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Wake of tandem cylinders near a wall

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Abstract

This article reports an experimental investigation on the near wake of two identical circular cylinders in crossflow arranged in tandem configuration in the streamwise direction and with the additional interference of the ground. The Reynolds number based on the cylinders diameter is 4.9×10^3 . The present study analyses the effect of longitudinal pitch-to-diameter ratios spanning the well known flow regimes of tandem cylinders in absence of ground effect, i.e. single bluff body, shear layer reattachment and vortex co-shedding of twin cylinders. Particle Image Velocimetry measurements are performed to extract first and second order statistics of the wake. The wake features have been analyzed and compared with the bluff-body wake models present in literature. Additionally, the flow fields are decomposed in Proper Orthogonal modes to characterize the main coherent structures. The time coefficients of the modes are analyzed to extract phase relations between vortical features.

Far enough from the wall the cylinders wake is symmetric, with Von Kármán vortices shed symmetrically in the wake with respect to cylinders centerline. The measured average wake characteristics and vortical structures are consistent with the data reported in the literature for tandem cylinders without ground effect. For a wall-to-cylinder gap equal to 1 diameter, the ground strongly influences the flow field, introducing asymmetry in the typical Von Kármán wake. The ground boundary layer is thickened past the downstream cylinder and a wall-jet appears, enclosed between the wall boundary layer and the cylinders wake. Eventually these flow structures are not distinguishable when the cylinders wake and the wall boundary layer are grown enough to merge. From POD analysis, vorticity blobs appear on the wall, paired with vortex shedding.

If the wall gap is decreased to 0.3 diameters, an extended low speed region appears close downstream of the cylinders. A POD mode similar to a vortex shedding is still present; however it cannot be associated to an alternate Von Kármán street. The shed structures show a shorter wavelength than the Von Kármán shedding in far-from-the ground cases and are accompanied by a flapping jet-like structure ejected from the wall gap between the cylinder and the wall. The jet strongly changes the features of the wake on the side opposite to that of the wall, thus suggesting a possible coexistence between jet oscillations and shedding.

Keywords: Crossflow cylinders, Wake flow, Bluff body flow, Particle Image Velocimetry, POD

1. Introduction

The periodic flow behavior generated by a cylinder or by a bluff-body in general, immersed in a cross flow has been widely studied [1]. The formation of periodic shedding of counter rotating vortices, known as Von Kármán vortices, gives rise to fluctuating forces and noise. The shedding occurs at a Strouhal number based on the cylinder diameter ($St = fD/V$ where f is the shedding frequency, D is the cylinder diameter and V is the free stream velocity) equal to about 0.2 in a range of Reynolds number $1300 < Re < 5000$ [2]. In this range of Reynolds numbers the shedding generally occurs with a laminar boundary layer, while transition to turbulent wake occurs further downstream with respect to the separation points.

The flow around arrays of cylinders immersed in a cross-flow is a typical engineering problem that can be found in many applications. Cylinder-like structures are typical elements, for example, of heat exchangers, cooling systems for nuclear power plants, offshore structures, buildings, chimneys, powerlines,

struts, grids, screens, and cables. Often these structures work in close proximity, thus introducing strong interaction effects between their respective wakes, thus affecting both amplitude and frequency of the periodic flow oscillations. Similarly, ground-structure interaction can effectively modify the behaviour of the flow surrounding these objects such as in the case of underwater pipelines or arrays of bluff-bodies in ground effect. Understanding the behaviour of tandem cylinders in ground effect is of paramount importance to design structures immersed in their wake and that are subjected to unsteady loads.

The flow around two cylinders in tandem configuration has been widely studied by [3] and [4]. A complete review of the subject is given by [5]. Depending on the Reynolds number and on the center-to-center longitudinal pitch (expressed in non dimensional form as L/D , abbreviated hereafter as the longitudinal pitch ratio) three main flow behaviors can be identified. At small longitudinal pitch ratios (approximately $1 < L/D < 1.2 - 1.8$ according to [4], or $1 < L/D < 2$ according to [6], depending on the Re range) the Kármán vortex shedding for the upstream cylinder is completely suppressed and the two cylinders act as a single bluff-body. Meneghini et al. [7] observed that vortex roll-up occurs closer to the downstream cylinder

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Figure 1: Flow behaviors for cylinders in tandem configurations: (a) bluff-body regime, (b) reattachment regime, (c) co-shedding regime.

with respect to the single cylinder case. This flow regime has been often defined as *extended-body* or *single bluff-body regime* as sketched in Fig. 1a. At intermediate longitudinal pitch ratios (approximately $1.2 - 1.8 < L/D < 3.4 - 3.8$ according to [4] or $2 < L/D < 5$ according to [6], depending on the range of Re) a complex flow behavior appears in the space between the cylinders. Even if in this regime the flow can show different behaviors, it can be mostly characterized by the reattachment of the separated free shear layers from the upstream cylinder on the surface of the downstream cylinder. This regime is referred as *reattachment regime* as sketched in Fig. 1b. At larger longitudinal pitch ratios (approximately $L/D < 3.4 - 3.8$ according to [4] or $L/D < 5$ according to [6], depending on the range of Re) both the cylinders develop a wake with the typical features of a Kármán street. This regime, sketched in Fig. 1c, is referred as *co-shedding regime*. In the co-shedding regime, both the cylinders shed vortices at the same frequency, with the upstream cylinder shedding triggering the downstream one (see [8]). The vortices shed from the downstream cylinder are larger in size but weaker in intensity than in the previous regimes [6].

The different regimes for various L/D are characterized also by the variation of the shedding frequency of the Von Kármán vortices in the cylinder wake; in particular Xu and Zhou [9] characterized the variation of the corresponding Strouhal number spanning through the tandem-cylinders regimes. For small longitudinal pitch-to-diameter ratios the Strouhal number is higher than 0.2 (shedding occurs at higher frequency than for an isolated cylinder). For higher longitudinal pitch ratios, the Strouhal number decreases, reaching values lower than 0.2 in the reattachment regime. A discontinuous jump occurs when the flow behavior passes from the *reattachment* to the *co-shedding regime*. For higher longitudinal pitch ratios the Strouhal number slowly approaches 0.2.

The effect of the ground proximity has been investigated for both circular [e.g. 10, 11, 12] and square [see 13, 14, 15, 16] single cylinders. The ground affects the pressure distributions, as well as the flow-induced vibration modes. The interaction of the cylinder with the wall has been also demonstrated to be a useful mean to control the flow in more complex arrangements [e.g. 17]. The flow behavior is controlled by the ratio between the cylinder-to-wall gap height and the cylinder diameter (G/D , abbreviated hereafter as the wall gap ratio). According to [18] a critical wall gap ratio between 0.3 and 0.5 the Von Kármán

vortex shedding is suppressed as the wall poses an irrotational constraint on the cylinder wake. According to [11], however, periodical release of vorticity is still present and its frequency depends on the Reynolds number for $Re < 2000$. Lin et al. [19] found evidence of the presence of a wall-jet flowing from the wall gap for wall gap ratios lower than 0.3, showing that this jet presents a self-similar velocity profile.

Little attention has been paid to the effect of the wall proximity in the case of two cylinders in tandem configuration. Bhat-tacharyya and Dhinakaran [20] investigated with 2D simulations the flow around two near-wall tandem square cylinders ($G/D = 0.5$, $L/D = 1.5 - 6$, $Re = 100 - 200$) with a shear velocity profile imposed upstream of them, and found that the cylinders wake resulted in a non-symmetric flow behavior: both non-uniformity of the flow and wall-induced vorticity weakens the lower separated shear layer with respect to the upper one. Harichandan and Roy [21] simulated the flow around two near-wall tandem circular cylinders ($G/D = 0.5 - 1$, $L/D = 2 - 5$, $Re = 100 - 200$): for $L/D = 5$ the shedding frequencies of both upstream and downstream cylinders are found to be equal. Moreover, the wake of tandem cylinders is less effective than the wake of a single cylinder in destabilizing the downstream wall boundary layer and causing separation. Wang et al. [22] measured with PIV the flow field around two square cylinders at $Re = 6300$ spanning a wide parametric space in both longitudinal pitch ratio ($L/D = 1.5 - 7$) and wall gap ratio ($G/D = 0.25 - 2$). Similarly to the case of the single cylinder for wall gap ratios below 0.5 the shedding is found to be suppressed. The interaction with the wall boundary layer is found to be relevant up to $G/D = 1$.

This work focuses on the near wake mechanisms downstream of tandem circular cylinders in ground effect. The flow field is measured by means of Particle Image Velocimetry (PIV, [23]) and reduced in principal components with Proper Orthogonal Decomposition (POD, [24]) to extract the dominant coherent structures. In particular this work uses the statistical information of POD to clarify how the shedding mechanisms are influenced by the multiple cylinders/wall interactions and whether (and how) shedding is suppressed for small wall gaps between the cylinder and the wall. In §2 the experimental setup and the test conditions are described. The data collected from each experiment are used to calculate first and second order statistics; the results are presented in §3 along with POD modes and are

Figure 2: Schematic of the experimental setup.

discussed and synthesized in §4.

2. Experimental methodology and processing techniques

2.1. Experimental setup and test procedure

The experimental setup is similar to that in [25]. The experimental study has been carried out in the closed loop wind tunnel of the Aerospace Engineering Group at the *Universidad Carlos III de Madrid*. This wind tunnel has a square test section of $0.4m \times 0.4m$ with a length of $1.5m$. The ceiling and the lateral walls of the tunnel are made in methacrylate in order to enable optical access. Free-stream turbulence intensity is estimated to be lower than 1% by means of PIV measurements in the empty tunnel. The tunnel is equipped with a plane splitter plate (750mm length) with a sharp leading edge. The splitter plate is used as ground (instead of a tunnel wall) in the attempt of obtaining a ground effect with relatively thin boundary layer. The boundary layer thickness on the plate without cylinders has been measured with PIV and resulted being smaller than 7mm at the position of the downstream cylinder and smaller than 11mm at the trailing edge of the splitter plate. Two PVC cylinders, with diameter 32mm and length 400mm (blockage ratio 0.08), are used as test models spanning the wind tunnel test section from side to side. Figure 2 shows a schematic of the experimental setup. The longitudinal position of the downstream cylinder is kept fixed with respect to the plate trailing edge for all the experiments, at a distance of $10D$. The longitudinal position of the upstream cylinder is changed to vary the longitudinal pitch ratio L/D . The minimum distance of the first cylinder to the leading edge of the splitter plate (i.e. for the case $L/D = 6$) is slightly less than $7.5D$. The vertical position of the plate with respect to the tandem cylinders is changed to vary the wall gap G/D . Experiments are carried out for L/D spanning from 1.5 to 6 and G/D in the range 0.3-3. The wind tunnel velocity is kept constant and equal to $2.3m/s$ for all the experiments in order to achieve a Reynolds number based on the cylinder diameter of about 4.9×10^3 .

Velocity field measurements are performed with digital Particle Image Velocimetry. The flow is seeded with Di-Ethyl-Hexyl-Sebacate droplets with diameter of approximately

$1\mu m$. The light source is a Big Sky Laser CFR400 ND:Yag ($230mJ/pulse$, pulse duration $3ns$). The acquisition is performed with a *TSI PowerView™ Plus 2MP* Camera (1600×1200 pixel array, $7.4\mu m \times 7.4\mu m$ sensor size) with a spatial resolution of about 7.2 pixels/mm. The sampling frequency of the PIV measurements is 10Hz. An ensemble of 2000 image couples is acquired for each experiment.

Image quality is improved by removing laser reflections and illumination background, allowing unbiased measurements near the wall.

The SPIV software, developed at University of Naples Federico II, is used to cross-correlate particle images and to calculate the velocity fields [26, 27, 28]. The interrogation strategy is an iterative multi-step image deformation algorithm, with final interrogation windows of 16×16 pixels, 50% overlap (final vector spacing 8 pixels, corresponding to $0.035D$). The vector validation is carried out with a universal median test [29] on a 3×3 vectors kernel and a threshold equal to 2 is used to identify invalid vectors. Discarded vectors are replaced with a distance-weighted average of neighbor valid vectors.

2.2. Proper Orthogonal Decomposition and extraction of shedding phase information

A modal analysis is conducted via POD to extract information on the coherent structures collecting the bulk of the turbulent kinetic energy. POD is a mathematical procedure which identifies a set of orthonormal basis functions computed as the solution of the integral eigenvalue problem known as Fredholm equation. The velocity field is decomposed according to the Reynolds decomposition as sum of mean and fluctuating velocity field.

$$\begin{aligned} \mathbf{U}(\mathbf{x}, t) &= \langle \mathbf{U}(\mathbf{x}) \rangle + \mathbf{u}(\mathbf{x}, t) \\ &\approx \langle \mathbf{U}(\mathbf{x}) \rangle + \sum_{n=1}^{N_m} a_n(t) \boldsymbol{\varphi}_n(\mathbf{x}, t). \end{aligned} \quad (1)$$

Without any loss of generality, let the symbols $\langle \mathbf{U}(\mathbf{x}) \rangle$ and $\mathbf{u}(\mathbf{x}, t)$ indicate the mean and the fluctuating part of the velocity field, respectively, with $\mathbf{x}$ and t being the spatial and time coordinates, respectively. The fluctuating part can be approximated as a linear combination of a set of basis function $\boldsymbol{\varphi}_n(\mathbf{x}, t)$, with coefficients $a_n(t)$ depending on time; the symbol N_m is used to indicate the number of modes, i.e. the rank of the algebraic space. Evidently, in the limit $N_m \rightarrow \infty$ the approximation becomes exact. Among the infinite sets of orthonormal basis functions that can be used to decompose the fluctuating velocity field in Eq. 1, POD aims to identify the set that is optimal in the least square sense, i.e. the one which minimizes the L^2 norm of the residual:

$$\min \left[\sum_{i=1}^{N_t} \left\| \mathbf{u}(\mathbf{x}, t_i) - \sum_{n=1}^{N_m} (\mathbf{u}(\mathbf{x}, t_i), \boldsymbol{\varphi}_n(\mathbf{x})) \boldsymbol{\varphi}_n(\mathbf{x}) \right\|^2 \right] \quad (2)$$

where N_t is the number of realizations and $(., .)$ indicates the inner product. The approach used in this work is the snapshot

method [25, 30]. Assuming that each realization can be treated as a N_p -dimensional vector, the data can be arranged in a $N_p \times N_t$ snapshot matrix:

$$U = \begin{bmatrix} u(x_1, t_1) & \cdots & u(x_1, t_{N_t}) \\ \vdots & \ddots & \vdots \\ u(x_{N_p}, t_1) & \cdots & u(x_{N_p}, t_{N_t}) \end{bmatrix}. \quad (3)$$

It can be demonstrated that the solution of the minimization problem of Eq. 2 corresponds to the computation of the Singular Value Decomposition (SVD) of the matrix U . The advantage of solving this problem via the SVD is that the eigenmodes will be sorted according to the value of the respective eigenvalues λ_i . The eigenvalues measure the relative importance of the different basis functions in terms of turbulent energy of the flow; consequently, the SVD determines a set of basis functions sorted by their energy contribution. The energetic optimality of POD ensures that the most energetically relevant turbulent features are contained in the first modes and that the energy content of the modes rapidly decays as the number of modes increases.

In case of phenomenologies characterized by a strong large scale shedding (such as the one in object), it is safe to assume that the first POD modes, apart from being orthogonal, might also show a strong harmonic relation such as in a Fourier decomposition of the field. Therefore, POD can be used to obtain phase information from non-time-resolved data and to extract a low order model of the phenomenon. For example, when the first two modes are dominant and indicative of a periodic shedding, the following decomposition can be used [31, 32]:

$$U_{LOR}(x, t) = \langle U(x) \rangle + a_1(\vartheta)\varphi_1(x) + a_2(\vartheta)\varphi_2(x) + \sum_{n=3}^{N_m} a_n(\vartheta)\varphi_n(x). \quad (4)$$

$$a_1 = \sqrt{2\lambda_1}\sin(\vartheta), \quad a_2 = \sqrt{2\lambda_2}\cos(\vartheta) \quad (5)$$

with ϑ being the period phase and λ_n being the n^{th} eigenvalue. In order to verify that the first two modes represent the coherent harmonics related to the shedding motion the scatter plot of the coefficients normalized with their respective eigenvalues $a_1/\sqrt{2\lambda_1}$ and $a_2/\sqrt{2\lambda_2}$ should be observed in search of Lissajous figures. If the points distribute in the neighborhood of a goniometric circle, the Eq. 4 is respected [31, 32].

In general, the scatter plot of the time coefficients might unlock information on the phase and frequency relation between higher order modes, thus shedding some light on the interconnection between the different flow features highlighted by the modal analysis. Assuming that the first POD modes are harmonically related and phase-shifted with respect to the 1st mode:

$$a_n = \sqrt{2\lambda_n}\sin(m_n\vartheta + \phi_n), f_n = m_nf_1 \quad (6)$$

where m_n is a positive integer and ϕ_n is the phase shift of the n^{th} mode with respect to the first mode. The procedure to individuate the harmonic relation and the phase shift for higher

order modes relies on the assumption that the first 2 modes are at the same frequency with $\pi/2$ phase shift (as in Eq. 5). It is possible to extract the period phase from the time coefficient of the first two modes:

$$\tan(\vartheta) = \sqrt{\frac{\lambda_2}{\lambda_1}} \frac{a_1}{a_2} \quad (7)$$

Subsequently, the positive integer and of the phase shift, that characterize the harmonic relation, are extracted from the minimization of:

$$\frac{a_n}{2\sqrt{\lambda_n}} - \sin(m_n\vartheta + \phi_n) \quad (8)$$

The phase relation is considered successful when the correlation coefficient between the normalized time coefficient vector and the harmonic relation is above 0.7.

3. Results

3.1. Time average fields and turbulence statistics

The velocity fields are processed to extract the distributions of the first and second order statistics. The extraction of several quantities of interest for a turbulent wake is straightforward. Analyzing the profiles of the wake deficit of momentum $u_1(x, y) = u(x, y) - u_{max}(x)$, where $u_{max}(x)$ is the local velocity maximum along the wall normal direction, the maximum deficit $u_{1,max}(x)$ and its position $y_{1,max}(x)$ are identified. This quantity is of paramount importance in order to properly scale the velocity profiles in the wake [33]. Following typical literature about turbulent wakes [34], the wake boundaries $y_{wake}(x)$ are defined as the locations in which $u_1(x, y_{wake})/u_{1,max}(x) = 0.5$. The wake width $b(x)$ is defined as the y -distance between the wake boundaries.

The velocity deficit profiles at various x -distances from the downstream cylinder for the case $L/D = 3$, $G/D = 3$ are illustrated in Fig. 3a. After rescaling the y -coordinate with the wake width b and the velocity deficit with $u_{1,max}$, the velocity deficit profiles tend to collapse on a single curve. Despite self-similarity cannot be assured at the tested short streamwise distance, velocity deficit profiles compare quite well with the theoretical asymptotic result given by Schlichting [33]:

$$u_1 = u_{1,max} \left(1 - \left(\frac{y}{b} \right)^{\frac{3}{2}} \right)^2 \quad (9)$$

Similar results have been found for almost all cases at $G/D = 3$ (see appendix Fig. A.1). This assures that the blockage effects on the presented results is minimal. Moreover, the absence of symmetry alteration in the profiles for $-1 < y/b < 1$ suggests that for all the cases with $G/D = 3$ the effect of the wall is negligible along the whole streamwise extension of the measured domain. This is also consistent with the symmetry of the distributions of the average streamwise velocity (see Fig. 4a,d,g,j) and of the turbulent kinetic energy (see Fig. 5a,d,g,j) and by the absence of rotation in the wake (in both figures red dashed and solid lines depict respectively the maximum defect location and the wake boundaries). As the cylinders approach the

Figure 3: Velocity deficit profiles of $L/D = 3$ cases for varying G/D . Red circles identify the self-similar solution by Schlichting [33].

wall (see Fig. 3b,c) the symmetry in the velocity deficit profiles is disrupted, showing a different shape with respect to the Schlichting profile in the wake portion on the wall side. On the wall side of the wake the Schlichting profile is substituted by a jet-like one that can be associated to the wall-jet described by [19]. Differently from [19], a proper scaling for this jet has not been found for our test, suggesting that the presence of multiple bluff-bodies may significantly change the characteristics of this jet. The wall-jet profile and the wake profile coexists until the jet maximum position approach the lower wake boundary; from this point on the given wake width definition does not appear a proper scaling for the velocity deficit profile (see appendix Fig. A.1b).

In Fig. 6 the wake width and the maximum velocity defect evolution along the x -axis are shown for the cases at $G/D = 3$. Fig. 6a depicts clearly the effect of different L/D on the wake: while the growth rate appears to be quite similar for all the cases past $x/D = 3$, the near wake ($x/D < 2$) shows the most remarkable differences. The single cylinder case presents a protracted region (about 3 diameters long) in which the wake expands before contracting again and finally spreading with higher growth rate similar to the tandem cylinders cases. This region can be associated with the distance required by the Kármán vortices to roll up and with the presence of an elongated recirculation bubble (refer to [1] for further details). In this region the maximum velocity deficit (see Fig. 6b) is nearly constant.

For $L/D = 1.5$ a relatively thin wake forms in close proximity of the downstream cylinder (20% smaller than the cylinder diameter, see Fig. 6a). The absence of the recirculation bubble is consistent with the picture of a *bluff-body regime* [4]. It can be conjectured that the shear layers are rolling up closer to the downstream cylinder with respect to the single cylinder case (thus outside of the measurement region). For $L/D = 3$ the wake width in proximity of the downstream cylinder is larger, being almost equal to one cylinder diameter. In contrast with the previous case, the wake width has a plateau for $x/D < 1.5$, and then starts growing, with slope similar to the other cases. This different wake behavior can be ascribed to the occurrence of a *reattachment regime* [4], with the wake of the upstream cylinder reattaching on the downstream one. Dif-

ferently from the *bluff-body regime*, in the *reattachment regime* the shear layer detaches directly from the surface of the downstream cylinder, thus showing some similarity to the case of the single cylinder. The velocity defect is stronger with respect to the case falling in the *bluff-body regime*, especially in the near wake ($x/D < 2$). The combined analysis of wake growth and maximum velocity deficit reveals a stronger drag being produced in this configuration, as found in literature for similar longitudinal pitch ratios [35]. For the case at $L/D = 6$ the wake is thicker than both the previous cases at $L/D = 1.5$ and $L/D = 3$, while the growth rate is similar to the one falling in the *bluff-body regime*. The maximum velocity defect dramatically increases with respect to both the other tandem cylinder cases throughout almost all the measurement domain. The effect of both increasing wake width and maximum velocity defect suggests a dramatic increase of the drag for the case with $L/D = 6$, depicting the scenario of a *co-shedding regime* [35].

The effect of reducing the wall gap ratio is more diversified (see Fig. 7) and seems to affect significantly the flow field regime. In most of the cases the wake width is relatively unaffected by the wall interference. For the case of the single cylinder the growth rate downstream of the recirculation bubble is unchanged when passing from $G/D = 3$ to $G/D = 1$, while the only appreciable effect seems to be a change of the bubble extension. For $G/D = 0.3$ the general characteristics of the wake width evolution along the streamwise coordinate are preserved approximately up to a point in which the cylinder wake on the wall side and the wall boundary layer merge (at about $x/D = 4$ as can also be appreciated in Fig. 4b). A similar behavior of the wake width evolution can be observed for almost all the cases. A very exemplifying case is represented by $L/D = 1.5$, $G/D = 1$: the wake growth rate is nearly unaltered with respect to the $G/D = 3$ case up to $x/D = 4$, while at this value it starts decreasing due to the merging of the wake with the boundary layer (see Fig. 4b). Starting from $x/D > 4$ the wake width b cannot be used as a suitable scaling parameter (see e.g. Fig. A.1b). The effect on the maximum velocity deficit (see Fig. 8) clearly indicates that as the cylinders approach the wall, the momentum defect tendentially increases, possibly due to the addition of the mass flow defect of the wall boundary

Figure 4: Contour maps of the average streamwise velocity component for all the longitudinal pitch ratios (L/D) and wall gap ratios (G/D) object of the study. x distance is measured from the downstream cylinder center; y distance is measured from the splitter plate. The red dashed curve represents the position of $u_{1,max}(x)$. The red continuous curves represent the wake boundaries for which the condition $u_1(x, y_{wake})/u_{1,max}(x) = 0.5$ is satisfied. Streamlines depict the average field behavior.

Figure 5: Contour maps of the Turbulent Kinetic Energy for all the longitudinal pitch ratios (L/D) and wall gap ratios (G/D) object of the study. x distance is measured from the downstream cylinder center; y distance is measured from the splitter plate. The red dashed curve represents the position of $u_{1,max}(x)$. The red continuous curves represent the wake boundaries for which the condition $u_1(x, y_{wake})/u_{1,max}(x) = 0.5$ is satisfied. Streamlines depict the average field behavior.

Figure 6: Comparison of a) wake width and b) maximum velocity defect for varying L/D and $G/D = 3$.

layer to the wake mass flow defect.

A better insight on the effect of the wall on the wake evolution is given by Fig. 4, that shows the contour plots of the average streamwise component of the velocity, along with the curves depicting the positions of the wake boundaries and the line of maximum velocity deficit. With respect to the symmetric cases (exemplified by wall gap ratio $G/D = 3$), all the cases at $G/D = 1$ and $G/D = 0.3$ present a more or less pronounced bending in the wake, showing a concavity towards the wall, as highlighted by the maximum deficit position and by the wake boundaries. The maximum velocity deficit line (red dashed line in the figures) is slightly rotated upwards for the closer portion of the wake. This behavior has been already observed for the single cylinder in [19], showing that the recirculation region rotates far from the wall following the wall jet. The farther portion of the wake, instead, appears to rotate towards the wall, suggesting the formation of a low pressure region on the wall that could draw the wake to the ground.

This rotation effect can be addressed as a weak interaction between the cylinders wake and the wall. As the wake lower boundary and the wall boundary layer (whose edge corresponds to the point of maximum velocity in the wall jet) merge, a strong interaction occurs: the upper wake boundary -that represent the only well-recognizable feature of the former wake- no longer spreads, remaining nearly parallel to the wall or shrinking. In this case no more distinction could be performed between wake and wall boundary layer: they completely merge in a single low speed region attached to the wall. It can be expected that this structure may eventually develop in a boundary layer, being the cylinders considered as an overtripping (e.g. see [36]). The geometrical configuration of the cylinders appears to be a major player in determining the position at which the two structures merge, controlling both wake characteristics -such as wake width and maximum velocity defect- and wall jet/boundary layer characteristics.

The contour maps of turbulent kinetic energy (TKE) are reported in Fig. 5, where $TKE = 3/4(\langle u'u' \rangle + \langle v'v' \rangle)$. For all the cases at $G/D = 3$ the TKE presents a symmetric pattern with respect to the wake centerline, in agreement with the previously

reported results on first order moments and wake behaviour. Significant differences between the different configurations can be appreciated for varying L/D . For $L/D = 1.5$ (*bluff-body regime*) the wake is characterized by a strong TKE production, especially localized on the wake centerline and close to the downstream cylinder. This TKE distribution resembles closely the one observed for the single cylinder, even if TKE values are consistently higher. Differently from the single cylinder case, the low turbulence intensity region associated with the recirculation bubble is not present, in agreement with the previously outlined results. For $L/D = 3$ (*reattachment regime*) the TKE in the wake is characterized by the presence of two elongated production regions, with some analogy with the shear layer development observed for the single cylinder case. This is again consistent with the view of a *reattachment regime*. For $L/D = 6$ (*co-shedding regime*) TKE values are slightly stronger and diffused than in the previous case, with peaks along the wake boundaries. This behavior appears to be consistent with the release of Kármán vortices from the upstream cylinder, as observed by [4]: the downstream cylinder is invested by a pulsating flow that convects the wake of the downstream cylinder; this determines a more intense spatial spreading of the TKE and an increase of the fluctuations in the most external part of the wake.

As the wall gap ratio is reduced (see Fig. 5e,h), the TKE distribution becomes asymmetric, increasing in intensity in the wall-facing portion of the wake. For low wall gap ratios (see Fig. 5b,c,f,i) a remarkable change in the spatial distribution of the TKE is present: in the most downstream portion of the wake the energy tends to redistribute from the centerline to the upper wake boundary as soon as the lower wake boundary and the wall boundary layer merge. This determines the formation of a low TKE region near the wall and the displacement of the maximum TKE region farther from the wall with respect to the wake centerline. Close to the downstream cylinder a strong asymmetry in the TKE distribution is still present, generally with an increase of TKE close to the wall and flowing out from the cylinder-wall gap. More than to turbulence production, this strong intensity of the fluctuating field can be addressed to a

(a) SINGLE CYLINDER

(b) $L/D=1.5$ (c) $L/D=3$ (d) $L/D=6$

Figure 7: Wake width comparisons for varying G/D . Solid lines represent traditional definition $b(x) = y_{wake,up}(x) - y_{wake,low}(x)$. Dashed lines represent $b_{up}(x) = 2(y_{wake,up}(x) - y_{1,max}(x))$.

(a) SINGLE CYLINDER

(b) $L/D=1.5$ (c) $L/D=3$ (d) $L/D=6$

Figure 8: Maximum velocity deficit comparisons for varying G/D .

periodical mean flow, suggesting the presence of a pulsating-jet-like flow in the wall gap between the downstream cylinder and the plate.

3.2. POD modes organization and mutual relation

The turbulent kinetic energy content of the first 20 POD modes is reported in Fig. 9 for the nine tandem cylinders configurations that have been tested (along rows, variable wall gap ratio; along columns, variable longitudinal pitch ratio). The case of a single cylinder is included for comparison in the last row. Similarly to the case of the single cylinder, at $G/D = 3$ the first two modes account for most of the turbulent kinetic energy in both the *bluff-body regime* (about 70% of the total energy) and *reattachment regime* (about 55% of the total energy). These modes, as will be clarified later in this section, are related to the first harmonic of the Von Kármán vortex shedding. A strong drop in the energy content of the first two modes occurs for the *co-shedding regime* (about 20% of the total energy), while energy redistributes over higher order modes, unveiling a more chaotic behavior. The effect of small wall gap ratios is evident for the case of the single cylinder: due to ground effect, the energy content of the first two modes strongly drops (passing from nearly 60% at $G/D = 3$ to about 35% at $G/D = 0.3$). This behavior is compatible with a flow not dominated anymore by the shedding. Similarly, for cylinders in tandem configuration, a more or less intense drop can be observed for all the cases when wall gap ratio decreases. Energy tends to redistribute from the first modes (strongly dominant at $G/D = 3$) over higher order modes that may be uncorrelated with the typical Von Kármán shedding behavior.

The most energetic POD mode is reported in Fig. 10 for all the cases under study. The case of a single cylinder in cross flow is also included for reference. Data are presented in terms of out-of-plane non-dimensional vorticity component maps to spotlight the vortical features. In all the cases the vorticity maps are dominated by the intermittent release of vorticity, coherently with the scenario of a shedding wake. This picture was expectable, and confirmed by the analogous vorticity distributions in the second mode (not included for conciseness). The dominance of a shedding is also confirmed by observing the scatter plot of the time coefficients of the first two POD modes (Fig. 11), normalized with twice the square root of the corresponding eigenvalue. The Lissajous curve obtained from low-order modelling of the scatter plot is indicated in red in Fig. 11. The analytical form of the Lissajous curve is:

$$a_1 = \sin(2\pi ft), \quad a_2 = \sin(2\pi ft \pm \pi/2) \quad (10)$$

In most of the cases under examination, the normalized time coefficients of the first two POD modes distribute with quite good fidelity on a circle with unitary radius. Notice that the agreement is more evident for the case of the *single-body regime* at the largest wall gap ratio. This is due to the different nature of the shedding. For the case of the *single-body regime* ($L/D = 1.5$, $G/D = 3$, see Fig. 10a), a regular shedding similar to that of a single cylinder (see Fig. 10j) is released, with concentrated blobs of vorticity on the centerline. It must be

highlighted that the vorticity blobs in the single cylinder case present a double tail, corresponding to the shear layers released from the cylinder in the vortex formation region. This shows a strong statistical correlation between the shear layers oscillations and the vortex shedding. A similar behavior cannot be observed for the *bluff-body regime*, as the vortices roll-up very close to the downstream cylinder. This difference is reflected also by scatter plots: while in the *single-body regime* there is a very weak scattering of the time coefficients from the Lissajous curve (see Fig. 11a), in the single cylinder case (see Fig. 11j) a stronger scattering appears, revealing a lower harmonic coherence due to the presence of the shear layers in the first two POD modes. For the case of the *reattachment regime* ($L/D = 3$, $G/D = 3$, see Fig. 10d) the vorticity blobs appear more stretched along the streamwise direction and mix up with the shear layer of the wake of the first cylinder. The two phenomena are statistically correlated but might be characterized by different characteristic frequencies, thus reducing the clearness of the scatter plot of the normalized time coefficients (Fig. 11d). In the *co-shedding regime* ($L/D = 6$, $G/D = 3$, see Fig. 10g) this process is further stressed, leading to the appearance of pairs of vortices with similar vorticity value released in the wake for the case of the largest distance from the wall. This is symptomatic of a phase-locked shedding of the two cylinders.

As the two cylinders approach the wall, intermittent release of vorticity is still clearly observed in all cases, revealing an underlying shedding behavior also for $G/D = 0.3$. At $G/D = 0.3$ (see Fig. 10c,f,i,l) the spatial frequency of the shedding strongly increases with respect to the case of $G/D = 3$. The shedding behavior is confirmed also by the circular pattern in the scatter plots of the time coefficients (see, in particular, Fig. 11c and 11f). Indeed, the not perfect agreement of the harmonic relation with time coefficients suggests a more chaotic behavior than in the far-from-the-wall cases, possibly characterized by the contamination of multiple frequencies. It must be also pointed out that the energy content of the first two modes (Fig. 9c and 9f) strongly drops for the close-to-the-wall cases, revealing that the shedding loses its dominance. For $L/D = 3$, $G/D = 0.3$, due to the energy drop, the shedding mode may be nearly completely covered by other oscillations in the wake. Similar shedding modes are present also for the single cylinder case at $G/D = 0.3$ (Fig. 10l). Despite most of the literature agrees with the observation that Von Kármán shedding from a cylinder should disappear at wall gap ratios equal to $G/D = 0.3$ (see, e.g. [10]), the presence of similar vortical structures in the wake has been already evidenced by [11] and [17]. For a single cylinder, the frequency at which this vorticity is released has been observed to be strongly dependent on the Reynolds number [11]. It must be highlighted that the shedding spatial frequencies observed for $L/D = 1.5$, $G/D = 0.3$ (Fig. 10c) and for $L/D = 3$, $G/D = 0.3$ (Fig. 10f) are similar to the one observed for the single cylinder. This suggests that the mechanism that promotes this shedding should be similar for both single and tandem cylinders. A possible explanation is that these oscillations are synchronized with the instabilities in the wall jet flow released from the wall gap: oscillations may then depend on the effective wall gap passage area -i.e. the equivalent pas-

Figure 9: Percentual turbulent kinetic energy content of the first 20 modes.

Figure 10: Contour maps of non-dimensional vorticity $\omega_z = D \left(\frac{\partial \phi_y}{\partial x} - \frac{\partial \phi_x}{\partial y} \right)$ of the 1st POD mode.

Figure 11: Scatter plot of time coefficients of the 2nd POD mode with respect to the time coefficients of the 1st mode. The red line indicate the Lissajous curve with analytical form $a_1 = \sin(2\pi ft)$; $a_2 = \sin(2\pi ft \pm \pi/2)$.

sage area once boundary layer mass defect has been subtracted. This justifies the different behavior for $L/D = 6$.

The picture of a strong interaction of the wall boundary layer with the shedding in the wake is further enforced by the asymmetry of POD modes for low wall gap ratio cases. A prominent feature of these cases is the production on the wall of intermittent vorticity patches synchronized with the shedding (see Fig. 10b, 10e and 10h) that, for the lowest wall gap ratio, tends to merge with the vorticity shed by the downstream cylinder (see Fig. 10c, 10f and 10g), thus introducing stronger asymmetry in the modes.

The case of $L/D = 6$, i.e. within the *co-shedding regime* for $G/D = 3$, is experiencing significant changes. While for the far-from-the-wall case a dual shedding wake has been observed, at $G/D = 1$ the wake assumes features which are more similar to that of the case of $L/D = 3$, i.e. to that of the *reattachment regime*. This would suggest that the effect of the wall is to push the wake towards the downstream cylinder, thus promoting reattachment. For the case of $L/D = 6$ and $G/D = 0.3$ (Fig. 10i) the scenario is further complicated, since the pure shedding is associated with a flapping jet-like ejection from the slot created by the second cylinder and the wall. This feature is not observed in the cases of $L/D = 1.5$ and $L/D = 3$ since in both cases the wake of the upstream cylinder do not have enough streamwise length to spread and merge with the wall boundary layer, producing blockage in the lower wall gap. However, even for the case of $L/D = 6$ and $G/D = 0.3$ (Fig. 10i) the intermittent release of vorticity of shedding wakes is still observable at $y/D = 1.5 - 2$, even though more diffused. This shedding is likely to be originating from the upstream cylinder, thus providing a picture of a shedding-attached regime, in which the wake of the downstream cylinder is strongly asymmetric, with an intermittent oscillating jet on the wall side and a shedding of vortices on the outer side.

Higher order modes present a more various set of behaviors for changing longitudinal pitch and wall gap ratios: same order modes can strongly differ from one case to the other, mostly because of the energy redistribution that interest these modes. In the cases where a pure vortex shedding is clearly the dominant behavior (namely for *bluff-body regime* at high wall gap ratios), higher order modes are prevalently characterized by intermittent release of vorticity at a spatial and temporal frequency that is an integer multiple of the principal shedding frequency. The vorticity distribution is in some cases symmetric with respect to the streamwise direction (see, for instance, for the 4th POD mode, the case of Fig. 12a and 12e), in some others asymmetric (see for example Fig. 12b and 12d). Symmetric modes tune the intensity and size of the vortices shed in the wake, and are related to odd multiples of the shedding frequency (see, for instance, Fig. 13a and 13e). Asymmetric modes, on the other side, model the cross-wise oscillation of the shedding wake, and are related to even multiples of the shedding frequency (see, for instance, Fig. 13b and 13d). For cases not characterized by a pure shedding (such as for *reattachment* and *co-shedding regimes*), oscillations not correlated with the shedding (Fig. 12c and 12f and Fig. 13c and 13f) can account for higher energy content than the higher order harmonics of the shedding.

4. Discussion and conclusions

An extensive investigation has been carried out to study the effect of the ground on the wake produced by two cylinders in tandem configuration at $Re = 4.9 \times 10^3$. A number of interesting cases have been selected on the basis of the existing literature about the topic: 3 different longitudinal pitch ratios ($L/D = 1.5, 3, 6$) have been selected in order to span the typical regimes found for cylinders in tandem configuration with no wall effect; 3 different wall gap ratios ($G/D = 3, 1, 0.3$) have been chosen to study the effect of the ground. As a reference, the cases of a single cylinder at the same wall gap ratios have been studied. The study has been performed by means of PIV measurements in the wake of the downstream cylinder. The non-time-resolved data have been used to retrieve information about the first and second order statistics of the flow fields. The wake characteristics have been identified, finding in almost all the cases a good agreement with the self-similar wake model proposed by Schlichting. A modal decomposition of the fluctuating flow field has been performed to identify the dominant oscillations of the wake. This analysis has been performed by means of Proper Orthogonal Decomposition.

In all the cases, decreasing the wall gap ratio, asymmetry is introduced in the average flow, resulting in a bent wake: the very near wake is tilted upwards due to the effect of a wall jet released from the wall gap; the wake is then pulled towards the ground, most likely due to the presence of a low pressure region on the wall. This effect is strengthened for the cases in which a strong and localized blockage occurs, such as in *reattachment regime*.

For moderate wall gap ratios, a still strong average flow is present through the gap. This results in the formation of a wall jet whose maximum velocity point can be considered as a boundary for the wall boundary layer. The wall jet appears to be strongly related to the width of the wake, resulting in a thickening of the boundary layer for thicker wakes, and thus in the merging of wake and boundary layer closer to the cylinders. Downstream of the merging, no more resemblance of the lower part of the former wake with the Schlichting model is found, suggesting that the low velocity region that is formed develops its own characteristics. Fluctuations in the wake of the cylinders are generally strengthened closer to the wall. For small wall gap ratios the average flow through the gap tends to be suppressed, due to the merging of wake and boundary layer close to the cylinder. Fluctuations tend to be further strengthened close to the wall gap, while fluctuations near the wall tend to disappear in the low velocity region formed. This behavior suggest the formation of a pulsating-jet-like flow in the gap and the periodical separation of the flow from the wall downstream with respect to the second cylinder.

The two most energetic modes from the decomposition revealed the presence in all the cases of study of an underlying shedding-like mode. For high and moderate wall gap ratios ($G/D = 3, 1$) these modes model the Von Kármán vortex shedding. Different characteristics in the shedding have been noticed at $G/D = 3$, depending on the L/D regime: in the *bluff-body regime* a simple and well localized shedding is observed;

Figure 12: Contour maps of non-dimensional vorticity $\omega_z = D \left(\frac{\partial \phi_y}{\partial x} - \frac{\partial \phi_x}{\partial y} \right)$ of POD modes.

Figure 13: Scatter plot of time coefficients of POD modes with respect to the time coefficients of the 1st mode.

in the *reattachment regime* vortices are stretched in the stream-wise direction and mix up with the shear layer coming from the cylinders; in the *co-shedding regime* the vortices are stretched along the crosswise direction and seem to be characterized by a double core. A reduction of the wall gap ratio to moderate values ($G/D = 1$) seems not to deeply influence the shedding, but a vorticity production paired with it appears at the wall. The only exception appears to be the case $L/D = 6$, $G/D = 1$ in which the regime seems to change from the *co-shedding regime* to the *reattachment regime* as the wall gap is reduced. This behavior may be explained by a bending of the shear layer released by the upstream cylinder due to the proximity of the wall and its reattachment on the surface of the downstream cylinder. For small wall gap ratio ($G/D = 0.3$) this vortex shedding appears to be at significantly higher frequency and strong interaction with the wall is highlighted by strong asymmetry in the vorticity map. This behavior suggests that this mode does not model anymore a Von Kármán vortex shedding. The authors suggest that this vorticity release may be related with pulsating jet instabilities coupled with shedding from the outer side of the cylinders. Also for $G/D = 0.3$ the case $L/D = 6$ appears to be an exception: in this case it is possible to clearly recognize a flapping jet that pairs with the shedding from the upstream cylinder.

The energy content of the first two POD modes is strongly reduced both for an increase of the longitudinal pitch ratio L/D and for a decrease of the wall gap ratio G/D . At the same time, higher order modes are generally interested by an energy redistribution and thus by a reordering in the POD spectrum. This suggests a shift of the flow towards less coherent behaviors and a reduction of the relevance of vortex shedding with respect to pure Von Kármán shedding cases. It must also be remarked herein that the introduction of the downstream cylinder in the *bluff-body regime* results in a strengthening of the shedding modes with respect to the single cylinder case, suppressing shear layers oscillations.

The analysis of higher order POD modes and of their time coefficients revealed the presence, in most of the cases, of modes harmonically related to the first 2 POD ones. These modes can be simply accounted as higher order harmonics of the vortex shedding and model vortex oscillations in both streamwise and crosswise directions. Due to the energy redistribution, for increasing L/D or for decreasing G/D these modes appear to be less energetically relevant and are often replaced by non harmonically related modes.

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Appendix

Figure A.1: Velocity deficit profiles