This is a postprint version of the following published document:

Marugán-Cruz, C., Flores, O., Santana, D., & García-Villalba, M. (2016). Heat transfer and thermal stresses in a circular tube with a non-uniform heat flux. *International Journal of Heat and Mass Transfer*, 96, 256-266

DOI <https://doi.org/10.1016/j.ijheatmasstransfer.2016.01.035>

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Heat transfer and thermal stresses in a circular tube with a non-uniform heat flux

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Abstract

This paper presents a thermal analysis of thin-wall pipes under non-uniform heat flux in the circumferential direction, with a turbulent flow in statistically stationary state inside. The temperature distribution in the solid and in the fluid is obtained using an spectral method that solves the conjugate heat transfer problem. Special attention is paid to the film temperatures and the thermal stresses on the solid, that are compared to predictions based on 1D models in which the the circumferential heat transfer is neglected. The comparison shows that, even if at sufficiently large Biot numbers ($Bi \gtrsim 0.2$) the 1D model gives a reasonable prediction of the film temperatures (less than 5% of error), the 1D model for the thermal stresses is only appropriate for very large Biot numbers ($Bi \gtrsim 10$), giving qualitatively wrong results for Biot numbers below 0.3.

Keywords: Temperature distribution, Thermal stresses, non-uniform heat flux, Biot number.

1. Introduction

Cumulative fatigue damage plays a key role in life prediction of tubular heat exchangers subjected to time varying heat fluxes. To estimate the tube thermal fatigue, the maximum thermal stress must be obtained for different operating cycles [1]. Therefore, the effect that the operating conditions and fluid properties have on the tube temperature profiles is of paramount importance from an engineering point of view.

There are many circumstances under which the tube thermal stress determination is straightforward. Such is the case of pipes under uniform thermal boundary conditions, where the temperature of the pipe is a function of the radial direction only. In such situations the thermal stress problem becomes a one-dimensional problem, and the effective stress can be obtained knowing the heat flux and the thermo-mechanical properties of the pipe [2].

On the other hand, there are applications where uniformity is not found. Concentrating solar power is a good example of such applications. In solar central receivers, for example, the front-tube section must withstand high heat flux (around 1 MW/m^2) whereas the rear-tube section is almost adiabatic, resulting in a circumferentially-varying heat flux on the outer wall of the pipe. That heat flux is transferred to a molten salt flowing inside the tube, resulting as well in a circumferentially-varying heat flux on the inner wall of the pipe. This asymmetry precludes, in principle, the use of one-dimensional solutions to obtain the peak thermal stress.

Similar working conditions can be found in other industrial applications, like parabolic trough collectors, space-vehicle heat-exchangers or in the cooling system of nuclear reactors. In all these cases, the non-uniform heat fluxes

produce radial and circumferential temperature gradients in the tube walls, which result in thermal stresses on the solid and circumferential variations of the fluid temperature at the wall (film temperature). Both, thermal stresses and film temperature, are extremely important from an engineering point of view, since the former can produce mechanical failure of the pipe and the latter controls the chemical degradation of the heat transfer fluid [3].

The prediction of heat transfer for non-uniform configurations has been the subject of many research studies in the past, using analytical methods, experiments and numerical simulations. First [4] and later [5] studied the case of a thermally developed flow with circumferential varying heat flux from an analytical point of view. They assumed a fully developed turbulent flow in the pipe, and prescribed a heat flux that was expressed as a sine or cosine series with zero mean value. The resulting temperature profiles in the fluid were obtained as a series of sines or cosines, whose coefficients vary with the Reynolds and Prandtl number of the flow. Comparing the film temperatures predicted by [4] for a sinusoidal heating with the film temperatures obtained from applying the Dittus and Boelter correlation yields a difference on the maximum film temperatures of about 50% for the typical working conditions of concentrating solar power receivers (Reynolds number $Re = 30000$, Prandtl number $Pr = 10$).

Painstaking experimental studies have also been carried out in the past. [6] investigated the local and average heat transfer coefficients of an air flow through a tube with circumferential varying wall heat flux. More recently, [7] measured the heat transfer performance of spiral tubes with molten salts as heat transfer fluid. The results from these investigations also show differences

between the experimental values of the Nusselt number and the correlations generally used in literature.

Finally other researchers have tried to tackle the problem of non-uniform heating from a numerical point of view, performing Direct Numerical Simulations (DNS) in more or less idealized configurations (fully developed turbulent flow, relatively low Reynolds and Prandtl numbers, constant fluid properties, etc.). Some of these studies focus on the evaluation of the eddy-diffusivity associated to the non-uniformity of the heat flux (see [8] or [9]). It is important to note that the thermal field in the tube and in the fluid are coupled, and that the heat transfer in the fluid strongly depends on the fluid motion (convective heat transfer). However, few studies consider the conjugate heat transfer problem, solving the temperature field in both the fluid and in the solid (see for instance [10]). There are also numerical studies of more applied cases, typically using Reynolds-Averaged Navier Stokes simulations to solve the heat transfer in the fluid. For instance, [11] reports an analysis of thermal stresses on the collector of a solar thermal plant for different configurations and [12] compares two simplified 2D models using a RANS calculation with a commercial software for the evaluation of film temperatures and thermal stresses on a solar external receiver.

In view of the above results, it is clear that the prediction of the thermal stress history for a tubular heat exchanger heated non-uniformly is a difficult problem. While simulations solving the conjugate heat transfer problem provide accurate results, they typically require additional simplifications regarding flow properties, geometries, etc. On the other hand, experimental measurement can be extremely expensive and time consuming. However,

the engineering approaches used to determine film temperatures and thermal stresses are usually based on empirical correlations derived from uniform heat flux conditions. The rationale for that simplification is that locally the conditions are uniform, with stronger temperature gradients in the radial direction than in the axial or circumferential directions. For instance, film temperatures are usually estimated using Nusselt number empirical correlations (e.g., [13] uses the Gnielinski correlation and [14] use the Dittus and Boelter correlation). Thermal stresses are customarily obtained using formulas derived for a circular tube with a radial temperature distribution (i.e., uniform heat flux in the circumferential direction), like those reported in [15], [16] and [17].

In principle, the parameter that controls whether a local approach is valid is the Biot number: the ratio between the characteristic heat flux in the radial direction and the characteristic heat flux in the circumferential direction. If we consider the conjugate heat transfer problem (solid + fluid), the heat fluxes in the radial direction (r) are controlled by the heat transfer in the solid-fluid boundary, and the heat fluxes in the circumferential direction (θ) by the conductivity of the solid. The definition of the Biot number is then

$$Bi = \frac{he}{\kappa_s}, \quad (1)$$

where h is the convective heat transfer coefficient between the fluid and the solid, e is the thickness of the tube wall and κ_s is the conductivity of the solid. For large values of Bi , the radial heat flux is dominant, and one can expect the correlations and methods based on uniform (radial) heat fluxes to be applicable locally. However, when the Bi is small these methods should be expected to fail.

The objective of the present work is to establish the range of Biot numbers where the local approach is applicable in pipes with non-uniform heating in the circumferential direction. When the local approach is applicable, the solution of the conjugate heat transfer problem can be avoided, thereby simplifying the stress analysis. To achieve this, the conjugate heat transfer problem for a circular pipe with a circumferentially varying heat flux and a fully developed turbulent flow inside is solved using a simple eddy diffusivity model and constant fluid properties.

Different fluid velocities, tube diameter and thickness have been studied, in order to analyze a range of Biot numbers spanning two orders of magnitude. Particular attention is paid to the effect of the Biot number on the film temperature and thermal stresses, and to our ability to estimate them using simple 1D models.

To some extent, the present numerical work can be considered as a continuation of previous works [4, 5, 18] which only considered the heat transfer in the fluid part. Here we add also the heat transfer problem on the solid part.

The paper is structured as follows. First, section 2 describes the formulation of the conjugate heat transfer problem, and the details of the numerical solver. Section 3 presents the results obtained in the calculations, discussing the film temperature (§3.2) and the thermal stresses on the solid (§3.3) in terms of the Biot number. Finally, conclusions are offered in section 4.

2. Physical, mathematical and numerical model

2.1. Physical model

We consider the conjugate heat transfer problem of a circular pipe subject to homogeneous heating along the axial/streamwise direction (x) and non-homogeneous heating in the circumferential direction (θ). The inner radius (diameter) of the pipe is denoted $R_i(D_i)$, while the outer radius (diameter) is denoted $R_o(D_o)$. The wall-thickness is $e = R_o - R_i$, and r is the radial coordinate.

A sinusoidal heat flux is imposed on one half of the wall of the pipe, while the other half is considered adiabatic, resulting in a heat flux on the outer surface

$$q_o(\theta) = \begin{cases} q_{max} \sin \theta & \text{if } 0 \leq \theta \leq \pi \\ 0 & \text{if } \theta > \pi \end{cases} \quad (2)$$

It is also convenient to define the total heat flux (per unit length) applied to the outer surface,

$$Q = \int_0^\pi q_o(\theta) R_o d\theta = 2q_{max} R_o. \quad (3)$$

As discussed in section 2.5, the total heat flux (Q) is kept constant for all the cases presented here, so that the maximum heat flux (q_{max}) is inversely proportional to the outer radius.

The fluid inside the tube is assumed to be flowing in a fully developed turbulent regime, both in terms of velocity and temperature. Moreover, it is assumed that the fluid has constant density (ρ), viscosity (μ), specific heat (C_p) and thermal conductivity (κ). Under these conditions, the hydrodynamic problem (fluid) is decoupled from the thermal problem (fluid + solid).

2.2. Mathematical formulation

In this section we briefly describe the mathematical formulation of the problem, which is similar to that used in [4] and [5], except that we consider the conjugate heat transfer problem to analyze the flux and temperature distributions in both the fluid and the tube.

With the assumptions detailed in the previous section (fully developed flow, constant fluid properties), the hydrodynamic problem in the fluid is decoupled from the thermal problem. In particular the mean velocity and the Reynolds stresses are functions of r , only, and depend on the Reynolds number, $Re = \rho U_b R_i / \mu$, where U_b is the bulk velocity (i.a. mean velocity averaged in time and in the cross-section of the pipe). For convenience, we use the analytic expressions given by [19] for the mean velocity profile ($U(r)$) and for the turbulent eddy viscosity (ν_T), although similar results are obtained when profiles obtained from DNS are used instead.

For a fully developed thermal problem, it is possible to show that the temperature of the fluid $T(x, r, \theta, t)$ can be decomposed in

$$T(x, r, \theta, t) = T_b(x) + \Theta(r, \theta) + \Theta'(x, r, \theta, t) = T_0 + x \frac{dT_b}{dx} + \Theta(r, \theta) + \Theta'(x, r, \theta, t) \quad (4)$$

where the bulk temperature, $T_b(x)$, is the time and cross-plane ($r - \theta$) averaged temperature, $\Theta(r, \theta)$ is the time averaged temperature fluctuation in the cross-plane, and $\Theta'(x, r, \theta, t)$ is the remaining fluctuation. For the case considered here, the bulk temperature increases linearly with x , and the bulk temperature gradient dT_b/dx is univocally determined by the total heat flux

received through the fluid-solid interface,

$$\rho C_p U_b \frac{dT_b}{dx} \pi R_i^2 = \int_0^{2\pi} \kappa \partial_r \Theta(R_i, \theta) R_i d\theta = Q, \quad (5)$$

which has to be equal to the the total heat flux received in the outer surface of the pipe, Q (defined in eq. 3).

If we model the turbulent heat flux using the concept of turbulent eddy conductivity, we can write the Reynolds-Averaged equation for the cross-plane temperature fluctuation

$$\rho C_p U(r) \frac{dT_b}{dx} = \nabla \cdot \kappa_T \nabla \Theta = \frac{1}{r} \frac{\partial}{\partial r} \left(r \kappa_T(r) \frac{\partial \Theta}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left(\frac{1}{r} \kappa_T(r) \frac{\partial \Theta}{\partial \theta} \right), \quad (6)$$

where κ_T is the turbulent eddy conductivity. In principle, κ_T is a function of r and θ . However, it has been shown that the effect of the circumferential variation of κ_T is small compared to its radial variation [20, 18, 9]. Hence, in the present paper we assume $\kappa_T = \kappa_T(r)$. Moreover, we assume that the turbulent Prandtl number $Pr_T = \rho C_p \nu_T / \kappa_T$ is equal to 1. [21] show that this approximation is appropriate when the Prandtl number $Pr = C_p \mu / \kappa$ is in the range 0.1 – 10, which is the case for the database presented here (see table 1). With these approximations, the turbulent viscosity model of [19] leads to the following expression for the turbulent eddy conductivity:

$$\frac{\kappa_T}{\kappa} = \frac{Pr}{2} \left[\left(1 + \frac{k^2 Re_\tau^2}{9} (1 - \eta^2)^2 (1 + 2\eta^2)^2 \left[1 - \exp \left(\frac{(\eta - 1) Re_\tau}{A} \right) \right]^2 \right)^{1/2} + 1 \right] \quad (7)$$

where $\eta = r/R_i$. The tunable parameters of the model are $k = 0.42$ and $A = 27$, whose values are taken from [19]. Note that the friction Reynolds

number Re_τ appearing in (7) is a known function of the Reynolds number (Re).

The temperature field in the solid $T^{(s)}(x, r, \theta, t)$ can also be decomposed in bulk ($T_b^{(s)} = T_b$), cross-averaged temperature ($\Theta^{(s)}$) and fluctuation ($\Theta^{(s)'}).$ Again, the energy conservation equation yields a linear variation of the bulk temperature with the axial coordinate, and the energy conservation in the cross plane yields the following equation for the cross-averaged temperature fluctuations,

$$0 = \nabla \cdot \kappa_s \nabla \Theta^{(s)} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \kappa_s \frac{\partial \Theta^{(s)}}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial \theta} \left(\frac{\kappa_s}{r} \frac{\partial \Theta^{(s)}}{\partial \theta} \right) \quad (8)$$

where κ_s is the thermal conductivity of the tube material.

In order to solve the conjugate heat transfer problem, we solve equation (6) for $\Theta(r, \theta)$ in $0 < r < R_i$ and equation (8) for $\Theta^{(s)}(r, \theta)$ in $R_i < r < R_o$. At the outer boundary of the solid pipe, the non-homogeneous heat flux conditions yields

$$\kappa_s \frac{\partial \Theta^{(s)}}{\partial r}(R_o, \theta) = q_o(\theta) \quad (9)$$

At the solid-fluid interface, the following coupling conditions apply

$$\kappa_s \frac{\partial \Theta^{(s)}}{\partial r}(R_i, \theta) = \kappa \frac{\partial \Theta}{\partial r}(R_i, \theta) \quad (10)$$

$$\Theta^{(s)}(R_i, \theta) = \Theta(R_i, \theta) \quad (11)$$

2.3. Numerical method

The coupled system of equations (6-11) is solved using a spectral method. We use a collocation method in which a Fourier expansion is used in the circumferential direction, and a Chebychev expansion in the radial direction. The method is implemented in MATLAB, using the formulation described

by [22] (chapter 11). We employ the Chebychev differentiation matrices from [23], that are more accurate than those from [22]. The resulting linear systems of equations are solved using the MATLAB backslash operator.

The code is validated using the data of a DNS of forced convection in a pipe [24], with uniform heating at $Re = 5300$ and $Pr = 0.7$. Using 20 modes in the circumferential direction and 51 modes in the radial direction, the error between the DNS data and the present method in the mean temperature distribution is smaller than 1%. We have also computed the Nusselt number (Nu) over a broad ranges of Re and Pr . Despite the strong assumptions in our method (constant fluid properties, simple model for the turbulent eddy diffusivity), the values obtained for Nu are in good agreement with the usual correlations found in the literature [25]: Gnielinski, Ditus-Boetter and Sieder-Tate. This result gives us confidence in the method. In addition, some of the cases computed by [4] for non homogeneous heating have been recomputed using the present method, obtaining good agreement. For the present calculations we have used 60 collocation points in the circumferential direction, and 150 collocation points in the radial direction. Further refinement of the grid did not result in any appreciable variation of the results.

2.4. Simplified 1D model

As noted in the introduction, we are interested in comparing the results obtained from solving the complete 3D conjugate heat transfer problem with the results obtained from a simplified model based on a local approach, that accounts only for conduction on the radial direction. For that model we will use the Gnielinski (experimental) correlation [26] for the convective heat

transfer coefficient, h :

$$Nu = \frac{h D_i}{\kappa} = \frac{f/8 (Re_{D_i} - 1000) Pr}{1 + 12.7 \sqrt{f/8} (Pr^{2/3} - 1)}, \quad (12)$$

where Nu is the Nusselt number and Re_{D_i} is the Reynolds number based on the bulk velocity and the inner diameter of the pipe. The Darcy friction factor, f , can be obtained using the Petukhov correlation [27]:

$$f = (0.790 \log(Re_{D_i}) - 1.64)^{-2}. \quad (13)$$

The determination of the model temperature Θ_{1D} is performed as follows. Once Nu is known from Re and Pr , it is possible to relate the film temperature in the fluid ($\Theta(R_i, \theta)$) to the heat flux in the outer wall ($q_o(\theta)$). To that end, we use the convection coefficient $h = q_i(\theta)/\Theta(R_i, \theta)$ appearing in the definition of Nu , where $q_i(\theta)$ is the heat flux through the fluid-solid interface. Note that the energy conservation in the solid implies that

$$\int_0^{2\pi} q_o(\theta) R_o d\theta = \int_0^{2\pi} q_i(\theta) R_i d\theta. \quad (14)$$

Then, the assumption of conductive heat transfer in the radial direction only implies that $q_o(\theta) R_o = q_i(\theta) R_i$, so that the the film temperature of the 1D model is

$$\Theta_{1D}(R_i, \theta) = \frac{q_i(\theta)}{h} = \frac{q_o(\theta) R_o / R_i}{h}, \quad (15)$$

with h given by eq. (12) and $q_o(\theta)$ by eq. (2).

Note that in equation (15) the variation of Θ_{1D} with the circumferential coordinate is given by the circumferential variation of $q_o(\theta)$, while the convective heat transfer coefficient h is constant. In that sense, the present 1D (o local) model can also be understood as a model based on the averaged heat transfer of the pipe.

2.5. Cases studied

We consider the heat transfer and absorption characteristics for different fluids (Pr number range from 0.7 to 10), different flow regimes (Re numbers vary from 2500 to 42000) and different wall thickness ($e/R_i = 0.085$ and 0.183). In all cases we consider a ratio of thermal conductivity between fluid and solid equal to $k_s/k = 37.5$, typical of concentrating solar power applications¹. As mentioned in the introduction, the total heat rate imposed on the outer wall has been kept constant in order to maintain the same bulk temperature gradient dT_b/dx in all cases (see eq. 5).

Table 1 shows the main parameters of all the cases computed. The table also includes the values of the *average* Bi number, where the h in equation (1) has been estimated using the 1D model (Gnielinski correlation for Nu [26]). Note that, for the range of Re , Pr and e/R_i considered here, the average Biot number Bi varies by roughly two orders of magnitude (between 0.019 and 1.44).

It is also possible to define a *local* Biot number, using the value of h obtained from the present calculations at the position of maximum incident heat flux,

$$Bi_{max} = \frac{eh|_{\theta=\pi/2}}{\kappa_s} = \frac{e}{\kappa_s} \left(\frac{\kappa}{\Theta(r, \theta)} \frac{\partial \Theta}{\partial r} \right)_{r=R_i, \theta=\pi/2} \quad (16)$$

The comparison between these two numbers is shown in figure 1, where it can be observed that the difference between them is relatively small (less than 30%) and decreases as Pr increases. Since from an engineering point

¹ $k_s/k = 37.5$ corresponds to using HITEC (molten nitrate salt, 7% NaNO_3 , 53% KNO_3 , 40% NaNO_2) as heat transfer fluid in a stainless steel pipe.

Figure 1: Ratio between the *local* Biot number, Bi_{max} , and the *average* Biot number, Bi . Black symbols, $Pr = 0.7$. Red symbols, $Pr = 3$. Blue symbols, $Pr = 10$. Circles, thick wall, $e/R_i = 0.183$. Squares, thin wall, $e/R_i = 0.085$. Closed symbols, $D_1 > 0$. Open symbols, $D_1 < 0$. See section 3.3 for the definition of D_1 . Reynolds numbers increase from left to right for each serie.

of view the average Bi number is more accessible (i.e., easier to compute), we will mainly use that Biot number in the rest of the paper. In any case, the use of Bi_{max} does not qualitatively change the results presented, nor the conclusions.

Case	Re	Re_τ	Pr	e/R_i	Bi
i	2532	180	0.7	0.085	0.019
ii				0.183	0.041
iii			3	0.085	0.034
iv				0.183	0.073
v			10	0.085	0.053
vi				0.183	0.114
vii	5815	360	0.7	0.085	0.038
viii				0.183	0.082
ix			3	0.085	0.074
x				0.183	0.160
xi			10	0.085	0.118
xii				0.183	0.255
xiii	8528	500	0.7	0.085	0.051
xiv				0.183	0.111
xv			3	0.085	0.103
xvi				0.183	0.223
xvii			10	0.085	0.167
xviii				0.183	0.361
xix	18863	1000	0.7	0.085	0.095
xx				0.183	0.205
xxi			3	0.085	0.202
xxii				0.183	0.437
xxiii			10	0.085	0.337
xxiv				0.183	0.729
xxv	41184	2000	0.7	0.085	0.174
xxvi				0.183	0.375
xxvii			3	0.085	0.389
xxviii				0.183	0.841
xxix			10	0.085	0.666
xxx				0.183	1.440

Table 1: Summary of the parameters of the simulations. $Re = \rho U_b R_i / \mu$ is the Reynolds number, $Re_\tau = \rho u_\tau R_i / \mu_\tau$ is the turbulent Reynolds number, $Pr = C_p \mu / \kappa$ is the Prandtl number, e/R_i is the wall thickness non-dimensionalized with the inner radius and Bi is the *average* Biot number, calculated using the correlation proposed by Gnielinski [26].

3. Results and discussion

3.1. Heat flux

As was previously explained, a sinusoidal heat flux was imposed on one half of the wall of the pipe (*front side*), while the other half was considered adiabatic (*rear side*) (see eq. 5). That heat flux is shown in Figure 2 (red solid line), non-dimensionalized with the maximum heat flux q_{max} . Note that this maximum heat flux depends on the case: for two cases with the same Reynolds and Prandtl number and different wall thickness, the magnitude q_{max} will be affected by a factor of R_i/R_o .

The heat flux distribution in the inner wall, $q_i(R_i, \theta)$, obtained from the present calculations is also shown in figure 2 for two cases with extreme Biot numbers: the blue empty-squares for case i ($Re_{D_i} \simeq 5000$, $Pr = 0.7$, thin wall $e/R_i = 0.085$, low $Bi = 0.019$), while the blue filled-squares correspond to the case xxx ($Re_{D_i} \simeq 82000$, $Pr = 10$, thick wall $e/R_i = 0.183$, large $Bi = 1.44$). It is worth noting that for the lowest Bi , the maximum heat flux through the inner wall is actually smaller than q_{max} , due to the redistribution of heat towards the adiabatic side of the tube. In the case of high Bi most of the heat transfer on the tube occurs in the radial direction, and the heat flux in the inner wall is larger by a factor close to $R_o/R_i = 1 + e/R_i \approx 1.2$.

Figure 2 also includes the heat flux distribution in the inner wall using the simplified 1D model (black circles in the figure), to compare it with the present results. As expected, assuming that there is no heat conduction on the circumferential direction leads to an overestimation of the maximum heat flux on the inner wall and to an underestimation of the heat flux on the opposite side. This difference is larger for the low Bi case.

Figure 2: Circumferential variation of heat flux on the outer and inner walls of the tube. The heat flux imposed on the outer surface, q_o/q_{max} , is the red solid line. The filled symbols correspond to the thicker tube (case xxx) and the empty symbols to the thinner tube (case i). The squares are the heat flux distribution in the inner wall obtained with the spectral method, $q_i(R_i, \theta)/q_{max}$, and the circles correspond to the inner distribution of the heat flux assuming no conduction in the circumferential direction in the tube wall, $q_{i,r}/q_{max}$.

Figure 3: Non-dimensional mean temperature, Θ/T_{ref} , in the cross plane for a) Case i $Re \simeq 2500$ ($Re_\tau = 180$), $Pr = 0.7$ and $e/R_i = 0.085$, $Bi = 0.038$ b) Case xxx $Re_R \simeq 41000$ ($Re_\tau = 2000$), $Pr = 10$ and $e/R_i = 0.183$, $Bi = 2.879$. Note that the scale is different in each panel.

3.2. Temperature profile

The importance of the Biot number on the temperature distribution is analyzed in this section, paying special attention to its effect on the film temperature.

Figure 3 shows the time averaged temperature fluctuation in the cross-plane in the fluid, Θ , and in the solid, $\Theta^{(s)}$. In both panels, the temperature fluctuations have been non-dimensionalized using the reference temperature proposed by [4],

$$T_{ref} = R_i \left. \frac{\partial \bar{\Theta}}{\partial r} \right|_{r=R_i} = \frac{R_i \bar{q}_i}{\kappa}, \quad (17)$$

where $\bar{\Theta}$ and $\bar{q}_i$ are the circumferentially averaged temperature fluctuation and heat flux at the inner wall, respectively.

As in figure 2, the left panel of figure 3 corresponds to the lowest Biot

number (case i, thin wall), whereas the right panel corresponds to the highest Biot number (case xxx, thick wall). The color scale is different in each panel, which for the same incident heat flux it would result in a higher temperature in the left panel (where the Re , and hence the flow velocity, is smaller).

In both cases the temperature on the solid is maximum where the incident heat is maximum in the outer wall. Also, there are important temperature gradients in the circumferential direction within the solid. However, for the high Bi case, those gradients are small compared to the temperature gradient in the radial direction (between the inner and outer wall of the pipe). Temperatures in the fluid are roughly uniform, except in the thermal boundary layers near the hot side of the pipe. It is clear from the figure that the thermal boundary layers are much larger in case i (figure 3a) than in case xxx (figure 3b), consistent with the variation in Re and Pr between the two cases. Since the heat flux is roughly the same in both cases, this yields a larger temperature jump in the fluid for the case with the thicker thermal boundary layer.

Figure 4 compares the film temperature from the present calculations (dashed lines) with the simplified model (solid lines), eq. (15). The difference between the film temperature, $T(r = R_i)$, and the bulk temperature, T_b , at that section, $T(r = R_i) - T_b$ has been non-dimensionalized with T_{ref} . It can be noticed that in general, the 1D model underestimates the film temperature on the hot side, while it overestimates it on the adiabatic side (note that for the 1D model, the film temperature in the adiabatic side has to be equal to the bulk temperature). The only exception are the cases with the lowest Reynolds numbers, $Re = 2532$, in which the film temperature is

overestimated by the 1D model also in the hot side.

Interestingly, figures 4(*a-c*) show that the difference between the film temperatures predicted by the 1D model and the spectral method decreases as the Re and Pr increases, even if the Gnielinski correlation is expected to work for the whole range of Re and Pr considered here [28]. Note that as Re and Pr increase, the Bi also increases, suggesting that the improvement of the agreement in the prediction of the film temperature is indeed related to the increasing dominance of the radial heat flux over the circumferential heat flux. This is confirmed in figure 4(*d*), where the relative differences between the peak film temperatures predicted by the spectral and 1D models are plotted for all the cases in table 1 as a function of the Biot number. It can be observed that for $Bi \gtrsim 0.2$ the error between the spectral model and the 1D model is smaller than 5%, which provides a rule of thumb for the applicability of 1D models to the prediction of film temperatures.

3.3. Thermal stresses

In order to evaluate the thermal stresses on the walls it is common in the recent literature [29, 30, 31, 32, 11] to use the expressions provided by [15] for a circular pipe with a radial temperature distribution (i.e., as when the heat flux is uniform in the circumferential direction). These thermal stresses tensor components are reproduced here for completeness :

$$\sigma_{rr} = \frac{\Gamma}{r^2} \left(\frac{r^2 - R_i^2}{R_o^2 - R_i^2} \int_{R_i}^{R_o} \Theta^{(s)} r dr - \int_{R_i}^r \Theta^{(s)} r dr \right), \quad (18)$$

$$\sigma_{\theta\theta} = \frac{\Gamma}{r^2} \left(\frac{r^2 + R_i^2}{R_o^2 - R_i^2} \int_{R_i}^{R_o} \Theta^{(s)} r dr + \int_{R_i}^r \Theta^{(s)} r dr - \Theta^{(s)} r^2 \right), \quad (19)$$

where $\Gamma = \alpha E / (1 - \nu)$, E is Young's modulus, ν is Poisson's ratio, α is the coefficient of thermal expansion and the cross term $\sigma_{r\theta}$ is zero.

Figure 4: (a - c) Circumferential variation of film temperature for different Reynolds number at a fixed thickness ($e/R_i = 0.0848$). (a) $Pr = 0.7$. (b) $Pr = 3$. (c) $Pr = 10$. Dashed lines correspond to the spectral method and solid lines to 1D model. (d) Error between the spectral model and the 1D model for the film temperature at the location of maximum temperature ($\theta = \pi/2$, $r = R_o$), as a function of the Bi number. Circles correspond to $e/R_i = 0.1833$, squares to $e/R_i = 0.0848$, dashed line correspond to a power law of Bi^{-1} .

The longitudinal stress, σ_{xx} , is obtained from (18) and (19) by imposing the condition of plane strain,

$$\frac{1}{E}[\sigma_{xx} - \nu(\sigma_{\theta\theta} + \sigma_{rr})] + \alpha\Theta^{(s)} = 0, \quad (20)$$

yielding²

$$\sigma_{xx} = \Gamma \left(\frac{2\nu}{R_o^2 - R_i^2} \int_{R_i}^{R_o} \Theta^{(s)} r dr - \Theta^{(s)} \right), \quad (21)$$

$$(22)$$

These expressions were originally developed for a temperature distribution which is a function of the radius only, that is $\Theta^{(s)} = \Theta^{(s)}(r)$, and they are often used in the literature. The rationale for the application of these formulas to non-uniform problem has already been explained in the introduction: if the circumferential gradients of temperature are small compared to radial gradients, a 1D model with conduction only in the radial direction might be applicable. However, it is apparent from figure 3 that in all the cases presented in table 1 the circumferential gradients are comparable to the radial gradients, which would prevent the application of these formulas.

A more general solution for the thermal stresses on a circular pipe was obtained by [33]. First, the temperature distribution on the pipe is expressed as a series of sines and cosines,

$$\begin{aligned} \Theta^{(s)}(r, \theta) = & A_0 + E_0 \log r + \sum_{n=1}^{\infty} (A_n r^n + C_n r^{-n}) \cos n\theta \\ & + \sum_{n=1}^{\infty} (B_n r^n + D_n r^{-n}) \sin n\theta, \end{aligned} \quad (23)$$

²Please note that the formula given by [15] for σ_{xx} contains a typo.

which corresponds to the most general solution of the Laplace equation in cylindrical coordinates. Then, the thermal stresses are obtained from the temperature distribution by solving the Duhamel-Neumann law, together with the equilibrium condition for the solid and the boundary conditions of no surface forces. [33] obtained the remarkable result that the thermal stresses only depend on the coefficients E_0 , C_1 and D_1 of the temperature distribution:

$$\begin{aligned} \sigma_{rr}(r, \theta) = & \frac{\Gamma E_0}{2} \left[\log \left(\frac{R_o}{r} \right) - \frac{R_i^2}{r^2} \frac{R_o^2 - r^2}{R_o^2 - R_i^2} \log \left(\frac{R_o}{R_i} \right) \right] \\ & - \frac{\Gamma}{2r^3} \frac{(R_o^2 - r^2)(r^2 - R_i^2)}{(R_o^2 + R_i^2)} (C_1 \cos \theta + D_1 \sin \theta), \end{aligned} \quad (24)$$

$$\begin{aligned} \sigma_{\theta\theta}(r, \theta) = & \frac{\Gamma E_0}{2} \left[\log \left(\frac{R_o}{r} \right) + \frac{R_i^2}{r^2} \frac{R_o^2 + r^2}{R_o^2 - R_i^2} \log \left(\frac{R_o}{R_i} \right) - 1 \right] \\ & - \frac{\Gamma}{2r^3} \frac{4R_i^2 r^2 - (3r^2 - R_o^2)(r^2 + R_i^2)}{(R_o^2 + R_i^2)} (C_1 \cos \theta + D_1 \sin \theta), \end{aligned} \quad (25)$$

$$\sigma_{r\theta}(r, \theta) = -\frac{\Gamma}{2r^3} \frac{(R_o^2 - r^2)(r^2 - R_i^2)}{(R_o^2 + R_i^2)} (C_1 \sin \theta - D_1 \cos \theta). \quad (26)$$

The computation of these coefficients (E_0 , C_1 and D_1) once the thermal field on the solid is known is trivial. Note also that for the present configuration, coefficient $C_1 = 0$ due to symmetry considerations. As with Faupel & Fisher's solution, the longitudinal stress σ_{xx} can be obtained from the cross-plane stresses by imposing the condition of plane strain, eq. (20).

Comparing eqs. (18-19) to (24-26), we can observe a few important differences. First, [33] gives a non-zero cross stress term, eq. (26), which implies the existence of bending moments on the wall of the pipe. Also, when C_1 and D_1 are zero, Gatewood's solution coincides with Faupel & Fisher's. That is the case when the temperature on the solid depends on the radial coordinate

only. Note that, if the temperature is n -periodic in the circumferential direction with $n \geq 2$, Faupel solution would yield the appropriate values for the stress if (and only if) the circumferentially averaged temperature is used in eqs. (18-21).

As an illustration, Fig. 5 shows the components of thermal stresses tensor calculated using the formulas proposed by [33] for cases ii (low Biot) and xxx (high Biot). The stresses are normalized with ΓT_{ref} , and the figure shows from top to bottom the components σ_{rr} , $\sigma_{\theta\theta}$ and $\sigma_{r\theta}$.

First, it is important to note that the scale is not the same for all the panels: the magnitude of the tangential component, $\sigma_{\theta\theta}$, is considerably larger than the other two. Second, the shape of the thermal stresses presents clear differences as the Bi number changes. When the Biot is small, the temperature gradients are larger in θ . As a consequence, the thermal expansion of the solid is roughly uniform in the radial direction. Under this condition, the dilatation of the front part of the pipe results in inward bending stresses in the front half of the pipe (compression inside, traction outside), and outward bending stresses on the rear side of the pipe (traction inside, compression outside).

On the other hand, when the Biot is large the radial temperature gradients are dominant. As a consequence, there is a large temperature difference between the outer and inner walls of the pipe, with the former suffering larger thermal expansion than the latter. This produces outward bending stresses in the front side of the pipe, and smaller thermal stresses on the rear side of the pipe.

In order to compare the thermal stresses obtained with the solutions of

Figure 5: Components of the thermal stresses tensor for different Biot numbers. (a) – (b) $\sigma_{rr}/\Gamma T_{ref}$. (c) – (d) $\sigma_{\theta\theta}/\Gamma T_{ref}$. (e) – (f) $\sigma_{r\theta}/\Gamma T_{ref}$. (a), (c), (e) Case ii. (b), (d), (f) Case xxx.

Faupel & Fischer and Gatewood, it seems convenient to use the effective stress defined as

$$\sigma_{ef} = \sqrt{(\sigma_{xx} - \sigma_{rr})^2 + (\sigma_{rr} - \sigma_{\theta\theta})^2 + (\sigma_{\theta\theta} - \sigma_{xx})^2 + 6(\sigma_{r\theta}^2 + \sigma_{rx}^2 + \sigma_{x\theta}^2)}. \quad (27)$$

Note that for the present configuration, the $\sigma_{\theta\theta}$ component of the stress tensor dominates the effective stress. Figure 6 shows this effective stress for three cases of the database, corresponding to low, intermediate and large values of the Biot number. The shape of the effective stress obtained with the Faupel & Fischer's solution, Fig. 6(b) – (d) – (f), is very similar for all Bi numbers, although their magnitude changes considerably. A large stress is predicted where the heating is applied and a negligible stress is predicted along the adiabatic part.

On the other hand, Gatewood's solution (fig. 6a – c – e) shows a strong dependence of the thermal stresses on the Biot number, as already observed in figure 5. The maximum thermal stresses occur on the hot side for high Bi , while they occur on the adiabatic side for low Bi . It is interesting to note that for intermediate Bi numbers ($Bi \approx 0.2$), a roughly homogeneous distribution of effective thermal stress is obtained.

In order to analyze in more detail the effective stresses from Gatewood's solution, let's consider the equation for the tangential stresses (eq. 25), the largest contributors to σ_{ef} , for the particular case of $C_1 = 0$:

$$\sigma_{\theta\theta} = E_0 f_0(r) + D_1 f_1(r) \sin \theta. \quad (28)$$

In this equation f_0 and f_1 are functions of r only. It is clear that the shape of the $\sigma_{\theta\theta}$ (and hence σ_{ef}) is governed by the sign of coefficient D_1 . When D_1 is

Figure 6: Effective thermal stress, $\sigma_{ef}/\Gamma T_{ref}$, (a),(c),(e) using Gatewood formulas. (b),(d),(f) using Faupel & Fischer formulas. (a) – (b) Case ii. (c) – (d) Case xvi. (e) – (f) Case xxx.

close to zero, the thermal stresses are essentially axisymmetric, as it happens for the intermediate Bi shown in Fig. 6(c). Note that $D_1 = 0$ does not imply that the temperature distribution is axisymmetric since $D_2, D_3, \dots, D_N$, might all be non-zero. When D_1 is positive the maximum effective stress tends to be in the adiabatic region, while when D_1 is negative the maximum effective stress occurs in the region where the heating is applied.

Figure 7(a) shows the dependence of D_1 with the Biot number, for all the cases considered here. We can observe that as Bi increases, D_1 decreases and eventually change signs, at $Bi \approx 0.1 - 0.2$. The plot suggest that the actual threshold depends on the tube thickness. This is clearer when D_1 is plotted against the *local* Bi (not shown here), based on the convective coefficient computed from the spectral simulations at the location of the peak heat flux.

An interesting consequence of the change of sign of D_1 is that the effective stress becomes minimal when D_1 is zero. This is hinted in figure 6, but becomes clear in figure 7(b), where the maximum σ_{ef} obtained with Gatewood's formula is plotted as a function of the Bi number. In general, the smallest thermal stresses are obtained for $Bi \approx 0.1 - 0.2$ (where D_1 crosses zero and the effective stresses on the pipe are uniform), although there are three cases with thin walls with minimum thermal stresses at $Bi \approx 0.05 - 0.1$.

Finally, the results of figure 6 show that for small Bi numbers Faupel & Fisher formulas predict qualitatively wrong thermal stresses. Moreover, even if at large Bi the thermal stresses seem to be qualitatively right, they are still quantitatively wrong: while $\sigma_{ef,max} = 3.9 \times 10^{-3} \Gamma T_{ref}$ in figure 6(e), it is $\sigma_{ef,max} = 5.3 \times 10^{-3} \Gamma T_{ref}$ in figure 6(f). This is emphasized in figure 8, where the relative difference between the $\sigma_{\theta\theta}$ obtained with Gatewood's and

Figure 7: (a) D_1 coefficient as a function of the Biot number, for all cases studied. Circles, thick wall, $e/R_i = 0.183$. Squares, thin wall, $e/R_i = 0.085$. (b) Maximum effective thermal stress at the hottest point in the wall ($\theta = \pi/2$ at the outer wall), as a function for the Bi number. Black symbols, $Pr = 0.7$. Red symbols, $Pr = 3$. Blue symbols, $Pr = 10$. Circles, thick wall, $e/R_i = 0.183$. Squares, thin wall, $e/R_i = 0.085$. Close symbols, $D_1 > 0$. Open symbols, $D_1 < 0$.

Figure 8: Relative difference between the tangential stresses at the point of maximum temperature ($\theta = \pi/2$ and $r = R_o$), computed using Faupel & Fisher formulas ($\sigma_{\theta\theta}^{(F)}$) and Gatewood's formulas ($\sigma_{\theta\theta}^{(G)}$). The relative error is plotted as a function of the Biot number. Black symbols, $Pr = 0.7$. Red symbols, $Pr = 3$. Blue symbols, $Pr = 10$. Circles, thick wall, $e/R_i = 0.183$. Squares, thin wall, $e/R_i = 0.085$. Close symbols, $D_1 > 0$. Open symbols, $D_1 < 0$.

Faupel & Fisher formulas is plotted as a function of the Biot number. It is clear that, even for $Bi \gtrsim 0.2$ (which was the limit for obtaining reasonable predictions for the film temperature with the local 1D model), the errors in the thermal stresses are large ($\approx 50\%$). The plot suggests that, for Faupel's formulas to provide an error smaller than 5%, the Bi must be at least ≈ 10 .

4. Conclusions

A conjugate heat transfer numerical study on thin-walled pipes subjected to a non-uniform heat flux has been carried out. Different operation conditions have been imposed to study the role of the Reynolds, Prandtl and Biot

number. The results of the present calculations have been compared to the estimations based on empirical correlations for the Nusselt number, where the heat transfer is assumed to occur only in the radial direction (1D model).

As expected, the higher film temperatures are found near the maximum heat flux. The agreement between the spectral method and the 1D model increases as the Bi number increases, although there is in general an underestimation of the maximum film temperature in the 1D model. As a rule of thumb, for $Bi \gtrsim 0.3$ the difference between spectral and 1D models is negligible (smaller than 5%).

From the point of view of the thermal stresses, Gatewood's formulas should be used to compute the thermal stresses in non-uniform configurations, when $\Theta^{(s)}$ depends on r and θ . The results show that the stress distribution on the pipe depends strongly on the Biot number: for $Bi < 0.1$, the maximum thermal stresses are concentrated in the adiabatic side of the pipe. For $Bi > 0.3$, the maximum thermal stresses appear near the peak heat flux, in the hot side of the pipe. For $Bi \approx 0.1 - 0.3$, the distribution of thermal stresses is roughly uniform in the circumferential direction. Indeed, for that range of Bi the maximum thermal stress seems to be minimum.

Finally, the thermal stresses obtained from Gatewood's and Faupel & Fisher's formulas are compared. It is concluded that Faupel & Fisher's formulas do not provide quantitatively correct results for $Bi \lesssim 10$, and provide qualitatively wrong results for $Bi \lesssim 0.3$. This implies that, while 1D (local) models can provide reasonable good estimations of the film temperature in the fluid, a proper estimation of the thermal stresses on the solid requires obtaining the temperature distribution on the whole tube and using Gate-

wood's formulas.

Acknowledgments

This work was supported by the Spanish Ministry of Economy and Competitiveness through the grant ENE2012-34255. O.F. and M.G.-V. were partially supported by grant TRA2013-41103-P of the Spanish Ministry of Economy and Competitiveness. The latter grant includes FEDER funding.

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