OUTLIERS ROBUST ECM COINTEGRATION TEST BASED ON THE TREND COMPONENTS

Miguel A. Arranz
Álvaro Escribano

00-87

WORKING PAPERS

Universidad Carlos III de Madrid

Working Paper 00-87
Statistics and Econometrics Series 44
December 2000

Departamento de Estadística y Econometría
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34) 91 624-98-49

OUTLIERS ROBUST ECM COINTEGRATION TESTS BASED ON THE TREND COMPONENTS

Miguel A. Arranz and Alvaro Escribano*

Abstract

The main goal of this paper is to analyze the behaviour of the ECM non–cointegration test when there are additive outliers in the time series under different co–breaking situations. We show that the critical values of the usual ECM test are not robust to the presence of transitory shocks and we suggest a procedure based on signal extraction to bypass this problem. These procedure renders ECM tests with a left tail of distribution under the null that is robust to the presence of additive outliers in the series. The small sample critical values and the empirical power of the test are analyzed by Monte Carlo simulations for several low frequency filters.

Keywords: Outliers; transitory co–breaks; cointegration testing; trend–component error correction models.

*Arranz, Department of Statistics and Econometrics, Universidad Carlos III de Madrid, e–mail: arranz@pvi.uc3m.es; Escribano, Department of Economics, Universidad Carlos III de Madrid, e–mail: alvaroe@eco.uc3m.es. This research was financial with Spanish DGICYT under grant PB98–0030 and TMR ERB–4061 PL87–094.

OUTLIERS ROBUST ECM COINTEGRATION TESTS BASED ON THE TREND COMPONENTS

MIGUEL A. ARRANZ AND ALVARO ESCRIBANO

ABSTRACT. The main goal of this paper is to analyze the behavior of the ECM non–cointegration test when there are additive outliers in the time series under different co–breaking situations. We show that the critical values of the usual ECM test are not robust to the presence of transitory shocks and we suggest a procedure based on signal extraction to bypass this problem. These procedure renders ECM tests with a left tail of distribution under the null that is robust to the presence of additive outliers in the series. The small sample critical values and the empirical power of the test are analyzed by Monte Carlo simulations for several low frequency filters.

1. INTRODUCTION

The properties of cointegration tests based on single equation error correction models (ECM test) are well known. The dependence of the critical values and the power of the ECM test on nuisance parameters is documented in Banerjee et al. (1986), Engle and Granger (1987), Kremers et al. (1992), and Banerjee et al. (1993) [1].

The effects of having breaks when applying unit root test, like Dickey and Fuller (1979) test, etc., are also well known, see Stock (1994). Perron (1989) is a good starting point to see those impacts. From Clements and Hendry (1999), a structural break essentially corresponds to an intermittent shock with a permanent effect on the series. If this permanent shock is not explicitly taken into account, standard unit roots tests might mistake the structural break with a unit root. The results of Hendry and Neale (1990) and Perron and Vogelsang (1992) indicate that a neglected shift in the mean also leads to spurious unit roots. Rappoport and Reichlin (1989) is probably the first reference to check if we want to know the impact of having segmented trends as an alternative to a unit root model. Andrés et al. (1990) extended the analysis of Rappoport and Reichlin to more that one break point in the trend. Other references on breaks and unit roots tests are Banerjee et al. (1992), Zivot and Andrews (1992), and Leybourne et al. (1998).

Key words and phrases. Outliers, transitory co–breaks, cointegration testing, trend–component error correction models.
This draft: December 15, 2000.
We thank A. Lucas for kindly providing us with the data for the empirical example.

[1] See Arranz and Escribano (1998, 2000) for a brief review of the main results.

From the applied point of view, the main nuisance is to add dummy variables for the structural breaks in order to apply valid unit root tests when the critical values obtained depend on the size and on the timing of the break. Again, the selection of the dummy variable is critical for the result of the test. A vast literature emerged searching for unknown break points using recursive or sequential tests (Andrews, 1993, Andrews et al., 1996, Bai, 1997, Vogelsang, 1997, Bai and Perron, 1998, Banerjee et al., 1998).

An important class of unusual events are additive outliers. These are events with a large, but temporary effect on the series. In certain cases, this effect dominates the remaining information contained in the series and biases unit root inference towards rejection of the unit root hypothesis even if the null hypothesis of a unit root is correct, as reported in Franses and Haldrup (1994) and Lucas (1995*a*,*b*).

With multiple time series the situation could be worse if the breaks are independent. Now we have to decide on the models that generate the anomalous observations (breaking trends, additive outliers, ...) taking into account that those irregularities need not occur simultaneously nor on all of the variables. Therefore, the multivariate analysis is generally more difficult. However, in some cases it can be more simple if there is partial co-breaking in the series.

In empirical applications the addition of dummy variables to obtain parameter "constant" models is more the rule than the exception. The effects of including dummy variables to capture structural breaks in ECM tests have been previously analyzed by Kremers et al. (1992), and Campos et al. (1996). Once again, critical values (C.V.) depend on the particular type of dummy variable included in the model and is a nuisance for empirical applications

One alternative to avoid the use of dummy variables is to use robust estimation techniques. This is the approach taken by Lucas (1995*a*,*b*) in the univariate case and Lucas (1997) and Franses and Lucas (1997*a*,*b*) in the multivariate case.

In this paper we follow a different route. The objective isto find robust modeling procedures to test for unit roots in the presence of structural breaks in an ECM context. Instead of including dummy variables in ECM models, we try to approximate those breaks by adding extra dynamic terms (lags), as determined by the SBIC criterion. In particular, we look at the critical values obtained with the overparameterized model. We study the size of the ECM test under different MA(1) errors, and analyze the power of the ECM test, using Monte Carlo simulations. We also investigate whether the robustness properties of the ECM test improve by following the same steps not on the observable variables, but on the trend components obtained from trend-cycle decompositions, as in Arranz et al. (2000). In particular, we study three filters, the Hodrick and

Prescott (1980, 1997) filter, the Baxter and King (1995, 1999) filter, HP and BK respectively from now on, and the median filter (see Wen and Zeng, 1999). Guay and St-Amant (1997) and Baxter and King (1995) provide some insights about the relationship between the HP and BK filters.

The structure of the paper is the following. In Section 2 we analyze the effects of having transitory breaks on alternative specifications of the ECM models, and in particular on the cointegrating relationship. Three types of cobreaking possibilities are studied in detail: full co–breaking, co–breaking in levels (not in differences) and co–breaking in differences (not in levels). We also study several cases without any cobreaking. Section 3 reviews the signal extraction filters that we apply. The trend component ECM models are introduced in Section 4. Section 5 presents the main results of the Monte Carlo simulation experiments. The usefulness of our approach is illustrated with an empirical application in Section 6. Finally, the conclusions.and some comments for further research directions are included in Section 7.

2. Error Correction Models with and without Simultaneous Co–Breaking

Consider the following conditional error correction model (ECM)

$$\Delta(y_t - \mu_{y,t}) = a\Delta(z_t - \mu_{z,t}) + b[(y_{t-1} - \mu_{y,t-1}) - \alpha(z_{t-1} - \mu_{z,t-1})] + u_{1t} \tag{2.1a}$$

$$\Delta(z_t - \mu_{z,t}) = u_{2t} \tag{2.1b}$$

Assume that $\ldots, y_{-1}, y_0 = 0$ and $\ldots, z_{-1}, z_0 = 0$, let $\mu_{y,t} = \tilde{\mu}_{y,t} + s_y\delta_t^y$, $\mu_{z,t} = \tilde{\mu}_{z,t} + s_z\delta_t^z$, where δ_t^y and δ_t^z are *iid* Bernouilli variables independent of $u_{1,t}$ and $u_{2,t}$

$$\begin{cases} \Pr(\delta_t = 1) = \Pr(\delta_t = -1) = \dfrac{\pi}{2} \\ \Pr(\delta_t = 0) = 1 - \pi. \end{cases}$$

The terms $\tilde{\mu}_{y,t}$ and $\tilde{\mu}_{z,t}$ include all possible deterministic components like: constant terms, deterministic trends, dummy variables, segmented trends, etc. Define B as the back-shift operator, $B^k y_t = y_{t-k}$, $\Delta = (1 - B)$ is the first differencing operator, and let $(1, -\alpha)$ be the cointegrating vector. Given valid initial conditions, the stochastic errors u_{1t} and u_{2t} are jointly, and serially uncorrelated with zero mean, and constant variances,say σ_1^2 and σ_2^2, respectively.

Model (2.1a)–(2.1b) can be written in terms of the observable variables y_t and z_t as follows,

$$\Delta y_t = c_t + a\Delta z_t + b(y_{t-1} - \alpha z_{t-1}) + u_{1t} \tag{2.2a}$$

$$\Delta z_t = \Delta \mu_{z,t} + u_{2t} \tag{2.2b}$$

$$c_t \equiv \Delta \mu_{y,t} - a\Delta \mu_{z,t} - b(\mu_{y,t-1} - \alpha \mu_{z,t-1}) \tag{2.2c}$$

$$\mu_{z,t} = \tilde{\mu}_{z,t} + s_z \delta_t^z \tag{2.2d}$$

$$\mu_{y,t} = \tilde{\mu}_{y,t} + s_y \delta_t^y \tag{2.2e}$$

In this paper we investigate the effects of having alternative generating models (with partial co–breaks, etc.) for the stochastic intercept c_t of (2.2a) given by (2.2c) on the ECM test for non–cointegration ($b = 0$).

Definition 2.1. We say that the time series y_t and z_t have *co-breaks in levels* if $\mu_{y,t} - \alpha\mu_{z,t} = c_l$, where c_l is a finite constant parameter.

Definition 2.2. We say that the time series y_t and z_t have *co-breaks in differences* if $\Delta\mu_{y,t} - a\Delta\mu_{z,t} = c_d$, where c_d is a finite constant parameter.

Definition 2.3. We say that the time series y_t and z_t have *simultaneous co-breaks* if $\Delta\mu_{y,t} - a\Delta\mu_{z,t} - b(\mu_{y,t} - \alpha\mu_{z,t}) = c_s$, where c_s is a finite constant parameter.

From definitions 2.1 and 2.2, it is clear that if y_t and z_t are *co-breaks in levels and in differences (full co-break)*, we have a particular case of simultaneous co-breaking.

Several possible intermediate cases are of interest in empirical applications and will be considered in the the simulation experiments later on.

Case 2.1. *Cobreak in levels but not in differences.* Cobreak in levels ($\mu_{y,t} - \alpha\mu_{z,t} = c_l$). Taking first differences, we have $\Delta\mu_{y,t} - \alpha\Delta\mu_{z,t} = 0$. But from equation (2.2c)

$$c_t = \Delta\mu_{y,t} - a\Delta\mu_{z,t} - bc_l = (\alpha - a)\Delta\mu_{z,t} - bc_l, \tag{2.3}$$

and equation (2.2a) becomes

$$\Delta y_t = -bc_l + (\alpha - a)\Delta\mu_{z,t} + a\Delta z_t + b(y_{t-1} - \alpha z_{t-1}) + u_{1,t} \tag{2.4}$$

Remark 2.1. cobreak in levels $\Rightarrow$ cobreak in differences if $a = \alpha$ (COMFAC restriction), for any value of c_t.

Case 2.2. *Cobreak in differences but not in levels.* Cobreak in differences: $\Delta\mu_{y,t} - a\Delta\mu_{z,t} = c_d$ implies that $\Delta\mu_{y,t} - \alpha\Delta\mu_{z,t} = (a - \alpha)\Delta\mu_{z,t} + c_d$. From recursive substitution $\mu_{y,t} - \alpha\mu_{z,t} =$

$(\mu_{y0} - \alpha\mu_{z0}) + c_d t + (a-\alpha)\mu_{z,t}$, and c_t becomes

$$c_t = c_d - b(\mu_{y,0} - \alpha\mu_{z,0}) - bc_d(t-1) - b(a-\alpha)\mu_{z_{t-1}} \tag{2.5}$$

and equation (2.2a) becomes

$$\Delta y_t = c_m + bc_d t - b(a-\alpha)\mu_{z,t-1} + a\Delta z_t + b(y_{t-1} - \alpha z_{t-1)} + u_{1,t}, \tag{2.6}$$

where c_m is a constant equal to $c_m = c_d - b(\mu_{y,0} - \alpha\mu_{z,0}) + bc_d$.

Remark 2.2. Assuming that $\mu_{y,0} - \alpha\mu_{z,0} = constant$, cobreak in differences $\Rightarrow$ cobreak in levels if $a = \alpha$ (COMFAC) and $c_d = 0$.

In general, without having any cobreak in levels or in differences, the most parsimonious representation in terms of unobservable variables is the conditional ECM model (2.1a). In terms of observable variables, the most parsimonious representation is (2.2a), because it only requires to add the regressors coming from the contemporaneous values of c_t. If we are interested in estimating the parameters a, α and b, we can estimate them by 1–step procedures (OLS or NLS) in ECM representation (2.2a), see Arranz and Escribano (2000) for details. However, to do that we need to know or to estimate $\mu_{y,t}$ and $\mu_{z,t}$, and this can incorporate arbitrary assumptions about unknown events (dummy variables, etc.), see Vogelsang (1999).

In this paper, we argue that in a non-stationary context $\mu_{y,t}$ and $\mu_{z,t}$ could include any possible combination of transitory breaks and outliers, which complicates the analysis. Therefore, we suggest to consider general detrending procedures to get rid of those unobserved non-stationary transitory elements. In particular, we recommend to specify error correction models in terms of the growth components variables (trend components) to obtain robust ECM test for non–cointegration ($b = 0$) in the presence of outliers, as will become clear in Sections 4 and 5.

2.1. **Error Correction Models under Simultaneous Cobreaking.** From equations (2.2a)–(2.2c) and the analysis of Escribano (1987) and Andrés et al. (1990), it is clear that any error correction model in terms of the observable variables and constant parameters should account for the joint effects of the following elements: $\Delta\mu_{y,t}$, $\Delta\mu_{z,t}$, $\mu_{y,t-1}$ and $\mu_{z,t-1}$.

Previous error correction models with certain cobreaks have been treated in Campos et al. (1996) and Clements and Hendry (1999). In this section we study models with simultaneous co–breaks so that $c_t = 0$. Under simultaneous co–breaks, (2.2a) and (2.2b) can be simplified to

$$\Delta y_t = a\Delta z_t + b(y_{t-1} - \alpha z_{t-1}) + u_{1t} \tag{2.7a}$$

$$\Delta z_t = \Delta\mu_{z,t} + u_{2,t} \tag{2.7b}$$

$$\mu_{z,t} = \tilde{\mu}_{z,t} + s_z\delta_t^z \tag{2.7c}$$

where (2.7a) has the form of the usual single equation error correction without a constant term since $c_t = 0$. The parameter s_z measures the size of the break, and δ_t^z is the additive outlier, see section 5 for more details.

From equations (2.7a) and (2.7b) it is clear that $y_t \sim I(1)$, $z_t \sim I(1)$, and that they are cointegrated in the sense of Engle and Granger (1987) with cointegration vector equal to $(1, -\alpha)$ for certain parameter values of b $(-2 < b < 0)$. Notice that we are allowing for transitory breaks in the 'exogenous' variable z_t that cobreak simultaneously with the endogenous variables y_t and therefore the breaks disappear from the conditional model, equation (2.7a).

2.2. **Error Correction Models without Simultaneous Co–breaking.** In the previous section we have discussed the case of joint cobreaking in levels and in differences. Our purpose now is to discuss several interesting intermediate cases, see Arranz and Escribano (2000) for a full discussion.

Case 2.1. *Co–breaks in levels but not co–breaks in differences.*

From equations (2.2a)–(2.2c) we have

$$\Delta y_t = -bc_l + (\alpha - a)s_z\Delta\delta_t^z + a\Delta z_t + b(y_{t-1} - \alpha z_{t-1}) + u_{1,t} \tag{2.8}$$

Therefore the breaks δ_t^z in the marginal process of Δz_t affect the error correction model unless the COMFAC restriction is satisfied $(a = \alpha)$. Later we analyze the effects of omitting the second term of the right–hand side of equation (2.8).

Case 2.2. *Co–breaking in differences but not co–breaking in levels.*

From equation (2.6)

$$\Delta y_t = c_m + bc_d t - b(a - \alpha)s_z\delta_{t-1}^z + a\Delta z_t + b(y_{t-1} - \alpha z_{t-1}) + u_{1,t} \tag{2.9}$$

Section 5 provides evidence of the effects of omitting the second and third terms of the right–hand side of (2.9)

Case 2.3. *Independent breaks*

The final possibility there is no cobreaking in levels nor in differences. This plausible empirical situation is the result of joining the effects of equations (2.8) and (2.9). In our simulation study, we will consider independent breaks on y_t and z_t, and also breaks on just one of the two series. Therefore, the term c_t from (2.2c) will be one of the following:

$$c_t = s_y \Delta\delta_t^y - a s_z \Delta\delta_t^z - b(s_y \delta_{t-1}^y - \alpha s_z \delta_{t-1}^z) \tag{2.10a}$$

$$c_t = -a s_z \Delta\delta_t^z + b\alpha s_z \delta_{t-1}^z \tag{2.10b}$$

$$c_t = s_y \Delta\delta_t^y + b s_y \delta_{t-1}^y \tag{2.10c}$$

and will analyze the impacts of considering c_t as a constant term in alternative modelling strategies.

3. Filters and Signal Extraction

The usual aim of a filter in macroeconomic time series is to extract particular components of the series: trend, cycle, irregular, etc. In this paper, we are interested in splitting an observed time series in two components,

$$y_t = y_t^g + y_t^c \tag{3.1}$$

where y_t^g is the growth component and y_t^c is the cyclical component. Two main possibilities are usually considered. First, define what the trend component is and the cycle is therefore the residual

$$y_t = y_t^g + (y_t - y_t^g) \tag{3.2a}$$

Second, define the cycle and the trend is the residual provided that the series are integrated $I(d)$, with $d \geq 1$

$$y_t = (y_t - y_t^c) + y_t^c \tag{3.2b}$$

3.1. **Baxter and King filter (BK) .** Most of the filters used in macroeconomic time series are two–sided infinite–order moving averages, as pointed out by King and Rebelo (1993) and Baxter and King (1995, 1999). In practice the filter has to be approximated by a two–sided MA(k)

$$y_t^\star = a_0 + \sum_{h=1}^{k} a_h (B^h + B^{-h}) y_t$$

The implications of the filters are clearly seen in the frequency domain by looking at the *frequency response function*. The frequency response function of the two–sided MA(∞), equation is given by

$$\beta(\omega) = \sum_{h=-\infty}^{\infty} b_h e^{-iwh} \tag{3.3}$$

while for the two–sided MA(k) the frequency response function is

$$\alpha(\omega) = \sum_{h=-k}^{k} a_h e^{-iwh}. \tag{3.4}$$

Baxter and King (1995) obtain optimal approximating filters by minimizing the mean square error of $\delta(\omega) \equiv \beta(\omega) - \alpha(\omega)$ with respect to a_h. The component y^c could be obtained by applying *trend reducing filters* (high–pass filters) based on symmetric MA(k) transformations. Baxter and King showed that when $\sum_{h=-k}^{k} a_h = 0$, y_t^* has no trend if the growing component of y_t was generated by deterministic trends (linear or quadratic) or by I(1) or I(2) processes.

Notice that the trend reduction condition, $\sum_{h=-k}^{k} a_h = 0$, implies that the frequency response function, equation (3.4) satisfies $\alpha(0) = 0$. The spectrum of y_t^c is zero at the zero frequency and it is associated with the business cycle component (y_t^c) and therefore ($y_t - y_t^c$) is the trend component (y_t^g). These trend reducing filters are called *high–pass filters* since they pass components of the data with frequency larger than a predetermined value $\underline{\omega}$ close to 0. That is $\beta(\omega) = 0$ for $|\omega| < \underline{\omega}$ and $\beta(\omega) = 1$ for $|\omega| \geq \underline{\omega}$.

On the other hand, *low–pass filters* are determined so that $\beta(\omega) = 0$ for $|\omega| > \underline{\omega}$ and $\beta(\omega) = 1$ for $|\omega| \leq \underline{\omega}$ and therefore low frequencies, (long term movements) remain unchanged while others are canceled out. In terms of the finite symmetric MA(k) filter, this means that low–pass filters must satisfy $\sum_{h=-k}^{k} a_h = 1$.

Baxter and King (1995) showed that an 'ideal' approximate *low–pass filters* could be obtained by choosing the coefficients of the two–sided MA(k) filter, equal to $a_0 = \frac{1}{\pi}\underline{\omega}$ and $a_h = \frac{1}{h\pi}\sin(hw)$ for $h = 1, 2, 3 \ldots$ Therefore, the complementary *high–pass filter* has coefficients $(1 - a_0)$ at $h = 0$ and $-a_h$ for $h = 1, 2, 3, \ldots$

When the filter passes frequencies between $\underline{\omega}$ and $\overline{\omega}$ of the spectrum where $0 < |\underline{\omega}| < |\overline{\omega}| < \pi$ it is called *band-pass filter* and can for example be obtained by subtracting two low–pass filters. Usually, the frequency interval is associated with the NBER business cycle duration as defined by Burns and Mitchell (1946) where $\underline{\omega}$ corresponds to 32 quarters (8 years) and $\overline{\omega}$ to 6 quarters (1.5 or 2 years). This band–pass filter is what we are calling the BK filter in the simulations.

3.2. **Hodrick and Prescott filter (HP).** The Hodrick and Prescott (1980, 1997) filter is widely used in macroeconomics to detrend series in order to study of the stylized facts of an economy along the business cycle. The basis of this filter is the following: starting from (3.1) they define the trend component as the solution to the following optimization problem

$$\min_{\{y_t^g\}} \sum_{t=1}^{T} \left[\left(y_t - y_t^g\right)^2 + \lambda\left(\Delta^2 y_{t+1}^g\right)^2 \right] \tag{3.5}$$

The first term of (3.5) might be regarded as a measure of the goodness of fit of the trend component to the observed series, while the second one imposes a penalty in order to get a smooth trend component. The values of the parameter λ suggested by Kydland and Prescott (1990) are $\lambda = 1600$ for quarterly data and $\lambda = 400$ for annual data, obtained as the ratios of volatility of the irregular components relative to the volatility of the growth components.

Expressing the problem in terms of the backward shift operator, B, the decompositions is written as

$$y_t = F(B)^{-1} y_t + C(B) y_t \tag{3.6}$$

with $y_t^g = F(B)^{-1} y_t$ and $y_t^c = C(B) y_t$, where the polynomials in B (filters) are

$$F(B) = \lambda B^2 - 4\lambda B + (6\lambda + 1) - 4\lambda B^{-1} + \lambda B^{-2} = \lambda(1-B)^2(1-B^{-1})^2 + 1$$

and

$$C(B) = \frac{\lambda(1-B)^2(1-B^{-1})^2}{\lambda(1-B)^2(1-B^{-1})^2 + 1},$$

which is the HP filter. Notice that $F(1) = 1$ and $C(1) = 0$.

A number of authors have studied the basic properties of the HP filter, see for example Harvey and Jaeger (1993) and King and Rebelo (1993).

3.3. **The median filter.** The HP and BK filters are examples of linear filters. For completeness, herein we will also consider a class of nonlinear filters, called the *median filter* (Wen and Zeng, 1999), that has been proven to be very useful in recent years in signal processing. Median filters have two interesting properties: edge (sharp change) preservation and efficient noise attenuation with robustness against impulsive-type noise. Neither can be achieved by traditional linear filtering techniques. To compute the output of a median filter, an odd number of sample values are sorted, and the middle or median value is used as the filter output. If the filter length is $2n+1$, the filtering procedure is denoted as

$$med\{y_{t-n}, y_{t-n+1}, \ldots, y_t, \ldots, y_{t+n}\}. \tag{3.7}$$

Frequency analysis and impulse response have no meaning in median filtering since the impulse response of a median filter is zero. Nonetheless, a very important property of the median filter is the so-called *root-convergence* property, namely, any finite sample time series contains a root signal set that is invariant to the median filtering. From an economic point of view, this invariant property is of interest because it makes possible that possible structural shifts of economic fundamentals be not disturbed by the filtering operation. See Wen and Zeng (1999) for further details. This property is particularly relevant for our purpose, as we will notice in the Monte Carlo experiments.

4. Trend Components ECM Test

Decomposing the series z_t as in (3.1) we get

$$z_t = z_t^g + z_t^c \tag{4.1}$$

If the actual series are $I(1)$ and $(y_t - \alpha z_t)$ is $I(0)$ they are cointegrated. In terms of the unobserved components we could write

$$y_t - \alpha z_t = (y_t^g - \alpha z_t^g) + (y_t^c - \alpha z_t^c) \tag{4.2}$$

Let $\bar{b}(B)$ be a general two–sided moving average filter where we impose some constraints in the $\bar{b}_k$ coefficients so that it is a low–pass filter (see section 3.1 for details), and call $y_t^g = \bar{b}(B)y_t$ the corresponding trend component. Then, multiplying equation (5.1a) by $\bar{b}(B)$ we get

$$\Delta\bar{b}(B)y_t = \bar{b}(B)c_t + a\Delta\bar{b}(B)z_t + b[\bar{b}(B)y_{t-1} - \alpha\bar{b}(B)z_{t-1}] + \bar{b}(B)u_{1,t} \tag{4.3}$$

which is an ECM model for the trend component

$$\Delta y_t^g = c_t^g + a\Delta z_t^g + b[y_{t-1}^g - \alpha z_{t-1}^g] + \bar{b}(B)u_{1,t} \tag{4.4}$$

Since $\bar{b}(B)u_{1,t}$ might have some autocorrelation, we can consider the more dynamic version of the ECM for the trend components given by

$$\phi_y(B)\Delta y_t^g = c_t^g + a\phi_z(B)\Delta z_t^g + b[y_{t-1}^g - \alpha z_{t-1}^g] + \eta_t \tag{4.5}$$

where η_t is considered white noise and the lags of $\phi_y(B)\Delta y_t^g$ and $\phi_z(B)\Delta z_t^g$ are determined by the SBIC criterion. We might expect that for significant smoothing, c_t^g can be approximated by a constant or a linear trend.

From equation (5.1a) we can write the ECM model for a general trend–cycle decomposition as,

$$\Delta(y_t^g + y_t^c) = c_t + a\Delta(z_t^g + z_t^c) + b[(y_{t-1}^g + y_{t-1}^c) - \alpha(z_{t-1}^g + z_{t-1}^c) + u_{1,t} \tag{4.6}$$

and grouping terms, equations (4.6) and (5.1d) can be written as

$$\Delta y_t^g = c_t^\star + a\Delta z_t^g + b(y_{t-1}^g - \alpha z_{t-1}^g) + u_{1,t} \tag{4.7a}$$

$$c_t^\star = (\Delta\mu_{y,t} - \Delta y_t^c) - a(\Delta\mu_{z,t} - \Delta z_t^c) - b[(\mu_{y,t-1} - \alpha\mu_{z,t-1}) - (y_{t-1}^c - \alpha z_{t-1}^c)] \tag{4.7b}$$

In practice, since we do not know where the breaks occur, we would like to approximate (4.7a)–(4.7b) by

$$\phi_y(B)\Delta y_t^g = c_0^\star + a\phi_z(B)\Delta z_t^g + b[y_{t-1}^g - \alpha z_{t-1}^g] + \epsilon_t \tag{4.8}$$

The question now is whether the ECM test based on the t-ratio ($t_{\hat{b}}$) of equations (4.8) is robust to the presence of outliers in the series.

5. Monte Carlo Simulation Experiment

Our data generating process (DGP) is based on several extensions of the one used by Kremers et al. (1992) and Campos et al. (1996). It is a linear first–order vector autorregression with normal disturbances, Granger causality in only one direction ($z \to y$), and a possible structural break in the strongly exogenous variables (Δz_t) for the parameters a and α of interest.

5.1. **The model.** Our DGP is based on

$$\Delta y_t = c_t + a\Delta z_t + b(y_{t-1} - \alpha z_{t-1}) + u_{1t} \tag{5.1a}$$

$$\Delta z_t = \Delta\mu_{z,t} + u_{2t} \tag{5.1b}$$

$$c_t = \Delta\mu_{y,t} - a\Delta\mu_{z,t} - b(\mu_{y,t-1} - \alpha\mu_{z,t-1}) \tag{5.1c}$$

$$\mu_{z,t} = \mu_{z,o} + s_z\delta_{j,t}^z \tag{5.1d}$$

$$\begin{cases} \Pr(\delta_t^z = 1) = \Pr(\delta_t^z = -1) = \dfrac{\pi}{2} \\ \Pr(\delta_t^z = 0) = 1 - \pi \end{cases} \tag{5.1e}$$

In order to get only cobreaks in differences, we impose that $\Delta\mu_{y,t} - a\Delta\mu_{z,t} = c_d = 0.5$. On the other hand, to simulate a set of series with only cobreaks in levels, we impose $\Delta\mu_{y,t} - \alpha\Delta\mu_{z,t} = 0$. The series will show full cobreaking when $\Delta\mu_{y,t} - a\Delta\mu_{z,t} - b(\mu_{y,t-1} - \alpha\mu_{z,t-1}) = c = 0$. In the case of no cobreaks, we add another shock to μ_y given by

$$\mu_{y,t} = \mu_{y,o} + s_y\delta_{j,t}^y \tag{5.1f}$$

where $\delta_{j,t}^y$ follows a stochastic process similar to $\delta_{j,t}^z$ in (5.1e) but mutually independent.

Based on (5.1a)–(5.1d), the critical values ($b = 0$), the size ($b = 0$ with Ma(1) structure in $u_{1,t}$) and the power ($b \neq 0$) of the test are obtained by Monte Carlo simulation experiments.

Without loss of generality, we take $\sigma_1^2 = 1$, $\alpha = 1$ and $\sigma_2 = s$, $\mu_{y,0} = \mu_{z,0} = 0$. Thus, the experimental design variables are the parameters a, b, s, and the sample size, T. The experiment is a full factorial design with:

$a = 0.0, 0.5, 1$ (contemporaneous correlation in first differences)

$b = 0.0$ (no cointegration), $b < 0$ (cointegration)

$\pi = 0$ (no breaks), 0.05, 0.1.

$s = 1$, 6, 16 (size of the breaks)

$T = 100$, 200, 500, 1000 (sample sizes)

This represents 144 experiments for each value of b . Notice that when $a = 1$ there is a common (COMFAC) restriction in the error correction model ($a = \alpha = 1$).

The Monte Carlo experiments are based on 2000 replications of each experiment where the first 50 observations of the simulated series are dropped to consider random initial conditions.

To obtain the empirical critical values we simulate the y_t and z_t series following the DGP (5.1a)–(5.1e) with $b = 0$ and we estimate the flollowing three models

$$\Delta y_t = c + a\Delta z_t + b(y_{t-1} - z_{t-1}) + u_{1t} \qquad \text{(Model 1)}$$

$$\phi(B)\Delta y_t = c + \theta(B)\Delta z_t + b(y_{t-1} - z_{t-1}) + u_{1t} \qquad \text{(Model 2)}$$

$$\phi(B)\Delta y_t^g = c + \theta(B)\Delta z_t^g + b(y_{t-1}^g - z_{t-1}^g) + u_{1t} \qquad \text{(Model 3)}$$

where we have imposed $\alpha = 1$, and the order of the polynomials $\phi(B)$ and $\theta(B)$ are chosen by means of the SBIC criterion. The variables y_t^g and z_t^g are the trend components obtained by Hodrick Prescott filter (HP10 and HP100), Baxter and king filter (BK) and the median filter (MD). See Section 3 for more details. The lower 5% tail of the distribution of the $t(\hat{b})$ statistic is the empirical critical value considered. The empirical power of the test is calculated analogously by simulating the series with the other parameter values of b ($b < 0$), and computing the percentage of rejections obtained using the previously obtained empirical critical values.

To impose simultaneous co–breaking we force $c_t = \Delta\mu_{y,t} - a\Delta\mu_{z,t} - b(\mu_{y,t} - \alpha\mu_{z,t}) = 0$. In order to get only co-breaks in differences, we impose that $\Delta\mu_{y,t} - a\Delta\mu_{z,t} = c_d = 0.5$. On the other hand, to simulate a set of series with only co-breaks in levels, we impose $\Delta\mu_{y,t} - \alpha\Delta\mu_{z,t} = 0$, see Arranz and Escribano (1998) for more details of the derivation.

Notice that we avoid the case $c_d = 0$ because the critical values would be the ones obtained in the simultaneous co–breaking case, since under $b = 0$ we have simultaneous co-breaking with $c_s = 0$. Furthermore, notice that in the case where $a = \alpha = 1$, co–breaks in differences would co–breaks in levels (full co-breaking). This comes from the fact that under the COMFAC restriction ($\alpha = a$), co–breaks in levels imply co–breaks in differences (full co–break).

5.2. **ECM test based on the trend components: No outliers.** Figure 1 represents the kernel density estimator of the whole empirical distribution of $t(\hat{b})$ for Model 2 (ECM test) and Model 3 using the HP10 filter, HP100 filter, BK and MD respectively. With all of the filters used we get similar results. It is important to remark that the best results are obtained for the left tail of the distributions. This implies that the critical values used to test the null hypothesis of no–cointegration against the cointegration alternative are robust to the type of filter used, see Table 1.

[Table 1 about here.]

[Figure 1 about here.]

[Figure 2 about here.]

In terms of power there is only a small loss when using the BK filter and even smaller when using the median filter, see Figure 2 and Table 2. The HP10 and HP100 filters display the lowest power.

[Table 2 about here.]

To evaluate the size of the test based on Models 1–3 we add an Ma(1) structure to the $u_{1,t}$ error term, i.e. $u_{1,t} + \theta u_{1,t-1} = v_t$ with $v_t \sim iid\, N(0,1)$, and the MA(1) parameter equal to $\theta = \pm 0.2$ and $\theta = \pm 0.5$. The empirical results for a 5% nominal size are included in Table 3

As expected, the largest size distortions are generated in small sample sizes and with $\theta = -0.5$. In the case of Model 2 with $T = 200$, $a = 1$, and $\theta = -0.5$ the 5% is transformed into 20%. This empirical size is reduced to 6.55% with the HP100 filter, 3.15% with HP10, 6.65% with BK and 6.8% with the MD filters. Once again, the most stable results in terms of the size are obtained by using the median filter (MD).

[Table 3 about here.]

5.3. **ECM test with additive outliers.** In order to evaluate the effects of the additive outliers on the ECM tests we evaluate the most favorable situation. That is, we allow for extra lags in the

ECM models to approximate the effect of the outliers and run the ECM test based on Model 2 when the orders of the polynomial $\varphi(B)$ and $\theta(B)$ were chosen by the SBIC criterion.

From Figure 3.1 we see that the effects of having 10% contamination of additive outliers are dramatic. The whole distribution is shifted to the left and therefore we will detect too much cointegration. this result supports previous evidence given in Franses and Haldrup (1994).

ECM test from Model 1 with additive outliers. The model in this case is

$$\Delta y_t = c + a\Delta z_t + b(y_{t-1} - z_{t-1}) + u_{1t} \qquad \text{(Model 1)}$$

which is correctly specified only under *simultaneous co-breaks.* The reason is that c_t from (5.1d) is equal to 0.

[Table 4 about here.]

Notice that the critical values (C.V.) are very stable for the simultaneous co-breaking case and only marginally affected by the sample size (T), the nuisance parameter a and the size of the break (s).

Co-breaks in differences, not in levels. In this case $c_t = c + bc_d t - b(a - \alpha)s_z\delta^z_{t-1}$. Therefore, in Model 1 there is a missing term even when $a = \alpha$ (COMFAC). The results of Table 4 show that the CV of $t(\hat{b})$ based on Model 1 are shifted to the right and the values change with T and only marginally with a and s (the size of the break). Notice that the values are close to the Normal distribution since there ia a missing trend.

Co-breaks in levels, but not in differences. Now $c_t = c + (\alpha - a)s_z\Delta\delta^z_t$, and Model 1 is misspecified. However, in this particular case, if $\alpha = a$ (COMFAC), $c_t = c$ and we are back to the simultaneous co-breaks case, which means that Model 1 is correctly specified. From Table 4, CV are now more stable with the sample size (T) but they span from -2.5 to-2.8 depending on the COMFAC restriction.

Independent shocks or shocks in only one of the variables. In this case c_t is always different from a constant, see equations (2.10a)–(2.10c), and therefore Model 1 is always seriously misspecified. Even for a sample size of 1000 the CV span form -3.09 to -16.3 depending on the size of the jump s, see Table 4. The question now is how much do we improve by adding extra lags in the ECM model.

ECM test from Model 2 with additive outliers. The model is

$$\phi(B)\Delta y_t = c + \theta(B)\Delta z_t + b(y_{t-1} - z_{t-1}) + u_{1t} \qquad \text{(Model 2)}$$

Under *simultaneous co-breaks* or with *co-breaks in levels, but not in differences* the CV are fairly stable, as before with Model 1. Similarly, with *co-breaks in differences but not in levels* the CV are shifted to the left but are more unstable with respect to the sample size (T) and fairly stable with respect to the other parameter values of a and s.

[Table 5 about here.]

[Table 6 about here.]

From Table 5 it is clear that the main improvements of Model 2 over Model 1 occurs when the *shocks are independent* or when they are in *only one of the variables.* However, this does not mean that they are always stable since for example, when the shocks are only in the variable y_t, the CV can span from -3 to -10.9 for a sample of 200 observations. therefore, adding extra lags helps, but it is not a satisfactory solution.

The power of the test depends heavily on the kind of co-break, see Table 6. In the case of *co-breaks in differences but not in levels* the test shows no power at all, which is explained by the fact that we are omitting a deterministic trend component. On the other hand the power of the test is high in the other two cases of co-breaks, namely *simultaneous co-breaks* and *co-breaks in levels but no in differences.* In the cases of *independent shocks* or *shock in one of the variables* the power depends on the kind and size of the shock as well as on the value of the parameter a.

ECM test from Model 3 with additive outliers. The model is

$$\phi(B)\Delta y_t^g = c + \theta(B)\Delta z_t^g + b(y_{t-1}^g - z_{t-1}^g) + u_{1t} \qquad \text{(Model 3).}$$

The intuition for expecting Model 3 to be a good approximation to the correctly specified model is the following. Equation (2.2a) is transformed into equation (4.7a) based on the trend components, where $c_t^\star$ is a stochastic intercept given by equation (4.7b). Since the additive outliers are transitory shocks they should mainly be part of the cycle in a trend-cycle decomposition and therefore the following elements should be free of outliers:

$$(\Delta\mu_{y,t} - \Delta y_t^g),\ (\Delta\mu_{z,t} - \Delta z_t^g),\ (\mu_{y,t-1} - y_{t-1}^g),\ (\mu_{z,t-1} - z_{t-1}^g)$$

The stochastic slope $c_t^\star$ from (4.7b) should be a stationary series which can be approximated by lags of Δy_t^g and Δz_t^g, and this is exactly what Model 3 does.

[Table 7 about here.]

[Table 8 about here.]

[Table 9 about here.]

[Table 10 about here.]

Tables 7–10 show the CV obtained under the six possible co–breaking and non–cobreaking situations previously analyzed. No matter what type of filter we use, the CV are stable for different sample sizes, different values of parameter a, different sizes of the jumps (s). We did the analysis for HP10, HP100, BK and MD filters, The only case where the CV of the ECM test depend seriously on the sample size (T) is when there is only co–breaks in differences but not in levels. Notice, however the robustness of those critical values to nuisance parameters such as s or s (the jump size), even for a 10% of outliers ($\pi = 0.1$).

[Table 11 about here.]

[Table 12 about here.]

[Table 13 about here.]

[Table 14 about here.]

The power of the test is analyzed in Tables 11–14. First, notice that in the case of co–breaks in differences not not in levels we get no power, as it happened with Model 2. It is remarkable that the power of the test depends crucially on the parameter a in most cases. As expected, the power of the test based on Model 3 is lower than the one obtained with Model 2, but this is not true in all cases. In particular, we whould notice that in the case of having shocks only in the variable y_t, with $a = 1$ and $s = 1, 16$, the most powerful test is the one based on Model 3 with the BK filter with $T = 100$. In the case of shocks in z_t, with $a = 1$ the most powerful test for $T = 100, 200$ is the one based on the MD filter. Another feature of the test based on Model 3 and the MD filter is that, apart from the case of co–breaks in differences but not in levels, the power of the test does not depend on the type of co–breaks considered. Furthermore, the test based on the MD filter yields the highest power among those tests based on Model 3.

5.4. **Robustness of critical values to the presence of outliers.** The question now is the following: Can we safely use the critical values of Table 1 to do cointegration tests, such as the ECM test, in the presence of additive outliers?

To answer this question we computed the empirical rejection frequencies obtained using the 5% critical values of Table 1 when the data generating process given in equations (5.1a)–(5.1f) is contaminated with the most dangerous case, that is, independent shocks in variables y_t and z_t under the null hypothesis $H_0 : b = 0$. If those critical values are robust, the percentage of rejections should be around 5%.

[Figure 3 about here.]

[Table 15 about here.]

Model 2 is clearly not robust since the percentage of rejection can reach 43.5% for a sample size of $T = 100$ with $s = 16$, see Figure 3.1. Using Model 3 the results improve dramatically, see Figures 3.3–3.5. With HP10 and BK the previous figure is reduced to 21.8% and 26.1%, but with HP100 and MD those values are 5.1% and 6.2%, respectively, see Table 15. Notice that we only mention the results for the worst contamination types. For most of the parameter values analyzed all of the filters perform quite well, but the best one in terms of robustness and power is the MD filter.

6. Empirical example

In this section we illustrate the practical usefulness of our procedure by performing the analysis on annual observations of the CPI–based US/Finland real exchange rates during the period 1980–88. This dataset has been previously studied in Perron and Vogelsang (1992), Franses and Haldrup (1994) and Franses and Lucas (1998). The question is whether the real exchange rate series is stationary or not, i.e does the purchasing power parity (PPP) hold? We proceed by testing if the nominal exchange rates and the CPI ratio are cointegrated.

First, we perform the analysis imposing the common factor restriction (COMFAC) with the following variables: y_t is the log of the nominal exchange rate and z_t is the log of CPI ratio. The cointegrating relationship is known and equal to the real exchange rate, i.e. $r_t = y_t - z_t$. The number of lags in the ECM model is chosen by means of the SBIC criterion. If the variables are cointegrated the term r_{t-1} should be significant in at least one of the two equations. We perform the ECM test by examinig the t–stats of the coefficient corresponding to r_{t-1} in both equations.

When the analysis is done on the observed variables, the values of the t–stats are -6.65 and -1.49, when the dependent variables are Δy_t and Δz_t, respectively. It is clear that the long run causality runs form z_t, the log of the CPI ratio, to y_t, the log of the nominal exchange rate, but it is unclear that the series are cointegrated. From Tables 1 and 5, the critical values would indicate that the series are cointegrated in the cases of partial cobreaks and shocks only in z_t, the log of the CPI rate. However, it is not clear that the series are cointegrated if there independent shocks or the shocks occur only in the variable y_t, the log of the nominal exchange rate, and this might well be the case. If we apply this procedure to the trend components ECM models obtained by the HP100 filter, the t–stats are -1.03 and -2.15, while with the BK filter the ECM test statistics are -1.84 and -0.47, and finally -2.16 and -0.37 for the MD filter. As we can see from the critical values in Tables 7, 9, and 10 the conclusion is that we cannot reject the null hypothesis that the series are not cointegrated, which agrees with the findings in Franses and Haldrup (1994) and Franses and Lucas (1998).

Notice, however, that imposing the COMFAC restriction reduces the power of the ECM test. Therefore, we apply the tests without imposing the COMFAC restriction, as we did in Section 5. In the case of the BK filter, the ECM test statistics are -3.63 and -0.86, and -6.71 and -1.42 for the MD filter. Since the absolute value of the second statistic is not significant, it suggests that the second variable is weakly exogenous for the long run parameter of interest. Therefore, we conclude that the series are cointegrated, using the critical values from Tables 7, 9, and 10, and that the real exchange rate follows a stationary process with outliers. This conclusion is consistent with the result of Vogelsang (1999) and Arranz et al. (2000).

7. Conclusions

In this paper we have analyzed the effects of having additive outliers in a multivariate context with cointegrated variables. Usual non-cointegration tests (like ECM tests) tend to find too much cointegration in this case. The problem is partially solved by using overparameterized models that include extra lags of the regression variables.

Different effects of the additive outliers have been analyzed in a multivariate context, running from simultaneous co-breaks, partial co-breaks and reaching independent shocks in each of the variables of the model. Obviously, the worst case is the one of independent shocks.

We suggest to approach this problem by doing the non-cointegration test on the ECM models based on the trend component instead of on the ECM model with the observed variables. We have analyzed by Monte Carlo simulation experiments different trend-cycle decompositions based on the Hodrick-Prescott filter (HP10, HP100), the Baxter and King filter (BK) and the median filter (MD). Most of them yield good results in terms of robustness, but the best ones are the HP100 and MD filters. On the other hand, in terms of power, the best results are provided by the BK and MD filters. Our results suggest that in the future we should pay more attention to nonlinear filters such as the median (MD) filter.

References

Andrés, J., A. Escribano, C. Molinas and D. Taguas (1990), *La inversión en España: Econometría con restricciones de equilibrio*, A. Bosch.

Andrews, D. (1993), 'Tests of parameter instability and structural change with unknown break point', *Econometrica* **61**, 821–856.

Andrews, D., W. Lee and W. Ploberger (1996), 'Optimal changepoints tests for normal linear regression', *Journal of Econometrics* **70**, 9–38.

Arranz, M. and A. Escribano (1998), Detrending procedures and cointegration testing: ECM tests under structural breaks, Working Paper 98–15, Universidad Carlos III de Madrid.

Arranz, M. and A. Escribano (2000), 'Cointegration testing under structural breaks: A robust extended error correction model', *Oxford Bulletin of Economics and Statistics* **61**, 23–52.

Arranz, M., A. Escribano and F. Marmol (2000), Effects of applying linear and nonlinear filters on tests for unit roots with additive outliers, Working paper, Dpt. of Statistics and Econometrics, Universidad Carlos III de Madrid.

Bai, J. (1997), 'Estimating multiple breaks one at a time', *Econometric Theory* **13**, 315–352.

Bai, J. and P. Perron (1998), 'Estimating and testing linear models with multiple structural changes', *Econometrica* **66**, 47–78.

Banerjee, A., J. Dolado, D. Hendry and G. Smith (1986), 'Exploring equilibrium relationships in econometrics through static models: Some Monte Carlo evidence', *Oxford Bulletin of Economics and Statistics* **48**, 853–877.

Banerjee, A., J. Dolado, J.W. Galbraith and D.F. Hendry (1993), *Cointegration, error-correction, and the econometric analysis of non-stationary data*, Oxford University Press.

Banerjee, A., R. Lumsdaine and J. Stock (1992), 'Recursive and sequential tests of unit root and trend break hypothesis: Theory and international evidence', *Journal of Business and Economic Statistics* **10**, 271–287.

Banerjee, A., S. Lazarova and G. Urga (1998), Bootstrapping sequential tests for multiple structural breaks, Discussion Paper 17–98, Center for Economic Forecasting, London Business School.

Baxter, M. and R. King (1995), Measuring business cycles: Approximate band–pass filters for economic time series, Working Paper 5022, NBER.

Baxter, M. and R. King (1999), 'Measuring business cycles: Approximate band–pass filters for economic time series', *Review of Economics and Statistics* **81**, 575–593.

Burns, A. and W. Mitchell (1946), *Measuring Business Cycles*, NBER.

Campos, J., N. Ericsson and D. Hendry (1996), 'Cointegration tests in the presence of structural breaks', *Journal of Econometrics* **70**, 187–220.

Clements, M. and D. Hendry (1999), *Forecasting Non-stationary Economic Time Series*, MIT Press, chapter 9.

Dickey, D. and W. Fuller (1979), 'Distribution of the estimators for autoregressive time series with a unit root', *Journal of the American Statistical Association* **74**, 427–431.

Engle, R. F. and C. W. J. Granger (1987), 'Cointegration and error-correction: Representation, estimation, and testing', *Econometrica* **55**, 251–276.

Escribano, A. (1987), Co–integration, time co–trends and error–correction systems: an alternative approach, Discussion Paper 8715, CORE.

Franses, P. and A. Lucas (1997*a*), Outlier robust cointegration analysis of Dutch interest rates, *in* 'Proceedings of the 1996 Meeting of the American Statistical Association', ASA, pp. 106–109.

Franses, P. and A. Lucas (1997*b*), Outlier robust cointegration tests, VU Research Memorandum 97–45, Vrije Universiteit Amsterdam.

Franses, P. and A. Lucas (1998), 'Outlier detection in cointegration analysis', *Journal of Business and Economic Statistics* **16**, 459–468.

Franses, P. and N. Haldrup (1994), 'The effects of additive outliers on unit roots and cointegration', *Journal of Business and Economic Statistics* **12**, 471–478.

Guay, A. and P. St-Amant (1997), Do the Hodrick-Prescott and Baxter-King filters provide a good approximation of business cycles?, Cahiers de Recherche 53, CREFE.

Harvey, A. C. and A. Jaeger (1993), 'Detrending, business cycles facts and the business cycle', *Journal of Applied Econometrics* **8**, 231–47.

Hendry, D. and A. Neale (1990), The impact of structural breaks on unit root tests, *in* P.Hackl and A.Westlung, eds, 'Economic Structural Change: Analysis and Forecasting', Berlin: IIASA and Springer–Verlag, pp. 95–119.

Hodrick, R. and E. Prescott (1980), Post–war U.S. business cycle: An empirical investigation, Technical report, Carnegie–Mellon University.

Hodrick, R. and E. Prescott (1997), 'Post–war U.S. business cycle: An empirical investigation', *Journal of Money, Credit and Banking* **29**, 1–16.

King, R. and S. Rebelo (1993), 'Low frequency filtering and real business cycles', *Journal of Economic Dynamics and Control* **17**, 207–31.

Kremers, J., N. Ericsson and J. Dolado (1992), 'The power of cointegration tests', *Oxford Bulletin of Economics and Statistics* **54**, 349–67.

Kydland, F. and E. C. Prescott (1990), 'Business cycles: Real facts and a monetary myth', *Federal Reserve Bank of Minneapolis Quarterly Review* **14**, 3–18.

Leybourne, S., T. Mills and P. Newbold (1998), 'Spurious rejections by Dickey–Fuller tests in the presence of a break under the null', *Journal of Econometrics* **87**, 191–203.

Lucas, A. (1995*a*), 'An outlier robust unit root tests with an application to the extended Nelson–Plosser data', *Journal of Econometrics* **66**, 153–173.

Lucas, A. (1995*b*), 'Unit roots tests based on M estimators', *Econometric Theory* **11**, 331–346.

Lucas, A. (1997), 'Cointegration testing using pseudo likelihood–ratio tests', *Econometric Theory* **13**, 149–169.

Perron, P. (1989), 'The great crash, the oil price shock and the unit root hypothesis', *Econometrica* **57**, 1361–1401.

Perron, P. and T. Vogelsang (1992), 'Nonstationarity and level shifts with an application to purchasing power parity', *Journal of Business and Economic Statistics* **10**, 301–320.

Rappoport, P. and L. Reichlin (1989), 'Segmented trends and nonstationary time series', *Economic Journal* **99**, 168–177.

Stock, J. (1994), Unit roots, structural breaks and trends, *in* R. F.Engle and D. L.McFadden, eds, 'Handbook of Econometrics', Vol. 4, North–Holland, chapter 46, pp. 2270–2381.

Vogelsang, T. (1997), 'Wald–type tests for detecting breaks in the trend of a dynamic time series', *Econometric Theory* **13**, 818–849.

Vogelsang, T. (1999), 'Two simple procedures for testing for a unit root when there are additive outliers', *Journal of Time Series Analysis* **20**, 173–192.

Wen, Y. and B. Zeng (1999), 'A simple nonlinear filter for economic time series analysis', *Economics Letters* **64**, 151–160.

Zivot, E. and D. Andrews (1992), 'Further evidence on the great crash, the oil price shock and the unit root hypothesis', *Journal of Business and Economic Statistics* **10**, 251–270.

Miguel A. Arranz, Dpt. of Statistics and Econometrics, Universidad Carlos III de Madrid

E-mail address: arranz@pvi,uc3m.es

Alvaro Escribano, Dpt. of Economics, Universidad Carlos III de Madrid

E-mail address: alvaroe@eco.uc3m.es

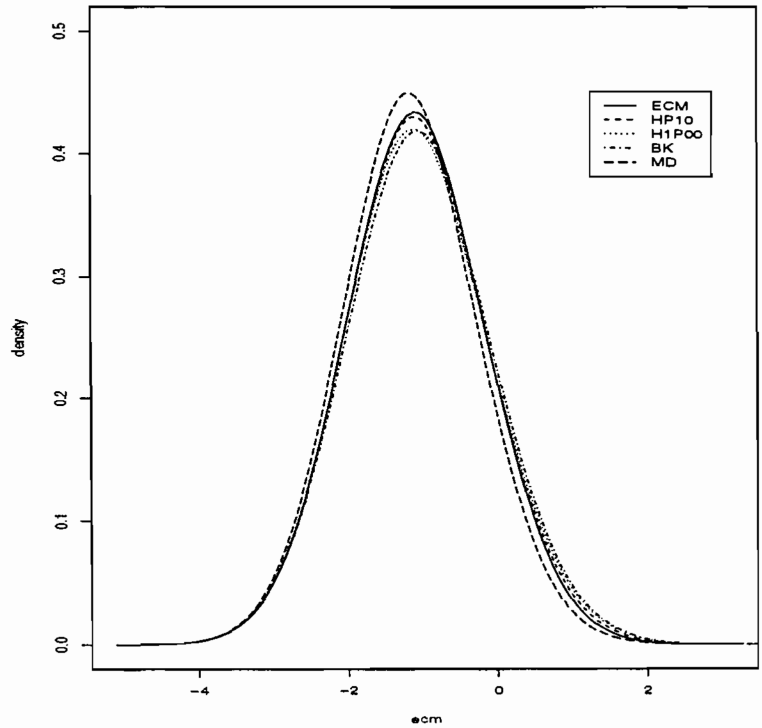

FIGURE 1. Kernel Density Estimator of the ECM test statistic (Model 2), and Low–Pass filter ECM test (Model 3) under the null hypothesis. $T = 1000$, $a = 0$. No outliers

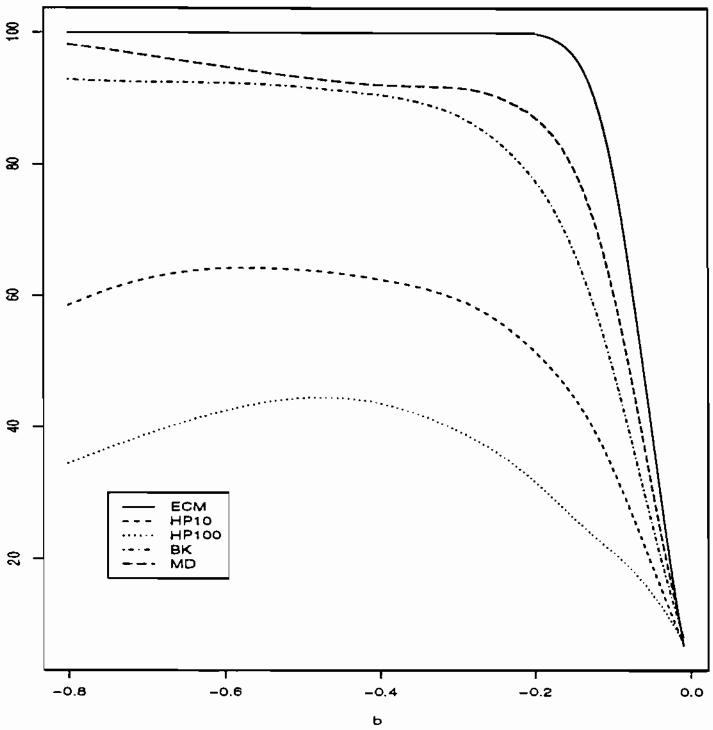

FIGURE 2. Comparison of powers of the ECM test (Models 2 and 3). $T = 100$, $a = 0$. No outliers

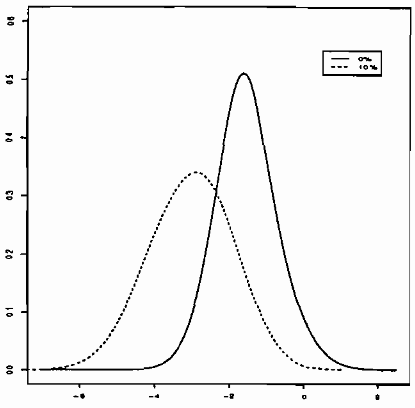

Figure 3.1: ECM

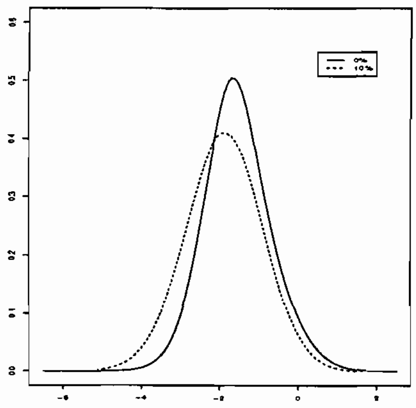

Figure 3.2: HP10

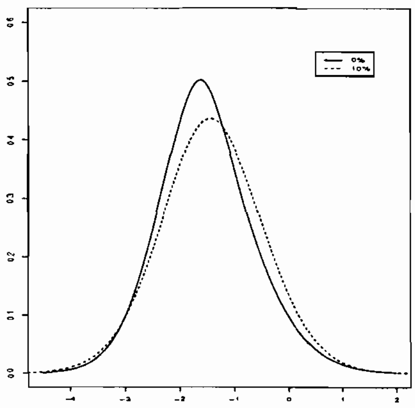

Figure 3.3: HP100

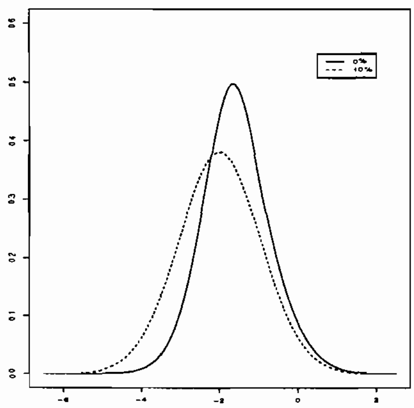

Figure 3.4: BK

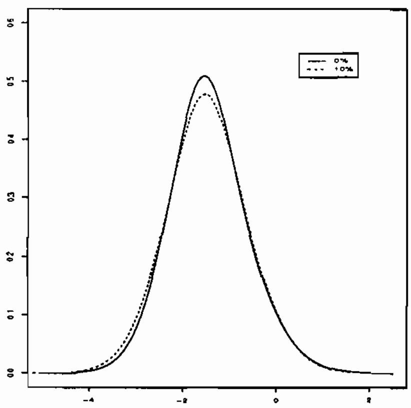

Figure 3.5: MD

FIGURE 3. Kernel Density Estimator of the ECM test (Model 2) and the Low–pass filtered data test (Model 3). $T = 1000$, $a = 1$, independent shocks in variables y_t and z_t, $s = 16$.

a	T=100	T=200	T=500	T=1000
		Model 1		
0.0	-2.624	-2.518	-2.654	-2.639
0.5	-2.769	-2.738	-2.758	-2.752
1.0	-2.875	-2.842	-2.878	-2.929
		Model 2		
0.0	-2.596	-2.495	-2.641	-2.652
0.5	-2.801	-2.787	-2.754	-2.789
1.0	-2.951	-2.868	-2.862	-2.949
		Model 3. HP10 Filter		
0.0	-3.017	-2.670	-2.668	-2.614
0.5	-3.272	-3.071	-2.859	-2.874
1.0	-3.481	-3.164	-2.940	-2.913
		Model 3. HP100 Filter		
0.0	-3.238	-2.726	-2.650	-2.608
0.5	-3.707	-3.071	-2.832	-2.797
1.0	-4.114	-3.085	-2.898	-2.891
		Model 3. BK Filter		
0.0	-2.889	-2.650	-2.667	-2.592
0.5	-3.069	-2.750	-2.676	-2.752
1.0	-3.021	-2.783	-2.889	-2.958
		Model 3. MD filter		
0.0	-2.749	-2.616	-2.654	-2.664
0.5	-2.816	-2.599	-2.561	-2.652
1.0	-2.859	-2.715	-2.736	-2.778

TABLE 1. Critical value of the tests. No outliers

a	T=100	T=200	T=500	T=1000
Model 1				
0.0	99.45	100.00	100.00	100.00
0.5	95.50	100.00	100.00	100.00
1.0	87.75	100.00	100.00	100.00
Model 2				
0.0	99.15	100.00	100.00	100.00
0.5	93.05	100.00	100.00	100.00
1.0	79.70	99.95	100.00	100.00
Model 3. HP10 Filter				
0.0	51.40	98.05	100.00	100.00
0.5	30.15	76.75	100.00	100.00
1.0	21.50	76.95	99.95	100.00
Model 3. HP100 Filter				
0.0	31.75	81.05	100.00	100.00
0.5	15.55	54.45	99.75	100.00
1.0	11.25	42.80	99.25	100.00
Model 3. BK Filter				
0.0	74.40	99.60	100.00	100.00
0.5	46.15	92.85	100.00	100.00
1.0	21.25	76.05	100.00	100.00
Model 3. MD filter				
0.0	88.10	99.65	100.00	100.00
0.5	58.35	97.55	100.00	100.00
1.0	30.60	85.00	100.00	100.00

TABLE 2. Power of of the tests. $b = -0.2$. No outliers

a	θ	T=100	T=200	T=500	T=1000
0.0	-0.5	2.40	2.60	2.05	1.95
	-0.2	5.00	4.80	4.20	3.90
	0.2	6.30	6.10	5.25	5.95
	0.5	8.10	8.00	5.55	6.10
0.5	-0.5	13.10	9.60	6.45	5.10
	-0.2	8.20	7.40	6.60	5.80
	0.2	4.25	4.95	5.30	5.45
	0.5	6.95	6.90	6.30	6.05
1.0	-0.5	31.20	20.05	12.50	8.55
	-0.2	11.70	11.40	7.50	5.35
	0.2	3.75	5.15	5.30	5.00
	0.5	6.75	6.50	4.70	3.75

Table 3.1: Model 2

a	θ	T=100	T=200	T=500	T=1000
0.0	-0.5	2.75	3.50	2.30	2.35
	-0.2	5.25	5.85	4.35	4.25
	0.2	4.95	5.75	5.50	5.30
	0.5	6.00	7.35	6.70	6.20
0.5	-0.5	2.55	2.80	4.35	4.35
	-0.2	5.00	5.55	6.95	5.90
	0.2	4.45	3.90	4.10	4.40
	0.5	4.05	3.80	4.20	4.25
1.0	-0.5	1.25	6.55	8.05	8.40
	-0.2	4.25	8.70	6.95	6.60
	0.2	2.75	3.90	3.60	4.00
	0.5	1.90	4.40	4.75	5.00

Table 3.2: Model 3. HP100

a	θ	T=100	T=200	T=500	T=1000
0.0	-0.5	2.05	2.80	2.40	2.55
	-0.2	3.25	4.85	4.15	4.05
	0.2	5.30	4.70	5.20	5.50
	0.5	5.65	6.10	5.95	6.05
0.5	-0.5	3.25	1.65	3.10	2.70
	-0.2	4.00	3.40	4.60	5.05
	0.2	5.65	4.10	3.95	3.85
	0.5	4.85	3.50	3.20	3.95
1.0	-0.5	2.45	3.15	4.75	5.75
	-0.2	4.10	3.75	6.35	6.85
	0.2	4.95	3.25	2.90	4.20
	0.5	2.95	2.10	3.75	4.55

Table 3.3: Model 3. HP10

a	θ	T=100	T=200	T=500	T=1000
0.0	-0.5	2.70	2.80	3.05	2.90
	-0.2	3.65	4.20	4.45	4.35
	0.2	5.45	5.30	5.35	6.25
	0.5	6.15	6.90	6.85	8.05
0.5	-0.5	5.20	5.45	5.05	3.50
	-0.2	5.50	4.95	5.05	4.80
	0.2	5.60	5.05	5.75	6.00
	0.5	5.45	7.85	9.25	7.90
1.0	-0.5	10.90	6.65	3.85	3.85
	-0.2	5.80	4.40	3.75	4.20
	0.2	4.90	7.00	6.25	5.25
	0.5	7.00	9.35	7.40	6.05

Table 3.4: Model 3. BK Filter

a	θ	T=100	T=200	T=500	T=1000
0.0	-0.5	5.05	4.70	3.05	2.15
	-0.2	5.30	4.65	4.30	3.35
	0.2	4.95	5.25	5.65	6.20
	0.5	5.95	5.90	6.15	7.55
0.5	-0.5	5.95	5.60	4.60	2.60
	-0.2	6.60	4.45	5.10	4.05
	0.2	4.80	5.05	6.10	6.00
	0.5	4.85	6.25	7.05	7.60
1.0	-0.5	8.90	6.80	4.75	4.85
	-0.2	5.15	5.40	4.85	5.20
	0.2	4.75	4.80	5.00	5.25
	0.5	5.00	4.85	5.45	5.45

Table 3.5: Model 3. MD Filter

TABLE 3. Size of the test. Simulated model with MA(1) disturbances.

T	$a=0.0$			$a=0.5$			$a=1.0$		
	s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
				Simultaneous cobreaks					
100	-2.631	-2.489	-2.350	-2.779	-2.694	-2.546	-2.883	-2.858	-2.849
200	-2.519	-2.481	-2.384	-2.742	-2.731	-2.649	-2.850	-2.841	-2.840
500	-2.652	-2.591	-2.533	-2.747	-2.748	-2.669	-2.879	-2.869	-2.869
1000	-2.640	-2.627	-2.544	-2.762	-2.774	-2.745	-2.935	-2.939	-2.944
				Cobreaks in differences, not in levels					
100	-2.010	-1.989	-1.997	-1.982	-1.982	-1.985	-1.970	-1.965	-1.967
200	-1.967	-1.923	-1.911	-1.936	-1.899	-1.897	-1.916	-1.902	-1.914
500	-1.813	-1.836	-1.852	-1.849	-1.831	-1.837	-1.845	-1.840	-1.839
1000	-1.763	-1.773	-1.772	-1.765	-1.760	-1.767	-1.766	-1.761	-1.758
				Cobreaks in levels, not in differences					
100	-2.577	-2.739	-2.792	-2.738	-2.765	-2.799	-2.883	-2.858	-2.849
200	-2.484	-2.754	-2.818	-2.735	-2.827	-2.844	-2.850	-2.841	-2.840
500	-2.578	-2.769	-2.827	-2.750	-2.814	-2.804	-2.879	-2.869	-2.869
1000	-2.544	-2.804	-2.891	-2.765	-2.885	-2.898	-2.935	-2.939	-2.944
				Independent shocks					
100	-2.650	-4.131	-5.426	-2.964	-4.751	-5.605	-3.293	-5.281	-5.756
200	-2.582	-4.856	-7.818	-2.971	-5.855	-8.405	-3.259	-6.660	-8.744
500	-2.683	-4.837	-9.457	-2.969	-5.994	-10.591	-3.253	-6.922	-11.229
1000	-2.667	-5.202	-11.207	-2.974	-6.437	-13.026	-3.281	-7.451	-13.953
				Shocks in y_t					
100	-2.624	-4.613	-7.632	-2.864	-5.467	-8.132	-3.044	-5.905	-8.437
200	-2.588	-5.108	-9.899	-2.890	-6.246	-11.059	-3.076	-6.878	-11.683
500	-2.692	-5.122	-11.485	-2.886	-6.235	-13.401	-3.081	-7.075	-14.573
1000	-2.667	-5.337	-12.553	-2.887	-6.643	-15.205	-3.089	-7.321	-16.323
				Shocks in z_t					
100	-2.631	-2.489	-2.350	-2.856	-2.967	-2.540	-3.080	-3.565	-2.857
200	-2.519	-2.481	-2.384	-2.822	-3.084	-2.839	-3.024	-3.708	-3.205
500	-2.652	-2.591	-2.533	-2.838	-3.127	-2.905	-3.060	-3.790	-3.433
1000	-2.640	-2.627	-2.544	-2.882	-3.203	-3.071	-3.094	-3.929	-3.724

TABLE 4. Critical values. Model 1 test, $\pi = 0.1$

T	$a=0.0$ s=1	$a=0.0$ s=6	$a=0.0$ s=16	$a=0.5$ s=1	$a=0.5$ s=6	$a=0.5$ s=16	$a=1.0$ s=1	$a=1.0$ s=6	$a=1.0$ s=16
Simultaneous cobreaks									
100	-2.601	-2.533	-2.407	-2.813	-2.736	-2.661	-2.934	-2.918	-2.899
200	-2.512	-2.481	-2.402	-2.772	-2.711	-2.657	-2.871	-2.874	-2.860
500	-2.650	-2.597	-2.512	-2.737	-2.757	-2.669	-2.853	-2.851	-2.840
1000	-2.664	-2.621	-2.531	-2.782	-2.801	-2.774	-2.955	-2.956	-2.954
Cobreaks in differences, not in levels									
100	-2.068	-2.072	-2.039	-2.042	-2.068	-2.029	-2.001	-1.999	-2.009
200	-1.928	-1.918	-1.903	-1.926	-1.916	-1.937	-1.897	-1.906	-1.910
500	-1.824	-1.829	-1.838	-1.819	-1.823	-1.830	-1.809	-1.826	-1.832
1000	-1.730	-1.736	-1.727	-1.729	-1.721	-1.720	-1.729	-1.707	-1.707
Cobreaks in levels, not in differences									
100	-2.587	-2.778	-2.820	-2.767	-2.800	-2.812	-2.934	-2.918	-2.899
200	-2.517	-2.786	-2.843	-2.787	-2.825	-2.843	-2.871	-2.874	-2.860
500	-2.590	-2.776	-2.821	-2.746	-2.808	-2.827	-2.853	-2.851	-2.840
1000	-2.612	-2.831	-2.885	-2.765	-2.882	-2.901	-2.955	-2.956	-2.954
Independent shocks									
100	-2.644	-2.695	-3.111	-3.009	-4.004	-3.516	-3.303	-4.997	-5.922
200	-2.571	-2.683	-3.563	-2.945	-3.593	-4.378	-3.253	-4.589	-4.919
500	-2.645	-2.481	-2.240	-2.893	-3.387	-3.060	-3.033	-4.455	-4.046
1000	-2.674	-2.509	-2.551	-2.954	-3.567	-3.597	-2.979	-4.108	-4.540
Shocks in y_t									
100	-2.624	-3.311	-7.673	-2.866	-5.336	-8.067	-3.063	-5.921	-8.309
200	-2.555	-2.780	-4.352	-2.891	-3.631	-6.740	-3.064	-4.249	-10.992
500	-2.662	-2.635	-3.281	-2.841	-3.308	-4.634	-2.999	-3.420	-5.473
1000	-2.670	-2.686	-3.437	-2.880	-3.180	-4.664	-3.029	-3.217	-4.967
Shocks in z_t									
100	-2.601	-2.533	-2.407	-2.850	-2.982	-2.591	-3.121	-3.504	-2.887
200	-2.512	-2.481	-2.402	-2.806	-3.023	-2.793	-3.029	-3.659	-3.205
500	-2.650	-2.597	-2.512	-2.806	-3.031	-2.902	-2.968	-3.582	-3.425
1000	-2.664	-2.621	-2.531	-2.866	-3.057	-3.051	-3.020	-3.430	-3.661

TABLE 5. Critical values. Model 2 test, $\pi = 0.1$

T	$a = 0.0$ s=1	$a = 0.0$ s=6	$a = 0.0$ s=16	$a = 0.5$ s=1	$a = 0.5$ s=6	$a = 0.5$ s=16	$a = 1.0$ s=1	$a = 1.0$ s=6	$a = 1.0$ s=16
				Simultaneous cobreaks					
100	99.65	99.90	100.00	94.75	97.65	99.85	84.95	85.95	86.25
200	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
500	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
				Cobreaks in differences, not in levels					
100	5.05	5.90	7.30	0.65	0.55	0.60	0.00	0.05	0.00
200	2.85	5.55	6.20	0.40	0.30	0.15	0.00	0.00	0.00
500	0.50	4.00	5.10	0.00	0.00	0.00	0.00	0.00	0.00
1000	0.05	2.00	5.25	0.00	0.05	0.10	0.00	0.00	0.00
				Cobreaks in levels, not in differences					
100	99.70	93.95	89.10	95.90	90.75	89.15	84.95	85.95	86.25
200	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
500	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
				Independent shocks					
100	99.65	90.20	44.80	94.55	52.80	14.30	84.95	48.05	3.70
200	100.00	99.95	95.60	100.00	98.85	69.00	99.85	92.80	51.15
500	100.00	100.00	100.00	100.00	100.00	98.40	100.00	100.00	89.05
1000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
				Shocks in y_t					
100	99.60	96.00	81.40	96.00	80.75	70.45	87.15	80.05	58.50
200	100.00	100.00	99.85	100.00	100.00	99.45	100.00	99.55	98.80
500	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
				Shocks in z_t					
100	99.60	93.10	40.10	94.65	65.95	19.45	85.10	38.05	10.25
200	100.00	99.95	85.20	100.00	98.90	48.30	99.95	89.50	23.95
500	100.00	100.00	99.80	100.00	100.00	92.50	100.00	100.00	69.70
1000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	97.05

TABLE 6. Size adjusted power of the test. Model 2 test, $\pi = 0.1$, $b = -0.2$.

T	$a = 0.0$			$a = 0.5$			$a = 1.0$		
	s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
Simultaneous cobreaks									
100	-3.264	-3.184	-3.086	-3.725	-3.671	-3.669	-4.114	-4.048	-3.894
200	-2.725	-2.763	-2.701	-3.058	-2.958	-2.875	-3.073	-3.021	-2.959
500	-2.657	-2.634	-2.652	-2.819	-2.777	-2.788	-2.893	-2.866	-2.865
1000	-2.611	-2.589	-2.587	-2.801	-2.795	-2.791	-2.898	-2.910	-2.881
Cobreaks in differences, not in levels									
100	-2.756	-2.732	-2.633	-2.915	-2.721	-2.625	-2.878	-2.690	-2.627
200	-2.187	-2.158	-2.143	-2.234	-2.169	-2.160	-2.203	-2.185	-2.132
500	-1.920	-1.920	-1.933	-1.923	-1.909	-1.902	-1.925	-1.931	-1.911
1000	-1.830	-1.824	-1.821	-1.832	-1.830	-1.818	-1.831	-1.832	-1.815
Cobreaks in levels, not in differences									
100	-3.477	-3.783	-3.677	-3.740	-3.834	-3.688	-4.114	-4.048	-3.894
200	-2.836	-2.952	-2.931	-3.035	-2.986	-2.924	-3.073	-3.021	-2.959
500	-2.733	-2.817	-2.820	-2.790	-2.828	-2.853	-2.893	-2.866	-2.865
1000	-2.701	-2.864	-2.873	-2.806	-2.888	-2.886	-2.898	-2.910	-2.881
Independent shocks									
100	-3.393	-3.119	-2.739	-3.725	-3.166	-2.886	-4.054	-3.526	-3.085
200	-2.782	-2.740	-2.650	-3.074	-2.981	-2.679	-3.318	-3.298	-2.952
500	-2.682	-2.707	-2.673	-2.945	-2.653	-2.675	-2.987	-2.907	-2.767
1000	-2.623	-2.607	-2.639	-2.873	-2.714	-2.740	-2.972	-2.891	-2.898
Shocks in y_t									
100	-3.403	-3.107	-3.142	-3.826	-3.248	-3.170	-4.069	-3.598	-3.317
200	-2.785	-2.746	-2.666	-3.109	-2.950	-2.779	-3.268	-3.059	-2.864
500	-2.668	-2.697	-2.673	-2.868	-2.792	-2.760	-2.951	-2.938	-2.813
1000	-2.629	-2.621	-2.624	-2.850	-2.867	-2.831	-2.971	-2.999	-2.956
Shocks in z_t									
100	-3.264	-3.184	-3.086	-3.655	-3.259	-3.179	-3.991	-3.657	-3.288
200	-2.725	-2.763	-2.701	-3.070	-2.721	-2.618	-3.166	-2.864	-2.755
500	-2.657	-2.634	-2.652	-2.887	-2.620	-2.463	-2.885	-2.776	-2.626
1000	-2.611	-2.589	-2.587	-2.838	-2.567	-2.477	-2.923	-2.790	-2.641

TABLE 7. Critical values. Model 3, HP100 test, $\pi = 0.1$

T	$a = 0.0$ s=1	s=6	s=16	$a = 0.5$ s=1	s=6	s=16	$a = 1.0$ s=1	s=6	s=16
Simultaneous cobreaks									
100	-2.978	-2.921	-2.785	-3.267	-3.204	-3.090	-3.442	-3.314	-3.248
200	-2.697	-2.661	-2.591	-3.058	-3.066	-3.028	-3.175	-3.148	-3.128
500	-2.662	-2.659	-2.576	-2.863	-2.818	-2.756	-2.948	-2.912	-2.901
1000	-2.608	-2.608	-2.612	-2.874	-2.805	-2.783	-2.906	-2.907	-2.900
Cobreaks in differences, not in levels									
100	-2.343	-2.362	-2.320	-2.402	-2.308	-2.277	-2.417	-2.297	-2.263
200	-2.037	-2.027	-2.007	-2.027	-1.999	-1.957	-2.006	-1.982	-1.933
500	-1.834	-1.839	-1.852	-1.820	-1.825	-1.840	-1.814	-1.819	-1.830
1000	-1.771	-1.776	-1.781	-1.773	-1.778	-1.771	-1.774	-1.772	-1.783
Cobreaks in levels, not in differences									
100	-3.054	-3.210	-3.160	-3.299	-3.264	-3.157	-3.442	-3.314	-3.248
200	-2.737	-3.113	-3.137	-3.085	-3.142	-3.133	-3.175	-3.148	-3.128
500	-2.711	-2.900	-2.915	-2.863	-2.888	-2.882	-2.948	-2.912	-2.901
1000	-2.770	-2.852	-2.873	-2.850	-2.911	-2.913	-2.906	-2.907	-2.900
Independent shocks									
100	-2.984	-2.836	-2.732	-3.266	-2.809	-2.946	-3.400	-2.953	-3.094
200	-2.725	-2.594	-2.661	-2.949	-2.616	-2.989	-3.123	-2.815	-3.140
500	-2.673	-2.656	-2.835	-2.807	-2.656	-3.180	-3.003	-2.941	-3.494
1000	-2.620	-2.668	-2.997	-2.859	-2.705	-3.486	-2.949	-2.970	-3.794
Shocks in y_t									
100	-2.979	-2.926	-2.851	-3.247	-3.130	-3.057	-3.412	-3.236	-3.292
200	-2.713	-2.596	-2.628	-3.037	-2.818	-2.974	-3.128	-2.879	-3.252
500	-2.674	-2.645	-2.792	-2.846	-2.692	-3.089	-3.020	-2.843	-3.420
1000	-2.621	-2.637	-2.775	-2.910	-2.782	-3.115	-2.985	-2.907	-3.324
Shocks in z_t									
100	-2.978	-2.921	-2.785	-3.294	-2.908	-2.666	-3.357	-3.251	-2.765
200	-2.697	-2.661	-2.591	-2.996	-2.675	-2.562	-3.155	-2.925	-2.673
500	-2.662	-2.659	-2.576	-2.847	-2.577	-2.474	-2.984	-2.667	-2.662
1000	-2.608	-2.608	-2.612	-2.882	-2.583	-2.541	-2.963	-2.752	-2.688

TABLE 8. Critical values. Model 3, HP10 test, $\pi = 0.1$

T	$a = 0.0$			$a = 0.5$			$a = 1.0$		
	s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
Simultaneous cobreaks									
100	-2.843	-2.712	-2.680	-3.073	-2.766	-2.682	-2.889	-2.812	-2.752
200	-2.654	-2.605	-2.563	-2.665	-2.717	-2.721	-2.773	-2.793	-2.722
500	-2.652	-2.682	-2.588	-2.670	-2.792	-2.832	-2.883	-2.927	-2.928
1000	-2.589	-2.566	-2.556	-2.752	-2.882	-2.887	-2.961	-3.044	-3.050
Cobreaks in differences, not in levels									
100	-2.471	-2.450	-2.416	-2.504	-2.474	-2.465	-2.487	-2.496	-2.480
200	-2.114	-2.124	-2.114	-2.212	-2.221	-2.221	-2.246	-2.234	-2.204
500	-1.962	-1.937	-1.929	-2.015	-1.991	-1.905	-1.985	-1.949	-1.896
1000	-1.854	-1.854	-1.853	-1.880	-1.764	-1.626	-1.755	-1.654	-1.624
Cobreaks in levels, not in differences									
100	-2.890	-2.797	-2.784	-2.990	-2.796	-2.728	-2.889	-2.812	-2.752
200	-2.712	-2.737	-2.771	-2.722	-2.778	-2.799	-2.773	-2.793	-2.722
500	-2.686	-2.944	-2.932	-2.740	-2.894	-2.909	-2.883	-2.927	-2.928
1000	-2.603	-2.999	-3.028	-2.837	-3.044	-3.111	-2.961	-3.044	-3.050
Independent shocks									
100	-2.815	-3.079	-4.165	-3.144	-3.201	-4.391	-3.242	-3.471	-4.578
200	-2.637	-2.720	-4.015	-2.868	-2.934	-4.553	-2.749	-3.033	-4.901
500	-2.689	-2.706	-2.500	-2.652	-2.740	-3.055	-2.813	-2.941	-3.578
1000	-2.587	-2.641	-2.720	-2.720	-2.863	-3.496	-2.882	-2.995	-4.089
Shocks in y_t									
100	-2.808	-2.990	-3.187	-3.175	-3.201	-3.410	-3.007	-3.362	-3.580
200	-2.629	-2.536	-2.938	-2.783	-2.770	-3.329	-2.770	-2.959	-3.539
500	-2.670	-2.615	-2.746	-2.700	-2.739	-3.105	-2.855	-2.867	-3.358
1000	-2.579	-2.598	-2.755	-2.728	-2.862	-3.286	-2.940	-2.879	-3.508
Shocks in z_t									
100	-2.843	-2.712	-2.680	-3.100	-2.762	-2.661	-3.148	-2.971	-2.757
200	-2.654	-2.605	-2.563	-2.783	-2.608	-2.532	-2.767	-2.877	-2.756
500	-2.652	-2.682	-2.588	-2.663	-2.646	-2.521	-2.867	-2.824	-2.713
1000	-2.589	-2.566	-2.556	-2.718	-2.687	-2.546	-2.894	-2.845	-2.802

TABLE 9. Critical values. Model 3, BK Filter, $\pi = 0.1$

T	$a = 0.0$			$a = 0.5$			$a = 1.0$		
	s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
	Simultaneous cobreaks								
100	-2.775	-2.813	-2.813	-2.810	-2.873	-2.850	-2.816	-2.729	-2.733
200	-2.609	-2.622	-2.626	-2.589	-2.622	-2.615	-2.718	-2.656	-2.638
500	-2.701	-2.658	-2.667	-2.580	-2.591	-2.594	-2.710	-2.666	-2.671
1000	-2.664	-2.663	-2.663	-2.655	-2.590	-2.598	-2.770	-2.767	-2.763
	Cobreaks in differences, not in levels								
100	-2.503	-2.520	-2.527	-2.543	-2.502	-2.353	-2.481	-2.284	-2.249
200	-2.171	-2.174	-2.174	-2.187	-2.231	-2.159	-2.196	-2.111	-2.111
500	-1.987	-1.980	-1.975	-1.973	-2.043	-2.183	-2.018	-2.136	-2.198
1000	-1.848	-1.858	-1.858	-1.865	-1.900	-1.887	-1.929	-1.934	-1.902
	Cobreaks in levels, not in differences								
100	-2.831	-2.727	-2.727	-2.835	-2.853	-2.850	-2.816	-2.729	-2.733
200	-2.611	-2.576	-2.576	-2.627	-2.615	-2.615	-2.718	-2.656	-2.638
500	-2.655	-2.599	-2.623	-2.574	-2.617	-2.560	-2.710	-2.666	-2.671
1000	-2.621	-2.538	-2.538	-2.651	-2.600	-2.598	-2.770	-2.767	-2.763
	Independent shocks								
100	-2.789	-2.769	-2.769	-2.845	-2.942	-2.942	-2.876	-3.078	-3.070
200	-2.611	-2.648	-2.648	-2.632	-2.737	-2.737	-2.739	-2.856	-2.831
500	-2.683	-2.641	-2.641	-2.584	-2.645	-2.666	-2.729	-2.802	-2.770
1000	-2.657	-2.634	-2.634	-2.668	-2.713	-2.713	-2.829	-2.855	-2.854
	Shocks in y_t								
100	-2.801	-2.760	-2.760	-2.818	-2.875	-2.873	-2.866	-2.958	-2.971
200	-2.637	-2.652	-2.652	-2.633	-2.672	-2.678	-2.705	-2.767	-2.785
500	-2.679	-2.661	-2.661	-2.600	-2.628	-2.631	-2.728	-2.744	-2.752
1000	-2.644	-2.632	-2.632	-2.664	-2.696	-2.696	-2.808	-2.807	-2.793
	Shocks in z_t								
100	-2.775	-2.813	-2.813	-2.828	-2.849	-2.854	-2.890	-2.956	-2.956
200	-2.609	-2.622	-2.626	-2.581	-2.684	-2.684	-2.741	-2.831	-2.831
500	-2.701	-2.658	-2.667	-2.558	-2.593	-2.595	-2.712	-2.762	-2.742
1000	-2.664	-2.663	-2.663	-2.668	-2.711	-2.710	-2.815	-2.830	-2.830

TABLE 10. Critical values. Model 3, MD test, $\pi = 0.1$

	$a = 0.0$			$a = 0.5$			$a = 1.0$		
T	s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
				Simultaneous cobreaks					
100	25.15	21.30	13.05	12.00	12.80	8.15	9.20	8.95	7.65
200	73.40	58.10	61.55	50.90	51.50	51.50	39.30	38.65	39.30
500	99.60	99.65	99.90	99.05	99.10	99.40	98.25	98.45	98.70
1000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
				Cobreaks in differences, not in levels					
100	2.20	3.90	6.00	1.40	2.15	3.25	1.70	2.10	1.55
200	1.25	2.35	4.50	0.70	1.15	1.95	0.55	0.70	0.50
500	0.05	0.25	1.55	0.05	0.00	0.25	0.05	0.05	0.05
1000	0.00	0.25	2.85	0.00	0.00	0.90	0.00	0.00	0.00
				Cobreaks in levels, not in differences					
100	21.05	11.90	11.55	13.20	12.25	11.05	9.20	8.95	7.65
200	68.90	46.45	40.45	51.70	43.80	41.50	39.30	38.65	39.30
500	99.50	97.00	96.60	98.80	98.05	97.95	98.25	98.45	98.70
1000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
				Independent shocks					
100	23.10	16.70	9.65	12.05	14.80	5.35	8.90	9.05	2.60
200	73.05	64.45	29.40	45.10	56.10	19.75	40.45	38.15	10.15
500	99.65	98.75	94.45	99.20	98.30	71.40	98.65	86.60	39.95
1000	100.00	100.00	100.00	100.00	100.00	99.95	100.00	100.00	98.85
				Shocks in y_t					
100	22.70	17.05	3.20	10.75	12.75	1.95	8.65	8.40	1.15
200	73.05	64.30	28.65	43.50	57.30	16.45	42.95	47.50	11.50
500	99.65	98.80	94.45	99.40	97.50	67.20	98.85	85.65	37.85
1000	100.00	100.00	100.00	100.00	100.00	99.90	100.00	100.00	98.75
				Shocks in z_t					
100	25.50	21.80	9.30	12.80	15.55	6.25	9.75	7.10	3.65
200	73.90	70.75	40.15	48.85	50.35	20.60	41.90	28.65	9.65
500	99.75	99.25	80.55	99.25	91.90	50.05	98.50	71.40	21.45
1000	100.00	100.00	99.95	100.00	100.00	94.70	100.00	99.85	63.60

TABLE 11. Size adjusted power of the test. Model 3 HP100 test, $\pi = 0.1$, $b = -0.2$.

T	$a = 0.0$ s=1	$a = 0.0$ s=6	$a = 0.0$ s=16	$a = 0.5$ s=1	$a = 0.5$ s=6	$a = 0.5$ s=16	$a = 1.0$ s=1	$a = 1.0$ s=6	$a = 1.0$ s=16
Simultaneous cobreaks									
100	52.25	48.15	47.45	28.10	29.60	34.60	22.20	26.25	27.80
200	97.05	94.90	96.15	78.80	84.30	86.35	76.65	78.95	78.95
500	100.00	100.00	100.00	100.00	100.00	100.00	99.95	99.95	99.95
1000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Cobreaks in differences, not in levels									
100	0.95	1.70	3.75	1.05	0.85	1.25	0.85	0.50	0.30
200	0.80	1.70	4.10	0.40	0.50	1.35	0.15	0.05	0.05
500	0.15	0.80	2.90	0.00	0.00	0.55	0.00	0.00	0.00
1000	0.05	0.55	3.50	0.00	0.00	0.60	0.00	0.00	0.00
Cobreaks in levels, not in differences									
100	50.70	32.65	32.35	27.55	29.50	32.40	22.20	26.25	27.80
200	97.15	82.15	78.75	82.30	80.10	78.90	76.65	78.95	78.95
500	100.00	100.00	99.95	100.00	99.95	99.95	99.95	99.95	99.95
1000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Independent shocks									
100	51.50	51.80	29.00	30.80	26.45	9.70	20.00	12.10	4.50
200	97.20	97.40	87.25	78.75	82.60	43.90	75.30	58.45	20.25
500	100.00	100.00	100.00	100.00	100.00	99.95	99.95	99.85	93.00
1000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Shocks in y_t									
100	52.30	63.95	60.80	30.95	42.45	41.25	22.05	29.05	26.85
200	97.20	99.25	99.70	76.80	93.25	96.90	78.55	81.85	90.75
500	100.00	100.00	100.00	100.00	100.00	100.00	99.95	100.00	100.00
1000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Shocks in z_t									
100	51.90	40.75	15.60	29.55	24.60	8.70	23.65	10.80	5.05
200	97.10	95.10	62.60	77.25	73.65	28.35	75.95	45.30	13.15
500	100.00	100.00	98.60	100.00	99.90	80.20	100.00	95.10	44.65
1000	100.00	100.00	100.00	100.00	100.00	99.50	100.00	100.00	85.45

TABLE 12. Size adjusted power of the test. Model 3 HP10 test, $\pi = 0.1$, $b = -0.2$.

T	a = 0.0			a = 0.5			a = 1.0		
	s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
	Simultaneous cobreaks								
100	76.65	83.15	94.90	44.55	51.55	73.70	24.30	24.15	26.40
200	99.80	99.50	99.70	94.60	96.00	98.20	76.15	74.70	78.85
500	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	Cobreaks in differences, not in levels								
100	1.80	3.00	5.40	0.80	0.70	1.05	0.15	0.10	0.05
200	1.35	3.50	6.15	0.20	0.15	0.80	0.00	0.00	0.00
500	0.40	1.75	5.20	0.05	0.05	1.10	0.00	0.00	0.00
1000	0.05	0.90	4.20	0.00	0.05	0.65	0.00	0.00	0.00
	Cobreaks in levels, not in differences								
100	73.35	41.15	27.60	39.00	30.25	28.55	24.30	24.15	26.40
200	99.60	92.70	82.70	92.25	84.35	77.00	76.15	74.70	78.85
500	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	Independent shocks								
100	77.70	77.75	27.30	43.50	47.90	9.65	28.25	19.00	3.65
200	99.80	99.45	91.85	93.70	93.75	75.85	84.65	71.65	47.75
500	100.00	100.00	100.00	100.00	100.00	99.00	100.00	99.45	80.40
1000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	Shocks in y_t								
100	78.35	83.40	98.15	45.45	62.95	97.10	26.45	66.00	96.25
200	99.70	99.85	99.95	93.95	98.85	99.85	82.75	97.95	99.95
500	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	Shocks in z_t								
100	76.90	70.05	27.15	45.95	35.70	12.65	25.95	14.40	5.75
200	99.85	99.25	79.45	93.20	85.00	40.50	80.60	50.95	13.90
500	100.00	100.00	99.95	100.00	100.00	89.05	100.00	96.90	52.45
1000	100.00	100.00	100.00	100.00	100.00	99.80	100.00	100.00	87.60

TABLE 13. Size adjusted power of the test. Model 3 BK test, $\pi = 0.1$, $b = -0.2$.

T	$a = 0.0$ s=1	s=6	s=16	$a = 0.5$ s=1	s=6	s=16	$a = 1.0$ s=1	s=6	s=16
Simultaneous cobreaks									
100	88.25	85.35	57.10	58.75	53.60	33.70	33.95	35.40	35.30
200	99.70	99.75	98.10	97.80	97.65	94.65	85.80	89.10	89.75
500	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Cobreaks in differences, not in levels									
100	3.20	3.35	3.25	1.25	1.55	1.10	0.50	0.35	0.25
200	2.05	2.20	2.15	0.80	0.75	0.55	0.05	0.05	0.05
500	0.30	0.35	0.35	0.05	0.20	0.15	0.00	0.00	0.00
1000	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Cobreaks in levels, not in differences									
100	86.90	86.60	86.50	56.45	54.25	54.25	33.95	35.40	35.30
200	100.00	99.95	99.95	97.10	97.45	97.45	85.80	89.10	89.75
500	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Independent shocks									
100	86.70	84.90	84.75	58.85	57.25	57.35	31.90	29.75	30.25
200	99.70	99.60	99.65	97.05	94.90	94.95	85.05	80.20	81.60
500	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
1000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Shocks in y_t									
100	87.05	87.25	87.25	59.60	57.60	57.80	30.70	32.00	32.15
200	99.65	99.60	99.60	96.95	96.45	96.35	85.90	86.20	86.10
500	100.00	100.00	100.00	100.00	100.00	100.00	99.95	99.95	99.95
1000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
Shocks in z_t									
100	88.30	86.30	86.25	57.10	59.05	58.65	31.15	30.75	30.90
200	99.65	99.75	99.75	97.75	96.50	96.50	85.25	80.55	80.50
500	100.00	100.00	100.00	100.00	100.00	100.00	100.00	99.95	99.95
1000	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

TABLE 14. Size adjusted power of the test. Model 3 MD test, $\pi = 0.1$, $b = -0.2$.

T	$a = 0.0$			$a = 0.5$			$a = 1.0$		
	s=1	s=6	s=16	s=1	s=6	s=16	s=1	s=6	s=16
				Model 2					
100	5.35	6.30	13.95	7.55	17.55	16.45	10.05	38.90	19.25
200	6.05	7.75	30.85	7.00	19.40	37.45	10.95	43.10	50.15
500	5.05	3.25	1.25	6.65	16.35	10.75	7.35	34.80	33.30
1000	5.35	3.40	3.60	6.45	16.80	20.45	5.50	21.75	43.55
				Model 3, HP10					
100	4.75	3.45	2.50	4.95	2.00	2.00	4.15	1.65	2.05
200	5.60	4.35	4.90	3.75	1.95	4.05	4.75	2.15	4.55
500	5.00	4.90	7.20	4.70	2.95	9.35	6.10	5.00	15.40
1000	5.10	6.00	11.00	4.75	3.35	13.80	5.50	5.40	21.85
				Model 3, HP100					
100	5.85	3.70	1.95	5.05	1.70	0.65	4.50	1.55	0.40
200	5.60	5.15	4.35	5.00	3.90	2.20	7.10	6.75	3.70
500	5.30	5.35	5.20	6.40	3.50	3.40	6.65	5.10	3.85
1000	5.35	4.95	5.35	6.05	4.15	4.45	6.15	5.00	5.15
				Model 3, BK filter					
100	3.95	7.20	34.75	5.60	7.00	41.10	6.65	10.00	48.55
200	4.65	5.80	22.15	5.90	6.85	34.15	4.65	9.35	45.70
500	5.05	5.25	3.75	4.80	5.80	11.35	3.85	5.55	18.65
1000	4.85	5.85	7.50	4.65	5.85	16.70	4.00	5.50	26.10
				Model 3, MD filter					
100	5.55	5.25	5.25	5.35	6.50	6.50	5.10	7.60	7.60
200	4.90	5.20	5.20	5.20	6.30	6.25	5.25	6.40	6.40
500	5.15	4.90	4.90	5.15	6.25	6.30	4.90	5.80	5.50
1000	4.85	4.35	4.35	5.25	5.90	5.90	5.40	6.15	6.25

TABLE 15. Robustness against outliers. Rejection frequencies when the model is simulated with DGP (5.1a)–(5.1f) with independent shocks in variables y_t and z_t with $\pi = 0.1$ under $b = 0$, using critical values obtained in Table 1.