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Beam-down linear Fresnel reflector: BDLFR

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Abstract

This paper presents a novel linear solar concentrating system named BDLFR, acronym of Beam-Down Linear Fresnel Reflector. A BDLFR system consists of a primary LFR array reflecting sunlight into a secondary hyperbolic –or elliptic– cylinder mirror that beams-downs reflected rays to its secondary focal line at ground level, where the receiver is placed. A model based on vector notation –and validated against SolTrace– predicts the optical behavior of BDLFR and generates layouts to avoid blockings. The edge ray approach is utilized to determine receiver aperture widths. As a function of beam-down curvature ratio, it is found out the BDLFR configurations that maximize solar collection. Concentration ratios about 80 are reached with a tertiary re-concentrator (CPC) coupled to the receiver aperture.

Keywords: Linear Fresnel reflector, Beam-down curvature, Flux concentration, Optical efficiency, Layout optimization, Compound Parabolic Concentrator

1. Introduction

Linear Fresnel Reflector (LFR) systems consist of several, in parallel arranged, mirror strips that reflect and concentrate solar beam radiation into a common focal line. Compared to parabolic trough collectors (PTC) –the other CSP linear technology–, LFRs are receiving great attention in recent years because of its lower costs, while achieving similar concentration ratios [1, 2, 3].

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Based on the original concept of LFR system, novel designs have been suggested to increase its functionality and performance. Mills and Morrison [4] proposed a Compact Linear Fresnel Reflector (CLFR) consisting in several receiver lines, so that significant spacing is saved between mirror rows alternatively aiming to the receivers while blockings are avoided. Using the CLFR concept, Collares-Pereira and Chaves [5, 6] improved its optical efficiency by means of a wave-shaped trough with variable mirror widths. LFRs with variable mirror widths were also widely researched by Abbas and Martínez-Val [7]. Huang et al. [8] designed an azimuth tracking LFR system. Another configuration with changeable number of mirror rows, named Scalable Linear Fresnel Reflector (SLFR), was suggested in Ref. [9]. A novel low-cost system, where LFR mirrors float on a water basin, was recently presented in Ref. [10].

In LFR optical systems, the linear receiver is placed elevated from the ground, so as to minimize blockings between Fresnel mirror rows. This LFR configuration limits its applicability to small and lightweight receivers, usually consisting of a single tube or a few of them. When dealing with large and heavy linear receivers, such as thermochemical reactors or fluidized beds, placement on top is not feasible.

Gómez-Hernández et al. [11] recently proposed a new LFR coupled with a linear beam-down (BD) reflector to irradiate a particle receiver placed on the ground. Taking advantage of beam-down optics [12], a secondary linear reflector –aligned with the primary focus of the LFR– redirects concentrated beam rays into its secondary focus. The linear receiver aperture is situated along such secondary focal line, allowing the receiver to rest on the ground. The present work presents and analyzes this novel concentrating solar system named Beam-Down Linear Fresnel Reflector, BDLFR by its acronym.

Beam-down optical systems present some issues, as clearly pointed out by Vant-Hull [13]. Secondary reflectors magnify the reflected spot by a factor equal to the ratio of hyperbola heights, later defined as f . In other words, the greater the curvature of the beam-down reflector, the more the reflected flux is dispersed. For this reason, beam-down systems require a final concentrator, usually CPC, at ground level. Despite this issue, easy operation and maintenance of ground reactors, which may also lead to cost savings, has promoted the research and industrial use of BD systems [14].

The application of BDLFR systems is straightforward in power generation. The use of particles as heat transfer fluid is a promising approach for CSP cost reduction [15]. In this sense, it is being researched a linear particle

receiver at ground level solarized by a BDLFR system [11]. Such receiver consists of a multi-stage fluidized bed where gas and particles could both reach temperatures around 900 °C [16].

BDLFRs are also suited to many solar thermochemical processes [14] ranging from hydrogen production or biomass gasification to processing of metals like zinc or aluminum. Thermochemical processes tested in small-scale central beam-down systems, such as water splitting [17], can be favorably adapted to the proposed linear beam-down. Linear solar reactors for pyrolysis of biomass [18] or waste tyres [19] are also good candidates to be integrated in a linear beam-down optic system.

The preliminary objective of the present work is to provide full knowledge on this novel optical system. In this regard, the following section introduces the mathematical background on the optics of BDLFR systems, and is followed by the edge ray approach to determine the receiver aperture widths.

The provision of tools to properly size BDLFR systems is another research goal. Sections 4 and 5 respectively deal with the generation of BDLFR layouts and the selection of the optimal configurations, which are those leading to the narrower receiver apertures.

Results on flux concentrations by the optimal BDLFR configurations are provided in Section 6, along with the validation of the model against Monte-Carlo Ray-Tracing (MCRT) simulations. Then, it is researched the option of coupling a Compound Parabolic Concentrator (CPC) at the receiver entrance. Last Section finally summarizes the conclusions from the whole study.

2. BDLFR basics

This section puts forward the mathematical background of the proposed beam-down linear Fresnel reflector system. Firstly, the BDLFR geometry is introduced. Using vector notation, relevant tracking formulae are later stated. Finally, the calculation of optical losses is outlined.

2.1. Geometry

The beam-down linear Fresnel reflector system consists of a primary stage of linear Fresnel reflector and a beam-down secondary. Taking advantage of the optical properties of conics, sunrays reflected by the mirror Fresnel array towards the focal line of a cylinder, whose transversal section is hyperbolic or elliptical, are redirected to the other focal line.

Previous research in point-focus beam-down systems showed better optical performance of hyperboloid compared to ellipsoid reflectors [12]. Hence, such favorable geometry results into a hyperbolic cylinder for a linear system, as herein considered.

Regarding the orientation of the LFR system, investigations such as that in Ref. [20], have shown preference of North-South over East-West. Thus, the system herein studied is oriented in the North-South direction axis, about which symmetry exists in the position of mirror rows. Each mirror is focused, having a slight curvature of radius equal to twice the distance to the primary focus [21]; i.e. on-axis alignment. Additionally, in the present study all the mirror rows have the same width, W_m .

The system of coordinates has origin in the secondary focus with X -axis pointing to the East, Y -axis aligned with the secondary focal line and Z -axis pointing to the zenith. According to this convention, YZ and XZ planes are respectively the longitudinal and transversal planes. The height of the primary focus, H , is then the distance between both focal lines. For the sake of convenience, the secondary focus is at the same height as the mirrors axes.

In the transversal plane, LFR systems have scaling properties over a point; say the origin of coordinates (Y -axis). All length parameters in the transversal plane can be expressed as non-dimensional parameters as a function of a scaling factor [6]. Herein it is taken the mirror width, W_m , as the scaling factor. This means that for $W_m = 1\text{ m}$, the remaining length parameters (i.e.: focal height, distance between mirror rows...) are also in meters. The notation throughout this document uses uppercase Latin symbols to represent dimensional parameters, and lowercase for the non-dimensional ones. For instance, non-dimensional focal height is represented with h , as defined as: $h = \frac{H}{W_m}$.

The beam-down reflector is a hyperbolic cylinder along the Y -axis. The geometric parameters of the hyperbola, as represented in Fig. 1, are: a main semiaxis, b secondary semiaxis, c linear eccentricity, and h focal distance, so that $h = 2c$. All the hyperbola parameters are correlated according to the following equation:

$$c = \sqrt{a^2 + b^2} = h/2 \quad (1)$$

The top hyperbola branch is considered as the single beam-down reflector. Since independent on y -coordinate –direction of cylinder axis–, the equation of the hyperbolic cylinder can be written in terms of the z -coordinate as a

Figure 1: Geometric parameters of hyperbolas.

function of the x -coordinate as follows:

$$z_{hyp} = \frac{a}{b} \sqrt{x_{hyp}^2 + b^2} + \frac{h}{2} \quad (2)$$

To consider the whole bundle of hyperbolas, it is defined f as the ratio of height of the hyperbola vertex to the total height h (Eq. 3), just like Segal and Epstein's Ref. [12]. f gives an idea on the openness or closeness of the hyperbola, thus it is also herein named as *curvature fraction* of the hyperbola. This f parameter ranges between 0.5 (flat horizontal) and 1.0 (infinite curvature, flat vertical). Fig. 2 represents the whole family of hyperbolas.

$$f = \frac{c+a}{2c} = 0.5 + \frac{a}{h} \quad (3)$$

2.2. Tracking equations

Using vector notation, this subsection defines the tracking formulae based on the central rays, whether incident or reflected. Vectors, reference points

Figure 2: Bundle of hyperbolas for f from 0.5 to 1.0, in steps of 0.05.

and system of coordinates are depicted in Fig. 3. Solar vector, $\mathbf{s} = [s_x \ s_y \ s_z]$, is the unit vector pointing to the center of the sun at each instant of time.

Given a mirror strip centered on M , with position vector $[M_x \ 0 \ 0]$, the main sunray reflected by the mirror is aimed to F , focal point with position vector $[0 \ F_y \ h]$. Naming $\mathbf{r}$ to the main reflected vector of unit norm, it leads to:

$$\mathbf{r} = \frac{\overline{MF}}{|\overline{MF}|} = \frac{[-M_x \ F_y \ h]}{\sqrt{M_x^2 + F_y^2 + h^2}} \quad (4)$$

Because of Snell reflection law, the mirror normal vector $\mathbf{m}$ is in the bisector of $\mathbf{s}$ incident and $\mathbf{r}$ reflected vectors, therefore:

$$\mathbf{m} = \frac{\mathbf{s} + \mathbf{r}}{|\mathbf{s} + \mathbf{r}|} \quad (5)$$

On the other hand, because of the single axis tracking in Fresnel mirrors, it follows that the mirror normal is in the XZ transversal plane, so that: $r_y + s_y = 0$. In other words, the y-component of the unit reflected vector equals to that of the sun vector with opposite sign. Combining that expression with Eq. 4, it is obtained F_y as:

$$F_y = -s_y \sqrt{\frac{M_x^2 + h^2}{1 - s_y^2}} \quad (6)$$

Figure 3: System of coordinates and main ray unit vectors.

from which can be obtained unit vectors $\mathbf{r}$ (Eq. 4) and $\mathbf{m}$ (Eq. 5), as well as the angle of incidence on the mirrors ϕ_m , or its cosine:

$$\cos \phi_m = \mathbf{s} \cdot \mathbf{m} = \mathbf{r} \cdot \mathbf{m} \quad (7)$$

Similarly, the equation of the reflected ray can be written in continuous form as follows:

$$\frac{r_x}{-M_x} = \frac{r_y - F_y}{F_y} = \frac{r_z - h}{h} \quad (8)$$

The intersection point, $I = [I_x \ I_y \ I_z]$, between the reflected ray (Eq. 8) and the hyperbolic cylinder (Eq. 2), is found by equalizing those equations. The x -component results from a quadratic equation whose solution is in Eq. 9.

$$I_x = \frac{-\beta - \text{sgn}(M_x) \sqrt{\beta^2 - 4\alpha\gamma}}{2\alpha} \quad (9)$$

where

$$\begin{aligned}\alpha &= \left(\frac{h}{M_x}\right)^2 - \left(\frac{a}{b}\right)^2 \\ \beta &= \frac{-h^2}{M_x} \\ \gamma &= \left(\frac{h}{2}\right)^2 - a^2\end{aligned}\tag{10}$$

By substituting I_x in Eq. 8, components y and z of I intersection point are retrieved, as stated in Eqs. 11 and 12.

$$I_y = F_y \left(1 - \frac{I_x}{M_x}\right)\tag{11}$$

$$I_z = h \left(1 - \frac{I_x}{M_x}\right)\tag{12}$$

The knowledge of I is important to determine not only the first slant range between mirror and hyperbola, $d_1 = |\overline{MI}|$, but also the ray reflected by the hyperbolic cylinder, $\mathbf{t}$. The slope of the hyperbola at the intersection point respect to the horizontal plane, z'_{hyp} , results from the derivation of z_{hyp} respect to x in Eq. (2), thus:

$$\left(\frac{dz}{dx}\right)_{hyp} = z'_{hyp} = \frac{aI_x}{b\sqrt{I_x^2 + b^2}}\tag{13}$$

from which it is gathered the normal of the hyperbolic cylinder at the intersection point, of unit vector $\mathbf{n}$, that holds that $n_y = 0$.

$$\mathbf{n} = \frac{\begin{bmatrix} z'_{hyp} & 0 & -1 \end{bmatrix}}{\sqrt{(z'_{hyp})^2 + 1}}\tag{14}$$

And applying again Snell reflection law, from Eq. 15 it is obtained the main reflected ray of unit vector $\mathbf{t}$ directed towards the secondary focus, with target point in T . The angle of incidence with the hyperbolic cylinder is straightforward from $\mathbf{r}$ and $\mathbf{n}$ (Eq. 16).

$$\mathbf{t} = \mathbf{r} + 2\mathbf{n} \cos \phi_{hyp}\tag{15}$$

$$\cos \phi_{hyp} = -\mathbf{r} \cdot \mathbf{n} \quad (16)$$

Because of the optical properties of the hyperbola, each $\mathbf{r}$ ray reflected by Fresnel mirrors aimed to the primary focal line will be reflected by the hyperbolic cylinder ($\mathbf{t}$ ray) towards the secondary focal line, centered on the receiver aperture. According to the reference system that matches Y -axis with the secondary focal line, coordinates of the target point are: $T = [0 \ T_y \ 0]$. Being d_2 the distance between I intersection point in the hyperbolic cylinder and T target point in the receiver, it is satisfied that $\overline{IT} = d_2 \mathbf{t}$. Then, d_2 and T_y are correspondingly gathered from Eqs. 17 and 18.

$$d_2 = \frac{-I_z}{t_z} \quad (17)$$

$$T_y = I_y + d_2 t_y \quad (18)$$

2.3. Optical losses

The optical efficiency of a linear concentrating system, η , is defined as the ratio of radiation intercepted on the receiver aperture to that incident on the collector field. Such efficiency is mathematically expressed in Eq. 19 where: $\bar{q}$ stands for the mean flux density incident on the receiver aperture, DNI is direct normal irradiation, N_m is the total number of Fresnel mirror rows, A_{ap} and A_m respectively stand for the areas of receiver aperture and mirror row, and W_{ap} and W_m stand for the widths of receiver aperture and mirror row.

$$\eta = \frac{\bar{q} \cdot A_{ap}}{DNI \cdot N_m \cdot A_m} \equiv \frac{\bar{q} \cdot W_{ap}}{DNI \cdot 2N_r \cdot W_m} \quad (19)$$

Because of symmetry in a North-South oriented LFR, rows of mirrors are equally numbered in east and west sides. Thus, N_r is half of the total number of mirror rows, which explains the $2N_r$ factor in Eq. 19 that accounts for all the mirrors. Having all the mirror rows the same width, W_m , the overall optical efficiency can be also expressed as the average of single mirror efficiencies:

$$\eta = \frac{\sum \eta_m}{2N_r} \quad (20)$$

Such global efficiency is smaller than one because of the several sources of optical loss taking place in BDLFRs. As expressed in Eq. 21, the optical

efficiency of a single mirror row, η_m , is the product of all the loss factors (ζ) below described [1].

$$\eta_m = \cos \phi_m \cdot \rho_m \cdot \rho_{hyp} \cdot \zeta_{sb} \cdot \zeta_{s,hyp} \cdot \zeta_{end} \cdot \zeta_{sp} \quad (21)$$

- Cosine loss, $\cos \phi_m$, is the reduction of solar collection area in Fresnel mirror rows because of the angle of incidence between direct solar beams, $\mathbf{s}$, and mirror normal, $\mathbf{m}$, as computed with Eq. 7. In linear systems this optical loss factor is usually quantified by the incidence angle modifier (IAM) which is gained experimentally or through MCRT simulations. For the purpose of this work, $\cos \phi_m$ is a good estimate of the IAM . It is important to note that the cosine of the incidence angle on the hyperbolic cylinder, $\cos \phi_{hyp}$ in Eq. 16, does not involve any optical loss as no reduction in solar collection takes place in the beam-down secondary reflector.
- Reflectivity, ρ , is the fraction of direct radiation actually reflected by the mirrors; this factor may also account for the mirror cleanliness. Both Fresnel and hyperbolic cylinder mirrors have its own reflectivity, ρ_m and ρ_{hyp} . As fixed values independent on BDLFR layout, this loss factor is neglected in the following ($\rho_m = \rho_{hyp} = 1$).
- Shading and blocking factor, ζ_{sb} , is the fraction of Fresnel mirror row neither shadowed nor blocked by neighbor rows. In North-South LFR orientation, no shading losses take place at noon time. As exposed in Section 4, the BDLFR systems in this study are designed to strictly avoid blockings at noon. Thus, taking equinox noon as design point, ζ_{sb} equals one at that instant of time.
- Shading by hyperbolic cylinder, $\zeta_{s,hyp}$. The beam-down reflector casts a shadow band on the ground, and definitely Fresnel mirror rows, of width equal to that of the hyperbolic cylinder, W_{hyp} . Neglecting end edge effects, $\zeta_{s,hyp}$ is equal to zero for a shadowed mirror row, and one otherwise.
- End factor, ζ_{end} , accounts for solar collection lost at the end of the receiver. For long enough LFRs, this loss factor can be neglected.
- Spillage factor, ζ_{sp} , quantifies the radiation finally intercepted at the receiver aperture. With a proper design of the receiver aperture width,

as proposed in the next Section, spillage losses become negligible. This loss factor is usually included in the aforementioned *IAM*.

At equinox noon design point and from the assumptions above stated, Eq. 21 is simplified into:

$$\eta_m = \cos \phi_m \cdot \zeta_{s,hyp} \quad (22)$$

3. Aperture widths: edge ray

Sun apparent size from the Earth causes a slight divergence of beams around the central ray. Having the sun disk an acceptance half angle $\theta_{sun} = 4.69 \text{ mrad}$, around 95% of the solar radiation is within the subtended cone [12]. The same applies to reflected beams, whether mirror slope and tracking errors are neglected.

To compute the width of the beam at the receiver aperture, we take advantage of the edge ray method [12, 22]. Tracing the solar cones of angle θ_{sun} reflected by each mirror, it is determined the spot size at each stage of the BDLFR system. Since the farthest mirror strip produces the largest spot sizes, edge rays traced from the center of the last mirror row defines the optimum width of the beam-down hyperbolic cylinder; wider hyperbola would result in unnecessarily larger shadow losses. Afterwards, applying the edge ray method to **t** ray reflected, it is set the width of the receiver aperture.

The application of the edge ray to determine aperture widths is graphically represented in Fig. 4. Central rays –as defined in previous Section– are depicted by dashed lines, while solid lines represent the highest and the lowest edge rays of each solar cone. It is worth mentioning that the angle between main and edge rays seen in the transversal view (Fig. 4) is not strictly θ_{sun} , which actually takes place in the vertical plane containing the main reflected rays.

In the following, it is stated the mathematical formulation to find out the widths of the beam-down reflector, w_{hyp} , and the receiver aperture, w_{ap} . Plus (+) and minus (−) subscripts denotes the highest and the lowest edge rays, respectively. Thus, from the focal points of topmost (F_+) and undermost (F_-) edge rays ($F_{\pm}$ for brevity), Eq. 23 sets the highest and lowest reflected unit vectors, equivalent to Eq. 4 in preceding Section.

$$\mathbf{r}_{\pm} = \frac{\overline{MF_{\pm}}}{|\overline{MF_{\pm}}|} = \frac{\begin{bmatrix} -M_x & F_y & h_{\pm} \end{bmatrix}}{\sqrt{M_x^2 + F_y^2 + h_{\pm}^2}} \quad (23)$$

Figure 4: Widths of hyperbolic cylinder and receiver aperture from edge rays (solid lines).

The outermost focal heights (Eq. 24) are gained from the height differences. Using the law of sines, such Δh_+ and Δh_- are gathered as in Eqs. 25 and 26, where ϵ_r stands for the elevation angle of the main reflected vector.

$$h_{\pm} = h \pm \Delta h_{\pm} \quad \begin{cases} h_+ = h + \Delta h_+ \\ h_- = h - \Delta h_- \end{cases} \quad (24)$$

$$\Delta h_+ = |\overline{MF}| \frac{\sin(\theta_{sun})}{\sin(\pi/2 - \epsilon_r - \theta_{sun})} \quad (25)$$

$$\Delta h_- = |\overline{MF}| \frac{\sin(\theta_{sun})}{\sin(\pi/2 + \epsilon_r - \theta_{sun})} \quad (26)$$

Following the same procedure as in previous Section, the intersection points of $\mathbf{r}_{\pm}$ edge rays with the hyperbolic cylinder are calculated with Eqs. 27, 28 and 29, for each $I_{\pm}$ component. Coefficients in $I_{x\pm}$ equation result from Eq. 30.

$$I_{x\pm} = \frac{-\beta_{\pm} - \text{sgn}(M_x) \sqrt{\beta_{\pm}^2 - 4\alpha_{\pm}\gamma_{\pm}}}{2\alpha_{\pm}} \quad (27)$$

$$I_{y\pm} = F_y \left(1 - \frac{I_{x\pm}}{M_x} \right) \quad (28)$$

$$I_{z\pm} = h_{\pm} \left(1 - \frac{I_{x\pm}}{M_x} \right) \quad (29)$$

$$\alpha_{\pm} = \left(\frac{h_{\pm}}{M_x} \right)^2 - \left(\frac{a}{b} \right)^2$$

$$\beta_{\pm} = \frac{-h_{\pm}(h_{\pm} \pm \Delta h_{\pm})}{M_x} \quad (30)$$

$$\gamma_{\pm} = \left(\frac{h_{\pm} \pm \Delta h_{\pm}}{2} \right)^2 - a^2$$

At this stage, the width of the hyperbolic cylinder mirror w_{hyp} can be determined from the intersection point of the highest edge ray reflected by the last mirror row. Taking advantage of LFR symmetry, such width is set as twice the x-coordinate of I_+ :

$$w_{hyp} = 2I_{x+}^{r=N_r} \quad (31)$$

Normal unit vectors at outermost intersections ($\mathbf{n}_{\pm}$) are computed as in Eq. 14, but now using $I_{\pm}$. Again applying Snell reflection law (Eq. 15), edge rays directed to the receiver ($\mathbf{t}_{\pm}$) are calculated. Being the receiver aperture at $z_{ap} = 0$ horizontal plane, slant ranges of edge rays ($d_{2\pm}$) are gathered by means of Eq. 17, but using $I_{z\pm}$. Then, it is satisfied that $\overline{I_{\pm}T_{\pm}} = d_{2\pm}\mathbf{t}_{\pm}$, from which can be determined the outermost target points $T_{\pm} = [T_{x\pm} \ T_{y\pm} \ 0]$ as stated in the following Equations. It is worth mentioning that, because of the varying curvature of the hyperbola, T_{x-} tends to be farther from the focal line than T_{x+} .

$$T_{x\pm} = I_{x\pm} - \frac{t_{x\pm}}{t_{z\pm}} I_{z\pm} \quad (32)$$

$$T_{y\pm} = I_{y\pm} - \frac{t_{y\pm}}{t_{z\pm}} I_{z\pm} \quad (33)$$

The width of the receiver aperture w_{hyp} can now be defined from the knowledge of the target points of edge rays. Because of the symmetry of LFRs about the vertical focal plane, and despite the fact that $|T_{x-}| \geq |T_{x+}|$, by locating the aperture window from $-T_{x+}$ to T_{x+} , it can be said that around 95% of the radiation reflected by such mirror row will be intercepted by the

receiver; as later confirmed by SolTrace validation. Since the sizing of the receiver aperture is based on the last mirror rows, and nearer rows produce smaller spot sizes, the spillage losses for a whole BDLFR system will be smaller 5%; the higher the number of rows, the lower the spillage losses will be. Therefore the criterion for sizing the receiver aperture is mathematically expressed as:

$$w_{ap} = 2T_{x+}^{r=N_r} \quad (34)$$

4. Layout generation

A proper design of optical systems is key to its successful performance. The design of LFR systems have been tackled by several authors, e.g. [7, 23]. The design optimization has been studied in the literature both using heuristics [24] and parametric optimization [25], as later herein adopted.

This Section puts forward the methodology to generate layouts of BDLFR. The position of each mirror row (determined by M_x^r) is such that satisfies two criteria (Fig. 5): (a) blocking by the previous row is strictly avoided at noon design point, and (b) coordinate of the first row is so that clears space for the receiver of w_{ap} width.

Strict avoidance of blockings means that the reflected ray by a mirror at its lower rim is tangent to the upper rim of the previous mirror strip. This first criteria is graphically represented in Fig. 5a for a 10+10 rows field (only East side is shown). Given the position of the first row (e.g. $M_x^{r=1} = 1.5$), precise determination of $M_x^{r=2}$ entails an iterative calculation procedure since $\mathbf{n}$ mirror orientation depends on M_x^r , consequently influencing the reflected ray from mirror lower edge (solid line in Fig. 5a). Once the position of the second row is determined, coordinate of third row is found in the same iterative way, and so on.

As BDLFR system, the receiver is placed on the ground centered along the secondary focal line (Y -axis). Space for receiver placement must be saved, thus the first mirror row must be settled in accordance. Receiver width is defined by Eq. 34, which actually depends on the position of the last row $M_x^{r=N_r}$. Therefore, compliance with this second criteria (represented in the layout in Fig. 5b) entails consequent positioning of the first row. For the sake of convenience, it was defined a set of $M_x^{r=1}$ coordinates starting from 1.5 and in unit steps.

Given for instance a hyperbolic cylinder of curvature fraction $f = 0.8$, for different focal heights Fig. 6(bottom) represents $T_{x\pm}$, x -coordinate of the

(a) Mirror row separation for strict blocking avoidance.

(b) Rearrangement of mirror position to allow for receiver width as estimated with the highest edge ray from the last row.

Figure 5: Layout generation.

outermost target points on the receiver plane by highest (+) and lowest (−) edge rays, depending on $M_x^{r=N_r}$ position of the last row. Because of sunrays divergence, it is evident that the farther the last row is, the higher $T_{x\pm}$ becomes.

As previously pointed out by Eq. 34, receiver aperture width is defined from such $T_{x+}^{r=N_r}$. Once w_{ap} is found, the position of the first row is the first available in the set $[1.5, 2.5, 3.5, \dots]$.

Figure 6: x -coordinate of first mirror row depending on that of the last row for $f = 0.8$ and several heights.

The top of Fig. 6 summarizes with grayscales the resulting x -coordinate of the first row depending on that of the last row (abscissa) and the focal height (ordinate). For example, if the last row is at 400 m and focus height is at 20 m, the first row must be at 5.5 m of the receiver axis, which is true for 1 m wide mirrors. With 2 m mirrors, for the same last row at 400 m

Figure 7: x -coordinates of the 30 rows in BDLFRs with $f = 0.8$ and several heights.

($M_x^{r=N_r} = 200$) and 20 m focal height ($h = 10$), the first row must be at 7 m ($M_x^{r=1} = 3.5$); taking advantage of the non-dimensional potential.

For a field of 60 mirror rows ($N_r = 30$) and hyperbolic cylinder of 0.8 curvature fraction, Fig. 7 depicts the x -coordinate of each row; non-dimensional focal heights of 10, 15, 20, 25 and 30 are considered. As a result of blocking avoidance criterion, the higher the primary focus is, the more compact the field comes. Also as expected, separation between rows gradually increases with row number, as highlighted for $h = 20$. It can be seen that the first row is at non-dimensional $M_x^{r=1} = 1.5$ for all the heights except for $h = 10$, where it is $M_x^{r=1} = 2.5$.

Parametric generation of BDLFR layouts, meeting the two above-explained criteria, was performed using the geometrical model developed up to this point. Dimensionless layouts (as divided by W_m) were obtained as a function of: N_r number of rows, f curvature fraction of the hyperbola, and h height of the primary focal line. The resulting database of BDLFR layouts is employed in the following to select optimal configurations.

5. Optimal configuration

BDLFR systems must satisfy the requirements set by the ground-placed linear receiver. A major requirement of solar receivers is to minimize radia-

tive losses to the surroundings, which is equivalent to minimize the receiver aperture, and hence its width w_{ap} .

Given a number of mirror rows and a BD curvature fraction, receiver aperture width (Eq. 34) ultimately depends on the height of the primary focus. Therefore, there might exist an optimal h focal height for which w_{ap} is minimum.

With the methodology explained in the previous Section, it was generated a database of BDLFR configurations. The three inputs were parametrically varied: half the number of rows N_r from 1 to 100 in unit steps; BD curvature fraction f from 0.5 (flat) to 0.98 in steps of 0.01; and BD focal height h from 1 to 200 in unit steps. Now, it is found for each N_r and f , the focal height that leads to the minimum aperture width.

For five selected curvature fractions, charts in Fig. 8 represent the aperture width as a function of focal height. As a general rule, both very high and, more evidently, very low secondary reflectors lead to wider receiver apertures. There is a height in between where w_{ap} is minimized for each f curvature fraction, as marked with a blue dot. As can be seen, the flatter the reflector, the higher its optimal focus is located.

The influence of the number of rows is also depicted in Fig. 8 by means of the color scale. Since farther rows produce wider apertures, it is clear that the more number of rows, the wider the receiver aperture is; which is also valid for the optimum height.

Results of the optimal focus height (h_{opt}) as a function of the curvature fraction and the number of rows are compiled in Fig. 9a. As can be observed, given a f , the optimal height follows a linear relationship with the number of rows. Straight lines fitting the points for each f and its corresponding linear equation are displayed in Fig. 9a.

The preceding Figure corresponds to a BDLFR located at 40° latitude. Since sun altitude angle (design point: equinox noon) depends on the location, Fig. 9b represents the same optimum height but at 30° latitude. Because of the higher sun elevation, the resulting h_{opt} is slightly lower at this latitude, for the same N_r and f . Results from now on correspond to 40° latitude.

Fig. 10 represents the minimum width of the receiver aperture, denoted as $w_{ap,opt}$. Again, there is a strong linear correlation with the number of rows. Subtle steps are due to the periodical change in the position of the first row to allow receiver placement, as explained in the previous Section. Regarding the influence of BD curvature fraction, flat horizontal reflector ($f = 0.5$) lead to the narrowest apertures. Increasing the curvature of the hyperbola

Figure 8: Receiver aperture width as a function of beam-down height, curvature fraction and number of mirror rows. Selection of optimal configurations.

progressively widens the receiver aperture. This magnification effect with beam-down curvature was formerly predicted by Vant-Hull [13].

In a similar way, Fig. 11 represents the width of the hyperbolic cylinder corresponding to the optimal configurations ($w_{hyp,opt}$). Regardless of the clear linear relationship with N_r , the influence of BD curvature is the opposite as with receiver aperture; low curvatures lead to wider reflectors.

From the results in this Section, it is possible to gather the optimal geometric configuration of BDLFR given a N_r and f . For instance, whether a 60+60 rows LFR with an almost flat BD of $f = 0.55$ is planned, the optimal geometric parameters are: $h = 109$, $w_{ap} = 1.35$ and $w_{hyp} = 56.4$. If mirror width is one meter ($W_m = 1$ m), those three parameters are directly in meters, this way taking advantage of the non-dimensional property. Even though, the receiver width in this example is quite narrow, promoting higher incident concentration, BD geometry becomes quite extensive, which can be a limitation to its practical application.

On the contrary, a very curved hyperbolic cylinder, for example $f = 0.95$,

(a) 40° latitude.

(b) 30° latitude.

Figure 9: Optimum beam-down focal height as a function of curvature fraction and number of rows.

with the same 60+60 rows ($W_m = 1$ m), generates: $H = 33$ m, $W_{ap} = 13.56$ m and $W_{hyp} = 6.13$ m. BD size is more acceptable now, however receiver aperture becomes too large, which negatively impacts on the incident flux and the receiver performance. These aspects are discussed in the following Sections.

Figure 10: Minimum aperture width of the receiver as a function of curvature fraction and number of rows.

Figure 11: Width of the hyperbolic cylinder in the optimal configurations as a function of curvature fraction and number of rows.

6. Flux concentration

In the end, the output from the BDLFR is a concentrated flux density incident on the receiver aperture. From the BDLFR model and the optimal configurations set up to this point, it can be obtained the mean flux density incident on the receiver $\bar{q}$. To make it independent of the instantaneous solar

DNI , it is preferred the mean concentration ratio of flux density, $\bar{C}$, which is non-dimensional as deducted from its definition:

$$\bar{C} = \frac{\bar{q}}{DNI} \quad (35)$$

In terms of $\bar{C}$, this Section provides results for the design point (equinox noon) at 40° latitude. At last, model results are validated against MCRT simulations.

6.1. Results

There is a relationship between $\bar{C}$, mean concentration ratio of flux density, and C_g , geometric concentration ratio. C_g is defined as the ratio of collector aperture to receiver aperture, i.e.:

$$C_g = \frac{N_m \cdot A_m}{A_{ap}} = \frac{2N_r \cdot W_m}{W_{ap}} \quad (36)$$

From the inspection of the definition of η optical efficiency in Eq. 19, the relation between $\bar{C}$ (Eq. 35) and C_g (Eq. 36) can be deduced. The averaged concentration ratio is equal to the geometric concentration affected by the optical efficiency:

$$\bar{C} = \eta \cdot C_g \quad (37)$$

For the optimal BDLFR configurations, the geometric concentration ratio is gained from Eq. 36. Depending on the curvature fraction (abscissa) and half the number of rows (color circles), Fig. 12 represents C_g . There is a fairly linear correlation with f , so that flatter reflectors lead to the highest geometric concentrations. As receiver aperture width is in the denominator of Eq. 36, narrower apertures (i.e. lower f) lead to the highest concentrations.

On the other hand, given a reflector curvature, the geometric concentration is constant regardless of the number of rows. This is an specific invariant of the optimal configurations, which somehow proves the scaling properties of BDLFR.

The overall optical efficiency (η) can be calculated as the mean optical efficiency of all the mirrors (Eq. 20), as long as they have the same width. In turn, the efficiency of each single mirror (η_m) is the product of all the optical loss factors (Eq. 21). This can be simplified into Eq. 22, according to the assumptions explained in Subsection 2.3. Therefore, the overall optical

Figure 12: Geometric ratio of concentration for the optimal configurations.

efficiency neglecting reflective losses, can be expressed as the product of the averaged cosine loss and the averaged shading loss by the BD reflector, thus:

$$\eta = \overline{\cos \phi_m} \cdot \overline{\zeta_{s,hyp}} \quad (38)$$

Figure 13: Mean shading loss by BD reflector for the optimal configurations.

Fig. 13 shows the mean loss by hyperbola shading ($\overline{\zeta_{s,hyp}}$) as a function of BD curvature fraction and number of rows. For very curved reflectors ($f > 0.92$), its width is so small that no shadow is projected into the mirror rows at noon, yielding $\overline{\zeta_{s,hyp}} = 1$. However, for flat secondary reflectors

($f = 0.5$), its width is almost half of the field width, so that this loss factor is equal to 0.5, or slightly above. The progression in between the extremes is linear, which is evident for high number of rows.

The other loss factor in Eq. 38, $\overline{\cos \phi_m}$, is moderately constant around 0.70, even though it slightly decreases with increasing f . The resulting optical loss of the BDLFR for the optimal configurations is depicted in Fig. 14. Regardless of the number of rows, η is quite linear from $f = 0.5$, with minimum around 0.4, to $f = 0.92$, with maximum in 0.7.

Figure 14: Optical efficiency for optimal BDLFR configurations.

Once the optical efficiency (Fig. 14) and the geometric concentration ratio (Fig. 12) are known, its product gives the mean concentration of flux density (Eq. 37). As can be seen in Fig. 15, $\bar{C}$ decreases when the curvature fraction increases. Thus, flux concentration is maximum for flat secondary reflectors, being $\bar{C}$ about 40.

According to these results, the highest concentration is achieved with a BD reflector consisting of a flat mirror at half the focal height. However, as pointed out in previous Section, in most of the cases a flat horizontal reflector is not pragmatic because of the required extensive size. The last Section explores the addition of a tertiary reflector.

6.2. Validation with MCRT

Comparing the results from the model with those from a Monte-Carlo Ray-Tracing software, it is verified the validity of the former. For this task, SolTrace tool by NREL [26] has been used.

Figure 15: Mean concentration ratio of flux density for the optimal configurations.

3D models of selected optimal configurations were introduced in SolTrace. For the BD secondary reflector, the hyperbolic cylinder surface was fitted into a Zernike series of eight order, which can be entered in SolTrace. To avoid edge effects along the length of the receiver, the mean flux concentration ratio (actually mean flux divided by 1 kW/m^2 of DNI) was considered in the central zone of the receiver aperture. Fig. 16 shows a snapshot of SolTrace model.

Figure 16: SolTrace 2D view of BDLFR east side for $N_r = 40$ and $f = 0.85$.

For BDLFR configurations with 40+40 rows, Table 1 specifies the mean flux concentration obtained both from the model and SolTrace for BD cur-

vature fractions from 0.55 to 0.95. The last column points out the relative difference between both of them. The model predicts $\bar{C}$ fairly well, with about 1% relative difference in most of the cases. Disregard of spillage losses by the model, explains that slight overestimation.

Table 1: Comparison of mean concentration ratio of flux density between the model and SolTrace.

f	H [m]	W_{ap} [m]	W_{hyp} [m]	$\bar{C}$		
				Model	SolTrace	Dev.
0.55	73	0.90	37.75	36.67	33.06	10.9%
0.65	39	1.18	29.12	33.26	32.74	1.6%
0.75	29	1.64	19.53	26.83	26.60	0.9%
0.85	25	2.86	11.24	17.71	17.57	0.8%
0.95	22	9.07	4.10	6.07	5.96	1.8%

Just for nearly flat reflectors, $f < 0.6$, the model certainly overestimates the mean flux concentration up to 12%. Long ray distances between reflections entails significant spillage losses in flat, and nearly flat, configurations, which are not accounted for by the edge ray approach. Such overvaluation by the model might be considered in the profile of $\bar{C}$ depending on f , as presented in Fig. 15.

7. BDLFR with CPC

From previous results (Fig. 15), it turns out that the highest flux concentration ratio of around 40 is achieved with flat beam-down reflectors. However, the required sizes, i.e. heights (Fig. 9a) and widths (Fig. 11), are excessive for $f = 0.5$, or slightly higher.

In the same way that point-focus beam-down systems [12], a final concentrator in the aperture of the receiver may be installed to re-concentrate incident radiation. This way, the wider flux distributions taking place with increasingly curved hyperbolas can be concentrated into narrower receiver widths.

The most common type of receiver concentrator is the Compound Parabolic Collector (CPC), which is originally attributed to Hinterberger and Winston [27]. CPCs consist of two branches of parabolic reflector whose foci lie on one each parabola endpoint.

In a 2D system as the BDLFR, the CPC becomes a cylinder with two parabolic branches, as depicted in Fig. 17. Thus, the CPC entrance aperture, with w_{ap} width, is centered and aligned with the secondary focus of the BD hyperbola, and the narrower CPC outlet, with $w_{ap,CPC}$ width, is now the receiver aperture.

Figure 17: BDLFR with CPC.

The geometric concentration ratio of a CPC is just the ratio of inlet and outlet widths. Based on the second law of thermodynamics, Rabl [28] showed that the maximum concentration ratio from a 2D CPC is the reciprocal of the sine of the acceptance half angle, as expressed in Eq. 39.

$$C_{g,CPC} = \frac{W_{ap}}{W_{ap,CPC}} = \frac{1}{\sin \theta_{ap}} \quad (39)$$

The CPC half angle of acceptance or aperture, θ_{ap} , is defined by the incidence angle (zenith) of the BD reflected ray from the latest mirror row. Main rays initially reflected in the last row are depicted in Fig. 17, along with θ_{ap} aperture angle.

For the optimal BDLFR configurations, Fig. 18 shows CPC acceptance angles as a function of hyperbola curvature fraction and half the number of rows. Such aperture angle is independent on N_r , but it is maximum for nearly flat BD (f around 0.62) and minimum for the most curved hyperbolas. Therefore, and according to Eq. 39, these narrower hyperbolic cylinders (high

Figure 18: CPC aperture angles.

Figure 19: CPC concentration ratios.

f) lead to the highest CPC concentrations, reaching up to 30 as represented in Fig. 19. On the contrary, BD hyperbolas of curvature fraction lower than 0.85 lead to $C_{g,CPC}$ smaller than 5.

From Eq. 39 it is gathered not only the geometric concentration ratio by the CPC, but also the final receiver width ($w_{ap,CPC}$), originally unknown. Fig. 20 shows the width as a function of half the number of rows and the curvature fraction. Flatter BD reflectors still result in narrower receivers, but widths have been significantly reduced for high f by coupling a CPC. Comparing the latter Figure with Fig. 10, it becomes clear the impact of the

Figure 20: Receiver aperture width for BDLFR with CPC.

re-concentrator in very curved hyperbolas. Given for instance $f = 0.9$ and 50+50 mirror rows ($W_m = 1$ m), receiver width changes from 5.48 m to 0.81 m by using the CPC.

Neglecting the reflectivity of the CPC reflector (i.e. $\rho_{CPC} = 1$), the corresponding mean concentration ratio of flux using the re-concentrator, $\bar{C}$ (*with CPC*), is just the product of both concentration ratios (Eq. 40): geometric of CPC and mean flux without CPC. Fig. 21 represents the resulting mean concentration ratio with CPC as a function of curvature fraction and number of rows. Again, the result is independent on N_r , but the profile of $\bar{C}$ is very distinct.

$$\bar{C} \text{ (with CPC)} = C_{g,CPC} \cdot \bar{C} \quad (40)$$

Compared to Fig. 15 (no CPC), flux concentrations are now larger regardless of f . Such $\bar{C}$ increment is more noticeable for greater curvature fractions, so that the profile is more uniform with an average $\bar{C}$ of around 80. Two peaks are identified: one for flat BD ($\bar{C} \approx 140$), and another for $f = 0.92$ producing $\bar{C} = 84$.

Because of size concerns regarding flat BD configurations, and despite its fairly better optical performance even with coupled CPC, it is concluded that the feasible optimal configuration of BDLFR with CPC is found for hyperbola curvature fraction equal to 0.92. Such BD geometry leads to mean concentration ratios of 84. This $\bar{C}$ actually sets a maximum at equinox noon,

Figure 21: Mean concentration ratio of flux density for the optimal configurations and including a CPC.

given that reflection losses have been disregarded in the analysis.

8. Conclusions

A novel CSP linear system, named Beam-Down Linear Fresnel Reflector, has been introduced in the present work. BDLFRs consist of a primary LFR array reflecting solar beam radiation into a secondary hyperbolic –or elliptic– cylinder mirror that beams-down reflected rays to its second focus at ground level, where the receiver is placed.

The mathematical background on BDLFR geometry and reflection optics has been thoroughly put forward in Section 2. Given the scaling properties of BDLFR systems respect to the receiver longitudinal axis, length parameters have been expressed as non-dimensional respect to the mirror width, assumed equal for all the rows. Vector notation has been used to determine reflected rays, as well as target points.

The edge ray approximation has been used in Section 3 to mathematically find out the receiver width, that collects the reflected beam radiation in the mirror rows, firstly, and the beam-down reflector, secondly.

It has been established a procedure to generate BDLFR layouts that avoids mirror blockings at noon and accounts for the space required by the ground-placed receiver. Using such methodology, a database of layouts has been parametrically generated. From such database, optimal BDLFR con-

figurations have been identified in Section 5 so that the receiver aperture is minimized.

The resulting model has been validated against SolTrace. In general, the model slightly overestimates the concentration ratio in around 1%, which is due to the spillage loss disregard assumed by the edge ray approach. For the flattest beam-down reflectors, relative error increases up to 10%.

Despite the significant shadow losses involving flat horizontal beam-down reflectors ($f = 0.5$), this configuration arises the highest concentration ratios ($\bar{C} \approx 40$), regardless the number of mirror rows. The better performance of flat beam-down mirror is explained by the magnification of the spot – and, consequently, the receiver width – proportional to beam-down curvature. However, the required size by a flat configuration is prohibitive (for instance with 40+40 mirror rows of 1 m each, flat beam-down width and height position respectively are 41.2 and 84.5 m), undermining its practical implementation.

As in point-focus beam-down systems, a third re-concentration stage has been researched. Coupling a CPC to the receiver entrance, the highest concentration ratio of flux is yet for the flat beam-down configuration. Nonetheless, a second maximum exists for very curved hyperbolic cylinder ($f = 0.92$) leading to $\bar{C} \approx 84$. Therefore, by means of workable configurations of BDLFR with CPC –i.e. non-horizontal flat secondary–, it can be obtained incident flux densities on the receiver up to around 84 kW/m² (with $DNI = 1$ kW/m²), regardless of the number of mirror rows. Since the receiver width has been precisely set by the number of rows, such number is finally determined by the target flux (in kW/m) required by each particular receiver.

Nomenclature

A	Area [m ²]
a	Hyperbola main semiaxis [-]
b	Hyperbola secondary semiaxis [-]
C_g	Geometric concentration ratio [-]
$\bar{C}$	Mean concentration ratio of flux density [-]
c	Hyperbola linear eccentricity [-]
D	Slant range [m]
d	Non-dimensional slant range [-]
DNI	Direct normal irradiation [W/m ²]

F	Focal point
f	Ratio of hyperbola heights (curvature fraction) [-]
H	Height of primary focus [m]
h	Non-dimensional height of primary focus [-]
I	Intersection point with hyperbolic cylinder
IAM	Incidence angle modifier [-]
M	Mirror (Fresnel) center
$\mathbf{m}$	Mirror (Fresnel) normal unit vector
N_m	Total number of mirror rows ($= 2N_r$) [#]
N_r	Half the total number of mirror rows [#]
q	Flux density incident on the receiver aperture [W/m ²]
$\mathbf{n}$	Hyperbolic cylinder normal unit vector
$\mathbf{r}$	Ray unit vector reflected in Fresnel mirror
$\mathbf{s}$	Unit vector pointing to the sun
T	Target point on the receiver aperture
$\mathbf{t}$	Ray unit vector reflected in hyperbolic cylinder
W	Width [m]
w	Non-dimensional width [-]
X, Y, Z	Cartesian coordinates axes
x, y, z	Coordinates in X, Y, Z directions, non-dimensional [-]

Greek symbols

α, β, γ	Coefficients of quadratic equation
ϵ	Elevation angle [rad]
η	Optical efficiency [-]
θ	Acceptance half angle [rad],[deg]
ρ	Reflectivity [-]
ϕ	Incidence angle [rad]
ζ	Loss factor [-]

Superscript

r	Row number
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Subscripts

+	Highest edge ray
−	Lowest edge ray
1	Primary reflection
2	Secondary reflection
ap	Receiver aperture
end	End loss
hyp	Hyperbolic cylinder
m	Mirror (Fresnel)
opt	Optimum
s,hyp	Shading by hyperbolic cylinder
sb	Shading and blocking
sp	Spillage

Acronyms

BD	Beam-Down
BDLFR	Beam-Down Linear Fresnel Reflector
CLFR	Compact Linear Fresnel Reflector
CPC	Compound Parabolic Concentrator
CSP	Concentrating Solar Power
LFR	Linear Fresnel Reflector
MCRT	Monte-Carlo Ray-Tracing
PTC	Parabolic Trough Collector
SLFR	Scalable Linear Fresnel Reflector

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