Analysis and Design of Cell-Free Massive MIMO Systems under Spatially Correlated Fading Channels

Alberto Álvarez Polegre

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Advisor/Tutor:
Prof. Ana García Armada

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A smooth sea never made a skilled sailor.

Abstract

Wireless communications have become a key pillar in our modern society. It can be hard to think of a service that somehow does not rely on them. Particularly, mobile networks are one of the most necessary technologies in our daily life. This produces that the demand for data rates is by no means stopping from increasing. The cellular architecture is facing a crucial challenge under limited performance by interference and spectrum saturation. This involves cell-edge users experiencing poor performance due to the close vicinity of base stations (BSs) using the same carrier frequency. Based on a combination of the coordinated multi-point (CoMP) technique and traditional massive multiple-input multiple-output (MIMO) systems, cell-free (CF) massive MIMO networks have irrupted as a solution for avoiding inter-cell interference issues and for providing uniform service in large coverage areas. This thesis focuses on the analysis and design of CF massive MIMO networks assuming a spatially correlated fading model. A general-purpose channel model is provided and the whole network functioning is given in detail.

Despite the many characteristics a CF massive MIMO system shares with conventional co-located massive MIMO its distributed nature brings along new issues that need to be carefully accounted for. In particular, the so-called channel hardening effect that postulates that the variance of the compound wireless channel experienced by a given user from a large number of transmit antennas tends to vanish, effectively making the channel deterministic. This critical assumption, which permeates most theoretical results of massive MIMO, has been well investigated and validated in centralized architectures, however, it has received little attention in the context of CF massive MIMO networks. Hardening in CF architectures is potentially compromised by the different large-scale gains each access point (AP) impinges on the transmitted signal to each user, a condition that is further stressed when not all APs transmit to all users as proposed in the user-centric (UC) variations of CF massive MIMO. In this document, the presence of channel hardening in this new architecture scheme is addressed using distributed and cooperative precoders and combiners and different power control strategies. It is shown that the line-of-sight (LOS) component, spatially correlated antennas, and clustering schemes have an impact on how the channel hardens. In addition, we examine the existent gap between the estimated achievable rate and the true network performance when channel hardening is com-

promised. Exact closed-form expressions for both a hardening metric and achievable downlink (DL) and uplink (UL) rates are given as well.

We also look into the pilot contamination problem in the UL and DL with different degrees of cooperation between the APs. The optimum minimum mean-squared error (MMSE) processing can take advantage of large-scale fading coefficients for canceling the interference of pilot-sharing users and thus achieves asymptotically unbounded capacity. However, it is computationally demanding and can only be implemented in a fully centralized network. Here, sub-optimal schemes are derived that provide unbounded capacity with much lower complexity and using only local channel estimates but global channel statistics. This makes them suited for both centralized and distributed networks. In this latter case, the best performance is achieved with a generalized maximum ratio combiner that maximizes a capacity bound based on channel statistics only.

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Published and submitted content

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This work is wholly included in Chapters 3 and 4.

The material from this source included in this thesis is not singled out with typographic means and references.

2. A. Álvarez Polegre, F. Riera-Palou, G. Femenias and A. García Armada, "Channel Hardening in Cell-Free and User-Centric Massive MIMO Networks With Spatially Correlated Ricean Fading," in IEEE Access, vol. 8, pp. 139827-139845, 2020, doi: 10.1109/ACCESS.2020.3012736.

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3. A. Álvarez Polegre, L. Sanguinetti and A. García Armada, "Pilot Decontamination Processing in Cell-Free Massive MIMO Networks," submitted to IEEE Communications Letters.

This work is wholly included in Chapters 3 and 5.

The material from this source included in this thesis is not singled out with typographic means and references.

Other research merits

The following works have also been published:

1. G. Femenias, F. Riera-Palou, A. Álvarez Polegre and A. García Armada, "Short-Term Power Constrained Cell-Free Massive MIMO Over Spatially Correlated Ricean Fading," in *IEEE Transactions on Vehicular Technology*, vol. 69, no. 12, pp. 15200-15215, Dec. 2020, doi: 10.1109/TVT.2020.3037009.
2. A. Álvarez Polegre, R. Pérez Leal, J. A. García García and A. García Armada, "Path Loss Model Validation for 5G NR based on Real Deployment Measurements," in *Proc. IRACON 12th MC Meeting and Final Workshop*, Louvain-la-Neuve, Belgium, 2020, pp. 1-4
3. A. Álvarez Polegre and A. García Armada, "User-Centric Massive MIMO Systems with Hardening-based Clusterization," in *Proc. WSA 2020; 24th International ITG Workshop on Smart Antennas*, Hamburg, Germany, 2020, pp. 1-5.
4. A. Álvarez Polegre, R. Pérez Leal, J. A. García García and A. García Armada, "Optimization of the Radio Access to Provide Vehicular Communications Based on Drive Tests," in *Proc. 2019 27th European Signal Processing Conference (EUSIPCO)*, A Coruña, Spain, 2019, pp. 1-5, doi: 10.23919/EUSIPCO.2019.8902598.
5. A. Álvarez Polegre, R. Pérez Leal, J. A. García García and A. García Armada, "Drive Tests-based Evaluation of Macroscopic Pathloss Models for Mobile Networks," in *Proc. 2019 European Conference on Networks and Communications (EuCNC)*, Valencia, Spain, 2019, pp. 501-505, doi: 10.1109/EuCNC.2019.8802061.
6. A. Álvarez Polegre, R. Corvaja and A. García Armada "Phase Noise Effects in Zero-Forcing Multi-User Massive MIMO Uplink and Downlink," in *Proc. XXXIII Simposium Nacional de la Unión Científica Internacional de Radio (URSI)*, 2018, Granada, Spain.

Contents

Abstract	iv
Published and submitted content	vii
Other research merits	viii
Acronyms and abbreviations	xiii
Notation	xvi
1 Introduction	1
1.1 Motivation	2
1.2 Research contributions	5
1.3 Thesis outline	6
2 The Pathway to a Cell-Less Architecture	7
2.1 Interference management techniques	7
2.1.1 Inter-cell interference coordination	8
2.1.2 Enhanced inter-cell interference coordination	8
2.1.3 Further enhanced inter-cell interference coordination	9
2.1.4 Coordinated multi-point	10
2.2 Key enabling technology: massive MIMO	12
2.3 State-of-the-art	13
3 System Model for Cell-Free Massive MIMO Networks	15
3.1 Network layout and frame structure	15
3.2 Channel model	16
3.2.1 LOS component	17
3.2.2 NLOS component	17
3.2.3 Path loss and shadow fading	18
3.3 UL training phase	19
3.3.1 Pilot assignment	19
3.3.2 Channel estimation	20
3.3.3 AP-MS linking	22
3.4 DL data payload phase	24
3.4.1 Data transmission and reception	24
3.4.2 Achievable data rate estimation	25
3.5 UL data payload phase	26
3.5.1 Data transmission and reception	26
3.5.2 Achievable data rate estimation	27
4 Channel Hardening in Cell-Free Massive MIMO	31
4.1 Spatially uncorrelated Rayleigh fading scenario	32
4.1.1 Precoding schemes and power allocation strategies	33
4.1.2 Channel hardening ratio computation	34
4.1.3 Achievable rate computation	36
4.1.4 Numerical results	37

4.1.5	Concluding remarks	41
4.2	Spatially correlated Ricean fading scenario	41
4.2.1	Distributed signal processing	41
4.2.2	Centralized signal processing	52
4.2.3	Numerical results	57
4.2.4	Concluding remarks	63
5	Reducing Processing Complexity under Pilot Contamination	65
5.1	Impact of local covariance matrices	66
5.2	Asymptotic behavior	69
5.3	Pilot decontamination processing	70
5.3.1	Reduced-complexity MMSE	72
5.3.2	Generalized MR	74
5.3.3	Precoder design	75
5.4	System performance	76
5.5	Concluding remarks	77
6	Conclusion and Future Research	79
6.1	Summary and conclusions	79
6.2	Future research work	81

List of figures

1.1	Different FRF options.	3
2.1	Inter-cell interference between two macro BSs	8
2.2	eICIC technique between a macro BS and a SC.	9
2.3	Comparison between ABS.	10
2.4	Example of CoMP functioning for the DL segment.	11
3.1	CF massive MIMO network layout.	16
3.2	TDD frame structure for a coherence interval.	16
3.3	CF massive MIMO network under the AP centric clustering architecture.	23
4.1	Channel hardening ratio for $K = 10$ and different N under CF and AP centric ($K_C = 5$) architectures.	39
4.2	Comparison between lowed bound and genie-aided rates for $M = 100$ under CF and AP-centric ($K_C = 5$) architectures.	40
4.3	Channel hardening ratio for CF with CB precoding and MF combining using uniform power allocation having each AP $N = 4$	57
4.4	CDF of the DL channel hardening ratio with CB precoding having each AP $N = 2$ and $K = 10$	59
4.5	DL channel hardening with CB precoding using uniform power allocation for single-antenna AP ($N = 1$) under different clustering approaches ($K_C = M_C = 5$).	61
4.6	DL channel hardening ratio with CB precoding and uniform power allocation under Rayleigh fading having each AP $N = 2$	62
4.7	Achievable rate for CF with CB precoding and MF combining using uniform power allocation having each AP $N = 4$	63
4.8	Average DL achievable rate with ZF precoding and low-complexity max-min power allocation having each AP $N = 2$	64
4.9	Average DL achievable rate with true and upper bounds under CB precoding and uniform power allocation having each AP $N = 4$	64
5.1	CDF of the UL average SE for different K under uncorrelated and correlated scenarios with MR and MMSE combining.	67
5.2	Average UL SE for both combiners with increasing M	70
5.3	CDF of the average SE for $K = 70$	76
5.4	Average UL SE for all combiners with $K = 4$	77

List of tables

4.1	Simulation default parameters for uncorrelated Rayleigh fading scenario.	38
4.2	Simulation default parameters for correlated Ricean fading scenario.	58
4.3	Average number of APs serving a MS and average number of MSs being served by an AP under different clustering strategies ($M = 100$ and $K_C = M_C = 5$). .	60

Acronyms and abbreviations

2G	2 nd generation
3G	3 rd generation
3GPP	3 rd Generation Partnership Project
4G	4 th generation
5G	5 th generation
ABS	almost blank subframe
ADC	analog-digital converter
AMPS	Advanced Mobile Phone System
AoA	angle of arrival
AP	access point
ASD	angular standard deviation
AWGN	additive white Gaussian noise
BS	base station
CB	conjugate beamforming
CDF	cumulative density function
CF	cell-free
CSI	channel state information
CSIR	CSI at receiver
CPU	central processing unit
CoMP	coordinated multi-point
DL	downlink
DFT	discrete Fourier transform
EDGE	Enhanced Data rates for GSM Evolution
eMBB	enhanced mobile broadband
eICIC	enhanced ICIC
FCC	Federal Communications Commission
FDD	frequency-division duplex
FDMA	frequency-division multiple access
feICIC	further eICIC

FRF	frequency reuse factor
GMR	generalized MR
GPRS	General Packet Radio Service
GSM	Global System for Mobile Communications
HetNet	heterogeneous network
ICIC	inter-cell interference coordination
IoT	Internet of things
LOS	line-of-sight
LS	least-squares
LTE	Long-Term Evolution
LTE-A	LTE Advanced
MF	matched filter
MIMO	multiple-input multiple-output
MMSE	minimum mean-squared error
mMTC	massive Machine Type Communications
mmWave	millimeter-wave
MR	maximum ratio
MS	mobile station
NCB	normalized CB
NLOS	non-line-of-sight
MU-MIMO	multi-user MIMO
NR	New Radio
OFDM	orthogonal frequency-division multiplexing
RC-MMSE	reduced-complexity MMSE
RF	radiofrequency
SC	small-cell
SE	spectral efficiency
SNR	signal-to-noise ratio
SINR	signal-to-interference-plus-noise ratio
SMS	short message service
SOC	second-order cone
TDD	time-division duplex
TDMA	time-division multiple access

UC	user-centric
UFR	universal frequency reuse
ULA	uniform linear array
UL	uplink
UMTS	Universal Mobile Telecommunications System
URLLC	Ultra-Reliable Low-Latency Communications
UatF	use-and-then-forget
VLC	visible light communications
ZF	zero-forcing

Notation

a	scalar
$\mathbf{a}$	vector
$\mathbf{A}$	matrix
$[\mathbf{a}]_n$	n -th element of $\mathbf{a}$
$[\mathbf{A}]_{m,n}$	m -th row and n -th column of $\mathbf{A}$
$\mathbf{A}^T$	transpose of $\mathbf{A}$
$\mathbf{A}^*$	conjugate of $\mathbf{A}$
$\mathbf{A}^H$	conjugate transpose (Hermitian) of $\mathbf{A}$
$\mathbf{A}^{-1}$	inverse of $\mathbf{A}$
$\text{tr}(\mathbf{A})$	trace of $\mathbf{A}$
$\text{diag}(\mathbf{a})$	diagonal matrix with the element of $\mathbf{a}$ on its main diagonal
$\text{diag}(\mathbf{A})$	column vector of diagonal elements of $\mathbf{A}$
$\text{diag}(\mathbf{A}_1, \dots, \mathbf{A}_n)$	block-diagonal matrix with square matrices $\mathbf{A}_1, \dots, \mathbf{A}_n$ on its main diagonal
$\mathbf{I}_N$	identity matrix of size $N \times N$
$\mathbf{0}_N$	zero matrix of size $N \times N$
$\odot$	element-wise product
$\mathbb{E}\{\cdot\}$	expectation operator
$\text{Var}\{\cdot\}$	variance operator
$\Re\{\cdot\}$	real part
$ \cdot $	absolute value
$\ \cdot\ $	Euclidean norm
$\ \cdot\ _F$	Frobenius norm
$\Gamma(\cdot)$	gamma function
$\mathcal{N}(\mu, \sigma^2)$	circularly-symmetric normal distribution with mean μ and variance σ^2
$\mathcal{CN}(\mu, \sigma^2)$	circularly-symmetric complex normal distribution with mean μ and variance σ^2
$\asymp$	asymptotic equality

Chapter 1

Introduction

If asked about the origins of mobile communications, few people would think of the year 1947. Back then, the Bell System company proposed an early version of the actual broadband mobile networks with the idea of providing radio service to a large area thanks to relatively low-power transmitters [1]. Unfortunately, the Federal Communications Commission (FCC) did not approve the usage of the requested spectrum band. It would take more than 30 years for the first commercial mobile communications system to appear. Among those early technologies, the Bell Laboratories developed the Advanced Mobile Phone System (AMPS) during the 1980s, which was based on analog frequency-division multiple access (FDMA) and came along with the concept of dividing the coverage area into *cells*. In addition, neighbor cells used different frequency slots so they would not interfere with each other. Thanks to the advancements made on digital processors, the next step for mobile communications came with the proposal of deploying fully digital cellular systems.

The year 1991 saw the launch of the Global System for Mobile communications (GSM) standard, the most successful 2nd generation (2G) technology [2]. Analog voice stepped aside for the new digital service and FDMA was replaced by time-division multiple access (TDMA). New features, such as short message service (SMS), were introduced. Enhancements were made with the introduction of the General Packet Radio Service (GPRS) and Enhanced Data rates for GSM Evolution (EDGE) standards [3]. However, the truly multimedia revolution arrived with the deployment of the Universal Mobile Telecommunications System (UMTS) technology, i.e. the 3rd generation (3G) of mobile communications [4]. Although the narrow 200 KHz carriers of GSM were taken to a 5 MHz width with UMTS to provide higher data rates, the pace of traffic demand continued to grow over the years. 3G technologies soon needed a successor.

With the aim of providing more robust and flexible communications while satisfying high data traffic requests, the almost-4th generation (4G) Long-Term Evolution (LTE) technology was standardized in late the 2000s and started its deployment shortly after [5]. LTE makes use of orthogonal frequency-division multiplexing (OFDM) and popularized the use of the MIMO

technique. Carrier bandwidth is no longer fixed and can vary from 1.4 MHz up to 20 MHz, which provides flexibility if different services are to be provided. More enhancements were introduced with LTE Advanced (LTE-A), the true 4G technology. LTE-A has continued to evolve over the years and has set the basis for the new 5th generation (5G) New Radio (NR) standard [6]. With this new standard it is not only a matter of enhancing mobile communications in terms of data rates, but also reducing latency, being more efficient in terms of energy consumption, managing more devices simultaneously, among other requirements.

In short, over the past four decades we have transitioned from primitive analog voice-only services under different technologies to a single and reliable standard that allows us to either stream high definition video or contact anyone in the world almost instantly. Together with many other telecommunication technologies, mobile networks have put together every place on Earth. Like Arthur C. Clarke would say, *the world is one* [7]. However, considering the ever-growing data traffic and the up-coming multimedia services, are we reaching the physical limit of the current paradigm of wireless networks? Should mobile systems continue to rely on the current frequency bands? And most importantly, is the cellular deployment concept (introduced more than 70 years ago) the answer for the challenges that are yet to come?

1.1 Motivation

The cellular deployment allows us to provide coverage to a large area by means of splitting each region into smaller sectors. It not only gives a much more uniform service to users, but also makes feasible deploying low-power BSs. For avoiding interference issues at cell edges, GSM deployments were planned so neighbor cells would not make use of the same carrier signal. The rate at which cells would reuse a specific carrier is denoted as frequency reuse factor (FRF). The 200 KHz carriers from GSM grant the possibility of setting a high FRF, even if having a small fraction of licensed spectrum only. However, if we think of this strategy when talking about LTE and beyond, we would need to acquire much more bandwidth from the corresponding administration considering the wider carriers this standard makes use of. Acquiring larger regions of the electromagnetic spectrum seems either too expensive or simply impossible due to current saturation of the sub-6 GHz frequency bands. A cheap and easy solution to implement is taking the FRF down to 1, i.e. every BS uses the same frequency band. This approach is also referred to as universal frequency reuse (UFR). This way there is no need to split the available bandwidth. Nevertheless, cell-edge users will suffer from high interference

Figure 1.1: Different FRF options.

due to close non-serving cells up to the point of completely denying service. So far, this is the most common solution for overcoming the spectrum saturation, though it is not the only one. Figure 1.1 illustrates the usage of different options of FRF under GSM and LTE.

Apart from particular configurations of the cellular layout, both the research community and the industry have come up with some other alternatives. If we take a look at the electromagnetic spectrum, we will easily notice that there are plenty of nearly unused frequency bands. A good option for enhanced mobile broadband (eMBB) communications is to allocate signals at the millimeter-wave (mmWave) band [8]. Compared to sub-6 GHz radiofrequency (RF) communications, the mmWave band can afford extremely wide bandwidths, small antenna arrays sizes due to the shorter wavelengths and very narrow beams. However, a mmWave network relies on LOS communications and suffers from atmospheric and meteorological issues, such as rain or air absorption. Another alternative is visible light communications (VLC) [9]. This technology may provide even larger bandwidths compared to mmWave and is far more secure when thinking about jamming and related threats. In addition, it does not interfere whatsoever with RF-based systems. Similar to mmWave, VLC also suffers from atmospheric absorption and it is very sensitive to background light sources. Either of these two technologies are fairly good options in very particular scenarios and will most probably play a complementary role in future wireless communications, but they are not core solutions.

Together with the first releases of LTE, some techniques for managing (or at the very least, mitigating) interference between cells were proposed. These techniques are aimed to alleviate

the undesired signal reception for cell-edge users, and we can summarize them into two different groups: one based on mitigating interference and another one based on a full coordination between BSs for allowing joint transmissions. All of the main interference management techniques will be presented on Chapter 2. It is worth mentioning that after several trials and field tests, neither of the proposed techniques have been fully deployed in current mobile networks.

At this point, mobile communication systems are facing a twofold challenge. On the one side, the extreme saturation of the current frequency bands makes it almost impossible to accommodate wider bandwidths without displacing other technologies. While mmWave and VLC-based networks are good proposals, they do not suit general purpose scenarios. On the other side, user terminals are not only demanding higher data rates for current and future multimedia applications, but they are also merging with different kind of networks, such as massive Machine Type Communications (mMTC) and Ultra-Reliable Low-Latency Communications (URLLC). In words, wireless communications are no longer homogeneous. With the idea of providing a solid uniform and interference-free service for any type of device, the research community recently proposed the CF massive MIMO networks [10–13] - the starting point of this thesis. Under CF massive MIMO, the cell concept is totally dropped and the coverage area acts as a whole. A large number of very low-power APs connected to a central processing unit (CPU) are deployed throughout the service region. Each AP may be equipped with either one or several antennas and provides service to every user in the network by means of coherent and joint transmission/reception. Ideally, the number of APs will be several times higher compared to the number of active users in the network, conforming this way a distributed massive MIMO system. Thanks to a uniform APs deployment and the almost complete mitigation of any source of interference, users will experience seamlessly service anywhere.

Taking the current research on CF massive MIMO as background, this thesis focuses on building a solid framework to analyze and proposes enhancements for future mobile communications. A thorough analysis on the channel hardening property is shown under different scenarios. This aspect of a massive MIMO-based network is essential when designing and optimizing the whole system and achieving a strong performance. Likewise, two competitive processing schemes are proposed taking advantage of a realistic channel model and the pilot contamination effect that these type of networks suffer from.

1.2 Research contributions

This thesis is focused on analyzing several properties of CF massive MIMO networks and proposing reduced-complexity and nearly optimal precoding and combining schemes for the DL and UL segments, respectively. All contributions can be summarized as follows:

- A general purpose CF massive MIMO system model is presented assuming spatial correlation between the antennas of each AP and the existence of a LOS component at the channel linking every transmitter and receiver for both the DL and UL segments. Furthermore, the employed path loss model is based on real field measurements and drive tests. A time-division duplex (TDD)-based communication is assumed in which the coherence interval is divided into three different phases: UL training for channel estimation, DL payload data transmission and UL payload data transmission. This model also supports cluster-based CF approaches, such as the UC scheme.
- A detailed analysis on the massive MIMO channel hardening property and its behavior on CF networks is provided by measuring the channel hardening ratio. First, a simple spatially uncorrelated channel model is adopted with maximum ratio (MR)-based precoders and combiners. Then, centralized processing schemes are introduced together with the more general channel model. Closed-form expressions for computing the channel hardening ratio and achievable rates are given for every scenario. Heuristic and signal-to-interference-plus-noise ratio (SINR) optimal power control strategies are assumed and compared regarding channel hardening.
- The effects of spatially correlated APs antenna arrays have on the system performance are addressed. Particular attention is given to pilot contamination and how to exploit its implication to turn it into a positive outcome by means of different processing approaches. It is shown that centralized schemes largely outperform the MR approach initially conceived for CF massive MIMO networks. Nevertheless, since centralized schemes are extremely demanding in terms of computational resources, a reduced-complexity and nearly optimal in the asymptotic regime processing scheme is proposed. Likewise, an enhanced approach of MR which is not limited by pilot contamination is also given. These strategies are particularly competitive under severe pilot contamination in largely crowded scenarios.

1.3 Thesis outline

This thesis is onward structured as follows:

- Given the current situation of mobile networks, Chapter 2 addresses the background this thesis is built on. Interference techniques in current standards are presented, as well as the basis that the CF massive MIMO paradigm relies on. Plus, a thorough CF massive MIMO state-of-the-art is given.
- Chapter 3 presents the baseline system model this whole thesis uses. Regardless the adopted combining and precoding schemes, this section introduces the communication frame structure for CF massive MIMO, the channel modeling, the channel estimation procedure and the UL/DL signal transmission and reception. Importantly, this chapter acts as a core for the remaining sections and contributions.
- The whole channel hardening analysis is given in Chapter 4. Under both the original CF massive MIMO scheme and several UC approaches, the behavior of the channel hardening ratio under different combining and precoding schemes together with power allocation strategies is shown.
- Chapter 5 first addresses how pilot contamination can be turned into a positive asset in CF massive MIMO. By exploiting spatial correlation a centralized and low-complexity combining scheme is given, which performs nearly as good as the optimal approach. Likewise, a precoding approach is presented based on similar principles together with a heuristic power control strategy.
- Finally, Chapter 6 sums up the presented work and the main contributions made. In addition, guidelines for future research are given having this thesis as starting point.

Chapter 2

The Pathway to a Cell-Less Architecture

Finding how to improve spectral efficiency (SE) in wireless communications has been of high importance for both the research community and the industry. Making a smarter use of the available spectrum is key from an economic perspective, and sometimes is the only possible option for achieving higher data rates. With each new 3rd Generation Partnership Project (3GPP) standard, we have seen how higher order modulation techniques have been introduced for increasing SE, but this comes at the cost of ramping the error rate up too, unless the SINR is increased. Along with many other advancements, the UFR adopted from LTE onward was also a good approach since it allows every BS to make use of the whole licensed bandwidth. However, as commented in the introduction, cell-edge users suffer now from high inter-cell interference and experience a considerable loss of performance. In this chapter, we will see how the current LTE standard interference management techniques try to fix the main drawback of UFR and how this was the very first milestone in the CF massive MIMO pathway.

2.1 Interference management techniques

From the early releases of LTE back in 2009 up to recently deployed 5G NR standard, the 3GPP has attempted at mitigating the presence of all kind of interference in mobile networks. We should not only think about macro cells interference with each other, but also with small-cells (SCs). SCs arrived as a promising technology for offloading macro stations and making the whole network able to manage a much denser traffic. However, the lack of any coordination between BSs can lead to very poor performance and, eventually, would make SCs useless. Those networks with mixed macro BSs and low-power SCs are referred to as heterogeneous networks (HetNets), which usually are composed by several other wireless technologies. Figure 2.1 depicts the macro BS inter-cell interference situation.

Through the next sections all of the main interference coordination techniques are addressed.

Figure 2.1: *Inter-cell interference between two macro BSs*

2.1.1 Inter-cell interference coordination

Together with the first release of LTE, the inter-cell interference coordination (ICIC) technique was presented. It is strictly oriented to enhance the performance of cell-edge users in macro coverage scenarios. This interference arises when users in different neighbor cells attempt at using the same frequency resource at the same time. In this case, the signal from the neighbor cell acts as a strong interferer. Both the desired signal and the interference signal are comparable to each other in terms of power. The corresponding user would not be able to distinguish one from the other. The ICIC technique takes advantage of the fronthaul connection between the BSs - the X2 interface. Through this interface, BSs can share information about the users they are serving and can allocate different radio resources to those located at cell edges. It is important to note that the different allocation in terms of frequency resources is only possible for data channels.

2.1.2 Enhanced inter-cell interference coordination

The main drawback of the ICIC technique is that it is not well suited for HetNet scenarios. Unlike data channels, control channels are not delivered through different frequency ranges, but distributed across the entire channel bandwidth instead. This makes it impossible to schedule control channels in different frequency locations. In a homogeneous network scenario, there is not much difference in terms of transmission power from neighbor cells, and hence no significant inter-channel interference by control channels is caused. In HetNets, on the other hand, the hosting macro cell has much higher transmission power than the SCs within its coverage range.

For solving this matter, a new version of ICIC, enhanced ICIC (eICIC), was developed. In this

Figure 2.2: *eICIC technique between a macro BS and a SC.*

case, interference management takes place both in time and power domain. The basic idea of the time domain techniques is to mute certain subframes of the macro cell in order to reduce the interference to the SCs. Ideally, the muted subframes (configured by the interferer cell) should be totally blank, however, broadcast channels and reference signals cannot be muted as they are needed to support important legacy operations. A subframe carrying no data is called almost blank subframe (ABS) since there is barely any information being carried. As said before, an ABS carries the minimum control information and hence the interference is reduced significantly but not completely. The eICIC procedure is shown in Figure 2.2.

2.1.3 Further enhanced inter-cell interference coordination

Under eICIC, ABS requires the macro cell to completely blank the transmit power on data radio resource elements in the blanked subframe. This effectively translates into resource loss. While a given SC could use eICIC and non-eICIC subframes, the macro cell could only use the non-eICIC subframes to schedule its users. A new technique that was introduced with the release of LTE-A, further eICIC (further eICIC (feICIC)), addresses this matter. The feICIC technique brings reduced-power-ABS, which does not require the macro cell to completely mute the blank subframes, as it could use these subframes with reduced power to serve central users. This requires advanced scheduling and coordination between the macro cell and the coordinated SCs group. The capacity gain from using reduced-power-ABS over zero-power-ABS depends on the ratio of eICIC subframes to non-eICIC subframes and the proper implementation of the scheduling. It is worth mentioning that fronthaul links connecting the macro BS and the SCs must have relatively low latency for making this technique viable. Figure 2.3 illustrates the reduced-power-ABS concept.

Further details on ICIC-based techniques can be found in [14].

Figure 2.3: Comparison between ABS.

2.1.4 Coordinated multi-point

Last but not least, the CoMP was introduced in the latest releases of LTE-A [15]. Rigorously speaking, CoMP is a set of various interference management techniques that not only mitigates undesired signals, but can also increase the overall system capacity and performance. We can divide CoMP into two major categories: joint processing and coordinated scheduling/beamforming. Joint processing occurs when there is a coordination between multiple entities, i.e. BSs, that are simultaneously transmitting to or receiving from the users. On the other hand, coordinated scheduling/beamforming is a form of coordination where a user is communicating with a single transmission or reception point, but the rest of the entities of the coordination area are aware of this communication.

Depending on whether the coordination is made on the DL or the UL, we can split CoMP into four different approaches. For the DL segment we may have:

- Joint transmission: Data is transmitted to the users simultaneously from several BSs. The aim of this approach is to improve the received signal quality and strength. It also actively cancels the interference from transmissions that are intended for other users. This type of CoMP places a high demand onto the fronthaul links.
- Coordinated scheduling/beamforming: Data is transmitted to a single user from one BS. The scheduling decisions as well as any beamforming are coordinated to control the interference that other BSs may generate. Fronthaul communication is much less critical than in joint transmission.

On the UL segment we have similar techniques:

Figure 2.4: Example of CoMP functioning for the DL segment.

- Joint reception: By joint reception all BSs from the coordination area receive the data transmitted by a given user. The signals received are then combined and processed to produce the final output signal. This technique allows signals that are very low in strength or affected by interference in some areas to be received with fewer errors. Large amount of data is required to be shared between BSs.
- Coordinated scheduling/beamforming: Similar to the DL scenario, the receiving elements are aligned for the reception and interference is minimized.

Figure 2.4 shows an example of the CoMP functioning with DL joint transmission and coordinated scheduling/beamforming.

CoMP presents a completely different philosophy if compared to the ICIC technique and its derived approaches. It brings the idea of a total coordination between BSs so interference is not mitigated, but turned into useful signal. Cell boundaries become much more foggy to users and the coordinated area provides uniform service. This kind of distributed network is the seed of CF massive MIMO. However, there is yet another technology that is needed: massive MIMO.

2.2 Key enabling technology: massive MIMO

The proposal of a multi-antenna system for enhancing data rates came in the early 1990s. It was intended to achieve wider bandwidth by combining different signals which were first spatially multiplexed. Later on, multi-antenna communications were denoted as MIMO. In the classical approach of a MIMO system, a transmitter and a receiver have both several antennas and make use of them for either having a more robust communication or a higher data rate [16]. In late 2000s, the concept evolved into a multi-antenna transmitter, e.g. a cellular BS, and several users. This new approach is referred to as multi-user MIMO (MU-MIMO) [17]. Compared to the original MIMO, with MU-MIMO computational complexity grows along the number of active users in the network and so do the training resources required to conduct the channel estimation procedure.

First introduced by Marzetta in [18], massive MIMO takes the MU-MIMO idea one step ahead. With massive MIMO, a given BS is equipped with a large number of antennas and serves a set of users. Under this new paradigm, and assuming a TDD protocol, only the BS is required to have channel estimates and simple linear processing is enough for both DL and UL segments. This linear processing allows the BS to serve all users coherently in the same time-frequency resource. Two aspects make massive MIMO a competitive technology: channel hardening and favorable propagation [19,20].

Thanks to channel hardening the small-scale fading effects of the propagation environment vanish and time/frequency-selective channels behave basically as additive white Gaussian noise (AWGN) channels. In other words, the channel gain of a given realization will match the averaged value of several channel realizations. Assuming $\mathbf{h}$ is the channel vector at realization t , channel hardening can be expressed as

$$\frac{\|\mathbf{h}\|^2}{\mathbb{E}\{\|\mathbf{h}\|^2\}} \rightarrow 1, \text{ as } N \rightarrow \infty, \quad (2.1)$$

where N is the number of antennas at the BS. In addition, channel hardening removes almost all of the variability of the DL channel. This makes the estimated achievable rates at the users sufficiently tight to the true data rate.

On the other side, with favorable propagation users are virtually independent to each other as their channels are orthogonal. This strongly minimizes the multi-user interference when using simple decoders that do not account for this negative effect. Assuming two users with channel

vectors $\mathbf{h}_1$ and $\mathbf{h}_2$, we may express favorable propagation as

$$\frac{\mathbf{h}_1^H \mathbf{h}_2}{\sqrt{\mathbb{E}\{\|\mathbf{h}_1\|^2\} \mathbb{E}\{\|\mathbf{h}_2\|^2\}}} \rightarrow 0, \text{ as } N \rightarrow \infty. \quad (2.2)$$

CF massive MIMO networks inherit these properties to some extent. Throughout this thesis we will see how the CF architecture, channel modeling and precoder/combiner design, just to name a few aspects, have a major impact on both channel hardening and favorable propagation. In the end, CF massive MIMO networks are, indeed, based on the same foundations as classical massive MIMO systems.

2.3 State-of-the-art

We may say that the combination of the CoMP philosophy and distributed massive MIMO gave birth to the CF massive MIMO concept. Cells exist no more and BSs are substituted by low-power APs. This architecture has been a hot topic in research for the past few years. In this section we will briefly expose some contributions from the research community to the topic. The following list is organized by particular contribution, though most of the referenced articles make contributions on more than one area.

- Baseline work [10, 12]: The seminal work on CF massive MIMO presented the concept of the technology itself and its basic functioning. It also proposes a particular power control and pilot assignment strategies based on optimization problems. It gives closed-form expressions for computing the DL and UL achievable rates with a simple precoding/combining scheme.
- Multi-antenna scenarios [21, 22]: The early contributions on CF massive MIMO assumed that both transmitters and receiver were equipped with one antenna only. In the cited contributions, system performance with multi-antenna devices is addressed.
- Correlated Rayleigh fading [23–25]: As a natural evolution of the multi-antenna proposals, a Rayleigh fading channel with spatially correlated antennas at each AP is assumed in these contributions.
- Ricean fading [26–28]: In previous works, all channel components were assumed to be equal on terms of average power. If having a dominant LOS component, a Ricean

distribution models the channel coefficients.

- Clustering strategies [29–35]: Despite the fact that in CF networks there are no longer boundaries or cells, some works have several proposals for implementing clustering strategies. Some contributions have posed a UC scheme in which only a set of APs serve a given user. On the other side, it can be conformed a particular region (smaller than the coverage area) as a cluster for APs cooperation. It is worth mentioning that clusterizing the coverage area is essential for making the system scalable in non-ideal networks.
- Signal processing and power control [11, 13, 36–38]: Even though massive MIMO-based systems are supposed to perform well under simple processing strategies, there is much to gain with more advanced precoders and combiners, specially under correlated fading channels. Several proposals address different processing schemes for making CF massive MIMO more competitive, together with heuristic or optimized power allocation strategies
- Pilot assignment strategies [39–42]: Pilot contamination is a well-known issue in massive MIMO, and so in CF. A good pilot assignment strategy is of cornerstone importance, specially when the number of users grows large. Cited works present fast and heuristic approaches for pilot assignment.
- Energy efficiency [43–45]: A core requirement for 5G and beyond 5G communications is a high energy efficiency. Computation of energy efficiency is key for developing strategies under which the system does not lose performance but achieves sufficient data rate.
- Channel hardening [46]: There is little contribution on this topic, particularly in CF massive MIMO. The cited reference addresses a channel hardening and favorable propagation analysis, but neglects the use of advanced processing schemes, complex channel modeling or power allocation strategies.
- Non-ideal hardware [47–50]: Several assumptions are made when addressing a CF massive MIMO system. From unlimited and error free fronthaul links to ideal analog-digital converters. The cited references address several kinds of non-ideal hardware issues and hardware impairments.

Though there is much more literature on CF massive MIMO, these are the main foundations in which this thesis is built on. Throughout this document some other references will be cited for either supporting claims, proving analytical expressions and/or justifying assumptions.

Chapter 3

System Model for Cell-Free Massive MIMO Networks

Throughout this chapter the baseline system model for CF massive MIMO networks that serves as the core pillar to the rest of the document will be addressed. Starting from the network structure and frame organization up to the DL/UL user data rate estimation, the mathematical expressions presented are valid for any transmit precoding or receive combining schemes. Likewise, the channel model composes a general approach that allows us to address different scenarios by assuming particular assumptions later on in this thesis.

3.1 Network layout and frame structure

Following the original CF massive MIMO proposal [10–12], our network deployment will consist of M uniformly distributed APs equipped with N antennas each, and K uniformly distributed single-antenna users, denoted as mobile stations (MSs), throughout a large coverage region. Each AP will be connected to a CPU thanks to an error free and unlimited capacity fronthaul link. The CPU acts as a local network controller and will manage every AP. Figure 3.1 depicts an example of a CF massive MIMO network layout. Unless stated otherwise, every AP will coherently serve all MSs at the same time-frequency resource.

Massive MIMO systems have the particular disadvantage of being questionably suited for frequency-division duplex (FDD) communications. The need for the users to estimate their DL channels from every serving antenna would produce huge training overhead. Same situation happens in the CF architecture. It would be not feasible for every MSs to estimate their channels linking to every APs. To overcome this issue, TDD is employed. To better understand the frame structure under the TDD operation, let us define at this point the coherence interval $\tau_c = B_c T_c$ as the time-frequency samples over which the channel response is considered to be constant and frequency-flat, having B_c as the coherence bandwidth and T_c as the coherence time. Both these latter parameters strongly depend on the propagation environment and transmitter/receiver relative velocity. Nevertheless, since APs are located at fixed positions, while

Figure 3.1: CF massive MIMO network layout.

Figure 3.2: TDD frame structure for a coherence interval.

MSs are assumed to be either static or moving at very low speed, it is reasonable to assume that τ_c would range from 200 to 500 samples [19].

By exploiting channel reciprocity, the TDD coherence interval will be divided into three phases of different duration: UL training phase (τ_p samples), DL data payload phase (τ_d samples) and UL data payload phase (τ_u samples). The duration of the whole TDD frame cannot be longer than the duration of the coherence interval, i.e. $\tau_c \geq \tau_p + \tau_d + \tau_u$. The TDD frame structure is shown in Figure 3.2.

3.2 Channel model

In this thesis, it is considered that the transmission/reception procedure takes place at any flat-fading OFDM subcarrier. Under this assumption, the UL $\mathbf{h}_{mk} \in \mathbb{C}^{N \times 1}$ channel vector linking the m -th AP and the k -th MS will be generically characterized as a Ricean fading channel

consisting of a LOS component added to a Rayleigh distributed component. That is,

$$\mathbf{h}_{mk} = \bar{\mathbf{h}}_{mk} + \mathbf{g}_{mk}, \quad (3.1)$$

where $\bar{\mathbf{h}}_{mk}$ is LOS component and $\mathbf{g}_{mk}$ is the non-line-of-sight (NLOS) component. Due to the different physical location of each AP, it is assumed that $\mathbf{h}_{mk}$ and $\mathbf{h}_{m'k}$ are totally uncorrelated, i.e. $\mathbb{E} \{ \mathbf{h}_{mk}^H \mathbf{h}_{m'k} \} = 0$. Following similar reasoning, $\mathbb{E} \{ \mathbf{h}_{mk}^H \mathbf{h}_{mk'} \} = 0$. In addition, $\mathbf{h}_{mk}$ at coherence interval t is independent from its counterpart at interval t' since slow-moving MSs are considered (block-fading channels). Thanks to the assumed channel reciprocity of TDD-based systems, the DL counterpart of (3.1) is $\mathbf{h}_{mk}^H$.

3.2.1 LOS component

Given the dense deployment of CF networks, it is reasonable to assume that some MSs would have a LOS link with those APs in close vicinity. Therefore, the LOS component $\bar{\mathbf{h}}_{mk}$ for a uniform linear array (ULA) whose elements are half-wavelength spaced is expressed as

$$\bar{\mathbf{h}}_{mk} = \sqrt{\frac{\kappa_{mk}}{\kappa_{mk} + 1}} \zeta_{mk} \left[1, e^{j\pi \sin \phi_{mk}}, \dots, e^{j(N-1)\pi \sin \phi_{mk}} \right]^T, \quad (3.2)$$

where ϕ_{mk} denotes the angle of arrival (AoA) between AP m and MS k , ζ_{mk} denotes the normalized-to-noise average channel gain accounting for path loss and shadow fading, and κ_{mk} denotes the Ricean κ -factor with $\kappa_{mk} = 0$ for NLOS propagation links and $10\log_{10}(\kappa_{mk}) \sim \mathcal{N}(\mu_\kappa, \sigma_\kappa^2)$ for LOS propagation links. Since APs are placed at fixed locations and MSs are considered to move at very low velocity, the AoA will not vary during a coherence interval and the phase shift $\pi \sin \phi_{mk}$ in (3.2) will be assumed to remain constant [26]. The link between AP m and MS k will toggle between LOS and NLOS with probability

$$P_{\text{LOS}}(d_{mk}) = \min \left(1, \frac{d_0}{d_{mk}} + \left(1 - \frac{d_0}{d_{mk}} \right) e^{-\frac{d_{mk}}{2d_0}} \right), \quad (3.3)$$

having d_{mk} as the distance of the mk AP-MS pair and d_0 a reference distance.

3.2.2 NLOS component

The NLOS component $\mathbf{g}_{mk}$ follows a $\mathcal{CN}(\mathbf{0}_N, \mathbf{R}_{mk})$ distribution, where $\mathbf{R}_{mk}$ represents the positive semi-definite spatial correlation matrix of the antenna array as seen from the k -th MS

for a particular angular standard deviation (ASD) with $\zeta_{mk} \triangleq \text{tr}(\mathbf{R}_{mk})/N$. Having a low ASD would translate into having a strong correlation between the antennas of the given AP. In contrast to the uncorrelated Rayleigh fading scenario in which the signal multi-path components arrive at the AP from any direction with the same probability, spatially correlated scenarios presumes some of these components may come more from some directions than the others. Moreover, the exhibited spatial correlation by a particular channel is also determined by the antenna arrays that most probably will not have uniform radiation patterns. The MIMO channel has been shown to be spatially correlated in practical scenarios [51–53], so, under multi-antenna APs, it is almost mandatory to assume a local covariance matrix at each AP if the channel model is to capture reality accurately.

In this thesis, a local scattering spatial correlation model is assumed to model each covariance matrix $\mathbf{R}_{mk}$ [20, Chapter 2], so the (r, q) -th component of the spatial correlation matrix is given by

$$[\mathbf{R}_{mk}]_{r,q} = \frac{\zeta_{mk}}{\kappa_{mk} + 1} \int_{-\infty}^{\infty} e^{j\pi(r-q)\sin(\phi_{mk}+\nu)} f(\nu) d\nu, \quad (3.4)$$

having $f(\nu)$ as the probability density function of the given multi-path component produced by the local scatterer. The angular deviation follows $\nu \sim \mathcal{N}(0, \sigma_\nu^2)$ with σ_ν the ASD around the nominal angle that conforms the scattering cluster at the MS. Though in practice correlation matrices vary every several coherence interval, throughout this work it is assumed that $\mathbf{R}_{mk}$ is fixed during the evaluated channel realizations. Nevertheless, there are several methods for obtaining spatial correlation matrices in massive MIMO systems (see for example [54, 55]).

Finally, considering both channel components, the channel coefficients are distributed as

$$\mathbf{h}_{mk} \sim \mathcal{CN}(\bar{\mathbf{h}}_{mk}, \mathbf{R}_{mk}). \quad (3.5)$$

3.2.3 Path loss and shadow fading

The 3-slope path loss model that was used in early CF massive MIMO works is actually based on macro-cell deployments and could lead to inaccurate conclusions. In this thesis, a simplified version of the 3GPP urban microcell model described in [56] and partially validated through field measurements in [57, 58], is used. It is expressed as

$$L_{mk} = L_0 + 10\alpha \log_{10}(d_{mk}) + \chi_{mk} - \sigma_n^2 \quad (3.6)$$

where L_0 is the path loss at reference distance of 1 m, a is the environment-dependent path loss exponent, and σ_n^2 is the noise variance. The shadow fading component χ_{mk} is modeled as a correlated log-normal random variable with variance σ_χ^2 as described in [12, (54)-(55)]. The channel gain results in $\zeta_{mk} = 10^{-\frac{L_{mk}}{10}}$.

3.3 UL training phase

Communication in any coherence interval of a TDD-based massive MIMO system invariably starts with the MSs sending their training pilot sequences to allow the channel to be estimated. During the UL training phase, all K MSs simultaneously transmit pilot sequences of τ_p samples to every APs. The $N \times \tau_p$ received UL training signal matrix at the m -th AP is given by

$$\mathbf{Y}_{p,m} = \sqrt{\tau_p \rho_p} \sum_{k=1}^K \mathbf{h}_{mk} \boldsymbol{\varphi}_k^T + \mathbf{N}_{p,m}, \quad (3.7)$$

where ρ_p is the signal-to-noise ratio (SNR) in the UL training signal, $\boldsymbol{\varphi}_k$ denotes the $\tau_p \times 1$ training sequence assigned to MS k with $\|\boldsymbol{\varphi}_k\|^2 = 1$, and $\mathbf{N}_{p,m} \in \mathbb{C}^{N \times \tau_p}$ is a noise matrix whose elements are distributed as $\mathcal{CN}(0, 1)$. The strategy for assigning the available pilot sequences is of key importance and strongly affects the whole system performance.

3.3.1 Pilot assignment

Pilot sequences should be designed to be pair-wisely orthogonal with each other. A good strategy is to build a discrete Fourier transform (DFT) matrix [59]. However, the length of the training should be limited in order not to lose too much performance due to training overhead. In most of the literature, this length ranges from 5 to 20 samples. This brings yet another issue when there is a considerably large amount of active MSs in the network, namely pilot contamination, a well studied phenomenon in the context of classical massive MIMO [19, 20]. Pilot contamination occurs when two or more MSs are using the same training sequence. This translates into a poor channel estimation and, thus, a performance degradation. Our baseline strategy for pilot assignment involves two possible scenarios:

- $K \leq \tau_p$: MSs are always assigned mutually orthogonal training sequences. No pilot contamination occurs.

- $K > \tau_p$: Unlike co-located massive MIMO, in CF networks MSs can be significantly distant due to the large coverage area. Even if two MSs are using the same sequence, pilot contamination could be nearly unnoticeable if those users are distant from each other thanks to their different channel gains. When there are not enough available training sequences, the *fingerprinting* pilot assignment strategy is used [40, Section V]. This technique ensures that pilot sequences are reused only among MSs located far apart from each other.

3.3.2 Channel estimation

Upon receiving all UL training signals, APs can either forward them to the CPU, or can compute their channel estimates locally. Which choice to make depends on the precoding and combining strategies adopted. Without loss of generality, it is assumed that local channel estimation is only needed if a local precoding/combining strategy is applied. In those scenarios in which a centralized strategy is adopted, it is a better option to perform the estimation procedure at the CPU due to scalability concerns [34]. Whenever channel estimation takes place at the CPU, each AP will need to forward a fixed-length signal of τ_p samples. However, after a local estimation, each AP will send K signals instead. This latter scenario can bring along a major issue in terms of capacity requirements at the fronthaul links if the number of MSs is large (which is a reasonable assumption in current and future wireless networks). Either way, the following channel estimation technique description will not mathematically vary.

Motivated by the previous mentioned assumption of having low-speed moving MSs, large-scale parameters are considered to be known beforehand and available at either the CPU or APs at anytime. This way, we define the projection of $\mathbf{Y}_{p,m}$ onto $\boldsymbol{\varphi}_k$ as

$$\begin{aligned}\check{\mathbf{y}}_{p,mk} &= (\mathbf{Y}_{p,m} - \mathbb{E}\{\mathbf{Y}_{p,m}\}) \boldsymbol{\varphi}_k^* \\ &= \sqrt{\tau_p \rho_p} \mathbf{h}_{mk} + \sqrt{\tau_p \rho_p} \sum_{k' \neq k}^K \mathbf{h}_{mk} \boldsymbol{\varphi}_{k'}^T \boldsymbol{\varphi}_k^* + \mathbf{N}_{p,m} \boldsymbol{\varphi}_k^*\end{aligned}\tag{3.8}$$

together with

$$\check{\mathbf{h}}_{mk} = \mathbf{h}_{mk} - \mathbb{E}\{\mathbf{h}_{mk}\},\tag{3.9}$$

and derive the MMSE estimate for the channel linking the m -th AP and the k -th MS as [26, 60]

$$\hat{\mathbf{h}}_{mk} = \bar{\mathbf{h}}_{mk} + \frac{\mathbb{E} \left\{ \check{\mathbf{y}}_{p,mk} \check{\mathbf{h}}_{mk}^H \right\}}{\mathbb{E} \left\{ \check{\mathbf{y}}_{p,mk} \check{\mathbf{y}}_{p,mk}^H \right\}} \check{\mathbf{y}}_{p,mk} = \bar{\mathbf{h}}_{mk} + \sqrt{\tau_p \rho_p} \mathbf{R}_{mk} \boldsymbol{\Psi}_{mk}^{-1} \check{\mathbf{y}}_{p,mk}, \quad (3.10)$$

where

$$\boldsymbol{\Psi}_{mk} = \tau_p \rho_p \sum_{k'=1}^K \mathbf{R}_{mk'} |\boldsymbol{\varphi}_{k'}^H \boldsymbol{\varphi}_k|^2 + \mathbf{I}_N, \quad (3.11)$$

is the covariance matrix accounting for pilot contamination and noisy pilots reception. The pilot contamination term does not only reduce the estimation quality, but it also makes the estimated to become correlated. Both effects deteriorate performance but only the second one is responsible for the coherence interference between MSs. The channel estimation coefficients are distributed as

$$\hat{\mathbf{h}}_{mk} \sim \mathcal{CN}(\bar{\mathbf{h}}_{mk}, \boldsymbol{\Gamma}_{mk}), \quad (3.12)$$

where

$$\boldsymbol{\Gamma}_{mk} = \tau_p \rho_p \mathbf{R}_{mk} \boldsymbol{\Psi}_{mk}^{-1} \mathbf{R}_{mk}. \quad (3.13)$$

We define at this point the channel estimation error as $\mathbf{e}_{mk} = \mathbf{h}_{mk} - \hat{\mathbf{h}}_{mk}$. A key property of the MMSE channel estimation is that $\hat{\mathbf{h}}_{mk}$ and $\mathbf{e}_{mk}$ are independent. The channel estimation error vector follows the distribution

$$\mathbf{e}_{mk} \sim \mathcal{CN}(\mathbf{0}_N, \mathbf{E}_{mk}) \quad (3.14)$$

with the error covariance matrix expressed as

$$\mathbf{E}_{mk} = \mathbb{E} \left\{ \mathbf{e}_{mk} \mathbf{e}_{mk}^H \right\} = \mathbf{R}_{mk} - \boldsymbol{\Gamma}_{mk}. \quad (3.15)$$

For later convenience, let us define the collective channel responses and estimates for MS k as $\mathbf{h}_k = [\mathbf{h}_{1k}^T, \dots, \mathbf{h}_{Mk}^T]^T$ and $\hat{\mathbf{h}}_k = [\hat{\mathbf{h}}_{1k}^T, \dots, \hat{\mathbf{h}}_{Mk}^T]^T$, respectively.

As in massive MIMO, the least-squares (LS) channel estimation is an alternative to the MMSE scheme. Channel estimates would be expressed instead as

$$\hat{\mathbf{h}}_{mk} = \bar{\mathbf{h}}_{mk} + \frac{1}{\sqrt{\tau_p \rho_p}} \check{\mathbf{y}}_{p,mk}. \quad (3.16)$$

Notice that with this scheme there is no need of having statistical knowledge. However, the LS estimator has been shown to produce a considerable performance loss in terms of SE when having severe pilot contamination in CF massive MIMO networks [31].

3.3.3 AP-MS linking

The research community proposed an alternate architecture for CF massive MIMO networks first in [29]. In this scheme, questionably referred to as UC, only a subset of MSs are served by the m -th AP. Each AP provides service to the K_C MSs whose instantaneous channel estimates are the strongest, i.e. the largest $\|\hat{\mathbf{h}}_{mk}\|^2$. This particular strategy implies that APs must compute a connectivity coefficient c_{mk} within each coherence interval to account for small-scale fading channel components, thus limiting its practical application. In this thesis, the main criteria for linking an AP and a MS under any clustering strategy will rely on large-scale coefficients ζ_{mk} only, which have been said to be known beforehand and whose variations takes several coherence intervals. The $M \times K$ connectivity matrix $\mathbf{C}$, which accounts for all c_{mk} coefficients, will depend on the relative variations of the MSs large-scale channel components and will have, in general, a variation pace even slower than ζ_{mk} . It will latter be shown that channel hardening clearly depends on the active number of APs and, thus, the design of $\mathbf{C}$ becomes of cornerstone importance. In addition, the fronthaul capacity requirements may be drastically reduced if a clustering strategy is implemented under distributed precoding and combining.

When considering any clustering approach we may express connectivity coefficients as [31]

$$c_{mk} = \begin{cases} 1, & \text{if AP } m \text{ serves MS } k \\ 0, & \text{otherwise.} \end{cases} \quad (3.17)$$

It is easy to check that the conventional CF scheme is attained by setting $c_{mk} = 1 \forall mk$. Note that c_{mk} coefficients are just a mathematical resource for easing the comprehension of the analysis throughout this work and they will be explicit when designing precoding and combining schemes.

We account for different ζ_{mk} -based clustering strategies explained as follows.

Figure 3.3: CF massive MIMO network under the AP-centric clustering architecture.

3.3.3.1 AP-centric

Similar to the initial proposal in [29], under this approach every AP will serve those K_C MS with largest ζ_{mk} . This strategy guarantees that no AP remains inactive in the network. Figure 3.3 depicts a CF massive MIMO network working under an AP-centric clustering architecture with $K_C = 3$.

3.3.3.2 AP-centric with master AP

In AP-centric scenario, though it is unlikely that any user will remain unserved if both AP and MS are uniformly deployed, we cannot take this situation for granted in practical scenarios. In order to avoid this unwanted situation, the AP-centric approach can be enhanced by assigning a *master* AP to each MS [34]. Under this approach every AP will serve K_C MS and will play the role of the master AP to a particular MS that may or may not be within the initial subset of K_C MS.

3.3.3.3 MS-centric

Inversely to the AP-centric approach, with the MS-centric strategy the user will select those M_C AP with stronger channels for being provided service with. In this scenario it is highly likely for several AP to remain inactive potentially resulting in a degradation of the overall system performance, so each AP will select the user with strongest channel to serve if it is not within

any of the subsets of M_C AP serving the MS.

3.3.3.4 AP-MS centric

As a combination of the AP centric and MS centric strategies, in the AP-MS centric approach every MS will select its M_C serving AP, while each AP will select its K_C MS subset. This mixed approach will generate much more fronthaul link data traffic than any other, but provides data rates close to the pure CF scheme. If compared to the rest of strategies, this scheme ensures a minimum of serving AP to a user (M_C) and guarantees that every AP serves at least K_C MS.

3.4 DL data payload phase

3.4.1 Data transmission and reception

After the acquisition of the channel estimates, a precoding vector can be constructed for transmitting the DL data payload. Regardless of having a centralized or distributed precoding strategy, let ϖ_{mk} be the $N \times 1$ beamforming vector assigned to the mk AP-MS pair. The DL payload data signal sent from the m -th AP is given by

$$\mathbf{x}_m = \sqrt{\rho_d} \sum_{k=1}^K \varpi_{mk} s_{d,k}, \quad (3.18)$$

where ρ_d denotes the DL SNR and $s_{d,k}$ is the data symbol intended to MS k which satisfies $\mathbb{E}\{|s_{d,k}|^2\} = 1$. The precoding vector must be designed so the DL signal fulfills the average power constraint

$$\mathbb{E}\{\|\mathbf{x}_m\|^2\} \leq \rho_d, \quad (3.19)$$

which needs to be satisfied over the long-term, i.e. accounting for several coherence intervals. Since single-antenna MSs are assumed, the estimated data symbol received at MS k is expressed as

$$\begin{aligned} \hat{s}_{d,k} &= \sum_{m=1}^M \mathbf{h}_{mk}^H \mathbf{x}_m + n_k = \sqrt{\rho_d} \sum_{m=1}^M \mathbf{h}_{mk}^H \left(\sum_{k'=1}^K \varpi_{mk'} s_{d,k'} \right) + n_k \\ &= \sqrt{\rho_d} \mathbf{h}_k^H \sum_{k'=1}^K \varpi_{mk'} s_{d,k'} + n_k, \end{aligned} \quad (3.20)$$

where $n_k \sim \mathcal{CN}(0, 1)$ accounts for the receiver thermal noise.

3.4.2 Achievable data rate estimation

For estimating the achievable data rate, and since there are not any DL pilots, MSs must rely on channel statistics, instead of channel state information (CSI) of each realization. As shown later on Chapter 4, if sufficient channel hardening is achieved, this estimation will be tight.

Let us rearrange (3.20) as

$$\hat{s}_{d,k} = \text{DS}_{d,k}s_{d,k} + \text{BU}_{d,k}s_{d,k} + \sum_{k' \neq k}^K \text{MUI}_{d,kk'}s_{d,k'} + n_k, \quad (3.21)$$

where the following definitions have been made:

$$\text{DS}_{d,k} \triangleq \sqrt{\rho_d} \mathbb{E} \left\{ \sum_{m=1}^M \mathbf{h}_{mk}^H \boldsymbol{\varpi}_{mk} \right\} = \sqrt{\rho_d} \mathbb{E} \{ \mathbf{h}_k^H \boldsymbol{\varpi}_k \}, \quad (3.22)$$

$$\text{BU}_{d,k} \triangleq \sqrt{\rho_d} \left(\sum_{m=1}^M \mathbf{h}_{mk}^H \boldsymbol{\varpi}_{mk}^H - \mathbb{E} \left\{ \sum_{m=1}^M \mathbf{h}_{mk}^H \boldsymbol{\varpi}_{mk} \right\} \right) = \sqrt{\rho_d} (\mathbf{h}_k^H \boldsymbol{\varpi}_k - \mathbb{E} \{ \mathbf{h}_k^H \boldsymbol{\varpi}_k \}), \quad (3.23)$$

and

$$\text{MUI}_{d,kk'} \triangleq \sqrt{\rho_d} \sum_{m=1}^M \mathbf{h}_{mk}^H \boldsymbol{\varpi}_{mk'} = \sqrt{\rho_d} \mathbf{h}_k^H \boldsymbol{\varpi}_{k'}, \quad (3.24)$$

where $\boldsymbol{\varpi}_k = [\boldsymbol{\varpi}_{1k}^T, \dots, \boldsymbol{\varpi}_{Mk}^T]^T$ is the collective precoding vector. These three latter definitions are the strength of the desired signal, the interference caused due to the beamforming uncertainty, and the multi-user interference caused by the transmission to the rest of the MSs, respectively.

Building on the work presented, for instance, in [12, 19], the SINR ratio can be derived as follows. In particular, the sum of the second, third and fourth terms in (3.21) are treated as *effective noise*. The additive terms constituting the effective noise are mutually uncorrelated, and uncorrelated to $s_{d,k}$. Therefore, both the desired signal and the so-called effective noise are uncorrelated indeed. Now, recalling the fact that uncorrelated Gaussian noise represents the worst case scenario, we may express the achievable DL rate measured in bps as

$$\mathcal{R}_{d,k} = B \frac{1 - \tau_p / \tau_c}{2} \log_2 (1 + \text{SINR}_{d,k}), \quad (3.25)$$

where B is the system bandwidth. The term $1 - \tau_p/\tau_c$ represents the losses due to the UL training phase overhead, while the $1/2$ factor accounts for the equal time-frequency split between DL and UL in the TDD frame. The SINR term is given by

$$\text{SINR}_{d,k} = \frac{|\text{DS}_{d,k}|^2}{\mathbb{E}\left\{|\text{BU}_{d,k}|^2\right\} + \sum_{k' \neq k}^K \mathbb{E}\left\{|\text{MUI}_{d,kk'}|^2\right\} + 1}. \quad (3.26)$$

The achievable rate in (3.25) represents a lower-bound and is often referred to as the *channel hardening bound*, since it relies on this characteristic for being tight to the true rate. In addition, note that it is not possible to compute an *instantaneous* SINR. The computation of (3.26) needs of several channel realizations through a Monte-Carlo procedure, unless a closed-form expression is provided.

DL training was shown to improved SE in [39]. However, single-antenna APs and fully orthogonal pilots were considered only. As later shown in this thesis, having several antennas at each AP will make (3.25) considerably tight by achieving a certain degree of channel hardening. In addition, the scenario presented in [39] is not realistic since, in practice, not every MS will have a pair-wisely orthogonal pilot. The general trend in the research community is not to use DL pilots in CF massive MIMO networks.

3.5 UL data payload phase

3.5.1 Data transmission and reception

Every MS will send its corresponding data symbol $s_{u,k}$ satisfying $\mathbb{E}\{|s_{d,k}|^2\} = 1$ to all APs. The transmission power will be weighted by the power control coefficient η_k subject to $0 \leq \eta_k \leq 1$. The received UL data payload signal at the m -th AP is expressed as

$$\mathbf{y}_m = \sqrt{\rho_u} \sum_{k=1}^K \sqrt{\eta_k} \mathbf{h}_{mk} s_{u,k} + \mathbf{n}_m, \quad (3.27)$$

where ρ_u is the UL SNR and $\mathbf{n}_m$ is a vector representing the thermal noise AP m whose entries are distributed as $\mathcal{CN}(0, 1)$. This signal can be processed at the APs with a distributed combining strategy, or it can be directly forwarded to the CPU for performing a centralized combining operation. In any case, we can express the decoded data symbol from the k -th MS

as

$$\begin{aligned}
 \hat{s}_{u,k} &= \sum_{m=1}^M \mathbf{v}_{mk}^H \mathbf{y}_m \\
 &= \sqrt{\rho_u \eta_k} \left(\sum_{m=1}^M \mathbf{v}_{mk}^H \mathbf{h}_{mk} \right) s_{u,k} + \sqrt{\rho_u} \sum_{k' \neq k}^K \sqrt{\eta_{k'}} \left(\sum_{m=1}^M \mathbf{v}_{mk}^H \mathbf{h}_{mk'} \right) s_{u,k'} + \sum_{m=1}^M \mathbf{v}_{mk}^H \mathbf{n}_m \\
 &= \sqrt{\rho_u \eta_k} \mathbf{v}_k^H \mathbf{h}_k s_{u,k} + \sqrt{\rho_u} \sum_{k' \neq k}^K \sqrt{\eta_{k'}} \mathbf{v}_k^H \mathbf{h}_{k'} s_{u,k'} + \mathbf{v}_k^H \mathbf{n},
 \end{aligned} \tag{3.28}$$

where $\mathbf{v}_k = [\mathbf{v}_{1k}^T, \dots, \mathbf{v}_{Mk}^T]^T$, being $\mathbf{v}_{mk} \in \mathbb{C}^{N \times 1}$ the combining vector, and $\mathbf{n} = [\mathbf{n}_1^T, \dots, \mathbf{n}_M^T]^T$ accounts for all noise contributions.

3.5.2 Achievable data rate estimation

Unlike the DL segment, data rate estimation for the UL data phase can be approached in two different ways. Since in this case channel estimates are available thanks to the UL training, they can be used for computing the instantaneous SINR. On the other hand, if estimates are forgone (such as in local combining strategies with global channel estimation), the use-and-then-forget (UatF) bound needs to be computed averaging several channel realizations. This name comes from the fact that the channel estimates are used for computing the receive combining vectors and then they are effectively forgotten before the signal detection takes place.

3.5.2.1 CSI available at the receiver

Assuming MMSE estimates are known and there is CSI at receiver (CSIR), an achievable rate expression is given by

$$\mathcal{R}_{u,k}^{\text{CSIR}} = B \frac{1 - \tau_p / \tau_c}{2} \mathbb{E} \left\{ \log_2 \left(1 + \text{SINR}_{u,k}^{\text{CSIR}} \right) \right\}, \tag{3.29}$$

where the SINR term is expressed as

$$\text{SINR}_{u,k}^{\text{CSIR}} = \frac{\rho_u \left| \mathbf{v}_k^H \hat{\mathbf{h}}_k \right|^2}{\rho_u \sum_{k' \neq k}^K \left| \mathbf{v}_k^H \hat{\mathbf{h}}_{k'} \right|^2 + \mathbf{v}_k^H \left(\rho_u \sum_{k'=1}^K \mathbf{E}_{k'} + \mathbf{I}_{MN} \right) \mathbf{v}_k}, \tag{3.30}$$

with $\mathbf{E}_{k'} = \text{diag}(\mathbf{E}_{1k'}, \dots, \mathbf{E}_{Mk'})$. Though this expression works for any combiner option, it is only valid if channel estimation is based on the MMSE estimator presented in (3.10) due to the necessarily independence of $\mathbf{h}_{mk}$ and $\hat{\mathbf{h}}_{mk}$. Proof can be found in [20]. This computation is typically addressed when centralized combining schemes, such as zero-forcing (ZF) or MMSE, are considered. Under such scenarios, channel estimates are needed to be available at the CPU for conforming the combining matrix. The availability of channel estimates makes it feasible to compute user data rate with the bound (3.29). Lastly, note that it is not possible to compute this bound in closed-form expression.

3.5.2.2 UatF bound

Similar to the DL data payload, the UatF bound computation relies on channel statistics. Following the same reasoning as in (3.21), we can rearrange (3.28) as

$$\hat{s}_{u,k} = \mathbf{D}\mathbf{S}_{u,k}s_{u,k} + \mathbf{B}\mathbf{U}_{u,k}s_{u,k} + \sum_{k' \neq k}^K \mathbf{M}\mathbf{U}_{u,kk'}s_{u,k'} + \mathbf{N}\mathbf{T}_{u,k}, \quad (3.31)$$

defining:

$$\mathbf{D}\mathbf{S}_{u,k} \triangleq \sqrt{\rho_d \eta_k} \mathbb{E} \left\{ \sum_{m=1}^M \mathbf{v}_{mk}^H \mathbf{h}_{mk} \right\} = \sqrt{\rho_u \eta_k} \mathbb{E} \{ \mathbf{v}_k^H \mathbf{h}_k \}, \quad (3.32)$$

$$\mathbf{B}\mathbf{U}_{u,k} \triangleq \sqrt{\rho_d \eta_k} \left(\sum_{m=1}^M \mathbf{v}_{mk}^H \mathbf{h}_{mk} - \mathbb{E} \left\{ \sum_{m=1}^M \mathbf{v}_{mk}^H \mathbf{h}_{mk} \right\} \right) = \sqrt{\rho_u \eta_k} (\mathbf{v}_k^H \mathbf{h}_k - \mathbb{E} \{ \mathbf{v}_k^H \mathbf{h}_k \}), \quad (3.33)$$

$$\mathbf{M}\mathbf{U}_{u,kk'} \triangleq \sqrt{\rho_d \eta_{k'}} \sum_{m=1}^M \mathbf{v}_{mk}^H \mathbf{h}_{mk'} = \sqrt{\rho_u \eta_{k'}} \mathbf{v}_k^H \mathbf{h}_{k'}, \quad (3.34)$$

and

$$\mathbf{N}\mathbf{T}_k \triangleq \mathbb{E} \left\{ \sum_{m=1}^M \mathbf{v}_{mk}^H \mathbf{h}_{mk} \right\} = \mathbb{E} \{ \mathbf{v}_k^H \mathbf{h}_k \}. \quad (3.35)$$

This latter term is referred to as the noise term. This way, a lower bound for the achievable rate is given by

$$\mathcal{R}_{u,k}^{\text{UatF}} = B \frac{1 - \tau_p / \tau_c}{2} \log_2 (1 + \text{SINR}_{u,k}^{\text{UatF}}), \quad (3.36)$$

with

$$\text{SINR}_{u,k}^{\text{UatF}} = \frac{|\mathbf{D}\mathbf{S}_{u,k}|^2}{\mathbb{E} \{ |\mathbf{B}\mathbf{U}_{u,k}|^2 \} + \sum_{k' \neq k}^K \mathbb{E} \{ |\mathbf{M}\mathbf{U}_{u,kk'}|^2 \} + \mathbf{N}\mathbf{T}_k}. \quad (3.37)$$

In contrast to the CSIR bound, (3.36) is mostly used when employing distributed combining schemes, such as MR. A totally distributed processing, i.e. channel estimation and combining operation, is the best option if the usage of the fronthaul links is limited.

Chapter 4

Channel Hardening in Cell-Free Massive MIMO

The concept of channel hardening first gained importance in the context of conventional (cell-based) massive MIMO, when it was shown that the time/frequency-selectivity of the channel vanishes as the number of antennas grows large [19, 20]. Under such condition, MSs can rely on channel statistics rather than in their instantaneous channel gains when estimating the DL achievable rates and conducting the optimization of certain network parameters (e.g., pilot sequence assignment, power control coefficients computation). This effect, together with the channel reciprocity given by TDD communications, eliminates the necessity of having any form of DL training. In a CF massive MIMO scenario, the total number of antennas (MN) is not the only factor that affects the channel hardening. As shown throughout this chapter, there are several parameters that contribute positively or negatively on how the channel hardens. First, let us generalize (2.1) for the DL segment as

$$\frac{|\mathbf{h}_k^H \boldsymbol{\varpi}_k|}{\mathbb{E} \{ |\mathbf{h}_k^H \boldsymbol{\varpi}_k| \}} \rightarrow 1, \text{ as } M, N \rightarrow \infty, \quad (4.1)$$

where the term $|\mathbf{h}_k^H \boldsymbol{\varpi}_k|$ represents the joint propagation channel gain. The designing of the beamforming vector $\boldsymbol{\varpi}_k$ involves a particular precoding, power coefficients allocation, and imperfect CSI. All these aspects have an impact on the hardening, apart from the channel nature itself.

From this point onward, and based on (4.1), the DL *channel hardening ratio* metric for an arbitrary MS k is defined as

$$\begin{aligned} \xi_{d,k} &= \text{Var} \left\{ \frac{|\mathbf{h}_k^H \boldsymbol{\varpi}_k|}{\mathbb{E} \{ |\mathbf{h}_k^H \boldsymbol{\varpi}_k| \}} \right\} = \frac{\text{Var} \{ |\mathbf{h}_k^H \boldsymbol{\varpi}_k| \}}{\mathbb{E} \{ |\mathbf{h}_k^H \boldsymbol{\varpi}_k| \}^2} \\ &= \frac{\mathbb{E} \{ |\mathbf{h}_k^H \boldsymbol{\varpi}_k|^2 \} - \mathbb{E} \{ |\mathbf{h}_k^H \boldsymbol{\varpi}_k| \}^2}{\mathbb{E} \{ |\mathbf{h}_k^H \boldsymbol{\varpi}_k| \}^2} = \frac{\mathbb{E} \{ |\text{BU}_{d,k}|^2 \}}{|\text{DS}_{d,k}|^2} \end{aligned} \quad (4.2)$$

In the last equality we have rearranged $\mathbb{E} \left\{ |\text{BU}_{d,k}|^2 \right\}$ as

$$\mathbb{E} \left\{ |\text{BU}_{d,k}|^2 \right\} = \rho_d \mathbb{E} \left\{ \left| \mathbf{h}_k^H \boldsymbol{\varpi}_k - \mathbb{E} \left\{ |\mathbf{h}_k^H \boldsymbol{\varpi}_k| \right\} \right|^2 \right\} = \rho_d \left(\mathbb{E} \left\{ |\mathbf{h}_k^H \boldsymbol{\varpi}_k|^2 \right\} - \mathbb{E} \left\{ |\mathbf{h}_k^H \boldsymbol{\varpi}_k| \right\}^2 \right) \quad (4.3)$$

The convergence of (4.2) to zero implies that the instantaneous joint propagation channel is close to its mean and thus a high degree of channel hardening is achieved. In other words, the channel hardening ratio essentially measures how deterministic a wireless link is, i.e. having a low hardening ratio would mean this link is quasi-deterministic. Furthermore, achieving a certain channel hardening has an impact on the accuracy of the estimated achievable rates, which will be addressed later on. Following similar computation as in (4.2), the channel hardening ratio for the UL segment can be expressed as

$$\xi_{u,k} = \text{Var} \left\{ \frac{|\boldsymbol{\vartheta}_k^H \mathbf{h}_k|}{\mathbb{E} \left\{ |\boldsymbol{\vartheta}_k^H \mathbf{h}_k| \right\}} \right\} = \frac{\mathbb{E} \left\{ |\text{BU}_{u,k}|^2 \right\}}{|\text{DS}_{u,k}|^2} \quad (4.4)$$

Up next the channel hardening ratio analysis and its impact are addressed for two scenarios: first, the commonly used Rayleigh fading channel model in which it is assumed that APs antennas are uncorrelated to each other, and second, the more general spatially correlated Ricean fading model introduced in Chapter 3.

4.1 Spatially uncorrelated Rayleigh fading scenario

For assuming totally NLOS channel links, let $\kappa_{mk} = 0$. On the other hand, having spatially uncorrelated antennas at each AP involves $\mathbf{R}_{mk} = \zeta_{mk} \mathbf{I}_N$. These assumptions result in uncorrelated Rayleigh fading distributed channel coefficients $\mathbf{h}_{mk} \sim \mathcal{CN}(0, \zeta_{mk} \mathbf{I}_N)$. Now, regarding the MMSE channel estimates, we can reformulate (3.10) as

$$\hat{\mathbf{h}}_{mk} = \frac{\sqrt{\tau_p \rho_p} \zeta_{mk}}{\tau_p \rho_p \sum_{k'=1}^K \zeta_{mk'} |\boldsymbol{\varphi}_{k'}^H \boldsymbol{\varphi}_k|^2 + 1} \mathbf{Y}_m \boldsymbol{\varphi}_k^*, \quad (4.5)$$

whose entries are distributed as $\mathcal{CN}(0, \gamma_{mk})$ with

$$\gamma_{mk} = \frac{\tau_p \rho_p \zeta_{mk}^2}{\tau_p \rho_p \sum_{k'=1}^K \zeta_{mk'} |\varphi_{k'}^H \varphi_k|^2 + 1}. \quad (4.6)$$

This latter term corresponds to the mean square of the channel estimates.

4.1.1 Precoding schemes and power allocation strategies

In this chapter, a multi-antenna version of the classical massive MIMO MR scheme will be assumed [19, 20], referred to as conjugate beamforming (CB) [10, 12]. In addition, a vector-normalized version of CB, namely normalized CB (NCB), is employed [12, 36]. These precoding schemes are mathematically expressed as

$$\varpi_{mk} = \begin{cases} c_{mk} \sqrt{\eta_{mk}} \hat{\mathbf{h}}_{mk}, & \text{for CB,} \\ c_{mk} \sqrt{\eta_{mk}} \frac{\hat{\mathbf{h}}_{mk}}{\|\hat{\mathbf{h}}_{mk}\|}, & \text{for NCB,} \end{cases} \quad (4.7)$$

where η_{mk} is the power control coefficient assigned to the link between the k -th MS and the m -th AP. Particularly, in the NCB beamformer case, and owing to the vector normalization, the small-scale channel coefficient effects vanish after averaging over the transmitted symbols. So, there is no need to average over the small-scale fading, thus allowing ρ_d in power constraint (3.19) to denote the available *instantaneous* transmit power. In this scenario, (3.19) turns into a short-term power constraint at AP m .

Now, let us define z_{mk} for further convenience as

$$z_{mk} \triangleq \mathbb{E} \left\{ \|\hat{\mathbf{h}}_{mk}\| \right\} = \mathbb{E} \left\{ \sqrt{\sum_{n=1}^N |\hat{h}_{mkn}|^2} \right\}, \quad (4.8)$$

being $\hat{h}_{mkn}$ the n -th entry of $\hat{\mathbf{h}}_{mk}$. Then

$$\mathbb{E} \{z_{mk}\} = \frac{2}{\Gamma(N) \gamma_{mk}^N} \int_0^\infty z^{2N} e^{-z^2/\gamma_{mk}} dz = \frac{\Gamma(N+1/2)}{\Gamma(N)} \sqrt{\gamma_{mk}}. \quad (4.9)$$

For later utility we define $\Gamma_N \triangleq \frac{\Gamma(N+1/2)}{\Gamma(N)}$.

The power control strategy is precoder-dependent. A uniform power control scheme has been

proven to be well matched to the CB precoding [37], while NCB precoding best suits a power allocation strategy where MSs with strongest channels are assigned more power [36]. These coefficients are then expressed as

$$\eta_{mk} = \begin{cases} \frac{1}{N \sum_{k'=1}^K c_{mk'} \gamma_{mk'}}, & \text{for CB,} \\ \frac{\gamma_{mk}}{\sum_{k'=1}^K c_{mk'} \gamma_{mk'}}, & \text{for NCB.} \end{cases} \quad (4.10)$$

Note that a different power allocation strategy (e.g. max-min fairness [12]) could be employed without compromising the up-coming analysis.

4.1.2 Channel hardening ratio computation

Recalling (4.1), closed-form expressions for computing the DL channel hardening ratio are given by the following propositions.

Proposition 1 *The DL channel hardening ratio that the k -th MS experiences in a CF massive MIMO network under a spatially uncorrelated Rayleigh fading channel when employing the CB precoding scheme, for any finite M , K , and N , is given by the closed-form expression*

$$\xi_{d,k} = \frac{\sum_{m=1}^M c_{mk} \eta_{mk} \gamma_{mk} \zeta_{mk}}{N \left(\sum_{m=1}^M c_{mk} \sqrt{\eta_{mk} \gamma_{mk}} \right)^2}. \quad (4.11)$$

Proof: This expression can be proven using the analytical tools used in [12] for computing the multi-antenna case of $|\text{DS}_{d,k}|^2$ and $\mathbb{E} \left\{ |\text{BU}_{d,k}|^2 \right\}$. ■

Proposition 2 *The DL channel hardening ratio that the k -th MS experiences in a CF massive MIMO network under a spatially uncorrelated Rayleigh fading channel when employing the NCB precoding scheme, for any finite M , K , and N , is given by the closed-form expression*

$$\xi_{d,k} = \frac{\sum_{m=1}^M c_{mk} \eta_{mk} (\zeta_{mk} + (N - \Gamma_N^2 - 1) \gamma_{mk})}{\left(\Gamma_N \sum_{m=1}^M c_{mk} \sqrt{\eta_{mk} \gamma_{mk}} \right)^2}. \quad (4.12)$$

Proof: First, recall that $\mathbf{h}_{mk} = \hat{\mathbf{h}}_{mk} + \mathbf{e}_{mk}$ and that $\hat{\mathbf{h}}_{mk}$ and $\mathbf{e}_{mk}$ are uncorrelated thanks to the MMSE estimator. Now, $|\text{DS}_{d,k}|^2$ is computed as

$$\begin{aligned} |\text{DS}_{d,k}|^2 &= \rho_d \mathbb{E} \left\{ \left| \mathbf{h}_k^H \boldsymbol{\varpi}_k \right| \right\}^2 = \rho_d \left(\sum_{m=1}^M c_{mk} \sqrt{\eta_{mk}} \mathbb{E} \left\{ \left| \mathbf{h}_{mk}^H \frac{\hat{\mathbf{h}}_{mk}}{\|\hat{\mathbf{h}}_{mk}\|} \right| \right\} \right)^2 \\ &= \rho_d \left(\sum_{m=1}^M c_{mk} \sqrt{\eta_{mk}} \mathbb{E} \left\{ \left| \left(\hat{\mathbf{h}}_{mk}^H + \mathbf{e}_{mk}^H \right) \frac{\hat{\mathbf{h}}_{mk}}{\|\hat{\mathbf{h}}_{mk}\|} \right| \right\} \right)^2 \\ &= \rho_d \left(\sum_{m=1}^M c_{mk} \sqrt{\eta_{mk}} \mathbb{E} \left\{ \|\hat{\mathbf{h}}_{mk}\| \right\} \right)^2 = \rho_d \left(\Gamma_N \sum_{m=1}^M c_{mk} \sqrt{\eta_{mk} \gamma_{mk}} \right)^2 \end{aligned} \quad (4.13)$$

Then, $\mathbb{E} \{ |\text{BU}_{d,k}|^2 \}$ is computed as

$$\begin{aligned} \mathbb{E} \{ |\text{BU}_{d,k}|^2 \} &= \rho_d \left(\mathbb{E} \{ |\mathbf{h}_k^H \boldsymbol{\varpi}_k|^2 \} - \mathbb{E} \{ |\mathbf{h}_k^H \boldsymbol{\varpi}_k| \}^2 \right) \\ &= \rho_d \left(\sum_{m=1}^M \sum_{m'=1}^M c_{mk} c_{m'k} \sqrt{\eta_{mk} \eta_{m'k}} \mathbb{E} \left\{ \left| \mathbf{h}_{mk}^H \frac{\hat{\mathbf{h}}_{mk}}{\|\hat{\mathbf{h}}_{mk}\|} \frac{\hat{\mathbf{h}}_{m'k}^H}{\|\hat{\mathbf{h}}_{m'k}\|} \mathbf{h}_{m'k} \right| \right\} \right. \\ &\quad \left. - \left(\Gamma_N \sum_{m=1}^M c_{mk} \sqrt{\eta_{mk} \gamma_{mk}} \right)^2 \right). \end{aligned} \quad (4.14)$$

Under uncorrelated Rayleigh fading channels, $\mathbf{e}_{mk} \sim \mathcal{CN}(0, (\zeta_{mk} - \gamma_{mk}) \mathbf{I}_N)$. If $m = m'$, we compute the expectation in (4.14) as

$$\begin{aligned} &\mathbb{E} \left\{ \left| \left(\hat{\mathbf{h}}_{mk}^H + \mathbf{e}_{mk}^H \right) \frac{\hat{\mathbf{h}}_{mk}}{\|\hat{\mathbf{h}}_{mk}\|} \frac{\hat{\mathbf{h}}_{mk}^H}{\|\hat{\mathbf{h}}_{mk}\|} \left(\hat{\mathbf{h}}_{mk} + \mathbf{e}_{mk} \right) \right| \right\} \\ &= \mathbb{E} \left\{ \left| \frac{\hat{\mathbf{h}}_{mk}^H \hat{\mathbf{h}}_{mk} \hat{\mathbf{h}}_{mk}^H \hat{\mathbf{h}}_{mk}}{\|\hat{\mathbf{h}}_{mk}\|^2} \right| \right\} + \mathbb{E} \left\{ \left| \mathbf{e}_{mk}^H \frac{\hat{\mathbf{h}}_{mk} \hat{\mathbf{h}}_{mk}^H}{\|\hat{\mathbf{h}}_{mk}\| \|\hat{\mathbf{h}}_{mk}\|} \mathbf{e}_{mk} \right| \right\} \\ &= \mathbb{E} \left\{ \|\hat{\mathbf{h}}_{mk}\|^2 \right\} + \mathbb{E} \left\{ \left| \frac{\hat{\mathbf{h}}_{mk}^H}{\|\hat{\mathbf{h}}_{mk}\|} \mathbb{E} \{ \mathbf{e}_{mk} \mathbf{e}_{mk}^H \} \frac{\hat{\mathbf{h}}_{mk}}{\|\hat{\mathbf{h}}_{mk}\|} \right| \right\} \\ &= N \gamma_{mk} + \zeta_{mk} - \gamma_{mk} = \zeta_{mk} + (N - 1) \gamma_{mk}. \end{aligned} \quad (4.15)$$

On the other side, if $m \neq m'$, due to the independence between $\hat{\mathbf{h}}_{mk}$ and $\hat{\mathbf{h}}_{m'k}$, it holds that

$$\mathbb{E} \left\{ \|\hat{\mathbf{h}}_{mk}\| \right\} \mathbb{E} \left\{ \|\hat{\mathbf{h}}_{m'k}\| \right\} = \Gamma_N^2 \sqrt{\gamma_{mk} \gamma_{m'k}}. \quad (4.16)$$

Finally, plugging (4.15) and (4.16) into (4.14) gives

$$\mathbb{E} \left\{ |\text{BU}_{d,k}|^2 \right\} = \rho_d \left(\sum_{m=1}^M c_{mk} \eta_{mk} (\zeta_{mk} + (N - \Gamma_N^2 - 1) \gamma_{mk}) \right). \quad (4.17)$$

■

4.1.3 Achievable rate computation

Estimating the DL achievable rate relies on (3.25) for both precoders. A closed-form expression for the CB scheme can be obtained straightforwardly extending the expression given in [10] for the multi-antenna APs case. As for the NCB precoder, the terms $|\text{DS}_{d,k}|^2$ and $\mathbb{E} \left\{ |\text{BU}_{d,k}|^2 \right\}$ were already given in (4.13) and (4.17), respectively, while for $\mathbb{E} \left\{ |\text{MUI}_{d,kk'}|^2 \right\}$ we have

$$\begin{aligned} \mathbb{E} \left\{ |\text{MUI}_{d,kk'}|^2 \right\} &= \rho_d \mathbb{E} \left\{ |\mathbf{h}_k^H \boldsymbol{\varpi}_{k'}|^2 \right\} \\ &= \rho_d \sum_{m=1}^M \sum_{m'=1}^M c_{mk'} c_{m'k'} \sqrt{\eta_{mk'} \eta_{m'k'}} \mathbb{E} \left\{ \left| \mathbf{h}_{mk}^H \frac{\hat{\mathbf{h}}_{mk'}}{\|\hat{\mathbf{h}}_{mk'}\|} \frac{\hat{\mathbf{h}}_{m'k'}^H}{\|\hat{\mathbf{h}}_{m'k'}\|} \mathbf{h}_{m'k} \right| \right\} \\ &= \rho_d \sum_{m=1}^M c_{mk'} \eta_{mk'} \underbrace{\mathbb{E} \left\{ \left| \mathbf{h}_{mk}^H \frac{\hat{\mathbf{h}}_{mk'}}{\|\hat{\mathbf{h}}_{mk'}\|} \frac{\hat{\mathbf{h}}_{mk'}^H}{\|\hat{\mathbf{h}}_{mk'}\|} \mathbf{h}_{mk} \right| \right\}}_{T_1} \\ &\quad + \rho_d \sum_{m=1}^M \sum_{m' \neq m}^M c_{mk'} c_{m'k'} \sqrt{\eta_{mk'} \eta_{m'k'}} \underbrace{\mathbb{E} \left\{ \left| \mathbf{h}_{mk}^H \frac{\hat{\mathbf{h}}_{mk'}}{\|\hat{\mathbf{h}}_{mk'}\|} \right| \right\} \mathbb{E} \left\{ \left| \frac{\hat{\mathbf{h}}_{m'k'}^H}{\|\hat{\mathbf{h}}_{m'k'}\|} \mathbf{h}_{m'k} \right| \right\}}_{T_2}. \end{aligned} \quad (4.18)$$

Now, if $\varphi_{k'} = \varphi_k$ then $\hat{\mathbf{h}}_{mk'} = \frac{\zeta_{mk'}}{\zeta_{mk}} \hat{\mathbf{h}}_{mk}$, and we compute T_1 and T_2 as in (4.15) and (4.16), respectively. If $\varphi_{k'} \neq \varphi_k$, instead we have $T_2 = 0$ and we rearrange T_1 for computing it as

$$T_1 = \mathbb{E} \left\{ \left| \frac{\hat{\mathbf{h}}_{mk'}^H}{\|\hat{\mathbf{h}}_{mk'}\|} \mathbb{E} \{ \mathbf{h}_{mk} \mathbf{h}_{mk}^H \} \frac{\hat{\mathbf{h}}_{mk'}}{\|\hat{\mathbf{h}}_{mk'}\|} \right| \right\} = \zeta_{mk}. \quad (4.19)$$

Plugging T_1 and T_2 for both cases in (4.18) gives

$$\begin{aligned} \mathbb{E} \left\{ |\text{MUI}_{d,kk'}|^2 \right\} &= \rho_d \sum_{m=1}^M c_{mk'} \eta_{mk'} \left(\zeta_{mk} + (N - 1) \gamma_{mk} |\varphi_k^H \varphi_{k'}|^2 \right) \\ &\quad + \rho_d \sum_{m=1}^M \sum_{m' \neq m}^M c_{mk'} c_{m'k'} \Gamma_N^2 \sqrt{\eta_{mk'} \eta_{m'k'} \gamma_{mk} \gamma_{m'k}} |\varphi_k^H \varphi_{k'}|^2. \end{aligned} \quad (4.20)$$

Having a low degree of channel hardening leads to a poor estimation of the achievable rate when using (3.25). Therefore, in order to contrast the information provided by the hardening coefficient, we define a data rate expression for a genie-aided MS that perfectly knows its instantaneous channel gain as

$$\mathcal{R}_{d,k}^{\text{GA}} = B \frac{1 - \tau_p/\tau_c}{2} \mathbb{E} \left\{ \log_2 \left(1 + \frac{\rho_d |\mathbf{h}_k^H \boldsymbol{\varpi}_k|^2}{\rho_d \sum_{k' \neq k}^K |\mathbf{h}_k^H \boldsymbol{\varpi}_{k'}|^2 + 1} \right) \right\}. \quad (4.21)$$

This expression provides a benchmark against which the lower bound achievable rate metrics can be measured. In those scenarios where there is sufficient hardening, (3.25) and (4.21) would tend to match each other when the total number of antennas grows large. On the contrary, when hardening is compromised, the achievable rate provided by (3.25) can dramatically differ from the upper bound. In addition, it would underestimate the true rate a MS can enjoy. Note, unfortunately, that (4.21) can only be estimated by simulation as it exhibits a dependency on the instantaneous CSI. Such a dependence precludes its practical use in governing power allocation or pilot assignment strategies that can be developed on the basis of achievable rate expressions. Consequently, it is desirable to establish conditions under which achievable rates are significantly close to the genie-aided user rates. We note that the gap between (3.25) and (4.21) also depends on the power allocation strategy since this has a large impact on the large-scale fading contribution that each MS experiences. In distributed massive MIMO-based systems like CF, there is also a dependency on the path loss propagation model as well as the spatial density of APs, thus compromising the achievability of channel hardening [46]. It seems obvious that in the UC approach it would be even more difficult to achieve a high degree of channel hardening since there are less transmitting elements to a particular MS. Intuitively, the degree of channel hardening can be improved by increasing the spatial density of APs and/or the number of antenna elements in each AP.

4.1.4 Numerical results

Following similar CF research, a squared coverage area with side D is considered. In order to avoid boundary effects, a wrapped-around technique is implemented [61]. We are considering the pure CF scheme and the AP-centric architecture for comparison. Simulation parameters are summarized in Table 4.1.

Parameter	Value
Carrier frequency (f_c)	2 GHz
System bandwidth (B)	20 MHz
Side of square coverage area (D)	1000 m
Noise figure at AP and MS	9 dB
Available average power at the AP	200 mW
Available average power at the MS	100 mW
AP antenna height (h_{AP})	15 m
MS antenna height (h_{MS})	1.65 m
Channel coherence interval length (τ_c)	200 samples
Training phase interval length (τ_p)	15 samples
Payload phase interval length ($\tau_d = \tau_u$)	$(\tau_c - \tau_p) / 2$ samples
Path loss parameters (L_0, a, σ_χ)	30, 3.67, 4 dB
Shadow fading decorrelation distance	9 m
Shadow fading correlation among APs	0.5

Table 4.1: Simulation default parameters for uncorrelated Rayleigh fading scenario.

4.1.4.1 Channel hardening ratio

The channel hardening ratio is depicted in Figure 4.1 with and without power control coefficients for different values of M using CB and NCB precoders. In agreement with [46], it is noticeable how for the single-antenna scenario under CB precoder, the hardening convergence is questionable, thus indicating that the true performance is likely to exceed the forecasted by the lower bound achievable rate. This would be the case experienced in most of the CF literature, specially in the seminal research works on this topic. However, by increasing the number of transmitting antennas at the APs, a much higher degree of hardening is achieved. Slight differences are shown between the CF and AP-centric approaches, however having $N \neq 1$ and large M almost makes those differences negligible. These results confirm that equipping the APs with more antennas always helps the channel to harden and makes the lower bound achievable rate expressions tighter regarding the upper bound. Confronting the results for CB with and without power control, it is worth noting that the power allocation coefficients significantly alter the hardening metric, in this case improving it. Assessing the impact other power allocation strategies may have on the hardening metric will be addressed later in this chapter. Turning now the attention to the NCB precoder, the hardening ratio decreases considerably when compared to the CB precoder. This fact can be attributed to the normalization step that the NCB scheme implements and suggests that NCB is a far more desirable precoding option than plain

(a) CB with power control.

(b) CB without power control.

(c) NCB with power control.

(d) NCB without power control.

Figure 4.1: Channel hardening ratio for $K = 10$ and different N under CF and AP centric ($K_C = 5$) architectures.

CB since it enforces APs to fulfill an instantaneous power constraint, it improves the hardening effect and, as it will be shown next, does not degrade the rate performance.

4.1.4.2 Lower bound achievable and genie-aided rates

A comparison between the lower bound achievable rate and the genie-aided rate is shown in Figure 4.2 for CB and NCB precoders. We refer to the AP-centric strategy just as UC for short. When having a large coverage area the gap between the two metrics is considerably high, while reducing this area makes the lower bound tighter, as Figures 4.2b and 4.2d show. This trend is present for both precoders, though the gap when using the NCB precoder, as shown in Figures 4.2c and 4.2d, is smaller; in line with the lower channel hardening ratio that NCB attains. In the case of the AP-centric approach, the behavior is similar but with slightly worse performance

 (a) $D = 1000$ m with CB.

 (b) $D = 100$ m with CB.

 (c) $D = 1000$ m with NCB.

 (d) $D = 100$ m with NCB.

Figure 4.2: Comparison between lowered bound and genie-aided rates for $M = 100$ under CF and AP-centric ($K_C = 5$) architectures.

(bear in mind that the original CF proposal constituted an upper bound in terms of average user rate). Increasing the number of antennas per AP entails reducing the hardening ratio and, thus, the gap between lower bound and genie-aided rates. This fact can be observed in the relative difference between rates defined as $(R_{d,k}^{GA} - R_{d,k})/R_{d,k}$ for different values of N , which can be seen to decrease with increasing N . As an example, in Figure 4.2a for the CF approach, the lower bound and genie-aided rates gap for $N = 1$ is 37%, while for $N = 2$ is 31.9% and for $N = 4$ is 27.18%. Lastly, it is worth noting that the sudden reduction of rate in $D = 100$ m for both precoders when $K > 20$ is due to the pilot contamination ($K > \tau_p$). This drop is present for every D value, however, it is much more pronounced when the area is small because of the close vicinity of MSs sharing the same pilot sequence.

4.1.5 Concluding remarks

This last section has studied the channel hardening effect in CF massive MIMO systems under realistic operational conditions for an uncorrelated Rayleigh fading scenario. Closed-form expressions for the channel hardening coefficient and achievable rates have been derived for the multi-antenna APs version of CB and vector NCB precoders when considering the use of imperfect CSI, pilot contamination and power loading. Numerical results have confirmed the advantage that the NCB precoder has over the CB scheme in terms of channel hardening. The effects of hardening in the recently proposed AP-centric approach have also been investigated revealing a rather modest degradation, both in achievable rate and hardening that becomes negligible as the number of APs and/or antennas per AP in the network increases. It has been shown that taking for granted the presence of channel hardening could lead to very poor estimation of the achievable rates, especially in large coverage areas with single-antenna APs. Up next we will encompass centralized precoding schemes, such as ZF precoder, while also considering the more general channel model able to include the effects of spatial correlation at the APs and the existence of LOS. In addition, the UL scenario will be also considered.

4.2 Spatially correlated Ricean fading scenario

Two different signal processing strategies will be assumed in this section, for both DL and UL. On the one side, a distributed processing strategy based on the previously introduced MR scheme. Recall MR precoding and combining operations are performed locally at each AP thanks to the previously computed local channel estimates. On the other side, a centralized approach, ZF, computed globally at the CPU will be addressed.

4.2.1 Distributed signal processing

The local CB precoding vector was already given in (4.7), while the local MR-based combiner vector (which will be referred to as matched filter (MF) in this section) is given by

$$\mathbf{v}_{mk} = c_{mk} \hat{\mathbf{h}}_{mk}. \quad (4.22)$$

In the following paragraph the computation of both channel hardening ratio and achievable rate are addressed.

4.2.1.1 Channel hardening ratio computation

Computing the channel hardening ratio again follows (4.2) and (4.4) for the DL and UL segments, respectively. With the correlated Ricean fading channels scenario the following propositions are given.

Proposition 3 *The DL channel hardening ratio that the k -th MS experiences in a CF massive MIMO network under an spatially correlated Ricean fading channel when employing the CB precoding scheme, for any finite M , K , and N , is given by the closed-form expression*

$$\xi_{d,k} = \frac{\sum_{m=1}^M c_{mk} \eta_{mk} \left(2 \bar{\mathbf{h}}_{mk}^H \mathbf{\Gamma}_{mk} \bar{\mathbf{h}}_{mk} + \text{tr}(\mathbf{R}_{mk} \mathbf{\Gamma}_{mk}) \right)}{\left(\sum_{m=1}^M c_{mk} \sqrt{\eta_{mk}} \left(\bar{\mathbf{h}}_{mk}^H \bar{\mathbf{h}}_{mk} + \text{tr}(\mathbf{\Gamma}_{mk}) \right) \right)^2}. \quad (4.23)$$

Proof: The computation relies on the following lemma [62].

Lemma 1 *Given $\mathbf{q} \sim \mathcal{CN}(\bar{\mathbf{q}}, \mathbf{R}_q)$ and a deterministic matrix $\mathbf{B}$, then*

$$\mathbb{E} \{ |\mathbf{q}^H \mathbf{B} \mathbf{q}| \} = \bar{\mathbf{q}}^H \mathbf{B} \bar{\mathbf{q}} + \text{tr}(\mathbf{R}_q \mathbf{B}), \quad (4.24)$$

and

$$\begin{aligned} \mathbb{E} \{ |\mathbf{q}^H \mathbf{B} \mathbf{q}|^2 \} &= |\bar{\mathbf{q}}^H \mathbf{B} \bar{\mathbf{q}}|^2 + |\text{tr}(\mathbf{R}_q \mathbf{B})|^2 + \text{tr}(\mathbf{R}_q \mathbf{B} \mathbf{R}_q \mathbf{B}^H) \\ &\quad + 2 \Re \{ \bar{\mathbf{q}}^H \mathbf{B} \bar{\mathbf{q}} \text{tr}(\mathbf{R}_q \mathbf{B}^H) \} + 2 \Re \{ \bar{\mathbf{q}}^H \mathbf{B} \mathbf{R}_q \mathbf{B}^H \bar{\mathbf{q}} \}. \end{aligned} \quad (4.25)$$

Now, we may compute $|\text{DS}_{d,k}|^2$ following a similar procedure as in (4.13) which results in

$$\begin{aligned} |\text{DS}_{d,k}|^2 &= \rho_d \left(\sum_{m=1}^M c_{mk} \sqrt{\eta_{mk}} \mathbb{E} \{ |\mathbf{h}_{mk}^H \hat{\mathbf{h}}_{mk}| \} \right)^2 = \rho_d \left(\sum_{m=1}^M c_{mk} \sqrt{\eta_{mk}} \mathbb{E} \{ |\hat{\mathbf{h}}_{mk}^H \hat{\mathbf{h}}_{mk}| \} \right)^2 \\ &= \rho_d \left(\sum_{m=1}^M c_{mk} \sqrt{\eta_{mk}} \left(\bar{\mathbf{h}}_{mk}^H \bar{\mathbf{h}}_{mk} + \text{tr}(\mathbf{\Gamma}_{mk}) \right) \right)^2, \end{aligned} \quad (4.26)$$

where in the last equality we have applied (4.24). The computation of $\mathbb{E} \{ |\text{BU}_{d,k}|^2 \}$ is per-

formed as

$$\begin{aligned} \mathbb{E} \left\{ |\text{BU}_{d,k}|^2 \right\} &= \rho_d \left(\sum_{m=1}^M \sum_{m'=1}^M c_{mk} c_{m'k} \sqrt{\eta_{mk} \eta_{m'k}} \mathbb{E} \left\{ \left| \mathbf{h}_{mk}^H \hat{\mathbf{h}}_{mk} \hat{\mathbf{h}}_{m'k}^H \mathbf{h}_{m'k} \right| \right\} \right. \\ &\quad \left. - \left(\sum_{m=1}^M c_{mk} \sqrt{\eta_{mk}} \left(\bar{\mathbf{h}}_{mk}^H \bar{\mathbf{h}}_{mk} + \text{tr}(\mathbf{\Gamma}_{mk}) \right) \right)^2 \right) \end{aligned} \quad (4.27)$$

A similar procedure to that addressed in (4.14) is followed for computing the expectation in (4.27) using Lemma 1. If $m = m'$, then

$$\begin{aligned} &\mathbb{E} \left\{ \left| \left(\hat{\mathbf{h}}_{mk}^H + \mathbf{e}_{mk}^H \right) \hat{\mathbf{h}}_{mk} \hat{\mathbf{h}}_{mk}^H \left(\hat{\mathbf{h}}_{mk} + \mathbf{e}_{mk} \right) \right| \right\} \\ &= \mathbb{E} \left\{ \left| \hat{\mathbf{h}}_{mk}^H \hat{\mathbf{h}}_{mk} \right|^2 + \mathbf{e}_{mk}^H \hat{\mathbf{h}}_{mk} \hat{\mathbf{h}}_{mk}^H \mathbf{e}_{mk} \right\} = \mathbb{E} \left\{ \left| \hat{\mathbf{h}}_{mk}^H \hat{\mathbf{h}}_{mk} \right|^2 \right\} + \mathbb{E} \left\{ \left| \mathbf{e}_{mk}^H \mathbf{\Gamma}_{mk} \mathbf{e}_{mk} \right| \right\} \\ &= \left| \bar{\mathbf{h}}_{mk}^H \bar{\mathbf{h}}_{mk} \right|^2 + |\text{tr}(\mathbf{\Gamma}_{mk})|^2 + 2\bar{\mathbf{h}}_{mk}^H \bar{\mathbf{h}}_{mk} \text{tr}(\mathbf{\Gamma}_{mk}) + 2\bar{\mathbf{h}}_{mk}^H \mathbf{\Gamma}_{mk} \bar{\mathbf{h}}_{mk} + \text{tr}(\mathbf{R}_{mk} \mathbf{\Gamma}_{mk}) \end{aligned} \quad (4.28)$$

In contrast, if $m \neq m'$, then

$$\begin{aligned} &\mathbb{E} \left\{ \left| \hat{\mathbf{h}}_{mk}^H \hat{\mathbf{h}}_{mk} \right| \right\} \mathbb{E} \left\{ \left| \hat{\mathbf{h}}_{m'k}^H \hat{\mathbf{h}}_{m'k} \right| \right\} \\ &= \bar{\mathbf{h}}_{mk}^H \bar{\mathbf{h}}_{mk} \bar{\mathbf{h}}_{m'k}^H \bar{\mathbf{h}}_{m'k} + \bar{\mathbf{h}}_{mk}^H \bar{\mathbf{h}}_{mk} \text{tr}(\mathbf{\Gamma}_{m'k}) + \text{tr}(\mathbf{\Gamma}_{mk}) \bar{\mathbf{h}}_{m'k}^H \bar{\mathbf{h}}_{m'k} + \text{tr}(\mathbf{\Gamma}_{mk}) \text{tr}(\mathbf{\Gamma}_{m'k}) \end{aligned} \quad (4.29)$$

Finally, plugging (4.28) and (4.29) into (4.27) gives

$$\mathbb{E} \left\{ |\text{BU}_{d,k}|^2 \right\} = \rho_d \sum_{m=1}^M c_{mk} \eta_{mk} \left(2\bar{\mathbf{h}}_{mk}^H \mathbf{\Gamma}_{mk} \bar{\mathbf{h}}_{mk} + \text{tr}(\mathbf{R}_{mk} \mathbf{\Gamma}_{mk}) \right) \quad (4.30)$$

■

Proposition 4 *The UL channel hardening ratio that the k -th MS experiences in a CF massive MIMO network under an spatially correlated Ricean fading channel when employing the MF combining scheme, for any finite M , K , and N , is given by the closed-form expression*

$$\xi_{u,k} = \frac{\sum_{m=1}^M c_{mk} \left(2\bar{\mathbf{h}}_{mk}^H \mathbf{\Gamma}_{mk} \bar{\mathbf{h}}_{mk} + \text{tr}(\mathbf{R}_{mk} \mathbf{\Gamma}_{mk}) \right)}{\left(\sum_{m=1}^M c_{mk} \left(\bar{\mathbf{h}}_{mk}^H \bar{\mathbf{h}}_{mk} + \text{tr}(\mathbf{\Gamma}_{mk}) \right) \right)^2}. \quad (4.31)$$

Proof: Follows same procedure as in the DL segment. ■

Confronting (4.23) and (4.31), it is important to note that whereas in the DL segment the hardening ratio depends on the power allocation strategy, the UL scheme shows no power control dependency since a MS assigns the same power coefficient to the communication with every AP.

We now introduce particular cases to compare with our baseline spatially correlated Ricean fading model. For brevity of exposition, we drop the UL segment in subsequent analysis since calculations are roughly the same.

Impact of LOS component For simplicity, let us consider an scenario with single-antenna APs and fully orthogonal noiseless pilots, i.e. $\mathbf{R}_{mk} = \mathbf{\Gamma}_{mk} = \frac{\zeta_{mk}}{1+\kappa_{mk}} \mathbf{I}_1$. Under this assumption, we may rewrite (4.23) as

$$\begin{aligned} \xi_{d,k} &= \frac{\sum_{m=1}^M c_{mk} \eta_{mk} \left(2|\bar{h}_{mk}|^2 \frac{\zeta_{mk}}{\kappa_{mk} + 1} + \left(\frac{\zeta_{mk}}{\kappa_{mk} + 1} \right)^2 \right)}{\left(\sum_{m=1}^M c_{mk} \sqrt{\eta_{mk}} \left(|\bar{h}_{mk}|^2 + \frac{\zeta_{mk}}{\kappa_{mk} + 1} \right) \right)^2} \\ &= \frac{\sum_{m=1}^M c_{mk} \eta_{mk} \left(\frac{\zeta_{mk}}{1 + \kappa_{mk}} \right)^2 (2\kappa_{mk} + 1)}{\left(\sum_{m=1}^M c_{mk} \sqrt{\eta_{mk}} \zeta_{mk} \right)^2}, \end{aligned} \quad (4.32)$$

where $\bar{h}_{mk}$ is the single-antenna case of (3.2) and $|\bar{h}_{mk}|^2 = \frac{\kappa_{mk}}{\kappa_{mk} + 1} \zeta_{mk}$. Given the Ricean factor dependency of the numerator, it is easy to check that this ratio decreases as K_{mk} increases, and it is maximum when $\kappa_{mk} = 0$, i.e. NLOS scenarios. Note that the loss of orthogonality between noisy pilots and the fact of having multiple antennas at the APs do not alter the Ricean factor. These simplifications are assumed for the sake of exposition, but the insights given by (4.32) will hold regardless the initial considerations. This way, the following key observation is given.

Remark 1 *The presence of a dominant LOS component in the wireless link between a given MS and the corresponding APs will always help the channel to harden, regardless the power coefficients and/or the particular clustering strategy.*

From an engineering perspective, one can guarantee a high probability of having a dominant LOS component by deploying APs at locations with good visibility. Moreover, deploying dense networks, such as CF massive MIMO and its derived approaches, would also assure LOS since the probability for a MS to be close to an AP is considerably high.

Impact of spatial correlation Without loss of generality, let us evaluate a multi-antenna APs scenario in which there is no dominant LOS component, i.e. $\kappa_{mk} = 0 \forall mk$, leading to a spatially correlated Rayleigh fading channel situation. Consequently, the terms $2\bar{\mathbf{h}}_{mk}^H \mathbf{\Gamma}_{mk} \bar{\mathbf{h}}_{mk}$ and $\bar{\mathbf{h}}_{mk}^H \bar{\mathbf{h}}_{mk}$ in the numerator and denominator of (4.23), respectively, vanish resulting in

$$\xi_{d,k} = \frac{\sum_{m=1}^M c_{mk} \eta_{mk} \text{tr}(\mathbf{R}_{mk} \mathbf{\Gamma}_{mk})}{\left(\sum_{m=1}^M c_{mk} \sqrt{\eta_{mk}} \text{tr}(\mathbf{\Gamma}_{mk}) \right)^2}. \quad (4.33)$$

In addition, consider again perfect CSI and no pilot contamination ($\mathbf{R}_{mk} = \mathbf{\Gamma}_{mk}$) giving

$$\xi_{d,k} = \frac{\sum_{m=1}^M c_{mk} \eta_{mk} \text{tr}(\mathbf{R}_{mk}^2)}{\left(\sum_{m=1}^M c_{mk} \sqrt{\eta_{mk}} \text{tr}(\mathbf{R}_{mk}) \right)^2} = \frac{\sum_{m=1}^M c_{mk} \eta_{mk} \zeta_{mk}^2 \text{tr}(\bar{\mathbf{R}}_{mk}^2)}{N^2 \left(\sum_{m=1}^M c_{mk} \sqrt{\eta_{mk}} \zeta_{mk} \right)^2}, \quad (4.34)$$

with $\bar{\mathbf{R}}_{mk} = \mathbf{R}_{mk} / \zeta_{mk}$. In order to have some insights on the impact that spatial correlation may have on the hardening ratio, let us particularize (4.34) to an uncorrelated arrays situation ($\bar{\mathbf{R}}_{mk} = \mathbf{I}_N$), giving a particularization of (4.11) expressed as

$$\xi_{d,k} = \frac{\sum_{m=1}^M c_{mk} \eta_{mk} \zeta_{mk}^2 \text{tr}(\mathbf{I}_N)}{N^2 \left(\sum_{m=1}^M c_{mk} \sqrt{\eta_{mk}} \zeta_{mk} \right)^2} = \frac{\sum_{m=1}^M c_{mk} \eta_{mk} \zeta_{mk}^2}{N \left(\sum_{m=1}^M c_{mk} \sqrt{\eta_{mk}} \zeta_{mk} \right)^2}. \quad (4.35)$$

Confronting (4.34) and (4.35), and owing to the fact that $\text{tr}(\bar{\mathbf{R}}_{mk}^2) \geq \text{tr}(\mathbf{I}_N) = N$ (with equality only for the uncorrelated case) since $\bar{\mathbf{R}}_{mk}$ is a positive semi-definite matrix (i.e. $\mathbf{r}^H \bar{\mathbf{R}}_{mk} \mathbf{r} \geq 0 \forall \mathbf{r} \in \mathbb{C}^{N \times 1}$), the hardening ratio in a spatially correlated scenario will always be higher than in the uncorrelated counterpart. The following remark is given.

Remark 2 *Under a multi-antenna APs scenario, the spatial correlation between the antennas conforming the array of a particular AP will imply a channel hardening degradation in comparison to the scenario with uncorrelated antennas.*

It is worth mentioning that, in particular scenarios with large K , spatial correlation can indeed have a positive effect in reducing multi-user interference [63]. Chapter 5 will deepen into this matter. On the other hand, and though the massive MIMO channel has been shown to be spatially correlated in most situations, a specific scenario in which this may not happen if APs are equipped with ULAs is a rich scattering location. In this latter situation, it would be reasonable to assume that all multi-path components arrive at a given AP with nearly same probability.

Impact of clustering approaches Regardless of any of the previous assumed scenarios, the hardening ratio will experience dependency on the connectivity coefficients. For simplicity, we take (4.35) as reference. The denominator grows faster than the numerator, so hardening will be enhanced when $c_{mk} = 1 \forall mk$, i.e. original CF scheme, while it will be degraded when employing any clustering approach, as stated as follows.

Remark 3 *Given the same network deployment with different clustering configurations, the pure CF scheme in which all APs provide service to all MSs will be the best case scenario in terms of channel hardening when compared to any other clustering strategy.*

The impact of the clustering strategies will be most dependent on the APs density. As it will be shown later, for the traditional $D = 1000$ m deployment area, only a small fraction of the total of APs in the CF scheme will effectively contribute in terms of power to the channel hardening.

To sum up, it has been shown that the total number of antennas in the network is far from being the only factor affecting the channel hardening ratio. There are many other factors that needed to be taken into account when designing a CF massive MIMO network if one is to assure a certain hardening degree.

4.2.1.2 Achievable rate computation

As in the uncorrelated Rayleigh fading channels scenario, the DL and UL achievable rates can be computed using (3.25) and (3.36), respectively. Closed-form expressions can be obtained by

computing the SINR term for the DL and UL segments, respectively, as

$$\text{SINR}_{d,k} = \frac{\rho_d \left(\sum_{m=1}^M c_{mk} \sqrt{\eta_{mk}} \mathcal{T}_{1,mk} \right)^2}{\rho_d \sum_{m=1}^M c_{mk} \eta_{mk} \mathcal{T}_{2,mk} + \rho_d \sum_{k' \neq k}^K \left(\sum_{m=1}^M c_{mk'} \eta_{mk'} \mathcal{T}_{3,mkk'} + \left| \sum_{m=1}^M c_{mk'} \sqrt{\eta_{mk'}} \mathcal{T}_{4,mkk'} \right|^2 \right) + 1}, \quad (4.36)$$

and

$$\text{SINR}_{u,k} = \frac{\rho_u \eta_k \left(\sum_{m=1}^M c_{mk} \mathcal{T}_{1,mk} \right)^2}{\rho_u \eta_k \sum_{m=1}^M c_{mk} \mathcal{T}_{2,mk} + \rho_u \sum_{k' \neq k}^K \eta_{k'} \left(\sum_{m=1}^M c_{mk'} \mathcal{T}_{3,mkk'} + \left| \sum_{m=1}^M c_{mk'} \mathcal{T}_{4,mkk'} \right|^2 \right) + \sum_{m=1}^M c_{mk} \mathcal{T}_{1,mk}}. \quad (4.37)$$

In these previous expressions the terms $\mathcal{T}_{1,mk}$, $\mathcal{T}_{2,mk}$, $\mathcal{T}_{3,mkk'}$ and $\mathcal{T}_{4,mkk'}$ are given by

$$\mathcal{T}_{1,mk} = \bar{\mathbf{h}}_{mk}^H \bar{\mathbf{h}}_{mk} + \text{tr}(\mathbf{\Gamma}_{mk}), \quad (4.38)$$

$$\mathcal{T}_{2,mk} = 2\bar{\mathbf{h}}_{mk}^H \mathbf{\Gamma}_{mk} \bar{\mathbf{h}}_{mk} + \text{tr}(\mathbf{R}_{mk} \mathbf{\Gamma}_{mk}), \quad (4.39)$$

$$\mathcal{T}_{3,mkk'} = \bar{\mathbf{h}}_{mk}^H \mathbf{\Gamma}_{mk'} \bar{\mathbf{h}}_{mk} + \left| \bar{\mathbf{h}}_{mk}^H \mathbf{R}_{mk}^{1/2} \bar{\mathbf{h}}_{mk'} \right|^2 + \bar{\mathbf{h}}_{mk'}^H \mathbf{R}_{mk} \bar{\mathbf{h}}_{mk'} + \text{tr}(\mathbf{R}_{mk} \mathbf{\Gamma}_{mk'}), \quad (4.40)$$

and

$$\mathcal{T}_{4,mkk'} = \bar{\mathbf{h}}_{mk}^H \bar{\mathbf{h}}_{mk'} + |\varphi_{k'}^H \varphi_k|^2 \text{tr}(\mathbf{R}_{mk} \mathbf{\Gamma}_{mk'} \mathbf{R}_{mk'}^{-1}). \quad (4.41)$$

Proof: The computation of $\mathcal{T}_{1,mk}$ and $\mathcal{T}_{2,mk}$ is performed as denominator and numerator of (4.2), respectively, while the computation of $\mathcal{T}_{3,mk}$ and $\mathcal{T}_{4,mk}$ follows similar computation as in (4.18) applying Lemma 1. Note that this proof is valid for either the DL or UL segments.

Assuming, for instance, the DL segment, we have

$$\begin{aligned}
 & \mathbb{E} \left\{ |\text{MUI}_{d,kk'}|^2 \right\} \\
 &= \rho_d \sum_{m=1}^M \sum_{m'=1}^M c_{mk'} c_{m'k'} \sqrt{\eta_{mk'} \eta_{m'k'}} \mathbb{E} \left\{ \left| \mathbf{h}_{mk}^H \mathbf{h}_{mk'} \hat{\mathbf{h}}_{m'k'}^H \mathbf{h}_{m'k} \right| \right\} \\
 &= \rho_d \sum_{m=1}^M c_{mk'} \eta_{mk'} \mathbb{E} \left\{ \left| \mathbf{h}_{mk}^H \mathbf{h}_{mk'} \hat{\mathbf{h}}_{mk'}^H \mathbf{h}_{mk} \right| \right\} \\
 &\quad + \rho_d \sum_{m=1}^M \sum_{m' \neq m}^M c_{mk'} c_{m'k'} \sqrt{\eta_{mk'} \eta_{m'k'}} \mathbb{E} \left\{ \left| \mathbf{h}_{mk}^H \hat{\mathbf{h}}_{mk'} \right| \right\} \mathbb{E} \left\{ \left| \hat{\mathbf{h}}_{m'k'}^H \mathbf{h}_{m'k} \right| \right\}.
 \end{aligned} \tag{4.42}$$

On the one side, following similar computation as in (4.28), we have that

$$\begin{aligned}
 & \mathbb{E} \left\{ \left| \mathbf{h}_{mk}^H \mathbf{h}_{mk'} \hat{\mathbf{h}}_{mk'}^H \mathbf{h}_{mk} \right| \right\} \\
 &= \left| \bar{\mathbf{h}}_{mk}^H \bar{\mathbf{h}}_{mk'} \right|^2 + \bar{\mathbf{h}}_{mk}^H \mathbf{\Gamma}_{mk'} \bar{\mathbf{h}}_{mk} + \left| \bar{\mathbf{h}}_{mk'}^H \mathbf{R}_{mk}^{1/2} \bar{\mathbf{h}}_{mk} \right|^2 + \bar{\mathbf{h}}_{mk'}^H \mathbf{R}_{mk} \bar{\mathbf{h}}_{mk'} + \text{tr}(\mathbf{R}_{mk} \mathbf{\Gamma}_{mk'}) \\
 &\quad + \left| \text{tr}(\mathbf{R}_{mk} \mathbf{\Gamma}_{mk'} \mathbf{R}_{mk'}^{-1}) \right|^2 \left| \boldsymbol{\varphi}_k^H \boldsymbol{\varphi}_{k'} \right|^2 + 2 \bar{\mathbf{h}}_{mk'}^H \bar{\mathbf{h}}_{mk} \text{tr}(\mathbf{R}_{mk} \mathbf{\Gamma}_{mk'} \mathbf{R}_{mk'}^{-1}) \boldsymbol{\varphi}_k^H \boldsymbol{\varphi}_{k'}.
 \end{aligned} \tag{4.43}$$

On the other side, following similar computation as in (4.29), we have that

$$\mathbb{E} \left\{ \left| \mathbf{h}_{mk}^H \hat{\mathbf{h}}_{mk'} \right| \right\} = \bar{\mathbf{h}}_{mk}^H \bar{\mathbf{h}}_{mk'} + \text{tr}(\mathbf{R}_{mk} \mathbf{\Gamma}_{mk'} \mathbf{R}_{mk'}^{-1}) \boldsymbol{\varphi}_k^H \boldsymbol{\varphi}_{k'}, \tag{4.44}$$

and that

$$\mathbb{E} \left\{ \left| \hat{\mathbf{h}}_{m'k'}^H \mathbf{h}_{m'k} \right| \right\} = \bar{\mathbf{h}}_{m'k'}^H \bar{\mathbf{h}}_{m'k} + \text{tr}(\mathbf{R}_{m'k'} \mathbf{\Gamma}_{m'k'} \mathbf{R}_{m'k'}^{-1}) \boldsymbol{\varphi}_{k'}^H \boldsymbol{\varphi}_k. \tag{4.45}$$

Plugging (4.43), (4.44) and (4.45) into (4.42) gives

$$\begin{aligned}
 & \mathbb{E} \left\{ |\text{MUI}_{d,kk'}|^2 \right\} \\
 &= \rho_d \left(\sum_{m=1}^M c_{mk'} \eta_{mk'} \underbrace{\left(\bar{\mathbf{h}}_{mk}^H \mathbf{\Gamma}_{mk'} \bar{\mathbf{h}}_{mk} + \left| \bar{\mathbf{h}}_{mk}^H \mathbf{R}_{mk}^{1/2} \bar{\mathbf{h}}_{mk'} \right|^2 + \bar{\mathbf{h}}_{mk'}^H \mathbf{R}_{mk} \bar{\mathbf{h}}_{mk'} + \text{tr}(\mathbf{R}_{mk} \mathbf{\Gamma}_{mk'}) \right)}_{\mathcal{T}_{3,mkk'}} \right) \\
 &\quad + \left| \sum_{m=1}^M c_{mk'} \eta_{mk'} \underbrace{\left(\bar{\mathbf{h}}_{mk}^H \bar{\mathbf{h}}_{mk'} + \left| \boldsymbol{\varphi}_{k'}^H \boldsymbol{\varphi}_k \right|^2 \text{tr}(\mathbf{R}_{mk} \mathbf{\Gamma}_{mk'} \mathbf{R}_{mk'}^{-1}) \right)}_{\mathcal{T}_{4,mkk'}} \right|^2.
 \end{aligned} \tag{4.46}$$

■

As in the uncorrelated scenario, (4.21) can be used for a benchmark comparison between the estimated user data rate and the maximum channel capacity. However, if one is to compare the lower bound (3.25) with the true network capacity (in which there is only statistical CSI), (4.21) cannot be used. Instead, let us now introduce another achievable rate bound, denoted as the true rate. This is given by

$$\mathcal{R}_{d,k}^{\text{true}} = B \frac{1 - \tau_p/\tau_c}{2} \mathbb{E} \left\{ \log_2 \left(1 + \frac{|\text{DS}_{d,k}|^2}{|\text{BU}_{d,k}|^2 + \sum_{k' \neq k}^K |\text{MUI}_{d,kk'}|^2 + 1} \right) \right\}. \quad (4.47)$$

This bound computes the actual rate that the k -th MS experiences. The gap between the lower bound and the true rate reduces as the hardening ratio becomes smaller, and it tends to match (4.21) as $|\text{BU}_{d,k}|^2 \rightarrow 0$. Though both (4.21) and (4.47) are specifically intended for the DL segment, conclusions are also valid for the UL segment MF combining.

4.2.1.3 Power control strategies

Depending on whether the DL or the UL segment are addressed and, particularly, on the processing scheme, power control coefficients may have certain impact on the channel hardening. As it will be shown later on this chapter, channel hardening in centralized processing schemes is agnostic of the power control strategy since power coefficients are only MS-dependent. Same situation occurs on the UL segment with the MF combining, as shown in (4.31). Nevertheless, the hardening ratio is sensitive to power control under CB precoding. Regardless the employed power allocation strategy, the power constraint (3.19) must be fulfilled. Power coefficients are allocated on a large-scale fading basis, i.e. these are computed once every several coherence intervals.

DL uniform power allocation Under the uniform power allocation scheme an AP assigns the same power coefficient to every user. This power control scheme has negligible complexity, while it does not lose too much performance in terms of average user data rate if compared with the max-min SINR optimal solution [12, 13]. Power coefficients for the DL segment are

computed as follows. From (3.19) we have

$$\begin{aligned}\mathbb{E}\left\{\|\mathbf{x}_m\|^2\right\} &= \rho_d \mathbb{E}\left\{|\boldsymbol{\varpi}_k^H \mathbf{h}_k|^2\right\} = \rho_d \mathbb{E}\left\{\left|\sum_{k=1}^K c_{mk} \eta_{mk} \hat{\mathbf{h}}_{mk}^H \mathbf{h}_{mk}\right|^2\right\} \\ &= \rho_d \sum_{k=1}^K c_{mk} \eta_{mk} \mathbb{E}\left\{\|\hat{\mathbf{h}}_{mk}\|^2\right\} = \rho_d \sum_{k=1}^K c_{mk} \eta_{mk} \left(\frac{\kappa_{mk}}{\kappa_{mk} + 1} N \zeta_{mk} + \text{tr}(\mathbf{\Gamma}_{mk})\right) \leq \rho_d,\end{aligned}\tag{4.48}$$

which leads to

$$\eta_{mk} = \eta_m = \frac{1}{\sum_{k=1}^K c_{mk} \left(\frac{\kappa_{mk}}{\kappa_{mk} + 1} N \zeta_{mk} + \text{tr}(\mathbf{\Gamma}_{mk})\right)}.\tag{4.49}$$

Note that whereas the UL power control coefficients are restricted to $0 \leq \eta_k \leq 1$, in the DL segment there is no restriction for η_{mk} , except for being positive. This computation can be performed locally at each AP, so there is no need to use the fronthaul link connection to the CPU for this particular purpose. This is a key aspect if fronthaul links are not supposed to have infinite capacity, which will be the case in practical scenarios.

DL max-min power allocation The max-min power allocation schemes developed in [12, 13] ensure that the minimum SINR is maximized for every user. In particular scenarios where a large number of users is present, this strategy will lead to a performance reduction of the overall network due to particular MSs with bad channel conditions, however, it provides a more uniform and fair service. The main drawback in this case is the considerable high computational overhead, which may result on making this power control scheme unfeasible in practical scenarios, while being mandatory to be performed at the CPU. Defining $\boldsymbol{\eta} = [\boldsymbol{\eta}_1^T, \dots, \boldsymbol{\eta}_M^T]^T$ and $\boldsymbol{\eta}_m = [\eta_{m1}, \dots, \eta_{mK}]^T$, the optimization problem is stated as

$$\begin{aligned}\max_{\boldsymbol{\eta}} \quad & \min_k \text{ SINR}_{d,k}, \\ \text{s.t.} \quad & \|\mathbf{x}_m\|^2 \leq \rho_d \quad \forall m.\end{aligned}\tag{4.50}$$

Using (4.36), (4.50) can be reformulated as

$$\begin{aligned}
 & \max_{\varsigma, \lambda} \min_k \frac{\left(\sum_{m=1}^M \varsigma_{mk} \mathcal{T}_{1,mk} \right)^2}{\sum_{m=1}^M \sum_{k'=1}^K \varsigma_{mk'}^2 \mathcal{Z}_{mkk'} + \sum_{k' \neq k}^K |\lambda_{kk'}|^2 + \frac{1}{\rho_d}}, \\
 & \text{s.t.} \quad \sum_{m=1}^M \varsigma_{mk'} \mathcal{T}_{4,mkk'} \leq \lambda_{kk'} \quad \forall k' \neq k, \\
 & \quad \sum_{k'=1}^K \varsigma_{mk'}^2 \mathbb{E} \left\{ \|\varpi_{mk'}\|^2 \right\} \leq 1 \quad \forall m, \\
 & \quad \varsigma_{mk} \geq 0 \quad \forall mk,
 \end{aligned} \tag{4.51}$$

where we have introduced the slack variables $\lambda_{kk'}$ and have used the definitions $\varsigma_{mk} = \sqrt{\eta_{mk}}$ and

$$\mathcal{Z}_{mkk'} = \begin{cases} \mathcal{T}_{2,mk}, & k' = k \\ \mathcal{T}_{3,mkk'}, & k' \neq k. \end{cases} \tag{4.52}$$

Problem (4.51) is a quasi-concave optimization problem that can be expressed in an equivalent form as

$$\begin{aligned}
 & \max_{\varsigma, \lambda, x} \quad x \\
 & \text{s.t.} \quad \sqrt{x} \left\| \boldsymbol{\mu}_{1k}, \dots, \boldsymbol{\mu}_{Mk} \bar{\boldsymbol{\lambda}}_k \frac{1}{\sqrt{\rho_d}} \right\|_F \leq \sum_{m=1}^M \varsigma_{mk} \mathcal{T}_{1,mk} \quad \forall k, \\
 & \quad \sum_{m=1}^M \varsigma_{mk'} \mathcal{T}_{4,mkk'} \leq \lambda_{kk'} \quad \forall k' \neq k, \\
 & \quad \sum_{k'=1}^K \varsigma_{mk'}^2 \mathbb{E} \left\{ \|\varpi_{mk'}\|^2 \right\} \leq 1 \quad \forall m, \\
 & \quad \varsigma_{mk} \geq 0 \quad \forall mk,
 \end{aligned} \tag{4.53}$$

where $\boldsymbol{\mu}_{mk} = [\varsigma_{m1} \sqrt{\mathcal{Z}_{mk1}}, \dots, \varsigma_{mK} \sqrt{\mathcal{Z}_{mkK}}]$ and $\bar{\boldsymbol{\lambda}}_k = [\lambda_{k1}, \dots, \lambda_{k(k-1)}, \lambda_{k(k+1)}, \dots, \lambda_{kK}]$. Problem (4.53) is a second-order cone (SOC) program that can be efficiently solved by using a conventional iterative bisection search algorithm. Specific details on the optimality, complexity and feasibility of these algorithms were fully commented by Ngo *et al.* in the seminal paper [12].

UL full power allocation Though there are several proposals for UL power control in the literature, we are assuming full power transmission, i.e. $\eta_k = 1$, since it has been shown to work rather well, even outmatching the max-min SINR optimal approach in terms of average sum-rate, as shown in [38, 64].

4.2.2 Centralized signal processing

If short-term cooperation between APs and the CPU is allowed, the classical MIMO ZF processing scheme, first introduced in the context of the CF architecture in [10, 13], is considered. Taking into account the connectivity matrix $\mathbf{C}$, the ZF precoding matrix is

$$\mathbf{W}_d = \hat{\mathbf{H}}_c^* \left(\hat{\mathbf{H}}_c^T \hat{\mathbf{H}}_c^* \right)^{-1}, \quad (4.54)$$

where $\hat{\mathbf{H}}_c = \hat{\mathbf{H}} \odot \mathbf{C}$ with $\hat{\mathbf{H}} = [\hat{\mathbf{h}}_1, \dots, \hat{\mathbf{h}}_K]$ being the $MN \times K$ matrix accounting for all channel estimates. Note that $\hat{\mathbf{H}}_c$ accounts for the specific AP-MS connectivity of the whole network¹. Let us reformulate (3.18) for having the overall $MN \times 1$ precoding vector as

$$\mathbf{x} = [\mathbf{x}_1^T, \dots, \mathbf{x}_m^T]^T = \mathbf{W}_d \mathbf{\Omega} \mathbf{s}_d, \quad (4.55)$$

where $\mathbf{s}_d$ is the $K \times 1$ vector of DL MSs information symbols and $\mathbf{\Omega} = \text{diag}([\sqrt{\omega_1}, \dots, \sqrt{\omega_K}])$ is a $K \times K$ diagonal power allocation matrix with each element ω_k representing the power coefficient assigned to MS k . As in the distributed CB/MF processing case, power allocation coefficients should be computed to satisfy the prescribed maximum power constraint shown in (3.19). The received symbol at the k -th MS can be rewritten as

$$\begin{aligned} \hat{s}_{d,k} &= \sqrt{\rho_d} \mathbf{h}_k^T \mathbf{x} + n_k \\ &= \sqrt{\rho_d} \left(\hat{\mathbf{h}}_k + \mathbf{e}_k \right)^T \hat{\mathbf{H}}_c^* \left(\hat{\mathbf{H}}_c^T \hat{\mathbf{H}}_c^* \right)^{-1} \mathbf{\Omega} \mathbf{s}_d + n_k \\ &= \sqrt{\rho_d \omega_k} s_{d,k} + \underbrace{\sqrt{\rho_d} \mathbf{e}_k^T \hat{\mathbf{H}}_c^* \left(\hat{\mathbf{H}}_c^T \hat{\mathbf{H}}_c^* \right)^{-1} \mathbf{\Omega} \mathbf{s}_d}_{\text{EN}_{d,k}} + n_k, \end{aligned} \quad (4.56)$$

where the first term is the desired signal, and the $\text{EN}_{d,k}$ term accounts for the imperfect CSI and errors derived from the fact of not inverting the whole channel matrix (as in clustering

¹Note that when using clustering approaches, matrix $\hat{\mathbf{H}}_c$ will typically be sparse, in which case efficient algorithms can be applied to compute the pseudo-inverse in (4.54) [65].

scenarios) treated as effective noise. Note that whereas CB precoding can be implemented both in a centralized (at the CPU) or a distributed manner (at the APs)², the ZF technique can only be applied at the CPU since the precoder calculation requires knowledge of the CSI between all of the MSs and their serving APs (in accordance to $\mathbf{C}$).

Turning the attention now to the UL segment, the collective signal received at the APs sent from all MSs can be expressed as a reformulation of (3.27) as

$$\mathbf{y} = \sqrt{\rho_u} \sum_{k=1}^K \sqrt{\theta_k} \mathbf{h}_k s_{u,k} + \mathbf{n} = \sqrt{\rho_u} \mathbf{H} \mathbf{\Theta} \mathbf{s}_u + \mathbf{n}, \quad (4.57)$$

where $\mathbf{\Theta} = \text{diag}([\sqrt{\theta_1}, \dots, \sqrt{\theta_K}])$ is a $K \times K$ diagonal power allocation matrix with each element θ_k representing the power coefficient assigned to MS k and $\mathbf{n} \in \mathbb{C}^{MN \times 1}$ is the vector accounting for thermal noise at all APs. The entries of $\mathbf{y}$ are forwarded from each AP to the CPU for detecting the corresponding set of data symbols thanks to the combining matrix $\mathbf{W}_u = \mathbf{W}_d^T$ as

$$\hat{\mathbf{s}}_u = \mathbf{W}_u \mathbf{y} = \sqrt{\rho_u} \mathbf{\Theta} \mathbf{W}_u \mathbf{H} \mathbf{s}_u + \mathbf{W}_u \mathbf{n} = \sqrt{\rho_u} \mathbf{\Theta} \mathbf{s}_u + \sqrt{\rho_u} \mathbf{\Theta} \mathbf{W}_u \mathbf{E} \mathbf{s}_u + \mathbf{W}_u \mathbf{n}, \quad (4.58)$$

If we define $\mathbf{e}_k = [\mathbf{e}_{1k}^T, \dots, \mathbf{e}_{mK}^T]^T$, we can express the overall channel estimation error matrix as $\mathbf{E} = [\mathbf{e}_1, \dots, \mathbf{e}_K]$. Then, each component of $\hat{\mathbf{s}}_u$ is combined as

$$\hat{s}_{u,k} = \sqrt{\rho_u \theta_k} s_{u,k} + \underbrace{\sqrt{\rho_u \theta_k} \mathbf{w}_{u,k} \mathbf{e}_k}_{\text{EN}_{u,k}} s_{u,k} + r_k, \quad (4.59)$$

where $\mathbf{w}_{u,k}$ the k -th row vector of matrix $\mathbf{W}_u$ and r_k is the k -th entry of the vector $\mathbf{W}_u \mathbf{n}$. In this expression the first term represents the useful signal and the second term represents the combined effects of imperfect CSI and the matrix inversion errors.

For the centralized ZF processing note the different notation for the power control coefficients (ω_k and θ_k) since they are computed differently from those for the CB/MF processing as it will be shown later on.

²When infinite capacity fronthaul links between the APs and MSs are considered, centralized and distributed implementation of the CB are mathematically equivalent. In contrast, when limited-capacity fronthauls are considered, the centralized and distributed implementations of these two algorithms differ dramatically.

4.2.2.1 Channel hardening ratio computation

An estimation for the hardening ratio under the case of ZF processing is shown up next. Recalling (4.56) we may define $\text{DS}_{d,k} \triangleq \sqrt{\rho_d \omega_k}$ and

$$\begin{aligned} \mathbb{E} \left\{ |\text{EN}_{d,k}|^2 \right\} &= \mathbb{E} \left\{ \left| \sqrt{\rho_d} \mathbf{e}_k^T \hat{\mathbf{H}}_c^* \left(\hat{\mathbf{H}}_c^T \hat{\mathbf{H}}_c^* \right)^{-1} \boldsymbol{\Omega} \right|^2 \right\} \\ &= \rho_d \text{tr} \left(\boldsymbol{\Omega}^2 \mathbb{E} \left\{ \left(\hat{\mathbf{H}}_c^T \hat{\mathbf{H}}_c^* \right)^{-1} \hat{\mathbf{H}}_c^T \mathbf{E}_{c,k}^* \hat{\mathbf{H}}_c^* \left(\hat{\mathbf{H}}_c^T \hat{\mathbf{H}}_c^* \right)^{-1} \right\} \right) \\ &= \rho_d \text{tr} \left(\boldsymbol{\Omega}^2 \mathbb{E} \left\{ \mathbf{W}^H \mathbf{E}_{c,k}^* \mathbf{W} \right\} \right) = \rho_d \sum_{k'=1}^K \omega_{k'} \varrho_{kk'}, \end{aligned} \quad (4.60)$$

with $\varrho_{kk'}$ being the k' -th entry in vector

$$\boldsymbol{\varrho}_k = \text{diag} \left(\mathbb{E} \left\{ \mathbf{W}^H \mathbf{E}_{c,k}^* \mathbf{W} \right\} \right), \quad (4.61)$$

and $\mathbf{E}_{c,k} = \text{diag} (\mathbf{E}_{c,1k}, \dots, \mathbf{E}_{c,Mk})$ with $\mathbf{E}_{c,mk} = c_{mk} \mathbf{E}_{mk}$. Note that $\boldsymbol{\varrho}_k$ represents the vector of interfering terms caused by the use of the channel estimates, rather than the true values, when designing the ZF matrix $\mathbf{W}$. As in [13], it is worth mentioning that the term $\mathbb{E} \left\{ |\text{EN}_{d,k}|^2 \right\}$ cannot be computed in closed-form and must be estimated via Monte-Carlo simulation. At this point we may define $\text{BU}_{d,k} \triangleq \sqrt{\rho_d \omega_k \varrho_{kk}}$ leading to the DL channel hardening ratio with ZF precoding expressed as

$$\xi_{d,k} = \frac{\mathbb{E} \left\{ |\text{BU}_{d,k}|^2 \right\}}{|\text{DS}_{d,k}|^2} = \frac{|\sqrt{\rho_d \omega_k \varrho_{kk}}|^2}{\rho_d \omega_k} = \varrho_{kk}. \quad (4.62)$$

Similar analysis follows for the UL segment. From (4.59) we define $\text{DS}_{u,k} \triangleq \sqrt{\rho_u \theta_k}$ and

$$\begin{aligned} \mathbb{E} \left\{ |\text{EN}_{u,k}|^2 \right\} &= \mathbb{E} \left\{ \left| \sqrt{\rho_u \theta_k} \mathbf{w}_{u,k} \mathbf{e}_k \right|^2 \right\} = \rho_u \text{tr} \left(\theta_k \mathbb{E} \left\{ \mathbf{E}^H \mathbf{w}_{u,k}^H \mathbf{w}_{u,k} \mathbf{E} \right\} \right) \\ &= \rho_u \text{tr} \left(\theta_k \mathbb{E} \left\{ \mathbf{W}_u \mathbb{E} \left\{ \mathbf{e}_k \mathbf{e}_k^H \right\} \mathbf{W}_u^H \right\} \right) = \rho_u \text{tr} \left(\theta_k \mathbb{E} \left\{ \mathbf{W}_u \mathbf{E}_{c,k} \mathbf{W}_u^H \right\} \right) \\ &= \rho_u \sum_{k'=1}^K \theta_{k'} v_{kk'}, \end{aligned} \quad (4.63)$$

with $v_{kk'}$ being the k' -th entry in vector

$$\mathbf{v}_k = \text{diag} \left(\mathbb{E} \left\{ \mathbf{W}_u \mathbf{E}_{c,k'} \mathbf{W}_u^H \right\} \right). \quad (4.64)$$

If we define $\text{BU}_{u,k} \triangleq \sqrt{\rho_u \theta_k v_{kk}}$, the UL channel hardening ratio with ZF combining is expressed as

$$\xi_{u,k} = \frac{\mathbb{E} \left\{ |\text{BU}_{u,k}|^2 \right\}}{|\text{DS}_{u,k}|^2} = \frac{|\sqrt{\rho_u \theta_k v_{kk}}|^2}{\rho_u \theta_k} = v_{kk}. \quad (4.65)$$

Similar to (4.62), there is no closed-form expression for (4.65), so the possible impact that spatial correlation or the LOS component may have on the hardening under ZF can only be deduced from simulation results. Nevertheless, we can point out that power control coefficients have no influence whatsoever on either DL or UL hardening with ZF processing.

4.2.2.2 Achievable rate computation

For comparison purposes, we will assume that the decoding entity has only statistical CSI knowledge. Then, the channel hardening lower bound achievable rate can be computed as in the distributed processing scenario through (3.25). The diagonalizing structure of the ZF precoder allows to compute the DL SINR term in (3.25) as [13]

$$\text{SINR}_{d,k} = \frac{|\text{DS}_{d,k}|^2}{\mathbb{E} \left\{ |\text{EN}_{d,k}|^2 \right\} + 1} = \frac{\rho_d \omega_k}{\rho_d \sum_{k'=1}^K \omega_{k'} \varrho_{kk'} + 1}, \quad (4.66)$$

where we have defined $\text{MUI}_{d,kk'} \triangleq \sqrt{\rho_d \omega_{k'} \varrho_{kk'}}$ for $k' \neq k$. As in the computation of the hardening ratio, the denominator of (4.66) can only be estimated by Monte-Carlo simulation.

For the UL segment, following the UatF bound in (3.36), we may express the SINR term as

$$\text{SINR}_{u,k} = \frac{|\text{DS}_{u,k}|^2}{\mathbb{E} \left\{ |\text{EN}_{u,k}|^2 \right\} + \text{NT}_k} = \frac{\rho_u \theta_k}{\rho_u \sum_{k'=1}^K \theta_{k'} v_{kk'} + \epsilon_k}, \quad (4.67)$$

where we have defined $\text{MUI}_{u,kk'} \triangleq \sqrt{\rho_u \theta_{k'} v_{kk'}}$ for $k' \neq k$ and $\text{NT}_k \triangleq \epsilon_k$ as the k -th entry in vector $\text{diag}(\mathbb{E} \{ \mathbf{W}_u \mathbf{W}_u^H \})$. Again, there is no closed-form expression for the UL SINR.

4.2.2.3 Power control strategies

Though power control coefficients have no impact on the channel hardening ratio, they are key for achieving good performance in terms of user data rate. We address two different schemes

intended for DL and UL segments separately.

DL low-complexity max-min power allocation As in the distributed precoding power allocation scenario, finding the optimal DL power coefficients that satisfy the max-min SINR criteria for cooperative precoding involves solving a similar optimization problem (though with less complexity since only K coefficients need to be computed), stated as

$$\begin{aligned} & \max_{\omega_1, \dots, \omega_K} \min \text{ SINR}_{d,k} \\ & \text{s.t.} \quad \|\mathbf{x}_m\|^2 \leq \rho_d \quad \forall m, \end{aligned} \quad (4.68)$$

where the transmitted power from the m -th AP is

$$\begin{aligned} \|\mathbf{x}_m\|^2 &= \rho_d \mathbb{E} \{ \mathbf{w}_{d,m}^T \mathbf{w}_{d,m}^* \} \\ &= \rho_d \mathbb{E} \left\{ \left(\hat{\mathbf{h}}_{c,m}^H \left(\hat{\mathbf{H}}_c^T \hat{\mathbf{H}}_c^* \right) \boldsymbol{\Omega} \right) \left(\hat{\mathbf{h}}_{c,m}^H \left(\hat{\mathbf{H}}_c^T \hat{\mathbf{H}}_c^* \right) \boldsymbol{\Omega} \right)^H \right\} \\ &= \rho_d \text{tr} \left(\boldsymbol{\Omega}^2 \mathbb{E} \left\{ \left(\hat{\mathbf{H}}_c^T \hat{\mathbf{H}}_c^* \right)^{-1} \hat{\mathbf{h}}_{c,m}^T \hat{\mathbf{h}}_{c,m}^* \left(\hat{\mathbf{H}}_c^T \hat{\mathbf{H}}_c^* \right)^{-1} \right\} \right) \\ &= \rho_d \sum_{k'=1}^K \omega_{k'} \delta_{kk'}, \end{aligned} \quad (4.69)$$

being $\hat{\mathbf{h}}_{c,m}$ the m -th row vector of matrix $\hat{\mathbf{H}}_c$, $\mathbf{w}_{d,m}$ the m -th row vector of matrix $\mathbf{W}_d$ and $\delta_{kk'}$ the k' -th entry of

$$\boldsymbol{\delta}_m = \text{diag} \left(\mathbb{E} \left\{ \left(\hat{\mathbf{H}}_c^T \hat{\mathbf{H}}_c^* \right)^{-1} \hat{\mathbf{h}}_{c,m}^T \hat{\mathbf{h}}_{c,m}^* \left(\hat{\mathbf{H}}_c^T \hat{\mathbf{H}}_c^* \right)^{-1} \right\} \right). \quad (4.70)$$

Note that power coefficients will differ only between users as a necessity for canceling the interference between MSs. The solution of (4.68) relies as well on the bisection method and has less online overhead as it would have in the distributed processing scenario since the power coefficient for the k -th MS will be the same regardless the AP. In the particular case of having an error-free estimation, problem (4.68) can be solved through a heuristic and low complexity approach having the power coefficient computed as [13]

$$\omega_1 = \dots = \omega_K = \frac{1}{\left(\max_m \sum_{k=1}^K \delta_{mk} \right)}. \quad (4.71)$$

Figure 4.3: Channel hardening ratio for CF with CB precoding and MF combining using uniform power allocation having each AP $N = 4$.

Channel estimation in this work is nevertheless imperfect and the solution obtained with this low complexity approach will differ from the optimal one. However, as shown in [13], this approach nearly matches the optimal max-min SINR solution while considerably reduces the computational overhead at the CPU making it suitable for real deployments.

UL full power allocation As in the distributed scenario, we use a full power allocation strategy for the UL segment under the cooperative combining scheme, i.e. $\theta_k = 1$.

4.2.3 Numerical results

The baseline scenario for simulation is built on the same deployment as in the uncorrelated Rayleigh fading setup. All simulation parameters will remain as in Table 4.2, unless stated otherwise in particular cases.

4.2.3.1 Channel hardening ratio

We first show the channel hardening ratio under the pure CF scheme in Figure 4.3 for both the DL and the UL segments, respectively. These results correspond to the CB/MF with the uniform power allocation strategy and $N = 4$. For comparison purposes, four different channel types are shown (uncorrelated Rayleigh fading, spatially correlated Rayleigh fading, uncorrelated Ricean fading and spatially correlated Ricean fading). To begin with, we point out the exact

Parameter	Value
Carrier frequency (f_0)	2 GHz
System bandwidth (B)	20 MHz
Side of square coverage area (D)	1000 m
Number of APs (M)	100
Noise figure at AP and MS	9 dB
Available average power at the AP	200 mW
Available average power at the MS	100 mW
AP antenna height (h_{AP})	15 m
MS antenna height, (h_{MS})	1.65 m
Channel coherence interval length (τ_c)	200 samples
Training phase interval length (τ_p)	15 samples
Payload phase interval length ($\tau_d = \tau_u$)	$(\tau_c - \tau_p) / 2$ samples
LOS reference distance (d_0)	20 m
Path loss parameters (L_0, α, σ_χ)	
- LOS, $d_{mk} \leq 150$ m	34, 2.2, 3 dB
- LOS, $d_{mk} > 150$ m	-5.17, 4, 3 dB
- NLOS	30, 3.67, 4 dB
Shadow fading decorrelation distance	9 m
Shadow fading correlation among APs	0.5
Ricean κ -factor distribution ($\mu_\kappa, \sigma_\kappa$)	9 dB, 5 dB
Distribution of the AoA deviation	$\mathcal{N} \sim (0, \sigma_v^2)$
ASD	10°

Table 4.2: *Simulation default parameters for correlated Ricean fading scenario.*

matching between the analytical and Monte-Carlo values for every case, thus confirming the accuracy of the hardening theoretical closed-form expressions for any scenario. While the UL is totally independent of the power coefficients as the hardening remains quasi-constant when varying K , this is not the case in the DL segment. This latter case shows a lower hardening ratio regardless the configuration when there are less active MSs, owing to the fact that the same amount of power needs to be shared among less users. When comparing the Rayleigh and Ricean fading, it is clear that the existence of a strong LOS component is always beneficial and boosts channel hardening in either segment. Bear in mind that Ricean fading channels are often considered to be quasi-deterministic channels. This fact agrees with both Figures 4.3a and 4.3b. In contrast, spatial correlation degrades the ratio, specially under Rayleigh fading. This behavior matches the insights derived in (4.23) and (4.31) and the subsequent analysis.

Figure 4.4: CDF of the DL channel hardening ratio with CB precoding having each AP $N = 2$ and $K = 10$.

In Figure 4.4 the cumulative density function (CDF) of the DL hardening ratio with CB precoding is shown for a set of $K = 10$ MSs, under uniform power allocation and max-min power allocation in Figures 4.4a and 4.4b, respectively. Figure 4.4a confirms what could already be seen in Figure 4.3a; hardening is considerably degraded under spatially correlated Rayleigh fading implying a major gap to the uncorrelated Rayleigh scenario. Again, minor differences are observed under the spatially correlated and uncorrelated Rician fading scenarios. As for the max-min power control, a much steeper CDF is shown, owing to the fact of ensuring that every MS will experience the same SINR irrespective of its location in the coverage area. In contrast to the uniform power allocation, the max-min power control satisfies stronger hardening while the differences between the correlated and uncorrelated scenarios are considerably smaller, specially under Rician fading.

In Figure 4.5 the DL hardening ratio is shown for the different clustering configurations and various coverage area sizes under a Rayleigh fading channel and $N = 1$. Note that conclusions drawn in this case do not vary no matter the fading type or having single or multi-antenna APs. CB precoding and uniform power allocation is considered. The classical CF scheme is shown for comparison purposes. Let us comment on every configuration, except for MS centric by now. Under the traditional $D = 1000$ m deployment area (Figure 4.5a), there are little differences between all architectures. With the classical CF configuration, the number of APs effectively contributing to the hardening ratio tends to match the number of active APs in any of the clustering configurations. In other words, the pure CF and the clustering architectures

Clustering scheme	$K = 20$		$K = 60$	
	APs serving a MS	MSs served by an AP	APs serving a MS	MSs served by an AP
CF	100	20	100	60
AP centric	25	5	8.3333	5
AP centric with master AP	25.001	5.0002	8.3609	5.0165
MS centric	7.2687	1.4538	5.2622	3.1573
AP-MS centric	25.1015	5.0203	9.2132	5.528

Table 4.3: Average number of APs serving a MS and average number of MSs being served by an AP under different clustering strategies ($M = 100$ and $K_C = M_C = 5$).

perform similarly with minor differences. However, if K grows large enough, a particular MS will be served by less APs as their service needs to be split among more users. The gap between the different clustering schemes is highly conditioned by the AP deployment density, which is strictly related to the size of the coverage area. As for the MS centric scenario, since the number of APs serving a given MS remains static, the gap with the pure CF scheme will be the same regardless of the number of active users in the network. To sum up, the channel hardening ratio that a given MS experiences will be inversely proportional to the number APs providing service to that user. In Table 4.3 the average number of APs serving a MS and the average number of MSs being served by an AP under different clustering strategies are shown. It can be clearly seen that the MS centric strategy has the least number of APs serving a particular MS (thus having the worst channel hardening ratio in accordance to the third remark previously given). There are little differences between the rest of the schemes in accordance to what is shown on Figure 4.5a. Now, turning our attention to Figure 4.5b, the gap between CF and clustering approaches slightly increases as more APs are effectively contributing in terms of hardening in the classical scheme. It is worth mentioning that, except for the MS centric architectures, all clustering schemes perform equally in terms of hardening since the selection of APs from a given MS, and vice-versa, tends to be the same. Though a good degree of channel hardening is achieved, user data rate could be severely degraded due to strong pilot contamination when the coverage area is small.

The impact that spatial correlation has on the channel hardening ratio as a function of the number of APs is shown in Figure 4.6. The hardening ratio for different values of the ASD plus an uncorrelated case for comparison is shown. Note that having 10° of ASD corresponds to strong correlation among antennas, while 40° implies a moderate correlation. As expected, hardening improves considerably as M grows large enough. Under Ricean fading, spatial correlation has

Figure 4.5: DL channel hardening with CB precoding using uniform power allocation for single-antenna AP ($N = 1$) under different clustering approaches ($K_C = M_C = 5$).

a minor impact on the hardening, even if antennas are strongly correlated. However, when turning the attention to Rayleigh fading, the difference between an uncorrelated scenario and strong correlation is substantial. This confirms that assuming uncorrelated Rayleigh fading channel in CF massive MIMO networks with multi-antenna APs could lead to misleading conclusions.

4.2.3.2 User data rates

Linking with Figure 4.3, in Figure 4.7 the achievable rates are presented for the same configurations. Again, a good matching is shown between Monte-Carlo simulations and the analytical closed-form expression values for any of the scenarios presented. Similarly to the hardening ratios, the LOS component of any of the two Ricean fading scenarios has significant impact on the system performance, while the correlation between AP antennas degrades it. It is worth mentioning that, as K grows, the spatial correlation enhances performance due to multi-user interference removal, thus reducing the gap between the correlated and uncorrelated curves. Anyhow, drawing conclusions of a CF massive MIMO network under the widely used uncorrelated Rayleigh fading channel model could be most misleading, as noted with the hardening ratios.

Regarding cooperative processing, Figure 4.8 shows the DL achievable rate with ZF precoding and the low-complexity max-min power control. As in Figure 4.7, we are showing all four

Figure 4.6: DL channel hardening ratio with CB precoding and uniform power allocation under Rayleigh fading having each AP $N = 2$.

channel configurations, i.e. Ricean/Rayleigh fading with spatially correlated/uncorrelated antennas. A good matching is shown between the proposed semi-analytical expressions and the pure Monte-Carlo simulations. Bear in mind that analytical expressions presented in this thesis for the ZF precoder need Monte-Carlo computation. Similar to the distributed processing scenarios, the LOS component boosts the achievable rates. In contrast, under ZF precoding user data rate exhibits a slight enhancement for spatially correlated scenarios with large K . This particular matter will be addressed in Chapter 5.

Lastly, in Figure 4.9 a comparison between the lower bound achievable rate, the true rate achieved in the absence of receiver CSI and upper bounds is shown. Focusing on the Rayleigh fading scenario, a remarkably large gap is observed between the achievable rate (which is always a lower bound in terms of performance) and the true network performance. This gap is strictly conditioned by the hardening ratio. A greater gap exists indeed if compared with the upper bound, which corresponds to the maximum channel capacity. If we now turn our attention to the Ricean fading scenario, where hardening is boosted thanks to the LOS component, the gap between bounds is reduced. These results prove that, even having multi-antenna APs, hardening is severely compromised in correlated Rayleigh scenarios and the true network performance is underestimated when relying on the closed-form achievable rates.

Figure 4.7: Achievable rate for CF with CB precoding and MF combining using uniform power allocation having each AP $N = 4$.

4.2.4 Concluding remarks

In this section CF and UC massive MIMO networks under a spatially correlated Ricean fading channel have been analyzed under different precoding and combining strategies. Results show how the dominant LOS component of the Ricean fading has a positive impact both on the hardening ratio and user data rates. On the contrary, spatially correlated antenna arrays imply a channel hardening degradation for distributed processing schemes. Interestingly, if a cooperative ZF processing is considered, spatial correlation entails an enhancement in terms of user data rate over the uncorrelated scenario when the number of users in the network is considerably large. It has been also shown how the newly proposed UC based approaches imply a slight degradation of the channel hardening under the traditional deployment area of 1000×1000 m, but this degradation may considerably worsen if the coverage area is small. The widely used uncorrelated Rayleigh fading channel model overestimates the system performance in terms of both hardening and user rate. If one assumes a more realistic scenario in which antennas from each AP are spatially correlated, channel hardening is compromised and the commonly used achievable rate may underestimate the true system performance. In general, it has been shown that, even in scenarios with multi-antenna APs, one cannot take for granted channel hardening in CF massive MIMO networks, since it has a strong dependency on the channel characteristics, power control coefficients, clustering strategies, and spatial density of APs. In this context, we have shown the existence of a gap between the true system performance and the estimated achievable rates in accordance with the degree of channel hardening that is achieved. Closed-

Figure 4.8: Average DL achievable rate with ZF precoding and low-complexity max-min power allocation having each AP $N = 2$.

Figure 4.9: Average DL achievable rate with true and upper bounds under CB precoding and uniform power allocation having each AP $N = 4$.

form expressions for both the channel hardening ratio and the achievable user rate were derived for CB precoding and MF combining, while semi-analytical expressions were given for ZF. These analytical expressions can be used to perform the design and optimization of key aspects in CF massive MIMO networks, such as scheduling algorithms or clustering strategies, though this is left for future work.

Chapter 5

Reducing Processing Complexity under Pilot Contamination

As commented in previous chapters, a major aspect when modeling a CF massive MIMO system is either to assume spatial correlation between antennas or not. In the latter case, the collective channel directions are uniformly distributed over the beamspace and they are totally independent of the channel gain. In those works where APs are considered single-antenna, presuming totally uncorrelated channels seems fairly realistic since each AP is located far apart from the rest. However, this will hardly occur if APs are equipped with more than one antenna, as shown by several research contributions mentioned earlier.

Throughout Chapter 4 we showed how spatially correlated antennas degraded both channel hardening and achievable rate for MR schemes, especially under Rayleigh fading channels and regardless the number of active MSs in the network. However, when assuming ZF precoding, the achievable rate was not degraded, but even slightly boosted for large K , under for Ricean and Rayleigh fading. This is due to the combination of the *ability* of ZF precoding for canceling multi-user interference (of which MR lacks) and the positive effect that pilot contamination has in this matter. Indeed, spatial correlation eases the multi-user interference cancellation even under severe pilot contamination [63]. Whatsmore, if pilot contamination is properly exploited in a spatially correlated scenario, massive MIMO-based systems can turn to be asymptotically unbounded in terms of SE [66], in contrast to the initial beliefs.

In comparison to co-located massive MIMO systems, CF networks are intended to provide service in larger areas, and thus, to a larger number of MSs. This makes the pilot contamination issue even more critical and highly desirable to be handled properly. A key question arises at this point. Do we need to rely on spatially correlated multi-antenna APs for providing unconstrained SE when suffering from pilot contamination? In this chapter, we introduce a reduced-complexity, though sub-optimal, MMSE-based combining scheme that also achieves unbounded capacity in high pilot-contaminated CF massive MIMO networks. The scheme can be implemented either in a centralized or a distributed fashion since only local channel esti-

mates are required, in addition to global channel statistics. An alternative, yet optimal, scheme is also proposed by using the more conservative UatF capacity bound that, as seen before, depends solely on statistical channel knowledge and provides an achievable rate when CSI is not available for final data detection. This is the case of a decentralized network in which the channels are estimated at the APs while data detection is performed at the CPU. These combiners are built based on the linear independence experienced by the global covariance matrices and the asymptotic behavior for a large number of APs, though they are valid under finite regime. Neither of these processing schemes are constrained by pilot contamination and both show a large reduction of computational complexity. Thanks to the UL-DL duality [20], precoding schemes can be obtained for the DL segment. These schemes are also valid for any of the clustering approaches derived from the original CF proposal.

5.1 Impact of local covariance matrices

Considering the training sequences are reused among the MSs, the multi-user interference cancellation is naturally strengthened by the possible existing orthogonality between the local spatial correlation matrices $\mathbf{R}_{mk}$. Under this circumstances, given a particular AP m and two MSs k and k' , SE is enhanced as

$$\text{tr}(\mathbf{R}_{mk}\mathbf{R}_{mk'}) \rightarrow 0 \quad (5.1)$$

Whenever the ASD of the spatial correlation model lowers, the antennas of the given AP become more correlated and the orthogonality of $\mathbf{R}_{mk}$ and $\mathbf{R}_{mk'}$ increases. In contrast, when assuming an uncorrelated channel model (5.1) would always be equal to N . Most of practical channels exhibit spatial correlation but not a *fully*-orthogonality situation among the covariance matrices $\mathbf{R}_{mk}$ [63].

By using processing schemes that account for multi-user interference, the overall system performance is enhanced when compared with MR processing. We now illustrate the impact that spatial correlation has on SE comparing MR and MMSE combiners. As in cellular massive MIMO (e.g., [38,63]), the SINR in (3.30) is a generalized Rayleigh quotient with respect to $\mathbf{v}_k$ and thus is maximized by the MMSE combining vector, i.e.,

$$\mathbf{v}_k = \rho_u \left(\sum_{k'=1}^K \rho_u \left(\hat{\mathbf{h}}_{k'} \hat{\mathbf{h}}_{k'}^H + \mathbf{E}_{k'} \right) + \mathbf{I}_{MN} \right)^{-1} \hat{\mathbf{h}}_k, \quad (5.2)$$

Figure 5.1: CDF of the UL average SE for different K under uncorrelated and correlated scenarios with MR and MMSE combining.

where $\mathbf{E}_k = \text{diag}(\mathbf{E}_{1k}, \dots, \mathbf{E}_{Mk}) \in \mathbb{C}^{MN \times MN}$ is the block-diagonal error correlation matrix. For simplicity, let us assume the Rayleigh fading version of the channel model by letting $\mathbf{h}_{mk} \sim \mathcal{CN}(\mathbf{0}_N, \mathbf{R}_{mk})$ for the next example. Nevertheless, the outcomes of this chapter would not vary if a Ricean component was assumed. In addition, the pure CF approach is considered only ($c_{mk} = 1 \forall mk$).

In the next example the parameters shown in 4.2 are assumed, except for those related to Ricean fading. In addition, τ_p is set to 5 for ensuring that pilot contamination occurs. The number of antennas per AP is fixed to $N = 4$. Instead of computing the user data rate as the key metric, we are showing the SE given by

$$\text{SE}_{u,k}^{\text{CSIR}} = \frac{1 - \tau_p/\tau_c}{2} \mathbb{E} \left\{ \log_2 (1 + \text{SINR}_{u,k}^{\text{CSIR}}) \right\}, \quad (5.3)$$

for the case of centralized combining (MMSE) and

$$\text{SE}_{u,k}^{\text{UatF}} = \frac{1 - \tau_p/\tau_c}{2} \log_2 (1 + \text{SINR}_{u,k}^{\text{UatF}}), \quad (5.4)$$

for distributed combining (MR).

Figure 5.1 shows the CDF of the average UL MSs SE under both MR and MMSE combiners assuming strong spatial correlation, i.e. $\text{ASD} = 10^\circ$, and no correlation, i.e. $\mathbf{R}_{mk} = \zeta_{mk} \mathbf{I}_N$. Full power transmission is considered.

In Figure 5.1a results for the given scenario are shown for a small number of active MSs $K = 10$. It is worth mentioning that in this situation there will be little pilot contamination since the coverage area is rather large ($D = 1000$ m) and the fingerprinting technique further reduces this effect. Under the MR combiner a considerable degradation of the average SE can be observed if antennas become spatially correlated. This observation matches the claim that assures spatial correlation has a negative impact on MIMO systems. However, when focusing on the MMSE combiner there is no such degradation. Instead, both the correlated and uncorrelated scenarios are roughly the same and differences are almost negligible. One may say that the centralized scheme is *agnostic* of spatial correlation under this case.

In Figure 5.1b same conditions as before are assumed, except for the number of users. The CF massive MIMO systems are supposed to be a key solution for ultra-dense networks so we are setting $K = 70$ to simulate such a scenario. Let us first set our attention on the MR combiner. It can be clearly seen a similar tendency when compared to the $K = 10$ case. In words, as spatial correlation increases, the performance worsens. However, the gap between the uncorrelated scenario and its correlated counterpart is reduced compared to the $K = 10$ situation. We should point out that the uncorrelated case is still an upper bound in terms of SE under MR combining. Now focus on the MMSE combiner. Under this combining strategy a pretty much different situation is observed. We see that the uncorrelated scenario is markedly outperformed by the correlated one. This involves a situation in which spatial correlation is not detrimental, but positive. We may clearly state that under a centralized combiner strategy and a crowded scenario, spatial correlation between APs antennas is beneficial for countering pilot contamination. However, the existent gap between the uncorrelated and the correlated lines is not particularly big.

The following key aspects can be drawn so far:

- The MMSE combiner outperforms notably the MR scheme under either scenario. In addition, the MMSE scheme can be implemented so it is scalable when K grows large, while the distributed approach of the MR scheme is not. Nevertheless, the MMSE scheme can only be implemented in a centralized fashion and requires more computational resources.
- Spatial correlation among APs antennas affects negatively to the MR combiner, though it becomes less harmful in crowded scenarios. On the other side, the MMSE scheme takes

advantage of this situation for suppressing multi-user interference, especially for a large set of MSs.

- In those scenarios in which K is large, pilot contamination can be hard to handle if one relies on managing the training sequences allocation only. Exploiting spatial correlation results in a fairly good approach for further enhancing SE, though with limited impact.

All in all, spatially correlated local covariance matrices have been shown not to have a big impact under the CF architecture, in contrast to cellular massive MIMO. Nevertheless, the most controversial aspect yet to address in CF massive MIMO is whether local spatial correlation is needed for achieving unbounded capacity or not.

5.2 Asymptotic behavior

The asymptotic behavior when M grows to infinite in CF massive MIMO has not been addressed so far. Notably, during initial research stages of spatially uncorrelated cellular massive MIMO networks, the coherent interference caused by the pilot contamination effect was set as the limiting factor at the asymptotic regime [19]. However, when assuming the more realistic spatially correlated channel model, SE can grow undoubtedly with M by employing the optimal MMSE combiner [20, 63, 66].

We will now analyze the asymptotic behavior of the SE in (5.3) with the MMSE combiner as $M \rightarrow \infty$. We assume that the *global* covariance matrices $\mathbf{R}_k = \text{diag}(\mathbf{R}_{1k}, \dots, \mathbf{R}_{Mk}) \in \mathbb{C}^{MN \times MN}$ are asymptotically linearly independent, as analytically defined in [66]. Under this condition, the following lemma follows.

Lemma 2 [66, Th. 1] *If MMSE combining is used and $\{\mathbf{R}_k : k = 1, \dots, K\}$ are asymptotically linearly independent, then the SE in (5.3) increases unboundedly as $M \rightarrow \infty$.*

Proof: It follows easily from [66, App. B] since the centralized CF case is basically a single-cell massive MIMO network with block-diagonal spatial correlation matrices. ■

What is most relevant from Lemma 2 is that it follows even for local uncorrelated covariance matrices and for single-antenna APs. The $N \times N$ matrices at the diagonal of $\mathbf{R}_k$ (or simply the scalars of its main diagonal in the case of $N = 1$) are subject to different channel gains, making

Figure 5.2: Average UL SE for both combiners with increasing M .

them linearly independent. This is a key difference between the CF architecture and classical single-cell massive MIMO systems.

To illustrate the performance of a CF massive MIMO network at the asymptotic regime, let us consider the same setup as in Figure 5.1, but assuming $\tau_p = 1$ and $K = 4$, i.e. full pilot reuse and maximal pilot contamination. Figure 5.2 shows the average SE per MS for both MMSE and MR combiners and $N = 4$ for correlated and uncorrelated scenarios. The optimal MMSE shows the best performance and confirms that CF massive MIMO networks can indeed achieve unbounded capacity in the asymptotic regime (as stated in Lemma 2), regardless the existence of spatial correlation among the set of antennas of each AP. On the contrary, MR combining stalls with increasing M .

5.3 Pilot decontamination processing

Even though the MMSE scheme achieves optimal performance in terms of SE, it comes at the cost of requiring a centralized implementation (and, therefore, tight synchronization between the APs and the CPU) and it demands high computational resources. Considering capacity-limited wired connections and scalability concerns, it may not be feasible to have such an implementation, thus limiting the utility of the CF architecture. In this thesis the possible synchronization issues derived from the imperfect fronthaul links are not addressed. However, we find the computation complexity of MMSE a controversial factor for dense deployments. Note that the number of complex multiplications per coherence block under such processing scheme grows cubically.

Before properly addressing this issue, let us reformulate a little bit the system model given in Chapter 3 for the sake of understanding. Likewise, consider single-antenna APs only ($N = 1$) and full pilot reuse (i.e. $\tau_p = 1$ and $\boldsymbol{\varphi}_k = \boldsymbol{\varphi} \forall k$) for a Rayleigh distributed channel coefficient $h_{mk} \sim \mathcal{CN}(0, \zeta_{mk})$. The MMSE channel estimate is rewritten as

$$\hat{h}_{mk} = \frac{\zeta_{mk}}{\psi_m} \left(\frac{1}{\sqrt{\rho_p}} \boldsymbol{\varphi}^H \mathbf{y}_{p,m} \right) \quad (5.5)$$

with $\mathbf{y}_{p,m}$ the single-antenna version of (3.7) and

$$\psi_m = \sum_{k=1}^K \zeta_{mk} + \frac{1}{\rho_p}. \quad (5.6)$$

The estimates and estimation errors $e_{mk} = h_{mk} - \hat{h}_{mk}$ are distributed as $\hat{h}_{mk} \sim \mathcal{CN}(0, \gamma_{mk})$ and $e_{mk} \sim \mathcal{CN}(0, \varepsilon_{mk})$. We have reformulated $\gamma_{mk} = \zeta_{mk}^2 / \psi_m$ and $\varepsilon_{mk} = \zeta_{mk} - \gamma_{mk}$. Now, the vector composition of channel coefficients and channel estimates are $\mathbf{h}_k = [h_{1k}, \dots, h_{Mk}]^T \in \mathbb{C}^{M \times 1}$, which follows a $\mathcal{CN}(\mathbf{0}_M, \mathbf{R}_k)$, and $\hat{\mathbf{h}}_k = [\hat{h}_{1k}, \dots, \hat{h}_{Mk}]^T \in \mathbb{C}^{M \times 1}$, which follows $\hat{\mathbf{h}}_k \sim \mathcal{CN}(\mathbf{0}_M, \mathbf{R}_k \boldsymbol{\Psi}^{-1} \mathbf{R}_k)$. The global covariance matrix is $\mathbf{R}_k = \text{diag}(\zeta_{1k}, \dots, \zeta_{Mk})$ and $\boldsymbol{\Psi} = \text{diag}(\psi_1, \dots, \psi_M)$ leading to $\mathbf{R}_k \boldsymbol{\Psi}^{-1} \mathbf{R}_k = \text{diag}(\gamma_{1k}, \dots, \gamma_{Mk})$. In order to comply with this reformulation of the system model let us rewrite (3.30) as

$$\text{SINR}_{u,k}^{\text{CSIR}} = \frac{|\boldsymbol{\vartheta}_k^H \hat{\mathbf{h}}_k|^2}{\sum_{k' \neq k} |\boldsymbol{\vartheta}_k^H \hat{\mathbf{h}}_{k'}|^2 + \boldsymbol{\vartheta}_k^H \mathbf{Z} \boldsymbol{\vartheta}_k}, \quad (5.7)$$

with $\mathbf{Z} = \text{diag}(z_1, \dots, z_M)$ being diagonal with elements

$$z_m = \sum_{k=1}^K \varepsilon_{mk} + \frac{1}{\rho_u}. \quad (5.8)$$

In the same fashion, (3.37) is rewritten as

$$\text{SINR}_{u,k}^{\text{UatF}} = \frac{|\mathbb{E} \{ |\boldsymbol{\vartheta}_k^H \mathbf{h}_k| \}|^2}{\sum_{k=1}^K \mathbb{E} \{ |\boldsymbol{\vartheta}_k^H \mathbf{h}_k|^2 \} - |\mathbb{E} \{ |\boldsymbol{\vartheta}_k^H \mathbf{h}_k| \}|^2 + \frac{1}{\rho_u} \mathbb{E} \{ \|\boldsymbol{\vartheta}_k\|^2 \}}. \quad (5.9)$$

Lastly, (3.26) is reformulated as

$$\text{SINR}_{d,k} = \frac{|\mathbb{E}\{|\mathbf{h}_k^H \boldsymbol{\varpi}_k|\}|^2}{\sum_{k=1}^K \mathbb{E}\{|\mathbf{h}_k^H \boldsymbol{\varpi}_k|^2\} - |\mathbb{E}\{|\mathbf{h}_k^H \boldsymbol{\varpi}_k|\}|^2 + \frac{1}{\rho_d}} \quad (5.10)$$

for computing the DL SE based on the channel hardening bound as

$$\text{SE}_{d,k} = \frac{1 - \tau_p/\tau_c}{2} \log_2(1 + \text{SINR}_{d,k}). \quad (5.11)$$

5.3.1 Reduced-complexity MMSE

Under the reformulated system model we can express the MMSE combining vector as

$$\boldsymbol{\vartheta}_k = \left(\sum_{k'=1}^K \hat{\mathbf{h}}_{k'} \hat{\mathbf{h}}_{k'}^H + \mathbf{Z} \right)^{-1} \hat{\mathbf{h}}_k. \quad (5.12)$$

Although this scheme is optimal and achieves unbounded capacity (as in cellular massive MIMO), it requires to compute the $M \times M$ matrix inverse in (5.12) in every coherence block, which may be too computationally demanding when the network size is large. To overcome this issue, we follow the similar approach as in [67], and consider the asymptotic regime where $M \rightarrow \infty$. This allows us to propose an alternative combiner $\boldsymbol{\vartheta}_k = [\vartheta_{1k}, \dots, \vartheta_{Mk}]^T$ with ϑ_{mk}

$$\vartheta_{mk} = \frac{1}{z_m} \sum_{k'=1}^K \left[\left(\mathbf{B} + \frac{1}{M} \mathbf{I}_K \right)^{-1} \right]_{kk'} \hat{h}_{mk'} = \frac{1}{\sum_{k'=1}^K \varepsilon_{mk'} + \frac{1}{\rho_u}} \sum_{k'=1}^K \beta_{kk'} \hat{h}_{mk'} \quad (5.13)$$

where $\beta_{kk'}$ is the (kk') -th element of the matrix $(\mathbf{B} + \frac{1}{M} \mathbf{I}_K)^{-1}$ with the entries of $\mathbf{B}$ given by

$$\begin{aligned} [\mathbf{B}]_{kk'} &= \frac{1}{M} \text{tr}(\mathbf{R}_k \boldsymbol{\Psi}^{-1} \mathbf{R}_{k'} \mathbf{Z}^{-1}) = \frac{1}{M} \sum_{m=1}^M \frac{\zeta_{mk} \zeta_{mk'}}{\psi_m z_m} \\ &= \frac{1}{M} \sum_{m=1}^M \frac{\zeta_{mk} \zeta_{mk'}}{\left(\sum_{i=1}^K \zeta_{mi} + \frac{1}{\rho_p} \right) \left(\sum_{k'=1}^K \varepsilon_{mk'} + \frac{1}{\rho_u} \right)}. \end{aligned} \quad (5.14)$$

Proof: Recall the definition $\hat{\mathbf{H}} = [\hat{\mathbf{h}}_1, \dots, \hat{\mathbf{h}}_K] \in \mathbb{C}^{M \times K}$ and use the matrix inversion lemma

to get (after multiplying and dividing by M)

$$\mathbf{v}_k \triangleq \frac{1}{M} \mathbf{Z}^{-1} \hat{\mathbf{H}} \left(\frac{1}{M} \hat{\mathbf{H}}^H \mathbf{Z}^{-1} \hat{\mathbf{H}} + \frac{1}{M} \mathbf{I}_K \right)^{-1} \mathbf{a}_k \quad (5.15)$$

where $\mathbf{a}_k$ is the canonical basis and from which $\mathbf{v}_{mk}$ is obtained as (by neglecting the factor $1/M$, since it does not affect the SINR in (5.7))

$$\mathbf{v}_{mk} \triangleq \frac{1}{z_m} \hat{\mathbf{h}}_m^T \left(\frac{1}{M} \hat{\mathbf{H}}^H \mathbf{Z}^{-1} \hat{\mathbf{H}} + \frac{1}{M} \mathbf{I}_K \right)^{-1} \mathbf{a}_k \quad (5.16)$$

with $\hat{\mathbf{h}}_m = [\hat{h}_{m1}, \dots, \hat{h}_{mK}]^T \in \mathbb{C}^{K \times 1}$. By exploiting the fact that, as $M \rightarrow \infty$,

$$\frac{1}{M} \left[\hat{\mathbf{H}}^H \mathbf{Z}^{-1} \hat{\mathbf{H}} \right]_{\ell j} = \frac{1}{M} \hat{\mathbf{h}}_\ell^H \mathbf{Z}^{-1} \hat{\mathbf{h}}_j \asymp \frac{1}{M} \text{tr}(\mathbf{R}_j \mathbf{\Psi}^{-1} \mathbf{R}_\ell \mathbf{Z}^{-1}) \quad (5.17)$$

the combiner ϑ_{mk} in (5.13) follows. ■

Since (5.13) reduces to (5.12) in the limiting regime $M \rightarrow \infty$, it follows that this new combiner also achieves unbounded capacity. Compared to the optimal MMSE, however, the computational complexity required by (5.13) is much lower. To see this, we use (5.5) to obtain

$$\vartheta_{mk} = \bar{\beta}_{mk} \left(\frac{1}{\sqrt{\rho_p}} \boldsymbol{\varphi}^H \mathbf{y}_m \right) \quad (5.18)$$

where

$$\bar{\beta}_{mk} = \frac{1}{\left(\sum_{k'=1}^K \zeta_{mk'} + \frac{1}{\rho_p} \right) \left(\sum_{k'=1}^K \varepsilon_{mk'} + \frac{1}{\rho_u} \right)} \sum_{k'=1}^K \beta_{kk'} \zeta_{mk'} \quad (5.19)$$

Under the assumption that coefficients $\bar{\beta}_{mk}$ are available, from (5.13) the complexity required for the computation of our proposal scales linearly with M , rather than as M^3 as with MMSE scheme. Therefore, we refer to (5.13) as reduced-complexity MMSE (RC-MMSE). Notice that, if the large-scale coefficients are locally available at AP m (which is a reasonable assumption since they change every several coherence intervals and can be broadcasted by the CPU), (5.13) can be locally computed. This implies that, unlike the MMSE combiner, it can also be adopted in a distributed network. In this case, however, the UatF bound in (5.4) must be used to compute an achievable SE since signal detection takes place without channel knowledge in a distributed network.

5.3.2 Generalized MR

The MMSE combiner is obtained as a matrix transformation of $\hat{\mathbf{h}}_k$. Inspired by this, we now assume that

$$\boldsymbol{\vartheta}_k = \mathbf{V}_k \hat{\mathbf{h}}_k \quad (5.20)$$

where $\mathbf{V}_k$ is an arbitrary matrix to be optimized. Unlike (5.2), we assume that this optimization can only be done on the basis of channel statistics (rather than of channel estimates). $\mathbf{V}_k$ can be precomputed at the CPU, and the computation of $\boldsymbol{\vartheta}_k$ in (5.20) reduces to only one matrix-vector multiplication. This is substantially lower computational complexity than MMSE combiner. The question is how to optimally design $\mathbf{V}_k$ in order not to incur in a significant loss in SE. Building on [67, 68], we can use the UatF bound in (5.4) for maximization. This yields

$$\vartheta_{mk} = \frac{1}{\psi_m} \sum_{k'=1}^K \left[\left(\mathbf{A} + \frac{1}{M} \mathbf{I}_K \right)^{-1} \right]_{kk'} \hat{h}_{mk'} = \frac{1}{\sum_{k'=1}^K \zeta_{mk'} + \frac{1}{\rho_p}} \sum_{k'=1}^K \alpha_{kk'} \hat{h}_{mk'} \quad (5.21)$$

where $\alpha_{kk'}$ is the (kk') -th element of the matrix $(\mathbf{A} + \frac{1}{M} \mathbf{I}_K)^{-1}$ with $\mathbf{A}$ having elements

$$\begin{aligned} [\mathbf{A}]_{kk'} &= \frac{1}{M} \text{tr}(\mathbf{R}_k \boldsymbol{\Psi}^{-1} \mathbf{R}_{k'} \boldsymbol{\Psi}^{-1}) = \frac{1}{M} \sum_{m=1}^M \frac{\zeta_{mk} \zeta_{mk'}}{\psi_m^2} \\ &= \frac{1}{M} \sum_{m=1}^M \frac{\zeta_{mk} \zeta_{mk'}}{\left(\sum_{k'=1}^K \zeta_{mk'} + \frac{1}{\rho_p} \right) \left(\sum_{k'=1}^K \zeta_{mk'} + \frac{1}{\rho_p} \right)}. \end{aligned} \quad (5.22)$$

We refer to (5.21) as the generalized MR (GMR) combining. As for MMSE, we can analyze its asymptotic behavior as $M \rightarrow \infty$. From [67, 68], we may state the following Lemma:

Lemma 3 *If GMR is used and $\{\mathbf{R}_k : k = 1, \dots, K\}$ are asymptotically linearly independent, then the SE in (5.4) increases unboundedly as $M \rightarrow \infty$.*

This lemma shows that GMR has the same scaling behavior of MMSE and RC-MMSE as $M \rightarrow \infty$. As with RC-MMSE, this is achieved by using only the channel statistics. Moreover, GMR has the same complexity of RC-MMSE as it follows by plugging (5.5) into (5.21) to obtain

$$\vartheta_{mk} = \bar{\alpha}_{mk} \left(\frac{1}{\sqrt{\rho_p}} \boldsymbol{\varphi}^H \mathbf{y}_m \right) \quad (5.23)$$

where

$$\bar{\alpha}_{mk} = \frac{1}{\left(\sum_{k'=1}^K \zeta_{mk'} + \frac{1}{\rho_p}\right) \left(\sum_{k'=1}^K \zeta_{mk'} + \frac{1}{\rho_p}\right)} \sum_{k'=1}^K \alpha_{kk'} \zeta_{mk'}. \quad (5.24)$$

The key difference with respect to RC-MMSE in (5.13) is the scaling factor $\bar{\alpha}_{mk}$, which does not depend on the variances $\varepsilon_{mk'}$ of the channel estimation errors but on the large-scale fading coefficients $\beta_{mk'}$. This is because (5.21) maximizes (5.4), which does not take into account the imperfect CSI in the decoding process. As RC-MMSE, GMR can also be used in a decentralized network.

5.3.3 Precoder design

A common heuristic approach for the precoder design relies on the UL-DL duality presented in [20], which holds between the UatF bound in (5.4) and hardening bound in (5.11). The duality states that the SE achieved in the UL can be achieved also in the DL, if the UL combining vectors are used as DL precoding vectors and the DL transmit power is allocated properly. Motivated by this, we select the DL precoding vectors as

$$\varpi_k = \frac{\vartheta_k}{\sqrt{\mathbb{E}\{|\vartheta_k^H \vartheta_k|\}}}. \quad (5.25)$$

Precoding vectors can be selected on the basis of RC-MMSE or GMR for either centralized or distributed networks. On the other hand, they can be selected based on the MMSE combiner for a centralized fashion only. The power control coefficients will be computed as follows. For centralized schemes a network-wide strategy can be implemented by

$$\eta_{mk} = \eta_k = \frac{1}{K}, \quad (5.26)$$

while for distributed precoders stronger channels will be assigned more power by letting [32]

$$\eta_{mk} = \frac{\sqrt{\zeta_{mk}}}{\sum_{k'=1}^K \sqrt{\zeta_{mk'}}}. \quad (5.27)$$

Regardless the given power allocation strategy, the asymptotic behavior of the proposed schemes will not be altered. Power coefficients can be selected for imposing a particular criterion, such as the max-min SINR fairness.

Figure 5.3: CDF of the average SE for $K = 70$.

5.4 System performance

For evaluating the system performance under the proposed processing schemes, we replicate the setup in Figure 5.1. The CDF for the UL average SE per MS is shown in Figure 5.3a. The number of APs is set to $M = 100$ and $K = 20$ randomly distributed MSs are dropped with $D = 250$ m. Both centralized and distributed versions of the RC-MMSE and GMR combiners are considered. As upper and lower benchmarks, the centralized MMSE and distributed MR combiners are also presented. Regarding centralized schemes, the proposed RC-MMSE scheme nearly matches the optimal combiner. This proves this new scheme is a very suitable alternative for lowering the required computational resources with little performance loss under severe pilot contamination. On the other side, the GMR shows a considerable gap. We can notably point out that for low SNR both RC-MMSE and GMR match each other. This is due to the fact that the RC-MMSE combiner relies on having good channel estimates, while GMR relies on channel coefficients instead. Focusing on the distributed approaches, the GMR combiner outperforms the RC-MMSE. This is reasonable since GMR is designed for maximization of the UatF bound.

The DL segment is shown in Figure 5.3b for the same setup. Thanks to the UL-DL duality, the channel hardening bound gives nearly the same performance in the DL as the UatF bound in the UL. This proves that RC-MMSE is unsuitable for this scenario since the MS must rely on statistics and have no instantaneous CSI.

Figure 5.4 shows the asymptotic behavior of the average UL SE per MS for $K = 4$ with in-

Figure 5.4: Average UL SE for all combiners with $K = 4$.

creasing M . The MSs are located in the center of the area being 5 m apart from each other with $D = 1000$ m. The optimal MMSE shows the best performance and confirms that CF massive MIMO networks can indeed achieve unbounded capacity in the asymptotic regime (as stated in Lemma 1) even with $N = 1$. Likewise, on both RC-MMSE and GMR centralized combiners, SE grows unboundedly with M . In accordance to Lemma 2, the distributed approach of GMR shows the same asymptotic behavior, though the UatF bound considerably underestimates its performance. Lastly, the poor performance of the RC-MMSE vouches that it is not an adequate scheme under statistical CSI.

5.5 Concluding remarks

Unlike traditional cellular massive MIMO systems, the CF architecture does not necessary need to rely on local spatial correlation matrices for suppressing pilot contamination and achieving unbounded capacity. Nevertheless, if a proper processing scheme is used, this aspect can help to slightly enhance SE. This chapter focused on pilot decontamination in CF massive MIMO networks. Building on the linear independence of correlation matrices, the RC-MMSE combining was proposed as a low-complexity alternative to the centralized MMSE scheme that almost matches the optimal performance and also achieves asymptotically unbounded capacity. An alternative scheme was also derived that has the same asymptotic behavior but maximizes the UatF. Both schemes are valid for a finite number of APs and have linearly increasing complexity (in contrast to the optimal MMSE combiner). By the UL-DL duality, the same performance can be achieved in the DL segment where the GMR alternative serves as an optimal approach

for MR-based precoders.

Chapter 6

Conclusion and Future Research

Though conceived in the 1940s, the cellular concept for wireless communications has lived up to our days. Generally speaking, this technology has evolved in the search of higher SE for accomplishing with the need of ever growing data rates. In the 4G and 5G era, the interference issue that comes along with cellular deployments has been the most limiting factor that might halt mobile communications from reaching even further milestones. Several innovations, such the use of the mmWave band or VLC, are fairly good alternatives for certain use cases and scenarios. However, the path for general purpose wireless systems leads to a more revolutionary network architecture that can cope with the technological challenges ahead.

The CF massive MIMO architecture seems as the natural evolution of mobile communications. It is built on the late CoMP concept and the well-studied massive MIMO technology. Even though we have already seen that it can be implemented in very different ways, the final purpose is to provide seamlessly service to a large number of users. This thesis addressed the modeling of such a network and addressed several key aspects and properties for its design. Likewise, we proposed alternative processing schemes for pilot contamination constrained scenarios with low computational demands. In this final chapter a summary of all contributions is presented together with conclusions. Some future research lines are also given having this work as a baseline.

6.1 Summary and conclusions

Throughout Chapter 2 the standardized interference management techniques were addressed. While the ICIC technique aims at mitigating interference in the frequency domain by allocating data channels in different slots, the eICIC coordinates a macro BS and the surrounding SCs in the time domain by means of the ABS. The feICIC slightly enhances eICIC by not having totally zero-power-ABS. Finally, the CoMP proposal goes one step further by having a total coordination between BSs. This chapter also introduces massive MIMO systems as a key technology for CF networks and gives a thorough state-of-the-art description of the latter one.

The system model for the whole document was presented in Chapter 3. The network layout and the TDD frame structure was first introduced. The channel model was given by a spatially correlated Ricean fading model assuming multi-antenna APs. During the UL training phase, MMSE channel estimation is performed. Several clustering options for AP-MS linking were also addressed. Lastly, both the DL and UL data payload phases were introduced. DL data rates estimation needed to rely on the channel hardening bound since there are no DL pilots. However, the UL data rates estimation could rely on the UatF bound or the CSI bound, depending on the channel knowledge available at data detection.

Chapter 4 was devoted to the analysis of channel hardening in CF massive MIMO networks by introducing the channel hardening ratio metric. On the one side, an uncorrelated Rayleigh fading was considered for the DL segment only. Closed-form expressions for both computing the channel hardening ratio and achievable data rates were given for the CB and NCB precoders. The NCB scheme experienced lower channel hardening ratio and capacity bounds were tighter to the true rate thanks to the normalization factor. On the other side, the Ricean fading model was considered for both segments and also with the centralized ZF processing scheme. Again, closed-form expressions were given for computing the channel hardening ratio and achievable data rates under the CB/MF precoder/combiner and semi-analytical expressions were given for the ZF case. The LOS component was shown to have a positive impact in achieving lower channel hardening ratio, while the spatial correlation between APs antennas deteriorated it. Notably, the ZF scheme was favored by spatial correlation since it helps with canceling coherent interference caused by pilot contamination for a large set of MSs. Several clustering strategies were addressed for every scenario.

Lastly, in Chapter 5 local covariance matrices were demonstrated to have a limited impact on the CF architecture. Multi-antenna APs are not required for achieving unbounded capacity with the optimal MMSE processing scheme, unlike cellular massive MIMO. For avoiding having high requirements on terms of computational resources, the RC-MMSE was proposed for the UL segment. Based on the linear independence of global correlation matrices and the behavior of the MMSE scheme in the asymptotic regime, the RC-MMSE proposal achieves almost the same SE as the optimal scheme under centralized processing. The alternative GMR was built on a similar basis as the best option for maximizing the UatF bound with a distributed network architecture. By means of the UL-DL duality, the same performance was achieved at the DL segment based on the channel hardening bound.

6.2 Future research work

Taking this thesis as background, the following research lines are proposed for future advancements:

- **Power control.** While CF massive MIMO systems have been shown to perform well with full power transmissions in the UL, the power coefficients are needed to be properly designed for the DL. The max-min SINR fairness proposed in [10, 12] guarantees almost the same performance for every MS, but it is computationally demanding. In contrast, heuristic schemes are fast-computing but neglect particularities that the network may have. In this thesis, power coefficients are shown to have a major impact in achieving certain degree of channel hardening and also are key for the UL-DL duality. More advanced strategies are required for considering a balance between average SE and fairness, latency and data rates, as well as distributed and centralized networks. Some proposals were already given in [69, Chapter 7].
- **RC-MMSE and GMR multi-antenna APs version.** The proposed RC-MMSE and GMR schemes were given for single-antenna APs, though multi-antenna APs is a general trend in the CF literature. Considering spatially correlated local covariance matrices will most probably slightly enhance the multi-user interference cancellation, and thus system performance. While it is not mathematically challenging, having multi-antenna APs will considerably complicate notation for these two schemes.
- **Hardware implementation.** The whole CF architecture seemed promising since it was first proposed. However, having such a dense deployment is, at the very least, complicated from an implementation perspective. There are already some proposals, such as the radio stripes architecture [70]. Still, CF massive MIMO real deployment is something that needs of thorough investigation. A key drawback that needs to be dealt with is the wired connections for all APs (the so-called *spaghetti monster*). A solution under current study is implementing fronthaul links by optical communications.
- **MSs mobility.** This thesis considered fixed or slowly-moving MSs. This eases channel modeling since there is no need to consider Doppler effect related issues. Likewise, large-scale fading coefficients and phase shifts are considered to remain unchanged during several coherence intervals. It is a fair assumption in various scenarios (walking humans or sensor networks, for instance), but one cannot expect it to be this way for

every use case. Fast-moving MSs would require tight channel estimation and large-scale fading components acquisition. In addition, if a clustering strategy is implemented, the connectivity matrix is also required to be updated faster.

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