Optimal-Probabilistic Method to Compute the Reach Settings of Distance Relays

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***Abstract*—This paper presents a novel method to optimize the settings of the resistive and reactive reaches of the zones of the distance relays. The method considers the probabilistic behavior of the variables that affect the apparent impedance seen by relays: prefault load flow, fault type, faulted line, distance up to the fault, fault resistance, and measurement errors. The optimization has been conceptually formulated as a multiobjective problem, with two objective functions: 1) minimize the probability of loss of sensitivity and 2) minimize the probability of loss of selectivity. To solve this problem, a preference function is defined, which is equal to the weighted sum of the two objective functions. A factor of weight represents the relative importance of selectivity regarding the sensitivity, and it is selected for each zone. This method was applied to a system with 7 busbars (3 of them with generation), where there are 18 distance relays with quadrilateral characteristic, and with an independent setting of the reactive and resistive reaches for the phase distance function and the ground distance function. The results obtained are compared with the results of other methods of adjustment.**

***Index Terms*—Distance relay setting.**

I. Introduction

THE methods to set the reach of distance relays can be classified in: 1) traditional [1]–[11]; 2) based on expert systems [12]–[15]; 3) adaptive [6]–[8], [15]–[19]; 4) based on optimization [7], [19]–[21]; and 5) probabilistic [22]. Traditional methods are based on simple rules, especially for the reactive reach on solid faults [1]–[5], [7]–[11]. Traditional methods seldom consider the independent setting of the resistive reach, and they usually do it with very simple rules, calculating a typical fault resistance and using it directly [1], [4], [9]. Expert systems have been used with predefined rules to coordinate the relay settings, applying an automated analysis of the relevant events of the electrical system. Adaptive methods assume that the relay settings will be adapted automatically, in real time, when there is an important change in the electrical system. Optimization techniques have been applied to maximize one of the desirable features of the protection (selectivity, sensitivity, and/or speed). The reviewed probabilistic method [22] keeps the probability of loss of one of the desirable features of the protection within limits.

This paper presents a novel method to optimally set the reach of distance relays, considering the probabilistic behavior of the variables that influence the impedance seen by the relay. The developed concept can be applied to all zones, but the specific problem solved here is for three zones (looking forward). A quadrilateral characteristic was used here, but this method can be adapted to any relay characteristic in the plane R-X.

It was assumed that the distance relays have algorithms to reduce transient measurement error, to determine the faulted phase, and to avoid undesirable operations by load conditions or power swings. Transmission lines with series capacitors, multiterminal lines, and mutual coupling between parallel lines were not considered. The transient change of the power system topology, which occurs when a breaker opens an end of the faulted line before the operation of the analyzed relay, was not taken into account. The existence of other relays, as a backup for the distance relays, is not considered, nor the possibility of changing the time delay settings of the zones to solve problems of selectivity. It was assumed that these and other aspects can be analyzed in the future. Despite this, the solved example in this paper enables showing the advantages of the developed method.

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II. Developed Method

A. Optimization Problem

The locus of the apparent impedance seen by distance relays depends on multiple factors and the relay characteristics are not adapted exactly to these loci. For example, the apparent impedance for faults out of the protected line could be inside the relay zone-1 (loss of selectivity) or the apparent impedance for faults within the protected line could be out of the relay zone-1 (loss of sensitivity).

The settings for the reactive and resistive reaches (Xr_1, Rr_1) of the quadrilateral characteristic for zone-1 might be carried out: 1) to minimize the loss of selectivity [Fig. 1(a)]; 2) to minimize the loss of sensitivity, for faults in a sector of the line [Fig. 1(b)]; and 3) with a compromise between selectivity and sensitivity [Fig. 1(c)]. Both selectivity and sensitivity are desirable features of the protection. Hence, the problem of finding the optimal settings is multiobjective: it is desirable to maximize the selectivity and to maximize the sensitivity of the protection, and the solution can imply a compromise between both objectives [Fig. 1(c)].

Let E be the space of all the faults of the power system (Fig. 2). There is a set G of faults that are relevant to determine the settings of each area of each relay. G is subdivided in F and D, the sets of the faults that are outside and inside the expected operation area of the relay, respectively. T is the subset of F for which the relay operates (faults with a loss of selectivity) and S is the subset of D for which the relay does not operate (loss of sensitivity). The problem has been formulated in terms of the

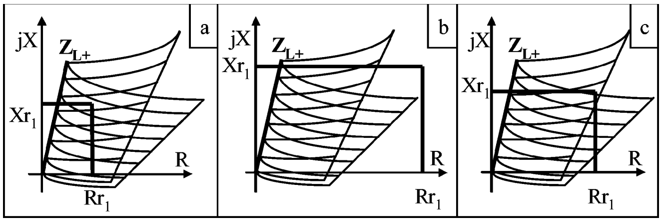

Fig. 1. Hypothetical options for setting the reaches (Xr_1, Rr_1).

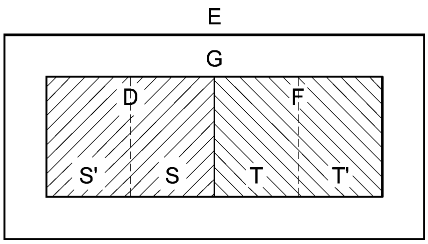

Fig. 2. Considered fault sets.

probability of loss of selectivity p(T) and of the probability of loss of sensitivity p(S). The total probability of incorrect operation p(I) is p(T) + p(S). The problem can be formulated to minimize p(I), but it might be preferable to use a different weight for p(T) and p(S). A weight (C, between 0 and 1) was defined for p(T), and a complementary weight for p(S). The function of preference (M) to be minimized is a weighted combination of p(T) and p(S)

$$\mathrm{M} = \mathrm{C} \cdot \mathrm{p(T)} + (1 - \mathrm{C}) \cdot \mathrm{p(S)}. \tag{1}$$

The optimization problem is to find the settings of the reaches that minimize the preference function M for each zone. The equality constraints are: the definition of p(T) and p(S) as a function of the decision variables (relay settings) and of the random variables that determine the impedance seen by the relay. These random variables are prefault load flow, fault type, faulted line, distance up to the fault, fault resistance, and measurement errors. Inequality constraints are used to limit the range of the results for the reaches of the three zones.

p(T) and p(S) were calculated considering the space E. $\mathrm{p(T\,|\,F)}$ and $\mathrm{p(S\,|\,D)}$ are the conditional probabilities of loss of selectivity and of sensitivity considering only the sets F and D, respectively, and they can be useful to analyze the results. $\mathrm{p(T\,|\,F)}$ and $\mathrm{p(S\,|\,D)}$ are computed in different bases; for this reason, they are not used for the preference function.

B. *Definition of p(T) and p(S)*

It is assumed that the probabilistic functions of the random variables are known. A probabilistic table for each variable was used. These tables are matrices with two columns: the discrete values of the variable and their probabilities of occurrence (the sum of these probabilities must be 1 [23]). For example, the probabilistic table for the fault resistance Rf has m discrete values $(\mathrm{Rf_1} \cdots \mathrm{Rf_m})$ and for each one, there is an occurrence probability $\mathrm{p(Rf_1)} \cdots \mathrm{p(Rf_m)}$

From the sequence networks, for each zone of each relay, the apparent impedances Zap are calculated for internal and external

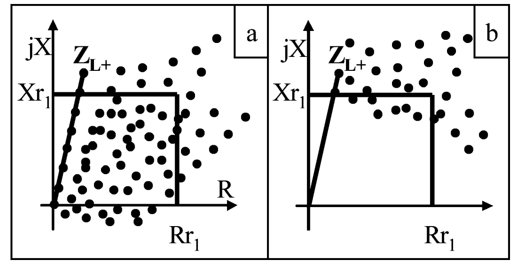

Fig. 3. Apparent impedances for faults (a) inside and (b) outside the line.

faults, using all combinations of the random variables (Fc, Tf, Lf, d, Rf, Rt, ξ). Since the random variables are independent, then the probability of occurrence of each computed apparent impedance p(Zap) is obtained by multiplying the probabilities of the considered random variables [23]. For example, for the fault g

$$\mathbf{Zap}_{\mathrm{g}} = f(\mathrm{Fc_h}, \mathrm{Tf_i}, \mathrm{Lf_j}, \mathrm{d_k}, \mathrm{Rf_m}, \mathrm{Rt_n}, \boldsymbol{\xi}_{\mathrm{p}}) \tag{2}$$

$$\mathrm{p}(\mathbf{Zap}_{\mathrm{g}}) = \mathrm{p(Fc_h)} \cdot \mathrm{p(Tf_i)} \cdot \mathrm{p(Lf_j)} \cdot \mathrm{p(d_k)} \cdot \mathrm{p(Rf_m)} \cdot \mathrm{p(Rt_n)} \cdot \mathrm{p}(\boldsymbol{\xi}_{\mathrm{p}}) \tag{3}$$

Fc	prefault load-flow case;
Lf	faulted line;
Rf, Rt	fault resistances;
Tf	fault type;
d	distance up to the fault;
$\boldsymbol{\xi}$	measurement error.

For a specific set of relay settings, p(T) is calculated by adding the probabilities p(Zap) of the external faults that are seen inside the relay characteristic, and p(S) is calculated adding the probabilities p(Zap) of the internal faults that are seen outside the relay characteristic. Fig. 3 illustrates the case of a quadrilateral zone-1 with a specific set of relay settings $(\mathrm{Xr_1}, \mathrm{Rr_1})$: $\mathrm{p(S)}$ is calculated by adding the probabilities of the apparent impedances that are out of zone-1 for faults in the line [Fig. 3(a)]; p(T) is calculated by adding the probabilities of the apparent impedances that are inside the zone-1 for faults outside the line [Fig. 3(b)].

The objective of zone-1 is to protect the line in study, without operating for faults at the remote busbar nor in adjacent lines. Therefore, $\mathrm{D_1}$ is the set of faults in the line in study. External faults are those which occur beyond the remote end and backward faults. Nevertheless, it was considered sufficient to evaluate a set of external faults $\mathrm{F_1}$, taking into account only the faults at the remote busbar and within a percentage of the beginning of the adjacent lines (for example, 20%).

The objective of zone-2 is to protect the region of the line that is not covered by zone-1. Zone-2 must not operate for faults that are outside zone-1 of the relays of the adjacent lines. To set zone-2, $\mathrm{D_2}$ only takes into account the faults in the line in study with apparent impedances outside of zone-1 of the same relay. $\mathrm{F_2}$ considers the faults in the adjacent lines at the remote end with apparent impedances outside zone-1 of the relays that protect these lines.

The objective of zone-3 is to protect the region of the adjacent lines that are not covered by zone-2 of the same relay. Zone-3 should not operate for faults that are outside zone-2 of the relays of the adjacent lines. To set zone-3, D_3 takes into account only the faults in the adjacent lines with apparent impedances outside zone-2 of the same relay. F_3 considers the faults in the adjacent lines to the adjacent lines with apparent impedances outside zone-2 of the relays that protect the adjacent lines.

C. *Optimization Method*

M was computed with all possible combinations of the relay settings in order to obtain its minimal value. The method is sequential: the settings of zone-1 of all relays are computed first, followed by the settings of zone-2 of all relays and, finally, the settings of zone-3 of all the relays are determined. The search of the optimal settings of the zones 2 and 3 of a relay needs the settings of the zones 1 and 2 of the adjacent relays, respectively. The algorithm is applied independently to the phase functions and ground functions.

III. System Used as an Example

A. *Power System*

The power system used as an example is described in a previous work [24]. The ground distance functions of all the relays are self polarized, as in the previously mentioned work. Fig. 4 indicates the nomenclature used for the relays (R11, R12...R91, R92). The first number identifies the line and the second one identifies the line terminal.

B. *Probabilistic Functions*

The probabilistic functions should represent the behavior of the random variables but their determination is out of the scope of this paper. As an example, a set of functions was assumed for this paper. For other systems, another realistic set of probabilistic functions could be assumed if there is not any better information: This is better than neglecting the effect of the randomness of these variables.

1) Prefault Load Flow: Twenty-four cases of prefault load flow (Fc) were used, combining three cases of load demand with eight cases of generated power in G1 and G2. (G7 is the slack bus).

The specified load values in the previous work [24] were assumed as the maximum demands. Three load cases were defined, assuming that the loads simultaneously change their demands without changing their power factor, and in each case, the demand of each busbar depends on a simple factor (Pd: the demand in per unit of the maximum demand value). The used values are $\mathrm{Pd} = [0.2\ 0.6\ 1]$, $\mathrm{p(Pd)} = [0.3\ 0.3\ 0.4]$.

The specified power values in the previous work for G1 and G2 [24] were assumed as the maximum values. The cases were defined with the factors $\mathrm{Pg_1}$ and $\mathrm{Pg_2}$ (generated power values in G1 and G2, in per unit of the maximum value). The values used for $\mathrm{Pg_1}$, $\mathrm{Pg_2}$ and their probabilities are

$$\begin{aligned}
\text{With Pd} = 0.2: \quad & \mathrm{Pg_1} = [0.1\ 0.2], \quad \mathrm{p(Pg_1)} = [0.3\ 0.7] \\
& \mathrm{Pg_2} = [0.1\ 0.2], \quad \mathrm{p(Pg_2)} = [0.3\ 0.7] \\
\text{With Pd} = 0.6: \quad & \mathrm{Pg_1} = [0.3\ 0.6], \quad \mathrm{p(Pg_1)} = [0.3\ 0.7] \\
& \mathrm{Pg_2} = [0.3\ 0.6], \quad \mathrm{p(Pg_2)} = [0.3\ 0.7] \\
\text{With Pd} = 1: \quad & \mathrm{Pg_1} = [0.5\ 1], \quad \mathrm{p(Pg_1)} = [0.3\ 0.7] \\
& \mathrm{Pg_2} = [0.5\ 1], \quad \mathrm{p(Pg_2)} = [0.3\ 0.7].
\end{aligned}$$

Two power factor values (fpg) were used for the generated power at G1 and G2, with the same values and the same probabilities: $\mathrm{fpg} = [0.85\ 0.96]$, $\mathrm{p(fpg)} = [0.5\ 0.5]$.

The probability of each case of prefault load flow $\mathrm{Fc_h}$ is $\mathrm{p(Fc_h)} = \mathrm{p(Pd_h)} \cdot \mathrm{p(Pg_{1,h})} \cdot \mathrm{p(Pg_{2,h})} \cdot \mathrm{p(fpg_h)}$.

For the purpose of this paper, the influence of the prefault load flow includes the changes in the network topology (e.g., the existence of out-of-service generators or out-of-service lines). However, these changes in the network topology were not numerically included in the example because their probability was assumed to be very low in comparison with the total annual time without these events.

2) Fault Type: It was assumed that: 1) the relays have an algorithm to detect the faulted phases and 2) ground distance function is only activated by single-phase faults ($\mathrm{Tf} = 1\phi$). To set this function, $\mathrm{p(Tf} = 1\phi) = 1$ was used. On the other hand, it was assumed that the phase distance function can detect faults between two or more phases ($\mathrm{Tf} = [2\phi\ 2\phi\mathrm{t}\ 3\phi]$), and the values $\mathrm{p(Tf)} = [0.6\ 0.2\ 0.2]$ were used.

3) Faulted Line: It was considered that the probability of faults in the line Lf depends on its length, and a specific consideration was included for faults at the remote busbar, as shown in (4) at the bottom of the next page.

p(fl)	probability of faults in lines $\mathrm{p(fl)} = 0.9$ was used;
p(fb)	probability of faults in busbar $\mathrm{p(fb)} = 0.1$ was used;
nl	number of lines of the system $(\mathrm{nl} = 9)$;
nb	number of busbars of the system $(\mathrm{nb} = 7)$;
length(j)	length of the transmission line j.

4) Distance up to the Fault: It was assumed that the fault in a line has equal occurrence probability in any part of its length.

$$\mathrm{p(Lf)} = \begin{cases} \mathrm{p(fl)} \cdot \mathrm{length(Lf)} / \sum_{\mathrm{j=i}}^{\mathrm{nl}} \mathrm{length(j)}, & \text{if the fault is in the line;} \\ \frac{\mathrm{p(fb)}}{\mathrm{nb}}, & \text{if the fault is at the remote bar.} \end{cases} \tag{4}$$

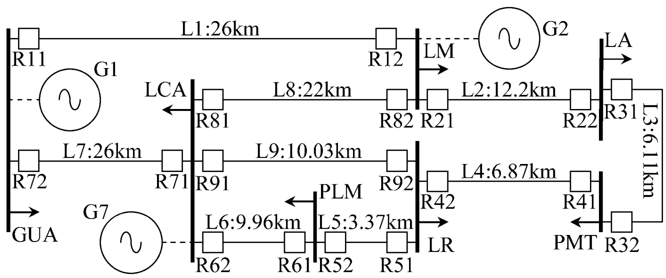

Fig. 4. Location of the relays in the system used as an example.

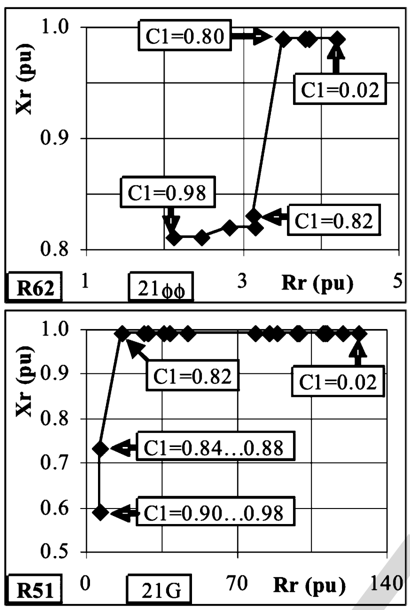

Fig. 5. Optimal settings for zone-1 of two relays (examples), varying C1. $21\phi\phi$: Phase; 21G: Ground.

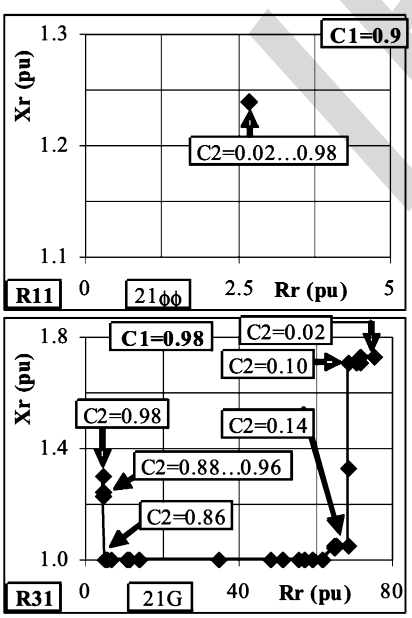

Fig. 6. Optimal settings for zone-2 of two relays (examples), varying C2. $21\phi\phi$: Phase; 21G: Ground.

The fault distance d (in per unit of the line length) has nine discrete values, with steps of 0.1 ($d = 0.1 \ldots 0.9$), each one with a fault probability of 1/9.

5) Fault Resistance: The following probabilistic distribution was assumed for the fault resistance between phases (Rf) and for the ground fault resistance (Rt):

$$\begin{aligned} \mathrm{Rf} &= [0\ 0.5\ 1\ 1.5\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10], \text{in ohms.} \\ \mathrm{p(Rf)} &= [0.0501\ 0.1460\ 0.1544\ 0.1460\ 0.1252\ 0.0834 \\ &\quad 0.0626\ 0.0501\ 0.0417\ 0.0375\ 0.0355 \\ &\quad 0.0342\ 0.0334] \\ \mathrm{Rt} &= [0\ 1.5\ 3\ 5\ 7\ 10\ 15\ 20\ 30\ 40 \\ &\quad 50\ 60\ 70\ 80\ 90\ 100], \text{in ohms.} \\ \mathrm{p(Rt)} &= [0.0407\ 0.0848\ 0.1188\ 0.1256\ 0.1188\ 0.1018 \\ &\quad 0.0848\ 0.0679\ 0.0509\ 0.0407\ 0.0346\ 0.0305 \\ &\quad 0.0275\ 0.0254\ 0.0241\ 0.0231]. \end{aligned}$$

6) Measurement Error: The measurement error (ξ) of the impedance is considered to be a complex number. The impedance calculated without the measurement error **Zapse** is used to calculate the possible apparent impedances seen by the relay $(\mathbf{Zap})$: $\mathbf{Zap} = \mathbf{Zapse} + \xi \bullet \mathbf{Zapse}$. The used values are

$$\begin{aligned} \boldsymbol{\xi} &= [0\ 0.1|\underline{-90^\circ}\ 0.1|\underline{0^\circ}\ 0.1|\underline{90^\circ}\ 0.1|\underline{180^\circ}] \\ p(\boldsymbol{\xi}) &= [0.5\ 0.125\ 0.125\ 0.125\ 0.125]. \end{aligned}$$

IV. Results

A. Sample of Results With the Proposed Method

The result of the optimal setting depends on the selected weight factor (C). This factor can be different for each zone of each relay and it can be different for the phase and ground distance functions $(21\phi\phi, 21\mathrm{G})$. Figs. 5–7 illustrate the variation of the optimal settings with regard to the selected value of C. C was changed in steps of 0.02 and the extreme values (0 and 1) were not included.

Fig. 5 shows two examples of the optimal setting of zone-1. In these examples, for low values of C (C1), the reactive optimal setting (Xr) tends to be equal to the imposed upper limit (0.99 p.u.; the base value for Rr and Xr is the reactance of the protected line). Increasing C1, the optimal value of Xr is lower. The optimal values of the resistive setting (Rr) tend to be greater for 21G since there is a greater probability of high values of fault resistance. Figs. 8 and 9 show the values of apparent impedance, optimal settings, and p(Zap) for the two examples of Fig. 5 (with an example of C1 for each case).

The example of $21\phi\phi$ in Fig. 6 only has an optimal solution, for any weight factor of zone-2 (C2). In this case, there is a separation between the regions of impedances seen by this relay for the faults in D and F, as is shown in Fig. 10. In these cases, there is a range of values for the possible optimal solutions. The lowest possible value of the settings (Rr and Xr) was selected in this paper, but the biggest possible value or an intermediate solution between these options might be selected.

The example of 21G in Fig. 6 shows how an increment of C2, from 0.02 up to 0.86, reduces the values of the optimal settings, but at the following step $(\mathrm{C2} = 0.88)$, there is an abrupt increase

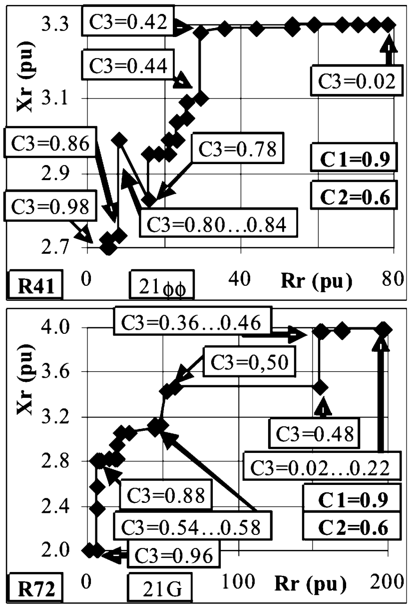

Fig. 7. Optimal settings for zone-3 of two relays (examples), varying C3. $21\phi\phi$: Phase; 21G: Ground.

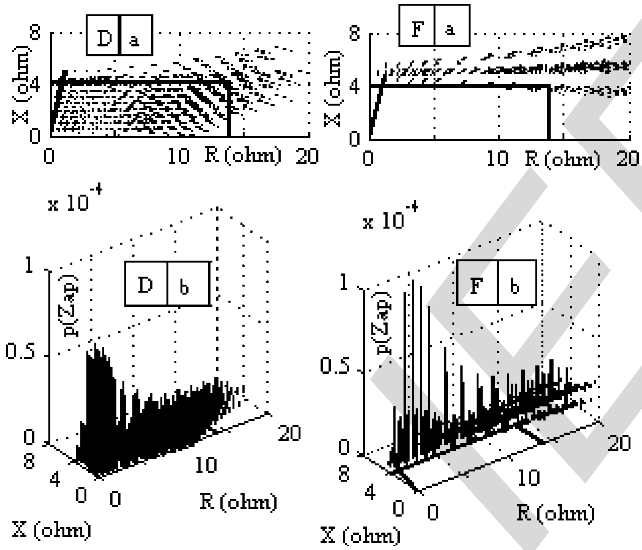

Fig. 8. Impedances seen by zone-1 of the phase function $(21\phi\phi)$ of R62, for faults inside (D) and outside (F) the line. (a) Optimal setting with $\mathrm{C}1 = 0.9$; (b) p(Zap).

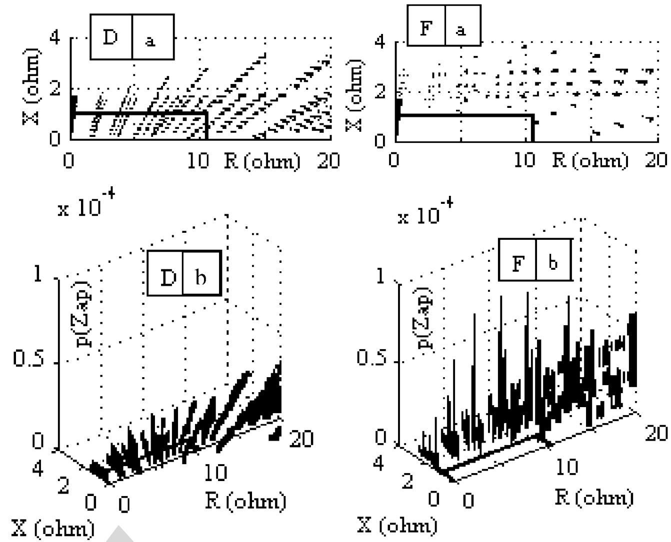

Fig. 9. Impedances seen by zone-1 of the ground function (21G) of R51, for faults inside (D) and outside (F) the line. (a) Optimal setting with $\mathrm{C}1 = 0.98$. (b) p(Zap).

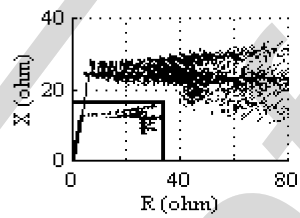

Fig. 10. Result for zone-2 of R11 $(21\phi\phi)$.

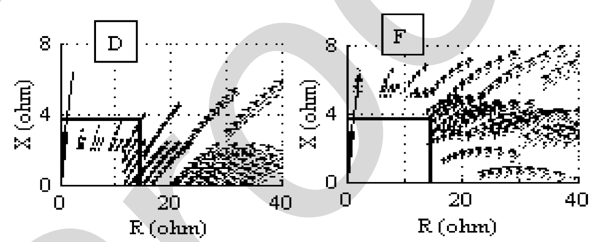

Fig. 11. Faults inside (D) and outside (F) the expected zone-2 for R31 (21G). Optimal setting with $\mathrm{C}2 = 0.88$.

of the optimal value of Xr. For $\mathrm{C}2 > 0.86$, the decrease of the optimal value of Rr enables the optimal value of Xr to enter the region of apparent impedances cleared by zone-1 of the relay of the adjacent line at the remote terminal, as is shown in Fig. 11. These abrupt jumps of the optimal solutions highlight the presence of discontinuities (the problem is highly nonlinear), which justifies the use of searching the optimal solutions by exploring the whole space of variables of decision (Rr and Xr).

Both examples in Fig. 7 show how an increment of the weight factor of zone-3 (C3) usually reduces the values of optimal settings. There can be exceptions, as in the example of $21\phi\phi$ with C3 from 0.78 to 0.8, or there can be sudden changes as at 21G with C3 from 0.48 to 0.5.

The shape of the graphs is different for each relay and Figs. 5–7 are only some examples. Fig. 12 shows the variation of the optimal value of the objective function (M) and the p(T) value in function of C1 for the $21\phi\phi$ function of R62 (it is a case of Fig. 5). For $\mathrm{C}1 > 0.8$, there are abrupt changes of the p(T) value and of the optimal value of the settings (Fig. 5), but there is not any abrupt change in the optimal value of the objective function (M). This is due to the fact that the problem is highly nonlinear.

Tables I and II show the obtained optimal settings for each relay, with a specific combination of C values. The values of C for 21G were chosen in order to have similar values of $\mathrm{p(T\,|\,F)}$ in comparison with the other two methods (Section IV-B). The values of C for $21\phi\phi$ were chosen in order to have similarity (and not equality) with those of 21G. The selection of C values

TABLE I
EXAMPLE OF RESULTS FOR THE $21\phi\phi$ FUNCTION WITH THE DEVELOPED METHOD ($C1 = 0.9, C2 = 0.6, C3 = 0.9$)

	Xr [pu]	Rr [pu]	M [%]	p(S) [%]	p(T) [%]	$p(S\mid D)$ [%]	$p(T\mid F)$ [%]	Xr [pu]	Rr [pu]	M [%]	p(S) [%]	p(T) [%]	$p(S\mid D)$ [%]	$p(T\mid F)$ [%]
Line	**Line end 1, Zone 1**							**Line end 2, Zone 1**						
L1	0.93	1.95	0.21	0.54	0.17	2.83	2.64	0.82	2.79	0.26	2.15	0.05	11.24	0.86
L2	0.82	2.99	0.16	1.24	0.04	13.86	1.82	0.82	3.61	0.23	1.58	0.08	17.66	0.98
L3	0.71	8.16	0.19	1.13	0.08	25.22	3.35	0.81	5.31	0.14	0.76	0.07	17.04	2.27
L4	0.65	5.40	0.27	2.08	0.06	41.18	1.90	0.81	4.13	0.13	0.85	0.05	16.88	2.05
L5	0.69	3.90	0.12	1.20	0.00	48.38	0.00	0.66	5.18	0.13	1.27	0.00	51.33	0.00
L6	0.99	4.80	0.20	1.72	0.03	23.47	0.34	0.82	2.83	0.13	1.07	0.03	14.57	1.41
L7	0.91	2.00	0.21	0.57	0.17	3.00	3.25	0.89	2.27	0.29	1.56	0.15	8.17	1.97
L8	0.95	2.18	0.19	0.36	0.17	2.24	2.46	0.82	4.10	0.29	2.02	0.10	12.53	1.22
L9	0.82	2.99	0.13	1.11	0.02	15.04	0.76	0.99	3.42	0.25	2.18	0.04	29.61	0.39
Line	**Line end 1, Zone 2**							**Line end 2, Zone 2**						
L1	1.24	2.67	0.00	0.00	0.00	0.00	0.00	1.20	4.26	0.00	0.00	0.00	0.00	0.12
L2	1.25	4.06	0.00	0.01	0.00	0.69	0.00	1.28	7.74	0.01	0.00	0.02	0.00	0.37
L3	1.57	6.12	0.09	0.24	0.00	20.84	0.03	1.64	7.13	0.00	0.00	0.00	0.08	0.00
L4	1.07	12.89	0.20	0.20	0.20	9.45	6.01	1.51	4.83	0.00	0.00	0.00	0.42	0.00
L5	2.03	13.40	0.00	0.00	0.00	0.09	0.00	2.92	14.45	0.01	0.03	0.00	2.45	0.06
L6	1.76	56.50	0.00	0.00	0.00	0.00	0.05	1.04	3.80	0.06	0.07	0.05	6.25	4.24
L7	1.19	2.92	0.00	0.00	0.00	0.00	0.00	1.40	4.76	0.00	0.00	0.00	0.00	0.00
L8	1.26	2.65	0.00	0.00	0.00	0.00	0.00	1.52	9.96	0.02	0.00	0.02	0.03	0.90
L9	1.08	4.05	0.06	0.06	0.05	5.33	2.57	1.57	49.46	0.00	0.00	0.00	0.00	0.00
Line	**Line end 1, Zone 3**							**Line end 2, Zone 3**						
L1	1.86	3.63	0.93	4.79	0.50	22.30	2.19	2.39	200.04	0.90	8.49	0.05	61.02	0.19
L2	1.67	3.26	0.04	0.40	0.00	18.51	0.02	3.75	30.94	1.75	16.53	0.11	50.78	0.31
L3	2.12	13.04	0.04	0.36	0.00	13.28	0.04	3.35	13.90	0.00	0.02	0.00	0.34	0.00
L4	2.70	5.94	0.54	5.06	0.04	55.43	0.35	2.31	6.24	0.00	0.01	0.00	0.65	0.00
L5	6.19	136.18	0.00	0.01	0.00	0.20	0.00	5.66	38.20	0.31	2.09	0.11	25.25	1.46
L6	4.10	200.08	0.53	4.47	0.09	90.23	0.18	1.83	4.76	0.01	0.05	0.00	2.42	0.00
L7	2.23	8.47	1.35	11.99	0.16	64.28	0.76	2.06	5.25	2.20	20.00	0.22	71.29	0.65
L8	2.41	5.42	0.69	6.30	0.06	25.64	0.39	2.40	81.96	1.79	17.02	0.10	82.59	0.34
L9	1.99	5.47	0.08	0.73	0.01	10.21	0.09	4.07	200.04	0.53	4.78	0.06	87.73	0.13

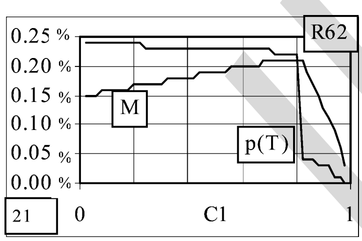

Fig. 12. Variation of the optimal value of M and p(T) as a function of C1 for R62 ($21\phi\phi$).

is outside the scope of this paper, since this should be performed by the person who does the coordination of the protections.

B. Comparison With Other Methods

Results for the 21G function are compared with those of two previous works [24], [25]. The methods are named: 1) method 1, the present work; 2) method 2 [24]; 3) method 3 [25]. In method 2, Xr is set with traditional rules, and Rr is set by analyzing the impedance seen by each relay. In method 3, Xr is set with traditional rules, different from method 2, and Rr is set in a simplistic way, multiplying the reactance value by $2 (\mathrm{Rr} = 2\,\mathrm{Xr})$. Method 2 considers the existence of out-of-service lines, and the existence of a very sensitive zone-4 for all of the relays. Method 3 uses only the base case of load flow, and it has a criterion for Xr of zone-2 that is more sensitive than the one used in method 2. Only a selected group of relays (R11, R22, R41, R52, R81, and R91) has zone-4 in method 3. The three methods have the same base case of load flow but there are slight differences in the considered premises.

A meticulous comparison of the results would need tables similar to Table II for methods 2 and 3. This will not be done here due to space limitations. Table III presents a synthesis of the results obtained with the three methods, with $\mathrm{C1} = 0.98$, $\mathrm{C2} = 0.68$, and $\mathrm{C3} = 0.94$. Methods 2 and 3 do not use these factors, but they are necessary to compute the results of M for each method. p(S) and p(T) were calculated with the premises of method 1. The values of C were selected in order to have a similar average of $\mathrm{p(T\,|\,F)}$ with the three methods.

For zone-1, method 2 has an average value of $\mathrm{p(T\,|\,F)}$ greater than the others. This occurs because the apparent impedance for faults outside the line tends to be inside zone-1 of the relays R41, R51, R22, and R82 for the load-flow cases in this paper. These load-flow cases were not considered when the settings of method 2 were computed.

The value of M is always lower with method 1 since it is the objective function. Consequently, method 1 produces less loss of sensitivity $\mathrm{p(S\,|\,D)}$ than the others and, by this, less probability of incorrect operation p(I). The right understanding of p(I) requires remembering its definition: for example, in zone-1, this

TABLE II
EXAMPLE OF RESULTS FOR THE 21G FUNCTION WITH THE DEVELOPED METHOD ($C1 = 0.98, C2 = 0.68, C3 = 0.94$)

	Xr [pu]	Rr [pu]	M [%]	p(S) [%]	p(T) [%]	p(S|D) [%]	p(T|F) [%]	Xr [pu]	Rr [pu]	M [%]	p(S) [%]	p(T) [%]	p(S|D) [%]	p(T|F) [%]
Line	**Line end 1, Zone 1**							**Line end 2, Zone 1**						
L1	0.74	5.18	0.15	6.58	0.02	34.48	0.37	0.71	2.13	0.19	9.32	0.00	48.80	0.06
L2	0.67	3.72	0.09	4.60	0.00	51.31	0.00	0.69	2.73	0.12	5.50	0.01	61.37	0.08
L3	0.67	3.79	0.06	3.17	0.00	70.60	0.00	0.56	4.65	0.06	2.90	0.00	64.58	0.00
L4	0.66	4.06	0.08	3.92	0.01	77.63	0.16	0.70	3.25	0.06	2.78	0.00	55.02	0.01
L5	0.59	6.37	0.04	1.80	0.00	72.92	0.00	0.68	5.95	0.04	1.86	0.00	75.27	0.00
L6	0.89	13.31	0.10	2.98	0.04	40.67	0.38	0.74	4.11	0.07	3.35	0.00	45.85	0.00
L7	0.71	2.17	0.18	8.38	0.01	43.88	0.20	0.79	2.53	0.19	8.33	0.02	43.62	0.33
L8	0.82	1.97	0.13	6.32	0.00	39.09	0.04	0.85	3.53	0.16	6.66	0.03	41.25	0.35
L9	0.75	4.35	0.07	3.34	0.00	45.28	0.00	0.89	6.99	0.10	3.69	0.03	50.11	0.29
Line	**Line end 1, Zone 2**							**Line end 2, Zone 2**						
L1	1.24	8.31	0.56	1.23	0.24	18.71	2.16	1.31	18.21	0.41	0.58	0.34	6.23	4.05
L2	1.01	18.08	0.47	0.88	0.28	19.11	8.96	1.25	16.69	0.57	1.42	0.17	25.89	1.09
L3	1.00	11.30	0.65	1.47	0.27	46.51	6.90	1.00	8.95	0.46	1.28	0.07	44.12	1.32
L4	1.28	14.34	0.62	1.81	0.07	46.25	1.30	1.00	25.85	0.34	0.62	0.21	22.26	7.20
L5	3.34	33.23	0.14	0.45	0.00	25.06	0.01	1.95	22.62	0.24	0.69	0.03	36.99	0.45
L6	2.84	200.08	0.03	0.04	0.02	1.36	0.13	1.11	18.93	0.45	1.06	0.15	31.67	8.31
L7	1.73	16.69	0.17	0.19	0.16	2.27	2.42	1.67	11.27	0.55	1.33	0.18	15.96	1.41
L8	1.18	9.97	0.41	0.86	0.20	13.57	1.47	1.83	45.66	0.34	0.33	0.35	4.98	2.31
L9	1.18	15.93	0.47	1.03	0.21	30.97	4.54	2.63	200.01	0.02	0.03	0.01	0.79	0.08
Line	**Line end 1, Zone 3**							**Line end 2, Zone 3**						
L1	1.97	4.24	0.90	11.61	0.22	51.72	1.28	2.66	200.03	0.65	5.33	0.35	46.71	1.34
L2	1.49	5.51	0.17	1.44	0.09	35.08	1.95	4.31	78.01	1.07	10.46	0.47	35.03	1.92
L3	2.14	13.91	0.12	2.00	0.00	42.66	0.00	3.19	26.96	0.10	1.62	0.00	18.47	0.00
L4	2.56	20.22	0.39	5.23	0.08	62.84	1.66	1.88	7.61	0.15	1.22	0.08	29.18	0.88
L5	9.38	200.26	0.02	0.27	0.00	9.36	0.00	6.08	59.89	0.29	3.75	0.07	34.32	1.13
L6	5.51	200.01	0.25	1.65	0.16	97.35	0.34	1.53	7.04	0.05	0.75	0.00	37.14	0.00
L7	2.23	37.69	1.61	10.30	1.06	73.33	4.71	2.37	6.52	2.66	22.76	1.38	87.37	3.89
L8	2.36	21.12	0.65	8.62	0.14	32.18	0.90	3.03	200.13	0.88	9.77	0.31	75.59	1.39
L9	2.11	11.08	0.12	1.99	0.00	30.19	0.05	5.47	200.20	0.22	1.96	0.11	96.91	0.25

TABLE III
AVERAGE OF THE RESULTS FOR THE 21G FUNCTION WITH THE THREE METHODS ($C1 = 0.98, C2 = 0.68, C3 = 0.94$)

Method	Zone	Xr (pu)	Rr (pu)	p(S)	p(T)	p(I)	M	p(S|D)	p(T|F)
1	1	0.73	4.49	4.75%	0.01%	4.76%	0.11%	53.43%	0.13%
	2	1.59	38.67	0.85%	0.16%	1.01%	0.38%	21.82%	3.01%
	3	3.35	72.25	5.60%	0.25%	5.85%	0.57%	49.75%	1.21%
2	1	0.80	10.29	5.32%	0.19%	5.51%	0.29%	54.96%	3.68%
	2	1.17	8.26	2.72%	0.09%	2.82%	0.93%	56.76%	2.25%
	3	2.20	16.58	8.75%	0.25%	9.00%	0.76%	78.15%	0.88%
3	1	0.80	1.60	5.99%	0.01%	6.00%	0.13%	68.72%	0.10%
	2	1.50	3.00	4.00%	0.17%	4.17%	1.40%	68.95%	3.87%
	3	2.42	4.84	10.22%	0.23%	10.45%	0.83%	82.95%	1.33%

definition implies considering any loss of sensitivity onto faults in the line as an incorrect operation, despite the existence of other zones.

The analysis of conditional probabilities is interesting. For example, in the case of zone-1, the average value of $p(T \mid F)$ is close to 0.1% for methods 1 and 3: this implies that zone-1 might trip in 1 of 1000 external faults. Another interesting result is the average value of $p(S \mid D)$ for zone-2, since it indicates the probability of line faults that are not seen by zone-2: the best result is close to 22% (method 1). This result demonstrates the importance of using methods as the developed one in the present work, in order to provide more sensitivity to the function 21G for resistive faults. The average value of $p(S \mid D)$ for zone-3 indicates the probability of nonoperation for faults in adjacent lines: the best result is close to 50% (method 1), which highlights the importance of having an additional zone-4 (more sensitive).

The average value of $p(T \mid F)$ of zone-2 is similar in the three methods, but it might be lower in method 1 if a specific value of C2 is selected for each relay. Table II shows that it would be sufficient to do this only for 5 relays since they have a value of $p(T \mid F)$, superior to the average (R21, R31, R91, R12, R42).

Although the average value of $p(T \mid F)$ for zone-3 is similar to the three methods, the average values of the settings are very different. This occurs because the loss of selectivity of zones

2 and 3 is associated with the settings of the respective zones 1 and 2 of the relays of the adjacent lines at the remote end, for each relay in study. For example, although the settings of method 3 are much lower than the settings of method 1, both have a similar average probability of loss of selectivity because this depends on the settings of the respective zones-2 (which are different). This implies that the main difference between the results with the three methods is sensitivity.

Finally, it is necessary to emphasize that the selection of weight factors greater than 0.5 for all of the zones indicates greater relative importance of selectivity compared to sensitivity. This occurs because the loss of sensitivity is usually corrected with a slower trip of other zones; however, the effect of the loss of selectivity cannot be corrected.

V. Conclusion

A novel method was developed to optimally set the reactive and resistive reaches for the zones of distance relays, considering the probabilistic behavior of the random variables that affect the apparent impedance seen by the relays.

The developed method was applied to a system with 18 relays with quadrilateral characteristics and with independent settings for the reactive and resistive reach of the phase and ground distance functions.

The results for the ground distance function were compared with the results obtained using two other setting methods. This enables showing the advantages of the developed method and to have numeric values for the relative importance that are usually given to the selectivity compared to the sensitivity.

This work can be complemented in the future in diverse ways. On the one hand, different details associated with the protection of lines with distance relays and/or with other protection functions might be included. On the other hand, the effect of having other probabilistic functions and/or of using thinner steps for the discrete variables might be studied. Also, the case of setting only a group of the relays of the system with the developed method might be studied, assuming that the remaining settings will not be changed, because this is a professional practice in some cases.

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