Risk premium: insights over the threshold

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The aim of this article is 2-fold: first to test the adequacy of Pareto distributions to describe the tail of financial returns in emerging and developed markets, and second to study the possible correlation between stock market indices observed returns and return's extreme distributional characteristics measured by Value at Risk and Expected Shortfall. We test the empirical model using daily data from 41 countries, in the period from 1995 to 2005. The findings support the adequacy of Pareto distributions and the use of a log linear regression estimation of their parameters, as an alternative for the usually employed Hill's estimator. We also report a significant relationship between extreme distributional characteristics and observed returns, especially for developed countries.

1. Introduction

Financial risk management is closely related with extreme events and so is highly concerned with profit and loss distribution's tail quantiles (e.g. the value of x such that $P(X > x) = 0.05$) and tail probabilities (e.g. $P(X > x)$, for a large value of x). No matter whether we are concerned with market, credit or other financial risks, one of the main challenges is to model the unusual but really damaging events to permit measurement of their consequences. Specifically in this article we deal with market risk, say the day-to-day determination of the losses we may incur on a trading book due to adverse market movements.

Recent literature (see for instance Embrechts *et al.*, 1997) claims that extreme value theory (EVT) holds promise for accurate estimation of extreme quantiles and tail probabilities of financial

asset returns, and hence could be a relevant tool for the management of extreme financial risks. This theory posits that methods based around assumptions of normal distributions are likely to underestimate tail risk. The key idea of EVT is that one can estimate extreme quantiles and probabilities by fitting a model to the empirical survival function (defined as $1 - F^*(x)$ where $F^*(x)$ is the empirical cumulative probability function) of a set of data using only the extreme event data rather than all the observed data available, thereby describing the tail and just the tail, which is the main concern in the extreme events risk management. However, Diebold *et al.* (1998) describe several pitfalls in the use of EVT in risk management. Also, up to now the question whether the market is compensating for extreme risk factors is still not answered.

In this article, we address one of those pitfalls related to the estimation by log linear regression of

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the parameters of the Pareto distribution and study if there is a risk-premium associated to extreme risk measures [Value-at-Risk (VAR) and Expected Shortfall (ES)], considering developed and emerging markets. We test the empirical model using daily data from 41 countries obtained from Morgan Stanley Capital International (MSCI). The findings support the use of a log linear regression estimation of the parameters, as an alternative for the Hill's estimator (Maximum Likelihood Estimator). Related to risk premium determinants, we present empirical evidence suggesting significant relationship between extreme distributional characteristics and observed returns, especially for developed countries.

The remaining part of this article is as follows. In Section II we provide a literature review of topics concerning VAR, ES and cross section of average returns' explanations. In Section III the theoretical framework is shown and in Section IV, we describe methodology analysing the main results. Section V concludes.

II. Literature Review

Already considered a stylised fact, asset prices in financial markets are characterized by presenting a higher probabilistic density on the tails of its returns' distribution than the Normal distribution and this effect is more salient in emerging markets, in comparison with developed markets; a phenomenon known as 'fat tails'. Extreme value theory has extensively been applied in such context in order to adjust the measure of those assets' VAR.

McNeil (1999) presents an overview of EVT as a method for modelling and measuring extreme risk, in particular showing how the Peaks-Over-Threshold (POT) method may be embedded in a stochastic volatility framework to deliver useful estimates of VAR and ES, a coherent alternative for measuring market risk.¹ Embrechts (2004) discusses the economic implication of studying EVT in its relation with risk management highlighting the need for further research on theoretical aspects of the empirical findings.

Among the empirical articles we find McNeil (1997,1998) that describe parametric curve-fitting methods for modelling extreme historical

losses; Longin and Solnik (2001) focus on extreme correlation and find that it increases in bear markets but not in bull markets; Delfiner and Girault (2002) analyse the 'fat tails' phenomena for emerging markets and posit that measures based on EVT explains returns better than the ones based on the normal hypothesis. Malevergne *et al.* (2006) evaluates the use of both Pareto and Stretched-Exponential distributions to describe the 'fat tails' and found mixed results related to the generalized extreme value (GEV) estimator and the generalized Pareto distribution (GPD) estimator, with the sample size influencing the convergence toward the asymptotic theoretical distribution. LeBaron and Ritirupa (2005) analyse the significance of booms and crashes in developed and emerging Markets. Their findings are consistent with prevalent notions of crashes being more salient in emerging markets than among developed markets. Additionally, Salomon and Groetveld (2003) study risk premium in a number of international markets and suggest that investors should focus more on downside risk instead of SDs.

Harvey (1995) reports that international CAPM fails when applied to emerging markets. During the 1970s some researchers developed the 'below-target semivariance capital asset pricing model' or better known as ES-CAPM (Hogan and Warren, 1972; Hogan and Warren, 1974; Nantell and Price, 1979) which is suggested to be useful when the distribution of returns is nonnormal and nonsymmetric as is usually the case in emerging markets. Bawa and Lindenberg (1977) generalize those results and show that their model explains the data at least as well as the CAPM. Finally, Harlow and Rao (1989) derive a mean-lower partial moment (MLPM) model for any arbitrary benchmark return, which incorporates the previous models as particular cases.² Faff (2004) tests the Fama and French three-factor model using Australian data and when estimated risk premia are taken into account, the support for the model is weak.

In a article close to ours, Harvey (2000) considered the VAR as one of his 18 risk factors for developed and emerging markets. He found evidence that an asset pricing framework that incorporate skewness has success in explaining average returns. In this line of arguments, Estrada (2000) estimated returns in emerging markets using a measure of downside risk finding significant results. More recently,

¹ Extreme value theory provides a natural approach to VAR and ES estimation and there is already a considerable literature on the subject. See Danielsson and de Vries (1997), Embrechts *et al.* (1997) and Diebold *et al.* (1998).

² Nawrocki (1999) presents a review of such literature.

Stevenson (2001) used downside risk measures to construct an optimal international portfolio while Estrada (2002) proposed the downside CAPM (D-CAPM) which considers the systematic downside risk as a way to estimate the required returns in emerging markets.

Our article contains four new contributions. First we discuss efficient procedures for estimating parameters of Pareto extreme value distribution and use these results to estimate ES. Second, from a range of 21 risk measures we study their statistical relationship with observed returns. Third, we implement a factor analysis to find different components, reaching a simplified but efficient empirical model; and fourth more recent price information is used with data until the end of 2005.

III. Theoretical Framework

Extreme value theory and expected shortfall

Measuring risk, in some sense, is to try to summarize its distribution with a number known as risk measure. For instance, VAR is a high quantile of the distribution of losses, typically the 95th or 99th percentile. VAR provides a sort of upper bound of losses, which is expected to be exceeded in very few occasions. In EVT, we are effectively concerned about those few moments in time where this limit is exceeded, which can be very harmful for the company or portfolio. The idea is to calculate ES given by the average expected loss exceeding VAR. It was first proposed by Artzner *et al.* (1997) who posit that it is a coherent risk measure, while VAR is not. Then ES is the conditional expectation of loss given that loss is beyond the VAR level; i.e. ES is defined as follows:

$$ES_\alpha(X) = E[X|X \geq \text{VAR}_\alpha(X)] \quad (1)$$

Observe that if the profit-loss distribution is Normal, the VAR and the ES essentially offers the same information, as assets with a higher VAR would also have a higher ES. VAR and ES would be just scalar multiples of the SD (Yamai and Yoshida, 2005). The problem arises when the distribution is not Normal, as tail risk may be present in the case of fat-tails and so high potential for large losses. In this case, it's crucial to identify the distribution function that best describes those extreme bad returns and one approach is to use EVT.

The modern approach to EVT is through POT models; these are models for all large observations,

which exceed a high threshold. McNeil (1999) posits that POT models are considered to be the most useful for practical applications, due to their more efficient use of the limited data on extreme values. Within POT class of models we will concentrate on those related to a Frechet distribution. Much of our discussion is related to the idea of tail estimation under a power law assumption, say, we assume that returns are in the maximum domain of attraction of a Frechet distribution, so that the tail of the survival function is a power law times a slowly-varying function:

$$P(X > x) = k(x)x^{-\alpha} \quad (2)$$

As pointed out by Diebold *et al.* (1998), often it is assumed that $k(x)$ is constant; in which case, attention is restricted to densities with tails of the form:

$$P(X > x) = kx^{-\alpha} \quad (3)$$

defined in the domain $x \geq k^{1/\alpha}$, with the parameters k and α to be estimated, respectively representing the scale and shape parameters of a Pareto distribution. In this case, the first moment is given by:

$$\mu = \left[\frac{\alpha}{\alpha - 1} \right] k^{1/\alpha} \quad (4)$$

There are several ways of estimating these parameters. One of the most popular is the Hill's estimator, which is based directly on the extreme values and proceeds as follows. Order the observations $x_{(1)} > x_{(2)} > \dots > x_{(m)}$ and form an estimator based on the difference between the m -th largest observation and the average of the m largest observations.

$$\hat{\alpha} = \left[\left(\frac{1}{m} \sum_{i=1}^m \ln(x_i) \right) - \ln(x_m) \right]^{-1} \quad (5)$$

The Hill's estimator is the maximum likelihood estimator and so has good theoretical properties: it can be shown that it is consistent and asymptotically normal, assuming iid data and that m grows at a suitable rate with sample size.³ A crucial problem in using the previous estimator, highlighted in Lux (2001), is to define the size of m , as there is an important bias-variance trade-off when varying m for fixed sample size: increasing m , we are using more data (moving toward the centre of the distribution), which reduces the variance but increases the bias, as the power law is assumed to hold just in the extreme tail. In this article, as our main purpose is to investigate the existence of a relative risk premium among country index returns, we adopt a procedure suggested by Yamai and Yoshida (2002) and select m

³ See, e.g. Embrechts *et al.* (1997, p. 348).

as the one day $\text{VAR}_{95\%}$ and $\text{VAR}_{99\%}$ for each country index.

In fact we use Hill's estimator as a reference point, but will privilege the estimation by linear regression. Simply note that

$$P(X > x) = kx^{-\alpha} \quad (6)$$

Implies,

$$\ln P(X > x) = \ln(k) - \alpha \ln(x) \quad (7)$$

so that α is the slope coefficient in a simple linear relationship between the log tail of the empirical survival function and the log extreme values and then we are able to estimate both k and α by a linear regression over the survival m observations.

The basic insight in this approach is that the essence of the tail estimation problem is fitting a log-linear function to a set of data, and so easy to handle and good for practical purposes. In order to verify the goodness of fit, we use mean square error (MSE), R^2 , comparing the results with the ones obtained using the Hill's estimator.

Moreover, in order to calculate the ES, we know that it is related to VAR by:

$$\text{ES}_\alpha = \text{VAR}_\alpha + E[X - \text{VAR}_\alpha | X > \text{VAR}_\alpha] \quad (8)$$

where the second term is the mean of the excess distribution over the threshold VAR_α . This has the probability distribution given by the power law assumption (Pareto distribution).

Risk premium

The framework proposed in this article to infer whether there is a risk premium associated to the VAR and to ES, follows APT's basic insight. Any required return can be thought of as having two components: a risk-free rate, and a set of risk premium. The first component is compensation required for the expected loss of purchasing power, which is demanded even for a riskless asset. The second component is extra compensation for bearing risk, which depends on the asset considered, say, expected returns should include rewards for accepting risk. APT provides an approximate relation for asset returns with an unknown number of unidentified factors.

We take the perspective of a US-based, internationally diversified investor. Thus the risk-free rate should compensate this investor for the dollar's expected loss of purchasing power, and the risk

premium should compensate the investor for bearing different risks (world market portfolio, extreme events, etc.). Our empirical model may be seen as an extension of Estrada (2000). We define observed return for country index i as:

$$\text{RR}_i - R_f = a_0 + a_{1i}(\text{RM}_{1i}) + \dots + a_{Ki}(\text{RM}_{Ki}) + \text{error} \quad (9)$$

where RR_i is the observed dollar return, R_f is the US risk free rate and RM_{ji} are risk factors $j=1, \dots, K$, and i indexes the markets. In the empirical application we use average daily observed returns in US dollars for country index i , which we express as RR_i .

From the several risk measures available in the literature (see for instance Harvey, 2000) we choose 21 different factors, focusing on extreme risk measures like VAR and ES. The measures are:

- (1) **SR** (systematic risk) measured by standard market's model beta and is estimated through the following 'world version' of a single factor model with R_{mt} denoting the return on the MSCI world index. We estimate the following regression⁴:

$$R_{it} - r_{ft} = \alpha_i + \beta_i[R_{mt} - r_{ft}] + e_{it} \quad (10)$$

where r_{ft} is the US 3-month Treasury bill rate and e_{it} is the residual.

- (2) **Down- β iw** is the β coefficient from market model (10) using observations when country returns and world returns are simultaneously negative.
- (3) **Down- β w** is the β coefficient from market model (10) using observations when world returns are negative.
- (4) **IR** (idiosyncratic risk) is the SD of residuals e_{it} , in model (10)
- (5) **TR** (total risk) is the SD of the country's index return.
- (6) **σ -garch(1, 1)** is the volatility forecast considering a GARCH (1,1) model to describe country index return's volatility.
- (7) **VAR95t** is the parametric VAR at 95% confidence level. The volatility considered is the one-day-ahead GARCH(1,1) forecast. Expected daily return is set equal to zero. One day $\text{VAR}_{95\%}$ is calculated for each index, using the expression (Jorion, 1995) given by:

$$\text{VAR}_{95\%} = z * PV * \sigma * \sqrt{\Delta t} \quad (11)$$

⁴We used Newey–West procedure in order to generate a covariance matrix that is consistent in the presence of both heteroskedasticity and autocorrelation of unknown form. Some recent articles like Cho and Engle (1999) Andersen *et al.* (2003) and Hwang and Salmon (2006) use this procedure.

which in our case, considering a unit monetary investment (present value (PV)=1), one day investment horizon⁵ and a confidence level of 95% ($z=1.6448$, assuming a Normal Distribution), is equivalent to:

$$\text{VAR}_{95\%} = 1.6448 * \sigma$$

$$\left\{ \frac{(\text{return at the 95th percentile level} - \text{mean return}) - (\text{mean return} - \text{return at the 5th percentile level})}{(\text{return at the 95th percentile level} - \text{return at the 5th percentile level})} \right\} \quad (15)$$

- (8) **VAR95d** is the empirical VAR, say the value of the observed return representing the 5th lowest percentile.
 (9) **VAR99t** is the parametric VAR similar to VAR95t but considering a confidence interval of 99%.

$$\text{VAR}_{99\%} = 2.3263 * \sigma$$

- (10) **VAR99d** is the empirical VAR, say the value of the observed return representing the 1st lowest percentile.
 (11) **ES95t** is the parametric ES, given by Equations 4 and 8, using as the threshold VAR95d.

$$\text{ES } 95t = \text{VAR } 95d + \left[\frac{\alpha}{\alpha - 1} \right] k^{1/\alpha} \quad (12)$$

- (12) **ES95d** is the sample average of returns below the 5th percentile level (VAR95d).

It's important to point out that we are assuming ES and VAR as measures of losses and so are represented by positive numbers. In order to consider risk factors related to semi-standard deviation, we used the following expression:

$$\text{Semi-}B = \sqrt{\frac{1}{T} \sum_{t=1}^T (R_t - B)^2}, \text{ for all } R_t < B \quad (13)$$

- (13) **Semi-Mean** is the semi SD with respect to the average return of the market. (B =average return of the market)
 (14) **Semi-0** is the semi SD with respect to 0. ($B=0$)
 (15) **Semi-rf** is the semi standard with respect to the risk free rate. (B =US risk free rate)
 (16) **Kurt** is the kurtosis of the return distribution.

- (17) **skew** is the unconditional skewness of returns. It's calculated by:

$$\text{skew} = \frac{\text{Mean}(e_i^3)}{\text{SD}(e_i)^3} \quad (14)$$

- (18) **skew5%** is given by:

- (19) **coskew1** represents coskewness definition 1 (Harvey and Siddique, 2000).

$$\text{coskew1} = \frac{\sum e_i e_m^2 / T}{\sqrt{((\sum e_i^2 / T) * (\sum e_m^2 / T))}} \quad (16)$$

where

$$e_{mt} = R_{mt} - \text{Avg}(R_{mt}) \quad (17)$$

- (20) **coskew2** represents coskewness definition 2 (Harvey and Siddique, 2000).

$$\text{coskew2} = \frac{\sum e_i e_m^2 / T}{(\sigma_{e_m})^3} \quad (18)$$

- (21) **Size** which is given by the natural logarithm of the market capitalization related to each country participation in the MSCI world index.⁶

For further reference, in this article, when we say beta related risk factors, we are considering the factors: SR, Down- β_{iw} and Down- β_w ; when we refer to distributional risk factors, we are considering: TR, σ -garch(1,1), VAR95t, VAR95d, VAR99t, VAR99d, ES95t, ES95d, ES99d, Semi-Mean and Semi-0; and when we refer to skewness factors we are considering skew, skew5%, coskew1 and coskew2.

So, first we use the time series of each country index to calculate their individual different risk factors (for instance, the beta is estimated using Equation 10) and then we implement a cross-sectional analysis (based on Equation 9) to identify the risk premium associated to each risk factor.

IV. Methodology and Results

In order to verify if there is a risk premium associated to the measures of risk previously discussed,

⁵ 'The Application of Basel II to Trading Activities and the Treatment of Double Default Effects' (BIS, 2005) suggests the use of a one-to-ten days horizon in the measurement of VAR.

⁶ Barry *et al.* (2002) found that mean returns for small firms exceed mean returns for large firms. Fama and French (1995) proposed size as a factor for their pricing model. This effect has been reversed in recent periods (Al-Rjoub *et al.* 2005).

we examined a sample of daily index returns (MSCI database) for the period from 4th April 1995 to 30th December 2005. We considered the following developed markets: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States; and the following emerging markets: Argentina, Brazil, Chile, Colombia, Hong Kong, India, Indonesia, Israel, Jordan, Malaysia, Mexico, New Zealand, Pakistan, Peru, Philippines, Poland, Singapore, South Africa, South Korea, Taiwan, Thailand, Turkey and Venezuela. The use of country indices is to avoid the thin trading effect – the returns data impacted by censoring, when using daily data (Brooks *et al.*, 2005).

So in the end we used 41 country indices daily dollar returns time series, the risk free rate time series and the time series of the world index also given by MSCI, representing 123 376 observations. We considered the 3-month Treasury Bill yield as the risk free rate. Once we had the time series of the observed daily returns it was possible to calculate the risk factors defined in the previous Section for each country. Tables 1 and 2 list observed returns and risk factors for each market.

Surprisingly, observed average daily dollar return for emerging markets (0.015%) is lower than for developed countries (0.034%). This is due to the negative result offered by most Asian emerging markets (Hong Kong, India, Indonesia, Israel, Jordan, Malaysia, Pakistan, Philippines, Singapore, Taiwan and Thailand). If we exclude those countries, average observed return rises to (0.038%). A similar result happens with beta related risk factors as they were in general higher for developed markets. It's important to notice that despite the fact that the average betas (SR) for both markets were lower than 1, if we weight each country's beta by its size, we would reach an overall beta close to 1. Observe that the US market represents almost 50% of the world market and has a beta of 1.141.

About the distributional risk factors, usual results are found as emerging markets present in general higher values for extreme risk measures. As usually found (Delfiner and Girault, 2002), kurtosis for both markets were higher than three pointing out to fat tails in return's distribution and this coefficient was higher for emerging markets. The skewness factors were negative for both groups, supporting previous results found in the literature (Harris *et al.*, 2004).

In order to highlight the fat tail phenomena, in Fig. 1 we may observe the QQ plot comparing observed daily returns for the world index to the normal distribution. It can be seen that for negative extreme tail, observed returns appear more often than what is predicted by the normal distribution, especially when returns are lower than -1.4% (aprox. equals to the VAR_{95d}). So the normal approximation seems to perform well for the central part of the distribution but offers bad results for the tails. Recall that the kurtosis is usually higher than 3 (for the world index, kurtosis is equals to 5.747). This fact raises the need for a better understanding of the tail distribution in order to predict extreme losses. Chung *et al.* (2004) using US equity data from 1930 to 1998 rejected normality of returns for daily, weekly, monthly, quarterly and semi-annual intervals. In the absence of normality, we expect investors to be concerned about the shape of the tails of the distribution of portfolio returns.

We then implemented the EVT analysis. We considered m as the number of failures in the VAR prediction, say when the observed return loss was greater than the empirical VAR measures (VAR_{95d} , VAR_{99d}). Then, we adjusted a power function to each of the instrument's excess loss time series. At this point we estimated the parameters by the two procedures described above: Hill (Maximum Likelihood Estimator) (Equation 5) and least square (LS) estimator (Equation 7). The average results for the parameters of the Pareto distribution considering all the individual stock indices are given in the Tables 3 and 4.

Figure 2 shows the graph of the Pareto Distribution considering the average LS parameters estimation for developed and emerging markets. It can be seen that the slope parameter (alpha) is higher for the developed countries' sample while the scale parameter (k) is lower. This supports the evidence that extreme losses should be higher for emerging markets. For instance, the probability that in the emerging market's sample we have a daily loss over 5% is around 40%, while this probability is just 14.5% for the developed countries' sample.

Table 7 provides the result of a t -test over the difference between the alpha parameters estimated by LS and Hill procedures. Observe that the difference is statistically different from zero,⁷ indicating that the estimation procedures are offering different results. In order to compare the estimation procedures we calculated the coefficient of determination (R^2)

⁷ Considering a significance level of 0.10.

ble 1. Summary statistics – developed markets (MSCI data)

ket	E[R]	SR	Down- β_{Dn}	Down- β_{Dn}	$\sigma\text{-garch}$ (1,1)	TR	IR	VAR95t	VAR95d	VAR99t	VAR95d	ES95t	ES95d	Semi- mean	Semi-0	Semi-rf	kurt	skew	skew5%	coskew1	coskew2	ξ
tralia	0.030	0.407	0.419	0.483	1.043	1.096	0.834	1.316	1.713	1.884	2.861	2.885	2.455	0.789	0.773	0.779	6.622	-0.188	-0.023	-0.153	-0.191	1
tria	0.035	0.371	0.425	0.377	1.011	1.057	0.807	1.268	1.634	1.818	3.068	2.897	2.489	0.764	0.747	0.752	5.212	-0.305	0.014	-0.130	-0.158	1
ium	0.030	0.766	0.675	0.729	0.999	1.185	0.751	1.158	1.897	1.670	3.280	3.157	2.784	0.846	0.832	0.836	7.551	0.079	-0.041	0.105	0.126	1
ada	0.046	0.953	0.883	1.105	0.861	1.171	0.687	1.041	1.866	1.509	3.280	3.587	2.803	0.868	0.846	0.852	8.805	-0.672	-0.046	-0.168	-0.173	1
mark	0.045	0.561	0.508	0.591	1.036	1.136	0.803	1.252	1.816	1.799	3.233	3.046	2.704	0.827	0.804	0.810	5.294	-0.315	-0.020	-0.069	-0.086	1
and	0.054	1.503	1.140	1.505	2.020	2.376	1.028	1.590	3.765	2.291	6.352	7.006	5.668	1.733	1.707	1.712	9.581	-0.475	-0.018	0.042	0.102	1
nce	0.031	1.057	0.919	1.082	0.955	1.299	0.767	1.205	2.114	1.728	3.759	3.377	3.013	0.938	0.922	0.927	5.198	-0.153	-0.034	0.012	0.014	1
many	0.026	1.227	0.990	1.202	1.038	1.457	0.752	1.175	2.438	1.687	4.173	3.847	3.489	1.054	1.042	1.046	5.521	-0.172	-0.036	0.085	0.106	1
nd	0.025	0.541	0.471	0.581	1.080	1.170	0.885	1.399	1.853	2.003	3.074	3.350	2.764	0.855	0.843	0.848	6.983	-0.440	-0.033	-0.038	-0.049	1
,	0.029	0.913	0.722	0.955	1.055	1.300	0.755	1.196	2.114	1.711	3.391	3.286	2.953	0.930	0.916	0.921	5.205	-0.105	-0.020	-0.011	-0.014	1
erlands	0.025	1.040	0.896	1.037	0.997	1.320	0.742	1.155	2.072	1.660	3.910	3.720	3.127	0.951	0.938	0.943	6.396	-0.187	-0.018	0.068	0.082	1
way	0.033	0.657	0.633	0.781	1.130	1.256	0.948	1.479	1.902	2.125	3.695	3.760	2.993	0.922	0.906	0.912	7.360	-0.430	-0.006	-0.161	-0.218	1
ugal	0.024	0.538	0.539	0.606	0.993	1.090	0.756	1.197	1.741	1.711	2.988	2.878	2.536	0.783	0.771	0.776	5.671	-0.190	-0.007	-0.137	-0.163	1
n	0.048	0.993	0.856	1.019	1.078	1.359	0.741	1.144	2.229	1.649	3.588	3.435	3.106	0.975	0.951	0.956	5.324	-0.066	-0.046	0.002	0.002	1
den	0.044	1.200	0.976	1.270	1.391	1.713	0.839	1.281	2.714	1.853	4.778	4.445	3.986	1.220	1.198	1.203	6.095	0.006	-0.034	0.032	0.053	1
zeland	0.035	0.788	0.642	0.797	0.921	1.131	0.746	1.168	1.787	1.677	3.055	2.983	2.605	0.809	0.792	0.797	6.548	-0.064	-0.034	-0.008	-0.009	1
zed	0.022	0.820	0.675	0.805	0.849	1.090	0.612	0.962	1.774	1.378	2.972	2.829	2.538	0.786	0.775	0.780	5.211	-0.156	-0.036	0.043	0.044	1
ingdom																						1
ed States	0.032	1.141	1.026	1.161	0.559	1.102	0.565	0.874	1.786	1.259	2.858	2.875	2.514	0.788	0.772	0.777	6.440	-0.115	-0.034	0.034	0.023	1
rage	0.034	0.814	0.705	0.847	1.001	1.227	0.738	1.150	1.959	1.653	3.385	3.335	2.870	0.886	0.870	0.875	6.054	-0.208	-0.025	-0.024	-0.027	1
1d Index	0.024	1.000	1.000	1.000	0.000	0.833	0.461	0.712	1.364	1.027	2.241	2.208	1.965	0.604	0.592	0.597	5.747	-0.196	-0.04553	-0.17409	-7.68E-17	1

1: Mean returns; **SR**: Systemic risk; **Down- β_{Dn}** : Downside beta calculated using observations when index and world index fall together; **Down- β_{Dn}** : Downside beta calculated using observations when world index falls; **IR**: Idiosyncratic risk; **$\sigma\text{-garch}(1,1)$** : volatility forecast considering a Garch (1,1) model to describe the country returns; **VAR**: Value at risk; **VAR95t** is the theoretical VAR considering a confidence interval of 95%; **VAR95d** is the empirical VAR, say the value of the return representing the 5th lowest percentile; **VAR99t** is the theoretical VAR similar to VAR95t but considering a confidence interval of 99%; **VAR99d** is the empirical VAR, say the value of the return representing the 1st lowest percentile; **ES95t** is the theoretical Expected Shortfall that follows the theoretical model previously described, using as the threshold the VAR95d; **ES95d** is the sample average of returns below the 5th percentile level (VAR95d); **ES99t** is the theoretical Expected Shortfall that follows the theoretical model previously described, using as the threshold the VAR99t; **ES99d** is the sample average of returns below the 1st percentile level (VAR99d); **Semi-0**: deviation with respect to mean; **Semi-0r**: semideviation with respect to 0; **Semi-rf**: semideviation with respect to Risk free rate; **kurt**: kurtosis; **skew**: skewness; **skew5%**: skewness calculated using formulas 14 and 15; **coskew1**: cosine deviation with respect to mean; **coskew2**: cosine deviation with respect to 0; **Size**: natural logarithm of the market capitalization.

Table 2. Summary statistics – emerging markets (MSCI data)

ket	E[R]	SR	Down- β_{lw}	Down- β_{hw}	IR	TR	$\sigma\text{-garch}(1,1)$	VAR95r	VAR95d	VAR99r	VAR95d	ES95r	ES95d	Semi-mean	Semi-0	Semi-rf	kurt	skew	skew5%	coskew1	coskew2
antina	0.021	0.957	0.872	1.212	2.262	2.398	1.537	2.436	3.341	3.483	6.275	9.135	5.521	1.792	1.783	1.788	56.665	-2.413	-0.013	-0.160	-0.435
al	0.040	1.265	1.228	1.522	1.929	2.198	1.469	2.291	3.534	3.292	6.225	6.284	5.262	1.597	1.578	1.583	8.301	-0.185	-0.058	-0.226	-0.524
e	0.009	0.623	0.487	0.713	1.052	1.173	0.899	1.455	1.809	2.068	3.162	3.154	2.699	0.838	0.833	0.839	6.607	-0.147	0.010	-0.159	-0.201
ambia	0.046	0.188	0.112	0.249	1.417	1.426	1.269	2.041	2.131	2.906	3.979	3.951	3.312	0.991	0.969	0.975	9.391	0.253	0.012	-0.131	-0.222
g Kong	0.011	0.664	0.674	0.776	1.536	1.633	0.683	1.076	2.469	1.542	4.432	4.980	3.847	1.156	1.151	1.156	13.792	0.112	-0.025	-0.090	-0.166
a	0.026	0.258	0.324	0.438	1.572	1.587	1.249	1.967	2.432	2.818	4.621	4.556	3.787	1.153	1.140	1.146	6.961	-0.294	-0.005	-0.111	-0.209
mesia	-0.018	0.492	0.523	0.418	3.250	3.275	1.265	1.996	4.184	2.858	9.114	17.798	7.910	2.385	2.392	2.396	32.719	-1.030	-0.014	-0.014	-0.055
al	0.034	0.855	0.639	0.933	1.389	1.561	1.000	1.569	2.482	2.251	4.708	4.642	3.816	1.128	1.112	1.117	7.896	-0.291	-0.021	0.014	0.023
an	0.042	0.017	0.097	0.090	0.958	0.957	1.620	2.664	1.337	3.768	2.584	3.281	2.205	0.651	0.634	0.639	13.920	0.207	0.055	-0.093	-0.107
aysia	-0.016	0.308	0.331	0.322	1.923	1.940	0.571	0.915	2.536	1.304	5.627	7.294	4.590	1.312	1.319	1.324	40.239	1.184	-0.029	-0.085	-0.195
ico	0.061	1.231	1.145	1.477	1.422	1.753	1.258	1.948	2.656	2.805	4.446	4.902	3.850	1.243	1.213	1.219	10.312	-0.095	0.006	-0.175	-0.299
Zealand	0.008	0.294	0.345	0.315	1.311	1.333	1.153	1.842	2.143	2.627	3.507	3.768	3.077	0.970	0.966	0.971	16.151	-0.552	-0.047	-0.006	-0.010
stan	0.006	0.056	0.052	0.135	2.060	2.061	1.215	1.972	3.297	2.800	5.825	6.309	5.121	1.509	1.506	1.511	9.302	-0.396	-0.046	-0.047	-0.117
i	0.035	0.441	0.560	0.499	1.363	1.412	1.265	2.019	2.069	2.881	3.978	4.234	3.233	0.997	0.980	0.985	9.002	-0.029	0.035	-0.191	-0.313
ippines	-0.040	0.307	0.375	0.382	1.747	1.765	1.246	2.053	2.676	2.903	5.011	4.878	4.138	1.198	1.218	1.223	19.291	1.097	-0.018	-0.070	-0.146
apore	0.002	0.550	0.558	0.546	1.344	1.420	0.546	0.879	2.213	1.252	3.776	4.023	3.262	0.996	0.997	1.002	5.613	-0.158	0.004	-0.093	-0.200
nd	0.037	0.714	0.683	0.764	1.797	1.893	1.368	2.180	2.982	3.112	5.119	5.017	4.331	1.353	1.335	1.340	10.644	0.185	-0.011	0.059	0.095
h Africa	-0.002	0.550	0.558	0.546	1.344	1.420	0.546	0.879	2.213	1.252	3.776	4.023	3.262	0.996	0.997	1.002	10.644	0.185	-0.011	0.059	0.095
h Korea	0.018	0.760	0.723	0.899	1.377	1.515	1.247	1.976	2.349	2.826	4.445	4.562	3.691	1.111	1.103	1.108	7.635	-0.514	-0.001	-0.179	-0.296
an	-0.008	0.375	0.440	0.483	1.746	1.774	1.179	1.906	2.880	2.709	4.560	4.538	4.008	1.238	1.242	1.248	5.498	0.016	0.036	-0.084	-0.176
iland	-0.036	0.543	0.274	0.508	2.234	2.279	1.014	1.654	3.533	2.345	6.369	5.843	5.184	1.528	1.546	1.552	10.753	0.784	-0.002	0.048	0.128
ey	0.042	0.681	1.134	0.896	3.202	3.251	1.706	2.712	4.994	3.874	9.251	9.354	7.703	2.317	2.297	2.302	8.011	-0.092	-0.035	-0.105	-0.403
zuela	0.012	0.525	0.688	0.800	2.662	2.698	2.479	4.015	3.309	5.704	6.671	22.017	5.891	2.019	2.094	2.098	182.880	-6.362	0.016	-0.107	-0.343
rage	0.015	0.535	0.535	0.633	1.757	1.833	1.184	1.894	2.722	2.701	5.041	6.361	4.279	1.310	1.303	1.308	21.235	-0.340	-0.007	-0.087	-0.185

[E]: Mean returns; **SR**: Systemic risk; **Down- β_{lw}** : Downside beta calculated using observations when index and world index fall together; **Down- β_{hw}** : Downside beta calculated using observations when world index falls; **IR**: Idiosyncratic risk; **$\sigma\text{-garch}(1,1)$** : volatility forecast considering a Garch (1,1) model to describe the country returns; **VAR**: Value at risk; **VAR95r** is the theoretical VAR considering a confidence interval of 95%; **VAR95d** is the empirical VAR, say the value of the return representing the 5th lowest percentile; **VAR99r** is the theoretical VAR similar to VAR95r but considering a confidence interval of 99%; **VAR99d** is the empirical VAR, say the value of the return representing the 1st lowest percentile; **ES95r** is the theoretical Expected Shortfall that follows the theoretical model previously described, using as the threshold the VAR95d; **ES95d** is the sample average of returns below the 5th percentile level (VAR95d); **ES99r** is the theoretical Expected Shortfall that follows the theoretical model previously described, using as the threshold the VAR99d; **ES99d** is the sample average of returns below the 1st percentile level (VAR99d); **Semi-0** is the deviation with respect to mean; **Semi-d**: semideviation with respect to 0; **Semi-rf**: semideviation with respect to Risk free rate; **kurt**: kurtosis; **skew**: skewness; **skew5%**: skewness calculated using formulas 14 and 15; **coskew1**: cosine definition 1; **coskew2**: cosine definition 2; **Size**: natural logarithm of the market capitalization.

Fig. 1. Quantile–quantile graph

Notes: This figure presents QQ plot comparing observed daily returns for the world index with the normal distribution.

Table 3. Summary statistics – Pareto distribution parameters (developed markets)

Threshold	Parameter	Mean	Min.	Max.	SD
VAR95 <i>d</i>	alpha (Hill)	3.007	2.574	3.393	0.234
	alpha (LS)	2.735	2.121	3.203	0.341
	<i>k</i> (LS)	9.919	3.881	27.609	7.183
VAR99 <i>d</i>	alpha (Hill)	4.441	2.977	6.531	0.947
	alpha (LS)	3.886	2.396	5.315	0.994
	<i>k</i> (LS)	362.450	16.202	2009.700	520.450

Table 4. Summary statistics – Pareto distribution parameters (emerging markets)

Threshold	Parameter	Mean	Min.	Max.	SD
VAR95 <i>d</i>	alpha (Hill)	2.712	1.977	3.443	0.363
	alpha (LS)	2.193	1.167	2.911	0.450
	<i>k</i> (LS)	13.963	1.754	51.001	13.219
VAR99 <i>d</i>	alpha (Hill)	3.581	2.274	6.217	0.841
	alpha (LS)	2.784	0.846	4.435	0.940
	<i>k</i> (LS)	352.340	4.197	3328.100	723.450

for the LS estimation and the MSE for both methodologies. Average results are presented in Tables 5 and 6.

If we consider as the threshold the VAR95*d*, we may observe that least squares estimator offered a better fit to the extreme losses data with a MSE much lower than the one provided by the Hill estimator. Also the R^2 for the regression used in the LS analysis was on average 0.96 again supporting the good adjustment of the Pareto distribution.

Taking VAR99*d* as the threshold, LS provides again better results than Hill. The R^2 greater than 0.90 supports the LS estimation of the Pareto distribution parameters. It's good to highlight that when we use the VAR99*d* as the threshold, we reduce the number of extreme losses of each return

time series. In our case, each extreme loss series had around 28 observations instead of 150 when we used the VAR95*d* as the threshold.

As the LS methodology offered satisfactory results, we used the LS estimators of α and k to define the survivor function of the extreme losses (Pareto Distribution). We calculated the parametric ES ES95*t* for each country following the discussed model (Equation 4). The summary results for developed and emerging markets are given in Tables 1 and 2. On average, the parametric ES ES95*t* measure for emerging markets was higher than the one for developed countries. Also, we may observe that the ES of the index is much lower than the one obtained just by averaging the ES for each market. For instance, ES95*t* is equal to 2.208 for the

Fig. 2. Pareto distribution for developed and emerging markets

Notes: This figure represents the Pareto Distribution considering the average parameters of α and k , for the developed and emerging markets samples.

Table 5. Pareto distribution parameter estimation (developed markets)

Threshold	Criteria	Mean	Min.	Max.	SD
VAR95d	R2 (LS)	0.956	0.922	0.991	0.026
	MSE (LS)	0.006	0.001	0.015	0.004
	MSE(Hill)	0.014	0.010	0.018	0.002
VAR99d	R2(LS)	0.953	0.875	0.989	0.031
	MSE (LS)	0.004	0.001	0.012	0.003
	MSE (Hill)	0.007	0.003	0.012	0.003

Table 6. Pareto distribution parameter estimation (emerging markets)

Threshold	Criteria	Mean	Min.	Max.	SD
VAR95d	R2 (LS)	0.960	0.920	0.995	0.019
	MSE (LS)	0.003	0.000	0.011	0.003
	MSE (Hill)	0.016	0.009	0.028	0.005
VAR99d	R2 (LS)	0.919	0.778	0.977	0.057
	MSE (LS)	0.005	0.001	0.011	0.003
	MSE (Hill)	0.010	0.002	0.020	0.005

Table 7. Pareto distribution parameter estimation α (LS) – α (Hill)

	Threshold	Mean	Correlation	t	p -Value
Developed markets	VAR95d	−0.272	0.552	−2.795	0.008
	VAR99d	−0.555	0.812	−1.715	0.095
Emerging markets	VAR95d	−0.519	0.716	−4.302	0.000
	VAR99d	−0.797	0.730	−3.029	0.004

world index (Table 1) and equals to 4.202 if we average this measure for all countries. This empirical evidence shows that the diversification effect holds for the ES.

Once we have all the risk measures defined and calculated over each market's time series, we begin

the cross section regression analysis and the starting point is to compute the Pearson correlation coefficient matrix among the considered risk variables (Tables 8 and 9). These tables show the correlation matrix for developed and emerging countries. A correlation coefficient higher, in absolute terms,

ble 8. Correlation matrix – developed markets

	E[R]	SR	Down- β_{lw}	Down- β_w	IR	TR	σ -garch(1,1)	VAR95 <i>r</i>	VAR95 <i>d</i>	VAR99 <i>r</i>	VAR95 <i>d</i>	ES95 <i>r</i>	ES95 <i>d</i>	Semi-mean	Semi-0	Semi-rf	kurt	skew	skew5%	coskew1	coskew2	S	
I	1.000	0.371	0.372	0.420	0.535	0.564	0.352	0.289	0.548	0.309	0.540	0.576	0.561	0.571	0.558	0.558	0.420	-0.302	-0.114	-0.106	-0.010	-	
m- β_{lw}	0.371	1.000	0.982	0.987	0.391	0.747	-0.014	-0.063	0.782	-0.048	0.742	0.688	0.740	0.740	0.741	0.740	0.296	0.081	-0.434	0.650	0.711	-	
m- β_w	0.372	0.982	1.000	0.981	0.289	0.670	-0.090	-0.144	0.703	-0.128	0.677	0.623	0.665	0.664	0.664	0.664	0.297	0.052	-0.425	0.582	0.637	-	
r	0.420	0.987	0.981	1.000	0.395	0.744	0.016	-0.037	0.774	-0.021	0.740	0.699	0.738	0.740	0.740	0.740	0.350	-0.006	-0.432	0.537	0.604	-	
r5%	0.535	0.391	0.289	0.395	1.000	0.900	0.833	0.808	0.873	0.816	0.885	0.903	0.902	0.903	0.903	0.903	0.468	-0.226	0.199	0.116	0.218	-	
r95 <i>r</i>	0.564	0.747	0.670	0.744	0.900	1.000	0.597	0.556	0.995	0.569	0.984	0.976	0.997	0.999	0.999	0.999	0.484	-0.138	-0.035	0.375	0.476	-	
r95 <i>d</i>	0.352	-0.014	-0.090	0.016	0.833	0.597	1.000	0.996	0.544	0.998	0.611	0.654	0.608	0.609	0.609	0.609	0.427	-0.373	0.406	-0.217	-0.156	-	
r99 <i>r</i>	0.548	0.782	0.703	0.774	0.873	0.995	0.544	0.503	1.000	0.516	0.976	0.959	0.992	0.993	0.993	0.993	0.442	-0.102	-0.090	0.424	0.524	-	
r99 <i>d</i>	0.309	-0.048	-0.128	-0.021	0.816	0.569	0.998	1.000	0.516	1.000	0.581	0.624	0.579	0.581	0.581	0.581	0.395	-0.366	0.431	-0.230	-0.173	-	
S <i>r</i>	0.576	0.688	0.623	0.699	0.903	0.976	0.654	0.611	0.959	0.624	0.969	1.000	0.969	0.989	0.984	0.985	0.985	0.457	-0.157	0.018	0.368	0.465	-
S <i>d</i>	0.561	0.740	0.665	0.738	0.902	0.997	0.608	0.566	0.992	0.579	0.989	0.984	1.000	0.999	0.999	0.999	0.506	-0.180	-0.022	0.366	0.469	-	
i-mean	0.571	0.740	0.664	0.740	0.903	0.999	0.609	0.568	0.993	0.581	0.984	0.984	0.999	1.000	1.000	1.000	0.506	-0.179	-0.029	0.352	0.454	-	
i-0	0.558	0.741	0.664	0.740	0.903	0.999	0.609	0.568	0.993	0.581	0.985	0.983	0.999	1.000	1.000	1.000	0.504	-0.176	-0.027	0.357	0.460	-	
i-rf	0.558	0.740	0.664	0.740	0.903	0.999	0.609	0.569	0.993	0.581	0.985	0.983	0.999	1.000	1.000	1.000	0.504	-0.176	-0.027	0.357	0.459	-	
t	0.420	0.296	0.297	0.350	0.468	0.484	0.427	0.380	0.442	0.395	0.457	0.624	0.506	0.506	0.504	0.504	1.000	-0.543	-0.133	-0.088	0.001	-	
/	-0.302	0.081	0.052	-0.006	-0.226	-0.138	-0.373	-0.362	-0.102	-0.366	-0.157	-0.319	-0.180	-0.179	-0.176	-0.176	-0.543	1.000	-0.228	0.577	0.506	-	
/5%	-0.114	-0.434	-0.425	-0.432	0.199	-0.035	0.406	0.441	-0.090	0.431	0.018	0.023	-0.022	-0.029	-0.027	-0.027	-0.133	-0.228	1.000	-0.464	-0.469	-	
ew1	-0.106	0.650	0.582	0.537	0.116	0.375	-0.217	-0.235	0.424	-0.230	0.368	0.261	0.366	0.352	0.357	0.357	-0.088	0.577	-0.464	1.000	0.989	-	
ew2	-0.010	0.711	0.637	0.604	0.218	0.476	-0.156	-0.180	0.524	-0.173	0.465	0.369	0.469	0.454	0.460	0.459	0.001	0.506	-0.469	0.989	1.000	-	
	-0.104	0.468	0.506	0.455	-0.448	-0.092	-0.675	-0.687	-0.047	-0.684	-0.136	-0.148	-0.117	-0.100	-0.099	-0.100	-0.074	0.232	-0.583	0.385	0.354	-	

∴ A correlation coefficient higher, in absolute terms, to 0.575 indicates that it is significant at a 1% confidence level; and if its value is between 0.456 and 0.575, it is significant at a 5% confidence level (2-tail test).

: Mean returns; **SR**: Systemic risk; **Down- β_{lw}** : Downside beta calculated using observations when index and world index fall together; **Down- β_w** : Downside beta calculated using observations when world index falls; **IR**: Idiosyncratic risk; **σ -garch(1,1)**: volatility forecast considering a Garch (1,1) model to describe the country returns; **VAR**: Value at risk; **VAR95*r*** is the theoretical VAR considering a confidence interval of 95%; **VAR95*d*** is the empirical VAR, say the value of the return representing the 5th lowest percentile; **VAR99*r*** is the theoretical VAR similar to VAR95*r* but considering a confidence interval of 99%; **VAR99*d*** is the empirical VAR, say the value of the return representing the 1st lowest percentile; **ES95*r*** is the theoretical Expected Shortfall that follows the theoretical model previously described, using as the threshold the VAR95*d*; **ES95*d*** is the sample average of returns below the 5th percentile level (VAR95*d*); **ES99*r*** is the theoretical Expected Shortfall that follows the theoretical model previously described, using as the threshold the VAR99*d*; **ES99*d*** is the sample average of returns below the 1st percentile level (VAR99*d*); **Semi-rf** is the theoretical Expected Shortfall that follows the theoretical model previously described, using as the threshold the VAR99*d*; **Semi-0** is the sample average of returns below the 1st percentile level (VAR99*d*); **Semi-ew1** is the theoretical Expected Shortfall that follows the theoretical model previously described, using as the threshold the VAR99*d*; **Semi-ew2** is the sample average of returns below the 1st percentile level (VAR99*d*); **skew**: skewness; **skew5%**: skewness calculated using formulas 14 and 15; **coskew1**: cosine deviation with respect to mean; **Semi-0**: semideviation with respect to 0; **Semi-rf**: semideviation with respect to Risk free rate; **kurt**: kurtosis; **skew**: skewness calculated using formulas 14 and 15; **coskew1**: cosine deviation with respect to mean; **coskew2**: cosine deviation with respect to market capitalization.

	E[R]	SR	Down- β_{HW}	Down- β_{HW}	TR	σ -garch(1,1)	VAR95 t	VAR95 d	VAR99 t	VAR95 d	ES95 t	ES95 d	Semi-mean	Semi-0	Semi-rf	kurt	skew	skew5%	coskew1	coskew2	
I	1.000	0.356	0.420	0.435	-0.230	-0.167	0.328	0.300	-0.118	0.308	-0.176	-0.179	-0.173	-0.130	-0.156	-0.157	-0.120	-0.118	0.150	-0.552	-0.513
-	0.356	1.000	0.901	0.977	0.141	0.269	0.057	0.019	0.329	0.030	0.249	0.080	0.250	0.281	0.270	0.270	-0.004	-0.127	-0.273	-0.369	-0.450
π - β_{HW}	0.420	0.901	1.000	0.925	0.286	0.397	0.250	0.211	0.447	0.223	0.375	0.210	0.376	0.418	0.405	0.405	0.091	-0.227	-0.267	-0.452	-0.616
π - β_{HW}	0.435	0.977	0.925	1.000	0.144	0.271	0.197	0.158	0.327	0.170	0.241	0.116	0.245	0.296	0.283	0.283	0.087	-0.233	-0.243	-0.480	-0.581
-	-0.230	0.411	0.286	0.144	1.000	0.991	0.398	0.389	0.942	0.392	0.980	0.777	0.987	0.982	0.984	0.984	0.390	-0.327	-0.350	0.132	-0.231
arch(1,1)	-0.167	0.269	0.397	0.271	0.991	1.000	0.400	0.385	0.960	0.390	0.988	0.768	0.994	0.993	0.994	0.994	0.376	-0.334	-0.380	0.074	-0.286
γ 95 t	0.328	0.057	0.250	0.197	0.398	0.400	1.000	0.999	0.327	0.999	0.332	0.596	0.356	0.462	0.453	0.452	0.633	-0.736	0.225	-0.355	-0.525
γ 95 d	0.300	0.019	0.211	0.158	0.389	0.385	0.999	1.000	0.310	1.000	0.316	0.592	0.340	0.446	0.438	0.437	0.641	-0.732	0.245	-0.334	-0.500
γ 99 t	-0.118	0.329	0.447	0.327	0.942	0.960	0.327	0.310	1.000	0.315	0.964	0.586	0.963	0.941	0.941	0.941	0.179	-0.182	-0.475	0.079	-0.284
γ 99 d	0.308	0.030	0.223	0.170	0.392	0.390	0.999	1.000	0.315	1.000	0.321	0.594	0.345	0.451	0.442	0.442	0.639	-0.733	0.239	-0.340	-0.508
γ 99 d	-0.176	0.249	0.375	0.241	0.980	0.988	0.332	0.316	0.964	0.321	1.000	0.705	0.996	0.973	0.974	0.974	0.278	-0.232	-0.436	0.089	-0.261
5 t	-0.179	0.080	0.210	0.116	0.777	0.768	0.596	0.592	0.586	0.594	0.705	1.000	0.737	0.814	0.816	0.816	0.817	-0.757	-0.070	0.054	-0.217
5 d	-0.173	0.250	0.376	0.245	0.987	0.994	0.356	0.340	0.963	0.345	0.996	0.737	1.000	0.986	0.986	0.986	0.314	-0.282	-0.430	0.085	-0.270
i-mean	-0.130	0.281	0.418	0.296	0.982	0.993	0.462	0.446	0.941	0.451	0.973	0.814	0.986	1.000	1.000	1.000	0.443	-0.431	-0.370	0.042	-0.322
i-0	-0.156	0.270	0.405	0.283	0.984	0.994	0.453	0.438	0.941	0.442	0.974	0.816	0.986	1.000	1.000	1.000	0.446	-0.426	-0.372	0.057	-0.307
i-rf	-0.157	0.270	0.405	0.283	0.984	0.994	0.452	0.437	0.941	0.442	0.974	0.816	0.986	1.000	1.000	1.000	0.445	-0.426	-0.372	0.057	-0.307
-	-0.120	-0.004	0.091	0.087	0.390	0.376	0.633	0.641	0.179	0.639	0.278	0.817	0.314	0.443	0.446	0.445	1.000	-0.886	0.116	-0.042	-0.237
/	-0.118	-0.127	-0.227	-0.23																	

: Mean returns; **SR**: Systemic risk; **Down-fwr**: Downside beta calculated using observations when index and world index fall together; **Down-fwr**: Downside beta calculated using observations when world index falls; **IR**: Idiosyncratic risk; **TR**: total risk; **$\sigma\text{-garch}(1,1)$** : volatility forecast considering a Garch (1,1) model to describe the country returns; **VAR95** is the empirical VAR considering a confidence interval of 95%; **VAR95d** is the empirical VAR considering a confidence interval of 95%; **VAR99** is the empirical VAR considering a confidence interval of 99%; **VAR99d** is the empirical VAR considering a confidence interval of 99%; **ES95** is the sample average of returns below the 5th percentile level (VAR95); **ES95d** is the sample average of returns below the 5th percentile level (VAR95d); **ES99** is the sample average of returns below the 1st percentile level (VAR99); **ES99d** is the sample average of returns below the 1st percentile level (VAR99d); **Semi-rf**: semideviation with respect to mean; **Semi-df**: semideviation with respect to 0; **Semi-rf**: semideviation with respect to Risk free rate; **kurt**: kurtosis; **skew**: skewness; **skew5%**: skewness calculated using formulas 14 and 15; **coskew1**: coskewness calculated using formulas 14 and 15; **coskew5%**: coskewness calculated using formulas 14 and 15; **Size**: natural logarithm of the market capitalization.

Table 10. Bivariate regressions – all markets

$$RR_i = c_0 + c_1 * Risk_i + u_i$$

Risk variable	c_0	p -Value	c_1	p -Value	R^2
SR	0.0035	0.6914	0.0286	0.0041	0.2017
Down- β_{iw}	0.0006	0.9524	0.0354	0.0058	0.2202
Down- β_w	0.0006	0.9480	0.0298	0.0024	0.2372
IR	0.0399	0.0000	-0.0113	0.0434	0.1144
TR	0.0401	0.0000	-0.0102	0.1038	0.0748
σ -garch(1,1)	0.0243	0.0014	-0.0008	0.9089	0.0002
VAR95 $_t$	0.0259	0.0005	-0.0015	0.7207	0.0018
VAR95 $_d$	0.0381	0.0009	-0.0059	0.2357	0.0465
VAR99 $_t$	0.0255	0.0006	-0.0008	0.7747	0.0011
VAR99 $_d$	0.0405	0.0001	-0.0038	0.1134	0.0788
ES95 $_t$	0.0323	0.0000	-0.0017	0.0279	0.0808
ES95 $_d$	0.0399	0.0000	-0.0043	0.1072	0.0744
Semi-mean	0.0379	0.0000	-0.0123	0.1283	0.0579
Semi-0	0.0390	0.0000	-0.0134	0.0966	0.0695
Semi-rf	0.0391	0.0000	-0.0134	0.0965	0.0696
kurt	0.0260	0.0000	-0.0002	0.0860	0.0450
skew	0.0230	0.0000	-0.0017	0.6408	0.0072
skew5%	0.0226	0.0000	-0.0540	0.6694	0.0037
coskew1	0.0209	0.0000	-0.0409	0.2300	0.0260
coskew2	0.0217	0.0000	-0.0157	0.4215	0.0131
size	-0.0004	0.9811	0.0028	0.0708	0.0617

to 0.575 (0.514) is statistically significant at a 1% confidence level; and if its value is between 0.456 (0.411) and 0.575 (0.514), it is significant at a 5% confidence level for developed (emerging) markets; considering a 2-tail test.

Taking these ranges into account we can observe some differences among both samples. Considering the correlations with the return we can see that the beta risk factors' coefficients were positive but not significant for developed markets. For emerging countries, except for SR which was not significant, the beta risk factors' coefficients were positive and significant. This initially indicates that CAPM models, based on SR, will not perform well in both samples. About the distributional risk factors, most of them offered positive and significant coefficients for developed countries, while they were negative and not significant for emerging markets. Coskewness was negative and significant for emerging markets, supporting the results found in Harvey (2000).

All previous results are in line with the general intuition that a higher risk should imply a higher return, however when we compare both samples we observe that emerging markets present higher risk and lower return when compared to the developed countries sample. At the same time the numbers suggest that the two groups of risk factors (beta-related and distributional factors) are positively

(and in general significantly) correlated within each group for both samples. This empirical fact could be considered as an evidence of 'home-bias'⁸ as market participants are investing and so requiring reward for the risk faced just within an asset group, say developed countries or emerging markets.

It's important to highlight the correlations between VAR and ES risk measures. The correlations among them in both samples were in general positive and significant, however far from one which would be the case if returns were following a Normal Distribution.

Regression analysis

We first implemented regressions considering each of the risk factors acting individually. These regressions examine the bivariate relation between the average returns and the average risk measures. Throughout all this section, we considered to be significant a p -value lower than 5% and we used White (1980) heteroskedasticity consistent covariance matrix. The results related to the all markets sample are summarized in Table 10.

We can observe that the beta-related risk factors were positive and significant, explaining around 22% of the average returns. The first regression is the classic CAPM and has an R^2 equals to 20.17%. Downside betas work well for the world sample

⁸ See French and Poterba (1991) or Shapiro (1999) for a review of the home bias effect.

Table 11. Bivariate regressions – developed markets

$$RR_t = c_0 + c_1 * Risk_t + u_t$$

Risk variable	c_0	p -Value	c_1	p -Value	R^2
SR	0.0244	0.0008	0.0113	0.1356	0.1374
Down- β_{iw}	0.0224	0.0060	0.0157	0.1341	0.1381
Down- β_w	0.0224	0.0028	0.0131	0.0878	0.1764
IR	0.0156	0.0113	0.0175	0.0007	0.2860
TR	0.0122	0.0193	0.0169	0.0000	0.3175
σ -garch(1,1)	0.0102	0.5373	0.0307	0.1603	0.1236
VAR95 $_t$	0.0145	0.4062	0.0161	0.2726	0.0838
VAR95 $_d$	0.0129	0.0225	0.0103	0.0002	0.3008
VAR99 $_t$	0.0132	0.4438	0.0120	0.2354	0.0952
VAR99 $_d$	0.0128	0.0230	0.0059	0.0003	0.2916
ES95 $_t$	0.0144	0.0012	0.0056	0.0000	0.3318
ES95 $_d$	0.0131	0.0076	0.0069	0.0000	0.3150
Semi-mean	0.0121	0.0134	0.0235	0.0000	0.3262
Semi-0	0.0128	0.0121	0.0232	0.0000	0.3108
Semi-rf	0.0127	0.0132	0.0232	0.0000	0.3111
kurt	0.0144	0.2011	0.0031	0.0739	0.1762
skew	0.0308	0.0000	-0.0151	0.2070	0.0912
skew5%	0.0323	0.0000	-0.0690	0.5735	0.0130
coskew1	0.0338	0.0000	-0.0112	0.6189	0.0113
coskew2	0.0341	0.0000	-0.0009	0.9648	0.0001
Size	0.0405	0.0051	-0.0007	0.5947	0.0107

Table 12. Bivariate regressions – emerging markets

$$RR_t = c_0 + c_1 * Risk_t + u_t$$

Risk variable	c_0	p -Value	c_1	p -Value	R^2
SR	-0.0007	0.9518	0.0287	0.0836	0.1266
Down- β_{iw}	-0.0040	0.7565	0.0345	0.0469	0.1765
Down- β_w	-0.0044	0.7152	0.0298	0.0284	0.1896
IR	0.0329	0.0452	-0.0096	0.2684	0.0527
TR	0.0289	0.0826	-0.0071	0.4010	0.0280
σ -garch(1,1)	-0.0113	0.4546	0.0215	0.0868	0.1074
VAR95 $_t$	-0.0088	0.5451	0.0122	0.1036	0.0901
VAR95 $_d$	0.0258	0.1816	-0.0037	0.5874	0.0140
VAR99 $_t$	-0.0096	0.5163	0.0088	0.0982	0.0951
VAR99 $_d$	0.0295	0.0934	-0.0027	0.4115	0.0309
ES95 $_t$	0.0221	0.0156	-0.0010	0.2317	0.0322
ES95 $_d$	0.0291	0.0837	-0.0031	0.3950	0.0299
Semi-mean	0.0254	0.1122	-0.0074	0.4988	0.0168
Semi-0	0.0274	0.0817	-0.0089	0.4127	0.0245
Semi-rf	0.0275	0.0820	-0.0089	0.4120	0.0245
kurt	0.0171	0.0097	-0.0001	0.2635	0.0144
skew	0.0145	0.0221	-0.0021	0.4870	0.0139
skew5%	0.0164	0.0067	0.1410	0.4120	0.0224
coskew1	-0.0024	0.7568	-0.1941	0.0020	0.3052
coskew2	-0.0008	0.9180	-0.0831	0.0024	0.2629
Size	0.0065	0.8066	0.0012	0.7268	0.0039

explaining around 23% and this result comes especially from emerging markets. The intercepts are not significantly different from zero. This seems to support the CAPM. The distributional risk factors came out to be in general negative but not significant. The results for the skewness risk factors

were negative but not significant. Size provided a positive but not significant result. The previous results give support to the use of the CAPM model in a world sample analysis. We performed the same analysis segregating developed and emerging markets (Tables 11 and 12).

Table 13. Multivariate regressions – all markets

$$RR_i = c_0 + c_1 * SR_i + c_2 * Risk_i + u_i$$

SR/Risk	c_0	p -Value	c_1	p -Value	c_2	p -Value	R^2
Down- β_{iw}	0.0007	0.9478	0.0068	0.7509	0.0279	0.3169	0.2219
Down- β_w	0.0001	0.9949	-0.0286	0.3876	0.0565	0.0915	0.2483
IR	0.0187	0.1021	0.0258	0.0075	-0.0090	0.0866	0.2732
TR	0.0207	0.0763	0.0293	0.0022	-0.0108	0.0645	0.2859
σ -garch(1,1)	-0.0040	0.7123	0.0304	0.0016	0.0062	0.3639	0.2125
VAR95 $_t$	-0.0029	0.7845	0.0303	0.0016	0.0033	0.4348	0.2096
VAR95 $_d$	0.0205	0.1109	0.0307	0.0017	-0.0074	0.1270	0.2748
VAR99 $_t$	-0.0032	0.7625	0.0303	0.0016	0.0024	0.4128	0.2104
VAR99 $_d$	0.0209	0.0806	0.0291	0.0022	-0.0040	0.0796	0.2870
ES95 $_t$	0.0121	0.2175	0.0274	0.0047	-0.0015	0.0506	0.2639
ES95 $_d$	0.0204	0.0803	0.0293	0.0022	-0.0046	0.0657	0.2847
Semi-mean	0.0188	0.2723	0.0296	0.0021	-0.0136	0.0712	0.2723
Semi-0	0.0196	0.0836	0.0294	0.0021	-0.0144	0.0563	0.2819
Semi-rf	0.0197	0.0830	0.0294	0.0021	-0.0144	0.0563	0.2820
Kurt	0.0063	0.4875	0.0274	0.0054	-0.0001	0.1373	0.2266
Skew	0.0033	0.7151	0.0284	0.0038	-0.0012	0.6529	0.2054
skew5%	0.0028	0.7342	0.0003	0.0003	0.1481	0.2626	0.2241
coskew1	-0.0036	0.7079	0.0325	0.0017	-0.0693	0.0239	0.2728
coskew2	-0.0015	0.8824	0.0311	0.0033	-0.0282	0.2424	0.2424
Size	0.0062	0.6683	0.0301	0.0020	-0.0004	0.7561	0.2026

From these tables, we have evidence that CAPM doesn't perform well in both samples taken apart. As pointed out in Estrada (2000), the lack of explanatory power of beta can be explained by: the markets are not fully integrated; the world-market portfolio is not mean-variance efficient; the model is misspecified; and finally, returns and betas may be uncorrelated if these two magnitudes are summarized by long-term averages but their true values change over time. In the multivariate regressions we could see that adding another risk variable (like VAR95 $_t$ for the case of developed markets) the SR (beta) became significant.

Considering the developed countries sample, distributional risk measures were in general positive and significant explaining up to 33.18% (in the case of ES95 $_t$) of the returns variation, indicating the relevance of downside extreme risk for sophisticated markets. VAR, ES and Semi-deviation measures are important and priced in developed countries. Related to skewness risk factors, just skew5%, which is correlated to distributional risk measures, offered a positive and significant coefficient.

In the case of emerging markets, downside betas and coskewness' measures were significant explaining in the case of coskew1 30.52%. Estrada (2000) already has shown the relevance of down-side beta for emerging markets. In terms of coskewness, as suggested in Harvey and Siddique (2000), if asset returns have systematic skewness, expected returns should include rewards for accepting this risk.

Emerging markets exhibit a more skewed return distribution and so we found significant coefficients for this risk measure. Also note that in a world where investors care about skewness of their portfolios, coskewness should count and skewness itself should not (this is similar to the roles played by beta and volatility in the classic CAPM). This is what we found for emerging markets.

On the other hand, IR and TR were not significant for emerging markets, however were priced in developed countries. Asset pricing theory predicts that only systematic variance should be priced but 31.75% of the developed countries' return was explained by total variance.

As the usual beta came out to be significant in the bivariate regression, considering the whole sample, we decided to investigate multivariate possibilities. The following regressions use two risk factors (SR + another risk measure). Tables 13–15 provide the results for the whole sample, developed markets and emerging markets, respectively.

When we considered all countries in the sample, the multivariate regressions don't seem to improve the CAPM analysis as we remain with an explanation (R^2) of around 27%, due to the beta and together with it, just coskew1 risk factor was negative and statistically significant. Kurtosis, skewness and size were negative but not significant. However when we segregate the sample we find better results for developed countries, with the R^2 raising up to around 34.67%. The distributional risk factors were

Table 14. Multivariate regressions – developed markets

$$RR_i = c_0 + c_1 * SR_i + c_2 * Risk_i + u_i$$

SR/Risk	c_0	p -Value	c_1	p -Value	c_2	p -Value	R^2
Down- β_{iw}	0.0232	0.0120	0.009	0.8917	0.0091	0.8500	0.1390
Down- β_w	0.0201	0.0102	-0.0539	0.0578	0.0675	0.0199	0.2546
IR	0.0132	0.0103	0.0058	0.3309	0.0151	0.0045	0.3168
TR	0.0119	0.0202	-0.0035	0.6679	0.0194	0.0036	0.3233
σ -garch(1,1)	0.0000	0.9991	0.0114	0.0491	0.0312	0.0259	0.2646
VAR95 t	0.0027	0.8249	0.0119	0.0483	0.0175	0.0655	0.2356
VAR95 d	0.0123	0.0236	-0.0046	0.6028	0.0125	0.0075	0.3097
VAR99 t	0.0018	0.8748	0.0118	0.0480	0.0127	0.0502	0.2441
VAR99 d	0.0126	0.0245	-0.0020	0.7950	0.0065	0.0087	0.2936
ES95 t	0.0145	0.0020	-0.0015	0.8335	0.0059	0.0005	0.3331
ES95 d	0.0129	0.0072	-0.0030	0.7011	0.0078	0.0024	0.3195
Semi-Mean	0.0119	0.0131	-0.0035	0.6555	0.0270	0.0016	0.3322
Semi-0	0.0126	0.0113	-0.0029	0.7167	0.0261	0.0025	0.3148
Semi-rf	0.0125	0.0123	-0.0028	0.7172	0.0261	0.0025	0.3150
kurt	0.0110	0.3041	0.0082	0.1997	0.0025	0.1026	0.2428
skew	0.0200	0.0016	0.0121	0.0628	-0.0168	0.0532	0.2483
skew5%	0.0247	0.0005	0.0120	0.1600	0.0350	0.8149	0.1401
coskew1	0.0126	0.0785	0.0232	0.0050	-0.0634	0.0041	0.3467
coskew2	0.0128	0.1012	0.0233	0.0106	-0.0470	0.0124	0.2892
size	0.0419	0.0016	0.0163	0.0208	-0.0022	0.0637	0.2355

Table 15. Multivariate regressions – emerging markets

$$RR_i = c_0 + c_1 * SR_i + c_2 * Risk_i + u_i$$

SR/Risk	c_0	p -Value	c_1	p -Value	c_2	p -Value	R^2
Down- β_{iw}	-0.0036	0.7838	-0.0097	0.7532	0.0434	0.1680	0.1792
Down- β_w	-0.0032	0.7685	-0.1217	0.0659	0.1307	0.0188	0.2945
IR	0.0194	0.3136	0.0319	0.0225	-0.0120	0.1646	0.2065
TR	0.0189	0.3274	0.0348	0.0146	-0.0121	0.1735	0.2013
σ -garch(1,1)	-0.0249	0.1056	0.0272	0.0789	0.02	0.0898	0.2213
VAR95 t	-0.0240	0.1154	0.0282	0.0684	0.0119	0.2128	0.2128
VAR95 d	0.0188	0.4015	0.0357	0.0160	-0.0083	0.2926	0.1888
VAR99 t	-0.0243	0.1124	0.0279	0.0713	0.0085	0.0970	0.2153
VAR99 d	0.0190	0.3422	0.0343	0.0156	-0.0043	0.2012	0.2012
ES95 t	0.0065	0.6413	0.0300	0.0507	-0.0012	0.1744	0.1701
ES95 d	0.0185	0.3388	0.0343	0.0158	-0.0050	0.1802	0.1997
Semi-mean	0.0156	0.4030	0.0343	0.0168	60.403	0.2035	0.1840
Semi-0	0.0171	0.3550	0.0346	0.0145	10.355	0.1656	0.1954
Semi-rf	0.0172	0.3533	0.0346	0.0144	30.0346	0.1653	0.1956
kurt	0.0011	0.9269	0.0286	0.0771	-0.0001	0.2707	0.1406
skew	-0.0008	0.9493	0.0279	0.0901	-0.0013	0.5846	0.1320
skew5%	-0.0021	0.8496	0.0345	0.0205	0.2509	0.1503	0.1924
coskew1	-0.0082	0.4139	0.0142	0.4063	-0.1713	0.0151	0.3319
coskew2	-0.0056	0.6153	0.0126	0.5240	-0.0717	0.0316	0.2825
size	0.0177	0.4961	0.0351	0.0285	-0.0029	0.3319	0.1451

positive and significant. Beta factor became significant when considered together with σ -garch(1,1), VAR95 t , VAR99 t , coskew1, coskew2 and Size.

In emerging markets, beta became positive and significant in most regressions, explaining around 20%, however, Down- β_w , coskew1 and coskew2

Table 16. Principal components

Risk factor	AII	Dev.	Emerg.
SR	2	1 or 2	3
Down- β_{iw}	2	1 or 2	1 or 3
Down- β_w	2	1 or 2	3
IR	1	1	1
TR	1	1	1
σ -garch(1,1)	1	1 or 2	1 or 2
VAR95 $_t$	1	1 or 2	1 or 2
VAR95 $_d$	1	1	1
VAR99 $_t$	1	1 or 2	1 or 2
VAR99 $_d$	1	1	1
ES95 $_t$	1	1	1
ES95 $_d$	1	1	1
Semi-mean	1	1	1
Semi-0	1	1	1
Semi-rf	1	1	1
kurt	1, 3 or 4	1 or 3	1, 2 or 4
skew	1, 3 or 4	2 or 3	1, 2 or 4
skew5%	2	2	2
coskew1	3 or 4	2 or 3	3 or 4
coskew2	1, 3 or 4	1, 2 or 3	1, 3 or 4
Size	1 or 2	2	2

offered better results and in these cases beta was not significant.⁹

As a last analysis, we addressed the several risk measures presented before and, using a principal components approach, tried to identify if they load in different factors or are offering the same information. We can observe from the Tables 8 and 9 that there is a high correlation among groups of risk factors, which would lead us to initially suppose the existence of two risk components, one related to the market risk and the other dealing with the distributional characteristics. An important result to highlight is the positive and significant correlation between the observed ES and the one using the model presented in this article, specially for developed countries (0.984), again supporting the use of the Pareto approximation to describe the tail distribution. When we run a principal components' analysis over the 21 risk factors we find the results presented in Table 16.

The principal components analysis offered four factors for the all countries sample; one related to beta, another related to a distributional characteristic and the remaining two linked to kurtosis and skewness measures. The results for developed and emerging markets are mixed reaching three and

four factors, respectively. Recall that distributional risk factors in general were not significant for the emerging market's sample.

Summing up, the empirical evidence suggests that returns extreme distributional characteristics have a risk-premium associated with them in developed markets. In the case of emerging markets, it is coskewness measures and, to a lower degree, downside betas which are correlated with observed returns.

V. Conclusions

In this article, we had four main goals: first discuss efficient procedures for estimating parameters of Pareto extreme value distribution and use these results to estimate ES; second, from a range of 21 risk measures we studied their statistical relationship with observed returns; third, we implemented a factor analysis to find different components, reaching a simplified but efficient empirical model; and fourth more recent price information was used with data until the end of 2005 and the results were compared with previous works (Estrada, 2000; Harvey, 2000). We used daily data from 41 emerging and developed countries, in the period from 1995 to 2005.

Surprisingly, over a 10 years period, observed average daily dollar return for emerging markets (0.015%) is lower than the one for developed countries (0.034%) whereas TR is higher in emerging (1.833%) than in developed (1.227%) countries. This fact challenges conventional wisdom. The findings support the use of a Pareto distribution to describe the tails and the log linear regression estimation of its parameters, as a better procedure if compared to the Hill's estimator (Maximum Likelihood Estimator). Related to the risk premium, we observed a significant relationship between observed returns and extreme distributional characteristics for developed countries, but not for emerging markets. CAPM seems to be supported when we consider the whole sample but fails when we segregate the markets. The results also support the use of coskewness as a risk measure in line of the Harvey and Siddique (2000) model, especially for emerging markets. The factor analysis for all countries indicated four components: one related to beta, another related

⁹ We also tried to improve the model introducing in the regression the size of each country's market. A three-factor model was fitted where two of them are beta and the natural logarithm of the market capitalization. The coefficient for size was negative in almost all regressions suggesting that small markets get a premium. However, due to colinearity problems results are of dubious statistical significance. Full results are available on request.

to distributional risk factors and the others mixed among the kurtosis and coskewness variables.

It is important to highlight that the explanation of the cross section of average returns remains a puzzle, as we were able to explain in the multivariate situation no more than 35% of the variation in returns. This level of explanation is usually what one gets when uses Capital Asset Pricing Model (CAPM) or Arbitrage Pricing Theory (APT) based models like us. So, an avenue of opportunities is open by extending these models to accept for instance behavioural aspects, relaxing the assumption about completely rational (risk-averse) market participants as suggested in Barberis and Thaler (2003).

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