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Buckling and Postbuckling of Dielectric Composite Beam Reinforced with Graphene Platelets (GPLs)

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Abstract

Buckling and postbuckling behaviours of graphene platelet (GPL) reinforced dielectric composite beams are investigated through theoretical formulation. The effective material properties of the GPL reinforced composite (GPLRC) as required for structural analysis, i.e. tensile modulus, dielectric constant and Poisson's ratio are obtained by using effective medium theory and rule of mixture. Governing differential equations for the composite beam are established through Timoshenko beam theory, von Kármán nonlinear strain-displacement relationship and principle of virtual work. Governing equations are numerically discretized and solved by employing differential quadrature method (DQM), through which several parameters affecting the buckling performances are quantitatively identified. The results demonstrate there exists a critical GPL concentration, above which the electrical field significantly affects the beam's buckling and postbuckling behaviours. The dielectric beam's buckling performances are very sensitive to AC (alternating current) frequency within a certain range. Moreover, it is found the dielectric beam's buckling and postbuckling performances comprehensively depend on the concentration and aspect ratio of GPLs. The present work is envisaged to provide guidelines to develop GPL-based smart composites and structures.

Keywords: Buckling; Graphene platelet; Composite beam; Dielectric property

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1. Introduction

Taking advantages of the excellent mechanical and physical properties of graphene and its derived materials, such as graphene platelet (GPL) and graphene oxide (GO), to develop high-performance and multifunctional composites and structures have attracted tremendous attention [1, 2]. Adding a small amount of the reinforcing fillers into a matrix can considerably enhance the mechanical performances of the composites. For example, Rafiee et al. [3] observed an increase of 31% in tensile modulus of epoxy composites with a loading of 0.1 wt% GPL by experiment. The ultimate strength and Young's modulus of PVA (Polyvinyl alcohol) nanocomposites fabricated by Zhao et al. [4] were increased by 150% and 10-folds with a dispersion of 1.8 vol% fully exfoliated graphene. Using molecular dynamics (MD) simulation, Rahman et al. [5] concluded the tensile modulus and shear modulus of graphene/epoxy composites can be improved by 24-40% with 1-3 wt% concentration compared to the ones of pure epoxy. Ji et al.'s [6] micromechanics modelling results also revealed that a small amount of graphene can obviously promote the effective elastic performances of the composites. Spanos et al.'s [7] multiscale finite element model evidenced improved mechanical performances of graphene reinforced composites are dependent on geometry and volume fraction of the fillers. More experimental and theoretical work on graphene reinforced composites can be found in [8-16].

Apart from mechanical properties, composites possessing excellent dielectric properties have shown great potential to develop smart materials and structures. Experimental and theoretical work has demonstrated embedding graphene nanofillers within polymer is an elegant solution to develop composites with increased dielectric properties. He et al.'s [17] experiments indicate that with a loading of 1.01 vol% of GPL, the dielectric constant of PVDF (polyvinylidene fluoride) composites can reach up to 200 and 2700, respectively, when the AC (alternating current) frequency is 1000 Hz and 100 Hz. Fan et al.'s [18] manufactured graphene/PVDF nanocomposite has a dielectric constant up to 7940 when the AC frequency is 100 Hz and the graphene concentration is 0.18 vol%. Xia et al.'s [19] developed theoretical model estimated the dielectric properties of polymer composites and evidenced that graphene significantly increased the dielectric permittivity. More work on dielectric properties of graphene reinforced polymer composites can refer to [20-23]. Apart from characterizing effective material properties, considerable efforts have been put on studying performances of various structural components reinforced by graphene, such as beam [24-27], plate [28-30] and cylindrical shell [31-34] and panel [35, 36]. The results suggest that the performances of the structures can be remarkably boosted by addition of a low concentration of graphene or its derivatives as reinforcing fillers.

As promising material candidates for developing smart structures, taking the intrinsic dielectric properties of graphene reinforced nanocomposite into account is essential. However, most previous studies are focused on analysis of structural behaviours with consideration of mechanical properties of the nanocomposites only. Wang and his co-workers' work[37, 38] investigated the static and dynamic behaviour of graphene reinforced composite beam with the consideration of dielectric permittivity. However, limited work has been found on the stability analysis incorporating the dielectric attributes of the graphene reinforced nanocomposites.

In present study, the buckling and postbuckling performances of a GPL reinforced composite (GPLRC) beam (as shown in Fig. 1) considering dielectric properties are investigated, in which polyvinylidene difluoride (PVDF) is selected as the polymer matrix. The length, width and thickness of the beam is L , b and h , respectively. To apply external electrical voltage, a thin layer of silver past can be coated on sides of the beam as the compliant electrodes. Since the mass and the stiffness of the silver past are much smaller than those of the beam, the effects of the compliant electrodes on the buckling behaviours of the structure will be neglected. To determine the parameters required for the structural analysis, effective-medium theory (EMT) is used for tensile modulus and dielectric permittivity of the GPL/PVDF composite while rule of mixture is employed for the approximation of Poisson's ratio. Through numerically solving governing equations, which are established on Timoshenko beam theory and von Kármán geometrical nonlinearity, parametric study on the buckling behaviour is conducted.

Fig. 1. Schematic configuration of GPLRC beam.

2. Governing Equations

The displacements of the beam can be given by Timoshenko beam theory as

$$\begin{cases} \tilde{U}(x, z, t) = U(x, t) - z\Psi(x, t) \\ \tilde{W}(x, z, t) = W(x, t) \end{cases} \quad (1)$$

where $U(x, t)$ and $W(x, t)$ are the displacements of the middle plane of the beam in x and z directions, respectively, $\Psi(x, t)$ is the cross section's rotation and t is time. Using von Kármán nonlinearity, the strains in the beam are

$$\begin{cases} \tilde{\varepsilon}_{xx} = \frac{\partial \tilde{U}}{\partial x} + \frac{1}{2} \left(\frac{\partial \tilde{W}}{\partial x} \right)^2 = \frac{\partial U}{\partial x} + \frac{1}{2} \left(\frac{\partial W}{\partial x} \right)^2 - z \frac{\partial \Psi}{\partial x} \\ \tilde{\gamma}_{xz} = \frac{\partial \tilde{U}}{\partial z} + \frac{\partial \tilde{W}}{\partial x} = -\Psi + \frac{\partial W}{\partial x} \end{cases} \quad (2)$$

The normal stress and shear stress are derived as

$$\begin{cases} \tilde{\sigma}_{xx} = \frac{E}{1-\nu^2} \tilde{\varepsilon}_{xx} = \frac{E}{1-\nu^2} \left[\frac{\partial U}{\partial x} + \frac{1}{2} \left(\frac{\partial W}{\partial x} \right)^2 - z \frac{\partial \Psi}{\partial x} \right] \\ \tilde{\tau}_{xz} = \frac{E}{1-\nu^2} \frac{1-\nu}{2} \tilde{\gamma}_{xz} = \frac{E}{1-\nu^2} \frac{1-\nu}{2} \left(-\Psi + \frac{\partial W}{\partial x} \right) \end{cases} \quad (3)$$

where E is the tensile modulus and ν is the Poisson's ratio of the material. Accordingly, the normal force, bending moment and transverse shear force in the beam are obtained as

$$\begin{cases} N_x = \int_{-h/2}^{h/2} \tilde{\sigma}_{xx} dz = A_{11} \left[\frac{\partial U}{\partial x} + \frac{1}{2} \left(\frac{\partial W}{\partial x} \right)^2 \right] - B_{11} \frac{\partial \Psi}{\partial x} \\ M_x = \int_{-h/2}^{h/2} \tilde{\sigma}_{xx} z dz = B_{11} \left[\frac{\partial U}{\partial x} + \frac{1}{2} \left(\frac{\partial W}{\partial x} \right)^2 \right] - D_{11} \frac{\partial \Psi}{\partial x} \\ Q_x = \int_{-h/2}^{h/2} \tilde{\tau}_{xz} dz = A_{55} \left(-\Psi + \frac{\partial W}{\partial x} \right) \end{cases} \quad (4)$$

where A_{11} , B_{11} , D_{11} , A_{55} are stiffness components given as

$$\begin{aligned}\{A_{11}, B_{11}, D_{11}\} &= \int_{-h/2}^{h/2} \frac{E}{1-\nu^2} \{1, z, z^2\} dz \\ \{A_{55}\} &= \int_{-h/2}^{h/2} \frac{E}{1-\nu^2} \frac{1-\nu}{2} k_s dz\end{aligned}\quad (5)$$

Then internal virtual work and external virtual work can be expressed as

$$\delta W_{\text{int}} = \int_V (\tilde{\sigma}_{xx} \delta \tilde{\varepsilon}_{xx} + \tilde{\tau}_{xz} \delta \tilde{\gamma}_{xz}) dV = \int_0^L \left\{ N_x \delta \left[\frac{\partial U}{\partial x} + \frac{1}{2} \left(\frac{\partial W}{\partial x} \right)^2 \right] - M_x \delta \frac{\partial \Psi}{\partial x} + Q_x \delta \left(-\Psi + \frac{\partial W}{\partial x} \right) \right\} dx \quad (6)$$

and

$$\delta W_{\text{ext}} = \delta \left\{ \frac{1}{2} \int_0^L \left[(P_m + P_e) \left(\frac{\partial W}{\partial x} \right)^2 \right] dx \right\} \quad (7)$$

where P_m is the axial loading applied on the beam per unit width. P_e in Eq. (7) is the axial loading per unit width induced by the through-thickness voltage determined as

$$P_e = \sigma_e h \quad (8)$$

where $\sigma_e = \varepsilon V^2 / h^2$ is the corresponding axial stress due to the dielectric property of the composites [39, 40], with ε , V and h being the dielectric permittivity of GPLRC, voltage of the electrical loading and beam thickness, respectively.

The governing equations for the buckling and postbuckling of the beam can be obtained by the principle of virtual work, i.e.

$$\delta W_{\text{int}} = \delta W_{\text{ext}} \quad (9)$$

Incorporating Eqs. (6) and (7) into Eq. (9) and setting the coefficients of δU , δW and $\delta \Psi$ as zero result in

$$\begin{cases} \frac{\partial N_x}{\partial x} = 0 \\ [N_x - (P_m + P_e)] \frac{\partial^2 W}{\partial x^2} + \frac{\partial Q_x}{\partial x} = 0 \\ \frac{\partial M_x}{\partial x} - Q_x = 0 \end{cases} \quad (10)$$

Substituting Eq. (4) into Eq. (10) gives

$$\begin{cases} A_{11} \frac{\partial^2 U}{\partial x^2} - B_{11} \frac{\partial^2 \Psi}{\partial x^2} + A_{11} \frac{\partial W}{\partial x} \frac{\partial^2 W}{\partial x^2} = 0 \\ A_{55} \frac{\partial^2 W}{\partial x^2} - A_{55} \frac{\partial \Psi}{\partial x} - (P_m + P_e) \frac{\partial^2 W}{\partial x^2} + A_{11} \frac{\partial U}{\partial x} \frac{\partial^2 W}{\partial x^2} + \frac{1}{2} A_{11} \left(\frac{\partial W}{\partial x} \right)^2 \frac{\partial^2 W}{\partial x^2} - B_{11} \frac{\partial \Psi}{\partial x} \frac{\partial^2 W}{\partial x^2} = 0 \\ -B_{11} \frac{\partial^2 U}{\partial x^2} + A_{55} \frac{\partial W}{\partial x} + D_{11} \frac{\partial^2 \Psi}{\partial x^2} - A_{55} \Psi - B_{11} \frac{\partial W}{\partial x} \frac{\partial^2 W}{\partial x^2} = 0 \end{cases} \quad (11)$$

The boundary conditions for clamped end (C) are given as

$$U = 0, W = 0, \Psi = 0 \quad (12)$$

The boundary conditions for hinged end (H) are given as

$$U = 0, W = 0, M_x = 0 \quad (13)$$

3. Material Properties of GPLRC

As seen from the governing equations, the effective properties of the composites need to be determined for structural analysis. In this following, rule of mixture and effective-medium theory (EMT) will be used for calculation of the effective materials properties of GPLRC as required.

3.1. Poisson's ratio

The Poisson's ratio of the GPLRC can be approximated via rule of mixture as

$$\nu_e = \varphi_m \nu_m + \varphi_f \nu_f \quad (14)$$

where the subscripts “e”, “m” and “f” denote the effective medium GPLRC, polymer matrix and GPL filler, respectively, ν is Poisson's ratio and φ represents volume fraction.

3.2. Tensile modulus and dielectric permittivity

Halpin-Tsai model was verified by Rafiee [3] as an effective method to predict the effective tensile modulus of GPLRC. However, this model can be hardly used to estimate the dielectric permittivity of GPLRC since it is difficult to incorporate the mechanisms of dielectric properties, including electron tunneling and Maxwell-Wagner-Sillars (MWS) polarization. Alternatively, EMT will be employed to estimate both the elastic and dielectric properties of the GPLRC.

For the two-phase GPLRC composites, the field of this homogeneous medium T_e can be estimated by using Maxwell's approach of far-field matching [41] as the summation of all far fields of scattered constituent components, i.e.

$$T_e = \sum_{i=1}^n \varphi_i T_i \quad (15)$$

where T_i and φ_i denote the field and the volume fraction of the i th scattered constituent component, respectively. Selecting a reference medium with modulus tensor being L_r , T_i can be written as

$$T_i = \left[(L_i - L_r)^{-1} + S_i L_r^{-1} \right]^{-1} \quad (16)$$

where L_i and S_i are the modulus and Eshelby's tensors of constituent i th component, respectively. Particularly, when the GPLRC itself is chosen as the reference medium, Maxwell's equation as shown in Eq. (15) becomes

$$\sum_{i=1}^n \varphi_i \left[(L_i - L_e)^{-1} + S_i L_e^{-1} \right]^{-1} = 0 \quad (17)$$

When GPLs are embedded in the matrix randomly and uniformly, Eq. (15) can be written as

$$\sum_{i=1}^n \varphi_i \left\langle \left[(L_i - L_e)^{-1} + S_i L_e^{-1} \right]^{-1} \right\rangle = 0 \quad (18)$$

where $\langle \rangle$ represents the average of the contribution of fillers over all orientations. For the two-phase GPL reinforced composites, Eq. (18) becomes

$$\varphi_m \left[(L_m - L_e)^{-1} + S_m L_e^{-1} \right]^{-1} + \varphi_f \left\langle \left[(L_f - L_e)^{-1} + S_f L_e^{-1} \right]^{-1} \right\rangle = 0 \quad (19)$$

Averaging over all orientations with considering symmetric attributes of the homogeneous medium, Eq. (19) is cast as a scalar equation, i.e.

$$\varphi_m \frac{L_m - L_e}{L_m + (1/3)(L_m - L_e)} + \frac{1}{3} \varphi_f \sum_{k=1}^3 \frac{L_k - L_e}{L_e + S_{kk} (L_k - L_e)} = 0 \quad (20)$$

where L_k is the component of the filler's modulus tensor in the k th direction.

Replacing L_m , L_e and L_k in Eq. (20) with the quantities of electrical properties, it becomes

$$\varphi_m \frac{\sigma_m^* - \sigma_e^*}{\sigma_e^* + (1/3)(\sigma_m^* - \sigma_e^*)} + \frac{1}{3} \varphi_f \sum_{k=1}^3 \frac{\sigma_k^* - \sigma_e^*}{\sigma_e^* + S_{kk} (\sigma_k^* - \sigma_e^*)} = 0 \quad (21)$$

where σ_m^* , σ_e^* and σ_k^* are the complex electrical conductivities of polymer, composites, and filler in the k th direction, respectively, and the subscript “*” denote the complex attribute. A complex electrical conductivity σ^* normally consists of two parts, i.e.

$$\sigma^* = \sigma + 2\pi f_{AC} \varepsilon j \quad (22)$$

with σ being DC (direct current) electrical conductivity and the constant of the imaginary part $2\pi f_{AC} \varepsilon$ being AC (alternating current) electrical conductivity. f_{AC} is AC frequency in Hertz and ε is the dielectric permittivity of the composite.

Taking GPLs as thin oblate spheroid, the components of its Eshelby's tensor are [42]

$$\begin{aligned} S_{11} = S_{22} &= \frac{\alpha}{2(1-\alpha^2)^{3/2}} \left[\arccos \alpha - \alpha(1-\alpha^2)^{1/2} \right] \\ S_{33} &= 1 - 2S_{22} \end{aligned} \quad (23)$$

where $\alpha = t_{GPL}/D_{GPL}$ with t_{GPL} and D_{GPL} being the thickness and diameter of the GPL, respectively.

To involve the imperfect bonding between GPLs and the polymer matrix, a surrounding interphase outside the GPL is usually introduced to form a coated GPL. Then the electrical conductivity and dielectric permittivity of the GPL in the k th direction, $\sigma_k^{(c)}$ and $\varepsilon_k^{(c)}$, are modified as [43]

$$\left\{ \begin{aligned} \sigma_k^{(c)} &= \sigma_0^{(int)} \left[1 + \frac{(1-\varphi_{int})(\sigma_k - \sigma_0^{(int)})}{\varphi_{int} S_{kk} (\sigma_k - \sigma_0^{(int)}) + \sigma_0^{(int)}} \right] \\ \varepsilon_k^{(c)} &= \varepsilon_0^{(int)} \left[1 + \frac{(1-\varphi_{int})(\varepsilon_k - \varepsilon_0^{(int)})}{\varphi_{int} S_{kk} (\varepsilon_k - \varepsilon_0^{(int)}) + \varepsilon_0^{(int)}} \right] \end{aligned} \right. \quad k = 1, 2, 3 \quad (24)$$

where $\sigma_0^{(int)}$ and $\varepsilon_0^{(int)}$ are the electrical conductivity and dielectric permittivity of the interphase, respectively. Then the quantities in Eq. (21) regarding the electrical properties of the GPL will be substituted by the ones in Eq. (24). The volume fraction of the introduced interphase in the coated GPL φ_{int} can be derived as

$$\varphi_{\text{int}} = 1 - \frac{\frac{4}{3}\pi\left(\frac{t_{\text{GPL}}}{2}\right)\left(\frac{D_{\text{GPL}}}{2}\right)\left(\frac{D_{\text{GPL}}}{2}\right)}{\frac{4}{3}\pi\left(\frac{t_{\text{GPL}}}{2} + t_{\text{int}}\right)\left(\frac{D_{\text{GPL}}}{2} + t_{\text{int}}\right)\left(\frac{D_{\text{GPL}}}{2} + t_{\text{int}}\right)} \quad (25)$$

where t_{int} is the thickness of the interphase.

Considering the interfacial tunneling and MWS polarization, the quantities $\sigma_0^{(\text{int})}$ and $\varepsilon_0^{(\text{int})}$ for the introduced interphase in Eq. (24) need to be modified by the following resistance-like function

$$\tau(\varphi_f, \varphi_f^*, \gamma) = \frac{F(1, \varphi_f^*, \gamma) - F(\varphi_f, \varphi_f^*, \gamma)}{F(1, \varphi_f^*, \gamma) - F(0, \varphi_f^*, \gamma)} \quad (26)$$

where φ_f^* is the percolation threshold of the GPLRC, $F(\varphi_f, \varphi_f^*, \gamma) = \frac{1}{\pi} \arctan\left(\frac{\varphi_f - \varphi_f^*}{\gamma}\right) + \frac{1}{2}$ is the so-called Cauchy's cumulative probabilistic function with γ being a scale factor denoting the variation of $F(\varphi_f, \varphi_f^*, \gamma)$ around the percolation threshold. Then electrical conductivity and dielectric permittivity of the interphase are written as

$$\begin{cases} \sigma_{\text{static}}^{(\text{int})} = \sigma_0^{(\text{int})} / \tau(\varphi_f, \varphi_f^*, \gamma_{\text{static}}^\sigma) \\ \varepsilon_{\text{static}}^{(\text{int})} = \varepsilon_0^{(\text{int})} / \tau(\varphi_f, \varphi_f^*, \gamma_{\text{static}}^\varepsilon) \end{cases} \quad (27)$$

where $\gamma_{\text{static}}^\sigma$ and $\gamma_{\text{static}}^\varepsilon$ are correspondingly scale factors and the subscript “static” denotes that the quantities involved are independent on AC frequency. Moreover, subjected to AC electrical loading, the electrical conductivity as well as dielectric permittivity of the composites also depends on AC frequency due to frequency facilitated electron tunneling and the formation of nanocapacitors. Therefore, these quantities can be further modified as functions of AC frequency as [19, 44]

$$\begin{cases} \sigma_{\text{frequency}}^{(\text{int})}(\omega_{\text{AC}}) = \sigma_{\text{static}}^{(\text{int})} p(\omega_{\text{AC}}) \\ \varepsilon_{\text{frequency}}^{(\text{int})}(\omega_{\text{AC}}) = \varepsilon_{\text{inf}}^{(\text{int})} + \frac{\varepsilon_{\text{static}}^{(\text{int})} - \varepsilon_{\text{inf}}^{(\text{int})}}{1 + \omega^2 t_\varepsilon^2} \end{cases} \quad (28)$$

where

$$\begin{cases} p(\omega) = \frac{\omega_{AC} t_\sigma \arctan(\omega_{AC} t_\sigma)}{\left[0.5 \ln \left(1 + (\omega_{AC} t_\sigma)^2\right)\right]^2 + \arctan^2(\omega_{AC} t_\sigma)} \\ \mathcal{E}_{inf}^{(int)} = \mathcal{E}_0^{(int)} / \tau(\varphi_f, \varphi_f^*, \gamma_{inf}^\varepsilon) \end{cases} \quad (29)$$

with t_σ being characteristic time of electron tunneling and t_ε being relaxation time. $\mathcal{E}_{inf}^{(int)}$ is dielectric permittivity of interphase at infinite AC frequency and γ_{inf}^ε is the corresponding scale factor at infinite AC frequency.

Selecting the same parameters in [17] as required, Xia et al. [19] verified that dielectric permittivity predicted by the effective-medium theory model agree excellently with the experimental results in [17]. Replacing the corresponding quantities in Eq. (20) with mechanical properties, the effective tensile modulus of the GPLRC, E_e , can be derived by

$$\varphi_m \frac{E_m - E_e}{E_e + (1/3)(E_m - E_e)} + \frac{1}{3} \varphi_f \sum_{k=1}^3 \frac{E_k - E_e}{E_e + S_{kk}(E_k - E_e)} = 0 \quad (30)$$

where E_k is the tensile modulus of the GPL in k th direction. Similar to the electrical property, the tensile modulus of the coated GPL is

$$E_k^{(c)} = E_0^{(int)} \left[1 + \frac{(1 - \varphi_{int})(E_k - E_0^{(int)})}{\varphi_{int} S_{kk}(E_k - E_0^{(int)}) + E_0^{(int)}} \right] \quad k = 1, 2, 3 \quad (31)$$

where $E_0^{(int)}$ is the tensile modulus of the interphase. The effective tensile modulus predicted by Eq. (29) agrees well with the experimental results [45].

4. Numerical Solution

Employing the dimensionless quantities below

$$\begin{aligned}\xi &= \frac{x}{L}, \quad \{u, w\} = \frac{\{U, W\}}{h}, \quad \psi = \Psi, \quad \{p_m, p_e\} = \left\{ \frac{P_m}{A_{110}}, \frac{P_e}{A_{110}} \right\} \\ \eta &= \frac{L}{h}, \quad \{a_{11}, a_{55}, b_{11}, d_{11}\} = \left\{ \frac{A_{11}}{A_{110}}, \frac{A_{55}}{A_{110}}, \frac{B_{11}}{A_{110}h}, \frac{D_{11}}{A_{110}h^2} \right\}\end{aligned}\quad (32)$$

where A_{110} is the value of A_{11} of pure polymer beam, Eq. (11) becomes the following dimensionless form

$$\begin{cases} a_{11} \frac{\partial^2 u}{\partial \xi^2} - b_{11} \frac{\partial^2 \psi}{\partial \xi^2} + \frac{a_{11}}{\eta} \frac{\partial w}{\partial \xi} \frac{\partial^2 w}{\partial \xi^2} = 0 \\ a_{55} \frac{\partial^2 w}{\partial \xi^2} - a_{55} \eta \frac{\partial \psi}{\partial \xi} - (p_m + p_e) \frac{\partial^2 w}{\partial \xi^2} + \frac{a_{11}}{\eta} \frac{\partial u}{\partial \xi} \frac{\partial^2 w}{\partial \xi^2} + \frac{1}{2} \frac{a_{11}}{\eta^2} \left(\frac{\partial w}{\partial \xi} \right)^2 \frac{\partial^2 w}{\partial \xi^2} - \frac{b_{11}}{\eta} \frac{\partial \psi}{\partial \xi} \frac{\partial^2 w}{\partial \xi^2} = 0 \\ -b_{11} \frac{\partial^2 u}{\partial \xi^2} + a_{55} \eta \frac{\partial w}{\partial \xi} + d_{11} \frac{\partial^2 \psi}{\partial \xi^2} - a_{55} \eta^2 \psi - \frac{b_{11}}{\eta} \frac{\partial w}{\partial \xi} \frac{\partial^2 w}{\partial \xi^2} = 0 \end{cases} \quad (33)$$

The governing equations as shown in Eq. (33) will be solved by differential quadrature method (DQM). Applying DQM, the displacement functions and their derivatives with respect to ξ can be discretized as

$$\begin{cases} \{u, w, \psi\} = \sum_{m=1}^N l_m(x) \{u_m, w_m, \psi_m\} \\ \left. \frac{\partial^k}{\partial \xi^k} \{u, w, \psi\} \right|_{\xi=\xi_i} = \sum_{m=1}^N c_{im}^{(k)} \{u_m, w_m, \psi_m\} \end{cases} \quad (34)$$

where $\{u_m, w_m, \psi_m\}$ is the displacement vector at point $\xi = \xi_m$, $l_m(x)$ is the Lagrange interpolation polynomials and $c_{im}^{(k)}$ is the weighting coefficients of the k -th derivative with respect to ξ at point $\xi = \xi_i$, and N denotes the total number of grid points distributed in the domain $\xi \in [0, 1]$. In present work, the distribution of grid points follows [46]

$$\xi_i = \frac{1}{2} \left[1 - \cos \frac{\pi(i-1)}{N-1} \right] \quad (i = 1, 2, \dots, N) \quad (35)$$

Incorporating Eq. (34) into Eq. (33) gives

$$\begin{cases} a_{11} \sum_{m=1}^N c_{im}^{(2)} u_m - b_{11} \sum_{m=1}^N c_{im}^{(2)} \psi_m + \frac{a_{11}}{\eta} \sum_{m=1}^N c_{im}^{(1)} w_m \sum_{m=1}^N c_{im}^{(2)} w_m = 0 \\ a_{55} \sum_{m=1}^N c_{im}^{(2)} w_m - a_{55} \eta \sum_{m=1}^N c_{im}^{(1)} \psi_m - (p_m + p_e) \sum_{m=1}^N c_{im}^{(2)} w_m + \frac{a_{11}}{\eta} \sum_{m=1}^N c_{im}^{(1)} u_m \sum_{m=1}^N c_{im}^{(2)} w_m + \frac{1}{2} \frac{a_{11}}{\eta^2} \left(\sum_{m=1}^N c_{im}^{(1)} w_m \right)^2 \sum_{m=1}^N c_{im}^{(2)} w_m - \frac{b_{11}}{\eta} \sum_{m=1}^N c_{im}^{(1)} \psi_m \sum_{m=1}^N c_{im}^{(2)} w_m = 0 \\ -b_{11} \sum_{m=1}^N c_{im}^{(2)} u_m + a_{55} \eta \sum_{m=1}^N c_{im}^{(1)} w_m + d_{11} \sum_{m=1}^N c_{im}^{(2)} \psi_m - a_{55} \eta^2 \psi_i - \frac{b_{11}}{\eta} \sum_{m=1}^N c_{im}^{(1)} w_m \sum_{m=1}^N c_{im}^{(2)} w_m = 0 \end{cases} \quad (36)$$

Correspondingly, the boundary conditions for clamped ends and hinged ends become

$$u = 0, w = 0, \psi = 0 \quad (37)$$

and

$$u = 0, w = 0, b_{11} \left[\sum_{m=1}^N c_{im}^{(1)} u_m + \frac{1}{2\eta} \left(\sum_{m=1}^N c_{im}^{(1)} w_m \right)^2 \right] - d_{11} \sum_{m=1}^N c_{im}^{(1)} \psi_m = 0 \quad (38)$$

Respectively.

Incorporating boundary conditions into Eq. (36), governing differential equations for beam buckling could be cast as

$$(K_L + K_{NL} - p_e K_e - p_m K_m) d = 0 \quad (39)$$

with K_L being the linear stiffness matrix and K_{NL} the nonlinear stiffness matrix depending on the displacement vector $d = \left\{ \{u_i\}^T, \{w_i\}^T, \{\psi_i\}^T \right\}^T$ ($i = 1, 2, \dots, N$). K_m and K_e are constant matrices corresponding to applied mechanical and electrical loadings, respectively.

Neglecting nonlinear stiffness matrix in Eq. (39), the eigenvalue buckling load and corresponding mode can be extracted. Employing the algorism detailed in [47], the dimensionless postbuckling buckling equilibrium path could be numerically obtained.

5. Results and Discussion

5.1. Convergence and Validation Study

For convergence study of the numerical solution, Table 1 tabulates the dimensionless buckling and postbuckling load of clamped-clamped (C-C) GPL/PVDF composite beams with different numbers of grid points. The GPL concentration in the composites is 1.5 wt%. The geometry of the beam is chosen as $L = 20$ mm and $h = 1.0$ mm, respectively. No external electrical loading is applied. The number of grid points N varies from 7 to 21. This table indicates that using 13 points is enough to achieve convergent results for critical buckling load p_{cr} at $w_m = 0$. For postbuckling, more grid points are required to achieve convergent results, e.g. $N = 17$ when $w_m = 0.5$ and $N = 19$ for $w_m = 1.0$. Considering computing efficiency with enough accuracy, 17 points will be adopted for the analysis in present work.

Table 1

Buckling and postbuckling load of GPLRC beam subjected to different bending deflections.

N	$w_m = 0.0$	$w_m = 0.5$	$w_m = 1.0$
7	0.021050566	0.021370123	0.023091276
9	0.021033483	0.024741095	0.035256336
11	0.021032525	0.025087423	0.03723737
13	0.02103254	0.025095354	0.037283297
15	0.02103254	0.02509551	0.037284405
17	0.02103254	0.025095509	0.037284417
19	0.02103254	0.025095509	0.037284418
21	0.02103254	0.025095509	0.037284418

To validate model, Table 2 compares the critical buckling loads for the first three modes of C-C beams with different geometries by present solution method and finite element package ABAQUS. The tensile modulus and Poisson's ratio are chosen as 30×10^6 psi and 0.4, respectively. Excellent agreement can be observed between the two kind of results.

Table 2

Dimensionless critical buckling loads of beams for the first two modes.

Geometry		Present			FEM		
		1 st	2 nd	3 rd	1 st	2 nd	3 rd
$h = 0.5$	$L = 10$	120052	237622	444361	120297	238613	447725
	$L = 20$	30631	62131	120052	30638	62162	120161
	$L = 30$	13666	27850	54166	13667	27857	54188
	$L = 40$	7697	15713	30631	7697.8	15715	30638
$h = 1.0$	$L = 10$	888722	1617336	2737454	891704	1627790	2765930
	$L = 20$	240105	475243	888722	240254	475879	8907700
	$L = 30$	108332	218313	418108	108362	218447	418561
	$L = 40$	61262	124262	240105	61272	124305	240254

To further validation the current for postbuckling behaviour, the postbuckling path for a C-C beam made of pure epoxy is compared with the result from previous work [46]. The tensile modulus and Poisson's ratio of the epoxy are selected as 3.0×10^9 Pa and 0.34, respectively. The length and thickness of the beam are 0.1 m and 0.01 m, respectively. The present postbuckling path agrees well with the previous one in reference again.

Fig. 2. Postbuckling path for clamped-clamped epoxy beam.

Following the convergence and validation study, buckling and postbuckling behaviours of GPL/PVDF dielectric beam will be comprehensively investigated. Unless otherwise stated, C-C beam will be adopted with fixed length and thickness, i.e. $L = 20$ mm and $h = 1$ mm. The concentration of GPL in the composite is 1.5 wt% and the diameter and thickness of the GPL are 50 nm and 8.1967 μm [19], respectively.

5.2. Buckling

As previously mentioned, the axial load induced by the electrical loading highly depends on thickness of beam. Fig. 3 plots the variation of dimensionless buckling load with the beam spam ratio $\eta = L/h$. As seen from this figure, the buckling load drops with the increase of the spam ratio as expected. It is found the electrical loading weakens the load-bearing capacity of the beam, especially for thinner beams. For example, a decrease of 69.43% in buckling load is observed when the DC voltage increases from 0 V to 200 V for a beam with spam ratio being 40. In contrast, for the same change in voltage, the buckling load only undergoes a 4.44% decrease for a beam with spam ratio being 20. The above phenomena can be explained by the following points. Firstly, the increase of the spam ratio deteriorates the stiffness of the beam, leading to smaller buckling load. However, the compressive axial load induced by through-thickness electrical loading decreases the stiffness of the beam due to alleviated axial stress. Such decrease is more significant for beam with larger spam ratio subjected to greater voltage. Therefore, larger discrepancy can be observed between the results with different voltages for beam with larger spam ratio while the results tend to converge for beams with sufficiently small spam ratio.

Fig. 3. Effect of spam ratio on dimensionless critical buckling load.

Fig. 4 demonstrates dependency of dimensionless buckling load on GPL weight fraction. When the DC voltage is 0 V, the dimensionless buckling load depends on the elastic property of the GPL/PVDF composite only. Considering the through-thickness electrical voltage, the variation of the buckling load with the GPL weight fraction exhibits a threshold phenomenon. For example, electrical voltage has very limited influence on dimensionless critical buckling load when GPL loading is below a threshold, i.e. 1.3 wt%. Beyond this threshold, the buckling load highly depends on the voltage. This phenomenon is ascribed to the GPL/PVDF composite's possession of percolation threshold in dielectric permittivity. When the GPL concentration is relatively small, i.e. before the percolation threshold, the electron hopping and polarization of the GPL fillers, which are the two main mechanisms for dielectric property, is negligible due to sufficiently large distance between neighboring fillers. Therefore, the buckling behaviours is not sensitive to the electrical voltage that can induce compressive axial force. In contrast, after percolation threshold, the dielectric permittivity of the composites highly dependent on GPL weight fraction and electrical voltage, leading to significant discrepancy among the results.

Fig. 4. Effect of GPL loading on dimensionless critical buckling load.

The effects of the diameter-to-thickness ratio of GPL on buckling load of the beam, which is subjected to different electrical voltage, is presented in Fig. 5. The thickness of the GPL is fixed as 50 nm while the diameter-to-thickness ratio varies between 100 and 2000. The GPL concentration is 1.5 wt%. The buckling load grows steadily and remains the same regardless of the value of the voltage as the GPL diameter-to-thickness ratio is smaller than a critical value, i.e. 140. However, as the GPL diameter-to-thickness ratio exceeds this critical value, the buckling load versus diameter-to-thickness ratio curves start to have different trends depending on the voltage. For example, the curve with voltage being 140 V drops as the GPL diameter-to-thickness ratio is larger than the critical value. This can be explained by the combined effects of the GPL diameter-to-thickness ratio on the tensile modulus and the dielectric permittivity of the composites. The elastic modulus of the composites grows gradually with the growth of the GPL diameter-to-thickness ratio due to increased larger surface, which improves the capacity of load transfer between polymer matrix and GPL additives. This leads to the increase of buckling load. Meanwhile, the percolation threshold for dielectric permittivity is also dependent on filler's geometry. The increase of GPL diameter-to-thickness ratio results in reduction of percolation threshold. In the current case, 1.5 wt% is below the percolation threshold if diameter-to-thickness ratio is approximately below 140. The change in the mechanical properties due to the variation of the GPL diameter-to-thickness ratio dominates the change of buckling load. When this ratio further increases, the percolation

threshold becomes smaller than 1.5 wt%. Then, the decrease of the beam stiffness due to dielectrical property becomes more significant when compared to the contribution of elastic Young's modulus.

Fig. 5. Effect of GPL diameter-to-thickness ratio on dimensionless critical buckling load.

The axial force induced by through-thickness electrical loading is also dependent on the DC voltage. Fig. 6 analyses the dependency of buckling load on the electrical voltage. The buckling load drops as the voltage increases. In addition, since the dielectric permittivity of GPL/PVDF composite is improved when more GPLs are added into the polymer matrix, the decrease is more significant for bigger GPL concentration. For example, the buckling load decreases by 7.17% as the DC voltage increases from 0 V to 400 V when the GPL concentration is 1.4 wt%. In contrast, for the same voltage change, a 27.73% decline is observed when the GPL concentration is 1.5 wt%.

Fig. 6. Effect of DC voltage on dimensionless critical buckling load.

Apart from the concentration and geometry of GPL, the dielectric permittivity varies with the AC frequency of the electrical field, which varies the buckling performance. Fig. 7 presents the variation of the buckling load with AC frequency. A transition range between 10 Hz and 1000 Hz is observed, within which the dimensionless critical buckling load undergoes a dramatic increase. This is attributed to the remarkable dependency of the dielectric permittivity on AC frequency within this range. When the AC frequency is smaller than this range, the dielectric permittivity of the composite converges to maximum regardless of the magnitude of the AC frequency. This results in more compressive axial load and smaller beam stiffness. In contrast, when the AC frequency is larger than this range, the dielectric permittivity converges to reach the minimum independent on the AC frequency. Correspondingly, less axial load is released due to the dielectric property, leading to larger beam stiffness and buckling load. In addition, within the shaded transition region, the variation of the buckling load is more significant for larger GPL concentration. For example, a 7.72 % increase is witnessed within the transition range when the GPL concentration is 1.4 wt% while an increase of 38.38 % when the GPL concentration is 1.5 wt%.

Fig. 7. Effect of AC frequency on dimensionless critical buckling load.

Without applying axial mechanical loading, Fig. 8 presented the critical buckling voltage for the GPL/PVDF composite beam. Fig. 8(a) shows that the buckling voltage decreases dramatically when the GPL weight fraction is approximately 1.3 wt%, especially for smaller AC frequency. In addition, the buckling voltage increases as the AC frequency increases. In Fig. 8(b), the buckling voltage demonstrates a percolation phenomenon against the AC frequency. For example, the buckling voltage suddenly increases as the AC frequency increases beyond approximate 200 Hz. This percolation behaviour becomes more obvious for greater GPL weight fraction. The mechanisms that underpin the above observations can be explained by the comprehensive dependency of axial stress on the attributes of the electrical field, i.e. the magnitude and AC frequency of the applied electrical field.

Fig. 8. Variation of critical buckling voltage of GPL/PVDF beam with: (a) GPL weight fraction, (b) AC frequency.

5.3. Postbuckling

In this section, the effects of influencing factors on postbuckling behaviours of GPL/PVDF beam are studied. The postbuckling equilibrium paths for the composite beam is presented in Fig. 9, in which the midspan deflection varies within the range $0 \leq w_m \leq 1$. For the same beam thickness, different electrical voltages are considered. As seen from this figure, the postbuckling load p_{NL} grows with the increase of the midspan deflection and the beam thickness as expected. The application of the electrical voltage will shift the postbuckling equilibrium path, which is more significant for thinner beams. For example, the postbuckling load of the beam with spam ratio being 40 undergoes an increase of 75.46 % as the midspan deflection increases from 0 to 1 when the DC voltage is 0 V while it increases by 248.17% when the DC voltage is 200 V. In comparison, for beam with spam ratio being 15, the variation of the voltage has limited effects on the postbuckling load.

Fig. 9. Effect of spam ratio on postbuckling equilibrium paths.

Fig. 10 demonstrates the dependency of postbuckling equilibrium paths on midspan deflection for beams subjected to different voltages, i.e. $V_{DC} = 0$ and 300 V. As shown in Figs. 10(a), the beams with higher GPL concentration have larger postbuckling loading. However, it is interesting to find from Fig. 10(b) that such trend is not applicable to the beam when subjected to 300 V, especially for smaller

midspan deflection. It is because that when the beam is subjected to smaller midspan deflection and higher voltage, the decrease of the beam stiffness due to the electrical voltage dominate the structural behaviours instead of the variation of the Young's modulus.

Fig. 10. Effect of GPL loading on postbuckling equilibrium paths: (a) $V_{DC} = 0$ V, (b) $V_{DC} = 300$ V.

The effect of the GPL aspect ratio on postbuckling behaviours is studied in Fig. 11. GPL concentration is fixed as 1.5 wt%. As afore mentioned, GPL fillers with larger diameter-to-thickness ratio are better reinforcements. Thus, postbuckling load is elevated as this ratio grows without the electrical voltage. From Fig. 11(a), it could be concluded that as the postbuckling load is enhanced as the GPL aspect ratio increases. Compared to Fig. 11(a), Fig. 11(b) shows a different trend. For example, the biggest GPL aspect ratio results in the smallest postbuckling load. This is because the postbuckling behaviours are highly dependent on the electrical voltage as that for buckling. The above phenomena again indicate that the postbuckling behaviours can be actively tuned by changing the electrical voltage, even the concentration and geometry of GPL are fixed.

Fig. 11. Effect of GPL diameter-to-thickness ratio on postbuckling equilibrium path: (a) $V_{DC} = 0$ V, (b) $V_{DC} = 200$ V.

The influence of the DC voltage on postbuckling equilibrium path is shown in Fig. 12. For relatively lower GPL concentration, i.e. 1.2 wt%, the electrical voltage has very limited effects. As the GPL concentration further increases, the increases of the voltage will induce lower postbuckling load, which is indicated by the significant shift of the postbuckling path as observed in Fig. 12(b). Such dependency of the postbuckling equilibrium path on the voltage and the GPL concentration can be explained by the percolation behaviour of the composites as discussed.

Fig. 12. Effect of DC voltage on postbuckling equilibrium paths: (a) $f_{GPL} = 1.2$ wt%, (b) $f_{GPL} = 1.5$ wt%.

The effects of AC frequency on the postbuckling behaviours are discussed in Fig. 13. Similar to the trend in Fig. 12, for lower GPL concentration, AC frequency has very limited effects on the postbuckling equilibrium path. As the GPL concentration further increases, the postbuckling equilibrium becomes dependent on the AC frequency. It is interesting to notice that the shift of postbuckling equilibrium path occurs during a certain range, i.e. $10 \text{ Hz} \leq f_{AC} \leq 1000 \text{ Hz}$, as suggested in Fig. 13(b). When the AC frequency falls outside the range, i.e. $f_{AC} > 1000 \text{ Hz}$ or $f_{AC} < 10$, the postbuckling equilibrium path converges to form either the lower or upper limit.

Fig. 13. Effect of AC frequency on postbuckling equilibrium path: (a) $f_{GPL} = 1.2 \text{ wt\%}$, (b) $f_{GPL} = 1.5 \text{ wt\%}$.

6. Conclusions

Buckling and postbuckling behaviours of GPL reinforced dielectric beam subjected to electrical voltage are investigated through employing Timoshenko beam theory and von Kármán nonlinearity. Governing differential equations are numerically solved by DQM. Through parametric study, it is found that buckling and postbuckling behaviours depends on combined effects of mechanical and dielectrical properties of the composites, which comprehensively rely on attributes of GPLs and the applied electrical field. There exists a percolation threshold, above which the voltage and the AC frequency of the electrical field significantly influence the buckling and postbuckling behaviours. GPLs with larger aspect ratio are found to be better reinforcements than their counterparts with smaller aspect ratio. Within a certain range of AC frequency, the buckling and postbuckling performances of the beam become highly susceptible to the variation of the AC frequency. The present analysis suggests that the

structural behaviours of the beam can be actively tuned by changing the attributes of the electrical field, which demonstrate great potential in developing smart materials and structures.

Conflict of interest statement

The authors declare that they have no conflict of interest.

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Figure Captions

Fig. 1. Schematic configuration of GPLRC beam.

Fig. 2. Postbuckling path for clamped-clamped epoxy beam.

Fig. 3. Effect of spam ratio on dimensionless critical buckling load.

Fig. 4. Effect of GPL loading on dimensionless critical buckling load.

Fig. 5. Effect of GPL diameter-to-thickness ratio on dimensionless critical buckling load.

Fig. 6. Effect of DC voltage on dimensionless critical buckling load.

Fig. 7. Effect of AC frequency on dimensionless critical buckling load.

Fig. 8. Variation of critical buckling voltage of GPL/PVDF beam with: (a) GPL weight fraction, (b) AC frequency.

Fig. 9. Effect of spam ratio on postbuckling equilibrium paths.

Fig. 10. Effect of GPL loading on postbuckling equilibrium paths: (a) $V_{DC} = 0$ V, (b) $V_{DC} = 300$ V.

Fig. 11. Effect of GPL diameter-to-thickness ratio on postbuckling equilibrium path: (a) $V_{DC} = 0$ V, (b) $V_{DC} = 200$ V.

Fig. 12. Effect of DC voltage on postbuckling equilibrium paths: (a) $f_{GPL} = 1.2$ wt%, (b) $f_{GPL} = 1.5$ wt%.

Fig. 13. Effect of AC frequency on postbuckling equilibrium path: (a) $f_{GPL} = 1.2$ wt%, (b) $f_{GPL} = 1.5$ wt%.

Table Captions

Table 1 Buckling and postbuckling load of GPLRC beam subjected to different bending deflections.

Table 2 Dimensionless critical buckling loads of beams for the first two modes.

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