The theory of contests: a survey

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1 Introduction

This paper provides an introduction to the theory of contests in a unified framework. In particular we present the basic model and study its main properties from which we derive various applications. The literature on this topic is vast and we make no attempt to cover all issues. Therefore many good papers and interesting topics are not covered. The interested reader can consult the surveys of Nitzan (1994) and Konrad (2006) for additional issues and references.

A part of economics (e.g., general equilibrium) studies situations where property rights are well defined and agents voluntarily trade rights over goods or produce rights for new goods. This approach has produced very important insights into the role of markets in resource allocation such as the existence and efficiency of competitive equilibrium, the optimal specialization under international trade, the role of prices in providing information to the agents, etc.

There are other situations, though, where agents do not trade but rather fight over property rights. In these situations agents can influence the outcome of the process by means of certain actions such as investment in weapons, bribing judges/politicians, hiring lawyers, etc. These situations are called *Contests.* The literature has developed

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from the seminal contributions by Tullock (1967, 1980) and Krueger (1974) who studied a specific contest, rent-seeking, and Becker (1983) who studied lobbying.[1] Lately, the framework was generalized to other situations. The example below refers to voting. Other examples are considered later on.

Example 1.1 Political competition: Two political parties value office in V_1 and V_2. To influence voters they use advertisement in quantities G_1 and G_2. The probability that party $i = 1, 2$ reaches office, denoted by p_i is

$$\begin{aligned} p_i &= \frac{G_i}{G_1 + G_2} \quad \text{if } G_1 + G_2 > 0, \\ p_i &= 1/2 \quad \text{if } G_1 + G_2 = 0. \end{aligned} \tag{1.1}$$

Expected monetary payments for party $i = 1, 2$ are,

$$\frac{G_i}{G_1 + G_2} V_i - G_i.$$

A *Contest* is defined by the following elements:[2]

- A (finite) set of agents, also called contenders, denoted by $N = \{1, 2, \ldots, n\}$.
- A set of possible actions (effort, investments) taken by agents before the prize is allocated. These actions determine the probability of obtaining the prize. They can be interpreted as the positions taken by agents before the conflict starts.
- A prize whose quantity may depend on the actions taken by agents.[3]
- A function, relating the actions taken by agents to the probabilities that they obtain the prize. This function is called *Contest Success Function*.
- A function that for each possible action yields the cost of this action. This function is called the cost function.[4]

Formally, let $p_i = p_i(G_1, \ldots, G_n)$ be the probability that agent i obtains the prize when actions are $(G_1, \ldots, G_n) \in \Re^n_+$. Another interpretation is that p_i is the fraction of the prize obtained by i. $V_i(G_1, \ldots, G_n)$ is the value of the prize as a function of the efforts made by agents and $C_i(G_i)$ is the cost attributed by i to her action G_i. If the valuations of the prize are independent of efforts they will be denoted by V_i and when they are identical for all agents, by V. Assuming that agents are risk-neutral with payoffs linear on the expected prize and costs, the payoff function of agent i, denoted by $\Pi_i(\,)$, is

$$\begin{aligned} \Pi_i(G_1, \ldots, G_i, \ldots, G_n) \equiv\ & p_i(G_1, \ldots, G_i, \ldots, G_n) \\ & \times V_i(G_1, \ldots, G_i, \ldots, G_n) - C_i(G_i). \end{aligned}$$

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1 See Tullock (2003) for his account of the development of the concept.

2 For a discussion of the concept of contest see Neary (1997) and Hausken (2005).

3 This may be due to the fact that agents value the effort made in the contest or because the investment increases the value of the prize, see Chung (1996) and Amegashie (1999a,b).

4 We assume implicitly that should expenses be publicly disclosed, contenders suffer no consequences. See Corchón (2000) for the case in which contenders can be legally prosecuted for accepting these expenses.

Thus, the definition of a contest has lead us to a game in normal form where payoffs are expected utilities and strategies are efforts/investments. For these games the less controversial concept of equilibrium is the one proposed by John Nash in 1950, generalizing an idea advanced by Cournot (1838): an equilibrium is a situation from which there are no unilateral incentives to deviate. Formally, we say that $(G_1^*, \ldots, G_i^* \ldots, G_n^*)$ is a Nash equilibrium (NE) if

$$\Pi_i((G_1^*, \ldots, G_i^*, \ldots, G_n^*) \geq \Pi_i(G_1^*, \ldots, G_i, \ldots, G_n^*), \quad \text{for all } G_i \in \Re_+, \\ \text{for each agent } i.$$

Now consider some more examples:

Example 1.2 Litigation/fight. In this case V_i's represent the value attached to some item, say, a piece of land, a state or a title of nobility. If the fight is conducted in the legal system G's are legal expenses. If the fight is a war, G's are costs of raising an army. G's could also be sabotage activities devoted to decreasing the efficiency of the opponent (Konrad 2000). The contest success function yields the probability of obtaining the item as a function of legal/military expenses or sabotage activities.

Example 1.3 Lobbying. In this case V_i's represent the value of a public policy like a law granting certain rights to some citizens, subsidies to agriculture or restrictions to enter a market, etc. The set of feasible policies is the interval [0, 1]. There are two agents that have opposite preferences over this issue (right and left, farmers and taxpayers, incumbent and entrant). p_i is the position taken on this issue and $p_i \; V_i$ is the payoff derived by i from this allocation.

Example 1.4 Awarding a prize. In this case V_i's represent the value of a grant, a prize or a patent. G's are the expenses made in order to participate and/or to influence the jury for a prize. The contest success function yields the probability of obtaining the prize as a function of efforts/expenses made in order to obtain merits/influence in the jury's eyes.

Example 1.5 Contracts. In this case, V_i's are the value of a contract for the public or the private sector or the value of hosting a public event, i.e., the Olympic Games. Expenses are made in order to present the case of each contender and/or to influence the jury. The contest success function yields the probability of obtaining the contract or the right to organize the event as a function of expenses.

Example 1.6 Cooperative production. The agents have preferences over pairs consumption/labor. Here $V(\;)$ is the production function, G_i is the labor i and $p_i(G_1, \ldots, G_n)$ is the share of i in the output. Thus $p_i V$ is is consumption.

In the following sections we will review several aspects of contests paying attention to both analytical results and applications.

Section 2 is concerned with the foundations of the success contest function.

The basic properties of equilibrium, existence, uniqueness and comparative statics, are amenable to a common analysis that encompasses Examples 1.1–1.5 above. Such an analysis is performed in Sect. 3, where we study the symmetric case and Sect. 4 where we are concerned with asymmetric contests.

Section 5 examines socially optimal policies under rent-seeking in well known problems; welfare losses due to monopoly and transaction costs as well as the impact of regulation. These problems correspond to Examples 1.2–1.3 above where the contest does not produce anything valuable for society.

In Sect. 6 we study the optimal design of a contest that produces something socially useful. This corresponds to Examples 1.4–1.5 above. A planner concerned with social welfare will simply stop many contests belonging to the class considered in Sect. 5, e.g., the fight for monopoly rights. On the contrary, the same planner, may subsidize many belonging to the second, e.g., R&D, etc.

2 Contest success functions

In this section we study the properties of contest success functions (CSF).

In order to be specific about the properties of an NE, it would be nice to have an idea of the form of CSF. Consider the following functional form:

$$p_i = \frac{\phi(G_i)}{\sum_{j=1}^{n} \phi(G_j)} \quad \text{if} \quad \sum_{j=1}^{n} \phi(G_j) > 0, \tag{2.1}$$

$$p_i = \frac{1}{n} \quad \text{otherwise.} \tag{2.2}$$

An intuitive interpretation of (2.1) is that $\phi(G_i)$ measures the impact of G_i in the contest, i.e., it summarizes the merits of i. Thus, in Example 1.1, $\phi(G_i) = G_i$ is the impact of advertisement on voters. The ratio $\phi(G_i)/\sum_{j=1}^{n} \phi(G_j)$ measures the relative impact (merit) of i. Hence, (2.1) says that the probability of an agent winning the prize equals the relative impact (merit) of that agent. Many papers dealing with contest models in the literature assume a CSF which is a special case of (2.1). For instance $\phi(G_i) = G_i^{\epsilon}$ which was introduced by Tullock (1980). If $\epsilon = 1$ we have the form considered in (1.1). If $\epsilon = 0$, the probability of success is independent of the effort made by the players. Another example is the logit form proposed by Hirshleifer (1989) where, given a positive scalar k, $\phi(G_i) = e^{kG_i}$.

Whenever the form (2.1) is postulated, the following properties are assumed.

i) $\phi(\,)$ is twice continuously differentiable in $\Re_{++}$.
ii) $\phi(\,)$ is concave.
iii) $\phi'(\,) > 0$.
iv) $\phi(0) = 0$, $\lim_{G_i \to \infty} \phi(G_i) = \infty$.
v) $G_i \phi'(G_i)/\phi(G_i)$ is bounded for all $G_i \in \Re_{+}$.[5]

Property ii) is helpful in the proof of the existence of a Nash equilibrium. iii) says that more effort by i increases the merit of i. The last two properties are technical. If $\phi(G_i) = G_i^{\epsilon}$ with $0 < \epsilon \leq 1$ all the above properties are fulfilled.

[5] When no confusion can arise, derivatives will be denoted by primes.

Let us present CSFs which are not special cases of the form (2.1). The first two consider the case of two contestants and build on the idea that only differences in effort matter. Baik (1998) proposed the following: Given a positive scalar σ,

$$p_1 = p_1(\sigma G_1 - G_2) \quad \text{and} \quad p_2 = 1 - p_1. \tag{2.3}$$

Che and Gale (2000) postulate a special form of $p_1(\,)$:

$$p_1 = max\left\{min\left\{\frac{1}{2} + \sigma(G_1 - G_2), 1\right\}, 0\right\} \quad \text{and} \quad p_2 = 1 - p_1. \tag{2.4}$$

These CSF are problematic because the winning probabilities depend on the units in which expenditures are measured (e.g., dollars or cents), see our discussion of property (H) later in this section. Alcalde and Dahm (2007) proposed the following CSF that circumvents this difficulty; Given a positive scalar α, suppose for simplicity that $G_j \geq G_{j+1}$. Then,

$$p_i = \sum_{j=i}^{n} \frac{G_j^\alpha - G_{j+1}^\alpha}{j \cdot G_1^\alpha}, \quad \text{for } i = 1, \ldots, n \ \text{ with } G_{n+1} = 0. \tag{2.5}$$

2.1 Axiomatics

Suppose that $p_i(\,)$ is defined for all subsets of N. Consider the following properties:

(P1) Imperfect discrimination: For all i, if $G_i > 0$, then $p_i > 0$.[6]

(P2) Monotonicity: For all i, p_i is increasing in G_i and decreasing in G_j, $j \neq i$.

(P3) Anonymity: For any permutation function π on the set of bidders we have

$$p(\pi \boldsymbol{G}) = \pi p(\boldsymbol{G}) \quad \text{for all } \ \boldsymbol{G} \equiv (G_1, \ldots, G_i, \ldots, G_n).$$

While these properties are standard, the next two properties are more specific and relate winning probabilities in contests to different sets of active contestants. Let $p_i^M(\boldsymbol{G})$ be contestant i's probability of winning a contest played by a subset $M \subset N$ of contestants with $\boldsymbol{G} \equiv (G_1, \ldots, G_i, \ldots, G_n)$.

(P4) Independence: For all $i \in M$, $p_i^M(\boldsymbol{G})$ is independent of G_j for all $j \notin M$.

(P5) Consistency: For all $i \in M$, and for all $M \subset N$ with at least two elements,

$$p_i^M(\boldsymbol{G}) = \frac{p_i(\boldsymbol{G})}{\sum_{j \in M} p_j(\boldsymbol{G})}, \quad \text{for all } \ \boldsymbol{G} \equiv (G_1, \ldots, G_i, \ldots, G_n).$$

[6] The name of this axiom refers to the fact that a contest can be interpreted as an *auction* where the prize is auctioned among the agents and efforts are *bids*. In standard auctions the higher bid obtains the prize with probability one. Here, any positive bid entitles the bidder with a positive probability to obtain the object, so it is as if the bidding mechanism did not discriminate perfectly among bids.

Together (P4) and (P5) imply that the CSF satisfies Luce's Choice Axiom (Clarke and Riis 1998) defined as follows: the probability that contestant i wins if player k does not participate is equal to the probability that i wins when k participates given that k does not win. This axiom holds for any subset of non-participating players. This is a kind of independence of irrelevant alternatives property.

Skaperdas (1996) proved the following result whose proof is omitted:

Proposition 2.1 (P1)–(P5) *are equivalent to assuming a CSF like* (2.1).

Properties (P1)–(P4) are reasonable. However, (P5) is debatable, as shown by the next example:

Example 2.1 There are three teams that play a soccer/basketball league. Teams have to play against each other twice. They obtain three, one or zero points if they win, draw or lose, respectively. Suppose efforts made by teams are given. There are two states of the world where each occurs with probability 0.5. In the first state results are:

Team 1 against Team 2: 1 obtains 4 points and 2 obtains 1 point.

Team 1 against Team 3: 1 obtains 0 points and 3 obtains 6 points.

Team 2 against Team 3: 2 obtains 6 points and 3 obtains 0 points.

In this state of the world Team 2 wins the league because it gets 7 points. Teams 3 and 1 get 6 and 4 points, respectively.

In the second state of the world results are identical except for the following:

Team 1 against Team 3: 1 obtains 6 points and 3 obtains 0 points.

In this state of the world Team 1 wins the league because it gets 10 points. Teams 2 and 3 obtain 7 and 0 points, respectively.

Hence, the probability that Team 1 wins the league is 0.5. However, if Team 3 does not play and the results of each match are independent Team 1 wins the league with probability 1. Thus we see that the ratio of probabilities of success between Teams 1 and 2 are altered when Team 3 does not play the league.

We now consider the following homogeneity property:

(H) $\forall i \in N$, $p_i(\,)$ *is homogeneous of degree zero, i.e.,* $p_i(G) = p_i(\lambda G), \forall \lambda > 0$.

(H) says that the probability of obtaining the prize is independent of units of measurement—i.e., whether effort is measured in hours or minutes, or investments in dollars or euros. If effort means attention or work quality, the interpretation is less clear. The forms (2.1) with $\phi_i = G_i^{\epsilon}$ and (2.5) fulfill (H). The form (2.1) with the logit specification, (2.3) and (2.4) do not fulfill (H).

Skaperdas (1996) proved that (P1)–(P5) and (H) imply the form (2.1) with $\phi(G) = G^{\epsilon}, \epsilon \geq 0$. An unpleasant implication of (H) is that if $p_i(\,)$ is continuous in $(0, 0, \ldots, 0)$, $p_i(\,)$ is constant (Corchón 2000) which contradicts (P2). Thus under (H), $p_i(\,)$ is discontinuous. Skaperdas (1996) also studied the logit form. He showed that this form is equivalent to (P1)–(P5) plus an additional property that says that the probability of success of a player only depends on the difference in the effort of players. Clarke and Riis (1998) extended Skaperdas' results dropping the anonymity assumption.

2.2 Other foundations

Hillman and Riley (1989) offer a model of the political process where the impact of effort is uncertain. They derive a CSF of the form $\phi(G_i) = G_i^{\epsilon}$ only for the case of two contestants. Fullerton and McAfee (1999) and Baye and Hoppe (2003) offer micro-foundations for a subset of CSFs of the form $\phi(G_i) = G_i^{\epsilon}$ for innovation tournaments and patent races. Finally, Corchón and Dahm (2007) derive arbitrary CSF for the case of two contentands who have incomplete information about the type of the contest administrator. They argue that with three or more players, the form (2.1) is not likely to occur. Here uncertainty comes from the fact that the decision-maker can be of multiple types, and in the other models it comes from the actions of the contestants. Corchón and Dahm also interpret CSF as *sharing rules* and establish a connection to bargaining and claims problems. They prove that a generalization of the class of CSF given in (2.1) can be understood as the weighted Nash bargaining solution where efforts are the weights of the agents.

3 Symmetric contests

From now on, unless stated otherwise, we keep the functional form (2.1) plus the properties i)–v) stated there. We assume that the cost function $C_i : \Re_+ \to \Re_+$ is twice continuously differentiable, convex, strictly increasing with $C_i(0) = 0$ and C_i' bounded. Notice that these assumptions are similar to those made about $p(\,)$.

Now we present the following assumption:

Assumption 1 **a)** All agents have the same cost function $C(\,)$.
b) $V = V_0 + a\sum_{j=1}^{n} \phi(G_j)$, $V_0 > 0$, $a \geq 0$.
c) There exist $(\bar{y}, \delta)$ such that, for all $y > \bar{y}$, $a\phi'(y) - C'(y) < \delta < 0$.

The interpretation of part **b)** of Assumption 1 (A1 in the sequel) is that i values the prize for two reasons. An intrinsic component V_0 and another component reflecting aggregate merit. The parameter a is the marginal rate of substitution between aggregate merit and intrinsic value of the prize. The case where merits do not add value to the prize corresponds to $a = 0$. Part **c)** of A1 implies that when effort is very large, the ratio C'/ϕ' is larger than a. The reason for this assumption is that if a or the marginal impact of the action (ϕ') is large or the marginal cost of the action is small, there are incentives to increase the effort without limit. This assumption eliminates that possibility.

3.1 Existence, uniqueness and comparative statics

We are now ready to prove our first result:

Proposition 3.1 *Under A1, there is a unique Nash equilibrium. This equilibrium is symmetric.*

Proof Assuming interiority, first order conditions of payoff maximization are,

$$
\begin{aligned}
\frac{\partial \Pi_i}{\partial G_i} &= a\phi'(G_i)\frac{\phi(G_i)}{\sum_{j=1}^n \phi(G_j)} + \left(V_0 + a\sum_{j=1}^n \phi(G_j)\right) \\
&\quad \times \frac{\phi'(G_i)\sum_{r\neq i}\phi(G_r)}{(\sum_{j=1}^n \phi(G_j))^2} - C'(G_i) = 0, \\
&\quad \text{or,}\ \ V_0\frac{\phi'(G_i)\sum_{r\neq i}\phi(G_r)}{(\sum_{j=1}^n \phi(G_j))^2} = C'(G_i) - a\phi'(G_i), \quad i = 1, 2, \ldots, n.
\end{aligned}
\tag{3.1}
$$

The second order condition is fulfilled because (3.1) can be written as

$$
\frac{\partial \Pi_i}{\partial G_i} = V_0\frac{\phi'(G_i)\sum_{r\neq i}\phi(G_r)}{(\sum_{j=1}^n \phi(G_j))^2} - C'(G_i) + a\phi'(G_i),
$$

and all terms in the right hand side of the equation are decreasing in G_i, hence $\frac{\partial^2 \Pi_i}{\partial G_i^2} \leq 0$. This implies that (3.1) corresponds to a maximum. Therefore the existence of a Nash equilibrium is equivalent to showing that the system (3.1) has a solution. We first prove that such a system can only have symmetric solutions. Let $G_i = \min_{r\in N} G_r$ and $G_j = \max_{r\in N} G_r$. If the solution is not symmetric, $G_i < G_j$. Since the right hand side of (3.1) is increasing in G_i, we have that,

$$
\begin{aligned}
V_0\frac{\phi'(G_i)\sum_{r\neq i}\phi(G_r)}{(\sum_{j=1}^n \phi(G_j))^2} &= C'(G_i) - a\phi'(G_i) \leq C'(G_j) - a\phi'(G_j) \\
&= V_0\frac{\phi'(G_j)\sum_{r\neq j}\phi(G_r)}{(\sum_{j=1}^n \phi(G_j))^2}.
\end{aligned}
$$

Also, since $\phi(\ )$ is concave, $\phi'(G_j) \leq \phi'(G_i)$. Hence the previous equation implies $\sum_{r\neq i}\phi(G_r) \leq \sum_{r\neq j}\phi(G_r)$, which in turn implies $G_i \geq G_j$, a contradiction.

Let $y \equiv G_i, i = 1, 2, \ldots, n$. Now (3.1) can be written as

$$
\phi'(y)\left(a + V_0\frac{n-1}{\phi(y)n^2}\right) - C'(y) = 0. \tag{3.2}
$$

Let the left hand side of (3.2) be denoted by $\Psi(y)$. If $y \to 0$, $\Psi(y) > 0$, and if $y \to \infty$, A1c) and property iv) of $\phi(\cdot)$ imply $\Psi(y) < 0$. Therefore the mean value theorem implies that (3.1) has a solution that—by the previous reasonings—is a Nash equilibrium. Since $\Psi(\ )$ is strictly decreasing equilibrium is unique.

Lastly let us consider the case in which the first order condition does not hold with equality, i.e., $G_k^* = 0$ and $G_i^* > 0$ for some k and i. In this case, from (3.1), the concavity of $\phi(\,)$ and the convexity of $C(\,)$ we have that

$$0 \geq \phi'(0)\left(V_0\frac{\sum_{r\neq k}\phi(G_r)}{(\sum_{j=1}^n \phi(G_j))^2}+a\right)-C'(0) \geq \phi'(G_i)\left(V_0\frac{\sum_{r\neq k}\phi(G_r)}{(\sum_{j=1}^n \phi(G_j))^2}+a\right)-C'(G_i)$$
$$> \phi'(G_i)\left(V_0\frac{\sum_{r\neq i}\phi(G_r)}{(\sum_{j=1}^n \phi(G_j))^2}+a\right)-C'(G_i)=0\left(\text{since }\sum_{r\neq k}\phi(G_r)>\sum_{r\neq i}\phi(G_r)\right).$$

To end the proof notice that $G_i^*=0, \forall i$ is impossible because if an agent increases effort by a small quantity, she wins the prize at a cost as close to zero as we wish (because $C(0)=0$ and $C(\,)$ is continuous). Thus, this situation cannot be an equilibrium. □

The previous result was obtained by Nti (1997) assuming $a=0$ and $C_i(G_i)=G_i$. Szidarovsky and Okuguchi (1997) generalized this result considering a CSF like

$$p_i=\frac{\phi_i(G_i)}{\sum_{j=1}^n \phi_j(G_j)} \quad \text{when } \sum_{j=1}^n \phi_j(G_j)>0 \text{ and } p_i=\frac{1}{n} \text{ otherwise}, \tag{3.3}$$

where each $\phi_i(\,)$ fulfils the properties attributed to $\phi(\,)$ in Sect. 2. Notice that the form (2.1) is a special case of (3.3). The next section is devoted to study asymmetric contests.

Example 3.1 Pérez-Castrillo and Verdier (1992) studied the case in which $\phi_i=G_i^\epsilon$ allowing for $\epsilon>1$, i.e., $\phi(\,)$ is not necessarily concave. If $\epsilon\leq 1$, $a=0$ and $C(G_i)=cG_i$, from (3.2) we can derive an explicit formula for the equilibrium value of the effort and payoffs, namely

$$G_i^*=\frac{\epsilon(n-1)V}{n^2c} \quad \text{and} \quad \Pi_i^*=\frac{V(n-\epsilon(n-1))}{n^2}.$$

The aggregate cost of effort is $cny=\epsilon(n-1)V/n$. Notice that the aggregate cost of effort increases with n and if n is not small it is, approximately, ϵV. In the case studied by Tullock (1980), i.e., $\epsilon=1$, this amounts to V, i.e., *rents are dissipated* because the value of the prize equals the aggregate value of efforts.[7] We will see that this fact has important consequences for social welfare.

Let us now concentrate on comparative statics. First, we notice that our game can be transformed into an aggregative game (Corchón 1994) in which payoffs of each player depend on the strategy of this player and the sum of all strategies. Indeed, since

[7] Rent dissipation also assumes that efforts are completely wasted and that they have a positive opportunity cost. When the action of rent-seekers increases the utility of someone else—e.g., bribes—rents are said to be *transferred*.

payoffs for i are

$$\frac{\phi(G_i)}{\sum_{j=1}^n \phi(G_j)}\left(V_0 + a\sum_{j=1}^n \phi(G_j)\right) - C_i(G_i)$$

setting $x_i \equiv \phi(G_i)$ the previous expression can be written as

$$\frac{x_i}{\sum_{j=1}^n x_j}\left(V_0 + a\sum_{j=1}^n x_j\right) - C_i\left(\phi^{-1}(x_i)\right) \equiv \Pi_i\left(x_i, \sum_{j=1}^n x_j\right).^8$$

Unfortunately, results obtained in this class of games are non applicable here. The reason is that they require monotonic best reply functions: either decreasing—i.e., strategic substitution, Corchón (1994)—or increasing—i.e., strategic complementarity, Vives (1990), Milgrom and Roberts (1990), Amir (1996).[9] But in Example 1.1 we see that if $C_i = G_i$, $V_1 = V_2 = 1$, the best reply of i is $G_i = \sqrt{G_j} - G_j$, which is neither increasing, nor decreasing. Thus, there is no hope that in the general case such properties hold. Fortunately, our symmetry assumption allows us to obtain comparative statics results.

Proposition 3.2 *Under A1, the value of effort/investment in the Nash equilibrium is strictly increasing in a and V_0 and strictly decreasing in n.*

Proof Write (3.2) as

$$0 = \phi'(y)\left(a + V_0\frac{n-1}{\phi(y)n^2}\right) - C'(y) \equiv \Psi(y, a, n, V_0). \tag{3.4}$$

where as we noticed before, $\frac{\partial\Psi}{\partial y} < 0$. Differentiating implicitly (3.4),

$$\frac{dy}{da} = \frac{\frac{\partial\Psi}{\partial a}}{-\frac{\partial\Psi}{\partial y}} = \frac{\phi'}{-\frac{\partial\Psi}{\partial y}} > 0.$$

A similar argument proves that $\frac{dy}{dV_0} > 0$. Finally, writing (3.2) as follows

$$V_0\frac{n-1}{n^2} = \left(\frac{C'(y)}{\phi'(y)} - a\right)\phi(y),$$

we see that the left hand side is strictly decreasing in n and the right hand side is strictly increasing in y. Therefore, y and n vary in opposite directions and, thus, y is strictly decreasing in n. □

[8] Notice that this payoff function is identical to a profit function in which inverse demand reads $\frac{V_0}{\sum_{j=1}^n x_j} + a$ and the cost function is $C_i(\phi^{-1}(x_i))$ (Szidarovsky and Okuguchi 1997).

[9] The concepts of strategic substitution and complementarity are due to Bulow et al. (1985).

The previous result generalizes Nti (1997) to the case of $a > 0$ and non linear cost functions.

3.2 The choice between productive and contest activities

So far we have assumed that the number of contenders is given. A possible mechanism for determining n is to assume that agents have the choice of either entering into a contest or performing a productive activity (Krueger 1974). Assume for simplicity that the productive activity yields a net return of ρ, with $\rho \leq V$, that each contender regards as given. Under the assumptions made in Example 3.1 above, the payoff of a potential contender is $V(n - \epsilon(n-1))/n^2$. Free entry in both activities equalizes net returns and yields the equilibrium number of contenders, namely

$$n^* = \frac{V(1-\epsilon) + \sqrt{(1-\epsilon)^2 V^2 + 4\epsilon\rho V}}{2\rho}.$$

The condition $V \geq \rho$ guarantees that $n^* \geq 1$. As intuition suggests, the number of contenders depends positively on the value of the prize and negatively on the productivity of the productive sector which is a measure of the opportunity cost of participating in the contest.

An application of the above mechanism is that if a positive shock increases the supply of productive activities such that ρ falls, rent-seeking is fostered. For instance if the supply of a natural resource increases, this is, in principle, good news because the economy now has more resources. However, the effect of this positive shock on social welfare is ambiguous because the increase in the supply of productive activities is matched by an increase in wasteful expenditure of the rent-seeking sector since these expenditures are increasing in n. Under some conditions, the second effect prevails (Baland and Francoise 2000; Torvik 2002) giving rise to the so-called “Dutch disease”.[10]

4 Asymmetric contests

In this section we study the case in which agents are different and, in general, Nash equilibrium is not symmetric. The reason for studying this case, other than increasing generality, is that there are situations that can only occur in asymmetric contests. For instance:

[10] The term originated as follows: In the 1960s the discovery of large reserves of gas in the North Sea raised the value of the Dutch currency. This increased imports and decreased exports negatively affecting the domestic industry. The use of the term was generalized later on to describe negative effects on real variables—GDP, etc.—of an increase in natural resources. It has also been translated to political science where the term “Political Dutch Disease” refers to the correlation between the size of oil reserves and the degree of authoritarianism.

1) Some agents might make zero effort in equilibrium, i.e., be inactive. Agents whose effort is positive in equilibrium will be called *active*.[11]
2) Agents with higher valuations/lower costs may obtain the prize with higher probability than the rest. This implies that in some cases—like the procurement example in Sect. 1—there is a positive relationship between rent-seeking and efficiency, a point to recall when discussing the social desirability of contests.
3) Some agents may be better off as a consequence of the contest. In a symmetric contest all contenders are better off if the contest is banished since they incur a positive cost simply to maintain the probability of obtaining the prize.

4.1 Basic properties of the model

In order to concentrate on the issues raised by asymmetries we will assume in this section that the value of the prize does not depend on efforts, that is $\alpha = 0$. Let us start by assuming that the CSF is of the form (3.3). Then,

$$\Pi_i = \frac{\phi_i(G_i)}{\sum_{j=1}^{n} \phi_j(G_j)} V_i - c_i(G_i).$$

Set $y_i \equiv \phi_i(G_i)$. Since $\phi_i(\,)$ is strictly increasing, it can be inverted. Set $c_i(\phi^{-1}(y_i)) \equiv Q_i(y_i)$. Then,

$$\Pi_i = \frac{y_i}{\sum_{j=1}^{n} y_j} V_i - c_i(\phi^{-1}(y_i)) = \frac{y_i}{\sum_{j=1}^{n} y_j} V_i - Q_i(y_i)$$

By a well-known result, NE are independent of linear transformations in payoffs. Dividing the previous expression by V_i and setting $\frac{Q_i(y_i)}{V_i} \equiv K_i(y_i)$, payoffs are now

$$\frac{y_i}{\sum_{j=1}^{n} y_j} - \frac{Q_i(y_i)}{V_i} = \frac{y_i}{\sum_{j=1}^{n} y_j} - K_i(y_i)$$

Thus, under (3.3) lack of symmetry in the contest success function can be translated to lack of symmetry in the cost function.

In the next result we will assume that the functions $K_i(\,)$'s are linear, see Cornes and Hartley (2005) for the non linear case.

Assumption 2 $K_i(y_i) = d_i y_i$, $d_i > 0$, $\forall i \in N$.

Notice that because $\alpha = 0$, A2 implies A1c). Without loss of generality set $d_1 \leq d_2 \leq \cdots \leq d_n$. There are two interpretations of A2. In the first one the CSF is $\phi(G_i) = G_i$ and agents have different costs/valuations reflected in different d's. In this case, $y_i = G_i$ and $d_i = c_i/V_i$ (see Hillman and Riley 1989). In the second

---

[11] If $\phi(\,)$ is not concave, Nash equilibrium may entail non active agents even under symmetry assumptions, see Pérez-Castrillo and Verdier (1992).

interpretation, the contest success function is a special case of the one proposed by Gradstein (1995), namely

$$
\begin{aligned}
p_i &= \frac{q_i \phi(G_i)}{\sum_{j=1}^{n} q_j \phi(G_j)} \quad \text{if } \sum_{j=1}^{n} q_j \phi(G_j) > 0 \\
p_i &= q_i \quad \text{if } \sum_{j=1}^{n} \phi(G_j) = 0.
\end{aligned}
\tag{4.1}
$$

where q_i can be interpreted as the prior probability that agent i wins the prize. Assume that $\phi(G_i) = G_i$ and agents are identical in costs and valuations. Denoting the marginal cost of effort by c we have that $d_i = \frac{c}{V q_i}$ and $G_i = \frac{y_i}{q_i}$.

Proposition 4.1 *Under A2 and (3.3) there is a unique Nash equilibrium. There is an $m \leq n$ such that all agents $i = 1, \ldots, m$ with $\sum_{j=1}^{m} d_j > d_i(m-1)$ are active and all agents $i = m+1, \ldots, n$ with $\sum_{j=1}^{m} d_j \leq d_i(m-1)$ are not active.*

Proof First notice that the set of agents for which $\sum_{j=1}^{m} d_j > d_i(m-1)$ has no "holes", i.e., if agent k belongs to this set, agent $k-1$ also belongs since $\sum_{j=1}^{m} d_j > d_k(m-1) > d_{k-1}(m-1)$, given that $d_{k-1} < d_k$.

Consider the following algorithm that begins with agent n and continues in decreasing order. If $\sum_{j=1}^{k} d_j \leq d_k(k-1)$, we go to agent $k-1$. If $\sum_{j=1}^{k} d_j > d_k(k-1)$, the algorithm stops and yields $m = k$. The algorithm stops before $k = 1$ because for $k = 2$, $d_1 + d_2 > d_2$. As we will see, this algorithm identifies active agents.

First order conditions of payoff maximization for $i = 1, \ldots, m$ are

$$
\frac{\partial \Pi_i}{\partial y_i} = \frac{\sum_{j \neq i} y_j}{(\sum_{j=1}^{m} y_j)^2} - d_i = 0, \quad \text{or} \quad \frac{\sum_{j \neq i} y_j}{(\sum_{j=1}^{m} y_j)^2} = d_i. \tag{4.2}
$$

It is easy to see that $\frac{\partial \Pi_i}{\partial y_i}$ is decreasing in y_i. Thus second order conditions hold.

Adding up (4.2) over 1 to m, we have that $(m-1)\sum_{j=1}^{m} y_j = (\sum_{j=1}^{m} y_j)^2 \sum_{j=1}^{m} d_j$. From there and (4.2) again we get that

$$
y_i^* = \frac{m-1}{\sum_{j=1}^{m} d_j}\left(1 - \frac{d_i(m-1)}{\sum_{j=1}^{m} d_j}\right), \quad i = 1, \ldots, m. \tag{4.3}
$$

which yields the effort of active agents. Notice that $y_i^* > 0$ because i belongs to the set for which $\sum_{j=1}^{k} d_j > d_k(k-1)$. For any other agent, say r the marginal payoff evaluated in $y_r = 0$ is

$$
\frac{\partial \Pi_r}{\partial y_r} = \frac{\sum_{j=1}^{m} y_j}{(\sum_{j=1}^{m} y_j)^2} - d_r = \frac{\sum_{j=1}^{m} d_j}{m-1} - d_r \leq 0. \tag{4.4}
$$

Thus, $y_r = 0$ is the optimal action of this agent.

We will now prove that the previous equilibrium is unique. Let us consider an arbitrary equilibrium. The first order condition is,

$$\frac{\partial \Pi_i}{\partial y_i} = \frac{\sum_{j \neq i} y_j^*}{(\sum_{j=1}^n y_j^*)^2} - d_i \leq 0 \quad \text{and if strict inequality holds, } y_i^* = 0.$$

Let $M \subseteq N$ be the set of active agents. For $i \in M$, we have that

$$\frac{\sum_{j \neq i} y_j^*}{(\sum_{j=1}^n y_j^*)^2} = d_i = \frac{\sum_{j=1}^n y_j^* - y_i^*}{(\sum_{j=1}^n y_j^*)^2}.$$

Again, we see that the set of active agents cannot have “holes” because if i is active and h is such that $d_h < d_i$ and $y_h^* = 0$, we had

$$\frac{\sum_{j=1}^n y_j^* - y_i^*}{(\sum_{j=1}^n y_j^*)^2} = d_i > d_h \geq \frac{\sum_{j=1}^n y_j^*}{(\sum_{j=1}^n y_j^*)^2}$$

which is impossible. Suppose now that there are two equilibria. In the first, agents 1 to k are active and in the second, agents 1 to h are active, with $h > k$. Thus agent h is not active in the first equilibrium but is active in the second. By the previous reasonings this implies

$$\frac{\sum_{j=1}^k d_j}{k-1} - d_h \leq 0 \quad \text{and} \quad \frac{\sum_{j=1}^h d_j}{h-1} - d_h > 0 \Rightarrow \frac{\sum_{j=k+1}^h d_j}{h-k} > d_h,$$

which is impossible because if agents are ordered in such a way that $d_i \leq d_{i+1}$, d_h is larger than the average of d's from d_{k+1} to d_h. Thus $k = m$. □

Under the first interpretation, recall that $y_i = G_i$ and $d_i = c_i / V_i$. Thus, from (4.3) and the form of the contest success function used here,

$$\begin{aligned} G_i^* &= \frac{m-1}{\sum_{j=1}^m c_j / V_j} \left(1 - \frac{c_i(m-1)}{V_i \sum_{j=1}^m c_j / V_j}\right), \\ p_i^* &= \frac{G_i^*}{\sum_{j=1}^n G_j^*} = 1 - \frac{c_i(m-1)}{V_i \sum_{j=1}^m c_j / V_j} \end{aligned} \tag{4.5}$$

Thus, agents who are more efficient (i.e., with lower c's, or larger V's) make more effort and have a greater probability of getting the prize than inefficient agents.[12]

Suppose $n = 2$ and $c_1 = c_2 = 1$. Expected payoffs for contender 1 in equilibrium are $\frac{V_1^3}{(\sum_{j=1}^2 V_j)^2}$. Since expected payoffs under no contest are $V_1/2$ the former are larger

[12] The equilibrium values of G_i's and p_i's depend on the ratio c_i/V_i and the harmonic mean of the ratios of cost/valuations defined as $\frac{m}{\sum_{j=1}^m c_i / V_j}$.

than the latter iff $V_1 > V_2(1+\sqrt{2})$. In this case the player who values the prize the most is better off as a consequence of the contest.

Under the second interpretation, recall that $d_i = c/(Vq_i)$ and $G_i = y_i/q_i$ Thus, from (4.3) and the form of the contest success function used here,

$$G_i^* = \frac{V(m-1)}{cq_i \sum_{j=1}^{m} 1/q_j}\left(1 - \frac{1/q_i(m-1)}{\sum_{j=1}^{m} 1/q_j}\right),$$
$$p_i^* = \frac{q_i G_i^*}{\sum_{j=1}^{n} q_j G_j^*} = 1 - \frac{1/q_i(m-1)}{\sum_{j=1}^{m} 1/q_j}. \tag{4.6}$$

Thus, more optimistic agents, (i.e., agents with large q_i's) make less effort and have a greater probability of getting the prize than pessimistic agents (i.e., those with small q_i's).[13]

If $n = 2$, $G_i^* = \frac{q_1 q_2 V}{c}$, $i = 1, 2$, i.e., Nash equilibrium is symmetric despite the fact that the contest success function is not. Moreover, $p_i^* = \frac{1/q_j}{\sum_{j=1}^{2} 1/q_j} = q_i$, i.e., prior and posterior probabilities coincide. We now study whether this result is generalizable to more general contest success function. Write $p_i = p_i(G_1, G_2, q_1, q_2)$. Assume a property that we discussed in Sect. 2, namely that $p_i(\cdot, \cdot, q_1, q_2)$ is homogeneous of degree zero in (G_1, G_2) and let d's be as in the first interpretation:

Proposition 4.2 *Under H, $n = 2$ and A.2, $G_1^* = G_2^*$ iff $d_1 = d_2$.*

Proof Consider first order conditions of payoff maximization for $i = 1, 2$:

$$\frac{\partial p_i}{\partial G_i} V_i - c_i = 0 \Leftrightarrow \frac{\partial p_1}{\partial G_1} - d_1 = 0 = \frac{\partial p_2}{\partial G_2} - d_2$$

From H, and $p_1 = 1 - p_2$ we get that

$$\frac{\partial p_1}{\partial G_1} G_1^* + \frac{\partial p_1}{\partial G_2} G_2^* = \frac{\partial p_1}{\partial G_1} G_1^* - \frac{\partial p_2}{\partial G_2} G_2^* = 0$$

From these two equations we obtain $G_1^* d_1 = G_2^* d_2$ and hence the result. □

Thus, if cost functions and valuations are identical for the two contenders, they make the same effort in the contest regardless of their priors or any other factor affecting the contest success function. Under the additional assumption that $p_i > q_i$ iff $G_1 > G_2$ (an assumption fulfilled by (2.1)) the previous argument shows that $p_1^* = q_1$ *iff* $d_1 = d_2$, see Corchón (2000).[14] Unfortunately, this result is not generalizable to games with more than two players. Recall that

$$p_i^* = 1 - \frac{1/q_i(m-1)}{\sum_{j=1}^{m} 1/q_j}.$$

[13] Here, equilibrium values of G_i's and p_i's depend on the harmonic mean of q_i's.

[14] In this paper it is shown that the conditions of Proposition 4.2 plus some mild requirements guarantee the existence of a Nash equilibrium for $n = 2$.

For instance, assuming $n = 3$ and $q = (0.375, 0.375, 0.25)$, $p^* = (0.43, 0.43, 0.14)$, i.e., prior and posterior probabilities do not coincide. However, from the formula above, we see that the ranking of prior and posterior probabilities is the same. In Corchón (2000) it is shown that this property holds in more general models. See Gradstein (1995), Baik (1998), Nti (1999) and Fang (2002) for further study of comparative statics when contest success functions are not symmetric.

4.2 Contests between groups

So far we have assumed that individual agents are the actors in the contests. But many times actors are associations of individuals who share a common objective, e.g., a law protecting the environment, a certain public decision, etc. In such a case the well-known free rider problem raises its ugly head: each member of the group will attempt to shift painful duties—effort, contributions—to other members in the same group. In some cases the group might be able to maintain discipline and enforce the optimal policy by means of punishments, ostracism, etc. But, in general, the optimal policy of the group will be difficult to maintain, because this maintenance will be a source of problems. Thus, let us adopt the point of view that inside each group, effort/money is supplied on a voluntary basis.

Let us present a model of a contest between two groups. The extension to more groups is straightforward from the formal point of view and not very relevant given that most conflicts in real life involve only two groups.

Let us add the following items to the previous notation. There are two groups denoted by $\mathcal{G}_1$ and $\mathcal{G}_2$ with n_1 and n_2 members, respectively. Total effort exercised by members of the first group will be denoted by $X \equiv \sum_{i \in \mathcal{G}_1} G_i$. Similarly, let the total effort made by the members of the second group be denoted by $Y \equiv \sum_{j \in \mathcal{G}_2} G_j$. The probability that group 1 wins the contest is denoted by $p(X, Y)$ where $p(\,)$ is increasing on X. Payoffs for an agent of group 1, say i, and an agent of group 2, say j, are $\Pi_i = p(X, Y)V_i - C_i(G_i)$ and $\Pi_j = (1 - p(X, Y))V_j - C_j(G_j)$. As before, a Nash equilibrium is a list of efforts such that each agent chooses effort to maximize her payoffs given the efforts decided by other agents, inside and outside her group. Let X^* and Y^* be the Nash equilibrium values of X and Y. We will not be concerned with existence or uniqueness of equilibrium (similar assumptions to those used before will do the job). Instead we will be concerned with the properties of equilibrium. These will be derived from first order conditions of payoff maximization that for active agents read:

$$\frac{\partial p(X^*, Y^*)}{\partial X} V_i = C_i'(G_i^*), i \in \mathcal{G}_1 \quad \text{and} \quad -\frac{\partial p(X^*, Y^*)}{\partial Y} V_j = C_j'(G_j^*), j \in \mathcal{G}_2. \tag{4.7}$$

In a classic contribution, Olson (1965) asserted that the free rider problem inside large groups is so acute that, in equilibrium, large groups exert *less* aggregate effort than small groups, which explains the success of the latter. We will examine his conjecture in the framework of our model.

We easily see in (4.7) that if costs are linear, X^* and Y^* do not depend on the number of agents inside each group. So, let us assume that $C_r'' > 0$, for all $r \in N$. We have seen that efforts in equilibrium depend on valuations and costs. So, in order to isolate the effect of the number of individuals in each group let us assume that valuations and cost functions are identical, denoted by V and $C(\,)$. From (4.7) it is clear that equilibrium is symmetric inside each group, so $G_i^* = X^*/n_1\ \forall i \in \mathcal{G}_1$ and $G_j^* = Y^*/n_2 \forall j \in \mathcal{G}_2$. Hence (4.7) can be written as

$$\frac{\partial p(X^*, Y^*)}{\partial X} V = C'\left(\frac{X^*}{n_1}\right) \quad \text{and} \quad -\frac{\partial p(X^*, Y^*)}{\partial Y} V = C'\left(\frac{Y^*}{n_2}\right) \tag{4.8}$$

Now we have the following:

Proposition 4.3 *Assume* (H)*, identical valuations and costs and* $C'' > 0$*. Then* $n_1 > n_2$ *implies* $X^* > Y^*$ *and* $G_i^* < G_j^*$ $\forall i \in \mathcal{G}_1$ *and* $\forall j \in \mathcal{G}_2$*.*

Proof Suppose that $X^* \leq Y^*$ and $n_1 > n_2$. Then, $X^*/n_1 < Y^*/n_2$ and given that $C'()$ is increasing $C'(X^*/n_1) < C'(Y^*/n_2)$. From (4.8) we get that

$$\frac{\partial p(X^*, Y^*)}{\partial X} < -\frac{\partial p(X^*, Y^*)}{\partial Y}$$

From (H), $p(\,)$ increasing in X and $X^* \leq Y^*$ we get that

$$\frac{\partial p(X^*, Y^*)}{\partial X} X^* = -\frac{\partial p(X^*, Y^*)}{\partial Y} Y^* \Rightarrow \frac{\partial p(X^*, Y^*)}{\partial X} \geq -\frac{\partial p(X^*, Y^*)}{\partial Y}$$

which contradicts the equation above. Thus $X^* > Y^*$.

Let us now prove the result regarding individual efforts. From (H) and $X^* > Y^*$ using (4.8) we obtain that

$$C'\left(\frac{X^*}{n_1}\right) = \frac{\partial p(X^*, Y^*)}{\partial X} V < -\frac{\partial p(X^*, Y^*)}{\partial Y} V = C'\left(\frac{Y^*}{n_2}\right)$$

which given that $C'(\,)$ is increasing, implies the desired result. □

Proposition 4.3 is due to Katz et al. (1990), see also Nti (1998). The conclusion is that, contrary to Olson's conjecture, the success of small groups cannot be traced to the larger effort made by their members. Our theory predicts that success in a contest is explained by large valuations, small costs or contest success functions that favor certain agents, see the discussion after Proposition 4.1. Esteban and Ray (2001) offer an interesting twist to the previous argument—and a partial vindication of Olson's conjecture—by assuming that $V_i = V/n_i^{\alpha}$, where $0 \leq \alpha \leq 1$. When $\alpha = 0$ the object is a pure public good—which is the case considered before—and when $\alpha = 1$ the object is a pure private good. Thus α is a measure of congestion ranging from no congestion—when the value of the prize is independent of the number of people in the winning group—to total congestion, where the private value of the prize is measured on a per capita basis. An example of the first is a law, and an example of the second is a monetary prize. Notice that, except when $\alpha = 0$, the smaller the group the larger the prize and—as the theory developed so far suggests—the larger the effort. Thus, this private good aspect of the prize generates a counterbalancing force to the one studied in the previous proposition. Esteban and Ray provided the conditions for this private good aspect to be strong enough to overcome the previous result.

Proposition 4.4 *Assume* (H) *and* $C_i = cG_i^{\beta}$ *with* $\beta \geq 1$. *Then, the smaller group makes more effort than the larger group if and only if* $\alpha + 1 > \beta$.

Proof First order condition of profit maximization read

$$\frac{\partial p(X^*, Y^*)}{\partial X} V_1 = c\beta \left(\frac{X^*}{n_1}\right)^{\beta-1} \quad \text{and} \quad -\frac{\partial p(X^*, Y^*)}{\partial Y} V_2 = c\beta \left(\frac{Y^*}{n_2}\right)^{\beta-1}.$$

From the equations above and (H) we get that

$$\frac{V_1 Y^*}{V_2 X^*} = \frac{\left(\frac{X^*}{n_1}\right)^{\beta-1}}{\left(\frac{Y^*}{n_2}\right)^{\beta-1}}.$$

Taking into account that $V_i = V/n_i^{\alpha}$ the equation above reads

$$\frac{n_2^{\alpha} Y^*}{n_1^{\alpha} X^*} = \frac{\left(\frac{X^*}{n_1}\right)^{\beta-1}}{\left(\frac{Y^*}{n_2}\right)^{\beta-1}} \iff \frac{Y^*}{X^*} = \left(\frac{n_1}{n_2}\right)^{\frac{\alpha-\beta+1}{\beta}}.$$

W.l.o.g. assume that $n_1 > n_2$. Then, from the previous equation, $X^* < Y^* \iff (\frac{n_1}{n_2})^{\frac{\alpha-\beta+1}{\beta}} > 1 \iff \alpha + 1 > \beta$ which proves the first claim. □

Proposition 4.3 corresponds to the case of $\alpha = 0$ (though under more general assumptions). In this case the necessary and sufficient condition above does not hold and hence the result. The most favorable case for the Olson conjecture is when $\alpha = 1$ (i.e., when the prize is a pure private good) but even in this case costs cannot have an exponent larger than two (i.e., quadratic). However if the actual contest is fought by external agents—lawyers, politicians—whose price per unit of effort is given, the cost function is linear—i.e., $\beta = 1$—and Olson conjecture holds for all values of α except for the extreme case of $\alpha = 0$.

Notice the key role of the elasticity of costs with respect to effort, β. Intuitively, it is clear that Olson's conjecture cannot hold if costs rise very quickly with effort: for instance if costs are zero up to a point, say $\bar{G}$ where they jump to infinity, all agents will make effort $\bar{G}$ and smaller groups will exert less effort than large ones.

Finally we notice that if the contest success function were symmetric, in the sense that the group that makes more effort wins the prize with greater probability, Proposition 4.4 implies that the smaller group has better chances of getting the prize, if and only if $\alpha + 1 > \beta$.

4.3.1 Litigation

Farmer and Pecorino (1999) compare British and American systems of financing legal expenditures. In the American system each party pays its own expenses in advance. In the British system the loser pays it all. They find that in the American system the equilibrium is symmetric, and prior and posterior probabilities of winning the trial coincide. This is a special case of Proposition 4.2, where we have seen that the result needs identical ratio of marginal costs/valuation. Under the British system payoffs look like

$$\Pi_i = p_i(G, q)V - (1 - p_i(G, q))(c(G_1) + c(G_2))$$

Computing equilibrium for suitable functional forms we find that, in general, prior and posterior do not coincide. Thus, the American system appears to be "less biased" than its British counterpart, at least in the case of identical costs/valuations.

4.3.2 Allocation of rights

Nugent and Sánchez (1989) discuss the conflict in Spain between migrant shepherds—organized in a syndicate called La Mesta—and agricultural settlers during the Middle Ages and beyond. The conflict involved the right of way and pasture of the shepherd. The Spanish crown systematically favored shepherds. Some historians link the decadence of Spain to this policy. Nugent and Sánchez (see also Ekelund et al. 1997) point out that if the allocation of way and pasture rights were a contest, the agent with the highest valuation spends more money and wins the contest with the highest probability, see our comments below (4.5). Indeed, it turns out that La Mesta channelled large quantities of gold into royal pockets. Thus, it can be argued that value added by shepherds was larger than the value added by agriculture and that the crown pursued the right policy.[15]

4.3.3 Insurrections and conflicts

Sánchez-Pagés (2006) has provided a twist to the argument against the futility of conflicts. He shows that conflict can enhance efficiency in the long run. The reason is that if current holders use a resource inefficiently—e.g., they over-exploit a natural resource—a group that would manage the resource more efficiently may have incentives to promote a conflict with current owners. From their point of view, conflict pays off because its costs are overcomed by the value of the resource and the high probability of winning as a consequence of the latter, see (4.6) above.

[15] This can be objected on two counts. First, the outcome may reflect the superior organization of shepherds with respect to farmers. Second, for reasons of their immediate needs, kings may have not taken into account the long run negative effect of shepherding on the environment.

Grossman (1991) has modeled insurrections as a contest where the probability of a revolution depends on the military might of the group in power and the number of insurrect. The former is financed by a tax paid by peasants. They can choose between joining the insurrection or staying as peasants. There is free entry, so in equilibrium, payoffs obtained in both activities must be equal. The group in power chooses the tax rate in order to maximize the probability of staying in power. The basic trade-off for the incumbent ruler is that high (resp. low) taxes allow for a powerful (resp. weak) army but they do (resp. do not) give incentives for insurrection because they lower (resp. raise) payoffs of peasants.

4.3.4 Divisionalized firms

Scharsftein and Stein (2000) studied rent-seeking in divisionalized firms. In these firms many decisions, like pricing, are taken by the managers of divisions and only long run decisions, like the internal allocation of capital, are taken by a central manager. Suppose that the internal allocation of capital depends on the rent-seeking activities made by the managers of divisions. Managers make effort in rent seeking and a productive activity. For simplicity, assume that the marginal net return of the latter, denoted by ρ_i, is exogenous. Efficient divisions have higher ρ_i's. The rational use of effort by the manager of division i is to equalize the marginal return of effort in both rent-seeking and productive activities, i.e.,

$$\frac{\partial \Pi_i}{\partial y_i} = \frac{\sum_{j \neq i} y_j}{(\sum_{j=1}^{m} y_j)^2} - d = \rho_i, \quad \text{or} \quad \frac{\sum_{j \neq i} y_j}{(\sum_{j=1}^{m} y_j)^2} = \rho_i + d \equiv d_i. \tag{4.9}$$

Equilibrium is identical to that in Proposition 4.1. Notice that (4.9) implies that managers with higher productivity have a higher cost of rent-seeking. Thus, if p_i is the fraction of funds allocated by the centre, divisions with high productivity receive fewer funds than those with low productivity, see (4.5). This points to a disturbing conclusion: in organizations where internal allocation of a resource is made by rent-seeking, productive agents will obtain less than unproductive ones.

4.4 Rent-seeking, institutions and economic performance

Suppose that there are two sectors: rent-seeking and production of a socially valuable item. Rent-seekers "prey" on producers by stealing, imposing taxes, etc. A free entry condition—which we have encountered in previous sections—determines the number of agents in each sector. Papers in this area differ in the mechanism of prey and fall into three categories.

1. *Random encounters with bandits*: Agents either produce a good or to steal those producing the good. The latter will be called bandits but they also could be interpreted as corrupted civil servants. Any producer may encounter a bandit in which case she looses a fixed part of her output. Let q be the proportion of bandits in the population. Expected returns of a producer, denoted by RP, are a decreasing function of q because when bandits are a few (resp. many) the probability of encounter one of them

is low (resp. high). Expected returns to a bandit, denoted by RB, are also a decreasing function of q because when there are many (resp. few) producers it is easy (resp. difficult) to find one. The proportion of bandits is in equilibrium when $RP = RB$. It is not difficult to obtain multiple equilibria because both functions have negative slope with respect to q (Acemoglu 1995). Murphy et al. (1991) showed that if talent is necessary for growth an economy can be trapped in a low growth path in which talented individuals work in rent-seeking activities. In these models two economies with the same basic data can be in equilibria that are very far apart.

These models formalize the idea that an economy may get into a poverty trap in which rent-seeking is determined by economic fundamentals. However, they imply that there is nothing virtuous in rich economies—e.g., Northern European countries-and nothing wrong in poor ones—Sub-Saharan countries. In fact all countries are essentially identical. It is simply a matter of being lucky or unlucky.

2. *Institutional rent-seeking*: The previous model does not pay sufficient attention to the question of institutions that make Northern European and Sub-Saharan countries so different. The background of the previous model is one of a weak government but this is not modelled. In contrast, the literature here emphasizes the connection between institutions, rent-seeking and economic performance.

North and Weingast (1989) discuss the events surrounding the Glorious Revolution in Great Britain in 1688. They argue that under absolute monarchy, it was "very likely…that the sovereign will alter property rights for his…own benefit" (id. p. 803). The methods were taxes unapproved by the Parliament, unpaid loans, sale of monopoly and peerage, purveyance or simply seizure. All these promoted rent-seeking activities that diverted potentially useful talents away from productive business. With a Parliament dominated by "…wealth holders, its increased role markedly reduced the king's ability to renege" (id. p. 804). Countries in which the Parliament was not strong, "…such as early modern Spain, created economic conditions that retarded long-run growth" (id, p. 808).[16]

3. *Governance and rent-seeking*: There is little doubt that in the case of seventeenth century Britain, Parliament played a prominent role in providing the basis for a sound economic performance. But according to Buchanan and Tullock (1962) and Olson (1982), parliaments can foster rent-seeking activities. Also, casual empiricism suggests that countries that experienced no institutional change dramatically altered their growth rates: Spain (1950–1959 vs. 1960–1974), India (1950–1992 vs. 1993–2005) and China, (1950–1975 vs. 1976–2005).[17] In these cases the *policies* pursued in the contrasting periods were very different but the basic institutions remained practically

[16] The question is why the Parliament "…would not then proceed to act just like the king?" (id. p. 817). On the one hand the coordination necessary for this made "…rent-seeking activity on the part of both monarch and merchants more costly" (Ekelund and Tollinson 1981). On the other hand, the legislative changes introduced by the Glorious Revolution made rent-seeking very difficult. Judges were elected from among prominent local people who had little incentive to punish those locals who defied monopoly laws selling goods at cheaper prices (Tullock 1992).

[17] Despite the similar experiences in terms of growth, these countries were politically very different: Spain was a right-wing dictatorship, India a democracy and China a left-wing dictatorship.

the same.[18] In other words, institutions do not determine policies univocally. This point has been made by Glaezer et al. (2004). They examine the existing empirical evidence and find little impact by institutions per se but a large impact by policies. See Gradstein (2004) for a dynamic model of evolution of a particular policy, namely that of protection of property rights.

Corchón (2007) offers a model where the connection between institutions and policies is explicitly addressed. There are two possible institutions: autocracy where taxes are set by the king and Parliament rule where taxes are decided by majority voting. Productive agents are taxed in order to finance the rent-seeking activities. Under parliament rule there is an equilibrium in which there are no rent-seekers. This equilibrium captures the idea that the Parliament wips out rent-seekers. Unfortunately under not implausible assumptions there is another equilibrium in which the Parliament is dominated by rent-seekers and the tax rate is identical to that under absolute monarchy. In this equilibrium the size of rent-seeking is larger than under autocracy. This cast doubts on the idea that "right" institutions necessarily promote good economic performance. Finally, it is shown that rent-seekers may be interested in overthrowing autocracy.[19]

5 Social welfare under rent-seeking

In this section we provide a new look to two well-known problems: welfare losses under monopoly and the Coase theorem with transaction costs. If property rights are undefined we have contests for monopoly and property rights. We show that classical welfare analysis is misleading because it does not consider the welfare loss due to this contest. We will see that these welfare losses may overwhelm welfare losses arising from standard misallocation.

5.1 The fight for a monopoly right

Tullock (1967) and Krueger (1974) pointed out that we have two kind of welfare losses associated with a distortion such as a monopoly, tariffs, quotas, etc. On the one hand the classical ones, measured by the welfare loss of the distortion. But once the prize is created there is a contest in which agents fight over it. This fight is costly and this cost must be added to the classical welfare loss in order to get a fair picture of the total costs produced by the distortion. This is of practical importance given the low estimates of welfare losses associated with monopoly that were found by Harberger (1954) and many subsequent papers.

We will present a simple example that highlights this point and generalizes results obtained by Posner (1975). We assume that in a market there is a single consumer

[18] The change in the growth rate was so sudden and permanent that these cases cast doubts on the theories of growth based on human capital.

[19] This conclusion can be applied to the process of decolonization and suggests a reason for local rent-seekers to fight against colonial powers.

with a utility function

$$U = \hat{a}x - \frac{b}{\alpha+1}x^{\alpha+1} - px, \quad \text{with } \hat{a} \geq 0, \alpha b > 0 \text{ and } \alpha > -1.$$

x and p are the output and the market price of the good.[20] The consumer maximizes utility taking p as given. Since $\frac{\partial^2 U}{\partial x^2} = -\alpha b x^{\alpha-1} < 0$, utility is concave on output. Thus, the first order condition of utility maximization yields the inverse demand function, namely $p = \hat{a} - bx^{\alpha}$. If $b > 0$ and $\alpha = 1$ this function is linear. If $\hat{a} = 0$, $b < 0$ and $\alpha < 0$ this function is isoelastic.

The Monopolist produces under constant marginal costs, denoted by k. Let $a \equiv \hat{a} - k$. The monopolist profit function reads $\pi = (a - bx^{\alpha})x$. This function is concave because $\frac{\partial^2 \pi}{\partial x^2} = -b\alpha x^{\alpha-1}(\alpha+1) < 0$. The first order condition of profit maximization yields the monopolist output and profits, namely

$$x^E = \left(\frac{a}{b(1+\alpha)}\right)^{\frac{1}{\alpha}} \quad \text{and} \quad \pi = \left(\frac{a}{b(\alpha+1)}\right)^{\frac{1}{\alpha}} \frac{a\alpha}{\alpha+1}.$$

The socially optimal allocation is found by maximizing social welfare defined as the sum of consumer and producer surpluses, i.e.,

$$W = U + \pi = \hat{a}x - \frac{b}{\alpha+1}x^{\alpha+1} - kx = ax - \frac{b}{\alpha+1}x^{\alpha+1}$$

This function is concave because $\frac{\partial^2 W}{\partial x^2} = -b\alpha x^{\alpha-1} < 0$. The first order condition of welfare maximization yields the optimal output

$$x^O = \left(\frac{a}{b}\right)^{\frac{1}{\alpha}}.$$

Evaluating social welfare in the optimum (W^o) and the equilibrium allocations (W^E) we obtain that

$$W^0 = \left(\frac{a}{b}\right)^{\frac{1}{\alpha}} \frac{a\alpha}{1+\alpha} \quad \text{and} \quad W^E = \left(\frac{a}{b(1+\alpha)}\right)^{\frac{1}{\alpha}} \frac{a\alpha(2+\alpha)}{(1+\alpha)^2}$$

Denoting by RM the relative welfare loss due to misallocation in the market of the good, we have that

$$RM \equiv \frac{W^O - W^E}{W^O} = 1 - \left(\frac{1}{1+\alpha}\right)^{\frac{1}{\alpha}} \frac{2+\alpha}{1+\alpha}.$$

[20] α is a measure of the curvature of demand function (inverse demand is concave iff $\alpha \geq 1$). b is an inverse measure of the size of the market since the maximum welfare is obtained when $x = ((\hat{a} - t)/b)^{\frac{1}{\alpha}})$. The slope of the demand function is determined by the sign of $-\alpha b$ and thus, it is negative.

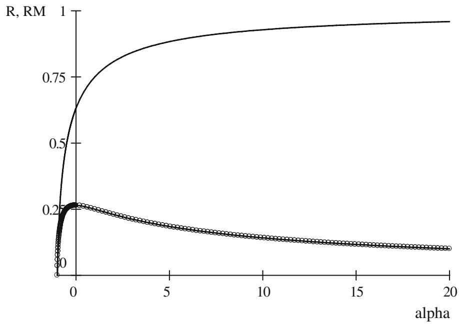

Fig. 1

The dotted line in Fig. 1 below plots the values of RM as a function of α. For instance, for values of $\alpha = 1$ (the case analyzed by Posner 1975) or $\alpha = -0.5$, $RM = 0.25$. See Hillman and Katz (1984) for the case of risk averse agents where risk aversion lowers efforts and welfare losses.

If the monopoly right is subject to rent-seeking, agents incur on unproductive expenses in order to obtain the prize. Assuming that rents are completely dissipated in wasted effort—recall our discussion in Sect. 2—profits equal unproductive expenses and thus become a welfare loss as well. Graphically, instead of the classical triangle—as in Harberger—welfare loss becomes a trapezoid—the so-called Tullock's trapezoid. Denoting the relative welfare loss by R we have that

$$R = \frac{\pi + W^O - W^E}{W^O}.$$

Notice that

$$\pi = \frac{W^O - W^E}{(1+\alpha)^{\frac{1}{\alpha}} - \frac{\alpha+2}{\alpha+1}}.$$

Manipulating the previous expressions we obtain the following:

Proposition 5.1 *In the example above and assuming complete wasteful rent dissipation, relative welfare loss associated with monopoly is*

$$R = \left(1 - \left(\frac{1}{1+\alpha}\right)^{\frac{1}{\alpha}} \frac{2+\alpha}{1+\alpha}\right)\left(\frac{(1+\alpha)^{\frac{1}{\alpha}} - \frac{1}{\alpha+1}}{(1+\alpha)^{\frac{1}{\alpha}} - \frac{\alpha+2}{\alpha+1}}\right).$$

The solid line in Fig. 1 above plots R as a function of α. For $\alpha = 1$ or $\alpha = -0.5$ welfare loss becomes, respectively, three times or twice the magnitude predicted by the classical theory. When $\alpha \to \infty$ relative welfare loss approaches one but the relative welfare loss due to misallocation of resources approaches zero! However, recall that rent-dissipation is by no means a general result. These calculations only illustrate the point that the classical theory may underestimate the magnitude of welfare losses.

5.2 The Coase theorem

Coase (1960), states that with well defined property rights and "zero transaction costs, private and social costs will be equal" (Coase 1988, p. 158). This result though, masks the fight for the property rights that may result in a wasteful conflict (Jung et al. 1995). For instance, suppose that two contenders fight for a property right that they value in v_1 and v_2 respectively with $v_1 > v_2$. After the property right has been allocated, agents can trade with probability r. r is an inverse measure of transaction costs that preclude a mutually beneficial transaction. There are two outcomes: In the first, agent 1 gets the property right and no trade results: Payoffs are $(v_1, 0)$. In the second, agent 2 gets the property right and with probability r sells the object to agent 1 for a price of $\frac{v_1+v_2}{2}$.[21] In this case expected payoffs are $(r\frac{v_1-v_2}{2}, r\frac{v_1+v_2}{2} + (1-r)v_2)$. Suppose that agents can influence the allocation of the right by incurring expenses G_1 and G_2. Denoting by p_1 the probability that agent 1 obtains the property right,

$$\Pi_1 = p_1 v_1 + (1-p_1) r \frac{v_1 - v_2}{2} - c(G_1)$$
$$\Pi_2 = (1-p_1)\left(r\frac{v_1+v_2}{2} + (1-r)v_2\right) - c(G_2)$$

Setting $V_1 \equiv v_1 - r\frac{v_1-v_2}{2}$ and $V_2 \equiv r\frac{v_1+v_2}{2} + (1-r)v_2$ the previous equations read

$$\Pi_1 = p_1 V_1 + r\frac{v_1 - v_2}{2} - c(G_1)$$
$$\Pi_2 = (1-p_1)V_2 - c(G_2)$$

Since agents take r as given the first payoff function is strategically equivalent to $p_1 V_1 - c(G_1)$. Suppose now that the contest probability function is like in (1.1) and that $c(G_i) = G_i$. Then, the conditions of Proposition 4.1 are met and in equilibrium, from (4.3)

$$G_i^* = \frac{V_i^2 V_j}{(V_1 + V_2)^2} \quad \text{and} \quad p_i^* = \frac{V_i}{(V_1 + V_2)}, \ i \neq j = 1, 2.$$

If rent-seeking expenses are totally wasteful the total expected welfare loss is

$$WL = \frac{V_1 V_2}{V_1 + V_2} + (1-r)(v_1 - v_2)(1 - p_1).$$

[21] This corresponds to the so-called standard solution in bargaining theory, see Mas-Colell et al. (1995, p. 846). For an analysis of the welfare losses yielded by different bargaining rules see Anbarci et al. (2002).

Notice that for $v_1 \cong v_2 = v$, say, the welfare loss due to transaction costs goes to zero but the welfare loss due to rent-seeking goes to $v/2$. Again the classical approach hides what might be the most significant welfare loss. But this is not the end of it. Since V_1 and V_2 are functions of r, WL can be written as $WL(r)$. We easily see that

$$WL(0) = \frac{v_1 v_2}{v_1 + v_2} + (v_1 - v_2)\frac{v_2}{v_1 + v_2} \quad \text{and} \quad WL(1) = \frac{v_1 + v_2}{4}.$$

We see that when $v_2 \simeq 0$, $WL(1)$ is *larger* than $WL(0)$, i.e., welfare loss can *increase* when transaction costs *decrease,* a complete reverse of what the classical approach asserts. This reversion is due to the fact that a decrease in transaction costs may exacerbate the contest for the object and, thus, rent-seeking expenses. Formally,

Proposition 5.2 *For some values of v_1 and v_2:* **a)** *The welfare loss associated with transaction costs tends to zero (i.e., when $v_1 \to v_2$) but the welfare losses due to rent-seeking can be arbitrarily large (i.e., when $v_1 \to \infty$ and $v_2 \to \infty$).* **b)** *Total welfare loss may increase when transaction costs decrease.*

6 The design of optimal contests

This section may sound paradoxical since many contests are totally wasteful because nothing socially valuable is produced (e.g., Examples 1.2–1.3 or the two cases considered in the previous section). In this case the best course from the social welfare point of view is to forfeit the contest. However, we have seen that in other cases contenders produce something valuable for society (e.g., Examples 1.4–1.6).[22] Moreover, certain parameters of the contest can be chosen prior to the actual contest is played: for instance in the case of selecting a host city for the Olympic Games, the Olympic Committee controls, at least to some extent, the form of the contest success functions and the number of contenders. Thus, the question of how the contest should be organized is a meaningful one.

6.1 Social objectives

Let us concentrate our attention on contests in which something valuable is produced. First, we must have a criterion by means of which the planner ranks the results in the contest. We have two classes of agents. On the one hand we have those that consume the prize and on the other hand we have those that participate in the contest. Following the example of the Olympic Games we will assume that consumers only care about the quality of the winner. This assumption is also reasonable in other cases, such as scientific or artistic prizes, etc. Following the interpretation given before, we assume that $\phi_i(G_i)$ measures the excellency/quality of the winner. Therefore, the expected excellence of the winner when m agents make efforts of $(G_1, \ldots, G_m)$ is

[22] In some cases, rent-seeking might increase social welfare if it diverts efforts from industries where there is too much effort (e.g., an industry characterized by negative externalities).

$\sum_{i=1}^{m} p_i(G)\phi_i(G_i)$. The payoffs obtained by contenders are $\sum_{j=1}^{m} p_i(G)V_i(G) - \sum_{j=1}^{m} C(G_j)$. We will assume the social welfare function is

$$W = \alpha \sum_{i=1}^{m} p_i(G)\phi_i(G_i) + (1-\alpha)\left(\sum_{j=1}^{m} p_i(G)V_i(G) - \sum_{j=1}^{m} C(G_j)\right), \quad \alpha \in [0,1] \tag{6.1}$$

where α can be interpreted as the proportion between consumers and contenders.

Notice that this social welfare function neither gives any weight to the quality of the losers—who could add prestige to the contest—nor embodies any distributional target. These are important points that we will ignore for the sake of simplicity. The case in which effort does not have a social merit—recall Example 1.2—can be dealt with by setting $\alpha = 0$.

6.2 Properties of the socially optimal contests

In this section we will assume A1, identical agents and that the optimum is symmetric. Denoting by y the common value of the efforts/investments (6.1) becomes

$$W = \alpha\phi(y) + (1-\alpha)(V_0 + an\phi(y) - nC(y)). \tag{6.2}$$

To find the optimal contest we choose $\phi(\ )$ and n in order to maximize W with the restriction that efforts are those made in a Nash equilibrium of the contest. In the case in which we only choose the number of contenders, we know that under A1 for each n we have a unique Nash equilibrium. We represent this by means of the function $y = y(n)$ which summarizes the restriction faced by the planner.

In this subsection and the next we will be concerned with the case in which $\alpha = 1$. This case may be a good approximation to a situation where the number of consumers is very large in relation to the number of contenders, as in the example of the Olympic Games. An implication of this assumption is that in the symmetric case optimality requires maximizing the effort per agent y.

First, let us look at the case in which the planner can choose the contest success function. Let us assume that this function is parametrized by a real number γ which belongs to an interval $[\underline{\gamma}, \bar{\gamma}]$. Hence, the function $\phi(\ )$ is now written $\phi(G_i, \gamma)$. We now assume that γ affects $\phi(\ )$ in the following way:

$$\frac{\partial\phi(G_i,\gamma)}{\partial G_i}\frac{G_i}{\phi(G_i,\gamma)} \text{ is increasing in } \gamma. \tag{6.3}$$

(6.3) means that γ raises the elasticity of $\phi(\ )$ with respect to G_i. For instance, if $\phi(G_i,\gamma) = G_i^{\gamma}$, $\gamma \in [0,1]$, we have that $\frac{\partial\phi(G_i,\gamma)}{\partial G_i}\frac{G_i}{\phi(G_i,\gamma)} = \gamma$. Hence (6.3) holds:

Proposition 6.1 *Under A*1, (6.3) *and* $a = 0$, *the optimal contest is* $\gamma = \bar{\gamma}$.

Proof Under our assumptions (3.4) reads

$$\frac{\partial \phi(y,\gamma)}{\partial G_i} \frac{V_0(n-1)}{\phi(y,\gamma)n^2} - C'(y) = 0.$$

Denote the left hand side of the previous equation by $\Psi(y,\gamma)$. $\Psi(\,)$ is decreasing in y (because $\phi(\,)$ is increasing and concave in y) and increasing in γ (by (6.3)). Since the right hand side of the above equation is non decreasing in y, differentiating implicitly we obtain that

$$\frac{dy}{d\gamma} = \frac{\frac{\partial \Psi(y,\gamma)}{\partial \gamma}}{\frac{d^2C(y)}{dy^2} - \frac{\partial \Psi(y,\gamma)}{\partial y}} > 0.$$

Hence y is maximized with the largest value of γ. □

To get a feeling for the previous result let us go back to the case where $\phi(G_i,\gamma) = G_i^{\gamma}$. Here, γ measures how the probability of getting the prize responds to efforts, for instance if $\gamma = 0$, this probability does not depend on the efforts. Thus, if we want to give incentives to agents to make the greatest effort possible, we must choose the largest γ. In this case this yields a linear $\phi(\,)$ (Dasgupta and Nti 1998 also proved—in a different context—that linear functions are optimal). However, in other cases a larger value of γ is optimal, provided that an equilibrium can be guaranteed.

Suppose now that the planner can choose the number of active contenders:

Remark 6.1 Under A1 the optimal number of active contenders is two.

Proof Maximizing $\phi(y)$ amounts to maximizing y which, according to Proposition 3.2, amounts to minimizing n.[23] □

The interpretation of this result is that competition is bad because it yields a low level of effort by the winner but monopoly is even worse because it yields no effort. Thus the optimal policy consists in choosing the smaller number of contenders.[24] This result may help to explain why in many sports finals are played by two teams or why the USA defence department chose two firms to compete in the so-called Joint Strike Fighter eliminating McDonell–Douglas which was the third contender. It could also be used to explain the so-called *Dual Sourcing* in which a firm demanding equipment chooses two companies as possible suppliers (Shapiro and Varian 1999, pp. 124–125).

This result does not hold when agents are either heterogeneous or when they have a different valuation for their own effort than for other people's. An example of the second situation is available under request from the author. Here there is an example of what may happen when agents are heterogeneous.

[23] An example where this result holds for $\alpha \neq 1$ is available from the author under request. See Chung (1996) for the case $a \neq 0$.

[24] Other examples in which an increase of competition may harm social welfare are markets with economies of scale (von Weizacker 1980) or with moral hazard (Scharsftein 1988).

Example 6.1 Assume $n = 3$ with $V_1 = V_2 = V_3 = 1$, $c_1 = 0.2$, $c_2 = 1$ and $c_3 = 1$. Social welfare is $W = \sum_{i=1}^{m} G_i^* p_i^*$. NE when there are only two agents is $p_1^* = 0.83$, $p_2^* = 0.17$, $G_1^* = 0.7$, $G_2^* = 0.14$, with $W^* = 0.6048$. NE with three agents is $p_1^* = 0.82$, $p_2^* = 0.09$, $p_3^* = 0.09$, $G_1^* = 0.745$, $G_2^* = 0.08$, $G_3^* = 0.08$, with $W^* = 0.62$.

The key to this example lies in the slope of best reply functions: If agent i is very efficient, i.e., she has a small c_i, her strategy increases with the strategies of the rest (strategic complementarity). Conversely, if i is very inefficient, i.e., c_i is large, her strategy decreases with the strategies of the rest (strategic substitution). The introduction of a third agent increases the effort of the efficient agents and decreases the effort of inefficient agents which is good from the point of view of social welfare: In the previous example with two agents $\sum_{j \neq 1} G_j = 0.14$ and $\sum_{j \neq 2} G_j = 0.7$, but with three agents $\sum_{j \neq 1} G_j = 0.16$ and $\sum_{j \neq 2} G_j = 0.825$, i.e., the introduction of a third agent increases G_1^* and decreases G_2^*.

We now turn our attention to the question posed by the statistician Francis Galton in 1902 regarding the optimal number of prizes. Suppose that there is a maximum of k prizes with values $V^1, V^2, \ldots, V^k$. Let M be the maximum amount of cash that can be spent on prizes, i.e., $M \geq \sum_{l=1}^{n} V^l$. We will also assume that all agents contend for all prizes (see Moldovanu and Sela 2001 for the case in which each agent can only receive one prize). Let p_i^l $l = 1, 2, \ldots k$ be the probability that agent i obtains prize l. We will assume that

$$p_i^l = \frac{G_i^{\epsilon l}}{\sum_{j=1}^{n} G_j^{\epsilon l}}, \quad \text{where } \epsilon l \in [0, 1]. \tag{6.4}$$

The planner has to choose the values $V^1, V^2, \ldots, V^k$ with the restriction $M \geq \sum_{l=1}^{n} V^l$ and taken as given n and $\epsilon l, l = 1, 2, \ldots, k$. Let $\epsilon^M \equiv \max_{l=1,\ldots,k}(\epsilon l)$ and $\epsilon_m \equiv \min_{l=1,\ldots,k}(\epsilon l)$ be respectively the maximum and the minimum values of ϵl.

Proposition 6.2 *Assume* A1a) *and* (6.4). *If* $\epsilon^M = \epsilon_m$ *any number of prizes is optimal. If* $\epsilon^M > \epsilon_m$, *the optimal number of prizes is one, namely prize* M.

Proof The first order condition of payoff maximization is

$$\frac{\epsilon 1 G_i^{\epsilon 1 - 1} \sum_{j \neq i} G_j^{\epsilon 1}}{\left(\sum_{j=1}^{n} G_j^{\epsilon 1}\right)^2} V^1 + \frac{\epsilon 2 G_i^{\epsilon 2 - 1} \sum_{j \neq i} G_j^{\epsilon 2}}{\left(\sum_{j=1}^{n} G_j^{\epsilon 2}\right)^2} V^2 + \cdots$$
$$+ \frac{\epsilon k G_i^{\epsilon k - 1} \sum_{j \neq i} G_j^{\epsilon k}}{\left(\sum_{j=1}^{n} G_j^{\epsilon k}\right)^2} V^k = C'(G_i)$$

Using methods like those used in Propositions 3.1 and 4.1 it can be shown that the second order condition holds and that there are no asymmetric equilibria. Thus, the previous equation can be re-written as

$$\frac{\epsilon 1(n-1)V^1}{n^2} + \frac{\epsilon 2(n-1)V^2}{n^2} + \cdots + \frac{\epsilon k(n-1)V^k}{n^2} = yC'(y) \equiv \Omega(y)$$

This equation yields the unique Nash equilibrium because $\Omega(\,)$ is strictly increasing and can be inverted, hence,

$$y = \Omega^{-1}\left(\frac{(n-1)}{n^2}\sum_{l=1}^{k} V^l \epsilon l\right).$$

Maximizing y yields the result. □

The interpretation of this result lies in the fact that ϵl's measure how the probability of getting the prize responds to efforts: If the planner wants to give incentives to agents to exert effort, she should choose the larger value of ϵl.

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