ m_g \geq d \quad \text{(NDg)} \]
\[ M_g \geq D \quad \text{(NDb')} \]
\[ M_b \geq D \quad \text{(NDg')} \]

Finally, \( e_g \) has to be such that sophisticated investors are willing to invest with g-funds:

\[ e_g \leq \frac{R_g - 1}{R_g}, \quad \text{(PIg)} \]

which will immediately hold since \( e_U \leq \frac{R_g - 1}{R_g} \) and \( e_g < e_U \).

An equilibrium will exist if all the inequality conditions (Pb, NIb', NIg', NDb, NDg NDb', NDg', and PIg) are satisfied simultaneously.

Given the relatively large number of parameters \((R_g, R_b, p, N, c, \gamma)\) and inequalities, we do not fully characterize the set of equilibria. Instead, we next show existence numerically.

Figures 1–2 show parameter regions for which this type of equilibrium exists.

**Proof of Proposition 4.**

Let \( e_p \) be the pooling fee and \( \bar{R} = pR_g + (1 - p)R_b \) denote the unconditional expectation of gross returns. We will assume that sophisticated investors’ beliefs are such that, if a fund sets \( e \in [\frac{c}{R_g}, \frac{c}{R_b}] \), it will be believed to be a g-fund, while for any \( e \geq \frac{c}{R_b} \) other than the equilibrium fee, it will be believed to be a b-fund.

For sophisticated investors to be willing to buy from a fund of unknown type:

\[ e_p \leq 1 - \frac{1}{\bar{R}}, \quad \text{(PCip)} \]

i.e., \( e_p \leq e_U \).

For b-funds to be willing to participate:

\[ e_p \geq \frac{c}{R_b} \quad \text{(PCbp)} \]

For a fund of type \( k \) not to be willing to deviate and serve the unsophisticated investor segment only:

\[ \frac{1}{N}(e_pR_k - c) \geq \frac{\gamma}{N}(e_U R_k - c), \text{ or} \]  
\[ e_p \geq \gamma e_U + (1 - \gamma) \frac{c}{R_k} \quad \text{(15)} \]
Since \( R_g > R_b \), it suffices to require
\[
e_p \geq \gamma e_U + (1 - \gamma) \frac{c}{R_b},
\]
which implies PCbp, since, by Assumption 3, \( e_U > \frac{c}{R_b} \).

Let \( m_g \) and \( m_b \) be the minimum fees that would make it profitable for a g- or a b-fund, respectively, to deviate.

\[
\left( (1 - \gamma) + \frac{\gamma}{N} \right) (m_g R_g - c) = \frac{1}{N} (e_p R_g - c)
\]
\[
\left( (1 - \gamma) + \frac{\gamma}{N} \right) (m_b R_b - c) = \frac{1}{N} (e_p R_b - c)
\]

If we let
\[
N^* \equiv N(1 - \gamma) + \gamma,
\]
it follows that:
\[
m_g = \frac{1}{N^*} e_p + \left( 1 - \frac{1}{N^*} \right) \frac{c}{R_g} < \frac{1}{N^*} e_p + \left( 1 - \frac{1}{N^*} \right) \frac{c}{R_b} = m_b,
\]
Now, let \( \xi \) be the maximum fee that would convince investors to shift to a fund believed to be bad, that is:
\[
(1 - \xi) R_b = (1 - e_p) \overline{R},
\]
or
\[
\xi = 1 - (1 - e_p) \frac{\overline{R}}{R_b}
\]
Therefore, if a fund deviates and sets \( d \equiv \max\{\xi, \frac{c}{R_b}\} \), it will capture the whole sophisticated market, so, for a g-fund not to be willing to deviate, it has to be the case that:
\[
m_g \geq d,
\]
which also implies that b-funds do not want deviate because \( m_b > m_g \).

**Case 1:** \( d = \xi \). Let us first look at the case in which \( d = \xi \), that is:
\[
1 - (1 - e_p) \frac{\overline{R}}{R_b} \geq \frac{c}{R_b}, \text{ or }
\]
\[
e_p \geq \tilde{e} \equiv \frac{\overline{R} - (R_b - c)}{\overline{R}}
\]
In this case the no-deviation condition for g-funds reads:

\[ m_g = \frac{1}{N^*} e_p + \left(1 - \frac{1}{N^*}\right) \frac{c}{R_g} \geq 1 - (1 - e_p) \frac{\overline{R}}{R_b} = \xi, \text{ or } \]

\[ e_p \leq \frac{N^* R_g (\overline{R} - R_b) + (N^* - 1) R_b c}{R_g (N^* \overline{R} - R_b)} \] \tag{19}

For an equilibrium of this sort to exist, thus, conditions (PCip), (NDu), (18) and (19) have to hold simultaneously.

First note that, given Assumption 2, (18) implies (NDu) for \( \gamma \) low enough. Inspection of the conditions also shows that for (PCip) and (18) to hold simultaneously it is necessary that

\[ R_b - c > 1 \] \tag{20}

If this condition holds, then it only rests to check that (18) and (19) can hold simultaneously. This requires:

\[ \frac{\overline{R} - (R_b - c)}{\overline{R}} < \frac{N^* R_g (\overline{R} - R_b) + (N^* - 1) R_b c}{R_g (N^* \overline{R} - R_b)} \] \tag{21}

After some algebra, this condition can be shown to be equivalent to:

\[ R_b < p \left( R_b + R_g \left( \frac{R_b - N^* c}{(N^* - 1) c} \right) \right) \] \tag{22}

Therefore, at least for \( \gamma \) low enough (so that we do not have to worry about condition (NDu)), if \( R_b > N^* c \), a pooling equilibrium will exist for high enough values of \( p \). Note, in particular, that existence does not require that \( \gamma > 0 \), so that this type of equilibrium would also exist in the absence of unsophisticated investors.

**Case 2:** \( d = \frac{c}{R_b} \). In equilibrium, \( d = \frac{c}{R_b} \) if and only if:

\[ e_p \leq \frac{\overline{R} - (R_b - c)}{\overline{R}} \] \tag{23}

For g-funds not to deviate, we need \( m_g \geq d = \frac{c}{R_b} \), i.e.,

\[ e_p \geq \frac{c}{R_g} + N^* c \frac{R_g - R_b}{R_g R_b} \] \tag{24}

Thus, for an equilibrium of this sort to exist, conditions (PCip), (NDu), (23), and (24) must hold. We need to consider two cases:
1. \( R_b - c > 1 \). In this case, condition (PCip) is implied by (23) and conditions (NDu) and (23) are always compatible for \( \gamma \) low enough since:

\[
\frac{\overline{R} - (R_b - c)}{\overline{R}} > \frac{c}{R_b} \Leftrightarrow \overline{R} R_b - R_b (R_b - c) > c \overline{R} \Leftrightarrow \\
\overline{R} (R_b - c) - R_b (R_b - c) > 0 \Leftrightarrow \overline{R} - R_b > 0,
\]

which is always true.

It rests to check that conditions (23) and (24) are compatible as well. This will happen if and only if:

\[
c \frac{R_g}{R_g} + N^* \frac{R_g - R_b}{R_b R_g} < \frac{\overline{R} - (R_b - c)}{\overline{R}} \quad (25)
\]

Rearranging this expression leads to inequality (22), so the same conditions as above guarantee existence of this type of equilibrium.

2. \( R_b - c < 1 \). Now, condition (23) is implied by (PCip). The latter condition will be consistent with (24) only if:

\[
1 - \frac{1}{\overline{R}} > \frac{c}{R_g} \left( 1 + \frac{N^* R_g}{R_b} - N^* \right) \quad (26)
\]

For fixed \( R_b \) and \( R_g \), the supremum of the left-hand side is \( 1 - \frac{1}{\overline{R}_g} \) (when \( p \to 1 \)). The infimum of the right-hand side is \( \frac{R_b - 1}{R_g} \left( 1 + \frac{N^* R_g}{R_b} - N^* \right) \) if \( R_b > 1 \) (when \( c \to 1 - R_b \)), and 0 if \( R_b < 1 \) (when \( c \to 0 \)). In the latter case, the above condition will hold. If \( R_b > 1 \), we must have:

\[
\frac{R_g - 1}{R_g} > \frac{R_b - 1}{R_g} \left( 1 + \frac{N^* R_g}{R_b} - N^* \right) \quad (27)
\]

Rearranging,

\[
(R_g - 1) R_b > (R_b - 1) (R_b + N^* R_g - N^* R_b) \Leftrightarrow \quad (28)
\]

\[
R_b < \frac{N^*}{N^* - 1} \quad (29)
\]

Therefore, if (29) holds and \( \frac{R_g - 1}{R_g} > \frac{c}{R_b} \), then there are pooling equilibria with \( R_b - c < 1 \).
References


Figures

Figure 1: Conditions for Existence of Separating Equilibria with Unsophisticated Investors. $R_b = 1.1$; $R_g = 1.3$; $p = 0.5$; N=2. The x-axis displays values of $\gamma$, while $c$ is displayed along the y-axis. The curves represent the minimum and maximum values of $c$ such that a separating equilibrium exists for each $\gamma$.

Figure 2: Conditions for Existence of Separating Equilibria with Unsophisticated Investors. $R_b = 1.1$; $R_g = 1.3$; $p = 0.5$; N=5. The x-axis displays values of $\gamma$, while $c$ is displayed along the y-axis. The curves represent the minimum and maximum values of $c$ such that a separating equilibrium exists for each $\gamma$. 
Figure 3: Conditions for Existence of Pooling Equilibria with Unsophisticated Investors. $R_b = 1.1$; $R_g = 1.3$; $p = 0.5$; $N=2$. The x-axis displays values of $\gamma$, while $c$ is displayed along the y-axis. The curves represent the minimum and maximum values of $c$ such that a pooling equilibrium exists for each $\gamma$.

Figure 4: Conditions for Existence of Pooling Equilibria with Unsophisticated Investors. $R_b = 1.1$; $R_g = 1.3$; $p = 0.5$; $N=2$. The x-axis displays values of $\gamma$, while $c$ is displayed along the y-axis. The curves represent the minimum and maximum values of $c$ such that a pooling equilibrium exists for each $\gamma$. 