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Deflection and Stresses in Solar Central Receivers

M. Laporte-Azcué*, P.A. González-Gómez, M.R. Rodríguez-Sánchez, D. Santana

Energy Systems Engineering Group (ISE), Department of Thermal and Fluid Engineering,
University Carlos III of Madrid, Av. Universidad 30, 28911.

* mlaporte@ing.uc3m.es

Abstract

The aim of the design of central solar receivers is to withstand the high non-uniform solar-heat-flux and temperature during the solar-power-plant lifetime. This high non-uniform tube temperature causes high thermal stress, producing creep and fatigue damage. Therefore, is necessary to obtain an accurate estimation of the tube stresses during the receiver operation. In the same way, to ensure the panel integrity, the frontal and lateral tube deflections must be obtained to avoid excessive panel bowing and warpage, respectively.

The huge number of simulations needed to perform the creep-fatigue analysis precludes the use of high time-consuming CFD-FEM simulations. To resolve this drawback, a reliable, accurate and fast procedure to obtain the tube stresses, using analytical stress estimation, is proposed. The procedure considers the temperature dependence of the thermo-mechanical properties.

The temperature-dependent hoop stress is estimated using the solution for constant mechanical properties whereas the radial stress is estimated taking constant the Young modulus only. The temperature-dependent axial-bending stress is obtained using the non-homogeneous beam equation subjected to the movement restriction produced by tube clips. When the tube displacement is restricted by tube clips, the equivalent stress difference is less than 2% taking temperature-dependent properties and slightly higher than 10% for constant properties. The proposed stress estimation is enough accurate to perform a reliable fatigue-creep analysis and two order of magnitude faster than the CFD-FEM simulations.

Finally, the tube deflection and displacement, restricted by tube clips, are derived straightforward using the temperature-dependent tube curvature and the beam theory.

Keywords: Solar power tower; External central receiver; Analytic method; Thermal stress; Deformation; Variable properties

Nomenclature

1	<i>Roman symbols</i>	10	I tube inertia moment
2	A, B, C, D Fourier series coefficients	11	L tube length (m)
3	A cross sectional tube area (m)	12	M bending moment (N·m)
4	a tube inner radius (m)	13	m number of axial divisions in the tube
5	b tube outer radius (m)	14	N number of reaction forces
6	D receiver diameter (m)	15	Np number of panels in the receiver
7	d separation between supports (m)	16	Nt number of tubes in the receiver
8	E Young modulus (GPa)	17	O coordinate system origin
9	H receiver height (m)	18	P force (N)

19	R reaction forces on the supports (N)	49	c analytical results with constant
20	r radial coordinate (m)	50	properties
21	T temperature (K)	51	FEM numerical results
22	$\bar{T}$ mean temperature (K)	52	M mechanical moments
23	th tube thickness (m)	53	MA free bending case
24	x,y cartesian coordinates	54	r radial component
25	z axial coordinate	55	T thermal moments
26	<i>Greek symbols</i>	56	v analytical results with variable
27	α linear thermal expansion coefficient	57	properties
28	Δz length of the axial cells (m)	58	VM Von Mises
29	δ tube deflection	59	x, y components in the cross section
30	θ circumferential coordinate (°)	60	xy product of inertia in xy axis
31	ν Poisson ratio	61	z axial component
32	ρ bending radius (m)	62	θ circumferential component
33	σ normal stress (MPa)	63	<i>Abbreviations</i>
34	τ shear stress(MPa)	64	CAE computer-aided engineering
35	<i>Superindexes</i>	65	CFD computational fluid dynamics
36	0 reference value	66	CGM coarse grid model
37	+ only positive values are considered	67	CSP concentrating solar power
38	* non-homogeneous properties	68	FEM finite element method
39	b forces and moments component	69	FGM fine grid model
40	e external surface	70	GPS generalized plane strain
41	i inner surface. Number of axial division	71	HHFM homogeneous heat flux model
42	j number of support	72	HTF heat transfer fluid
43	r radial distribution contribution	73	HTM homogeneous wall temperature
44	θ circumferential distribution	74	model
45	contribution	75	MA moment annulment
46	ν Hook law component	76	PPA power purchase agreements
47	<i>Subscripts</i>	77	RDA receiver design algorithm
48	b bending	78	SPT solar power tower

79 1. Introduction

80 Concentrating solar power (CSP) technologies are becoming more relevant in the renewable
81 energy landscape, being one of the most growing alternatives to the traditional energy
82 sources for power production. CSP has increased its capacity worldwide over 600% in the

last ten years (IRENA, 2019), only behind solar photovoltaic and offshore wind energy. The great advantage that CSP presents is the high degree of flexibility that allows the thermal storage of the heat transfer fluid (HTF).

In this CSP category, Solar Power Tower (SPT) plants operating with molten salts as HTF are expected to become the dominant technology due to the advantages that brings its higher working temperatures, compared to systems such as parabolic trough and Fresnel (IRENA, 2012). Among these benefits are the high steam cycle efficiency and greater thermal storage capacity, which is the main advantage of the use of molten salts as HTF in SPTs, becoming a relevant renewable option for the baseload supply.

It must be considered though that SPT plants entail a great initial capital investment. While the initial costs in other CSP systems have decreased due to the technology improvement and the apparition of economies of scale in their components manufacturing, the penetration of SPTs has been lesser, resulting in an unclear trend (Lilliestam et al., 2017). However, according to (IRENA, 2016), SPT installed costs are expected to drop around 37% by 2025. Recent instances such *Aurora* (Australia) and *Dewa IV* (Dubai) invite to optimism, being two revolutionary projects in terms of their extraordinary low power purchase agreements (PPA) (Lilliestam and Pitz-paal, 2018). In the case of *Dewa IV*, this low PPA is directly related to its 35-year duration, which is larger than the usual 20 years. This means that the integrity and correct operation of the plant must be ensured for a longer period of time.

One of the most critical aspects of the SPTs are the receiver tubes due to the demanding working conditions that they have to endure. The advantages that come with the high working temperatures in SPTs have also counterpoints that compromise the integrity of such tubes. In *Solar One*, (Radosevich and Skinrood, 2015) reported that, in less than one year after the power production phase started, cracks due to thermal stresses caused by the temperature gradients in the tubes and in the tube support clips had already appeared. Those cracks in the tubes caused leaks, being the main issue affecting the schedule of the plant. Therefore, the bowing and warpage deformations definitely reduce the receiver life of the SPTs, being the cracks the most critical outcome. The back to front thermal gradients cause the bowing in such direction, while the lateral thermal gradients do the same in the respective direction; the clips are placed periodically to prevent the bowing of the tubes, but they do not completely avoid it and the stresses in the tubes increase due to the mechanical restrictions they impose (Montoya et al., 2018). The tubes also suffer from cyclic fatigue, as consequence of the nightly periods of inactivity and the clouds interference in the solar flux incident on the receiver during the daytime operation (Liao et al., 2014).

The important role of the tube temperature makes essential to study the thermal and mechanical behaviour of the tubes in order to improve the receiver design and solve the inconveniences that cause the early receiver failure. This has lead numerous authors to develop a variety of models to analyse it. In that process, authors have had to deal with the issue of high computational costs while trying to obtain the most precise approximations of the tube behaviour. Two types of models can be distinguished: the detailed ones, based on CFD and FEM simulations and the simplified models, analytic codes.

In the field of thermal models, the detailed ones are computationally expensive so the simulation of the whole receiver is avoided. Among the vast variety of instances of the application of these models in the SPTs field, (Yang et al., 2012) studied the heat transfer occurring in a single tube receiver, noticing the importance of the temperature in the tube inner wall to prevent the salts decomposition and the uneven temperature distribution of the tube wall and salt. (Fritsch et al., 2017) developed a FEM model of a single receiver tube with constant heat transfer coefficients and 1D fluid elements, obtaining a good agreement with a more detailed CFD model. (Maytorena and Hinojosa, 2019) studied the direct steam generation in a tubular receiver of a SPT plant affected by non-uniform concentrated solar

radiation using a CFD model to analyse one tube of the receiver. CFD models have also been used to simulate new receiver geometries, like the design presented by (Garbrecht et al., 2013), consisting in hexagonal pyramid shaped elements.

Several progresses have been made in the field of simplified thermal models, whose lesser computational costs makes them ideal to simulate the whole receiver. The model developed by (Jianfeng et al., 2010) considered only a single pipe of an external receiver under unilateral concentrated radiation to study its efficiency. (Singer et al., 2010) developed an iterative model, *receiver design algorithm* (RDA), that calculates the receiver efficiency matrix, evaluating the losses at the effective radiation temperature considering its variation in the axial coordinate, but neglecting the circumferential ones. Then (Xu et al., 2011) modified the model by (Singer et al., 2010) in order to design a molten salt cavity receiver, focusing on the exergy analysis.

In their work, (Rodríguez-Sánchez et al., 2014a) developed two analytical models that included circumferential divisions of the tubes in addition to the axial ones, allowing a much more precise characterization of the tube temperature distribution. These two models are the *homogeneous wall temperature model* (HTM) and the *homogeneous heat flux model* (HHFM), both named after the initial hypothesis considered in the discretized tube cells. They take into account adjacent tubes but only one tube per panel was considered, assuming that all of the tubes in the same panel receive the same heat flux. This work was later further developed to consider every tube in the receiver (Rodríguez-Sánchez et al., 2018), resulting in the *fine grid model* (FGM) that has been found to be even more precise than the previous two when it comes to the tube temperature prediction. Since this model takes into account the fact that the incident flux in the tubes is not symmetrical, the FGM allows to study the bending of the tubes not only in the radial direction, allowing to analyse the possible collisions between adjacent tubes.

As for the mechanical studies, there has been a great interest on analysing the thermal stress and deformation problem in cylinders. (Gatewood, 1940) discussed the thermal stresses and displacements in long hollow cylinders subjected to a bidimensional temperature gradient. (Timoshenko and Goodier, 1951), starting from the plane strain case, studied the thermal stresses and displacements in unidimensional and bidimensional problems in different geometries. After developing the plane strain results, they presented the results for generalized plane strain (GPS) by changing the boundary conditions. Then (Goodier, 1957) compiled some of the cases presented in (Timoshenko and Goodier, 1951). Goodier stated that the variations with temperature of the linear thermal expansion coefficient need to be considered. (Noda et al., 2003) studied the thermal stresses in beams, including the problem of non-homogeneous properties. For quick thermal stresses estimations, (Babcock & Wilcox Company, 1984) presented a simplified equation that takes into account the thermal gradients across the cross section and the temperature difference between two sides of the tubes.

That knowledge of the thermal stresses and displacements in hollow cylinders has later been applied in the CSP field to aid the design of the receiver. In these works, different boundary conditions for the tubes have been studied. With an uniform heat flux in a billboard panel, (Conroy et al., 2018) studied the thermal stresses for different tube diameters and materials as part of a creep-fatigue analysis. The tubes were allowed to expand axially and to bend freely. Intermediate supports influence is not taken into account. They also analyzed the stresses due to the internal pressure that the tubes withstand, which were considerably low compared to the thermal stresses. (Logie et al., 2018) consider that the tubes cannot bend freely, studying the case of GPS. (Marugán-Cruz et al., 2016) studied as well thermal stresses in the plane strain scenario using a numerical model, concluding that the temperature distribution is needed for the whole tube, discarding the 1D models, and finding variations in the stress results depending on the Biot number. (Uhlig et al., 2018)

used a FEM model to analyse the stresses in the inlet panel of a central receiver under the conditions of the cases of the filling process and the solar operation. In the field of parabolic through collectors, (Khanna et al., 2014) also studied analytically the stresses and bending of the absorber tube supported along its length in different spots, considering two different supporting arrangements. These tubes are placed horizontally and are typically longer than the ones in the SPT receiver, but their further work (Khanna and Sharma, 2015) is useful to remark how the number of supports considered affects tube deflection, which they also tested experimentally (Khanna et al., 2016). The importance of considering the periodic supports of the tubes in the STP receiver is presented by (Montoya et al., 2018). They studied the stresses with a 3D thermo-elastic model, concluding that the mechanical restrictions affect the axial stress component, causing it to be up to three times higher than the one appearing in a non-restricted tube.

With everything presented above, it is seen the need to create an analytical model to study the stresses due to thermal gradients and mechanical constraints in external solar receivers. In the first Section of this work, a simplified analytic method to obtain the thermal stresses and displacements in the receiver tubes is presented, resulting in a low computational cost, considering the thermal gradients, the periodical supports of the tube and temperature dependent properties for the tube material. In Section 3, the characteristics of the receiver that will be studied are described. In Section 4 the analytical method is verified with the numerical one for a single axial division of a tube under free bending conditions. For the analytical method, the cases of temperature dependant properties and temperature independent properties have been compared. After that, a whole tube of the receiver is studied with the analytical model. The deflection of the tube and the maximum equivalent stress in it are obtained. The chosen boundary conditions that constrain the receiver tubes recreate their real operating conditions: they are allowed to expand axially and they have their bending prevented in certain spots by a series of equally spaced supports, called clips, placed along their length. These periodical supports reduce the overall bending of the tube but induce mechanical stresses due to the reaction forces that appear in them when the tube is trying to bend in these spots. A real heat flux distribution is considered by using the SPTFlux¹ software presented in (Sánchez-González et al., 2018). The temperature distribution has been obtained using the FGM (Rodríguez-Sánchez et al., 2018), being asymmetrical in the cross section and suffering from variations in z , appearing bowing in both x and y directions. Finally, Section 5 discusses the conclusions of this work.

2. Stress calculation in cylindrical tubes

An analytical methodology for the calculation of the stresses for a single cylindrical tube, of inner radius a and outer radius b is presented. The tube is under the boundary condition of free thermal axial expansion and its bending is restricted in certain spots along its length by the periodic placement of supports, so-called clips. These clips allow the tube displacement in z and the rotation of the tube cross-section in the plane of the support, Figure 1.

¹ <http://ise.uc3m.es/research/solar-energy/fluxspt/>

Figure 1. Support arrangement and boundary conditions.

224 The tube studied is discretized in vertical divisions, along the z axis, and every one of these
 225 divisions is discretized in radial r and circumferential θ cells, as depicted in Figure 2.a.

Figure 2. a) Grid of calculation in the cross-section and coordinates origins in the centroid (x, y, z) and in the neutral axis (x^*, y^*, z) . b) Deflection of a tube axial division.

226 Since the case of study is a tube, most of the equations presented in this work are expressed
 227 in cylindrical coordinates. However, there are equations that are typically written in
 228 cartesian coordinates. Given that the tube studied has non-homogeneous properties, two
 229 coordinate origins were selected: the one corresponding to the centroid (x, y, z) , and the one
 230 related to the neutral axis (x^*, y^*, z) . The neutral axis is the tube axis along which there are
 231 no longitudinal strain, and therefore the length of each vertical division (Δz) remains the
 232 same, during bending. It is presented in Figure 2.b delimited by the origins of its coordinate
 233 system in two different cross sections: O^* and O'^* . The bending radius of each axial division
 234 can be written in terms of the neutral axis, ρ_{y^*} (and ρ_{x^*} for the x^* axis), and in terms of the
 235 centroid, ρ_y (ρ_x for the x axis).

236 2.1 Yield criterion

237 Among the criteria to determine the admissible stresses in ductile materials, to prevent
 238 failure and serve as an indicator of a good design, the most commonly used are the Von
 239 Mises yield criterion (Maximum Distortion Energy Theory) and the Tresca yield criterion
 240 (Maximum Shear Stress Theory).

241 The Von Mises or equivalent stress in the tubes, studied in this work, is calculated in
 242 cylindrical coordinates as:

$$\sigma_{VM} = \sqrt{\frac{(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_z - \sigma_r)^2}{2}} + 3 \cdot \tau_{r,\theta}^2 \quad (1)$$

243 Since $\tau_{r,z}$ and $\tau_{z,\theta}$ are equal to zero due to the assumption of GPS. This null shear stresses
 244 situation is not altered by the addition of supporting clips.

245 The Tresca criterion, more conservative than the Von Mises one, takes into account only the
 246 maximum shear stress. Therefore, it is obtained as:

$$\sigma_{Tresca} = \max(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|) \quad (2)$$

247 Thus, in order to guarantee the integrity of the tubes of the receiver by using these
 248 criterions, it is necessary to obtain the different stress components that the tubes withstand.

249 2.2 Radial and hoop thermal stress

250 Due to a steady heat flux, the tube is subjected to a temperature field that depends on the r
 251 and θ coordinates but remains constant in the z one for a certain small-sized cell. The
 252 temperature profile in the cross section can be split in the temperature variation in r , $T_r(r)$,
 253 and the temperature variation in θ , $T_\theta(r, \theta)$,

$$T(r, \theta) = T_r(r) + T_\theta(r, \theta) = T_r(r) + \sum_{n=1}^{\infty} (A_n r^n + \frac{B_n}{r^n}) \cos n\theta + (C_n r^n + \frac{D_n}{r^n}) \sin n\theta \quad (3)$$

254 where $T_r(r)$ is the mean temperature of all the cells at r . Using this and the general
 255 expression, in cylindrical coordinates, for the thermal conduction in a cylinder whose
 256 thermal conductivity is constant and under a steady state, obtained from the Laplace
 257 equation (Necati Özisik, 1993), the temperature profile can be expressed as a harmonic
 258 Fourier series for each set of cells in a certain axial division, see Appendix A.

259 Splitting the temperature distribution in the addition of $T_r(r)$ and $T_\theta(r, \theta)$, the thermal
 260 stress problem can be studied as the superposition of both contributions (Goodier, 1957).
 261 It is an unidimensional problem while working with T_r and a bidimensional one while doing
 262 so with T_θ . Such methodology is also valid if only the inner and outer temperature profiles
 263 are known, see Appendix A.

264 The stresses due to the radial temperature field are:

$$\sigma_r^r(r, \theta) = \frac{E}{1-\nu} \left[\left(1 - \frac{\alpha^2}{r^2} \right) \frac{1}{2} \alpha \bar{T} - \frac{1}{r^2} \int_a^r \alpha_r T_r r \, dr \right] \quad (4)$$

$$\sigma_\theta^r(r, \theta) = \frac{E}{1-\nu} \left[\left(1 + \frac{\alpha^2}{r^2} \right) \frac{1}{2} \alpha \bar{T} + \frac{1}{r^2} \int_a^r \alpha_r T_r r \, dr - \alpha_r T_r \right] \quad (5)$$

265 Here, α is the linear thermal expansion coefficient and can be evaluated for the temperature
 266 of each cell, E is the Young modulus, and ν is the Poisson coefficient. In the development of
 267 such equations, the Young modulus and the Poisson coefficient have been considered
 268 constant for a whole set of cells in a certain axial division in order to obtain the stresses

expressions due to T_r . The subscript r in α indicates that it has been evaluated for the radial temperature field. The mean product of α and the temperature field is:

$$\overline{\alpha T} = \frac{2}{b^2 - a^2} \int_a^b \alpha T r dr \quad (6)$$

On the other hand, the stresses due to the circumferential temperature profile are:

$$\sigma_r^\theta(r, \theta) = \frac{r}{a^2 + b^2} (B_1 \cos \theta + D_1 \sin \theta) \frac{\alpha E}{2(1 - \nu)} \left(1 - \frac{a^2}{r^2} \right) \left(1 - \frac{b^2}{r^2} \right) \quad (7)$$

$$\sigma_\theta^\theta(r, \theta) = \frac{r}{a^2 + b^2} (B_1 \cos \theta + D_1 \sin \theta) \frac{\alpha E}{2(1 - \nu)} \left(3 - \frac{a^2 + b^2}{r^2} - \frac{a^2 b^2}{r^4} \right) \quad (8)$$

$$\tau_{r,\theta}(r, \theta) = \frac{r}{a^2 + b^2} (B_1 \sin \theta - D_1 \cos \theta) \frac{\alpha E}{2(1 - \nu)} \left(1 - \frac{a^2}{r^2} \right) \left(1 - \frac{b^2}{r^2} \right) \quad (9)$$

The stresses σ_r^θ , σ_θ^θ and $\tau_{r,\theta}$ are obtained assuming that the mechanical properties are constant. This is not the case in real applications, being more common to work with materials with temperature dependent properties, which will result in a small deviation in the results, depending on how influential such properties variations are. However, these divergences are admissible since these stresses are an order of magnitude lower compared to the axial stress. Also, the complexity of the general expressions of these equations, obtained from the Navier equations for three-dimensional thermoelastic problems (Eslami et al., 2013), makes the previous simplification preferable.

Superimposing the solutions presented above, σ_r and σ_θ are:

$$\sigma_r(r, \theta) = \sigma_r^r + \sigma_r^\theta \quad (10)$$

$$\sigma_\theta(r, \theta) = \sigma_\theta^r + \sigma_\theta^\theta \quad (11)$$

2.3 Axial thermal stresses

For a certain axial division of the tube, the stresses in the z axis can be expressed as:

$$\sigma_z(r, \theta) = \sigma_z^b + \sigma_z^\nu \quad (12)$$

The first term is related to the stresses due to the expansion in the axial direction and the bending of the tube, that in this case is understood as a beam. It is directly tied to the boundary conditions of the tube studied.

The second term comes from the application of the Hook law. It is related to the stresses in the tube cross-section obtained in Section 2.2, as presented by (Goodier, 1957).

The tube is considered to be non-homogeneous, thus the properties of the beam material are temperature dependent. See Appendix B for homogeneous properties. In case of non-homogeneous beams, is more appropriate to work with the neutral axis coordinate system (x^*, y^*, z) , instead of the one placed in the centroid (x, y, z) , see Figure 2. Its placement is such that takes into account the weighted section quantities, where E_0 is the reference Young modulus, which can be regarded as the one for the homogeneous properties case:

$$A^* = \int_A \frac{E}{E_0} dA \quad (13)$$

$$x^* = x + \bar{x} = x + \frac{1}{A^*} \int_A \frac{E}{E_0} x dA \quad y^* = y + \bar{y} = y + \frac{1}{A^*} \int_A \frac{E}{E_0} y dA$$

294 The first term in (12), σ_z^b , is introduced by (Noda et al., 2003) for a free bending beam so
 295 an additional stress in the expression they present must be considered, the one due to the
 296 beam supports. The complete term for the case studied in this work results as:

$$\sigma_z^b(r, \theta) = -\alpha ET + \frac{E}{E_0} \frac{P_M + P_T}{A^*} + \frac{E}{E_0} \left(y^* \frac{E_0}{\rho_{y^*}} + x^* \frac{E_0}{\rho_{x^*}} \right) \quad (14)$$

297 In this case, the properties are evaluated for the temperature of each cell. The term σ_z^b
 298 shown in (14) includes:

- 299 - The stresses due to the local thermal expansion.
- 300 - The stresses due to mechanical forces, P_M , trying to expand the tube. In this case,
 301 they are null due to the free axial expansion hypothesis.
- 302 - The stresses due to a virtual axial force P_T that appears if the tube is free to expand
 303 in order to counteract the expansion axial force.
- 304 - The stresses due to the bending moments in the tube.

305 Being:

$$P_T = \int_A \alpha ET dA \quad (15)$$

$$\rho_{x^*} = \frac{I_x^* I_y^* - I_{xy}^{*2}}{M_y^* I_x^* - M_x^* I_{xy}^*} E_0 \quad \rho_{y^*} = \frac{I_x^* I_y^* - I_{xy}^{*2}}{M_x^* I_y^* - M_y^* I_{xy}^*} E_0 \quad (16)$$

$$I_x^* = \int_A \frac{E}{E_0} y^{*2} dA \quad I_y^* = \int_A \frac{E}{E_0} x^{*2} dA \quad (17)$$

$$I_{xy}^* = \int_A \frac{E}{E_0} x^* y^* dA$$

306 The moment inducing axial stresses via the bending radii (16) is the addition of a thermal
 307 bending moment and a mechanical bending moment:

$$M^* = M_T^* + M_M \quad (18)$$

308 It presents two components, one in the x^* axis (or x), M_x^* , and one in the y^* axis (or y), M_y^* .
 309 Therefore, this moment results in the following stresses:

- 310 - The stresses due to a virtual thermal bending moment that considers the free
 311 rotation of the ends of the tube, being able to bend freely. It constitutes the moment
 312 annulment (MA) when the tube is not restricted. This is valid as long as the
 313 Bernoulli-Euler hypothesis is true, fulfilling that the normal plane to the bending
 314 radius of the neutral axis before the deformation remains plane and normal to the
 315 neutral axis after the deformation.

316 The thermal bending moment can be calculated as:

$$M_{T,x}^* = \int_A \alpha E T y^* dA \quad M_{T,y}^* = \int_A \alpha E T x^* dA \quad (19)$$

317 - The stresses produced by the mechanical bending moment due to the reaction
 318 forces in the supports of the tube. Notice that the mechanical moment has been
 319 written with the centroid coordinate system, since it is independent of the
 320 coordinate system chosen due to the fact that is caused by a force contained in the
 321 cross section plane. It allows us to consider the halfway scenarios between:

- 322 • The free thermal bending of the tube, that would result in this part of the
 323 term being zero, given the absence of intermediate supports.
- 324 • The tube being kept completely straight, GPS. This part of the term would
 325 counteract completely the stresses due to the virtual thermal bending
 326 moment.

327 The mechanical moments due to the supports are obtained as:

$$M_{M,x}(z) = \sum_{j=1}^N [R_{j,y}(z - (j - 1)d)^+] \quad M_{M,y}(z) = \sum_{j=1}^N [R_{j,x}(z - (j - 1)d)^+] \quad (20)$$

328 Here + indicates that only the positive values are considered, being the parenthesis result
 329 substituted by zero in the rest of the cases. There are N reaction forces R_j , equal to the
 330 number of supports, including the ones placed at both ends. These forces, just as the
 331 mechanical moments, are independent from the coordinate system chosen since the z
 332 coordinate at which the reactions occur is common to both the centroid system and the
 333 neutral axis system.

334 The reaction forces R_j can be obtained knowing that the displacement at the supports, which
 335 are placed spaced a distance d from each other, is null. The equation governing the
 336 displacement is calculated as:

$$\frac{d^2 \delta(z)}{dz^2} = -\frac{1}{\rho(z)} \quad (21)$$

337 Solving such differential equation, the tube displacement is obtained (22), which is zero at
 338 the supporting points. Each axial division of the tube is subject to thermal and mechanical
 339 moments that differ from one to another, so the number of different bending radii (m) in
 340 the tube is equal to the number of axial divisions considered to study it. Like the equation
 341 for the mechanical moments, the expression for deformation will present two forms, one
 342 for each axis of the cross section: δ_x and δ_y .

$$\delta(z) = - \left[\frac{z^2}{2\rho_{T,1}} + \sum_{i=2}^m \frac{(z - (i - 1)\Delta z)^{2,+}}{2} \left(\frac{1}{\rho_{T,i}} - \frac{1}{\rho_{T,i-1}} \right) + \sum_{j=1}^N \frac{R_j(z - (j - 1)d)^{3,+}}{6} \right] + K_1 z + K_2 \quad (22)$$

343 Here ρ_T is the thermal bending radius depending only on the thermal bending moments,
 344 not the mechanical ones, see equations (23). To represent consistently the displacement in
 345 the whole beam, the thermal bending radius has been referred to the centroid (x, y) of the
 346 cross section instead of the neutral axis (x^*, y^*) and is calculated as:

$$\begin{aligned}\rho_{T,x} &= \rho_{T,x}^* - \bar{x} = \frac{(I_y^* I_x^* - I_{xy}^{*2}) E_0}{I_x^* M_{T,y}^* - I_{xy}^* M_{T,x}^*} - \frac{1}{A^*} \int_A \frac{E}{E_0} x \, dA \\ \rho_{T,y} &= \rho_{T,y}^* - \bar{y} = \frac{(I_y^* I_x^* - I_{xy}^{*2}) E_0}{I_y^* M_{T,x}^* - I_{xy}^* M_{T,y}^*} - \frac{1}{A^*} \int_A \frac{E}{E_0} y \, dA\end{aligned}\quad (23)$$

Note that the thermal bending radii ρ_T corresponds to the bending radii ρ in the case of MA, as studied by (Conroy et al., 2018). In the case of the GPS, with the tube bending prevented, the bending radius is infinite since it remains straight, (Logie et al., 2018).

The constants K_1 and K_2 in (22) are calculated by adding two more equations to the system. These two equations are the ones for the null sum of reaction forces and null sum of mechanical moments. Additionally, once the reactions R_j and the constants K_1 and K_2 are known, the displacement along the whole tube can be obtained.

$$\begin{aligned}\delta(z) &= 0 \text{ in } z = 0, d, 2d, \dots, (N-1)d \\ \sum_{j=1}^N R_j &= 0 \\ \sum_{j=1}^N R_j (N-j)d &= 0\end{aligned}\quad (24)$$

On the other hand, the second part of equation (12), the stresses in the tube cross-section is:

$$\sigma_z^v(r, \theta) = v(\sigma_r + \sigma_\theta) \quad (25)$$

The stresses in the r and θ coordinates were obtained separately in the previous section, equations (10) and (11). With all of this, the axial stresses considering variable properties of the tube can be expressed as:

$$\sigma_z(r, \theta) = v(\sigma_r + \sigma_\theta) - \alpha ET + \frac{E}{E_0} \frac{P_T}{A^*} + \frac{E}{E_0} \left(y^* \frac{E_0}{\rho_y^*} + x^* \frac{E_0}{\rho_x^*} \right) \quad (26)$$

3. Central receiver with a non-symmetrical temperature distribution

The receiver analysed (Fig. 3), with a nominal thermal power of 120 MW, is a cylindrical external tubular receiver of 10 m high (H) and 8.4 m of diameter (D). The cylindrical frame is surrounded by 18 vertical panels (Np), each one of them being constituted by 62 tubes (Nt) that are connected by an inlet and outlet collector. The tube external diameter (d_e) is 2.21 cm, with a thickness (th) of 1.2 mm. The separation between tubes belonging to the same panel is the 8% of their external diameter. This distance falls inside the range of typical values, between 1 to 2.5 mm according to (Litwin, 2002). They are manufactured in Inconel 625, whose properties are obtained from the ASME BPVC Code (ASME, 2010) and they are covered with a black Pyromark coating, to increase the insolation absorptance (Persky and Szczesniak, 2008). The tubes are guided by a series of clips, placed periodically along its whole length in order to avoid an excessive bending and warpage. The separation between clips is 2 m, making it a total of 6 supports in each tube.

Figure 3. Schematic of the receiver studied.

372 The HTF used in this receiver is solar salt, with an inlet temperature of 290 °C and an outlet
 373 temperature of 565° C. The properties of the HTF have been calculated with the equations
 374 presented by (Zavoico, 2001). The mass flow rate of the solar salt is a fixed value since the
 375 receiver thermal power and inlet and outlet temperatures for the salt have been restricted.
 376 The solar salt is divided into two flow paths, one oriented to the east and the other to the
 377 west. In both paths the salt changes its flow direction alternatively from panel to panel (the
 378 red panels in Figure 3 represent the upwards flow and the green ones the downwards flow),
 379 with a cross between paths at the sixth panel. The solar noon of the Spring Equinox has been
 380 chosen as the design point.

Figure 4. Schematic temperature profile evolution of the HTF. The HTF is heated as it flows through the panels tubes, from panel a to panel b.

The operating conditions in a SPT plant imply that the tubes are subject to a non-symmetrical temperature field with respect the y axis. This is due to the fact that the heat flux being reflected from the heliostats field to the receiver tubes is not uniform on the surface of any of them and will cause them to bend not only in the y axis direction but also in the x , which may lead to undesired contact between adjacent tubes of a panel. The temperature profile asymmetry is greater in the first and last tubes from each panel. This is especially significant when studying the inlet vertical division of the last tube or the outlet division from the first tube of a panel. Such temperature asymmetry for those axial cells is not only due to the non-symmetrical heat flux received but also because the HTF flows in the opposite direction in adjacent panels, as depicted in Figure 4. The HTF goes through panel a, then through panel b. When referring to the last (62a) and initial (1b) tubes of two adjacent panels, on one side they find a tube from the same panel (61a for tube 62a or 2b for tube 1b), with the HTF flowing in the same direction and with a temperature evolution very similar. However, on the other side they find a tube from another panel with the HTF flowing in the opposite direction (1b and 62a) and very different temperature.

4. Results and discussion

The results of the analytical method and the numerical simulations are compared for a non-uniform heat flux over the receiver design that was presented in Section 3. The case of free bending is studied in just one axial division in order to verify the behaviour of the analytical model during operation. Then, once the correct performance of analytical method has been proven in that scenario, the influence in the stresses of the support clips guiding the tubes is studied, as well as the bending of the tube.

The tube selected in both cases is the final one of the first panel, facing north, belonging to the eastern or western flow paths indistinctly, since the design point selected presents N-S symmetry. This panel is chosen due to the fact that the two northern panels are the ones presenting the most asymmetrical heat flux with respect the y axis for the design point of this study. As stated in Section 3, among the tubes of a same panel, the initial and final are the ones suffering for a greater temperature asymmetry. Since the first tube of this particular panel finds the tube belonging to the adjacent panel at the same thermal conditions (N-S symmetry), the last tube of the panel has been preferred.

The non-symmetrical heat flux scenario is studied for a receiver similar to the Gemasolar one, using the thermal FGM developed by (Rodríguez-Sánchez et al., 2018). This allows us to obtain the different stresses and displacements that such heat flux causes in the receiver tubes. Evolving from the *coarse grid model* (CGM) presented in (Rodríguez-Sánchez et al., 2014b) -that included vertical and circumferential divisions of the tubes but only takes into account one representative tube per panel-, (Rodríguez-Sánchez et al., 2018) developed a *fine grid model* (FGM) including all of the tubes in the receiver. The initial data for the model are the geometry of the receiver, the ambient conditions, the incident radiation in each cell, calculated with FluxSPT, and the mass flow rate and inlet temperature of the salts. An initial estimation for the wall temperature of the tube is required. Calculating the radiation losses, the external and internal convection losses and the fouling resistance, the mean temperature of the fluid in each axial division of the tube can be obtained. The outlet salts temperature must be checked since the limit is set at 565°C. The wall temperature is recalculated and this process is repeated until the salts leave the receiver at the desired temperature.

4.1 Initial verification: analytical vs numerical model

In order to compare the results of the analytical and numerical methods, a single tube under free bending conditions (with the stress term of MA) is initially analysed. In this case, since

the effect of the clips is not considered, only one axial division of 0.5 m will be selected, Figure 5, instead of the whole tube. This is so for simplification reasons, necessary when working with the numerical method.

Figure 5. Axial division of the receiver tube under free bending conditions.

The Abaqus model was built with 8-node linear solid elements. The calculation was carried out by means of a static analysis using Abaqus/standard. Only thermal loads were applied introducing the temperature field obtained from the FGM to Abaqus. The default boundary conditions were used since the tube was allowed to freely expand. A mesh sensitivity analysis was conducted for a 2D tube case varying the number of elements through the tube thickness from 4 to 24 and keeping the aspect ratio equal to 1. Based on this analysis the mesh size was set to 12 elements through tube thickness, which obtained a stress variation lower than 1% compared to the finest mesh size.

When deciding what vertical division to study, two possibilities were presented: the one with the greater asymmetry, providing bending moments in both x and y axis, or the one with the greater temperature gradient, so the maximum stresses are studied, Figure 6. Since this is a verification of the stresses and the goal is not yet to study the bending of the tube in both x and y directions, the section presenting the greater temperature gradient, placed at the middle length of the tube (Figure 6), is selected as the representative case.

Figure 6. Average temperature difference between the right and left halves of the tube cross section along the tube length, representing the asymmetry (blue solid line). Maximum temperature gradient in the cross section along the tube length (orange solid line).

Using the outer and inner temperature distribution obtained from the thermal model, the whole temperature profile in the selected z section of such tube is calculated (Fig. 7). It is

obtained both for the analytical expressions and FEM simulations considering that the thermal conductivity of the tube differs among cells of a different θ coordinate but is constant in the radial cells, $k=k(\theta)$. This allows us to use the stress equations developed in Section 2 under the assumption of constant thermal conductivity, because the heat flux, modifying the temperature field, is considered to have only in the radial component at a certain θ coordinate.

Figure 7. Cross sectional temperature distribution

For the temperature distribution of Figure 7, the different stresses have been calculated with the analytical and numerical method. Using the analytical method, these stresses have been obtained for temperature dependent (Case a) properties of the tubes material according to (ASME, 2010) with the considerations regarding certain properties depending on the stress component presented in Section 2 and using equations (9), (10), (11) and (12). The temperature independent properties scenario (Case b) has also been studied, using equations in Appendix B, evaluating the material properties in the whole cross-section at its mean temperature. In this case the temperature distribution differs slightly from the one presented in Figure 7, a maximum of 5.3 K, since the thermal conductivity is considered constant in the whole cross section. On the other hand, the numerical results have been calculated using temperature dependent properties.

The analytical results for both scenarios are compared with the numerical results by depicting the difference between the numerical results (FEM) and the analytical ones (subscripts v for variable properties and c for constant properties) over the maximum stress in each case obtained with the numerical method. These results are depicted in Figures from 8 to 11. The maximum stress in each case are: 2.37 MPa for σ_r , 78.74 MPa for σ_θ , 182.82 MPa for σ_z and 165.64 MPa for σ_{VM} .

Figure 8. σ_r stress for a) variable properties and b) constant properties.

Figure 9. σ_{θ} stress for a) variable properties and b) constant properties.

Figure 10. $\sigma_{z,MA}$ stress for a) variable properties and b) constant properties.

Figure 11. $\sigma_{VM,MA}$ stress for a) variable properties and b) constant properties.

It can be seen that the analytical method using variable properties, Case a, allows a precise calculation of the tube axial and equivalent stresses, since the results barely differ from the ones obtained with the numerical method. However, the simplification of using constant properties in the analytical model, Case b, should be discarded given that for the most critical stress for the tube endurance (σ_z and σ_{VM}) the deviation is four times higher than the one resulting of taking into account temperature dependant properties: the difference over the maximum stress is just above a 5% with temperature dependent properties and over a 20% when they are temperature independent. Such high difference is undesirable especially when obtaining the rupture time to estimate the receiver lifespan, since it depends on the logarithmic stress as presented by (Eno et al., 2008).

The small deviations in Case a are due to the fact that the expressions for the stress in the cross-section, σ_r , σ_θ and τ , consider most of the properties involved as constant, with just the exception of the thermal expansion coefficient and thermal conductivity in the stress due to the radial temperature profile, σ^r , likewise was presented in Section 2.2. However, the axial stress σ_z^b does indeed takes into account the temperature dependence of the mechanical and thermal properties of the material, as developed in Section 2.3. Given the accuracy of the axial stress obtained with temperature dependent properties, the Von Mises stress is not affected by the errors in the cross section stress since the axial component is the most prominent.

Besides the accuracy obtained with the analytical method with variable properties, the computational cost is considerably less than in the FEM simulations: 5.5 seconds against to 1800 seconds. The analytical simulations were performed with a standard PC of 8 GB of RAM and Intel® Core™ i7-7700 CPU @ 3.6 GHz 3.6 GHz processor, while the numerical ones were performed using a workstation of 128 GB @ 2666 GHz of RAM and Intel® Xeon® Silver 4114 CPU \ 2.2 GHz 2.2 GHz processor.

Therefore, when considering the variable properties proposed in this work, the analytical model is a fast and reliable alternative to study the stress in the receiver, which would simplify a further creep-fatigue analysis and allow a more precise prediction of the receiver lifespan.

4.2 Guided tube

Once the accuracy of the analytical expressions has been proven, the whole tube selected in Section 4 can be studied using the analytical method. This will save us a considerable time with respect to the numerical simulations for the whole tube. It also allows us to obtain the

stresses and displacements accurately, as checked in the previous Section.

The tube is now considered to be guided with the clip supports, emulating the ones in real life operating receivers of SPTs. The number of clips selected is 6. The receiver is subject to the non-symmetrical temperatures field discussed in Sections 3 and 4, so bending in both cross-section axis is expected for the tube studied. This asymmetry is greater at tube inlet, as discussed in Section 3. The temperature difference between the right half of the tubes cross section (from 0° to 180°, with the sun at 0°) and the left half is illustrated in Figure 12.a for the first axial division. The decrease of such asymmetry with the z coordinate can be seen in Figure 12.b, that depicts the temperature difference between the right half of the tube outer wall and the left one, for the tubes whole length.

Figure 12. a) Temperature difference between the right and left halves of the first axial division cross section and b) Temperature difference between the right and left halves of the tube outer wall.

However, the temperature field asymmetry and the temperature dependent properties considered for the material do not result in a remarkable shift of the neutral axis respect to the centroid axis. The distances between axes of both coordinate systems are: $\bar{x} = -0'54 \cdot 10^{-19}$ and $\bar{y} = 0'31 \cdot 10^{-18}$. Hence, in this case, the neutral axis coincides with the centroid axis.

4.2.1 Tube displacement

Figure 13. a) Tube bending in the y axis. b) Tube bending in the x axis.

The displacements in axis x and y are depicted in Figure 13 for both constant and variable properties of the tube material. Notice that the use of constant properties of the tube translates in an underestimation of the bending which should be avoided specially when trying to determine if a collision between adjacent tubes can occur studying δ_x , and therefore warpage.

For the deviation in axis y , δ_y , the inclusion of clips dramatically reduces the bending, going from 1.5 m when the tube is only fixed at its ends to 2.71 mm in the case of clips every 2 m (Fig. 13 a). This diminishing is especially relevant to avoid undesirable great deformations of the tubes.

Regarding the deflection in axis x , as stated in Section 3, the distance between adjacent tubes in a panel is 1.8 mm. The maximum this tube bends towards its neighbour in the same panel is 0.02 mm (Fig. 13 b). As for the contact with tube in the next panel, the tube studied bends towards it a maximum of 0.08 mm in the lower part of its length, so contact does not occur on the right side of the tube cross-section either. Therefore, and since the tube with the maximum asymmetry is the one studied, it can be concluded that with this configuration of clips and this aiming strategy there is no contact between tubes for the heat flux distribution coming from the heliostats studied.

4.2.2 Maximum equivalent stress

Following the two main criteria to determine the equivalent stress in ductile materials, Von Mises and Tresca, the maximum ones for each axial division are presented in this section.

The maximum Von Mises equivalent stress has been calculated for the whole tube, both using variable properties and constant properties, Figure 14. It can be seen that the maximum stress is obtained with an undesirable underestimation in the case of constant properties, following the tendency present in the calculation of the displacements of the tube. The maximum stresses appear at the tube middle length, where the thermal gradients are greater, as depicted earlier, and correspond to the side of the tube facing the heliostats field.

Figure 14. Maximum Von Mises stress for the tube with clips, considering variable and constant properties. Maximum von Mises stress in the case of GPS and MA.

The addition of the mechanical constraints of the clips increases the Von Mises equivalent stress from the MA case, at which the tube bends freely, being the extreme scenario the GPS one, where the tube is kept completely straight with an infinite number of clips. For the distribution of clips considered, the maximum Von Mises stress resembles to the GPS one, being lesser at some points but higher in others due to the high reaction forces appearing in the support spots. This close resemblance to the case of the tube being kept completely straight matches the displacements presented in section 4.2.1, which are small compared to the length of the tube.

As it can be seen in equations (9), (10) and (11), τ , σ_r and σ_θ do not vary with the different boundary conditions scenarios (GPS, MA and clips). However, the inclusion of the clips increases the axial stress, equation (14), which is the main responsible for the value of the

Von Mises stress, given its entity in comparison to the cross section stress. Since the increment of the axial stress is due to the reaction forces introduced by the supports, such calculation can be expected to be the same in the FEM simulations and in the analytical ones. Therefore, the overall difference between the results of the numerical and analytical simulations is considerably less in the case of the tube with clips, Figure 15, than what it was for the free bending tube. The maximum difference is less than a 2% with respect the maximum stress in the case of temperature dependent properties and just over a 10% for the temperature independent scenario. Also, in comparison to the MA case, the location of the maximum Von Mises stress shifts from the inner lateral face of the tube to the outer front (facing the heliostats), while the maximum axial stress shifts from the inner lateral face to the inner rear side (facing the receiver structure).

Figure 15. $\sigma_{VM,Clips}$ stress for a) variable properties and b) constant properties.

To obtain the Tresca equivalent stress, the stresses σ_r , σ_θ and σ_z have been used since the shear stress $\tau_{r,\theta}$ is at least one order of magnitude lower than the normal stresses. With a stress tensor of such characteristics, the principal stresses σ_1 , σ_2 and σ_3 in equation (2), are virtually equivalent to the normal stresses in the cross section, σ_r , σ_θ and σ_z . For the Tresca equivalent stress, Figure 16, the difference between the case of dependent and independent properties is also noticeable. As expected, it is a more restrictive criterion than the Von Mises one. The tendencies in all the cases studied for the Tresca equivalent stress are the same that the ones obtained for the Von Mises stress though, presenting the maximum values at the centre of the tube.

Figure 16. Maximum Tresca stress for the tube with clips, considering variable and constant properties. Maximum Tresca stress in the case of GPS and MA.

The underestimation of the equivalent stress resulting from considering temperature independent properties of the material would induce great receiver overestimation-lifetime. The creep analysis depends on the Von Mises equivalent stress (Eno et al., 2008) while the fatigue analysis does the same with the Tresca equivalent stress (González-Gómez et al., 2019). Therefore, the error obtained with the use of constant properties is relevant in both cases. Error that would result in a more optimistic prediction of the receiver lifetime, which may lead to an early unexpected failure.

5. Conclusions

An analytical method to study the thermal stresses and bending of the tubes of an external tubular receiver of SPT plants has been presented. With these analytical equations the stress and displacement calculations can be performed under different assumptions: from the free bending case (MA) to the GPS. The interest of these analytical expressions lies in the consideration of temperature dependant properties and the inclusion of the clips without needing a high computational time.

The analytical model proposed in this work constitutes a fast and accurate alternative to numerical models, which are more complex and time-consuming. The comparison for the moment annulment case showed that the hoop, radial and axial stress difference with respect the numerical results, using the proposed estimation, is around 10%, 30%, 5%, respectively when considering temperature dependent properties. The high difference in the cross section stress does not affect greatly the equivalent stress since the highest component is the axial one, which presents the smallest difference. The difference for the Von Mises equivalent stress is just above a 5%. As a consequence of this, a more complete creep-fatigue analysis can be easily undertaken. On the other hand, the simplification of temperature independent properties in the analytical model presents severe deviations, therefore its use is ill-advised, at least for the case analysed.

The boundary conditions for SPTs of external tubular receivers is the intermediate between the free bending and the GPS, where the tube is guided with a series of supports periodically placed along its length, called clips. These supports prevent the displacement in the x and y coordinates, but allow the axial displacement. Therefore, the clip supports reduce greatly the tube deflection, preventing the collision with adjacent tubes and their permanent

deformation. Again, taking into account variable properties is crucial in order to not underestimate these displacements in comparison to keeping the properties as temperature independent.

The maximum Von Mises stresses with clips gravitates towards the values obtained for the GPS case. In this scenario, the maximum Von Mises stress still differs greatly when using constant properties, resulting in lesser values along the whole tube. However, there is an improvement, from a 20% of difference in the MA case to a 10% in the clips case. The difference with respect to the numerical method is also reduced using variable properties, with the difference being lesser than a 2%, in comparison to the MA results since the stress due to the clips are coincident in the numerical and analytical simulations. Such small deviation with the analytical method and temperature dependent properties makes it suitable to study the stress in external tubular receivers.

The differences observed for the Von Mises stress between when using temperature dependent and independent properties are also present in the Tresca equivalent stress, a more conservative criterion than the Von Mises one. Since the creep and fatigue analysis depend on these two equivalent stresses, the use of constant properties is discouraged as it would lead to an overestimation of the receiver lifespan.

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631 **Appendix A**

632 The coefficients of the Fourier series (3), calculated as presented by (Bijlaard et al.,
633 1968), result:

$$B_1 = \frac{4}{\pi} \frac{\int_a^b \int_0^\pi T_\theta \left(r^2 - \frac{b^2 + a^2}{2} \right) \cos \theta \, dr d\theta}{b^2 - a^2 - (b^2 + a^2) \ln(b/a)} \quad (\text{A.1})$$

$$A_1 = \frac{2}{b^2 - a^2} \left[\frac{2}{\pi} \int_a^b \int_0^\pi T_\theta \cos \theta \, dr d\theta - B_1 \ln(b/a) \right] \quad (\text{A.2})$$

$$D_1 = \frac{4}{\pi} \frac{\int_a^b \int_0^\pi T_\theta \left(r^2 - \frac{b^2 + a^2}{2} \right) \sin \theta \, dr d\theta}{b^2 - a^2 - (b^2 + a^2) \ln(b/a)} \quad (\text{A.3})$$

$$C_1 = \frac{2}{b^2 - a^2} \left[\frac{2}{\pi} \int_a^b \int_0^\pi T_\theta \sin \theta \, dr d\theta - B_1 \ln(b/a) \right] \quad (\text{A.4})$$

634 When the inner (A.5) and outer (A.6) temperature profiles are known,

$$T_i(\theta) = A_0' + \sum_{n=1}^{\infty} A_n' \cos n\theta + B_n' \sin n\theta \quad (\text{A.5})$$

$$T_e(\theta) = A_0'' + \sum_{n=1}^{\infty} A_n'' \cos \theta + B_n'' \sin n\theta \quad (\text{A.6})$$

635 where A_0' and A_0'' are the mean values of the internal temperature, $\bar{T}_i$ (at $r=a$),
636 and external, $\bar{T}_e$ (at $r=b$), respectively, calculated as:

$$\bar{T}_i = \frac{1}{2\pi} \int_0^{2\pi} T_i(\theta) \, d\theta \quad (\text{A.7})$$

$$\bar{T}_e = \frac{1}{2\pi} \int_0^{2\pi} T_e(\theta) \, d\theta \quad (\text{A.8})$$

637 The equivalence of the coefficients in equation (3) with the ones in equations (A.5) and
638 (A.6) can be easily obtained by substituting $r=a$ and $r=b$ in the circumferential temperature
639 profile Fourier series (3).

$$\left(A_1 a + \frac{B_1}{a} \right) \cos \theta + \left(C_1 a + \frac{D_1}{a} \right) \sin \theta = A_1' \cos \theta + B_1' \sin \theta$$

$$A_1' = A_1 a + \frac{B_1}{a} \quad (\text{A.9})$$

$$B_1' = C_1 a + \frac{D_1}{a} \quad (\text{A.10})$$

$$\left(A_1 b + \frac{B_1}{b} \right) \cos \theta + \left(C_1 b + \frac{D_1}{b} \right) \sin \theta = A_1'' \cos \theta + B_1'' \sin \theta$$

$$A_1'' = A_1 b + \frac{B_1}{b} \quad (\text{A.11})$$

$$B_1'' = C_1 b + \frac{D_1}{b} \quad (\text{A.12})$$

$$B_1'' = C_1 b + \frac{D_1}{b}$$

640 Appendix B

641 Since the thermal conductivity is considered constant in the whole vertical division of the
642 tube, the stresses (4) and (5) can be rewritten as:

$$\sigma_r^r(r) = K_r \frac{\alpha_r E}{2(1-\nu)} \left[-\ln\left(\frac{b}{r}\right) - \frac{a^2}{b^2 - a^2} \left(1 - \frac{b^2}{r^2}\right) \ln\left(\frac{b}{a}\right) \right] \quad (\text{B.1})$$

$$\sigma_\theta^r(r) = K_r \frac{\alpha_r E}{2(1-\nu)} \left[1 - \ln\left(\frac{b}{r}\right) - \frac{a^2}{b^2 - a^2} \left(1 + \frac{b^2}{r^2}\right) \ln\left(\frac{b}{a}\right) \right] \quad (\text{B.2})$$

643 Here K_r is:

$$K_r = \frac{(\bar{T}_i - \bar{T}_e)}{\ln(b/a)} \quad (\text{B.3})$$

644 As for the circumferential temperature field contribution, equations (7), (8) and (9) can be
645 simplified as:

$$\sigma_r^\theta(r, \theta) = K_\theta \frac{\alpha E}{2(1-\nu)} \left(1 - \frac{a^2}{r^2}\right) \left(1 - \frac{b^2}{r^2}\right) \quad (\text{B.4})$$

$$\sigma_\theta^\theta(r, \theta) = K_\theta \frac{\alpha E}{2(1-\nu)} \left(3 - \frac{a^2 + b^2}{r^2} - \frac{a^2 b^2}{r^4}\right) \quad (\text{B.5})$$

$$\tau_{r,\theta}(r, \theta) = K_\tau \frac{\alpha E}{2(1-\nu)} \left(1 - \frac{a^2}{r^2}\right) \left(1 - \frac{b^2}{r^2}\right) \quad (\text{B.6})$$

646 Being K_θ and K_τ two constants dependent on the coefficients of the Fourier series, see
647 Appendix A, for the complete temperature field (3):

$$K_\theta = \frac{r}{a^2 + b^2} (B_1 \cos\theta + D_1 \sin\theta) \quad (\text{B.7})$$

$$K_\tau = \frac{r}{a^2 + b^2} (B_1 \sin\theta - D_1 \cos\theta) \quad (\text{B.8})$$

648 Therefore, the contribution of the stresses in the cross section for the axial stress is, which
649 corresponds to the second term of equation (12):

$$\sigma_z^v(r, \theta) = K_r \frac{\nu \alpha_r E}{2(1-\nu)} \left[1 - 2\ln\left(\frac{b}{r}\right) - \frac{2a^2}{b^2 - a^2} \ln\left(\frac{b}{a}\right) \right] + K_\theta \frac{\nu \alpha E}{(1-\nu)} \left(2 - \frac{a^2 + b^2}{r^2}\right) \quad (\text{B.9})$$

650 Now that the tube is considered to be homogeneous, with properties independent of the
651 temperature, the virtual force P_T that appears when the tube is free to expand axially is

652 constant $P_T(z) = \int_A \alpha E T dA = E \overline{\alpha T}$. Thus, the first term in equation (12) can be written
 653 as:

$$\sigma_z^b(r, \theta) = E(\overline{\alpha T} - \alpha T) + E \left(r \cos \theta \frac{E}{\rho_y} + r \sin \theta \frac{E}{\rho_x} \right) \quad (\text{B.10})$$

654 For the bending radius, since E is assumed constant instead of temperature dependent, the
 655 numerator is the product EI, where I is the tube inertia moment, resulting from integrating
 656 $\int_A r^2 dA$. For a cylindrical tube:

$$I_x = I_y = \frac{\pi(b^4 - a^4)}{4} \quad (\text{B.11})$$

$$\rho_x = \frac{I_y E}{M_y} \quad \rho_y = \frac{I_x E}{M_x} \quad (\text{B.12})$$

657 Here M_x and M_y are the moments on the tube, constituted by the thermal moments and the
 658 mechanical moments. The mechanical term is the one presented in (20) and remains
 659 unchanged since it does not depend on the tube material. The thermal moment working
 660 with constant properties changes though, being obtained as:

$$M_{T,x}(z) = \int_A \alpha E T r \cos \theta dA \quad M_{T,y}(z) = \int_A \alpha E T r \sin \theta dA \quad (\text{B.13})$$

661 With all of this, the final stresses in z can be rewritten as:

$$\begin{aligned} \sigma_z(r, \theta) = & K_r \frac{\nu \alpha_r E}{2(1-\nu)} \left[1 - 2 \ln \left(\frac{b}{r} \right) - \frac{2a^2}{b^2 - a^2} \ln \left(\frac{b}{a} \right) \right] + \\ & + K_\theta \frac{\nu \alpha E}{(1-\nu)} \left(2 - \frac{a^2 + b^2}{r^2} \right) + E(\overline{\alpha T} - \alpha T) + E \left(r \cos \theta \frac{E}{\rho_y} + r \sin \theta \frac{E}{\rho_x} \right) \end{aligned} \quad (\text{B.14a})$$

662 And since the thermal expansion coefficient is constant, it can be regrouped as:

$$\begin{aligned} \sigma_z(r, \theta) = & K_r \frac{\alpha E}{2(1-\nu)} \left[1 - 2 \ln \left(\frac{b}{r} \right) - \frac{2a^2}{b^2 - a^2} \ln \left(\frac{b}{a} \right) \right] + K_\theta \frac{\nu \alpha E}{2(1-\nu)} \left(2 - \frac{a^2 + b^2}{r^2} \right) \\ & - E \alpha T_\theta + E \left(r \cos \theta \frac{E}{\rho_y} + r \sin \theta \frac{E}{\rho_x} \right) \end{aligned} \quad (\text{B.14b})$$

663

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