This is a postprint version of the following published document:

Wang, Y., Feng, C., Wang, X., Zhao, Z., Romero, C. S. & Yang, J. (2019). Nonlinear free vibration of graphene platelets (GPLs)/polymer dielectric beam. *Smart Materials and Structures*, 28(5), 055013.

DOI: [10.1088/1361-665x/ab0b51](https://doi.org/10.1088/1361-665x/ab0b51)

© 2019 IOP Publishing Ltd.

This work is licensed under a [Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License](https://creativecommons.org/licenses/by-nc-nd/4.0/).

Nonlinear free vibration of graphene platelets (GPLs)/polymer dielectric beam

Yu Wang¹, Chuang Feng^{1,4} , Xinwei Wang², Zhan Zhao¹, Carlos Santiuste Romero³ and Jie Yang¹

¹ School of Engineering, RMIT University, Bundoora, VIC 3083 Australia

² State Key Laboratory of Mechanics and Control of Mechanical Structures, Nanjing University of Aeronautics and Astronautics, Nanjing, People's Republic of China

³ Department of Continuum Mechanics and Structural Analysis, Universidad Carlos III de Madrid, Avda de la Universidad 30, E-28911 Leganés, Spain

E-mail: chuang.feng@rmit.edu.au and j.yang@rmit.edu.au

Abstract

Nonlinear free vibration of graphene platelets (GPLs) reinforced dielectric composite beam subjected to electrical field is analysed. Effective medium theory is adopted to approximate the overall Young's modulus and dielectric permittivity of the GPLs reinforced composite. The Poisson's ratio and mass density of the composites are estimated by rule of mixture. Based on Timoshenko beam theory, governing equations for beam vibration are established by using Hamilton's principle and nonlinear von Kármán strain–displacement relationship. Numerical solution to the governing equations is obtained through differential quadrature method. The effects of GPL concentration and size, and the electrical voltage and AC (alternating current) frequency upon the nonlinear vibration of the GPL reinforced composite beam are investigated. The results demonstrate that there exists a threshold for GPL weight fraction in the polymer matrix, above which the electrical field plays a dominant role on the vibration behaviours. Increasing the voltage of the electrical field will enhance the ratio of nonlinear frequency to linear frequency. A transition region for the AC frequency is observed, within which the vibration characteristics varies dramatically. The analysis conducted in present work is envisaged to provide guidelines for designing GPL reinforced smart composites and structures.

Keywords: nonlinear vibration, differential quadrature method, graphene platelets, dielectric beam

1. Introduction

Owing to the increasing demand in developing smart materials and structures, extensive efforts have been devoted into studying the materials with high dielectric permittivity/constant (high- κ materials) [1–3]. Among the material candidates interested, certain polymers, i.e. dielectric elastomer, have demonstrated great potentials for engineering applications due to their tunable material properties, flexibility, easy processing and biological compatibility. Considerable work, including theoretical and experimental work [4–6], has demonstrated

such polymers' unique features in engineering applications. Unfortunately, these polymers normally have relatively low dielectric permittivity, i.e. below 10, which significantly restricts the polymer's performances and applications. To intensify these polymers' dielectric permittivity/constant, an effective solution is to introduce conductive particles into the polymer matrix. For example, carbon black [7, 8] and carbon nanotubes [9–11] have been widely used as typical additives in the polymer matrix for developing high- κ composites.

Recently, graphene and its derivatives have attracted great attentions from academic and industrial communities to develop high performance polymer composites, including the ones with high dielectric permittivity/constant [12–14].

⁴ Author to whom any correspondence should be addressed.

Polymer composites enhanced by graphene possess both excellent mechanical properties and impressive electrical and dielectric properties. For example, extensive theoretical and experimental work has demonstrated the mechanical performances of these composites can be remarkably enhanced by adding graphene fillers with low concentration [15–21]. At the same time, graphene can also substantially improve the dielectric properties of these composites. Fan *et al* [13] manufactured highly flexible graphene/polyvinylidene fluoride (PVDF) nanocomposites and observed that the lowest percolation threshold is 0.18 vol%. With 1.77 vol% of graphene filler, the dielectric constant of the composites can reach up to 7940 at 100 Hz of AC (alternating current) frequency. He *et al* [22] manufactured graphene platelet (GPL) reinforced PVDF nanocomposites and demonstrated that with 1.01 vol% loading of GPL, the dielectric constant is more than 200 and 2700 at 1000 Hz and 100 Hz of AC frequency, respectively. Chen *et al* [12] prepared graphene/polymer composites and observed that the dielectric constant is five times larger than pure polymer. Xia *et al* [23] developed a theoretical model to estimate the dielectric properties of polymer composites reinforced with graphene and evidenced significant improvement in the dielectric permittivity.

In addition to the work on composite materials, performances of structures made of polymer composites enhanced by graphene and its derivatives have been extensively explored. Kitipornchai *et al* [24] analysed the vibration and buckling behaviours of GPL reinforced functionally graded (FG) porous beams. Feng *et al* [25, 26] theoretically researched the vibration and bending of GPL/polymer composite beams considering geometry nonlinearity. Yang *et al* [27] investigated the buckling behaviours of beams made of FG GPL reinforced composite with multilayers in the thickness direction. Kiani *et al* [28–32] explored the stability and dynamic performances of graphene reinforced composite structures in thermal environment. Dynamic instability of graphene reinforced FG beams in thermal environment is studied by Wu *et al* [33]. Shen's team [34, 35] explored the buckling, bending and vibration of graphene reinforced laminated beams and plates. Arani *et al* [36–38] analysed the structural response of graphene sheets under longitudinal magnetic field. Zhao *et al* [39] and Wang *et al* [40, 41] conducted finite element analysis on the bending, vibration and buckling behaviours of GPL/polymer composite plates and cylindrical shells, respectively.

Although plenty of work is carried out on graphene and its derivatives reinforced composite materials and structures, most of previous studies are either concentrated on characterizing materials properties or on structural analysis without considering the intrinsic dielectric property of the composites. However, for engineering application of high- κ composites as smart structures, it is of necessity to examine the structural behaviours of the dielectric components subjected to electric field whereas limited studies have been found on such topic. In present work, free vibration of GPL/polymer dielectric beam subjected to applied electrical voltage (as shown in figure 1) will be investigated with the consideration of geometry nonlinearity, in which the dimensions of the beam are denoted as L , b and h ,

respectively. For case study, PVDF is selected to be the polymer matrix and GPLs as the reinforcing fillers for the composite material of the beam. Based on Timoshenko beam theory, governing equations for the beam are established and solved numerically. To accommodate the electrical field, compliant electrodes are applied on sides of the beam. Since the Young's modulus and mass of the electrodes are much smaller than these of the beam, the effects of the electrode on the vibration behaviours can be neglected. Through parametric study, the effects of several influencing factors on the nonlinear free vibration behaviours of the GPL/PVDF dielectric beam are identified.

2. Effective material properties of GPL/polymer composites

2.1. Effective medium theory (EMT)

To carry out structural analysis of GPL/polymer composite beam with considering dielectric properties, the effective mechanical and physical properties (i.e. tensile modulus, dielectric permittivity, mass density and Poisson's ratio) need to be known parameters. For mechanical properties of the composites, Halpin–Tsai has been justified as an effective method with accuracy to approximate the overall tensile modulus of polymer composites reinforced by GPLs [20]. However, it is challenging to adopt Halpin–Tsai model to predict the dielectric property of the composites, attributed to the difficulty in incorporating electrical field. In contrast, EMT is capable of predicting mechanical properties as well as integrating electrical field into the model for the dielectric permittivity. Therefore, EMT is used to predict material properties of GPL reinforced polymer composites in present work.

For multi-phase composites with uniform dispersion of reinforcing fillers, Maxwell's approach [42] can be adopted to get expressions of EMT. This approach assumes that the field of homogenous effective medium, i.e. GPL reinforced polymer composites, is equivalent to the sum of all far fields of scattered constituent components, which can be expressed as

$$\mathbf{T}_e = \sum_{i=1}^n \varphi_i \mathbf{T}_i, \quad (1)$$

where $\mathbf{T}_e$ is the field of effective medium, and φ_i and $\mathbf{T}_i$ are the volume fraction and field of i th scattered constituent component at far distance, respectively. The field $\mathbf{T}_i$ in a reference medium can be written as $\mathbf{T}_i = [(\mathbf{L}_i - \mathbf{L}_r)^{-1} + \mathbf{S}_i \mathbf{L}_r^{-1}]^{-1}$, in which $\mathbf{S}_i$ represents the Eshelby's tensor for i th constituent component, and $\mathbf{L}_i$ and $\mathbf{L}_r$ are the moduli tensors of the i th constituent and the reference medium, respectively. The moduli tensors here can be tensors of stiffness, electrical conductivity or dielectric permittivity depending on the problem involved.

Choosing the effective medium itself as the reference medium, Maxwell's approach becomes the EMT and equation (1) reduces to

$$\sum_{i=1}^n \varphi_i [(\mathbf{L}_i - \mathbf{L}_e)^{-1} + \mathbf{S}_i \mathbf{L}_e^{-1}]^{-1} = 0, \quad (2)$$

Figure 1. Schematic configuration of graphene platelets reinforced beam.

Figure 2. GPL filler with local Cartesian coordinates.

where L_e is the modulus tensor for the effective medium. For effective medium reinforced with randomly distributed fillers, equation (2) becomes

$$\sum_{i=1}^n \varphi_i \langle [(\mathbf{L}_i - \mathbf{L}_e)^{-1} + \mathbf{S}_i \mathbf{L}_e^{-1}]^{-1} \rangle = 0, \quad (3)$$

where $\langle \rangle$ denotes term averaging over all orientations of the fillers in the effective medium. For GPL/polymer composites, equation (3) is simplified as

$$\varphi_m [(\mathbf{L}_m - \mathbf{L}_e)^{-1} + \mathbf{S}_m \mathbf{L}_e^{-1}]^{-1} + \varphi_f \langle [(\mathbf{L}_f - \mathbf{L}_e)^{-1} + \mathbf{S}_f \mathbf{L}_e^{-1}]^{-1} \rangle = 0. \quad (4)$$

The subscripts 'm' and 'f' in equation (4) refer to the polymer matrix and reinforcing fillers, respectively. After averaging over all orientations, equation (4) becomes a scalar equation as

$$\varphi_m \frac{L_m - L_e}{L_m + (1/3)(L_m - L_e)} + \frac{1}{3} \varphi_f \sum_{k=1}^3 \frac{L_k - L_e}{L_e + S_{kk}(L_k - L_e)} = 0, \quad (5)$$

where L_k ($k = 1, 2, 3$) is the moduli component of fillers in the k th directions in their local coordinate system, i.e. GPL filler as shown in figure 2.

2.2. Effective dielectric permittivity

Equation (5) holds true for complex electrical conductivity of the effective medium. Therefore, one can have

$$\varphi_m \frac{\sigma_m^* - \sigma_e^*}{\sigma_e^* + (1/3)(\sigma_m^* - \sigma_e^*)} + \frac{1}{3} \varphi_f \sum_{k=1}^3 \frac{\sigma_k^* - \sigma_e^*}{\sigma_e^* + S_{kk}(\sigma_k^* - \sigma_e^*)} = 0 \quad (6)$$

where σ_m^* and σ_e^* represent the complex electrical conductivities of the polymer and effective medium, respectively, and σ_k^* represents complex electrical conductivity of the reinforcing filler in k th direction. In general, the complex electrical conductivity σ^* has two components, i.e. $\sigma^* = \sigma + \omega_{AC} \varepsilon j$ where $\omega_{AC} = 2\pi f_{AC}$, where f_{AC} and ε are the AC frequency in Hertz and dielectric permittivity of the composites, respectively. For the complex electrical conductivity, the real part is the so-called DC (direct current) conductivity and the imaginary part is the AC conductivity, which is dependent on the frequency of applied electrical field.

In present study, treating GPL as thin oblate spheroid, the elements of its Eshelby's tensor are expressed as [43]

$$S_{11} = S_{22} = \frac{\alpha}{2(1 - \alpha^2)^{3/2}} [\arccos \alpha - \alpha(1 - \alpha^2)^{1/2}]$$

$$S_{33} = 1 - 2S_{22}, \quad (7)$$

where $\alpha = t_{GPL}/D_{GPL}$ denotes the thickness-to-diameter ratio of the GPL. t_{GPL} and D_{GPL} are the thickness and diameter of the GPL, respectively.

When GPLs are dispersed in polymer matrix, the imperfect bonding/interaction between the fillers and the matrix can deteriorate the material performances of the composites. To include the influences of such imperfect bonding, a surrounding interlayer (as shown in figure 2) is introduced to coat the GPL filler. Assuming $\sigma_0^{(int)}$ and $\varepsilon_0^{(int)}$ are the electrical conductivity and dielectric permittivity of the interlayer, respectively, the effective material properties of the GPL filler with interlayer are [23, 44]

$$\begin{cases} \sigma_k^{(c)} = \sigma_0^{(int)} \left[1 + \frac{(1 - \varphi_{int})(\sigma_k - \sigma_0^{(int)})}{\varphi_{int} S_{kk}(\sigma_k - \sigma_0^{(int)}) + \sigma_0^{(int)}} \right] \\ \varepsilon_k^{(c)} = \varepsilon_0^{(int)} \left[1 + \frac{(1 - \varphi_{int})(\varepsilon_k - \varepsilon_0^{(int)})}{\varphi_{int} S_{kk}(\varepsilon_k - \varepsilon_0^{(int)}) + \varepsilon_0^{(int)}} \right] \end{cases} \quad k = 1, 2, 3, \quad (8)$$

where $\varepsilon_k^{(c)}$ and $\sigma_k^{(c)}$ are the dielectric permittivity and electrical conductivity of the coated GPL, respectively, which will

substitute σ_k , ε_k in equation (6). φ_{int} represents the volume fraction of the interlayer, which is derived as

$$\varphi_{\text{int}} = 1 - \frac{\frac{4}{3}\pi\left(\frac{t_{\text{GPL}}}{2}\right)\left(\frac{D_{\text{GPL}}}{2}\right)\left(\frac{D_{\text{GPL}}}{2}\right)}{\frac{4}{3}\pi\left(\frac{t_{\text{GPL}}}{2} + t_{\text{int}}\right)\left(\frac{D_{\text{GPL}}}{2} + t_{\text{int}}\right)\left(\frac{D_{\text{GPL}}}{2} + t_{\text{int}}\right)}, \quad (9)$$

where t_{int} is the thickness of the interphase.

Considering the electron tunnelling together with Maxwell–Wagner–Sillars polarization in interlayer, $\sigma_0^{(\text{int})}$ and $\varepsilon_0^{(\text{int})}$ in equation (8) are modified as [23]

$$\begin{cases} \sigma_{\text{static}}^{(\text{int})} = \sigma_0^{(\text{int})} / \tau(\varphi_f, \varphi_f^*, \gamma_{\text{static}}^\sigma) \\ \varepsilon_{\text{static}}^{(\text{int})} = \varepsilon_0^{(\text{int})} / \tau(\varphi_f, \varphi_f^*, \gamma_{\text{static}}^\varepsilon) \end{cases}, \quad (10)$$

where $\tau(\varphi_f, \varphi_f^*, \gamma_{\text{static}}^\sigma)$ is a resistance like function, φ_f^* denotes the percolation threshold of the GPL/polymer composites, $\gamma_{\text{static}}^\sigma$ and $\gamma_{\text{static}}^\varepsilon$ are scale factors characterizing the electron tunnelling and nanocapacitors formation in the composites at static state, respectively. According to previous studies [23, 44–46], the function $\tau(\varphi_f, \varphi_f^*, \gamma_{\text{static}}^\sigma)$ can be determined as

$$\tau(\varphi_f, \varphi_f^*, \gamma) = \frac{F(1, \varphi_f^*, \gamma) - F(\varphi_f, \varphi_f^*, \gamma)}{F(1, \varphi_f^*, \gamma) - F(0, \varphi_f^*, \gamma)}, \quad (11)$$

where $F(\varphi_f, \varphi_f^*, \gamma) = \frac{1}{\pi} \arctan\left(\frac{\varphi_f - \varphi_f^*}{\gamma}\right) + \frac{1}{2}$ is the so-called Cauchy's cumulative probabilistic function.

Experimental study also reveals that electrical conductivity together with dielectric permittivity depends on AC frequency of the applied electrical field. To consider such dependence, $\sigma_{\text{static}}^{(\text{int})}$ and $\varepsilon_{\text{static}}^{(\text{int})}$ are further modified as functions of the electrical field frequency [23, 47], i.e.

$$\begin{cases} \sigma_{\text{frequency}}^{(\text{int})}(\omega) = \sigma_{\text{static}}^{(\text{int})} p(\omega) \\ \varepsilon_{\text{frequency}}^{(\text{int})}(\omega) = \varepsilon_{\text{inf}}^{(\text{int})} + \frac{\varepsilon_{\text{static}}^{(\text{int})} - \varepsilon_{\text{inf}}^{(\text{int})}}{1 + \omega^2 t_\varepsilon^2} \end{cases}, \quad (12)$$

where

$$\begin{cases} p(\omega) = \frac{\omega t_\sigma \arctan(\omega t_\sigma)}{[0.5 \ln(1 + (\omega t_\sigma)^2)]^2 + \arctan^2(\omega t_\sigma)}, \\ \varepsilon_{\text{inf}}^{(\text{int})} = \varepsilon_0^{(\text{int})} / \tau(\varphi_f, \varphi_f^*, \gamma_{\text{inf}}^\varepsilon) \end{cases}, \quad (13)$$

where t_σ and t_ε denote the characteristic time of electron tunnelling and relaxation time, respectively. $\varepsilon_{\text{inf}}^{(\text{int})}$ is the dielectric permittivity of the interphase when electric field frequency is infinite. $\gamma_{\text{inf}}^\varepsilon$ is the scale factor describing the formation of nanocapacitors at sufficiently large AC frequency.

To validate the above model, PVDF and GPLs are chosen as the polymer matrix and the reinforcing fillers, respectively. The parameters required for the above EMT to predict the dielectric properties are selected as the same ones in [22]. As verified by Xia *et al* [23], the dielectric constant obtained by employing the above EMT together with the available experimental data in [22]. Excellent agreement between the two results has been observed [23]. Unless stated otherwise, the parameters involved for parametric study thereafter will be chosen as the ones in table 1.

2.3. Effective Young's modulus

For mechanical properties, equation (5) is written as the following form

$$\varphi_m \frac{E_m - E_e}{E_e + (1/3)(E_m - E_e)} + \frac{1}{3} \varphi_f \sum_{k=1}^3 \frac{E_k - E_e}{E_e + S_{kk}(E_k - E_e)} = 0, \quad (14)$$

where E_e is the homogeneous medium's effective Young's modulus. Similarly, to include the influences of the interlayer, the corresponding Young's modulus of GPLs in equation (14) is modified as

$$E_k^{(\text{c})} = E_0^{(\text{int})} \left[1 + \frac{(1 - \varphi_{\text{int}})(E_k - E_0^{(\text{int})})}{\varphi_{\text{int}} S_{kk}(E_k - E_0^{(\text{int})}) + E_0^{(\text{int})}} \right] \quad k = 1, 2, 3, \quad (15)$$

where $E_k^{(\text{c})}$ denotes the effective Young's modulus of coated GPLs which will substitute E_k in equation (14) while $E_0^{(\text{int})}$ represents the Young's modulus of the interlayer. It is found that the effective Young's modulus of GPL reinforced polymer composite obtained from equation (14) agrees well with the experimental results reported in [48]. The Poisson's ratio and mass density for the composites as required for structural analysis could be evaluated with rule of mixture as

$$\begin{cases} \nu_e = \varphi_m \nu_m + \varphi_f \nu_f \\ \rho_e = \varphi_m \rho_m + \varphi_f \rho_f \end{cases}, \quad (16)$$

where ν_f and ν_m denote Poisson's ratios of the GPL filler and polymer matrix, respectively, and ρ_f and ρ_m are the corresponding mass densities.

3. Governing equations

For Timoshenko beam, the following expressions can be used as the displacement field

$$\begin{cases} \tilde{U}(x, z, t) = U(x, t) - z\Psi(x, t) \\ \tilde{W}(x, z, t) = W(x, t) \end{cases}, \quad (17)$$

where $U(x, t)$ and $W(x, t)$ represent the displacements of the neutral plane of the beam in longitudinal and transverse directions, respectively, $\Psi(x, t)$ denote the rotation of the cross-section of the beam and t is the time. The strains in the beam with considering geometry nonlinearity can be written as

$$\begin{cases} \tilde{\varepsilon}_{xx} = \frac{\partial \tilde{U}}{\partial x} + \frac{1}{2} \left(\frac{\partial \tilde{W}}{\partial x} \right)^2 = \frac{\partial U}{\partial x} + \frac{1}{2} \left(\frac{\partial W}{\partial x} \right)^2 - z \frac{\partial \Psi}{\partial x} \\ \tilde{\gamma}_{xz} = \frac{\partial \tilde{U}}{\partial z} + \frac{\partial \tilde{W}}{\partial x} = -\Psi + \frac{\partial W}{\partial x} \end{cases}. \quad (18)$$

Table 1. Parameters for the prediction of dielectric permittivity of GPL/polymer composites [22, 23].

Parameters	Values
Thickness of GPL, t_{GPL} (m)	5×10^{-8}
Diameter of GPL, D_{GPL} (m)	8.1967×10^{-6}
Electrical conductivity of PVDF, σ_m (S m^{-1})	3.5×10^{-9}
Dielectric permittivity of PVDF, ε_m (F m^{-1})	4.425×10^{-11}
In-plane electrical conductivity of GPL, σ_1, σ_2 (S m^{-1})	8.32×10^4
Out-plane electrical conductivity of GPL, σ_3 (S m^{-1})	8.32×10^1
In-plane dielectric permittivity of GPL, $\varepsilon_1, \varepsilon_2$ (F m^{-1})	1.3275×10^{-10}
Out-plane dielectric permittivity of GPL, ε_3 (F m^{-1})	8.89425×10^{-11}
Thickness of interlayer, t_{int} (m)	3×10^{-9}
Electrical conductivity of interlayer at $\varphi_f = 0$, $\sigma_0^{(\text{int})}$ (S m^{-1})	1.5×10^{-5}
Dielectric permittivity of interlayer $\varepsilon_0^{(\text{int})}$ at $\varphi_f = 0$ (F m^{-1})	2.655×10^{-11}
Scale factor for electronic tunnelling at static state, $\gamma_{\text{static}}^{\sigma}$	0.002
Scale factor for formation of nanocapacitors at static state, $\gamma_{\text{static}}^{\varepsilon}$	2.8×10^{-13}
Scale factor for formation of nanocapacitors at infinite frequency, $\gamma_{\text{static}}^{\varepsilon}$	1.5×10^{-7}
Characteristic time of electron tunnelling, t_{σ} (s)	10^{-4}
Relaxation time of Debye theory, t_{ε} (s)	2×10^{-3}

The normal stress and shear stress are obtained as [49]

$$\begin{cases} \tilde{\sigma}_{xx} = \tilde{\sigma} + \frac{E}{1-\nu^2} \tilde{\varepsilon}_{xx} \\ \tilde{\sigma}_{xx} = \tilde{\sigma} + \frac{E}{1-\nu^2} \left[\frac{\partial U}{\partial x} + \frac{1}{2} \left(\frac{\partial W}{\partial x} \right)^2 - z \frac{\partial \Psi}{\partial x} \right] \\ \tilde{\tau}_{xz} = \frac{E}{1-\nu^2} \frac{1-\nu}{2} \tilde{\gamma}_{xz} = \frac{E}{1-\nu^2} \frac{1-\nu}{2} \left(-\Psi + \frac{\partial W}{\partial x} \right) \end{cases}, \quad (19)$$

where E and ν are the tensile modulus and Poisson's ratio and Young's modulus of the material, respectively. The axial stress $\tilde{\sigma}$ in the beam as used in equation (19) can be expressed as [50, 51]

$$\tilde{\sigma} = \tilde{\sigma}_0 - \varepsilon \frac{V^2}{h^2}, \quad (20)$$

where $\tilde{\sigma}_0$ is the pre-applied tensile stress in the axial direction of the beam, V and ε are the electrical voltage applied and dielectric permittivity of the material. On right-hand side of equation (20), the second term denotes the electrostatic stress induced by the applied electrical field. Here it should be noted the applied electrical field across the beam thickness normally has two components, i.e.

$$V = V_{\text{DC}} + V_{\text{AC}} \sin(\omega_{\text{AC}} t), \quad (21)$$

where V_{DC} and V_{AC} are the amplitudes of DC and AC voltage, respectively. Since the applied voltage V_{AC} is much smaller compared to V_{DC} . Under such circumstances, the effects of V_{AC} on the induced electrostatic stress can be neglected.

The normal force, bending moment and transverse shear force in the beam are derived as

$$\begin{cases} N_x = \int_{-h/2}^{h/2} \tilde{\sigma}_{xx} dz = A_{00} \tilde{\sigma} + A_{11} \left[\frac{\partial U}{\partial x} + \frac{1}{2} \left(\frac{\partial W}{\partial x} \right)^2 \right] - B_{11} \frac{\partial \Psi}{\partial x} \\ M_x = \int_{-h/2}^{h/2} \tilde{\sigma}_{xx} z dz = B_{00} \tilde{\sigma} + B_{11} \left[\frac{\partial U}{\partial x} + \frac{1}{2} \left(\frac{\partial W}{\partial x} \right)^2 \right] - D_{11} \frac{\partial \Psi}{\partial x} \\ Q_x = \int_{-h/2}^{h/2} \tilde{\tau}_{xz} dz = A_{55} \left(-\Psi + \frac{\partial W}{\partial x} \right) \end{cases}, \quad (22)$$

where A_{00} , B_{00} , A_{11} , B_{11} , D_{11} , A_{55} are stiffness components defined as

$$\begin{cases} \{A_{00}, B_{00}\} = \int_{-h/2}^{h/2} \{1, z\} dz \\ \{A_{11}, B_{11}, D_{11}\} = \int_{-h/2}^{h/2} \frac{E}{1-\nu^2} \{1, z, z^2\} dz \\ \{A_{55}\} = \int_{-h/2}^{h/2} \frac{E}{1-\nu^2} \frac{1-\nu}{2} k_s dz. \end{cases} \quad (23)$$

Then the virtual strain energy of the beam is determined as

$$\begin{aligned} \delta \Pi_S &= \int_V (\tilde{\sigma}_{xx} \delta \tilde{\varepsilon}_{xx} + \tilde{\tau}_{xz} \delta \tilde{\gamma}_{xz}) dV \\ &= \int_0^L \left\{ N_x \delta \left[\frac{\partial U}{\partial x} + \frac{1}{2} \left(\frac{\partial W}{\partial x} \right)^2 \right] - M_x \delta \frac{\partial \Psi}{\partial x} + Q_x \delta \left(-\Psi + \frac{\partial W}{\partial x} \right) \right\} dx. \end{aligned} \quad (24)$$

The kinetic energy of the beam is

$$\begin{aligned}\Pi_T &= \frac{1}{2} \int_V \left[\rho \left(\frac{\partial \tilde{U}}{\partial t} \right)^2 + \rho \left(\frac{\partial \tilde{W}}{\partial t} \right)^2 \right] dV \\ &= \frac{1}{2} \int_0^L \left\{ I_1 \left[\left(\frac{\partial U}{\partial t} \right)^2 + \left(\frac{\partial W}{\partial t} \right)^2 \right] \right. \\ &\quad \left. - 2I_2 \left(\frac{\partial U}{\partial t} \right) \left(\frac{\partial \Psi}{\partial t} \right) + I_3 \left(\frac{\partial \Psi}{\partial t} \right)^2 \right\} dx,\end{aligned}\quad (25)$$

where I_1, I_2, I_3 are inertial components defined as

$$\{I_1, I_2, I_3\} = \int_{-h/2}^{h/2} \rho \{1, z, z^2\} dz. \quad (26)$$

The governing equations of nonlinear free vibration for the beam can be obtained by applying the following Hamilton's principle

$$\int_0^t (\delta \Pi_S - \delta \Pi_T) dt = 0. \quad (27)$$

Combining equations (24), (25), (27) and setting the coefficients of the variations of the displacements, i.e. δU , δW and $\delta \Psi$, as zero yields

$$\begin{cases} \frac{\partial N_x}{\partial x} = I_1 \frac{\partial^2 U}{\partial t^2} - I_2 \frac{\partial^2 \Psi}{\partial t^2} \\ \frac{\partial \left(N_x \frac{\partial W}{\partial x} \right)}{\partial x} + \frac{\partial (Q_x)}{\partial x} = I_1 \frac{\partial^2 W}{\partial t^2} \\ \frac{\partial M_x}{\partial x} - Q_x = I_2 \frac{\partial^2 U}{\partial t^2} - I_3 \frac{\partial^2 \Psi}{\partial t^2} \end{cases} \quad (28)$$

Substituting equation (22) into (28) gives

$$\begin{cases} A_{11} \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial W}{\partial x} \frac{\partial^2 W}{\partial x^2} \right) - B_{11} \frac{\partial^2 \Psi}{\partial x^2} = I_1 \frac{\partial^2 U}{\partial t^2} - I_2 \frac{\partial^2 \Psi}{\partial t^2} \\ A_{00} \bar{\sigma} \frac{\partial^2 W}{\partial x^2} + A_{11} \left[\frac{\partial^2 U}{\partial x^2} \frac{\partial W}{\partial x} + \frac{\partial U}{\partial x} \frac{\partial^2 W}{\partial x^2} + \frac{3}{2} \left(\frac{\partial W}{\partial x} \right)^2 \frac{\partial^2 W}{\partial x^2} \right] \\ - B_{11} \left(\frac{\partial^2 \Psi}{\partial x^2} \frac{\partial W}{\partial x} + \frac{\partial \Psi}{\partial x} \frac{\partial^2 W}{\partial x^2} \right) + A_{55} \left(-\frac{\partial \Psi}{\partial x} + \frac{\partial^2 W}{\partial x^2} \right) \\ = I_1 \frac{\partial^2 W}{\partial t^2} \\ - B_{11} \left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial W}{\partial x} \frac{\partial^2 W}{\partial x^2} \right) + D_{11} \frac{\partial^2 \Psi}{\partial x^2} + A_{55} \left(-\Psi + \frac{\partial W}{\partial x} \right) \\ = -I_2 \frac{\partial^2 U}{\partial t^2} + I_3 \frac{\partial^2 \Psi}{\partial t^2} \end{cases} \quad (29)$$

The boundary conditions for clamped end (denote as C) can be given as

$$U = 0, \quad W = 0, \quad \Psi = 0. \quad (30)$$

The boundary conditions for hinged end (denote as H) can be given as

$$U = 0, \quad W = 0, \quad M_x = 0. \quad (31)$$

4. Solution

The governing equations can be cast in dimensionless form by introducing the following quantities

$$\begin{aligned}\xi &= \frac{x}{L}, \quad \{u, w\} = \frac{\{U, W\}}{h}, \quad \psi = \Psi, \\ \tau &= \frac{t}{L} \sqrt{\frac{A_{110}}{I_{10}}}, \quad \eta = \frac{L}{h} \\ \{a_{00}, a_{11}, a_{55}, b_{00}, b_{11}, d_{11}\} &= \left\{ \frac{A_{00}}{A_{110}}, \frac{A_{11}}{A_{110}}, \frac{A_{55}}{A_{110}}, \frac{B_{00}}{A_{110}h}, \right. \\ &\quad \left. \frac{B_{11}}{A_{110}h}, \frac{D_{11}}{A_{110}h^2} \right\} \\ \{\bar{I}_1, \bar{I}_2, \bar{I}_3\} &= \left\{ \frac{I_1}{I_{10}}, \frac{I_2}{I_{10}h}, \frac{I_3}{I_{10}h^2} \right\},\end{aligned}\quad (32)$$

where I_{10} and A_{110} are the corresponding values of I_1 and A_{11} the pure polymer beam, respectively, equation (29) becomes the following dimensionless form

$$\begin{cases} a_{11} \left(\frac{\partial^2 u}{\partial \xi^2} + \frac{1}{\eta} \frac{\partial w}{\partial \xi} \frac{\partial^2 w}{\partial \xi^2} \right) - b_{11} \frac{\partial^2 \psi}{\partial \xi^2} = \bar{I}_1 \frac{\partial^2 u}{\partial \tau^2} - \bar{I}_2 \frac{\partial^2 \psi}{\partial \tau^2} \\ a_{00} \bar{\sigma} \frac{\partial^2 w}{\partial \xi^2} + a_{11} \left[\frac{1}{\eta} \frac{\partial^2 u}{\partial \xi^2} \frac{\partial w}{\partial \xi} + \frac{1}{\eta} \frac{\partial u}{\partial \xi} \frac{\partial^2 w}{\partial \xi^2} + \frac{3}{2} \frac{1}{\eta^2} \left(\frac{\partial w}{\partial \xi} \right)^2 \frac{\partial^2 w}{\partial \xi^2} \right] \\ - b_{11} \left(\frac{1}{\eta} \frac{\partial^2 \psi}{\partial \xi^2} \frac{\partial w}{\partial \xi} + \frac{1}{\eta} \frac{\partial \psi}{\partial \xi} \frac{\partial^2 w}{\partial \xi^2} \right) + a_{55} \left(-\eta \frac{\partial \psi}{\partial \xi} + \frac{\partial^2 w}{\partial \xi^2} \right) = \bar{I}_1 \frac{\partial^2 w}{\partial \tau^2} \\ - b_{11} \left(\frac{\partial^2 u}{\partial \xi^2} + \frac{1}{\eta} \frac{\partial w}{\partial \xi} \frac{\partial^2 w}{\partial \xi^2} \right) + d_{11} \frac{\partial^2 \psi}{\partial \xi^2} + a_{55} \left(-\eta^2 \psi + \eta \frac{\partial w}{\partial \xi} \right) \\ = -\bar{I}_2 \frac{\partial^2 u}{\partial \tau^2} + \bar{I}_3 \frac{\partial^2 \psi}{\partial \tau^2} \end{cases} \quad (33)$$

The governing equations as shown in equation (33) will be solved by differential quadrature method (DQM). Applying DQM, the displacement functions and their derivatives can be discretized as

$$\begin{cases} \{u, w, \psi\} = \sum_{m=1}^N l_m(x) \{u_m, w_m, \psi_m\} \\ \left. \frac{\partial^k}{\partial \xi^k} \{u, w, \psi\} \right|_{\xi=\xi_i} = \sum_{m=1}^N c_{im}^{(k)} \{u_m, w_m, \psi_m\} \end{cases}, \quad (34)$$

where $\{u_m, w_m, \psi_m\}$ is the displacement vector at point $\xi = \xi_m$, $l_m(x)$ is the Lagrange interpolation polynomials and $c_{im}^{(k)}$ is the weighting coefficients of the k th derivative with respect to ξ at point $\xi = \xi_i$, and N denotes the total number of grid points distributed within domain $\xi \subseteq [0, 1]$ as required for DQM. In present work, the distribution of grid points

Table 2. Dimensionless linear natural frequency for different boundary conditions.

N	1st mode			2nd mode			3rd mode		
	C-C	C-H	H-H	C-C	C-H	H-H	C-C	C-H	H-H
7	0.278 79	0.134 45	0.056 01	2.246 82	1.369 40	0.844 15	8.205 48	6.088 01	3.978 89
9	0.278 98	0.134 72	0.055 97	2.006 93	1.358 00	0.868 55	7.191 94	5.564 04	4.202 01
11	0.278 98	0.134 72	0.055 97	2.006 70	1.353 81	0.865 76	7.197 38	5.543 11	4.155 55
13	0.278 98	0.134 72	0.055 97	2.006 63	1.353 95	0.865 85	7.195 85	5.542 93	4.158 56
15	0.278 98	0.134 72	0.055 97	2.006 63	1.353 95	0.865 85	7.195 90	5.542 93	4.158 42
17	0.278 98	0.134 72	0.055 97	2.006 63	1.353 95	0.865 85	7.195 90	5.542 93	4.158 43

follows

$$\xi_i = \frac{1}{2} \left[1 - \cos \frac{\pi(i-1)}{N-1} \right] \quad (i = 1, 2, \dots, N). \quad (35)$$

Incorporating equation (34) into (33) gives

$$\begin{cases} a_{11} \left(\sum_{m=1}^N c_{im}^{(2)} u_m + \frac{1}{\eta} \sum_{m=1}^N c_{im}^{(1)} w_m \sum_{m=1}^N c_{im}^{(2)} w_m \right) - b_{11} \sum_{m=1}^N c_{im}^{(2)} \psi_m = \bar{I}_1 \ddot{u}_i - \bar{I}_2 \ddot{\psi}_i \\ a_{00} \bar{\sigma} \sum_{m=1}^N c_{im}^{(2)} w_m + a_{11} \left[\frac{1}{\eta} \sum_{m=1}^N c_{im}^{(2)} u_m \sum_{m=1}^N c_{im}^{(1)} w_m + \frac{1}{\eta} \sum_{m=1}^N c_{im}^{(1)} u_m \sum_{m=1}^N c_{im}^{(2)} w_m + \frac{3}{2} \frac{1}{\eta^2} \left(\sum_{m=1}^N c_{im}^{(1)} w_m \right)^2 \sum_{m=1}^N c_{im}^{(2)} w_m \right] \\ - b_{11} \left(\frac{1}{\eta} \sum_{m=1}^N c_{im}^{(2)} \psi_m \sum_{m=1}^N c_{im}^{(1)} w_m + \frac{1}{\eta} \sum_{m=1}^N c_{im}^{(1)} \psi_m \sum_{m=1}^N c_{im}^{(2)} w_m \right) + a_{55} \left(-\eta \sum_{m=1}^N c_{im}^{(1)} \psi_m + \sum_{m=1}^N c_{im}^{(2)} w_m \right) = \bar{I}_1 \ddot{w}_i \\ - b_{11} \left(\sum_{m=1}^N c_{im}^{(2)} u_m + \frac{1}{\eta} \sum_{m=1}^N c_{im}^{(1)} w_m \sum_{m=1}^N c_{im}^{(2)} w_m \right) + d_{11} \sum_{m=1}^N c_{im}^{(2)} \psi_m + a_{55} \left(-\eta^2 \psi_i + \eta \sum_{m=1}^N c_{im}^{(1)} w_m \right) = -\bar{I}_2 \ddot{u}_i + \bar{I}_3 \ddot{\psi}_i \end{cases} \quad (36)$$

Correspondingly, the boundary conditions for clamped end become

$$u = 0, \quad w = 0, \quad \psi = 0. \quad (37)$$

The boundary conditions for hinged end are

$$\begin{aligned} u = 0, \quad w = 0, \quad b_{00} \eta + b_{11} \left[\sum_{m=1}^N c_{im}^{(1)} u_m + \frac{1}{2\eta} \left(\sum_{m=1}^N c_{im}^{(1)} w_m \right)^2 \right] \\ - d_{11} \sum_{m=1}^N c_{im}^{(1)} \psi_m = 0. \end{aligned} \quad (38)$$

Incorporating above boundary conditions into equation (36), the governing equation for the vibration of the composite beam is simplified as following

$$(\mathbf{K}_L + \mathbf{K}_{NL}) \mathbf{d} = \mathbf{M} \ddot{\mathbf{d}}, \quad (39)$$

where $\mathbf{K}_L$ and $\mathbf{K}_{NL}$ are the linear stiffness matrix and the non-linear stiffness matrix depending on the displacement vector $\mathbf{d} = \{\{u_i\}^T, \{w_i\}^T, \{\psi_i\}^T\}^T$, respectively, and $\mathbf{M}$ denote the mass matrix.

Using the direct iterative algorithm as discussed in [49], equation (39) can be numerically solved by the following steps: (1) extract eigenvalue and associated eigenvector of the linear vibration system by neglecting the nonlinear stiffness

matrix $\mathbf{K}_{NL}$; (2) scale up the obtained eigenvector till the vibration amplitude equals to a given value $w_{\max}$; (3) calculate the nonlinear stiffness matrix $\mathbf{K}_{NL}$ using the obtained eigenvector; (4) extract eigenvalue and associated eigenvector of the updated vibration system with stiffness matrix being

$\mathbf{K}_L + \mathbf{K}_{NL}$; (5) scale up the obtained eigenvector till the vibration amplitude equals to a given value $w_{\max}$; (6) repeat step (2) till the eigenvalue and eigenvector converge.

5. Results and discussion

5.1. Convergence and validation study

Table 2 tabulates the dimensionless linear natural frequency (the first three modes) of GPL/PVDF composite beams with different numbers of grid points and boundary conditions. The beam dimensions are fixed, i.e. $L = 10$ mm and $h = 0.5$ mm. No initial axial stress and external electrical field are applied on the beam. The concentration of GPL in the composites is 1.5 wt%. The total number of grid points increases from 7 to 17. As can be seen, more grid points are required to achieve convergent results for higher order modes. For example, convergent results are obtained at $N = 9$ for the 1st mode. 17 grid points are needed for convergent results for all boundary conditions for 3rd mode. To ensure convergence, 17 grid points will be used for parametric study thereafter.

Table 3. Comparison of the first three dimensionless linear natural frequencies of composite beams.

	Present			Feng [25]		
	C-C	C-H	H-H	C-C	C-H	H-H
1st mode	0.512 537 055	0.356 173 296	0.229 563 805	0.492 88	0.338 46	0.215 42
2nd mode	1.374 597 239	1.129 127 544	0.902 946 783	1.328 17	1.078 55	0.852 26
3rd mode	2.603 061 971	2.284 607 743	1.978 820 815	2.5325	2.198 67	1.882 92

Table 4. Comparison of ratio ω_{NL}/ω_L for various vibration amplitudes.

w_{max}/Θ	Present	Mei [52]	Raju <i>et al</i> [53]
0.0	1.0	1.0	1.0
1.0	1.0610	1.0613	1.0607
3.0	1.4531	1.4617	1.4573
5.0	1.9634	2.0378	2.0289

Comparison of first three dimensionless linear natural frequencies with various boundary conditions is presented in table 3. It is found that the natural frequencies obtained by the current model agree well with the ones by Feng [25].

To further validate the model and the solution procedure, as shown in table 4, the ratio ω_{NL}/ω_L of an H-H homogeneous beam with varying amplitude w_{max}/Θ is compared with the results obtained by Mei [52] and Raju *et al* [53]. ω_{NL} and ω_L are the dimensionless nonlinear and linear natural frequency, respectively. Θ is the radius of gyration of the beam with respect to y axis, which is given by $\Theta = \sqrt{I/A}$ with A being the area of cross-section and I being the moment of inertia of the beam, respectively. It is observed the present results match excellently with previously reported data.

5.2. Linear free vibration

Figure 3 investigates the dependence of dimensionless natural frequency ω_L on GPL weight fraction for a C-C beam. A pre-stress of 10 MPa is applied on the axial direction of the beam before applying boundary conditions on the two ends of the beam. In figure 3(a), the AC frequency of the electrical field is $f_{AC} = 0$ Hz, which indicates only DC electrical component is applied on the beam. The dimensionless natural frequency ω_L increases slightly as more GPLs are added into PVDF when GPL concentration is smaller than a critical value (percolation threshold), i.e. $f_{GPL} = 1.30$ wt%. When the GPL weight fraction is smaller than this critical concentration, the dielectric permittivity can be neglected, which does not induce obvious electrostatic stress. As a result, the dimensionless natural frequency is independent on the applied voltage. However, when the GPL weight fraction increases beyond the critical concentration, the dielectric permittivity increases dramatically with the increase of GPL weight fraction. Therefore, the dimensionless natural frequency becomes highly dependent on the electrical voltage. For example, when the applied voltage is 0 V, the dimensionless natural

frequency ω_L increases slightly from 0.556 14 to 0.636 65 when the GPL weight fraction increases from 1.3% to 2%. In contrast, when the applied voltage increases to 150 V, the dimensionless natural frequency ω_L undergoes a sharp decrease from 0.556 13 to 0.288 18. The main reason resides in the fact that when the GPL weight fraction is relatively small, the variation of ω_L is ascribed to the change of Young's modulus of the GPL/polymer composites. However, when the GPL concentration becomes sufficiently large, i.e. larger than the percolation threshold for dielectric properties, the dimensionless natural frequency will decrease significantly due to the significant drop in axial stress released by applied electrical field.

In figure 3(b), the applied voltage is fixed as 100 V while the AC frequency varies from 0 Hz and 10 MHz. A similar trend is found as observed in figure 3(a). Before a critical GPL concentration, the variation of the frequency of electrical field has limited effects on the vibration frequency. However, after the critical concentration, a snap-through phenomenon is observed for the vibration frequency. For example, when the electrical frequency $f_{AC} \leq 10$ Hz, the curves converge to form the lower limit for the dimensionless natural frequency. The curve for the vibration frequency jumps suddenly when the electrical frequency increases to 100. Then all curves converge again and form the upper limit when the electrical frequency $f_{AC} \geq 10^3$ Hz.

Figure 4 studies the dependence of the dimensionless natural frequency ω_L on GPL diameter-to-thickness ratio of a C-C beam subjected to electrical field with different AC frequencies. The thickness of GPLs is set as 50 nm and the GPL diameter-to-thickness ratio changes between 100 and 2000. The initial axial stress is set as $\sigma_0 = 10$ MPa. The applied voltage is fixed as 0 V, 25 V, 50 V and 75 V, respectively while the AC frequency varies. When the applied voltage is 0 V, the dimensionless natural frequency versus GPL diameter-to-thickness ratio curves overlap with each other. When the external electrical field is applied, these curves diverge as the GPL diameter-to-thickness ratio grows, indicating that the dimensionless natural frequency is more sensitive to AC frequency for beams reinforced by GPLs with larger diameter-to-thickness ratio. This phenomenon becomes more evident as the applied voltage enhances. It is also notable that when the AC frequency is sufficiently large, i.e. $f_{AC} \geq 10^3$ Hz, the curves converge and form the upper limit. The dimensionless natural frequency decreases as the AC frequency decreases. When AC frequency is sufficiently small, i.e. $f_{AC} \leq 10$ Hz, the curves converge again and forms

Figure 3. Effect of GPL concentration on dimensionless natural frequency ω_L : (a) $f_{\text{AC}} = 0$ Hz and (b) $V_{\text{DC}} = 100$ V.

Figure 4. Effect of GPL diameter-to-thickness ratio on dimensionless natural frequency ω_L of GPL/PVDF composite beam: (a) $V_{\text{DC}} = 0$ V, (b) $V_{\text{DC}} = 25$ V, (c) $V_{\text{DC}} = 50$ V and (d) $V_{\text{DC}} = 75$ V.

Figure 5. Effect of applied voltage on dimensionless natural frequency ω_L of GPL/PVDF composite beam.

the lower limit. When the voltage increases to 75 V (figure 4(d)), it is found the dimensionless natural frequency ratio first increases and then decreases when the AC frequency is small. This phenomenon could be explained by the competition of two influencing factors. As expected, the dimensionless natural frequency depends on elastic modulus of the composite. In the meantime, the dimensionless natural frequency is highly related to the axial stress, which depends on the dielectric property of the composite materials. The dielectric permittivity of the GPL/polymer composites, which is small at the beginning, increases dramatically and converges as the GPL aspect ratio increases. When the applied voltage is small, the change of Young's modulus dominates the value of dimensionless natural frequency. However, when the applied voltage is large enough, the variation of the natural frequency of the composite beam is the result of the combined effects of the two influencing factors.

Figure 5 plots the dimensionless natural frequency versus the electrical voltage for a C–C beam with different GPL concentrations. As can be seen, the dimensionless natural frequency weakly depends on the applied voltage when the GPL concentration is relatively small (i.e. 1.1, 1.2, 1.3 wt%). It is because the GPL/polymer nanocomposites have small dielectric permittivity as the GPL weight fraction is below the percolation threshold, causing limited electrostatic stress in the axial direction regardless of the magnitude of the voltage. The dimensionless natural frequency decreases significantly when the GPL concentration further increases. It is notable that the decreasing rate enhances as GPL concentration increases. For example, the dimensionless natural frequency decreases from 0.567 71 to 0.503 88 when the electrical voltage increases from the 0 to 400 V when the GPL concentration is 1.4 wt%. In comparison, the dimensionless natural frequency decreases from 0.579 29 to 0.221 47 when the electrical voltage increases from the 0 to 400 V when the GPL concentration is 1.5 wt%.

Figure 6. Free vibration response of GPL/PVDF composite beam with various amplitudes.

Figure 7. Frequency ratio versus dimensionless amplitude of GPL/PVDF composite beam with different boundary conditions.

5.3. Nonlinear free vibration

Figure 6 plots the free vibration response of a C–C beam, in which different maximum dimensionless amplitudes are considered. As expected, the curves with larger amplitude shift to left compared to the ones with smaller amplitude. This corresponds to the increase of natural frequency of beam with larger vibration amplitude due to nonlinear effects. Figure 7 shows the frequency ratio versus dimensionless amplitude curves of the GPL/PVDF composite beam. Three different boundary conditions, i.e. C–C, C–H and H–H, and two different initial axial stresses, i.e. $\sigma_0 = 0$ MPa and $\sigma_0 = 10$ MPa, are considered. Similar to the trend as observed in figure 6, the frequency ratio increases with dimensionless amplitude. As expected, the composite beams with fully constraints in both ends of beam and larger axial stress have smaller frequency ratio.

Figure 8. Effects of beam thickness on frequency ratio ω_{NL}/ω_L .

Figure 9. Effects of initial axial stress on frequency ratio ω_{NL}/ω_L .

As indicated in equation (21), the effect of the applied electrical field highly depends on the thickness of a C–C beam. Figure 8 shows the dependence of the ratio ω_{NL}/ω_L on the thickness of the beam. This ratio always increases with the thickness of the beam for lower electrical voltage (i.e. $V_{DC} = 0$ and 100 V). However, for higher voltage (i.e. $V_{DC} \geq 200$ V), the frequency ratio first decreases and then increases as the thickness of the beam increases. This can be attributed to the competition of the nonlinear effects from beam geometry and electrostatic axial stress. On one hand, it is well known that increasing the beam thickness will contribute to the nonlinearity (i.e. elongation of neutral plane). On the other hand, the increase of the beam thickness results in the decrease of the electrostatic stress released by electrical field and increase of the axial stress, making the beam stiffer. This factor is particularly obvious when the beam thickness is small. When the thickness of the beam is sufficiently large, this factor has limited influences on the nonlinearity and can be ignored. For smaller voltage, limited electrostatic axial

stress is released by the electrical field. Under such circumstance, the beam geometry is dominating the nonlinearity. Therefore, the frequency ratio increases with the thickness of the beam as expected. However, for higher voltage, the frequency ratio comprehensively depends on the geometry and electrostatic axial stress of the beam. As the thickness of the beam is relatively small, the effect of electrostatic axial stress outweighs the effect of beam geometry, leading to the decrease of the nonlinear frequency ratio. However, as the beam thickness further increases, the effect of the beam geometry overwhelms the effect of electrostatic axial stress. Under such circumstance, the nonlinearity of the beam once again is controlled by the geometry, which is indicated by the convergence of all curves.

Figure 9 shows the dependence of the ratio ω_{NL}/ω_L on the initial axial stress. The frequency ratio decreases with the initial axial stress. This agrees with the analysis for the observations in figure 8. Moreover, the increase of the electrical voltage increases with the frequency ratio as observed. It is because higher voltage will release more electrostatic axial stress and soften the beam, leading to the enhancement of nonlinearity of the beam.

Figure 10 plots the change of the ratio ω_{NL}/ω_L with GPL weight fraction for a C–C beam. An initial axial stress of 10 MPa is applied on the beam. In figure 10(a), the AC frequency of the electrical field is $f_{AC} = 0$ Hz, which indicates only DC electrical field is applied on the beam. The frequency ratio ω_{NL}/ω_L increases as more GPLs are added into polymer. It is found that there exists a critical GPL weight fraction in this figure. Below this critical concentration, the frequency ratio weakly depends on the GPL concentration regardless of applied voltage. In contrast, as the GPL weight fraction is greater than the critical concentration, the ratio ω_{NL}/ω_L dramatically increases with the applied voltage. For example, the frequency ratio has a sharp increase from 1.135 99 to 1.619 16 when the weight fraction varies from 1.3% to 2% when a voltage of 150 V is applied. In figure 10(b), the applied voltage is fixed while the AC frequency changes from 0 to 10^7 Hz. Similar to figure 10(a), the frequency ratio ω_{NL}/ω_L increases as the GPL weight fraction grows steadily for all AC frequency at first. Then there exist snap-through phenomenon and lower and upper limits for the frequency ratio as observed in linear free vibration.

Figure 11 investigates the influence of diameter-to-thickness ratio of GPL upon the frequency ratio ω_{NL}/ω_L of a C–C beam. The thickness of GPLs is chosen as $t_{GPL} = 50$ nm and the GPL diameter-to-thickness ratio increases from 100 to 2000. The initial axial stress is fixed as $\sigma_0 = 10$ MPa. Seen from figure 11(a), the frequency ratio ω_{NL}/ω_L increases when the diameter-to-thickness ratio increases. The increase of applied voltage enhances the increase of the frequency ratio, especially for smaller GPL diameter-to-thickness ratio and larger voltage. Similar to figure 11(a), the frequency ratio in figure 11(b) increases with the GPL diameter-to-thickness ratio for different AC frequency. The frequency ratio ω_{NL}/ω_L converges when the AC frequency $f_{AC} \leq 10$ Hz or $f_{AC} \geq 1000$ Hz. This indicates that the frequency ratio is very sensitive to the AC frequency around 100 Hz. A small

Figure 10. Effect of GPL weight fraction on frequency ratio ω_{NL}/ω_L : (a) $f_{AC} = 0$ Hz and (b) $V_{DC} = 100$ V.

Figure 11. Effect of GPL diameter-to-thickness ratio on frequency ratio ω_{NL}/ω_L of GPL/PVDF composite beam: (a) $f_{AC} = 0$ Hz and (b) $V_{DC} = 50$ V.

increase or decrease in the AC frequency will induce significant variation in the frequency ratio. The above observation suggests that the nonlinear vibration can be designed and actively tuned by changing the attributes of dispersed GPLs and the applied electrical fields.

The influence of applied electrical voltage on the frequency ratio ω_{NL}/ω_L of a C–C beam is displayed in figure 12. As demonstrated, the applied voltage has limited effects on the frequency ratio when the GPL concentration is relatively small, i.e. less than 1.3 wt%. This can be ascribed to the very small dielectric permittivity of the composites as the GPL weight fraction is smaller than the percolation threshold, resulting in limited electrostatic stress in the beam regardless of the magnitude of the voltage. When the GPL concentration further increases, the frequency ratio increases significantly. For example, this frequency ratio is increased from 1.139 24

to 1.783 58 when the applied voltage increases from 0 to 500 V for 1.5 wt%.

The influence of the AC frequency upon the ratio ω_{NL}/ω_L of a C–C beam is presented in figure 13. The initial axial stress is 10 MPa and the amplitude is set as $w_{max} = 1$. The applied electrical voltage is $V_{DC} = 300$ V with the AC frequency varies from 10^{-7} to 10^7 Hz. As can be observed, the frequency ratio is independent on the AC frequency when the GPL concentration is low, i.e. 1.1, 1.2 and 1.3 wt%. For higher GPL concentrations, i.e. 1.4 and 1.5 wt%, there exists a transition region (shaded area), within which the frequency ratio decreases substantially as the AC frequency increases. Within the transition region, the ratio ω_{NL}/ω_L is very sensitive to the variation of the AC frequency for beams with higher GPL concentration. The frequency ratio becomes less dependent when the AC frequency falls outside of the transition region.

Figure 12. Effect of applied voltage on frequency ratio ω_{NL}/ω_L of GPL/PVDF composite beam.

Figure 13. Effect of AC frequency on frequency ratio ω_{NL}/ω_L of GPL/PVDF composite beam.

6. Conclusions

Nonlinear free vibration behaviours of GPL/PVDF dielectric beam are comprehensively investigated. The Young's modulus and dielectric permittivity of GPL/PVDF composites are predicted using EMT with consideration of an interlayer outside GPL. Governing equations for beam vibration are established and numerically solved by DQM. The influences of several factors on the beam vibration are identified through parametric study, from which the following conclusions can be obtained:

- (1) The frequency ratio ω_{NL}/ω_L depends on both the GPL concentration and the applied electrical field. Adding more GPLs into PVDF increases the frequency ratio. However, there exists a critical GPL concentration. When the GPL weight fraction is smaller than this

critical concentration, the effect of applied electrical field can be ignored. Once the GPL weight fraction is greater than the critical value, the applied electrical field demonstrated prominent effect on nonlinear behaviours of the beam.

- (2) The increase of GPL diameter-to-thickness ratio is beneficial to the increase of the frequency ratio ω_{NL}/ω_L . The increase of this frequency ratio will be enhanced when the electrical voltage increases.
- (3) The AC frequency of electrical field has significant effects on nonlinear behaviours of the beam. There exists a transition region, within which the variation of the frequency induces significant change in the ratio ω_{NL}/ω_L . As the AC frequency goes beyond the transition region, the frequency ratio tends to be convergent.

Acknowledgments

The work conducted in present paper is fully supported by a research grant from the Australian Research Council under Discovery Early Career Researcher Award (DECRA) scheme (DE160100086). The authors also thank the financial support from the Australian Research Council under Discovery Project scheme (DP160101978).

Conflict of Interest

The authors declare no conflict of interests.

ORCID iDs

Chuang Feng <https://orcid.org/0000-0002-1879-1950>

Jie Yang <https://orcid.org/0000-0001-9147-1489>

References

- [1] Chen T, Qiu J H, Zhu K J, He X Y, Kang X and Dong E L 2014 Poly(methyl methacrylate)-functionalized graphene/polyurethane dielectric elastomer composites with superior electric field induced strain *Mater. Lett.* **128** 19–22
- [2] Liu S T, Tian M, Yan B Y, Yao Y, Zhang L Q, Nishi T and Ning N Y 2015 High performance dielectric elastomers by partially reduced graphene oxide and disruption of hydrogen bonding of polyurethanes *Polymer* **56** 375–84
- [3] Javey A, Kim H, Brink M, Wang Q, Ural A, Guo J, McIntyre P, McEuen P, Lundstrom M and Dai H 2002 High-kappa dielectrics for advanced carbon-nanotube transistors and logic gates *Nat. Mater.* **1** 241–6
- [4] Feng C, Jiang L Y and Lau W M 2011 Dynamic characteristics of a dielectric elastomer-based microbeam resonator with small vibration amplitude *J. Micromech. Microeng.* **21** 095002
- [5] Wissler M and Mazza E 2005 Modeling and simulation of dielectric elastomer actuators *Smart Mater. Struct.* **14** 1396–402

- [6] Feng C, Yu L and Zhang W 2014 Dynamic analysis of a dielectric elastomer-based microbeam resonator with large vibration amplitude *Int. J. Non-Linear Mech.* **65** 63–8
- [7] Selvi M, Devaraju S, Sethuraman K and Alagar M 2014 Carbon black-polybenzoxazine nanocomposites for highK dielectric applications *Polym. Compos.* **35** 2121–8
- [8] Calberg C, Blacher S, Gubbels F, Brouers F, Deltour R and Jerome R 1999 Electrical and dielectric properties of carbon black filled co-continuous two-phase polymer blends *J. Phys. D: Appl. Phys.* **32** 1517–25
- [9] Wang L and Dang Z M 2005 Carbon nanotube composites with high dielectric constant at low percolation threshold *Appl. Phys. Lett.* **87** 042903
- [10] Ameli A, Nofar M, Park C B, Potschke P and Rizvi G 2014 Polypropylene/carbon nanotube nano/microcellular structures with high dielectric permittivity, low dielectric loss, and low percolation threshold *Carbon* **71** 206–17
- [11] Ren J W, Yu D M, Feng L H, Wang G L and Lv G W 2017 Nanocable-structured polymer/carbon nanotube composite with low dielectric loss and high impedance *Composites A* **98** 66–75
- [12] Chen Y, Zhuang Q, Liu X, Liu J, Lin S and Han Z 2013 Preparation of thermostable PBO/graphene nanocomposites with high dielectric constant *Nanotechnology* **24** 245702
- [13] Fan P, Wang L, Yang J, Chen F and Zhong M 2012 Graphene/poly(vinylidene fluoride) composites with high dielectric constant and low percolation threshold *Nanotechnology* **23** 365702
- [14] Tian M, Zhang J, Zhang L, Liu S, Zan X, Nishi T and Ning N 2014 Graphene encapsulated rubber latex composites with high dielectric constant, low dielectric loss and low percolation threshold *J. Colloid Interface Sci.* **430** 249–56
- [15] Feng C, Wang Y, Kitipornchai S and Yang J 2017 Effects of reorientation of graphene platelets (GPLs) on Young's modulus of polymer nanocomposites under uni-axial stretching *Polymers* **9** 532
- [16] Skountzos E N, Anastassiou A, Mavrantzas V G and Theodorou D N 2014 Determination of the mechanical properties of a poly(methyl methacrylate) nanocomposite with functionalized graphene sheets through detailed atomistic simulations *Macromolecules* **47** 8072–88
- [17] Feng C, Wang Y and Yang J 2018 Effects of reorientation of graphene platelets (GPLs) on Young's modulus of polymer composites under bi-axial stretching *Nanomaterials* **8** 27
- [18] Spanos K N, Georgantzinos S K and Anifantis N K 2015 Mechanical properties of graphene nanocomposites: a multiscale finite element prediction *Compos. Struct.* **132** 536–44
- [19] Sun R, Li L L, Feng C, Kitipornchai S and Yang J 2018 Tensile behavior of polymer nanocomposite reinforced with graphene containing defects *Eur. Polym. J.* **98** 475–82
- [20] Rafiee M A, Rafiee J, Wang Z, Song H, Yu Z Z and Koratkar N 2009 Enhanced mechanical properties of nanocomposites at low graphene content *ACS Nano* **3** 3884–90
- [21] Ji X Y, Cao Y P and Feng X Q 2010 Micromechanics prediction of the effective elastic moduli of graphene sheet-reinforced polymer nanocomposites *Model. Simul. Mater. Sci. Eng.* **18** 045005
- [22] He F, Lau S, Chan H L and Fan J T 2009 High dielectric permittivity and low percolation threshold in nanocomposites based on poly(vinylidene fluoride) and exfoliated graphite nanoplates *Adv. Mater.* **21** 710
- [23] Xia X D, Wang Y, Zhong Z and Weng G J 2017 A frequency-dependent theory of electrical conductivity and dielectric permittivity for graphene-polymer nanocomposites *Carbon* **111** 221–30
- [24] Kitipornchai S, Chen D and Yang J 2017 Free vibration and elastic buckling of functionally graded porous beams reinforced by graphene platelets *Mater. Des.* **116** 656–65
- [25] Feng C, Kitipornchai S and Yang J 2017 Nonlinear free vibration of functionally graded polymer composite beams reinforced with graphene nanoplatelets (GPLs) *Eng. Struct.* **140** 110–9
- [26] Feng C, Kitipornchai S and Yang J 2017 Nonlinear bending of polymer nanocomposite beams reinforced with non-uniformly distributed graphene platelets (GPLs) *Composites B* **110** 132–40
- [27] Yang J, Wu H L and Kitipornchai S 2017 Buckling and postbuckling of functionally graded multilayer graphene platelet-reinforced composite beams *Compos. Struct.* **161** 111–8
- [28] Kiani Y and Mirzaei M 2018 Enhancement of nonlinear thermal stability of temperature dependent laminated beams with graphene reinforcements *Compos. Struct.* **186** 114–22
- [29] Kiani Y 2018 NURBS-based isogeometric thermal postbuckling analysis of temperature dependent graphene reinforced composite laminated plates *Thin Wall Struct.* **125** 211–9
- [30] Kiani Y 2018 Isogeometric large amplitude free vibration of graphene reinforced laminated plates in thermal environment using NURBS formulation *Comput. Methods Appl. Mech. Eng.* **332** 86–101
- [31] Kiani Y 2019 Buckling of functionally graded graphene reinforced conical shells under external pressure in thermal environment *Composites B* **156** 128–37
- [32] Mirzaei M and Kiani Y 2017 Isogeometric thermal buckling analysis of temperature dependent FG graphene reinforced laminated plates using NURBS formulation *Compos. Struct.* **180** 606–16
- [33] Wu H L, Yang J and Kitipornchai S 2017 Dynamic instability of functionally graded multilayer graphene nanocomposite beams in thermal environment *Compos. Struct.* **162** 244–54
- [34] Shen H S, Xiang Y and Lin F 2017 Nonlinear bending of functionally graded graphene-reinforced composite laminated plates resting on elastic foundations in thermal environments *Compos. Struct.* **170** 80–90
- [35] Shen H S, Xiang Y and Lin F 2017 Nonlinear vibration of functionally graded graphene-reinforced composite laminated plates in thermal environments *Comput. Methods Appl. Mech. Eng.* **319** 175–93
- [36] Arani A G, Haghparast E and Zarei H B 2016 Nonlocal vibration of axially moving graphene sheet resting on orthotropic visco-Pasternak foundation under longitudinal magnetic field *Physica B* **495** 35–49
- [37] Arani A G and Jalaei M H 2017 Investigation of the longitudinal magnetic field effect on dynamic response of viscoelastic graphene sheet based on sinusoidal shear deformation theory *Physica B* **506** 94–104
- [38] Jalaei M H and Arani A G 2018 Size-dependent static and dynamic responses of embedded double-layered graphene sheets under longitudinal magnetic field with arbitrary boundary conditions *Composites B* **142** 117–30
- [39] Zhao Z, Feng C, Wang Y and Yang J 2017 Bending and vibration analysis of functionally graded trapezoidal nanocomposite plates reinforced with graphene nanoplatelets (GPLs) *Compos. Struct.* **180** 799–808
- [40] Wang Y, Feng C, Zhao Z and Yang J 2018 Buckling of graphene platelet reinforced composite cylindrical shell with cutout *Int. J. Struct. Stab. Dyn.* **18** 1850040
- [41] Wang Y, Feng C, Zhao Z, Lu F Z and Yang J 2018 Torsional buckling of graphene platelets (GPLs) reinforced functionally graded cylindrical shell with cutout *Compos. Struct.* **197** 72–9
- [42] Weng G J 2010 A dynamical theory for the Mori–Tanaka and Ponte Castañeda–Willis estimates *Mech. Mater.* **42** 886–93

- [43] Taya M 2005 *Electronic Composites: Modeling, Characterization, Processing, and MEMS Applications* (Cambridge: Cambridge University Press)
- [44] Wang Y, Shan J W and Weng G J 2015 Percolation threshold and electrical conductivity of graphene-based nanocomposites with filler agglomeration and interfacial tunneling *J. Appl. Phys.* **118** 065101
- [45] Wang Y, Weng G J, Meguid S A and Hamouda A M 2014 A continuum model with a percolation threshold and tunneling-assisted interfacial conductivity for carbon nanotube-based nanocomposites *J. Appl. Phys.* **115** 193706
- [46] Hashemi R and Weng G J 2016 A theoretical treatment of graphene nanocomposites with percolation threshold, tunneling-assisted conductivity and microcapacitor effect in AC and DC electrical settings *Carbon* **96** 474–90
- [47] Dyre J C 1985 A simple-model of AC hopping conductivity in disordered solids *Phys. Lett. A* **108** 457–61
- [48] Layek R K, Samanta S, Chatterjee D P and Nandi A K 2010 Physical and mechanical properties of poly(methyl methacrylate)-functionalized graphene/poly(vinylidene fluoride) nanocomposites piezoelectric beta polymorph formation *Polymer* **51** 5846–56
- [49] Kitipornchai S, Ke L L, Yang J and Xiang Y 2009 Nonlinear vibration of edge cracked functionally graded Timoshenko beams *J. Sound Vib.* **324** 962–82
- [50] Dubois P, Rosset S, Niklaus M, Dadras M and Shea H 2008 Voltage control of the resonance frequency of dielectric electroactive polymer (DEAP) membranes *J. Microelectromech. Syst.* **17** 1072–81
- [51] Popov E P 1968 *Introduction to Mechanics of Solids* (New Jersey: Prentice-Hall)
- [52] Mei C 1973 Finite element displacement method for large amplitude free flexural vibrations of beams and plates *Comput. Struct.* **3** 163–74
- [53] Raju I S, Rao G V and Raju K K 1976 Effect of longitudinal or inplane deformation and inertia on large-amplitude flexural vibrations of slender beams and thin plates *J. Sound Vib.* **49** 415–22