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Estimation of time resolved turbulent fields through correlation of non-time resolved field measurements and time-resolved point measurements

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Abstract

A method for the estimation of time-resolved turbulent fields from the combination of non-time-resolved field measurements and time-resolved point measurements is proposed. The approach poses its fundamentals on a stochastic estimation based on the Proper Orthogonal Decomposition (POD) of the field measurements and of the time-resolved point measurements. The correlation between the temporal modes of the field measurements and the temporal modes of the point measurements at synchronized instants is evaluated; this correlation is extended to the “out-of-sample” time instants for the field measurements, i.e. those in which field data are not available. In the “out-of-sample” instants, POD modes time coefficients are estimated and the flow fields are reconstructed. The proposed method extends the work by Hosseini *et al.* (Experiments in fluids, 56, 2015) by proposing a truncation criterion which allows removing the uncorrelated part of the signal from the reconstruction of the flow fields. The truncation is fundamental in case of turbulent flow fields, in which a great wealth of scales is involved, thus reducing the correlation between the probe signal and the field measurements. The threshold selection poses its basis on the random distribution of the uncorrelated signal. Additionally, the selection of the probe timespan to perform the POD analysis on the probe signal is discussed. The method is validated with a synthetic test case and an experimental one. A Direct Numerical Simulation database of a channel flow is selected since its spectral richness is expected to represent a significant challenge for this method. This dataset allows isolating the effects of correlation between field measurements and point measurements, removing issues connected to noise contamination or to the finite spatial resolution which would inevitably affect experimental data. The quality of the dynamic estimation is found to be affected by the noise contamination of the data and by the poor convergence of the POD modes, which add on the effect of the probe location, i.e. on the correlation between probe events and flow features. The squared correlation coefficient between reconstructed data and in-sample data is proposed as an assessment of the flow fields estimation quality. The use of the squared correlation coefficient directly on in-sample data is allowed by the truncation itself.

Keywords: Proper Orthogonal Decomposition, Linear Stochastic Estimation, Dynamic Estimation, PIV

1. Introduction

In the last decades experimental fluid mechanics has experienced the flourishing of field measurement techniques, disclosing the access to the instantaneous distribution of field quantities such as velocities, concentrations, temperatures or pressures as in Particle Image Velocimetry [1], thermographic phosphors [2], Infrared thermography [3] or pressure sensitive paints [4]. Unfortunately due to limitations related to data rate or sensor technology, it often occurs that such measurement techniques can not be applied to provide information on the flow dynamics, such as for instance in moderate to high Reynolds number turbulent flows. For example, for the case of Particle Image Velocimetry, although the most recent technological developments have led in the

last decades to powerful high speed light sources and fast high-resolution scientific cameras, which have enabled the flourishing of novel approaches exploiting time resolution (see, e.g. the reviews in [5] and [6]), time-resolved PIV remains of real practical use prevalently for low Reynolds numbers flows.

When the real-flow dynamics are not accessible due to hardware limitations, dynamic estimation of coherent structures is an extremely appealing option. Dynamic estimation often relies on low order models, and has proven to be a prominent candidate to elaborate flow control strategies (see, e.g. [7, 8]). Low order models often truncate small scales and aim to model prevalently the dynamics of large scale structures, thus losing detail of description in high Reynolds number flows. Nonetheless, large scale structures are widely recognized to be the main actors in momentum transport in turbulent flows. For example in wall-bounded flows large scale structures carry the bulk of the kinetic energy and

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contribute largely to the production of Reynolds stresses (see for instance the very large scale structures in pipe flows [9], or the interaction between large scale structures in the outer layer and near wall features in adverse-pressure-gradient boundary layers [10, 11]). Among the possible solutions, one relevant candidate is based on modal analysis. Coherent structures are often detected through Proper Orthogonal Decomposition (POD, [12]) and then, through further analysis, their dynamics are identified. In case of shedding-dominated phenomena, POD has been widely used to extract phase coherent information for the first and higher order harmonics, both in 2D (see, e.g. [13, 14, 15]) and 3D [16, 17]. This is however only possible when a dominant phase relation between POD modes can be identified from the scatter plots of the time coefficients and it does not allow extracting any absolute frequency information. Through some physical assumptions on convection velocities it is however possible to identify also modes temporal information as done by Jaunet *et al.* [18].

The modes frequency information can be retrieved with the help of additional time-resolved data by making use of Linear Stochastic Estimation (LSE). Originally developed to provide an approximation of a conditional average (see [19]), LSE allows the estimation of the most probable value of a certain quantity (e.g. a POD time coefficient), given another measured quantity. By using a time-resolved probe, LSE allows thus estimating the most probable instantaneous flow field given a certain value measured by the probe. This appears extremely appealing, especially for flow control applications. The first attempts of dynamic estimations of flow fields were based on the information from only one time instant and are referred to as single-time-delay techniques (see, e.g. [20]). The word *delay* refers to the fact that a time shift, typically corresponding to a given convective time, was imposed between the value to be estimated and the value used to perform the estimation.

The temporal coherence of time-resolved probes was later exploited more intensively by means of multi-time-delay LSE. Of course, multi-time delays are beneficial since they allow reducing the effect of noise contamination on the probe measurements and are able to deal with several convective time scales. Depending on the application of interest, multi-time-delay LSE has been preferred in the spectral (e.g. for aeroacoustics applications [21]) or temporal domain (for flow field reconstruction from PIV data or flow control applications [22]). An assessment of stochastic estimation methods performances and of the effect of the sensor location is reported in [23].

Most recent developments for LSE-based dynamic estimations of flow fields include the method proposed by Tu *et al.* [24] who defined a linear model of the flow field and incorporated a Kalman smoother to improve the estimation and minimize the effects of noise contamination. Baars and Tinney [25] proposed a significant improvement of spectral LSE providing a POD-based method ex-

ploiting higher order coherences and thus able to identify mutual interaction between coherent structures. This aspect is especially interesting when dealing with turbulent flows in which more energetic coherent structures affect the convective velocity of less energetic ones; a higher order representation thus appears necessary to account for the multiple interaction of coherent structures.

An interesting alternative approach is based on the Extended POD (EPOD). Borée [26] has shown that, when dealing with multiple synchronized measurements, the correlation between them can be ascertained through the evaluation of the *extended POD modes* which are estimated through the projection of the snapshot matrix of a given quantity on the temporal basis corresponding to another one. This technique reduces to the LSE if all the POD modes of the probe data are retained in the reconstruction. A recent improvement of this technique for shedding wake flows has been proposed by Hosseini *et al.* [27], which performed the EPOD projection using the correlation between temporal coefficients of the POD of both velocity fields and probe data. The advantage is that the spatio-temporal correlation of the EPOD approach is replaced by solely temporal correlations. Additionally, history effects are taken into account by adding “virtual” probes using time-shifted probe data (with the support of the Taylor’s hypothesis of uniform convection). A similar probe data organization was previously proposed in [28] to extract the conditional flow structure downstream of a reattaching flow region. Hosseini *et al.* [27] argued that only the modes for which the correlation with the velocity is non-vanishing are considered, i.e. the method is self-tuning. It has to be underlined that Hosseini *et al.* [27] applied their method to three kind of modes: symmetric, antisymmetric and non-periodic low-frequency modes in the wake of a wall-mounted pyramid. It will be shown later that, for turbulent flows with a richer variety of scales, the self-adjustment to only non-vanishing modes is not straightforward and uncorrelated modes might introduce spurious effects. In this case the method converges to a Linear Stochastic Estimation, losing the advantage of the Extended POD which allows minimizing the contamination of the reconstructed signal due to uncorrelated signal which results in random noise.

Additionally, it has to be noted that in all the aforementioned cases both LSE and EPOD have been extensively applied to shedding-dominated phenomena. The scenario of a turbulent flow with full rank of frequencies appears intuitively more complex. For instance Kerhervé *et al.* [29] report the dynamic description of very-large-scale motions in high Reynolds number turbulent boundary layers, based on synchronized low-repetition-rate two-dimensional stereo-PIV measurements and hot-wires. The experiments by Kerhervé *et al.* made use of a two-dimensional rake of 143 single probes sampled at 30 kHz to obtain a satisfactory reconstruction of the flow dynamics using LSE. The large number of probes needed might possibly be due to the inability of LSE to deal with

optimal truncation of the uncorrelated part of the signal.

The objective of this work is to demonstrate and assess a modified EPOD procedure to deal with the dynamic estimation of field quantities in turbulent flows. The modification of the procedure consists in an automated simple truncation criterion which allows removing the noisy and uncorrelated part of the signal. Additionally, while for shedding-dominated fields the selection of the optimum timespan for multi-time delay approaches is straightforward (for instance Hosseini *et al.* [27] used the shedding period, and tune the number of virtual sensors to describe a clearly periodic phenomenon), this choice is not trivial for turbulent flows and will be discussed in the following.

The paper first presents in section 2 a mathematical description of the proposed methodology. Later the method is validated through the application on a synthetic DNS database of a turbulent channel flow which is chosen to ensure sufficient spectral richness. The synthetic test case is not affected by noise contamination or by modulation effects due to finite spatial resolution. It is thus the ideal scenario to investigate the effect of the proposed truncation method. An experimental assessment is performed on the wake of a stalled airfoil; this test case, although being shedding-dominated, exhibits a significant spectral richness and noise contamination. The presence of the shedding, however, provides a scenario in which the correlation between probe signal (with probes immersed in the shedding wake) and the flow field is greater than in the previous test case. It is thus possible to assess the effects of noise contamination and poor convergence of the POD.

2. Extended POD for turbulent flows

2.1. Extended POD approach

POD is widely used to identify modes which contain the dominant structures in turbulent flows [12]. In fact POD modes are optimal in the least square sense, i.e. the first n_m POD modes form an orthogonal basis which minimizes the Frobenius norm of the flow field reconstruction error. This implies that POD provides flow field modes ordered by their *energy* content. Suppose that a set of n_t field snapshots, each containing n_p points, is acquired. Each snapshot of the fluctuating velocity field can be rearranged into a one-dimensional vector (of length n_p) and then all the snapshots can be arranged in a data matrix ($\underline{U}$ of size $n_t \times n_p$) of which the rows are formed by the data pertaining to each corresponding snapshot. When dealing with a dataset with $n_t < n_p$, a quick and efficient computation of the POD modes is done with the snapshot approach [30]. It has to be noted here that when a snapshot contains only one quantity (e.g. a temperature) n_p corresponds to the number of measurements points, while when dealing with multiple quantities (e.g. 2 or 3 components velocity vectors at each point) n_p is equal to the product of the snapshot points times the

number of available quantities. For regularly spaced data the snapshot data matrix can be decomposed with an economy-size Singular Value Decomposition (otherwise grid weights have to be taken into account, see, e.g. [7]):

$$\underline{U} = \underline{\Psi} \underline{\Sigma} \underline{\Phi} \quad (1)$$

In the decomposition of equation (1) the matrix $\underline{\Psi}$ has size $n_t \times n_t$ (if, as common for field measurement data, $n_t < n_p$) and its columns contain the temporal modes $\underline{\psi}_i$ of $\underline{U}$; $\underline{\Phi}$ has size $n_t \times n_p$ and its rows contain the spatial modes $\underline{\phi}_i$ of $\underline{U}$; $\underline{\Sigma}$ is a square diagonal matrix which contains the singular values σ_i of $\underline{U}$, i.e. the norms of the contributions of each mode to the snapshot matrix.

Suppose now that the signal of s synchronized fast probes is sampled with a rate q times faster than the sampling rate of the field measurements. Therefore, data from the synchronized fast probes is available such that $n_{tt} = s \times q$ independent probe samples are recorded for each field snapshot and $n_{pr} = s \times q \times n_t$ probe samples are recorded in total. For example, a low repetition rate PIV system capturing flow fields at 10Hz for 10s would provide $n_t = 100$ PIV samples, and three hot-wires capturing at 1kHz for the same time would lead to $n_{pr} = s \times q \times n_t = 30000$ total samples and $n_{pr}/n_t = 300$ independent samples per snapshot.

The field snapshots can be rearranged in vectors and then in a data matrix as described previously. Also the probe data can be rearranged in a matrix $\underline{U}_{pr}$, with the rows being formed by n_{tt} “virtual” probe data pertaining to each corresponding snapshot (thus having n_t rows as the snapshot data matrix, and n_{tt} columns). Without leading the generality of the problem, we will assume that the probe snapshot matrix is built considering $n_{tt} = n_{pr}/n_t$, i.e. assigning to each field snapshot a number of virtual probes equal to the captured probe data in the time interval between the field snapshots. In general for turbulent flows the choice of this time interval depends on the average convection velocity, as it will be shown in Section 3.

The proposed approach for virtual probes construction is analogous to that proposed by Sicot *et al.* [28] and Hosseini *et al.* [27] which generated virtual probes under the Taylor’s hypothesis assumption. This probe snapshot matrix can be decomposed with a POD snapshot approach [30] as:

$$\underline{U}_{pr} = \underline{\Psi}_{pr} \underline{\Sigma}_{pr} \underline{\Phi}_{pr} \quad (2)$$

The modes $\underline{\phi}_{pr,i}$ forming the rows of $\underline{\Phi}_{pr}$ constitute a pseudo-spatial base of the probe *snapshot* vectors. $\underline{\Psi}_{pr}$ (with columns $\underline{\psi}_{pr,i}$) represents the corresponding temporal base and $\underline{\Sigma}_{pr}$ is again a diagonal matrix containing the singular values $\sigma_{pr,i}$. The probe snapshot vectors can thus be reconstructed using n_m modes as:

$$\underline{u}_{pr}(t) \approx \sum_{i=1}^{n_m} \psi_{pr,i}(t) \sigma_{pr,i} \underline{\phi}_{pr,i} \quad (3)$$

Although the rows of $\underline{\Phi}_{pr}$ would be a basis for the n_{tt} -vector space only for $n_t = n_{tt}$, if the experiment is ergodic and the decomposition of the $\underline{U}_{pr}$ matrix contains a dataset sufficiently large and rich (i.e. it contains enough snapshots which are taken over a sufficiently long sampling time) to reach statistical convergence, the modes $\underline{\Sigma}_{pr} \underline{\Phi}_{pr}$ are a basis sufficiently large and well-converged to reconstruct any $\underline{u}_{pr}$ vector with a minimal residual error. A generic probe acquisition $\underline{u}_{pr}^+$ at a generic time instant t^+ can be used to estimate the vector of time coefficients $\underline{\psi}_{pr}^+$ which allows approximating the probe acquisition as a linear combination of the probe snapshot modes $\sigma_{pr,i} \underline{\phi}_{pr,i}$

$$\underline{\psi}_{pr}^+(t^+) = \underline{u}_{pr}^+(t^+) \underline{\Phi}_{pr}^T \underline{\Sigma}_{pr}^{-1} \quad (4)$$

where the superscript T refers to a transpose. It has to be noted that, if the experiment is ergodic, whatever probe sequence $\underline{u}_{pr}^+(t^+)$ can also be acquired outside (either before or after) of the acquisition from which the modes were estimated. For a square snapshot matrix ($n_t = n_{tt}$) the rows of $\underline{\Phi}_{pr}^T$ form a basis in the n_{tt} vector space and the reconstruction in equation (3) is exact for any $\underline{u}_{pr}^+$.

For the problem under investigation, the *extended POD modes* [26] corresponding to the field measurements can be estimated as:

$$\underline{\psi}_{pr}^T \underline{U} = \underline{\Sigma}_e \underline{\Phi}_e = \underline{\psi}_{pr}^T \underline{\Psi} \underline{\Sigma} \underline{\Phi} = \underline{\Xi} \underline{\Sigma} \underline{\Phi} \quad (5)$$

where the subscript e refers to extended POD modes and the matrix $\underline{\Xi} = \underline{\psi}_{pr}^T \underline{\Psi}$ is a matrix containing information about the temporal correlation between fields and probe modes.

Given equation (5), the probe time coefficient $\underline{\psi}_{pr}^+(t^+)$ can be used to reconstruct the part of the field snapshots which is correlated with the probe signal through the extended POD modes. For computational efficiency, it is preferable to operate in the space of POD modes [27]. If the dataset is sufficiently large to reach statistical convergence, it is possible to assume that the matrix $\underline{\psi}_{pr}^T \underline{\Psi}$ represents a good estimate of the dyadic product $\underline{\psi}_{pr}^+ \underline{\psi}_{pr}^{+T}$ which embeds the relation between the time coefficient of the probe snapshot at t^+ and the time coefficient $\underline{\psi}_{pr}^+$ of the out-of-sample estimate of the field measurement. Through a linear transformation between the probe time coefficients and the flow fields time coefficients, the dynamic estimation of the latter starting from $\underline{\psi}_{pr}^+(t^+)$ is obtained as:

$$\underline{\psi}_{pr}^+(t^+) \approx \underline{\psi}_{pr}^+(t^+) \underline{\Psi}^T \underline{\Psi} \quad (6)$$

The time coefficients can then be used to reconstruct the flow field snapshot at the time instant t^+ .

This procedure can be carried out for any time instant t^+ at which probe data are available, providing an estimation of the field measurements at the same sampling frequency of the probes.

$$\underline{u}_{DYN}^+(t^+) \approx \sum_{i=1}^{n_m} \psi_{DYN,i}^+(t^+) \sigma_i \underline{\phi}_i \quad (7)$$

Following the previous example of n_t field snapshots synchronized with s probes sampling q times faster than the field measurements, this would lead to obtain up to qn_t fields, increasing the sampling rate by a factor q .

2.2. Truncation of the time coefficients correlation matrix

The i^{th} row of $\underline{\Sigma}_e \underline{\Phi}_e$ (equation 5) which corresponds to the i^{th} extended POD mode contains solely and entirely the part of $\underline{U}$ which is correlated to the i^{th} probe mode [26]. Consequently, extended POD has been used in the past to correlate flow field modes with other measured quantities, e.g. OH concentrations in flames [31] and wall pressures [28]. As shown on the right hand side of equation (5), an extended POD mode is the combination of all the POD modes of the snapshot field. Consequently, the estimation of the time coefficients of a certain mode (equation 7) depends on all the probe time coefficients, accounting also for modes interaction through the matrix $\underline{\Xi} = \underline{\psi}_{pr}^T \underline{\Psi}$. This estimation method removes the need of multiple-time delays and accounts for the variation of the modes time delays due to modal interaction.

Nevertheless, this modal interaction might result in the noise contamination of even the highly energetic POD modes. In fact, although in PIV measurements noise has typically a spectrally-white content and has a greater impact on low-energy modes (see, e.g., [32], which report the noise distribution through the POD modes in turbulent flows) the contamination from higher order modes can produce spurious effect also in the first extended POD modes.

The larger (in term of number of samples) and richer (in term of statistical convergence) is the dataset with which the fields and probe times coefficients and modes are estimated, the greater will be the accuracy of the estimation of the modes mutual interaction. Nevertheless, the columns of both $\underline{\psi}_{pr}$ and $\underline{\Psi}$ are orthonormal vectors and form 2 bases in the n_t -dimension vector space; consequently $\underline{\Xi}$ is also composed by columns forming a basis. This implies that higher order modes which might be mutually uncorrelated are taken into account for the dynamic estimation in equation (7) which coincides with a LSE for $n_m = n_t$.

The error in the estimation of a generic snapshot $\underline{u}_{DYN}^+(t^+)$ is dependent on the correlation between probe and field modes, on the convergence in the estimation

of the modes and on the noise content of the measurements. In particular, the noise content has to be taken into account carefully. According to Hosseini *et al.* [27] the vanishing elements of Ξ will not contribute in building the dynamic estimated field, thus this approach is, in this sense, self-adjusting. However, while for a spatially persistent shedding the correlation between the velocity and probe data is dominating, in case of a richer range of scales this correlation might become weaker, giving rise to significant noise contamination. Reminding that all rows/columns of Ξ have unitary norm and supposing that at a certain i^{th} probe mode (j^{th} field mode) is uncorrelated with the field modes (the snapshot modes) the i^{th} row (j^{th} column) of Ξ will be composed of randomly distributed elements, with unitary norm and zero mean, thus standard deviation equal to $\sigma = 1/\sqrt{n_t}$.

This consideration can be easily exploited to reduce the spurious contamination due to uncorrelated residual components, thus enabling a powerful truncation. In fact, for a normal distribution (such as that of the rows and columns corresponding to uncorrelated POD modes) 99.7% of the data is within three standard deviations of the mean [33]. Consequently it is possible to remove practically all the uncorrelated and noisy contribution from u_{DYN}^+ by truncating the matrix Ξ with the 3σ rule, i.e:

$$-\frac{3}{\sqrt{n_t}} \leq \Xi_{ij} \leq +\frac{3}{\sqrt{n_t}} \Rightarrow \Xi_{ij} = 0, \quad i, j = 1, \dots, n_t \quad (8)$$

It will be shown that the proposed truncation allows obtaining higher quality data with a smaller number of probes, as well as to provide a useful strategy to obtain a reconstruction accuracy estimate for experimental data.

3. Method validation

The method is applied to synthetic and experimental data using a Direct Numerical Simulation (DNS) dataset and an experimental PIV test case. The DNS dataset includes fields from numerical simulations in a channel at friction Reynolds number $Re_\tau \sim 1000$ [34, 35]. This dataset has been chosen since the spectrum of a fully turbulent wall-bounded flow without any dominant shedding frequency poses an extremely challenging environment for the present technique. Moreover, the absence of noise contamination and the high spatial resolution of the DNS database allows assessing the sole effect of the correlation between field measurements and point measurements. The assessment on experimental data is carried out using data of a time-resolved PIV experiment. Time-resolved probes are extracted considering localized grid points as pseudo-probes, while a non-time resolved PIV sequence is obtained via down-sampling (as done by [24]). The PIV snapshots are collected in the wake of a NACA0012 2D wing at angle of attack equal to 15°

Figure 1: Schematic representation of the sub-domains extracted from the Channel Flow DNS spatial domain for the synthetic PIV experiment. Different positions in the $x-y$ plane are accounted with different letters (A, B, C and D). The velocity probes (not included in the figure for clarity) are located at the downstream end of each slice, uniformly separated between the wall and the channel centerline.

Figure 2: Energy spectrum and cumulative energy contribution of the flow field modes of the channel simulation. Arrows indicate the reading scale of the curves.

and chord-based Reynolds number equal to 1800. This dataset is characterized by a clear vortex shedding (even though still involving a range of scales after the break-up of the wake vortices), which is well resolved with the PIV, thus allowing for a strong correlation between the field measurements and the probe measurements. The limited number of snapshots supposes a further challenge due to the poor statistical convergence of the dataset and the noise contamination of the modes.

3.1. DNS test case

The Channel Flow DNS database from the John Hopkins Turbulence Databases (<http://turbulence.pha.jhu.edu/>) includes a complete channel flow velocity field with bulk velocity U_b equal to 1 over a domain of 2 half-channel-heights h from wall to wall, 3π channel half heights in the span-wise direction and 8π channel half heights in the stream-wise direction. The DNS sequence duration is of one channel flow-through time $8\pi h/U_b$ with a DNS time step of $\delta t = 0.0014h/U_b$. In order to

Figure 3: North-West corner of the time coefficients correlation matrix Ξ for a probe acquisition time of 1 convective time h/U_b : (a) 3 probes and (b) 11 probes. The symbol i indicates probe POD mode number, while j indicates field measurement POD mode number.

simulate a virtual stereo-PIV experiment, the three components of the velocity field were extracted over a square $h \times h$ domain (going from the wall to the centerline) in the $x-y$ plane. Snapshots were extracted one each convective time h/U_b in order to reduce the temporal correlation between them. Due to the limited temporal duration of the simulation, a maximum of 24 uncorrelated snapshots in the same spatial position can be extracted, which is not enough to reach the statistical convergence of the dataset. In order to obtain a well-converged dataset, the snapshots were extracted at different spatial positions, exploiting problem symmetry and statistical homogeneity. In particular, slices were extracted in 80 span-wise locations at the beginning and at half channel stream-wise length, both at the bottom and upper side of the channel over 24 instants. The spacing in both space and time between snapshots has been chosen to minimize their correlation. A total of $n_t = 7680$ snapshots with 88×88 points (along the channel height and in the stream-wise direction) were used, i.e. $n_p = 7744$. The size of the slices, from the wall up to half channel height, allows identifying adequately the main flow structures present in the flow field under study. A sketch of the slices extracted from the DNS domain is given in Figure 1.

The probe data were obtained probing the three velocity components at 11 points equally spaced from the wall up to the channel centerline between $0.05 h$ and $0.95 h$. For each snapshot the corresponding probes were placed at the same span-wise locations as the corresponding slice and at a stream-wise location immediately downstream of the slice domain. For each flow field snapshot, data of each probe included 880 time samples during the 1.25

convective times h/U_b starting after the time instant corresponding to the flow field snapshot. Consequently the probe frequency was arbitrarily set in order to obtain $q = 704$ time instants for each turnover time, i.e. with time spacing approximately equal to the DNS time step. The resulting probe snapshot matrix U_{pr} contains $7680 \times (s \times L)$ elements, being L the number of samples allocated for each probe for each snapshot (i.e. the number of virtual probes corresponding to each real probe).

The fluctuating velocity fields were used to compute the snapshot modes $\underline{\Sigma} \underline{\Phi}$. Figure 2 reports both the modes energy distribution ($\lambda_i = \sigma_i^2$) and the cumulative energy content for the flow field snapshots. High energy contribution is due to the first modes corresponding to the large scale motions [36] while following modes have a lower energy contribution. Owing to the wide range of spatial scales, a high number of modes (about 400) is needed to reach 90% of the cumulative energy spectrum.

The probe signal was used to estimate the probe modes $\underline{\Sigma}_{pr} \underline{\Phi}_{pr}$ and the time-coefficients correlation matrix Ξ . The effect of the number of probes and of the temporal length of the sequence was analyzed using all 11 probes and a restricted set of $s = 3$ probes (using the probes located closer to the wall, closer to the channel centerline and in the center of the domain) over 1 convective time h/U_b . The effect of the probe snapshot length is further addressed using 11 probes over 0.75, 1 and 1.25 convective times h/U_b (i.e. having $L = 528, 774, 880$).

Figure 3 reports the contour plot of the north-west corner of the matrix Ξ estimated with the probe signal from 3 (Figure 3a) and 11 probes (Figure 3b). The flow field es-

Figure 4: Temporal evolution of the time coefficients of the first mode estimated with 3 probes, using a sampling length of one convective time. No truncation is performed on the time-coefficients correlation matrix.

timisation task would be straightforward and noise-free if the matrix Ξ was diagonal, i.e. if the problem was characterized by a biunivocal correspondence between probes and field modes. The matrix, however, exhibits a diagonal dominance in the north-west corner, corresponding to high energy probes and field modes; moving through the diagonal towards the matrix center (for increasing number of modes) this dominance disappears. Using only 3 probes the diagonal dominance of the Ξ matrix is lost beyond the 100th probe mode while a wider range of diagonal dominance is attained for the case with 11 probes. This has to be ascribed to the better spatial sampling of the 11 probes case which captures modes with smaller wavelength.

A time sequence with a length of 25 convective times h/U_b was acquired at a fixed location for both the probes and the slices. The chosen region is different from any of those used to estimate the modes. It is located at a stream-wise coordinate $x/h = 0$ and at a span-wise location equal to $z/h = 0.1$. The region is sketched in Figure 1 and labelled with E. Therefore all fields have to be considered as “out-of-sample”. Time resolved probes were used to perform the flow field estimation at each instant of the time sequence and the available exact velocity fields were used as ground truth for comparison. The effects of the poor diagonal behavior shown by the matrix Ξ for the 3 probes case are evident from Figure 4 which reports the time coefficients of the first mode $\psi_{1,DYN}(t)$ estimated from three probes without any truncation on the matrix Ξ . The dynamically-estimated time coefficients are compared with the true time coefficients, computed by projection as $\psi_{1,true}(t) = \underline{u}(t)\phi_1^T\sigma_1^{-1}$ with $\underline{u}(t)$ being the true velocity field at the time t (and represented as a vector with size $1 \times n_p$). The estimation of the time coefficients appears to be noise-dominated. The flow field estimation, in fact, here converges to a LSE which is not able to capture the flow evolution since spurious effects due to the contamination of uncorrelated higher-order modes

Figure 5: Squared correlation coefficient of the reconstructed time coefficient series with the ground truth solution for the case of sampling length equal to h/U_b and for different number of probes.

affects even the first mode, which contains the highest energy content (subsequent modes are omitted for brevity).

This phenomenon can be synthesized through the representation of the squared correlation coefficient R^2 of the modes time coefficients, defined as:

$$R^2(i) = \frac{\langle (\psi_{i,DYN}(t)\psi_{i,true}(t)) \rangle^2}{\langle (\psi_{i,DYN}(t))^2 \rangle \langle (\psi_{i,true}(t))^2 \rangle} \quad (9)$$

where angular brackets indicate ensemble averaging. The squared correlation coefficient is reported in Figure 5 against the mode number for 3 and 11 probes both with and without truncation on the uncorrelated elements of Ξ . Here, again, the true time coefficients are estimated as done for the first mode. Increasing the number of probes significantly improves the squared correlation coefficient and thus the quality of the flow field estimation, thus justifying the choice by Kehrervé *et al.* [29] of using more than 100 probes. However, it is remarkable that, by means of the truncation on Ξ , acceptable results are recovered also for the set with only 3 probes.

When using 11 probes the already satisfactory estimation is significantly improved by the truncation, allowing for a squared correlation coefficient greater than 0.5 for the 100th mode. A visual representation of the estimation quality is assessed in Figure 6 which reports the time coefficients estimated with 3 and 11 probes against the true solution. Data are reported for 3 probes only for the truncated version of Ξ while for 11 probes both the time coefficient for truncated and non-truncated Ξ are reported. Both with 3 and 11 probes the signal reconstruction is quite satisfactory; however, the non-truncated estimation with 11 probes exhibits spurious high frequency content, probably ascribed to contamination from uncorrelated modes. The truncated case with 3 probes, instead, presents a smoother time coefficient history with only minor errors.

Figure 7 provides a visual representation of the flow field reconstruction quality attained with 11 probes and

Figure 6: Temporal evolution of the time coefficients of the first 6 modes obtained with sampling time equal to h/U_b for different number of probes.

truncating Ξ . The contour of the reconstruction of the fluctuating stream-wise velocity component with superimposed vector arrows is reported for one single snapshot (Figure 7a). The true flow field is reported for comparison (Figure 7c), along with the exact reconstruction of the flow field truncated at 200 modes (Figure 7b). The LOR truncation at 200 modes can be assumed to be comparable to the filtering on Ξ , since the squared correlation coefficient falls below 0.3 for modes of order higher than 200 (as shown in Figure 5).

The error maps reported in Figure 7d and Figure 7e show that the method correctly estimates the large scale features located sufficiently far from the wall (for $y/h > 0.4$ the error is about $\pm 2\%$ of the bulk velocity). Small scales located nearby the wall are instead estimated with

a larger error (for $y/h < 0.4$ the errors reach peaks of the order of $\pm 10\%$). The higher error for small wall-near scales is only marginally affecting the quality of the proposed reconstruction, which is mainly focused to the estimation of large-scale structures in the flow. The poorer estimation nearby the wall might be ascribed to three reasons:

1. the lower convective velocity of near-wall flow structures which might not reach the probe location during the probes sampling time;
2. the highly three-dimensional flow field nearby the wall which might cause these flow features to never be convected towards the probe location since they exit out of the flow field slice;

Figure 7: Contour plot of fluctuating stream-wise velocity component with superimposed (down-sampled) vector arrows. Top: comparison of the dynamic reconstruction of the flow field against the exact flow field and the exact low order reconstruction with the same mode truncation as the dynamically estimated flow field. Bottom: reconstruction error distributions in percentage with respect to the LOR (d) and to the original field (e).

3. smaller structures are characterized by smaller turnover times and their convective velocities are affected by the larger scales, thus reducing the correlation between field measurements and point measurements. In this sense recent works have shown that the Taylor's hypothesis is not applicable for near-wall structures [37].

While the issue of three-dimensionality can be tackled as done by Kehrervè *et al.* [29] by placing the probes in multiple span-wise planes, the fact that different eddies have different convective velocities poses a question on the most suitable choice for the probe sampling time. This issue was solved by Hosseini *et al.* [27] for cases in which a clear shedding is identified by choosing for the virtual probes a timespan equal to the vortex shedding period. For turbulent flows in which no dominating frequencies are present a more suitable choice appears to be connected with the convection time through the field measurement domain. In particular Figure 8 reports the squared correlation coefficient of estimated modes time coefficients against the mode number for varying probe sampling time equal to 0.75, 1 and 1.25 h/U_b . The re-

Figure 8: Squared correlation coefficient of the reconstructed time coefficient series with the ground truth solution for the case with 11 probes and different probe sampling time equal to 0.75, 1 and 1.25 h/U_b .

sults show that it is fundamental to select a probe timespan equal to at least the time needed for all the information in the field to pass through the probe location. It has to be underlined that a timespan slightly larger than one simple convective time might be partly beneficial since it might improve the correlation between field data and probes close to the wall, where the local convective velocity is lower. Adaptive approaches might be suggested using different time series lengths according to the local value of the time-average velocity.

Finally, it should be highlighted that in the present validation the probe signal is constituted of velocity point measurements, which simulate hot-wire probes inside the flow. The use of velocity probes is not the most common configuration for both flow sensing/control, preferring a more suitable (from a purely experimental point of view) configuration with wall-mounted pressure probes. The estimation of the flow field dynamics through wall-mounted pressure probes is possible for flows with a significant imprint of large-scale structures on the wall. Few examples are the test cases proposed by Sicot et al. [28], investigating the separating/reattaching flow on a thick plate with sharp leading edge, and by Hosseini et al. [27] with the shedding wake of a wall-mounted pyramid obstacle. For large scale motion estimation in wall-bounded flows (such as the channel flow here reported) a similar setup is not optimal since the correlation of the wall pressure fluctuations with the large scale structures in the outer boundary layer is expected to be rather weak. As a matter of fact, in wall-bounded flows, near-wall features are mostly characterized by an amplitude modulation of high frequency events rather than by a linear relation between large wavelength events (see e.g. the seminal works by Marusic et al. [38] and by Mathis et al. [39]). This suggests that, perhaps, in this case the extended POD should be performed between the spatial modes of the velocity fields and modes of the near wall fluctuations expressed in the Fourier space (analogously to what done for instance by Baars and Tinney [25]).

3.2. Experimental test case

The experimental database is generated using time-resolved PIV data collected in the wake of a NACA0012 2D wing in stall conditions. The experiment is carried out in the recirculating water tunnel of the Universidad Carlos III de Madrid. The tunnel has a rectangular test section (0.5 x 0.55 m) with a length of 2.5m, and it is designed to have a turbulence intensity lower than 1%. The wing chord is $c=30\text{mm}$ and spans the test section from side-to-side to minimize 3D flow effects. The free-stream velocity is tuned to 0.06m/s , which results in a Reynolds number, based on the wing chord, of 1800. The wing is set to an angle of attack of 15° to promote the formation of a complex separated flow in the wake including a dominant vortex shedding with additional smaller-scale turbulence. The observation region extends from

Figure 9: Sketch of the PIV experimental setup.

approximately 1.5 to 5.5 chords in the stream-wise direction (measured from the wing leading edge) and from -1.2 to 2 chords in the crosswise direction.

A schematic of the PIV experimental setup is reported in Figure 9. The particles used for seeding the flow are neutrally-bouyant polyamide particles with $56\text{ }\mu\text{m}$ diameter. A dual cavity pulsed Nd:Yag Quantel Evergreen laser ($200\text{ mJ}=\text{pulse}$ at 15 Hz), reshaped in a sheet with 1 mm thickness, illuminates the observation region. The recording device is a 5.5 Mpix Andor sCMOS camera, with $6.5\text{ }\mu\text{m}$ pixel pitch. The camera is equipped with a 100mm Tokina objective set at $f/\# = 11$. The magnification is approximately 0.06 , thus leading to a resolution of about 9.1 pix/mm . A set of 27000 PIV snapshots is collected at a frequency of 30 Hz by setting the time separation of the two laser heads at $1/30\text{ s}$ (which represent the limit sampling frequency for the PIV system employed). With the adopted imaging, flow speed and sampling rate, the particle displacement in the images is of almost 20 pixels and allows for time-resolved estimation of the velocity in the airfoil wake. The maximum displacement of 20 pixels results in strong particles displacement gradients, thus reducing the correlation quality and resulting in velocity fields with significant noise contamination [40]. The relatively large displacement is due to repetition rate limitations of the used laser source. Nonetheless the consequent signal degradation is exploited to isolate the effects of the noise contamination and poor convergence in a test case in which there is a less reach wealth of scales with respect to the synthetic validation (being the wake dominated by vortex shedding), and thus the correlation between probe signal and velocity fields is much stronger (provided that the probes are located within the shedding wake).

The quality of the PIV images is improved in the pre-processing phase by removing laser reflections and illumination background by means of the POD-based PIV background removal technique reported in [41]. The PIV

Figure 10: Energy spectrum and cumulative energy contribution of the flow field modes of the experimental test case. Arrows indicate the reading scale of the curves.

software developed at University of Naples Federico II is used to calculate the velocity fields through digital cross-correlation analysis of the particle images [42]. The interrogation procedure consists of an iterative multi-grid/multi-pass [43] image deformation algorithm [44], with final interrogation windows size of 64×64 pixels with 75% overlap (which results in 17 vectors/chord). B-spline interpolation schemes are used to improve the accuracy of the PIV processing [45]. The vector validation to identify invalid vectors is carried out with the universal median test [46] on a 3×3 vectors kernel and an error threshold equal to 2. Outlier vectors are replaced with a distance-weighted average of the neighbouring valid vectors.

From the full set of 27000 PIV snapshots, 334 velocity field snapshots are extracted, equally spaced in time, down-sampled by a factor of 80 with respect to the original time-resolved data. The set of 334 velocity field snapshots is assumed to be a virtual field measurement employed for the dynamic estimation of the turbulent field. Pseudo-probes are located in the wake of the wing and generated by extracting velocity vectors with the original time resolution (i.e. $q = 80$ and $n_{tt} = s \times 80$). The original 27000 PIV snapshots are used as a ground truth to assess the quality of the dynamic estimation of the flow fields. Two pseudo-probes ($s = 2$) are used, located at $y/c = 0.3$ and $y/c = 0.7$, at the same stream-wise position $x/c = 5.25$. The choice of the downsampling rate and of a timespan for the probe snapshot matrix equal to 80 PIV snapshots is ascribed to the convective velocity in the wake. Assuming a displacement in the wake of approximately 15 pixels between each PIV snapshot, a timespan of 80 snapshots leads to a displacement of 1200 pixels in the stream-wise direction, which is the stream-wise length of the observed region. The selected timespan allows us to include in the rows of the probe snapshot matrix all the information included in the corresponding PIV snapshots. This choice is following the guideline obtained in Section 3.1. The resulting probe snapshot matrix U_{pr} contains $334 \times (s \times q)$ elements, being $s = 2$ and

$q = 80$.

The spectral distribution of the energy of the POD modes is shown in Figure 10. The first two modes, corresponding to the vortex shedding, are energetically dominant. The modes spectrum falls rapidly in intensity, until reaching nearly constant eigenvalues due to the strong noise contamination [32].

The first 6 flow field spatial modes (evaluated over only 334 snapshots in order to simulate a real field measurement with insufficient time resolution) are reported in Figure 11. While the first two modes describe the shedding in the wake of the stalled airfoil, higher order modes have not yet reached a full statistical convergence. In particular modes 4 and 5 are poorly converged as they seem to represent a higher frequency component of the shedding phenomenon (i.e. higher order harmonics, similarly to what observed in the wake of a cylinder in [15]) although partially contaminated by cross-talk with higher order modes.

Figure 12 reports the squared correlation coefficient of the estimated POD time coefficients with respect to the true temporal variation of the POD times coefficients. The squared correlation coefficient is evaluated using the reconstructed time coefficient of the entire sequence, including both in-sample (i.e. used for the POD calculation) and out-of-sample (i.e. not used for the POD calculation) snapshots. The improvement using the 3σ approach is remarkable, allowing the use of up to 25 modes if a threshold of 0.2 is set to the squared correlation coefficient (while for the unfiltered approach only 3 modes should be included under the same criteria).

It is important to underline that in a real experiment a ground truth is not available for out-of-sample snapshots. In-sample snapshots do not allow a direct application of the squared correlation coefficient as a metric for reconstruction accuracy quantification, since indeed using all modes would provide a perfect reconstruction for the in-sample snapshots, i.e. $R^2 = 1$. The available alternative with experimental data is to eliminate from the sequence part of the samples, and use them as fictitious out-of-sample snapshots for comparison. In some cases this approach might pose convergence issues (consider for instance short duration experiments due to facility limitations). The results of Figure 12 show that the squared correlation coefficient applied on in-sample snapshots using the 3σ filtering is still a very indicative figure of merit, thus providing a useful assessment instrument also for short sequences. Nonetheless, it has to be underlined that the squared correlation coefficient obtained with in-sample data is still slightly larger than the real one, thus requiring a stricter cutoff on the number of modes to reduce noise contamination of the reconstructed time coefficients history.

The reconstruction quality can be assessed through the observation of Figure 13 and Figure 14. Figure 13 reports the estimated POD time coefficients (with black markers and black line) against the original values (in blue

Figure 11: Contour plot of the stream-wise velocity component of the first 6 flow fields modes $\phi_i(\underline{x})$ with superimposed (down-sampled) vector arrows.

Figure 12: Squared correlation coefficient between dynamically estimated time coefficient and original time coefficients of the PIV sequence.

line) and the samples used to build the original snapshot dataset (in red markers connected through red lines). The red lines indicate the output of a reconstruction based on linear interpolation of the time coefficients. The proposed methodology is able to reconstruct events with a frequency far beyond the Nyquist limit and results are well satisfactory both for time-periodic modes (modes 1 and 2) and non-periodic modes (modes 3-6). As expected, the estimation quality drops with respect to the channel DNS database due both to the decrease of the number of snapshots (and the consequent increase of $1/\sqrt{n_t}$), which imposes a stronger truncation on the matrix Ξ , and to the increase of the measurement noise which causes a poorer statistical convergence of the POD modes. It is possible to argue that a longer time-sequence would increase the estimation quality. In fact, as discussed in Raiola *et al.* [32] noise contamination can be reduced by increasing

the number of snapshots, here limited by memory storage restriction when working with time-resolved sequences.

A visual representation of the reconstruction quality can be seen in Figure 14 which reports the reconstruction of one single snapshot (Figure 14a) against the true flow field (Figure 14c) and the exact reconstruction of the flow field truncated at 20 modes (Figure 14b). The errors with respect to the truncated LOR and the original field reported in Figure 14d,e, respectively, show that the error is mostly due to the truncation of the smaller scales of the shedding (the error with respect to the original field which contains the small scales is indeed significantly higher than the error with respect to the LOR). The poor convergence of the modes 4 and 5 and consequently their poor dynamic estimation is the cause of the errors in the quantification of the velocity fluctuations in the wake. Although the shedding frequency is perfectly identified, the lack of convergence of higher order modes causes a modulation in the estimation of the wake deformation.

4. Conclusions

A robust and easy-to-implement method for the estimation of flow fields using synchronized acquisition with fast probes (i.e. hotwires) and non-time resolved field measurement techniques (such as PIV) for turbulent flows has been assessed. The method poses its foundations on the work by Hosseini *et al.* [27], which is improved here by truncating the time correlation matrix between temporal modes of the flow field measurements and of the probe measurements. The rationale behind the truncation is in the random distribution of noise and uncorrelated field-probe measurements. Using a standardized 3σ criterion, a robust approach is proposed, which

Figure 13: Time coefficients of the true measured signal, of the dynamic estimation and of the subsampled PIV sequence.

is applicable for flows involving a significant spectral richness.

The proposed method has been validated with a synthetic test case, based on DNS data of a turbulent channel flow, and with an experimental test case of the wake of an airfoil in stall conditions. The proposed dynamic estimation method has proved to be able to properly capture the dynamics of the large-scale structures via estimating the time coefficients of the most energetic POD modes. Two aspects arose from the validation of the filtering approach:

1. When building the probe snapshot matrix, the choice of the timespan of the probe data corresponding to each field snapshot has to be similar to the time needed to transport all the information contained in

the field snapshots to the probe location. This depends on the probe location (either past the field or embedded in it). In case of a preferential direction of the mean flow this timespan might be trivially equal to the length of the domain divided by the mean convection velocity. In case of strong mean velocity gradients or in absence of a preferential direction of the mean flow, an adaptive-probe timespan might be a suitable solution. This aspect might be object of future studies;

2. When dealing with turbulent flows, thus without any dominant shedding frequency, truncation of the time correlation matrix allows reducing significantly the time-jittering in the rebuilt history of the time coefficients, and to remarkably improve the reconstruc-

Figure 14: Contour plot of stream-wise velocity component with superimposed (down-sampled) vector arrows. Top: comparison of the dynamic reconstruction of the flow field against the exact flow field and the exact low order reconstruction with the same mode truncation (20 modes) as the dynamically estimated flow field. Bottom: reconstruction error distributions in percentage with respect to the LOR (d) and to the original field (e).

tion quality also for a small number of probes. This filtering is fundamental for turbulent flows, where uncorrelated small scale features contribute in contaminating the time correlation matrix, and consequently, the fields estimation. While this filter is of simple implementation for the extended POD case, it is not trivial to include it in LSE, which is instead strongly dependent on a proper convergence of the correlation between the quantities to be measured.

This second point reflects directly in the experimental test case, where a small set of snapshots is used for the dynamics reconstruction in the separated wake of a stalled airfoil. Considering the poor convergence of the POD modes, the time correlation matrix filtering with the 3σ criterion has shown to significantly improve the quality of the reconstructed history of the time coefficients. Additionally the filtered data can be used to directly evaluate the squared correlation coefficient of the time coefficients directly on the in-sample data. This is potentially useful in case of short time sequences, in which in-sample snapshots can not be extracted to benchmark the reconstruction without significantly affecting the convergence itself.

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