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EQUITY AND ECONOMIC THEORY: REFLECTIONS ON METHODOLOGY AND SCOPE

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Abstract

This paper provides an introduction to the recent literature on ordinal distributive justice. Its objetive is to explain the *process* of the mathematical analysis of fairness and to consider its *potential* for solving real allocative problems by means of several illustrative examples.

Keywords: Fairness, equity, distributive justice.

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1. Introduction

The basic question I wish to address is: how can modern economic theory help us understand the issue of fairness? In the course of the discussion, I hope to accomplish two things: first, to explain the *process* of the mathematical analysis of fairness — how this research is conducted and why — and, second, to evaluate the *potential* of this methodology for solving real allocative problems.

I will begin by considering the separate roles played by theoretical and applied research. And in doing so, I will include various reflections on methodology, modelling strategies, objectives, etc. that I hope will provide some perspective on what may seem like an arcane exercise.

But the focus of the paper will be on the second objective, on recent techniques that have been developed expressly for the purpose of providing practicable tools. Among the concepts to be discussed, the common feature is that they do not require information other than that which is readily available or easy to obtain, and thus, in principle, they are capable of being used in practice. In particular, they expressly avoid interpersonal comparisons of utility or psychological well-being. That is, they avoid such statements as: Person A should receive more than B because A likes it better.[1]

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[1]Of course, I do not wish to suggest that interpersonal comparisons of all sorts are untenable, but only those of utility. Nor do I wish to suggest that we can or should avoid utility comparisons altogether. There are some circumstances under which such comparisons are feasible (for example, if A wants more and B does not), and there are other circumstances under which they are inevitable (in sharing dessert, spouses will continue to consider intensity of preference in spite of the fact that this might not stand the rigors of logic). But for the standard case in which neither A nor B is satisfied with their portion, the statement "A likes it better" is logically insupportable since there is no common measure of wants.

Note, however, that the issue of interpersonal comparisons of well-being is by no means resolved. For recent discussions, see Elster and Roemer (1991) and D'Aspremont (1994).

I refer to the concepts presented here as *ordinal theories of distributive justice*. And, while ultimately some interpersonal utility comparisons may prove necessary, the immediate question concerns the extent to which allocative problems can be resolved fairly without relying on such comparisons, thereby removing one source of subjectivity.

In spite of the fact that equity has been one of the great overarching themes in the formal discipline of Economics since its inception in the late 18th century, the ordinal approach has emerged only within the last 25 years. It is thus relatively new. But, as I hope to show, it is a promising development. And I anticipate that as we become more aware of the limits to growth as a panacea, our attention will turn more to the question of how to share our resources.

The plan for the paper is the following. First, I will discuss methodological issues in the analysis of fairness. I will then describe various notions of fairness that have been proposed in the literature. But rather than include an exhaustive survey, I will simply provide examples of the main paradigms.[2] Each of these, I believe, represents a significant departure from conventional views both within the profession and outside. I will begin by describing notions of equitable *outcomes* or *end-states*. However, while economists have focussed primarily on this domain, the public has been largely concerned that the *procedures* used to allocate resources should be fair and that people should have *equal opportunities*. Subsequently, therefore, I will discuss some theoretical contributions on procedural fairness. Formal analysis of opportunity inequality has begun only recently, and I will describe preliminary work in this area as well.

[2]See Arnsperger (1994), Thomson (1990b), and Thomson and Varian (1985) for complete surveys.

I will then consider several areas in which the above ideas might be fruitfully applied. These include the division problem with single-peaked preferences, cost sharing, common ownership, and privatization. I will conclude by highlighting some of the important issues that have arisen in the course of this research and by suggesting an agenda for the future.

2. Theory and Application in the Study of Fairness

I believe the ultimate goal of most practitioners in this area is to identify equitable solutions to real allocative problems. The purpose of theoretical research is to try to make this statement precise: what is an equitable solution? Thus, theory and application are related endeavors. However, neither can be seen as the sole legitimate research focus. There are several clear reasons for this.

In one direction, we may come to realize through the theoretical analysis of fairness that an essential ingredient is immeasurable and thus impracticable. For example, it may be impossible to fully disentangle the social contribution to the development of talents (in the form of education, liberty to pursue one's interests, etc.) and yet this may be necessary in order to identify just rewards. In economic terms, we may wish to recoup the returns to public investment in training, allowing each individual to retain only that portion of their reward that accrues to their individual contribution, and yet we may only be able to observe an amalgam of effort, innate talent, and training. It is precisely by means of theoretical research that we come to identify such intricate webs. This legitimizes the pursuit of understanding independent of applicability.

Also, a particular application requires that we specify the details of the problem at hand. In contrast, theoretical analyses tend to focus on general principles and thus may be applicable to entire classes of problems. Thus, an additional benefit of theoretical research is that it may facilitate applications.

Finally, experimenting on a grand scale with various social institutions may be prohibitively costly, and theoretical research affords a way to compare alternative institutional structures without incurring such costs.

In the other direction, there are many real allocative problems that must be resolved equitably and cannot wait for the development of a solid theoretical foundation. For example, as a humane society, we must maintain a social safety net in the form of welfare programs in spite of the fact that we do not yet fully understand all of the consequences of doing so (in terms of incentives, administrative costs, etc.) — perhaps we can identify the *direction toward* a just society even if we cannot see the final result.

And so theoretical and applied research are both necessary; they play independent but complementary roles. The next question is how does one do theoretical research. The canonical procedure is the following. First, the physical and/or social environment usually provides the source for analytical problems.[3] Theoretical work begins by trying to isolate the essential ingredients in determining actual phenomena. This is done by constructing a mathematical model and attempting to replicate the phenomena. Indeed, the first important test of a model is the "fit" between its observable implications and actual data. Then, given

[3]Of course, along the way, numerous questions arise concerning the analytical techniques themselves. And the development of analytical/mathematical tools has generated entire fields within Economics.

a sufficiently well-fitting model, the final stage of the analytical process is to investigate its unforeseen implications. In other words, assuming the model adequately describes the actual process that generated the data, what can we forecast or project beyond that which is known?

This is typically the manner in which *positive*, or descriptive, economic theory is conducted. However, most theoretical research on fairness is not of a positive nature at all, but is *normative*, or prescriptive. Here, the objective is not to infer the underlying economic structure from observed data, but rather to consider and contrast alternative solutions to economic problems. In terms of the aforementioned schema, the process is much the same, with one essential difference. Rather than evaluate the model by considering its fit with actual data, we now appeal to our own introspection or intuition for validation. For instance, there may be certain "test cases," particular examples in which we have very strong preconceptions of what is fair, and we can apply the model to these to ensure that it generates solutions consistent with our basic intuition. Having passed such tests, we can then apply the model to more difficult problems where our intuition is a less reliable guide and see what the model recommends.

Consider the following example. Suppose there are two small children and two toys, and, as is often the case, each child wants to play with both toys. This is an example of an exchange problem in which given quantities of two "commodities" are to be allocated between two individuals. In this case, however, it is not the toys *per se* that are to be allocated but the amount of time each child gets to play with them. And a solution is a sharing rule which specifies such an allocation. We then ask: what is a fair solution? If we assume that both children have equal claims to the toys, then clearly one equitable solution would be to let each child play with each toy for half the time, that is, to divide the resources equally.

One test case for any concept of fairness is that in which the individuals are identical in every respect. Intuition suggests that in such cases agents should be treated equally. And, indeed, "fairness as equal division" behaves accordingly. However, if the agents have different tastes, then it is no longer obvious that *only* equal division is fair. Returning to the example, even though both children want to play with both toys, that does not mean they have the same preferences nor that they like both toys equally. Suppose, for instance, that one child has a taste bias for the first toy and the other child has a taste bias for the second toy.[4] Then both children could be better-off if the toys were shared unequally. Thus, equal division seems too restrictive a definition in even the simplest case involving heterogeneous agents. In the subsequent discussion, I will describe alternative notions of fairness that take taste differences into account.

2.1. The Definitional vs. Axiomatic Approach

The previous example demonstrates what I would call the *definitional approach* to fairness. Here, one directly proposes a definition of fairness (such as equal division) and proceeds to evaluate how well it works by applying it to various cases. But as we saw in the example, the same notion of fairness might work well for some problems (i.e., provide reasonable answers) and not for others. This suggests that we should not only distinguish between different concepts of fairness, but also between different classes, or *domains*, of

[4]Note that admitting taste differences and directly comparing utilities are quite distinct. It is possible to include the former in such a way as to avoid the latter.

problems.

Formally, let $\mathscr{E}$ represent the set of all allocation problems. One can think of a notion of fairness as a rule F defined on a class or subset of problems $\mathscr{E}^o$ that associates with each problem in the class, one or more fair solutions. And one must define both an appropriate rule F and an appropriate domain $\mathscr{E}^o$, and these are inextricably linked.

Some examples of different classes of problems are the following: (1) 2-agent problems, such as in the previous example, versus n-agent problems, which involve any number of individuals,[5] (2) exchange problems in which the aggregate quantities of resources are fixed versus production problems in which the aggregate quantities can vary; and (3) problems in which agents have the same tastes and/or talents versus those in which agents are heterogeneous. Ideally, the goal is to formulate a "good" F for as large a class $\mathscr{E}^o$ as possible. However, I believe it is now generally accepted that no single formulation is appropriate for all problems.[6]

The definitional approach is complicated by the fact that distinguishing fair from unfair solutions is only the first and simplest step in a hierarchy of analytical objectives. The goal of most theoretical research on fairness has been to formulate an appealing and, as mentioned, practicable definition. But this does not tell us which of two unfair solutions is <u>more</u> fair. That would require an *ordinal ranking* of solutions. And in light of the fact that full equality may be unattainable, such comparisons are necessary. Finally, we might wish to know whether a social state is very inequitable or only slightly unfair. For this, we would

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[5]I discuss the value of concentrating on the 2-agent case below.

[6]See Baumol (1986, p.5), Binmore (1994, p.11), Thomson and Varian (1985, p.125), and Young (1994, p.162), to name only a few.

need a *cardinal measure* which would assign a specific value to the degree of fairness of each social state.

This hierarchy is not simply semantic. As Thomson and Varian (1985) discuss, at least a full ordinal ranking is required for second best analyses, or analyses of incremental improvements in fairness.

An alternative methodology is the *axiomatic approach*.[7] Here, one begins not with a particular formulation of fairness, but rather with a list of general properties, or *axioms*, that an appropriate formulation should satisfy. One example of such a property is the *equal treatment of equals* described earlier. More generally, one might require that a suitable notion of fairness should differentiate between individuals only on an appropriate basis — it should not be capricious. One way to capture this intuition is to insist that, were the agents to trade places in all relevant aspects (and only those), then the solution should change accordingly: if you were awarded x and I was awarded y on the basis of certain characteristics, then if we were to trade characteristics, I should receive x and you y. This ensures that no other characteristics matter. This property, referred to as *anonymity*, logically implies the aforementioned equal treatment property. Another property is that of *resource monotonicity*, introduced in this context by Roemer (1986). To capture the intuition that people are jointly entitled to a pool of common resources, resource monotonicity requires that if the aggregate quantity of resources were to increase, then all agents should end up better-off. Alternatively, *individual rationality from equal division* means that if all

[7]Yaari and Bar-Hillel (1984) contains an excellent description of this method.

agents have equal claims to given resources, then each should be at least as well-off as if he or she were to receive an equal share. A final example, intimated earlier, is that, in addition to allocating resources equitably, one might want to allocate them *(Pareto) efficiently* as well, that is, it should be impossible to make someone better-off without making someone else worse-off.

Given a list of such axioms, the central issue is whether there are elements that satisfy them.[8] If there are, the next step is to identify what they are — to *characterize* or completely describe the class.[9] (I will provide examples of such axiomatic characterizations below.) If there are no such elements, then the axioms are logically inconsistent and we have identified a necessary tradeoff: one cannot achieve all of the properties and must therefore choose among them. For example, Moulin and Thomson (1988) have shown that resource monotonicity, individual rationality from equal division, and Pareto efficiency are logically incompatible. That is, there is no allocation procedure that satisfies all three properties at once. Such "negative" or "impossibility" results are instructive in that they help us identify logical inconsistencies in our moral intuition; if our goals are unrealistic, then we must refine our sensibilities and our expectations of fairness.

2.2. Additional Remarks on Methodology

I will conclude my discussion on methodology with a few additional remarks.

[8]A note on terminology. In the sequel, I will discuss various objects for a notion of fairness — outcomes, procedures, net trades, opportunities. For this reason, I use the generic term "element" to refer to any of the above.

[9]Generally, the axiomatic approach is not useful for evaluating and contrasting alternative (feasible) policies.

2.2.1. Income inequality

First, one might ask: why in market economies is it necessary to discuss fairness in terms other than the distribution of income or wealth? That is, since money is the means of acquisition, and since it is comparable across individuals, why should a "practical" theory of fairness consider anything else? Moreover, the focus on income inequality is particularly attractive since it reduces the relevant comparisons to a single dimension and there is a well-developed body of theory on the subject.

Unfortunately, however, a theory of fairness which concentrates solely on income differences is inadequate for several reasons.[10] First, income or money is a means to an end, namely, the acquisition of tradable commodities, and it is only appropriate within a market context. Thus, the analysis of income inequality does not help us understand the basic meaning of fairness, but only its manifestation in a market economy. In particular, it does not tell us *why* a state of the economy is fair. And it is precisely this "why" which enables us to apply similar reasoning in non-market settings. Thus, for example, income inequality analysis is irrelevant when considering the allocation of nontradable goods such as voting rights among an electorate or schooling among children or transplant organs among potential recipients. Yet these are real allocative problems that must be resolved even in market-oriented societies.

From a broader perspective, if we begin within a particular institutional setting, such as that of a market economy, then we cannot compare alternative institutions. Which is more likely to achieve a fair outcome? And at what expense?

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[10]I do not mean to imply that the analysis of income inequality is either unimportant or misdirected, but simply that it should be part of a more extensive research program.

Finally, income inequality comparisons may be unsuitable for problems involving a small number of individuals where the assumption of price-taking behavior is inappropriate.

2.2.2. Micro versus macro justice

Generally, this work falls under the rubric of microeconomic analysis. Indeed, we often use models such as those mentioned earlier involving only two or three agents and several commodities. But it is not the fact that we use small models that distinguishes this research as a branch of microeconomics; microeconomics comprises a set of tools, whereas (contemporary) macroeconomics comprises a set of topics — inflation, unemployment, growth. Nor does the fact that the analysis is suitable for small problems (such as children sharing toys or firm partners sharing profits) preclude its relevance for large issues (such as the distribution of national wealth). The principle advantage of studying small models of this sort is that they often present the problem in its most basic form; larger models often add little to our understanding and yet come at great analytical cost. A disadvantage of small models, however, is that they may actually include factors which are unimportant at the aggregate level. For example, with many agents, strategic behavior (the consideration of the consequences of one's actions on other agents and vice versa) may be unimportant. (In contrast, as mentioned above, price-taking behavior may be inappropriate with only a few agents.)

2.2.3. Equity and other objectives

Finally, I would point out that equity is rarely posed as the sole social objective.

Otherwise, one might easily create a "just" society by disposing of a sufficient quantity of resources so that all people might be equally impoverished. The more interesting issue, therefore, concerns the possibility of achieving an equitable state that also satisfies other social criteria such as meeting peoples' needs, utilizing resources efficiently, providing proper incentives, or encouraging growth. I shall focus primarily on efficiency. Thus, in formulating a notion of fairness, the fundamental issue will be whether there exist equitable and efficient social states.

3. Ordinal Distributive Justice

As Amartya Sen (1992) has argued, "the central question in the analysis and assessment of equality is ... 'equality of what?'" I.e., there are many variables which might serve as the focus of our attention. One might define equality in terms of income or consumption or rights or achievements or opportunities or .... In the subsequent discussion, I will first distinguish between *outcome* or *end-state* notions of fairness versus *procedural* notions. The former ascribe equity as a property of the allocation of resources and the latter as a property of the procedure or mechanism by which resources are allocated.[11] Thus far, the study of fair outcomes has received the most attention, and I will refer to this as the *classical domain*.

[11]The terms *outcome* and *end-state* are slight misnomers since they might equally apply to the initial position rather than the final one. However, in that the initial allocation might be the outcome under autarky, I will retain the nomenclature.

3.1. Equitable Outcomes: Two Definitions

There are currently two main paradigms in the classical tradition, and a number of variants within each. I will call the first the *no-envy* approach and the second the *egalitarian-equivalence* approach, and I will describe them in turn.

3.1.1. No-envy

The first fully ordinal equity criterion was introduced by Foley (1967).[12] According to this criterion, an allocation is *envy-free* if no agent prefers the consumption bundle of another agent.[13] Formally, let $u^i(x^i)$ denote person i's utility from consuming the bundle x^i.[14] Then an allocation $\mathbf{x}=(x^1,\ldots,x^n)$ is envy-free if $u^i(x^i) \geq u^i(x^j)$, for all pairs i,j.

There are several attractive features of this notion of fairness. First, as mentioned previously, it does not require interpersonal utility comparisons. Rather than compare person i's utility to person j's utility, it entails comparing i's utility from consuming x^i to i's utility from consuming x^j. Clearly, it is possible to ask person i which of two consumption bundles he or she prefers. Also, Varian (1974) and Svennson (1983) have shown that envy-free and efficient allocations exist in exchange economies under very general conditions. For instance, if we were to divide resources equally and allow people to trade at market-determined prices, then the outcome would be envy-free and efficient. (In fact, this

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[12]See Baumol (1986), Thomson (1990b), Arnsperger (1994), or Young (1994) for historical notes.

[13]A strong form of this criterion was proposed by Tinbergen (1953), but this relies on nonordinally comparable information. See also Steinhaus (1948).

[14]I use a utility function rather than an ordinal preference relation to describe agents' tastes solely for convenience. Here, as in the remainder of the paper, only ordinal properties of the utility function are used. That is, $u^i(x^i) > u^i(x'^i)$ means person i prefers x^i to x'^i, and no significance is attributed to the cardinal value of $u^i(x^i)$.

describes a procedure for generating an equitable and efficient outcome which itself has various appealing properties. I will return to this point below.) Indeed, envy-free and efficient allocations exist under even weaker conditions than those needed to ensure the existence of a competitive/market equilibrium.

However, there are several problems and paradoxes involving the envy-free criterion as well. For example, Kolm (1972) has shown that there may be envy-free allocations that allow for mutual welfare improvements (and are thus inefficient), and yet all such improvements introduce envy! Indeed, Goldman and Sussangkarn (1978) established that this phenomenon is entirely general and not limited to a few isolated examples. But the most disturbing feature of this notion of fairness was demonstrated by Pazner and Schmeidler (1974). They showed that in economies with production, when people have different abilities as well as different tastes, envy-free and efficient allocations may not exist.

To address this and other difficulties with the envy-free criterion, numerous modifications were introduced in the 1970s and 1980s. These include *balanced envy-freedom* [Daniel (1975)] (everyone should envy and be envied by the same number of agents), *per-capita fairness* [Pazner (1977)] (no one should prefer the average bundle in the economy to his or her own), *average envy-freedom* [Thomson (1982)] (no one should prefer the average bundle among the other agents), *coalitional fairness* [Jaskold-Gabszewicz (1975) and Vind (1972)] (no group of agents should prefer the aggregate resources of any other group of the same size[15]), *income-fairness* [Pazner and Schmeidler (1978a)[16]] (all agents should have

[15]A group of agents prefers one bundle of resources to another if every member of the group can be made better-off with the former than with the latter.

[16]Although published in 1978, Pazner and Schmeidler (1978a) was written in 1974.

the same wealth at efficiency prices), and *wealth-fairness* [Varian (1974)] (no one should prefer the consumption-output bundle of another agent, when one compares the amount of time he or she would have to work to match the other agent's output). All of these are discussed in the surveys mentioned earlier.[17]

3.1.2. Egalitarian-equivalence

An alternative notion of fairness representing an entirely different methodological approach is that of egalitarian-equivalence introduced by Paznar and Schmeidler (1978b). To motivate the definition, note that an obvious focal point in identifying a fair outcome is equal division (the egalitarian allocation), in which all agents receive exactly the same quantities of all commodities. Indeed, in the earlier example of the two children, I did not suggest that equal division is unfair but only that it is generally inconsistent with Pareto efficiency. But if equal division is fair, then, one might argue, so is any other allocation in which everyone is indifferent between their bundle and the egalitarian one. In fact, we need not consider only the egalitarian allocation of the existing resources; we might agree that the egalitarian allocation of *any* bundle of resources is fair. Then, in reference to the *actual* bundle, an allocation is *egalitarian-equivalent* if each person is indifferent between their present component and an egalitarian allocation in some, possibly fictitious, reference economy. Thus, one might argue that the present allocation of welfare or utility *could have arisen*

[17]Rather than modify the envy-free criterion for use in production economies, an alternative might be to restrict the use of this criterion to only those production problems in which envy-free and efficient allocations do exist. Piketty (1994) has identified sufficient conditions to ensure existence, although his conditions are quite limited. Also, Varian (1974) established that if all agents have either identical tastes or identical productivities, then such allocations exist.

under strict egalitarianism in an appropriately defined reference economy. Formally, a feasible allocation $\mathbf{x}=(x^1,\ldots,x^n)$ is egalitarian-equivalent if there exists a bundle x such that $u^i(x^i)=u^i(x)$, for all i. Here, x is interpreted as the per capita bundle at the egalitarian allocation $(x,\ldots,x)$. In spite of the fact that this identifies a fair distribution of welfare, it is operationally devoid of interpersonal utility comparisons.

Pazner and Schmeidler established that egalitarian-equivalent and efficient allocations exist under very mild restrictions even in production economies. But there are other, more significant, problems with this notion. Most importantly, it is possible for one agent to get more of all commodities than another agent at an egalitarian-equivalent allocation. In fact, it is even possible for one agent to get everything and another nothing. In my opinion, this is an unacceptable feature for a notion of fairness.[18]

3.2. Alternative Domains: Procedures, Net Trades, and Opportunities

Each of the aforementioned notions proposes a definition of a fair outcome. But while this has been the focus of most theoretical work on the subject, the public seems largely concerned that economic institutions should be fair and that people should have equal opportunities. In the United States, for example, there is legislation to protect against discrimination in housing, education, and employment. But one does not see legislative initiatives to effect equal outcomes.[19] Also, in describing the experimental evidence on

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[18]The equivalence methodology has also generated several variants including *envy-free equivalence* [Pazner (1977)], *equal-opportunity equivalence* [Thomson (1994)], and *constant returns equivalence* [Mas-Colell (1980)] discussed below.

[19]Redistributive taxation can be viewed either as a means to affect outcomes or as a means to affect opportunities. In any event, there are no initiatives to entirely level the distribution of wealth.

people's perceptions of fairness, Miller (1992) concludes, "popular opinion gives a central place to dessert [i.e., compensation for effort and achievement] in thinking about justice."

In this section I will describe some of the work that has been done on procedural fairness and on the new analysis of equitable opportunities.

3.2.1. Procedural fairness

First, I would distinguish between a fair procedure versus a procedure designed to bring about a fair outcome. For the latter, any procedure that guarantees the correct result is satisfactory. Whereas for the former, any outcome is satisfactory providing it was obtained by an acceptable procedure. For example, if one wishes to obtain an envy-free allocation among two people, then a procedure one might use is the *divide-and-choose method*: let one person divide the resources and the other choose their most preferred portion.[20] But while this will ensure that the outcome is envy-free, the mechanism is systematically biased in favor of the divider in that both people would prefer that role.[21]

And so a fair outcome can be obtained by a biased or unfair procedure. Conversely, a fair procedure might generate very unfair outcomes. For example, in the exchange context, Schmeidler and Vind (1972) applied the concept of no-envy to *net trades*, or to changes in one's holdings, rather than to consumption bundles. Thus, if agent i is endowed

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[20]See Kolm (1972) and Crawford (1977). Also, generalizations of the divide-and-choose method to include additional agents were discussed by Steinhaus (1948), Singer (1962), and Kuhn (1967).

[21]They can do at least as well, and often better, in the role of divider as in the role of chooser.

with the bundle ω^i and yet consumes x^i, then its net trade is $t^i = x^i - \omega^i$.[22] A list of net trades is envy-free if no agent prefers the net trade of another agent, or if $u^i(\omega^i + t^i) \geq u^i(\omega^i + t^j)$, for all pairs i,j. Now, suppose agents trade from the initial allocation at market-determined prices. These "competitive trades" would indeed be envy-free. However, if we operate this procedure from a grossly inequitable starting point, then, of course, the outcome will be inequitable as well because the procedure does not change the distribution of wealth.

What if we were to operate a fair procedure from a fair initial allocation?[23] For example, the process I described earlier in which we first divide resources equally and then allow people to trade at market-determined prices will necessarily result in an envy-free and efficient allocation in an exchange problem. (In fact, I believe this was the intuition behind the Czech and Hungarian privatization schemes, although the designers were probably unaware of the precise sense in which this is fair.) Such an outcome, called a *competitive equilibrium from equal division*, has several appealing properties the most compelling of which is that they are the only envy-free and efficient outcomes in large, diverse populations.[24] However, in economies with production, a competitive equilibrium from equal endowments need not be envy-free due to differences in the productivities of the agents.

[22]The *l*th component of t^i is positive if i has acquired additional units of commodity *l* and consumes more than its endowment, and negative if it has reduced its holdings of *l* and consumes less than its endowment.

[23]Thomson (1983) contains a systematic discussion of consistency between equity criteria applied to the initial position, the rules of exchange, and the final position.

[24]To be precise, an envy-free and efficient allocation must be a competitive equilibrium from equal division in an economy with a continuum of agents among whom tastes vary continuously. See Varian (1976, p.254).

An alternative, axiomatic analysis of procedural fairness in production economies is contained in Kranich (1994). There, I addressed the following question. In an economy with diverse tastes and talents, is it possible to design an allocation procedure with three properties: (1) agents are free to work any number of hours they choose, (2) if agents work equal numbers of hours, they receive equal rewards regardless of their productivities, and (3) the outcome is Pareto efficient?

To answer this, I considered the simplest formulation of the joint production problem in which two people, A and B, each contribute labor to produce a single output, y. The technology is given by $y=f(L^A,L^B)$, where L^i denotes the labor input of person i, and f is known by both agents. Also, person i's preferences are represented by a well-behaved utility function of the form $u^i(L^i,y^i)$, which is decreasing in its first argument and increasing in its second.

In this context, a *division procedure* is a rule ϕ which specifies a division of the product for each pair of labor inputs, i.e., $\phi(L^A,L^B)=(\phi^A(L^A,L^B),\phi^B(L^A,L^B))$ such that $\phi^A(L^A,L^B)+\phi^B(L^A,L^B)=f(L^A,L^B)$, for all L^A,L^B. I then imposed the following requirement on ϕ that I called the *equal-division-for-equal-work (EDEW) principle*: if $L^A=L^B$, then $\phi^A(L^A,L^B)=\phi^B(L^A,L^B)$, that is, if the agents work the same number of hours, they should divide the output evenly.

At first glance, it would seem that by ignoring productivities, a procedure that satisfies the EDEW principle, an *EDEW procedure*, would fail to provide the necessary incentives to achieve an efficient outcome. However, I showed in the paper that that is often not the case; there is a large class of problems in which an EDEW division rule can indeed be used to achieve an efficient allocation.[25]

The extent to which this result can be generalized to include additional agents and/or commodities is an open question. Although even at this stage, it may be applicable to problems such as operating a jointly owned enterprise or partnership.

3.2.2. Opportunities

The issue of equitable opportunities has been discussed extensively in Social Philosophy and Ethics, but relatively little within Economics. And, in particular, there has been almost no formal analysis. Many authors have suggested that agents should have the same opportunities. [Cf. Kolm (1973) and Thomson (1994).] Then any differences that might emerge would reflect the idiosyncracies of tastes or individual effort. But what if opportunities are not identical, how should we compare them? In terms of our earlier distinction, the *definition* of equitable opportunities might be straightforward, but what is required is an *ordinal ranking* of distributions of opportunities. In other words, given a group of n individuals, and given two alternative lists, or *profiles*, $\mathbf{O}=(O^1,\ldots,O^n)$ and $\mathbf{O}'=(O'^1,\ldots,O'^n)$ specifying the opportunity sets for each of the individuals, which is more equitable? This is the question I posed in Kranich (1993a). Before discussing this, however,

[25]To be precise, I assumed the agents take the division rule as given, that is, as an institutional feature beyond their control. Then since the reward of each person depends on the labor contribution of the other person, agents can behave strategically. That is, each division rule induces a game in which the agents choose their labor supplies and the outcome is determined by the rule. I consider the Nash equilibria of such games, or the pairs of strategies which each represent a best response to the other. And I establish sufficient conditions under which there exists an EDEW division rule with an efficient Nash equilibrium allocation. The proof, in which I construct an appropriate ϕ, is based on the following observation. Under an EDEW division rule, and given the choice of L^B, agent A's strategies include the bundles $(L^A,\phi^A(L^A,L^B))$ and $(L^B,\frac{1}{2}f(L^B,L^B))$. That is, agent A has the option of working as much as agent B and dividing the resulting output equally. However, in equilibrium A chooses not to.

I would note that a recent paper by Fleurbaey (1995) contains the seeds of a similar inquiry.[26]

Fleurbaey addresses, somewhat abstractly, the appropriate domain of social responsibility. He proposes the following:

> My proposal would be to draw a list of all relevant outcomes which depict an individual's fate (health, living standard, educational level, career, family life, sense development, preference satisfaction, cheerfulness, etc.).... Then for each of these outcomes ... it must be decided what decision centers (government, local authorities, associations, family and friends...and last but not least, the individual herself) should assume some responsibility for the achievement obtained by the individual.... Now, the particular responsibility of social institutions would be to achieve equality across individuals, in the outcomes for which these institutions are responsible.

While this contains an explicit call for equality of opportunity (at least in those dimensions that are the responsibility of social institutions), by identifying the components of "an individual's fate," it implicitly provides a method of comparing profiles: devise a measure of distance for all relevant components. How does access to health care vary across individuals? Or the standard of living? Etc. Perhaps the distance between components can be used as a measure of inequality.

In contrast, in Kranich (1993a) I directly addressed the issue of ranking profiles of opportunity sets. There, the objective was to demonstrate by means of an example that the axiomatic approach is tractable in this context, that is, it is possible to specify criteria which enable us to rank <u>all</u> profiles of opportunity sets. Generally, such criteria take the form of "independence" requirements which specify changes in the components that <u>should</u> <u>not</u> affect

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[26]Here, I am extrapolating beyond the contents of the paper. Although the author did not pursue the question of ranking alternative profiles, his formulation may be amenable to such an extension.

the ranking, and "monotonicity" requirements which specify changes that should affect the ranking and how. For example, such a ranking should be anonymous. Thus, in the 2-agent case, (O^1,O^2) and (O^2,O^1) should be judged equally. Alternatively, suppose O^1 is a subset of O^2. Then one might argue that agent 2's opportunites are greater than agent 1's, and any further expansion of O^2 should decrease fairness. Or, intuitively, if the "rich" get richer without any change in the circumstances of the "poor," then fairness should decrease. In Kranich (1993a), I established that these and similar axioms imply a unique way to rank profiles of finite opportunity sets, namely, one profile is more equitable than another if and only if the difference in size between the opportunity sets in the former is less than that in the latter.

This previous result is naïve for several reasons, but it demonstrates that the axiomatic framework is viable, and it suggests that further research in this direction might be fruitful. In fact, there have already been several attempts by myself and others to extend the analysis by considering alternative axioms [see Kranich (1993b) and Herrero, Iturbe-Ormaetxe, and Nieto (1995)] and by investigating the compatibility of equitable opportunites with other social objectives [see Kranich (1995)]. While I am very optimistic about the potential usefulness of this work, it is too recent to have generated applications.

3.3. Some Limited Applications[27]

Next, I will describe some limited applications of ordinal theories of justice.

[27]In addition to those mentioned here, Baumol (1986) discusses several applications in Industrial Organization and Public Finance. See also Young (1994).

3.3.1. Division with single-peaked preferences

To be concrete, I will consider a particular example. A common problem in divorce proceedings is that of determining how much time children should spend with each parent. The formal structure of this problem is quite simple. It involves allocating a single, divisible commodity (the childrens' time) between the two parents, each of whom has preferences defined over the various allocations. Typically, each parent has a most preferred share and is worse-off if they receive either more or less than this amount. Such preferences are called "single-peaked." I will describe two possible solution procedures for such a problem.

First, suppose both parents were asked to state their most preferred allocation. For instance, suppose the father suggests that the children spend 1/2 of their time with each parent, and the mother suggests that they stay with her 3/4 of the time and with the father for 1/4. The two procedures I will describe recommend different solutions.

First, the *Uniform Rule* would proceed to allocate the childrens' time equally to both parents as long as there is enough available and until one parent has obtained their most preferred quantity. At this point, more time would be allocated to the other parent until he or she received their most preferred quantity. If there is more time to be allocated, that would again be divided equally. In the present example, each parent would receive 1/2.[28]

The Uniform Rule focuses on how much time each parent wants. Conversely, one could focus on the amount of time each parent would give the other. According to the *Contested Garment Principle*, which was described in the Babylonian Talmud, the body of

[28]One test case for an arbitration scheme is that in which the participants fully agree, and an appropriate scheme should recommend this outcome. In the example, for instance, suppose the father would like to have the children for ¼ of the time and the mother would like them for ¾. Both the Uniform Rule and the subsequent procedure would recommend this division.

Jewish legal commentary from the first five centuries A.D., the father should get 3/8 and the mother 5/8. According to this principle, in announcing their preferred allocations, the father conceded 1/2 to the mother and the mother 1/4 to the father. Thus, what is at issue is the remaining 1/4, and the Contested Garment Principle recommends that this be divided equally.

Now that we have seen two reasonable procedures for solving this problem, which is better? In one respect, at least, the Uniform Rule behaves better. Consider that this problem is somewhat different from those discussed earlier. Here, although we are not comparing directly the father's utility to the mother's, we are comparing their preferred divisions, and since this is not verifiable, they could lie. That is, knowing how the arbitration is to be resolved, both parents might announce a division other than their most preferred in order to influence the outcome in their favor. For example, if the mother knew that the decision was to be made by means of the Contested Garment Principle, then by suggesting more than 3/4 for herself, she could be allotted more than 5/8. In this way, the Contested Garment Principle can be manipulated by strategic agents. However, the Uniform Rule does not have this problem: Sprumont (1991) established that under the Uniform Rule, one cannot be better-off by recommending an allocation other than one's true most preferred division. We say the procedure is *strategy-proof*. Moreover, Sprumont showed that the outcome under this procedure will necessarily be envy-free and Pareto efficient *and it is the only procedure with these three properties!*[29]

Both the Uniform Rule and the Contested Garment Principle can be generalized to

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[29]Sprumont also established that the Uniform Rule is the only solution to satisfy anonymity (see p.8), strategy-proofness, and efficiency. For an alternative characterization see Ching (1994).

include any number of agents. Also, they clearly apply to a larger class of problems than our simple example. The salient features of such problems are (i) they involve the allocation of a single, infinitely divisible commodity between two or more agents, (ii) all agents have equal claims to the resources, and (iii) the agents' preferences are single-peaked.[30]

In this third feature, the example is somewhat atypical of the traditional economic problem in which agents are assumed to want as much of each resource as they can possibly obtain. A more traditional problem is the following.

3.3.2. Cost sharing

Consider the problem of sharing the general maintenance costs among the co-tenants of an apartment building, or that of allocating the construction costs of a shared water facility between municipalities. These are examples of cost sharing problems. The general structure of such problems involves two or more agents who demand different quantities of a homogeneous output, and the central question is how to divide the required inputs, either financial or real.

In the simplest formulation of the cost sharing problem, the output demands are given, and thus the aggregate cost is fixed. A more complete analysis would determine the output demands as well. In practice, a variety of algorithms for allocating costs have been employed, some of which are discussed in Moulin (1987). Here, I will discuss a procedure introduced by Shenker (1990) called Serial Cost Sharing. Rather than describe the general procedure, I will again consider a numerical example.

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[30] Feature (ii) is not essential. In fact, Klaus, Peters and Storcken (1994) recently extended the Uniform Rule to problems with possibly different initial claims.

As suggested above, suppose three towns are to install a joint water supply network. Suppose further that the cost of producing a network of size Q is given by the function $c(Q)=Q^2$. The towns demand $q_A=10$, $q_B=20$, and $q_C=30$, respectively. Thus, the total quantity to be produced is 60. The Serial Cost Sharing formula would allocate the total production cost of 3600 such that town A pays 300, town B pays 1100, and town C pays 2200. The reasoning is as follows. Since each of the towns demands at least 10, the first 30 units are "demanded equally." Hence, the production costs (900) should be shared equally. The next 20 units, however, are demanded only by towns B and C. Therefore the incremental cost (2500−900) should be shared equally among B and C only. The final 10 units are demanded only by C and thus it should incur the full incremental cost (3600−2500).

The Serial Cost Sharing formula is more complicated than others such as allocating costs equally (in terms of the example: 1200,1200,1200) or in proportion to demand (600,1200,1800). Why then should we be interested in this mechanism? The answer is that it has at least two attractive features. First, like the Uniform Rule for problems with single-peaked preferences, it is strategy-proof; if the agents know that costs will be allocated by means of the Serial Cost Sharing formula, then they can do no better than to announce their true demands. Hence, this procedure directly determines the output demands in that it provides the incentives for correct revelation. Also, the outcome under Serial Cost Sharing will be envy-free, that is, no agent will prefer the input/output combination assigned to any other agent. Unfortunately, however, for the class of problems in question, Serial Cost Sharing is *not* necessarily Pareto efficient. In fact, Moulin (1990a) has shown that for this class of problems, there is *no* solution procedure that generates envy-free and Pareto efficient outcomes and, in addition, satisfies the following condition known as the *stand alone property*. Notice that in the previous example, the technology for producing Q exhibits decreasing returns to scale. This means that as the quantity produced increases so do the unit costs of production. Thus, when the municipalities cooperate to build a larger facility, they drive up the unit production costs for each of them relative to the unit costs if each were to produce their own facility using the same technology.[31] Hence, they impose an externality, of sorts, on each other. The stand alone property requires that none of the three should gain as a result of this externality, or that they should all contribute to the loss.

The fact that no mechanism is envy-free, efficient, and satisfies the stand alone property identifies another necessary tradeoff in our reasoning about social objectives.

3.3.3. Common ownership and privatization

The reciprocal problem of cost allocation is surplus division. For example, consider the case in which two or more individuals own a productive enterprise in common. If the enterprise is ongoing, there is the question of how to divide the periodic profits or losses. Or if the enterprise is to be dissolved, they must distribute the joint assets. The former is complicated by the fact that partners need not contribute equally. And the latter is particularly relevant in today's climate of privatization of collective enterprises.

In fact, we have seen one formulation of this problem already in the discussion of the

[31]This does not mean that they should necessarily build three separate facilities. First, there is the question of whether, individually, they have access to the technology for producing Q. Also, here we are considering the cost of producing the water supply network rather than the cost of operating it. The latter may exhibit economies of scale.

EDEW principle, where a single output was produced from heterogeneous labor. There, the technology might be considered the common property of the two agents, and they are jointly entitled to its product. To be precise, it concerns the common ownership of one productive resource (the technology itself) and the private ownership of another (the labor inputs). This issue has been the subject of several recent investigations by Moulin, Roemer and Silvestre.[32] [See Silvestre (1994) for a survey.] Their approach has been axiomatic as well. For example, as an expression of public ownership of the means of production, one might insist that all agents should gain from an improvement in the technology. Or, if labor is privately owned, one might require that no agent should be worse-off as a result of an increase in his or her productivity, *ceteris paribus*. Using such axioms in conjunction with Pareto efficiency, Roemer and Silvestre (1989, 1993) and Moulin (1990b) characterized three solution concepts: the *equal benefits solution*, which assigns equal profit shares of a joint technology; the *proportional solution*, which assigns profit shares proportional to the value of labor inputs; and the *constant returns equivalent solution* [Mas-Colell (1980)], in which the current utility allocation could have been generated by some, possibly fictitious, constant returns to scale technology.[33]

How do these solutions to the mixed ownership problem compare to the analysis of the equal-division-for-equal-work principle? Obviously, one difference concerns the axioms. But a further difference concerns the disposition of labor. The EDEW analysis explicitly

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[32]Moulin (1990a, 1990b), Roemer (1986), and Roemer and Silvestre (1989, 1993).

[33]Also, Moulin and Roemer (1989) characterized the *welfare egalitarian mechanism*, which associates with each economy those allocations at which normalized utilities are equal across agents, as an expression of "public ownership of the external world and private ownership of self." But this obviously requires interpersonally comparable utilities.

addressed the strategic/incentive effects of the labor supply decision, whereas the analyses of the other solutions have not. That is, they do not analyze the optimal behavior of the agents when they are aware of the fact that their labor supply decisions will affect their share of the profits.[34]

3.3.4. Potential applications of opportunity rankings

Since formal analysis of opportunity fairness is just beginning, there have been no applications of these techniques as of yet. So, rather than review past accomplishments, I will comment briefly on the potential of this approach.

There are at least two motivations for advocating equitable opportunities rather than equitable outcomes or procedures: first, to allow for individual differences in the desire or motivation to exploit one's opportunities; and, second, to distinguish between those differences that are the responsibility of the individual and those that are not, and to remedy the latter. Several of the most controversial issues in the United States today concern this distinction between personal and societal responsibility. Two obvious examples are affirmative action, the preferential treatment of minorities and women, and public funding of private and religious education.

Regarding affirmative action, the fundamental questions are: When should society remediate for past injustices? And when is remediation no longer necessary? Currently, there is a great deal of controversy over the merits of such programs. There are those who believe that equity has been achieved (or is not worth achieving). And there are those who

[34]Moreover, attempting to incorporate such considerations could be problematic in that the induced game of labor choice may have no (pure strategy) Nash equilibria without additional restrictions.

believe that the legacy of racism is still pervasive. In my opinion, this controversy stems from the fact that there are no adequate tools for assessing what, if any, progress has been made. To evaluate the success of affirmative action programs requires that we can rank profiles in the manner described above. An even more difficult question concerns the extent of progress, but this would require a cardinal measure of opportunity fairness.

Concerning the issue of public funding of private education, there are those who feel that this would introduce competition between public and private schools which would improve the quality of the public school system. But I believe the issue is more basic and concerns the taxes versus benefits of those who wish to send their children to private schools. Currently, such individuals are required to pay taxes to support the public school system, and yet they receive no direct benefits. Indeed, they must pay an additional amount for similar benefits provided privately. The simplest case that can be made in support of the current system is that this transfer/subsidy is warranted on the grounds that it improves the distribution of opportunities. However, to properly evaluate the efficacy of this claim would again require that we can compare such distributions.

Similar issues arise in the context of housing, employment, and health care — whether to assess the effects of discrimination or to evaluate the need for remediation, it is necessary to compare distributions of opportunities.

4. Conclusion and an Agenda for the Future

In this paper, I have attempted to convey some of the flavor of the present state of theoretical research on fairness. In particular, I have concentrated on ordinal theories and their potential applications. However, in providing a selective review, I naturally overlooked many factors (such as equity in economies with indivisible commodities[35] or with public goods[36]), and I touched on others only in passing (such as incentives and implementation). In conclusion, I would like to highlight some of the issues raised and to offer some suggestions for future research.

First, let me reiterate that formulating an appropriate definition of fairness is only the first and simplest analytic objective. For the vast majority of applications, simply dividing solutions into those that are fair and those that are not is insufficient. It is often necessary to select among unfair alternatives. And to do so requires either an ordinal ranking or a cardinal measure of fairness. Thus, even within the classical domain of outcome fairness, there is much work to be done.

Next, I would emphasize the role of incentives and strategic behavior in the design of fair allocation mechanisms. A common concern, particularly in the context of production, is that greater equality will undermine incentives. For example, in their survey of beliefs about inequality, Kluegel and Smith (1986) report that 85% of their respondents agree with the proposition that "giving everybody about the same income regardless of the type of work

[35]For recent examples, see Alkan, Demange, and Gale (1991) and Aragones (1992) and the references cited therein.

[36]See Diamantaras (1992).

they do would destroy the desire to work hard and do a better job."

Some of the concepts I discussed above incorporate strategic considerations and some do not. For instance, the analysis of EDEW division procedures explicitly addressed the strategic effects of the labor supply decision. Also, the Uniform Rule and the Serial Cost Sharing mechanism are both strategy-proof, and, in this sense, they are robust to manipulative behavior. But other sharing formulas are not immune to incentive effects. For example, in the surplus sharing problem, proportional division (which Aristotle took to be the very definition of fairness) is subject to strategic manipulation, i.e., if agents know that output will be divided in proportion to their individual contributions, then this will distort their contributions. In fact, for some of the notions I mentioned, the strategic properties have yet to be investigated. Overall, the literature contains very few examples of fair procedures and most of those do not consider incentives.

Another area that requires further investigation is that of designing simple, practical procedures for attaining fair outcomes. The divide-and-choose method, the Uniform Rule, and Serial Cost Sharing each result in an envy-free allocation. Also, Crawford (1979) and Demange (1984) have proposed variations of divide-and-choose that result in egalitarian-equivalent and efficient outcomes.[37] Similarly, one might devise other procedures that are capable of achieving other notions of fairness. In light of the present consensus that different

[37]The general question of whether or not there exists a procedure that will "implement" a specific solution concept is the subject of a rather abstract literature in applied game theory. [See Maskin (1985) for a survey of the theory of Nash implementation and Thomson and Varian (1985, Sec.5) for a general discussion of the problem of implementing equitable allocations.] Suffice it to say that the general procedures advanced in this literature are complex and quite synthetic. Such procedures are designed to achieve an arbitrary social objective; in effect, they are shells that can be applied with great generality. This explains the lack of specificity. A recent attempt to construct simple procedures to solve the general problem based on the "divide-and-permute" principle is contained in Thomson (1993).

notions may be suitable in different contexts, such alternative procedures are necessary.

The final agenda item I would mention is the issue of opportunites. The work that has been done in this area so far is very preliminary, but the direction seems promising, both for theoretical research as well as for eventual applications. Ironically, this area, which represents the greatest concern to those outside the profession, has received the least attention within.

References

ALKAN, Ahmet, DEMANGE, Gabrielle, and GALE, David (1991), "Fair Allocation of Indivisible Objects and Criteria of Justice". *Econometrica*, Vol.59, no.4, pp.1023-1039.

ARAGONES, Enriqueta (1992), "A Solution to the Envy-Free Selection Problem in Economies with Indivisible Goods". Northwestern University, mimeo

ARNSPERGER, Christian (1994), "Envy-Freeness and Distributive Justice: A Survey of the Literature". *Journal of Economic Surveys*, Vol.8, no.2, pp.155-186.

BAUMOL, William (1986), *Superfairness*. Cambridge, Mass.: MIT Press.

BINMORE, Ken (1994), *Game Theory and the Social Contract, Volume 1: Playing Fair*. Cambridge, Mass.: MIT Press.

CHING, Stephen (1994), "An Alternative Characterization of the Uniform Rule". *Social Choice and Welfare*, Vol.11, pp.131-136.

CRAWFORD, Vincent (1977), "A Game of Fair Division" *Review of Economic Studies*, Vol.44, no.2, pp.235-247.

________________ (1979), "A Procedure for Generating Pareto-Efficient Egalitarian-Equivalent Allocations". *Econometrica*, Vol.47, no.1, pp.49-60.

DANIEL, Terrance (1975), "A Revised Concept of Distributional Equity". *Journal of Economic Theory*, Vol.11, no.1, pp.94-100.

D'ASPREMONT, Claude (1994), "Welfarism and Interpersonal Comparisons". *Investigaciones Económicas*, Vol.13, no.1, pp.3-17.

DEMANGE, Gabrielle (1984), "Implementing Efficient Egalitarian Equivalent Allocations". *Econometrica*, Vol.52, no.5, pp.1167-1177.

DIAMANTARAS, Dimitrios (1992), "On Equity with Public Goods". *Social Choice and Welfare*, Vol.9, no.2, pp.141-157.

ELSTER, Jon and ROEMER, John E. (eds.) (1991), *Interpersonal Comparisons of Well-Being*, Cambridge, U.K.: Cambridge University Press.

FLEURBAEY, Marc (1995), "Equal Opportunity or Equal Social Outcome". *Economics and Philosophy*, Vol.11, no.1, pp.25-55.

FOLEY, Duncan (1967), "Resource Allocation and the Public Sector". *Yale Economic Essays*, Vol.7, no.1, pp.45-98.

GOLDMAN, Steven M. and SUSSANGKARN, Chal (1978), "On the Concept of Fairness". *Journal of Economic Theory*, Vol.19, no.1, pp.210-216.

HERRERO, Carmen, ITURBE-ORMAETXE, Iñigo, and NIETO, Jorge (1995), "Fair Equality of Opportunity". Universidad de Alicante, mimeo.

JASKOLD-GABSZEWICZ, Jean (1975), "Coalitional Fairness of Allocations in Pure Exchange". *Econometrica*, Vol.43, no.4, pp.661-668.

KLAUS, Bettina, PETERS, Hans, and STORCKEN, Ton (1994), "Strategy-Proof Division with Single-Peaked Preferences and Initial Endowments". University of Limburg, mimeo.

KLUEGEL, James and SMITH, Eliot (1986), *Beliefs about Inequality*. New York: Aldine de Gruyter.

KOLM, Serge-Christophe (1972), "Super-Équité". *Kyklos*, Vol.26, no.4, pp.841-843.

____________________ (1973), *Justice et Équité*. Paris: Editions du Centre National de la Recherche Scientifique.

KRANICH, Laurence (1993a), "Equitable Opportunities: An Axiomatic Approach". Universidad Carlos III de Madrid, mimeo.

____________________ (1993b), "Equitable Opportunities in Economic Environments". Universidad Carlos III de Madrid, mimeo.

________________ (1994), "Equal Division, Efficiency, and the Sovereign Supply of Labor". *American Economic Review*, Vol.84, no.1, pp.178-189.

________________ (1995), "The Distribution of Opportunities: A Normative Theory". Universidad Carlos III de Madrid, mimeo.

KUHN, Harold W. (1967), "On Games of Fair Division", in SHUBIK, Martin (ed.), *Essays in Honor of Oskar Morgenstern*. Princeton, New Jersey: Princeton University Press.

MAS-COLELL, Andreu (1980), "Remarks on the Game Theoretic Analysis of a Simple Distribution of Surplus Problem". *International Journal of Game Theory*, Vol.9, pp.125-140.

MASKIN, Eric (1985), "The Theory of Implementation in Nash Equilibrium: A Survey", in HURWICZ, Leonid, SCHMEIDLER, David, and SONNENSCHEIN, Hugo (eds.), *Social Goals and Social Organization: Essays in Memory of Elisha Pazner*. Cambridge, U.K.: Cambridge University Press.

MILLER, David (1992), "Distributive Justice: What the People Think". *Ethics*, Vol.102, no.2, pp.555-593.

MOULIN, Hervé (1981), "Implementing Just and Efficient Decision Making". *Journal of Public Economics*, Vol.16, no.2, pp.193-213.

____________ (1987), "Equal or Proportional Division of a Surplus, and Other Methods". *International Journal of Game Theory*, Vol.16, pp.161-186.

____________ (1990a), "Fair Division under Joint Ownership: Recent Results and Open Problems". *Social Choice and Welfare*, Vol.7, no.2, pp.149-170.

____________ (1990b), "Joint Ownership of a Convex Technology: Comparison of Three Solutions". *Review of Economic Studies*, Vol.57, no.3, pp.439-452.

MOULIN, Hervé and ROEMER, John E. (1989), "Public Ownership of the External World and Private Ownership of Self". *Journal of Political Economy*, Vol.97, no.2, pp.347-367.

MOULIN, Hervé and SHENKER, Scott (1992), "Serial Cost Sharing". *Econometrica*, Vol.60, no.5, pp.1009-1037.

MOULIN, Hervé and THOMSON, William (1988), "Can Everyone Benefit from Growth? Two Difficulties". *Journal of Mathematical Economics*, Vol.17, no.4, pp.339-345.

PAZNER, Elisha A. (1977), "Pitfalls in the Theory of Fairness". *Journal of Economic Theory*, Vol.14, no.2, pp.458-466.

PAZNER, Elisha A. and SCHMEIDLER, David (1974), "A Difficulty in the Concept of Fairness". *Review of Economic Studies*, Vol.41, no.3, pp.441-443.

______________ and__________________ (1978a), "Decentralization and Income Distribution in Socialist Economies". *Economic Inquiry*, Vol.16, no.2, pp.257-264.

______________ and__________________ (1978b), "Egalitarian Equivalent Allocations: A New Concept of Economic Equity". *Quarterly Journal of Economics*, Vol.92, no.4, pp.671-687.

PIKETTY, Thomas (1994), "Existence of Fair Allocations in Economies with Production". *Journal of Public Economics*, Vol.55, no.3, pp.391-405.

RAWLS, John (1971), *Theory of Justice*. Cambridge, Mass.: Harvard University Press.

ROEMER, John E. (1986), "Equality of Resources Implies Equality of Welfare". *Quarterly Jounal of Economics*, Vol.101, no.4, pp.751-784.

ROEMER, John E. and SILVESTRE, Joaquim (1989), "Public Ownership: Three Porposals for Resource Allocation". University of California, Davis, mimeo.

______________and _________________ (1993), "The Proportional Solution in Economies with both Private and Public Ownership". *Journal of Economic Theory*, Vol.59, no.2, pp.426-444.

SCHMEIDLER, David and VIND, Karl (1972), "Fair Net Trades". *Econometrica*, Vol.40, no.4, pp.637-642.

SEN, Amartya (1992), *Inequality Reexamined.* Cambridge, Mass.: Harvard University Press.

SHENKER, Scott (1990), "Making Greed Work in Networks: A Game-Theoretic Analysis of Gateway Service Disciplines". Xerox Research Center, mimeo.

SILVESTRE, Joaquim (1994), "Economic Analysis of Public Ownership". *Investigaciones Económicas*, Vol.18, no.1, pp.19-66.

SINGER, E. (1962), " Extension of the Classical Rule of 'Divide and Choose'". *Southern Economic Journal*, Vol.38, pp.391-394.

SPRUMONT, Yves (1991), "The Division Problem with Single-Peaked Preferences: A Characterization of the Uniform Rule". *Econometrica*, Vol.59, no.2, pp.509-519.

STEINHAUS, H. (1948), "The Problem of Fair Division". *Econometrica*, Vol.16, pp.101-104.

SVENNSON, Lars-Gunnar (1983), "On the Existence of Fair Allocations". *Journal of Economics*, Vol.43, no.3, pp.301-308.

THOMSON, William (1982), "An Informationally Efficient Equity Criterion". *Journal of Public Economics*, Vol.18, pp.243-263.

_______________ (1983), "Equity in Exchange Economies". *Journal of Economic Theory*, Vol.29, no.2, pp.217-244.

_______________ (1990a), "The Consistency Principle," in ICHIISHI, Tatsuro, NEYMAN, Abraham, and TAUMAN, Yair (eds.), *Game Theory and Applications*, San Diego, CA: Academic Press.

_______________ (1990b), *The Theory of Equitable Allocation*. University of Rochester, manuscript.

_______________ (1993), "'Divide and Permute' and the Implementation of Solutions to the Problem of Fair Division". University of Rochester, mimeo.

_______________ (1994), "Notions of Equal, or Equivalent, Opportunities". *Social Choice and Welfare*, Vol.11, no.2, pp.137-156.

THOMSON, William and VARIAN, Hal R. (1985), "Theories of Justice Based on Symmetry," in HURWICZ, Leonid, SCHMEIDLER, David and SONNENSCHEIN, Hugo (eds.), *Social Goals and Social Organization: Essays in Memory of Elisha Pazner*. Cambridge, U.K.: Cambridge University Press.

TINBERGEN, Jan (1953), *Redelijke Inkomensverdeling*. Haarlem, NL: N.V. DeGulden Pers.

VARIAN, Hal R. (1974), "Equity, Envy, and Efficiency". *Journal of Economic Theory*, Vol.9, pp.63-91.

____________ (1976), "Two Problems in the Theory of Fairness". *Journal of Public Economics*, Vol.5, no.3,4, pp.249-260.

VIND, Karl (1972), "Lecture Notes in Mathematical Economics". Stanford University, mimeo.

YAARI, Menachem E. and BAR-HILLEL, Maya (1984), "On Dividing Justly". *Social Choice and Welfare*, Vol.1, no.1, pp.1-24.

YOUNG, H. Peyton (1994), *Equity in Theory and Practice*. Princeton, New Jersey: Princeton University Press.