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On the double transition in the failure mode of polycarbonate

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Highlights

- Two transition points have been found in the failure mode of PC.
- Failure mode transition is explored both experimentally and numerically.
- A global damage model has been proposed, calibrated and validated.
- Proposed damage model is capable of reproducing and predicting these transitions.
- Transitions depend on a combination of the specimen thickness and strain rate.

On the double transition in the failure mode of polycarbonate

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Abstract

In the present work, the transition in the mode of failure from brittle to ductile, observed in certain polymeric materials, is explored both experimentally and numerically, focusing on polycarbonate, a polymer of wide industrial use. The limit between both behaviours depends on several intrinsic factors, such as temperature and deformation rate, and extrinsic factors such as notch radius and specimen thickness. The parameters that have been explored in this work are the thickness of the specimen and the offset of the initial notch from symmetry. We explore this transition through experiments on polycarbonate and numerical simulations using a global damage model. In order to accomplish this, a VU-MAT user subroutine has been developed in the finite element commercial code ABAQUS/Explicit, which takes into account both failure criteria (brittle and ductile), independently. Thus, it has been possible to reproduce the transition in the failure mode of polycarbonate specimens subjected to three point bending dynamic fracture tests. These numerical results have allowed to observe that a double transition may occur, depending on the thickness and strain rate.

Keywords: Fracture, Polycarbonate, Failure Mode Transition, Hopkinson Bar, Three Point Bending, Finite Element Method

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1. Introduction

The use of polymeric materials in industry, for the manufacture of mechanical or structural components has increased in recent years, since there is good compromise between their impact strength, low weight and cost; furthermore, some of them can be used as substitutes for glass because of their transparency. Many of these components may be subjected to impulsive or impact loads throughout their service life, and it is therefore essential to be able to predict their failure conditions, such as the moment at which the fracture will occur, as well as its velocity and direction of propagation of cracks. For these reasons, the study of dynamic fracture and mechanical behaviour of polymeric materials subjected to high deformation rates has received a great deal of attention from the scientific community in recent decades [1, 2, 3, 4, 5].

With this aim, many authors have analyzed the failure mode in which dynamic fracture occurs for this type of polymeric materials, such as polymethylmethacrylate (PMMA) [6, 7] or polycarbonate (PC) [8, 9, 10]. One of the main characteristics of the fracture of some of these polymers is the transition that occurs in their failure mode, being able to break in a brittle way dominated by a crazing mechanism [11], or in a ductile form by a process called shear yielding [12], related to the formation of shear bands. A similar transition has also been observed and extensively studied in metallic materials [13, 14, 15, 16, 17, 18].

The appearance of this transition phenomenon in failure mode depends on several *intrinsic* factors such as temperature and strain rate that directly influence which deformation and failure mechanisms are triggered, and *extrinsic* factors such as specimen thickness and notch radius which influence the specimen constraint and therefore indirectly influence the deformation and failure mechanisms. In the case of PMMA, the transition in the failure mode only occurs in the presence of very high confining pressures [19], whereas for the PC, this transition is observed under normal conditions of pressure and temperature, without the need to confine the specimen. For this reason, PC is the material

chosen for the development of this study.

This phenomenon of transition in failure mode was previously observed in metals, like steel C-300 [13, 20, 21] or martensitic steel 250 [18], as reported in the work of Kalthoff [13, 22]. In this work, the failure mode was studied in double-notched flat steel specimens and subjected to dynamic loads according to a deformation mode II, when these specimens were impacted with a projectile with a diameter equal to the spacing between the two notches.

Since the first experimental results published by Kalthoff [13], this problem has been studied by numerous researchers from an analytical [23], numerical [20, 18, 14, 24] and experimental [8, 9, 21, 25, 26] point of view. The type of specimen used in these investigations differs from the one used by Kalthoff [13], since they are three-point bending tests specimens [26] or only have one notch instead of two [20, 18]. This latter difference in the number of notches allows the end of the crack to be subjected to the loads induced by the incident pulse of the impact during a longer duration, thus avoiding the effects of the reflections of waves produced by the neighboring notch [9].

In the case of polymeric materials, the mechanisms of deformation that give rise to the two modes of fracture discussed above are *crazing* and *shear yielding*. The mechanism of *crazing* [11] is the one associated with the breakage of the material, producing the failure in direction perpendicular to the maximum principal stress. This process is driven by the appearance of crazes, which can be considered as microcracks, but differ from these in that crazes contain small fibers of material that still hold the two faces of the crack together, being able to maintain some degree of stress. The existing separations between these fibers generate the appearance of microvoids that produce an increase of the volume of the material. Thus, when these fibers oriented perpendicular to the plane of the craze are broken, they give rise to a crack (see Fig.1(a)).

Figure 1: Mechanisms of deformation in polycarbonate

The mechanism that gives rise to the ductile fracture of the material is called *shear yielding*, which is related to the appearance of plastic deformations in the form of shear bands, resulting in changes of shape of the material without an associated change of volume [29, 30]. This mechanism of deformation has been studied and characterized mainly in metals [13, 15, 16, 17]. One of the main conclusions that was obtained from these works is that the appearance of a thermal softening was necessary for these shear bands to occur [14]. However, a new approach has recently emerged in terms of its formation process [31], in which it is established that thermal softening plays a more secondary role, whereas it is the dynamically stored cold working energy that drives the formation of these shear bands. This kind of behavior is not only restricted to metals, but has also been observed in polymeric materials, with the great majority of the work related to the appearance of shear bands in polymers focused on three amorphous materials: polymethylmethacrylate, polycarbonate and polystyrene, probably due to its transparency [6, 8, 9, 32, 33, 34], as can be seen in Fig.1(b).

Whether brittle or ductile fracture of a polymer is triggered depends on a large number of critical parameters that establish a boundary between both behaviours. For the particular case of PC, this transition has been observed with temperature [35, 36], with aging [37], strain rate [9, 38, 39], notch radius [10, 40, 41, 42] and thickness [43]. As a result of this, the numerical prediction

of the failure mode seems rather complicated, and the constitutive models used to reproduce the behaviour of this type of materials involves a large number of adjustable parameters, even for those models derived from micromechanics [29, 44, 45].

Numerous authors have proposed different damage models with the intention of capturing this observed transition in the failure mode. Many of these models [3, 46, 47, 48] focus exclusively in the brittle failure derived from *crazing*, while the shear band dominated deformation and failure mechanisms are implicitly included in the constitutive model, without the establishment of a ductile failure criterion.

Recently, models that sought to predict the fracture from an alternative point of view have been proposed, combining constitutive models based in physics, with failure criteria set at the continuum level [49, 50]. This approach implies that the microstructural characteristics of the fracture process are taken into account only on an averaged basis within a representative volume element. These damage models are established from an invariant value of stress or strain, which reaches a critical value from which the failure process begins. Unlike those previously mentioned, these models do establish a criterion of ductile fracture. Specifically, the model proposed by Gearing and Anand [49, 50] defines a brittle failure criterion based on a critical value of hydrostatic stress σ_m^c , and a ductile failure criterion based on a critical value of plastic elongation λ_p^c . These failure criteria are incorporated into the proposed constitutive model by Anand and Gurtin [51]. This damage model has served as a basis for other works, as for example the one presented by Kattekola et al. [43] in which the ductile failure criterion is established from a critical opening stress value. Another of the works based on the model of Gearing and Anand is the published by Torres and Frontini [5], in which the critical parameters that define the failure criteria are considered dependent on triaxiality (σ_m/σ_{vm}), where σ_{vm} the Von Mises equivalent stress. This dependency with triaxiality was already studied by Sternstein and Ongchin [52], who present a dependence of the beginning of the crazing with this triaxiality.

Finally, it is necessary to highlight the damage model presented by Dolinski et al. [18], which considers a direct competition between a brittle and a ductile mode of failure. This damage model was initially proposed for use in metals, but due to this failure mode transition that has also been observed in polymers, the possibility of extending its applicability to polymeric materials has also been considered.

This paper is organized as follows: In Section 2, an integrated damage model which takes into account the competition between a brittle and a ductile fracture criterion, both acting at all times independently and simultaneously, is proposed. The development of a full three-dimensional numerical implementation of the proposed constitutive and damage model, and the calibration of this model through a comparison to experimental results available from the literature are described in Section 3. These simulations indicate that the model described in Section 2 is able to capture the failure mode transition in PC from brittle to ductile with increasing strain rates. Validation of the model is considered in Section 4; new experiments, performed on polycarbonate specimens of three different thicknesses, two different notch locations, and different impact speeds that provide a rich data set of experimental results, and their corresponding numerical simulations are described in this section. These results show that there is a transition from ductile to brittle failure mode that occurs intrinsically with increasing strain-rates, but is significantly influenced by extrinsic factors such as the specimen thickness and the notch geometry; this dependence is examined in Section 5 through a detailed examination of the simulations.

2. Constitutive and failure model for polycarbonate

In this section, the constitutive and damage models used for PC are presented. It is important to emphasize that the main objective of this work is to formulate an integrated damage model to be able to predict the transition in the failure mode that has been evidenced in polymeric materials. For that reason, a relatively simple constitutive model that takes into account the main

mechanical characteristics of the PC, but allows reduced computational cost in numerical simulations, is chosen.

Both the constitutive and the damage models have been implemented in the finite element commercial code ABAQUS/Explicit (v 6.11) [53], through a VUMAT user subroutine following the procedure presented by Zaera and Fernandez-Saez [54], and taking into account a return mapping algorithm [55, 56].

2.1. Constitutive model

As a simplifying hypothesis it is considered that the material behaves according to the true stress-strain law shown in Eq.(1), which is a modification of the one presented by Ravi-Chandar et al. [9] in quasi-static regime.

$$\sigma(\varepsilon, \dot{\varepsilon}, \theta) = \begin{cases} \sigma_1 & \text{for } \varepsilon \leq \frac{\sigma_y(\theta, \dot{\varepsilon})}{E} \\ \sigma_2 & \text{for } \frac{\sigma_y(\theta, \dot{\varepsilon})}{E} \leq \varepsilon \leq 0.81\varepsilon_h + \frac{\sigma_y(\theta, \dot{\varepsilon})}{E} - \varepsilon_0 \\ \sigma_3 & \text{for } 0.81\varepsilon_h + \frac{\sigma_y(\theta, \dot{\varepsilon})}{E} - \varepsilon_0 \leq \varepsilon \leq \varepsilon_h + \frac{\sigma_y(\theta, \dot{\varepsilon})}{E} - \varepsilon_0 \\ \sigma_4 & \text{for } \varepsilon \geq \varepsilon_h + \frac{\sigma_y(\theta, \dot{\varepsilon})}{E} - \varepsilon_0 \end{cases} \quad (1)$$

with functions σ_i defined as

$$\sigma_1 = E\varepsilon \quad (2)$$

$$\sigma_2 = \sigma_y(\theta, \dot{\varepsilon}) + A \left(\varepsilon - \frac{\sigma_y(\theta, \dot{\varepsilon})}{E} \right) \quad (3)$$

$$\sigma_3 = \sigma_y(\theta, \dot{\varepsilon}) + A(0.81\varepsilon_h - \varepsilon_0) + B \left(\varepsilon - 0.81\varepsilon_h - \frac{\sigma_y(\theta, \dot{\varepsilon})}{E} + \varepsilon_0 \right) \quad (4)$$

$$\sigma_4 = \sigma_y(\theta, \dot{\varepsilon}) + A(0.81\varepsilon_h - \varepsilon_0) + 0.19B\varepsilon_h + E \left(\varepsilon - \varepsilon_h - \frac{\sigma_y(\theta, \dot{\varepsilon})}{E} + \varepsilon_0 \right) \quad (5)$$

where E is the Young's modulus of the material, ε is the strain, $\dot{\varepsilon}$ is the strain-rate, θ is the temperature, A and B are two constants that determine the slope of the different elasto-plastic sections, $\sigma_y(\theta, \dot{\varepsilon})$ is the initial yield strength, ε_0 is the strain at the beginning of the plastification and ε_h the strain at which the process of molecular orientation begins. The numerical values of the material parameters used are shown in Table 1.

Table 1: Physical and mechanical properties of PC

Young's modulus [9]	E	2.4 GPa
Density [9]	ρ	1160 kg/m ³
Poisson's coefficient [9]	ν	0.34
Constitutive law constant	A	5 MPa
Constitutive law constant	B	100 MPa
Quasi-static yield strength [9]	σ_{y0}	50.4 MPa
Quasi-static yield strain [9]	ε_0	0.021
Strain at the beginning of molecular orientation [9]	ε_h	0.521
Coefficient of thermal softening [9]	β	0.25 MPa/K
Specific heat [9]	c_v	1170 J/(kg · K)
Thermal conductivity [57]	k	0.21 W/(K · m)
Coefficient of thermal expansion [9]	α	$6.5 \cdot 10^{-5} K^{-1}$
Quinney-Taylor coefficient [5]	η	0.6
Reference temperature [9]	θ_0	293 K
Parameter set [58]	$2k\Theta/V$	187 MPa
Characteristic strain rate [58]	$\dot{\varepsilon}^*$	$1.7 \cdot 10^6 s^{-1}$
Material constant [58]	n	5.18

The influence of temperature and strain rate on the behavior of these types of materials is strong, and for this reason the yield strength of the material is represented as:

$$\sigma_y(\theta, \dot{\varepsilon}) = \sigma_{y0} - \beta(\theta - \theta_0) + \frac{2k\Theta}{V} \sinh^{-1} \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}^*} \right)^{1/n} \quad (6)$$

where both, the thermal [9] and strain rate [58] components, have been obtained from the literature. In Eq.(6), θ_0 is the reference temperature, k is the Stefan-Boltzmann constant, Θ is the absolute temperature (which, for simplicity, has been considered as constant in this expression), V is the activation volume, n is a material parameter and $\dot{\varepsilon}^*$ a characteristic strain rate related to the activation energy. The values of all these parameters are also shown in Table 1.

The Quinney-Taylor coefficient was established as 0.6 but we anticipated that higher values will only influence the quantitative details of the transitions.

The uniaxial stress-strain response of this model is shown in Fig.2; this is generalized to a multiaxial stress state through a standard von Mises J2 incremental theory of plasticity.

Figure 2: Constitutive behaviour considered in this work for PC

2.2. Damage model

Failure is modeled through specific damage models for brittle and ductile modes. This damage model simultaneously accounts for the two modes of failure observed in this material; for the brittle fracture mode, damage is defined in terms of the maximum principal stress, and for the ductile fracture mode, damage is defined in terms of the strain energy density. In this way, the failure of the material is determined by the competition between both criteria. Each of these two failure criteria is described below.

2.2.1. Brittle fracture criterion

Brittle fracture is defined as a local fracture criterion based on the maximum principal stress σ_I . Thus, when σ_I reaches a critical value σ_I^{crit} at a material point, brittle fracture occurs and the material loses its ability to carry stress:

$$\sigma_I = \sigma_I^{crit}(\dot{\epsilon}) \quad (7)$$

As can be seen in Eq.(7), the value of σ_I^{crit} has been considered dependent on equivalent strain rate $\dot{\epsilon}$, following the same methodology used by other authors [18, 26]. For this reason, it is necessary to obtain the strain rate dependence of σ_I^{crit} .

In order to calibrate this brittle damage (fracture) criterion, it is necessary to know the stress-strain state of the material just at the moment and the surrounding area of fracture, for different values of strain rate. In the absence of these experimental results for PC, results published in the scientific literature are used.

Ravi-Chandar et al. [9] carried out experiments of asymmetric impact of a cylindrical projectile against a single notched specimen (both made of PC). A complete three-dimensional finite element model (including both the projectile and the specimen) which reproduces these tests was developed. Both the first principal stress and the strain rate were obtained from an area surrounding the vicinity of the notch, which comprises a characteristic distance d_c ($1 - 1.5 \text{ mm}$) from the end of the notch, where the first principal stress reaches its critical value σ_I^{crit} causing the fracture of the material. This is similar to the RKR criterion [59] for metallic materials and it is also supported by the experimental observations of nucleation of a crack at a distance of d_c ($1 - 1.5 \text{ mm}$) from the notch tip which will be reported later. The relationship between the maximum principal stress and the strain rate at the moment of fracture was then obtained from numerical simulations of tests at different impact velocities.

Once the values of critical first principal stress σ_I^{crit} and equivalent strain rate $\dot{\epsilon}$ at the end of the notch in the instants of fracture are related, the relation

defined in Eq.(7) can be established. This relationship is shown graphically in Fig.3, where it can be seen that there is a lower strain-rate limit around $\dot{\epsilon} = 27000 \text{ s}^{-1}$, below which the critical first principal stress remains constant and equal to 160 MPa .

Figure 3: Relation between $\dot{\epsilon}$ and σ_I^{crit} in the brittle fracture criterion

This data is in accordance with the one published by Dorogoy and Rittel [60], where a tensile strength of 160 MPa is established for the PC. In the present work, the tensile strength associated with the maximum principal stress is set to 100 MPa , which is of the same order of magnitude. This difference could be due to the different type of PC used. Above this strain rate limit value, the curve can be optimally approximated ($R^2 \simeq 1$) by a third degree polynomial function. Thus, the damage criterion in Eq.(7), can be written as:

$$\sigma_I^{crit}(\dot{\epsilon}) = \begin{cases} \sigma_{I,quasi}^{crit} & \text{for } \dot{\epsilon} \leq 27000 \text{ s}^{-1} \\ \sigma_{I,dynam}^{crit}(\dot{\epsilon}) & \text{for } \dot{\epsilon} > 27000 \text{ s}^{-1} \end{cases} \quad (8)$$

with:

$$\sigma_{I,quasi}^{crit} = 160 \text{ MPa} \quad (9)$$

$$\sigma_{I,dynam}^{crit}(\dot{\epsilon}) = 1.622 \cdot 10^{-11} \dot{\epsilon}^3 - 1.3274 \cdot 10^{-6} \dot{\epsilon}^2 + 3.6544 \cdot 10^{-2} \dot{\epsilon} - 178.27 \text{ MPa} \quad (10)$$

2.2.2. Ductile fracture criterion

The ductile fracture criterion consists of establishing (i) a critical level of total strain energy density, W_{crit} , at which the degradation of the material will begin, and (ii) a limit value of this strain energy density, W_{frac} at which the final fracture occurs. This criterion has been used widely as a damage indicator in fatigue failure, sheet metal forming and other applications; Dolinski et al. applied this damage indicator to study adiabatic shear bands in metals [18, 61, 62, 63]. A schematic representation of this ductile fracture criterion is given in Fig.4.

Figure 4: Schematic representation of ductile fracture criterion

The critical value strain energy density, W_{crit} , is established according to the following expression:

$$W_{crit} = \int_0^{\alpha} \sigma_{ij} d\epsilon_{ij} \quad (11)$$

where σ_{ij} and ε_{ij} are the stress and strain tensor components respectively. The upper integration limit α is fixed as the critical limit of equivalent strain $\bar{\varepsilon}_{crit}$ from which the structural strength begins to deteriorate gradually:

$$\bar{\sigma} = \bar{\sigma}^*(1 - D^b) \quad (12)$$

with $\bar{\sigma}^*$ denoting the equivalent stress without degradation at the current time for a given equivalent strain $\bar{\varepsilon} \geq \bar{\varepsilon}_{crit}$; and b an exponent which allows to establish the degree of the relationship between $\bar{\sigma}^*$ and $\bar{\sigma}$. D determines the level of damage in the considered location and can acquire values between 0, if the element is intact, and 1, if completely damaged, leading to its elimination. This parameter D is defined as:

$$D = \frac{W - W_{crit}}{W_{frac} - W_{crit}} \quad (13)$$

Note that $D = 0$, if $W < W_{crit}$ and that D cannot decrease. In the case of the polymeric material considered, a difference is observed between failure under tension and under compression. For this reason, the ductile damage criterion is developed using the third invariant of the stress tensor, I_3 , and thus defining different values of W_{crit} and W_{frac} in tension and compression.

In the work published by Dolinski et al. [18], it is indicated that the parameters defining this ductile damage model, W_{crit} , W_{frac} and b , can be obtained in metals from dynamic compression tests in cylinders [64]. For the PC, it was not possible to find such parameters in the literature, and they have been obtained in an indirect manner from the asymmetric impact tests on simple notched flat specimens made by Ravi-Chandar et al. [9], using the finite element model to be detailed in the following section. These values are shown in the Table 2.

As a starting hypothesis, these parameters have been considered as constants, but it would be interesting to study the possibility that they could vary with strain rate or triaxiality.

The values of W_{crit} and W_{frac} are smaller in tension rather than compression, according to experimental evidence for this type of materials. The value

Table 2: Values of the parameters used in the ductile fracture criterion for the PC

Ductile fracture criterion parameters	
W_{crit}^{trac}	$= 50 \text{ MJ/m}^3$
W_{frac}^{trac}	$= 70 \text{ MJ/m}^3$
W_{crit}^{comp}	$= 140 \text{ MJ/m}^3$
W_{frac}^{comp}	$= 170 \text{ MJ/m}^3$
b	$= 1$

of $W_{crit}^{trac} = 50 \text{ MJ/m}^3$ corresponds to an equivalent stress of the same order as the value of the tensile quasi-static yield strength σ_{y0} , while the value of $W_{crit}^{comp} = 140 \text{ MJ/m}^3$ corresponds to an equivalent stress of the order of twice the quasi-static yield strength in compression, which is considered to be 1.13 times higher than the tensile one [65].

In addition, a deformation limit value, $\varepsilon_{max} = 1.6$ [26], has been included in order to avoid possible numerical errors derived from the excessive distortion that appears in elements close to its elimination, causing convergence problems. This value is much greater than the uniaxial failure strain of 0.6 for the PC, but this is used only for avoiding numerical errors at large strains.

3. Calibration of the constitutive and damage models

In this section, the finite element model used to calibrate the constitutive model will be presented together with the results obtained in the calibration process. As stated above, the experimental results used for the calibration were those carried out by Ravi-Chandar et al. [9], consisting on the asymmetric impact of a cylindrical projectile against a flat plate with single notch, both made in PC.

3.1. Finite element model

A complete three-dimensional numerical model (see Fig.5(a)), including the specimen and the projectile has been developed, unlike other works existing in the literature [8, 9, 18], that model the asymmetric impact of the projectile against the single notched specimen as two-dimensional, imposing the impact velocity as a boundary condition. In addition, the configuration of the test allows the simplification of the model by using a plane of symmetry, defined by a plane perpendicular to the notch and passing through the center of the specimen, finally reducing the model to 1/2 of the initial geometry. This simplification allows to decrease the number of nodes and elements that will compose the finite element model, reducing the total number of degrees of freedom in the simulation and hence the associated computational cost.

Another simplification related to the reduction of computational time is the modelling of the PC projectile as linear elastic, after verifying that the stress levels were low enough to not cause inelastic deformations. For the specimen, the thermoviscoplastic material model described in Section 2 was used through a VUMAT.

Figure 5: Three-dimensional finite element model for calibration tests

One of the most critical features in the modeling of problems involving crack propagation is the definition of the mesh. In both solids (projectile and specimen) a mesh formed by hexahedral elements with reduced integration (C3D8R) of structured type has been considered. In the area closest to the end of the notch the mesh is chosen as random, avoiding its influence in the propagation path of the crack.

The damage model used will be sensitive to mesh size; however, by considering small element sizes that can represent the high gradients encounters and fixing the mesh size through a calibration to the global response leads to reproducible predictive results. Thus, due to the high stress gradients near the notch, it is necessary to increase the number of elements in this area, imposing

a minimum element size of 0.04 mm (see Fig.5(b)). The crack width is 0.3 mm and its length is 12.7 mm , resulting in a ratio between the minimum element size and these two quantities of 0.13 and $3.15 \cdot 10^{-3}$ respectively. For the same reason, a change in the concentration of the elements from the plane of symmetry ($z = 0$) to the free surface of the specimen ($z = B/2$) has been imposed along the thickness B , since around the free surface the stress gradient is more pronounced. Regarding the projectile, a higher element concentration has been considered at the specimen-projectile contact zone. As a result of the meshing process, there are a total of 165030 elements in the specimen and 3220 in the projectile.

As for the boundary conditions, in addition to the logical conditions of symmetry in the model, the vertical movement on the underside of the specimen has been inhibited, as indicated in [9]. Finally, an initial velocity is imposed on the projectile and an initial temperature on the specimen is prescribed.

3.2. Calibration results

The comparison between the experimental and numerical results of the calibration process are shown in Table 3, together with the corresponding error. It is important to highlight the few available data to carry out the calibration, since the quantitative results provided in [9] were: the starting time (time between projectile-specimen contact and the onset of fracture) and the propagation angle for a brittle crack as result of an impact at $v_{imp} = 40\text{ m/s}$, the total length and the propagation angle for a ductile crack as result of an impact at $v_{imp} = 60\text{ m/s}$, and the projectile velocity at which the transition in the failure mode occurs. It can be observed that the average error is about 8%, and the maximum being 25%, corresponding to the total length of the ductile crack.

In a more qualitative way, a comparison of the experimental and numerical results for brittle and ductile fracture modes can be observed in Figs.6 and 7, respectively. For the brittle mode of failure, as shown in Fig.6, two cracks appear, both numerically and experimentally, denominated with the letters B_c and L_c . In both the experiments and numerical simulations, the main crack

Table 3: Results of the model calibration

	Experimental [9]	Numerical	Error
Brittle crack start time	$50 \mu s$	$46 \mu s$	8 %
Brittle crack prop. angle	70°	68°	2.8 %
Transition velocity	$55 m/s$	$58 m/s$	5 %
Ductile crack total length	$10 mm$	$7.5 mm$	25 %
Ductile crack prop. angle	10°	10°	0 %

labeled B_c appears due to the initial impact of the projectile with the specimen, while the secondary crack labeled L_c appears as a result of the interaction of the reflected waves at the edges of the specimen, with the end of the initial notch.

(a) Brittle fracture obtained experimentally [9] (b) Brittle fracture obtained numerically

Figure 6: Comparison between experimental and numerical results for a brittle crack, $v_{imp} = 40 m/s$

Regarding the ductile mode of fracture, it can be observed in Fig.7 that the angle of propagation of the crack is the same, 10° , both experimentally and numerically. However, numerically the crack stops its propagation slightly sooner than it does in the experiments.

(a) Ductile fracture obtained experimentally [9]

(b) Ductile fracture obtained numerically

Figure 7: Comparison between experimental and numerical results for a ductile crack, $v_{imp} = 60 \text{ m/s}$

Based on these comparisons, we believe that the proposed model has been calibrated reasonably well to a small set of experimental results. It is important to highlight here that the objective of this calibration process is not to predict the experimental results reported in [9], but to use those experimental results to establish the parameters that define both failure criteria, and which are shown in Eq.(8) for the brittle criterion and Table 2 for the ductile criterion. Once the parameters of the damage model have been set, its validation will be explored next by the experimental and numerical reproduction of newly performed dynamic three-point bending tests.

4. Validation procedure

As mentioned before, the validation of the proposed constitutive and damage approach has been carried out through the comparison between a new set of experimental results from dynamic three point bending tests in a modified Hopkinson bar and their corresponding numerical simulations. In this section,

both the experimental procedure and the numerical model developed for the validation of the constitutive and damage model described in Section 2 will be discussed, together with the results of such validation process.

4.1. Experimental procedure

Dynamic three-point bending tests which generate relatively high deformation rates were performed using a compression split Hopkinson pressure bar (SHPB), a device also known as a Kolsky bar. The objective of these tests is to deform pre-notched specimens in both mode I and mixed mode (through the use of eccentric notch) conditions, with the aim of capturing the brittle to ductile failure mode transition of the PC. The instant of time at which the crack propagation initiates is obtained, and the propagation process of the crack is analyzed, calculating its direction and propagation velocity.

Blunt-notched specimens of PC were machined by waterjet cutting from plates of different thickness. The geometry of these specimens is shown in Fig.8.

Figure 8: Geometry of the specimens

Three different thicknesses $B = 5, 10$ and 20 mm , and two different notch positions $d = 0$ and 15 mm were considered; the other geometric parameters that remain fixed in all tests are: $L = 100\text{ mm}$, $W = 20\text{ mm}$, $l = 5\text{ mm}$ and $w = 2\text{ mm}$.

The experimental device used is based on a modification of the Hopkinson Bar [66, 67]. In this case, a cylindrical projectile of length 330 mm and diameter 22 mm impacts against the incident bar of length 1 m , and same diameter. Both the incident bar and the projectile are made of SAE 0-1 steel. The end of the

incident bar is in contact with the specimen and presents a similar shape to a Charpy pendulum nose tip (EN10045/1,1990).

The clamping and positioning of the specimen is done with a three point bending tool made of steel F125 [68], that takes advantage of its suitable properties against fatigue and impact. This tool is shown in Fig.9(a); the specimen placed on it and supported by two rollers of 10 mm of diameter spaced 80 mm from each other. This tool also has a set of screws and springs that hold the specimen in a vertical position, since this is the position of the specimens to be tested.

Due to the high crack propagation velocity in this kind of material, two *PHOTRON FASTCAM SA-Z* high-speed monochrome digital cameras capable of recording up to 2.1 million frames per second synchronously are used. Both cameras are arranged as shown in Fig.9(b). Camera 1 focuses on the area of the crack-tip in an oblique view, allowing the detection of the location and the instant of time at which the damage begins to grow in the interior of the specimen. Camera 2 focuses perpendicularly to the free surface of the specimen, encompassing the contact point with the incident bar, and recording the advance of the crack during the propagation process.

(a) Three point bending tool (b) Image acquisition system

Figure 9: Elements used in the SHPB experimental device

As discussed above, three thicknesses of specimen and two positions of notch were considered. For each geometric configuration, three impact velocities, reproducing a minimum of three repetitions for each case, were tested. In the end, approximately 90 tests were performed.

To identify the different specimens, a nomenclature which includes the thickness B of the specimen, the position of the notch d , the pressure supplied to the projectile P and the number N of the specimen, is used. The identification $B10d0P2N3$ will refer to the specimen number $N = 3$, of thickness $B = 10\text{ mm}$, with a notch at $d = 0\text{ mm}$ and tested at a pressure of $P = 2\text{ bar}$. If it is necessary to refer generically to all specimens whose eccentricity is $d = 15\text{ mm}$, irrespective of its thickness and the test pressure, shall be identified as: $d15$.

The crack propagation velocity was obtained from the crack length at each instant of time. For that, the crack-tip coordinates and the horizontal coordinate of the notch upper lip end, which will be taken as reference point as shown in Fig.10, were obtained for each frame. Note that the pixelation observed in this figure is due to the resolution of the high speed camera; 1 pixel equals 0.27 mm . The rigid body motion of the specimen is practically negligible as can be seen from the sequence of frames shown in Fig.11.

Figure 10: Reference points used for the crack length calculation

Figure 11: Frames sequence obtained during the crack propagation process of a *B20d0P1.75* specimen

The total crack length increment Δa_{i+1} can be obtained from:

$$\Delta a_{i+1} = \sqrt{(\Delta u_{i+1})^2 + (\Delta v_{i+1})^2} \quad (14)$$

where Δu_{i+1} and Δv_{i+1} are the crack position increments in the horizontal and vertical directions respectively. The crack propagation velocity $\dot{a}(t)$, in each instant of time is then calculated as:

$$\dot{a}(t) = \frac{a(t + \Delta t) - a(t - \Delta t)}{2\Delta t} \quad (15)$$

where Δt is the time-lapse between two consecutive frames. The characteristic propagation velocity value of each test $\dot{a}$, will be obtained by averaging the values provided by Eq.(15) through the entire propagation process.

In Table 4, a summary of the different thicknesses and launching pressures used in the whole experimental study is presented.

Table 4: Summary of launching pressures used for the different thicknesses

$B = 5\text{ mm}$	$B = 10\text{ mm}$	$B = 20\text{ mm}$
$P_1 = 1.00\text{ bar}$	$P_1 = 1.50\text{ bar}$	$P_1 = 1.75\text{ bar}$
$P_2 = 1.50\text{ bar}$	$P_2 = 1.75\text{ bar}$	$P_2 = 2.25\text{ bar}$
$P_3 = 1.75\text{ bar}$	$P_3 = 2.25\text{ bar}$	$P_3 = 2.50\text{ bar}$

4.2. Experimental results

The experimental results obtained for each fracture mode, are discussed in this section.

4.2.1. Ductile mode of fracture

The ductile mode of fracture mode was observed for all $B5$ specimens regardless of the notch position and velocity impact, and for the majority of the $B10$ specimens. For the latter thickness, brittle fracture appeared only in the case of a centered notch and a launching pressure $P_3 = 2.25\text{ bar}$; more specifically, of the five specimens tested under these conditions, three broke in ductile way and two in brittle one, clearly indicating that these conditions are just at the onset of the ductile to brittle transition.

Fig.12 shows some of the $B5$ and $B10$ tested specimens, in which it can be observed that the surface fracture exhibits typical characteristics of a ductile breakage, such as the reduction of the thickness due to high plastic deformation, and the presence of crack fronts produced by the intermittent advance of the crack during its propagation process.

(a) Fracture surface for ductile breakage

(b) Ductile fracture for *B5d0* specimen (c) Ductile fracture for *B10d15* specimen

Figure 12: Ductile fracture

Moreover, in the ductile mode of fracture, the path of propagation of the crack always remains in the plane of the notch, both, for the *d0* (which is expected) and for the *d15* specimens (which appears to be not intuitive).

The average crack propagation velocity versus the projectile impact velocity is shown in Fig.13, for the specimens *B5d0*, *B5d15*, *B10d0* and *B10d15*; the values for each test and the average values corresponding to the same launching pressure are shown. The crack speed in such ductile fracture mode is only in the range of a few m/s , significantly below the Rayleigh critical propagation velocity C_R , which establishes an theoretical upper limit for the crack propagation velocity in solids; for PC, this is approximately $C_R = 1000 m/s$ [69].

(a) B5 specimens

(b) B10 specimens

Figure 13: Experimental crack propagation velocities versus impact velocity - Ductile fracture

For both notch eccentricities it is observed that as the impact velocity increases, the crack propagation velocity also becomes higher. In addition, it is seen how the eccentrically-notched specimens need a higher impact velocity in order to reach the failure conditions, due to the stresses at the end of the notch being smaller than in the specimens with center notch for the same impact ve-

locity. Regarding the differences between specimens of different thicknesses, it can be seen that for the same impact velocity, crack propagation velocities are higher in the smaller thickness specimens.

4.2.2. Brittle mode of fracture

The brittle mode of fracture was observed in two of the five *B10d0P2.25* specimens tested, as indicated earlier, and also in all the *B20* specimens both with center and eccentric notches.

Fig.14 shows two of the *B20* specimens tested, one with a centered notch (Fig.14(a)) and the other with eccentric notch (Fig.14(b)). It can be seen that for this mode of fracture, the crack begins to propagate from the center of the specimen, and no crack fronts or thickness reduction are observed. In addition, and contrary to what happened in the case of the ductile fracture mode, when the notch is eccentric the crack propagates following the path in which the local loading conditions correspond to a mode *I* of deformation (see Fig.14(b)).

(a) Centered notch specimen *B20d0*

(b) Eccentric notch specimen *B20d15*

Figure 14: Brittle fracture

The crack propagation velocity values obtained from all the performed tests

on *B20* specimens are shown in Fig.15, and it can be seen that there is no marked trend in the relationship between the impact velocity and this crack propagation velocity, for the case of brittle fracture, with the crack speed being nearly constant at about 250 m/s, and well below the Rayleigh wave speed.

Figure 15: Experimental crack propagation velocities versus impact velocity - Brittle fracture

As a summary, the fracture modes observed in the dynamic three point bending experimental tests are shown in Fig.16, for each of the specimen thicknesses, depending on the impact velocity. It can be seen that the thickest specimens (*B20*) suffered a brittle fracture mode over the range of impact velocities considered, while the thinnest specimens (*B5*) broke always in a ductile fracture mode, and for the intermediate-thickness specimens (*B10*) it was observed that the fracture produced was of a ductile type, except for the highest impact velocity ($v_{imp} \simeq 18 \text{ m/s}$) at which for some of the specimens the fracture was brittle. This points to extrinsic effects introduced by the specimen geometry; we will explore this through numerical simulations.

Figure 16: Summary of the fracture modes observed experimentally, as a function of the impact velocity of the projectile

4.3. Finite element model

The developed three-dimensional numerical model is a complete thermoviscoplastic model, including the projectile, the incident bar, the specimen and the support system, as can be seen in Fig.17(a). One-half symmetry model is used to reduce computational cost (see Fig.17(b)).

Figure 17: Numerical model for the dynamic three point bending tests

The projectile and the incident bar material (steel SAE 0-1), as well as the support system material (steel F125), have been modeled as linear elastic, since they are expected to behave elastically throughout the impact process. The PC specimen is assumed to follow the behavior defined in Section 2, implemented in the finite element commercial code ABAQUS/Explicit, using a VUMAT user subroutine.

All the meshing was done with hexahedral elements of reduced integration (C3D8R). Due to the different geometries of specimens, it was necessary to model each of the six configurations; Fig.18 show the meshes for the two different crack locations. As in the numerical model developed for the calibration process, in the areas where there is a greater stress gradient a mesh with smaller elements has been used, with a minimum of 0.05 mm at the edge of the notch (see Fig.18(c)). This minimum element size provides a ratio of 0.025 and 0.01, respectively, with the width and length of the notch. In addition, the areas in which contact is expected were also refined: between projectile and incident

bar, between incident bar and specimen, and between specimen and supports.

Figure 18: Meshing of the three point bending tests specimens

The number of elements considered along the width of the specimen has been varied according to its thickness; for specimens with thickness of 5 mm , 10 mm and 20 mm , 10, 12 and 15 elements respectively across the thickness were used.

As for the initial conditions, the temperature of the specimen and the velocity of the projectile have been imposed; while the boundary conditions considered are those corresponding to the symmetry of the model and clamping of the rear part of the supports.

4.4. Validation results

For the validation of the model, the experimental and numerical results, regarding the morphology of the fracture and the propagation velocity of the crack are compared for each mode of fracture.

4.4.1. Ductile mode of fracture

With regard to numerical results, the ductile mode of fracture was also obtained for all the $B5$ and $B10$ specimens, reproducing what was seen in the Hopkinson Bar experimental results, independently of the position of the initial notch.

Fig.19 shows, by way of example, the result of an offset notched specimen, being able to verify that morphologically the results are very similar, since in both cases the path of propagation of the crack remains in the plane of the notch. In an equivalent way, it can be seen in Fig.20 the accurate results for the comparison of the thickness reduction observed in the ductile mode of fracture.

Figure 19: Comparison for $B = 5\text{ mm}$ and $d = 15\text{ mm}$ specimen

Figure 20: Comparison of thickness reduction

As for the comparison of crack propagation, the evolution of crack length versus time is also shown for two specific specimens by way of example, both for $B5$ and $B10$ thicknesses (Figs.21(a) and 21(b) respectively).

(a) $B = 5 \text{ mm}$ specimen

(b) $B = 10 \text{ mm}$ specimen

Figure 21: Numerical and experimental comparison of crack length versus time, from the onset of crack propagation, for ductile mode of fracture

The summary of the results obtained for the ductile mode of fracture regarding experimental and numerical average crack propagation velocity, for different projectile launch pressures, is shown in Tables 5 and 6, for $B5$ and $B10$ speci-

mens respectively. Based on the relative error values obtained for the different cases (around 12% average), it can be assumed that the numerical model reliably reproduces the ductile fracture mode.

	Pressure [bar]	Exp. vel. [mm/s]	Num. vel. [mm/s]	Rel. error
$d = 0\text{ mm}$	1	1391.53 ± 171.82	1558.51	11%
	1.5	2815.45 ± 155.73	3265.92	14%
	1.75	3136.22 ± 92.86	3481.20	10%
$d = 15\text{ mm}$	1	Does not break	Does not break	-
	1.5	4393.24 ± 208.29	5008.29	12%
	1.75	6079.59 ± 207.30	6626.75	8%

Table 5: Comparison of the crack propagation velocities for $B = 5\text{ mm}$ specimens

	Pressure [bar]	Exp. vel. [mm/s]	Num. vel. [mm/s]	Rel. error
$d = 0\text{ mm}$	1.5	1925.56 ± 45.80	2375.01	19%
	1.75	2331.85 ± 188.24	2732.95	15%
	2.25	3376.93 ± 59.77	3797.60	11%
$d = 15\text{ mm}$	1.5	Does not break	Does not break	-
	1.75	3219.76 ± 113.17	3778.41	15%
	2.25	3608.43 ± 207.30	3897.10	8%

Table 6: Comparison of the crack propagation velocities for $B = 10\text{ mm}$ specimens

4.4.2. Brittle mode of fracture

As in the case of experimental results, the brittle mode of fracture has been faithfully reproduced for all $B20$ specimens, no matter the position of the initial notch. Fig.22 shows a comparison of the morphology after breakage for eccentrically notched specimens. It can be verified in both cases, the direction

of crack propagation for the brittle mode of failure varies trying to find the path according to which the local loading conditions correspond to a mode I of deformation.

Figure 22: Comparison for $B = 20 \text{ mm}$ and $d = 15 \text{ mm}$ specimen

From the video obtained with the high-speed cameras and the surface of fracture, it can be experimentally observed that in the brittle mode, the failure begins in the middle plane of the specimen and propagates through the thickness to the free surfaces. Clearly, the onset of fracture occurs at a certain distance from the edge of the notch, as observed by other authors [10]. Quite remarkably, the numerical simulations successfully reproduced interior nucleation of damage, as can be seen in Fig.23.

Figure 23: Starting point of brittle fracture

The comparison between the numerical and experimental values of this brittle fracture nucleation distance is shown in Table 7. The relative errors for the three pressures of launch of the projectile, and for the two notch eccentricities considered are quite small, indicating excellent agreement between the experiments and simulations.

	Pressure [bar]	Exp. distance [mm]	Num. distance [mm]	Rel. error
$d = 0 \text{ mm}$	1.75	1.4 ± 0.3	1.5	6.7%
	2.25	1.2 ± 0.2	1.3	7.7%
	2.5	0.8 ± 0.3	0.8	0%
$d = 15 \text{ mm}$	1.75	1.2 ± 0.2	1.4	14%
	2.25	1.4 ± 0.2	1.6	12.5%
	2.5	1.1 ± 0.3	1.2	8.3%

Table 7: Comparison between experimental and numerical brittle fracture starting distance in $B = 20 \text{ mm}$ specimens

Regarding the comparison of crack propagation process, the crack length versus time for a particular $B20$ case is shown in Fig.24. The agreement between the measured crack position and the simulated position is very good. It is clear that in the first ten to fifteen microseconds, the crack propagates at speeds in the range of about 330 m/s , approximately $0.33 C_R$, which is below

the most commonly observed limiting crack speed for brittle dynamic fracture. Beyond ten microseconds, the crack slows down to an average crack speed of about 160 m/s , resulting in a total average value of about 250 m/s throughout the entire propagation process. Similar crack position history and agreement between experiments and simulations was observed for all experiments.

Figure 24: Experimental crack propagation velocities versus impact velocity from the onset of crack propagation – Brittle fracture

Quantitative comparison of the averaged crack propagation velocities for the brittle mode of fracture is set out in Table 8

	Pressure [bar]	Exp. vel. [m/s]	Num. vel. [m/s]	Rel. error
$d = 0\text{ mm}$	1.75	247.40 ± 21.45	220.70	11%
	2.25	272.72 ± 8.51	268.94	1.4%
	2.5	271.82 ± 18.95	282.43	3.7%
$d = 15\text{ mm}$	1.75	251.37 ± 14.97	262.51	4.2%
	2.25	242.65 ± 9.94	222.86	8.1%
	2.5	237.72 ± 7.32	221.62	6.8%

Table 8: Comparison of the crack propagation velocities for $B = 20\text{ mm}$ specimens

It can be seen that the crack propagation velocities in the case of a brittle fracture mode are about two orders of magnitude higher than those corresponding to a ductile fracture mode.

Taking into account the excellent agreement between the simulation and experimental results corresponding to ductile fracture mode and brittle fracture mode, it can be concluded that the constitutive and failure model described in Section 2, and calibrated in Section 3 with comparisons to the experiments of Ravi-Chandar et al. [9], is able to reproduce the failure mode transitions observed in the present set of experiments performed in a different geometrical configuration over a range of strain rates. The numerical results are now explored in greater detail in the next section to explore the origins of the double transition in the failure mode.

5. Numerical analysis of the transition in the failure-mode

In order to analyze the influence of the specimen thickness on the failure mode transitions, additional simulations were performed with the computational model described above, with projectile velocities between 4 and 40 m/s, reaching the physical limits of the Hopkinson bar model used. In these simulations, the temporal evolution of the parameters that govern the fracture of the specimen

(the maximum principal stress σ_I in the brittle criterion, and the critical strain energy density W_{crit} in the ductile criterion) in the area surrounding the notch, for different strain rates, was obtained, permitting the assessment of which of the two damage criteria was activated corresponding to each loading (or strain) rate. An example is illustrated in Fig.25, where the time variation of the maximum principal stress σ_I (normalized by σ_I^{crit}) at the location where eventual failure was triggered as well as the absolute maximum of W (normalized by the critical value W_{crit}) at every time step in the entire domain is shown. It is noted that in these simulations, the failure criteria are suppressed to permit the calculation of these two quantities for a stationary crack. Clearly, in the case shown in Fig.25(a), the brittle fracture criterion will be activated ($\sigma_I/\sigma_I^{crit} = 1$, at about $270 \mu s$) prior to the ductile failure criterion becoming critical and hence brittle crack propagation will occur. In the case of Fig.25(b), while the maximum principal stress is high at early times, it does not reach the critical state, being the ductile fracture criterion the one that is activated first ($W/W_{crit} = 1$, at about $1100 \mu s$), resulting in a ductile crack growth. It should be noted that corresponding to the large plastic deformations, there is an associated temperature increase of about $50^\circ C$ that enables further plastic flow in regions where plastic deformation is concentrated.

(a) Brittle criterion triggered. $v_{imp} = 22 m/s$

(b) Ductile criterion triggered. $v_{imp} = 15 m/s$

Figure 25: Time variation of the normalized fracture parameters for a $B10d0$ specimen

In order to assess these failure mode transitions, we plot in Fig.26, the maximum principal stress σ_I^{max} attained in simulations corresponding to different strain rates (impact speeds) in comparison to the brittle fracture initiation criterion (circular symbols and a linear fit). This is for the case of $B = 5 mm$, $d = 0 mm$. This figure also shows the critical maximum principal stress from which the brittle fracture occurs, as a function of the strain rate (Eq.7 - solid line). It is important to highlight that the strain rate values marked with circu-

lar symbols were obtained for each impact speed and calculated in a small area of elements surrounding the element in which the failure criterion was triggered.

Figure 26: Brittle fracture condition ($18000 \text{ s}^{-1} \leq \dot{\epsilon} \leq 41000 \text{ s}^{-1}$) for $B = 5 \text{ mm}$ and $d = 0 \text{ mm}$ specimens

The shaded area in Fig.26 indicates the range of strain rates at which a brittle fracture of the specimen would occur, $18000 \text{ s}^{-1} \leq \dot{\epsilon} \leq 41000 \text{ s}^{-1}$, since the maximum principal stress at the end of the notch exceeds the critical value defined by the damage criterion.

For other values of $\dot{\epsilon}$ below the lower end of the previous range, the specimen could break in a ductile mode, or not break at all, but not exhibit a brittle mode of fracture. It should be noted that the range of strain rates would depend on the specimen thickness and in-plane dimensions. In order to determine which of the two options is chosen, the maximum values of the strain energy of deformation W^{trac} reached in the notch end surrounding area, as well as the value of W_{crit}^{trac} defined in the ductile damage model (see Table 2), are represented in Fig.27.

Figure 27: Ductile fracture condition ($\dot{\epsilon} \geq 3450 \text{ s}^{-1}$) for $B = 5 \text{ mm}$ and $d = 0 \text{ mm}$ specimens

The results shown in Fig.27 indicate that for all values of strain rate greater than 3450 s^{-1} and the corresponding impact speed, a ductile fracture of the specimen would occur, and below that strain rate the specimen would not break in the present geometrical/loading configuration.

Therefore, by combining the data of Figs.26 and 27 in a single graph (Fig.28), it is possible to establish the strain rate ranges for which the $B = 5 \text{ mm}$ thickness and center notched specimens break in a ductile way, in a fragile way, or not break at all.

Figure 28: Modes of fracture as function of $\dot{\epsilon}$ for $B = 5 \text{ mm}$ and $d = 0 \text{ mm}$ specimens

In view of Fig.28 it can be seen that below $\dot{\epsilon} \simeq 3450 \text{ s}^{-1}$ no breakage occurs. When this strain rate is exceeded, the specimen breaks in a ductile way, till a value of $\dot{\epsilon} \simeq 18000 \text{ s}^{-1}$, when the brittle fracture criterion is fulfilled first. It is important to keep in mind that in those borderline cases where both criteria predict the failure of the specimen, the fracture mode which comes to the fore is not perfectly determined, being in these extreme cases a process that could be defined in a stochastic way. Therefore, around $\dot{\epsilon} \simeq 18000 \text{ s}^{-1}$ a first transition in the failure mode occurs, going from a ductile fracture mode to a brittle fracture mode. This transition has been observed experimentally in the dynamic three point bending tests for the $B = 10 \text{ mm}$ specimens.

For even higher strain rates, a brittle fracture continues to occur, until the value of $\dot{\epsilon} \simeq 41000 \text{ s}^{-1}$ is reached. At this strain rate a new transition in the failure mode occurs, from brittle to ductile in this case. This second transition is observed experimentally in the tests conducted by Ravi-Chandar et al. [9], but it has not been possible to corroborate experimentally in the dynamic three point bending tests due to the impossibility of reaching the corresponding impact velocities in the Hopkinson's Bar used.

After obtaining the strain rate ranges that define the different fracture modes for the $B = 5 \text{ mm}$ specimens, the influence of this thickness in these strain rate ranges is explored. For this purpose, the ranges for which the different failure modes were obtained for the three different thicknesses analyzed are represented in Fig.29.

Figure 29: Comparison of the ranges of $\dot{\epsilon}$ that establish the different modes of fracture for each thickness

In Fig.29 the two transitions observed in the PC failure mode are clearly shown. In addition, as the thickness of the specimen increases, the tendency to trigger ductile mode of fracture is reduced and the mode of failure that becomes predominant is the brittle mode, even in the complete disappearance of the first transition (ductile-brittle) in the specimens of greater thickness.

6. Conclusions

In this work, the failure mode transition that occurs in certain polymeric materials, and specifically in polycarbonate, has been analyzed numerically and

experimentally. A global damage model capable of reproducing and predicting this transition has been proposed, calibrated and validated.

As a consequence, a good correlation between numerical and experimental results has been achieved, both qualitatively and quantitatively. The validation has been carried out by performing dynamic three-point bending tests in a compression split Hopkinson pressure bar, varying both the thickness and the eccentricity of the initial notch of the test specimens used. As a result of these tests, it has been proven that there is a transition in the failure mode of polycarbonate, which occurs under normal conditions of pressure and temperature. In addition, it has been shown that crack propagation starts earlier in the cases of brittle fracture than in those of ductile fracture, and that crack propagation velocities are two orders of magnitude lower in the latter case.

Moreover, it is important to highlight that two transition points have been found in the failure mode of PC, analyzing the ranges of strain rates for which a brittle or a ductile fracture mode occurs in this loading configuration. A first transition from ductile to brittle fracture mode occurs as the strain rate increases (the one observed experimentally in the dynamic three-point bending tests), while a second transition from brittle to ductile fracture mode for even higher strain rates (this is the transition observed in the tests conducted by Ravi-Chandar et al. [8]). Both transitions depend not only on the thickness of the specimen, but also on a combination of thickness and strain rate; i.e. there is neither an independent critical value of thickness, nor an independent critical value of strain rate that establishes the boundary between failure modes, but it is a combination of both parameters that determines the mode of fracture that will occur.

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Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.