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Influence of ply orientation on free-edge effects in laminates subjected to in-plane loads.

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Abstract

An analysis of the influence of fibre orientation on the interlaminar stresses near the free edge of composite laminates was carried out. The study of these stresses is relevant due to the fact that they contribute to a number of failure mechanisms, such as delamination and shear stress cracking at the laminate edge. The presence of a displacement gradient was demonstrated using a numerical model, which showed that the maximum value is dependent on the orientation of the plies in the laminate. A solid correlation of the model predictions with experimental results was obtained. A classic finite difference scheme similar to that proposed by Pipes and others was implemented, though on this occasion modified via the use of non-dimensional variables. The displacement gradient is related to interlaminar stresses, which in turn are dependent on fibre orientation. In order to study the dependence of the out-of-plane stresses on these two parameters, an analysis of symmetric laminates with varying stacking sequences and ply orientations was carried out.

Keywords: A. Lamina/ply; B. Stress concentrations; C. Laminate mechanics; C.

Numerical Analysis; Edge effect.

1. Introduction

Laminate composites are particularly sensitive to interlaminar stresses due to the fact that the strength and stiffness properties in the laminate thickness direction are inferior to the in-plane properties [1, 2]. Out-of-plane stresses appear even if in-plane loads are applied to the laminate, for example, in points near a free edge. These stresses appear due to the need to verify the boundary conditions between two adjacent plies with different orientations at the free edge [3, 4]. As a result of interlaminar stresses, a gradient of longitudinal displacements along the width appears in laminate plates subjected to uniaxial tensile loads. Pipes and Pagano [5] verified the existence of this gradient experimentally using optical techniques based on the Moiré principle. Regarding the effect of free edge, there exist two failure modes, namely delamination and shear failure, which are controlled by different components of the stress tensor. Delamination is one of the most critical failure modes in laminates, and one that is principally controlled by the out-of-plane normal stress and/or by the interlaminar shear stresses [6-8]. This failure mode is responsible for a significant loss of structural strength when subjected to compression loads [8, 9]. Shear failure is the failure mode that causes transverse cracks and it is controlled by the interlaminar shear components of the stress tensor. Estimation of the out-of-plane stresses is, therefore, of great relevance in determining the failure mode in the laminate [4, 10]. Several techniques have been applied in an effort to reduce the possibility of damage at the free edge due to interlaminar stresses, including the use of stitched fibres through the laminate thickness at the edge [11], the inclusion of an adhesive layer between plies at the edge [12-14] or physical modifications to the edge of the laminate [6]. Another alternative is to define a laminate stacking sequence that reduces the interlaminar

stresses at the edge [15], thereby increasing the laminate strength. In all cases an accurate estimation of these stresses is required.

The use of Laminate Theory does not allow for the estimation of out-of-plane stresses; as a result, damage may appear due to the edge effect. For this reason, more complex 2D models have been applied to the problem, including displacement/stress-based equivalent single-layers or layer-wise theories. D'Ottavio et al. [16] analysed the accuracy of both theories and demonstrated the greater accuracy of the latter in terms of capturing the edge effect, though problems arise regarding accurate estimation of the interlaminar stresses.

The limitations of 2D models have resulted in the use of 3D models. The application of 3D Elasticity equations to arbitrary problems is a complex issue, the truth being that no closed form solutions exist for obtaining the stresses at the free edge in general cases [17]. Several analytic approaches to solving the 3D equations of the problem have been proposed and the stress-function-based solution first proposed by Lekhnitskii has been applied on occasion [18, 19]. Zhang et al. [20] used a Fourier series expansion of the field variables in the in-plane direction to analyse the free-edge effect in laminates subjected to tensile loads. An alternative is to estimate the interlaminar stresses by means of equilibrium considerations, which can be simplified in some problems involving symmetric laminates and some load cases, such as, for example, uniaxial loads.

In the resolution of three-dimensional equations the use of the Finite Element Method is very common [21-23], though it requires a high level of mesh refinement in order to be able to analyse the stress concentration phenomenon and presents some problems due to the singularity that appears at the free edge [14]. Other methodologies used in this

problem are the Boundary Element Method [24] and the Finite Differences Method [25].

The problem of out-of-plane stresses associated with the edge effect has been studied since the pioneering work of Pipes and Pagano in 1970 [25] and continues to the present day [10, 17, 26]. A review of the state of the art of this subject can be found in [27] and [28].

Several stacking sequences and laminate lay-ups have repeatedly appeared in studies, such as 0/90, 90/0, +45/-45, or 10/-10 [3, 4, 10, 17, 21, 22, 24, 26, 29]. The results of these works show that interlaminar stresses near the free edge are significantly modified by the particular stacking sequence and fibre orientation of each ply. However, a direct comparison between these studies cannot be made since they feature different materials, geometries and load conditions. A systematic study of different fibre orientations and laminate lay-ups is therefore if more general conclusions are to be obtained.

In this work the interlaminar stresses near a free edge in symmetrical laminates were analysed and the influence of fibre orientation was evaluated in a series of lay-ups. A finite difference-based model was employed and the results of the model were correlated with those from other scientific studies obtained using different methodologies [21]. The gradient of longitudinal displacement in the laminate near the free edge was analysed as an indicator of the interlaminar stresses, thereby confirming its variation according to fibre orientation. The interlaminar stresses components were determined using four laminate lay-ups - two with four plies and two with six plies, the fibre orientation varying between 0° and 90°.

2. Model formulation

The problem examined in this work consists of a symmetric laminate composed of N plies with thickness h_0 . The laminate width is $2b$, this being inferior to the length L of

the laminate (Fig. 1). Two types of axes are defined, these being global, or geometric axes in which the plane xy is the laminate plane, and local, or orthotropic axes for each ply in which direction 1 is the fibre direction. The orthotropic axes are at θ degrees to the geometric axes (θ being the fibre orientation in each ply). In the geometric axes, the plane $z = 0$ is the symmetry plane of the laminate. The laminate is subjected at its edges to a uniform uniaxial strain ε_0 in the x direction.

In the problem analysed no body forces were considered, meaning the equilibrium relations can be defined as:

$$\begin{aligned}\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} &= 0 \\ \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} &= 0 \\ \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} &= 0\end{aligned}\quad (1)$$

The problem is solved assuming small displacements and small strains. Using this hypothesis the compatibility relations can be written as:

$$\begin{aligned}\varepsilon_{xx} &= u_{,x} \\ \varepsilon_{yy} &= v_{,y} \\ \varepsilon_{zz} &= w_{,z} \\ \gamma_{yz} &= w_{,y} + v_{,z} \\ \gamma_{xz} &= u_{,z} + w_{,x} \\ \gamma_{xy} &= u_{,y} + v_{,x}\end{aligned}\quad (2)$$

Where u , v , and w are the components of the displacement vector in each ply point.

Assuming linear elastic behaviour for each ply, the constitutive law can be expressed in the geometric axes as:

$$\sigma = C \cdot \varepsilon \quad (3)$$

$$\begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{pmatrix} = \begin{pmatrix} \overline{C}_{11} & \overline{C}_{12} & \overline{C}_{13} & 0 & 0 & \overline{C}_{16} \\ \overline{C}_{12} & \overline{C}_{22} & \overline{C}_{23} & 0 & 0 & \overline{C}_{26} \\ \overline{C}_{13} & \overline{C}_{23} & \overline{C}_{33} & 0 & 0 & \overline{C}_{36} \\ 0 & 0 & 0 & \overline{C}_{44} & \overline{C}_{45} & 0 \\ 0 & 0 & 0 & \overline{C}_{45} & \overline{C}_{55} & 0 \\ \overline{C}_{16} & \overline{C}_{26} & \overline{C}_{36} & 0 & 0 & \overline{C}_{66} \end{pmatrix} \begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{pmatrix} \quad (4)$$

Where σ and ε are the stress and strain tensors in Voigt notation, and C is the orthotropic constitutive tensor of each ply. The components of this tensor are functions of the nine material elastic constants in orthotropic axes and the angle θ . The relations between $\overline{C}_{ij}$ coefficients and the material elastic constants are described in Appendix 1.

Under application of a uniaxial loading state, stress tensor components tend to be independent of the x coordinate. According to Saint-Venant's principle, this stress state will hold in laminate regions that are sufficiently distanced from load application areas. The stress independence of x also implies strain independence. Moreover, geometric symmetry of the laminate and load state allow the integration of Eq. 2 to obtain:

$$\begin{aligned} u &= \varepsilon_0 x + U(y, z) \\ v &= V(y, z) \\ w &= W(y, z) \end{aligned} \quad (5)$$

Where U , V and W are displacement functions that are independent of x . These functions lead to a functional dependence on stress components, which can be expressed as:

$$\sigma_{ij} = \sigma_{ij}(U, V, W) = \sigma_{ij}(y, z) \quad (6)$$

Under these considerations, equilibrium relations assume a reduced form:

$$\begin{aligned} \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} &= 0 \\ \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} &= 0 \\ \frac{\partial \sigma_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} &= 0 \end{aligned} \quad (7)$$

It is therefore possible to reduce the region of interest to just one quadrant, using symmetry hypothesis, as is shown in Fig. 2:

In boundary lines, free stress boundary conditions must be imposed, these being:

$$\begin{aligned} \sigma_{yy} = \sigma_{yz} = \sigma_{xy} &= 0 \quad (y = b) \\ \sigma_{zz} = \sigma_{yz} = \sigma_{xz} &= 0 \quad \left(z = \frac{H}{2} \right) \end{aligned} \quad (8)$$

In order to solve the problem, some dimensionless variables were considered:

$$\bar{y} = \frac{y}{b}; \quad \bar{z} = \frac{z}{h_o}; \quad \bar{U} = \frac{U}{h_o}; \quad \bar{V} = \frac{V}{b}; \quad \bar{W} = \frac{W}{h_o}; \quad R = \frac{b}{h_o} \quad (9)$$

The functional form of the displacements given in Eq.4 can be used to obtain strains components by substituting in Eq. 2. Moreover, stresses obtained through constitutive law (Eq. 4) should comply with the equilibrium equations in Eq. 7 that can be expressed in terms of displacement components in each material point. By using the non-dimensional variables given in the Eq. 9, the equilibrium governing partial equations can be written as follows:

$$\begin{aligned} R\bar{C}_{26}\bar{V}_{,\bar{y}\bar{y}} + R(\bar{C}_{36} + \bar{C}_{45})\bar{W}_{,\bar{y}\bar{z}} + \bar{C}_{66}\bar{U}_{,\bar{y}\bar{y}} + R^3\bar{C}_{45}\bar{V}_{,\bar{z}\bar{z}} + R^2\bar{C}_{55}\bar{U}_{,\bar{z}\bar{z}} &= 0 \\ R\bar{C}_{22}\bar{V}_{,\bar{y}\bar{y}} + R(\bar{C}_{23} + \bar{C}_{44})\bar{W}_{,\bar{y}\bar{z}} + \bar{C}_{26}\bar{U}_{,\bar{y}\bar{y}} + R^3\bar{C}_{44}\bar{V}_{,\bar{z}\bar{z}} + R^2\bar{C}_{45}\bar{U}_{,\bar{z}\bar{z}} &= 0 \\ \bar{C}_{44}\bar{W}_{,\bar{y}\bar{y}} + R(\bar{C}_{45} + \bar{C}_{36})\bar{U}_{,\bar{y}\bar{z}} + R^2(\bar{C}_{23} + \bar{C}_{44})\bar{V}_{,\bar{y}\bar{z}} + R^2\bar{C}_{33}\bar{W}_{,\bar{z}\bar{z}} &= 0 \end{aligned} \quad (10)$$

On the other hand, displacement components at the boundary material points should comply with the boundary condition (Eq. 8). These tensile free-boundary conditions can also be rewritten in terms of displacements components derivatives (using Eq. 2 and Eq. 4) and using the non-dimensional variables given in the Eq.10:

$$\begin{aligned} \bar{C}_{22}\bar{V}_{,\bar{y}} + \bar{C}_{23}\bar{W}_{,\bar{z}} + \frac{1}{R}\bar{C}_{26}\bar{U}_{,\bar{y}} &= -\bar{C}_{12}\varepsilon_o \\ \frac{1}{R}\bar{C}_{44}\bar{W}_{,\bar{y}} + R\bar{C}_{44}\bar{V}_{,\bar{z}} + \bar{C}_{45}\bar{U}_{,\bar{z}} &= 0 \\ \bar{C}_{26}\bar{V}_{,\bar{y}} + \bar{C}_{36}\bar{W}_{,\bar{z}} + \frac{1}{R}\bar{C}_{66}\bar{U}_{,\bar{y}} &= -\bar{C}_{16}\varepsilon_o \end{aligned} \quad (11)$$

$$\begin{aligned} \bar{C}_{23}\bar{V}_{,\bar{y}} + \bar{C}_{33}\bar{W}_{,\bar{z}} + \frac{1}{R}\bar{C}_{36}\bar{U}_{,\bar{y}} &= -\bar{C}_{13}\varepsilon_o \\ \frac{1}{R}\bar{C}_{44}\bar{W}_{,\bar{y}} + R\bar{C}_{44}\bar{V}_{,\bar{z}} + \bar{C}_{45}\bar{U}_{,\bar{z}} &= 0 \\ \frac{1}{R}\bar{C}_{45}\bar{W}_{,\bar{y}} + R\bar{C}_{45}\bar{V}_{,\bar{z}} + \bar{C}_{55}\bar{U}_{,\bar{z}} &= 0 \end{aligned} \quad (12)$$

Field equations (Eq. 10) subjected to boundary conditions (Eqs. 11 and 12) were solved using a finite difference scheme defined on a map of discrete material points. This method allows us to obtain discrete values of displacement components, strain and stress components in the discrete material points. In order to preserve continuity of the out-of-plane stress components along the interfaces between two plies with different

orientations, discretization is carried out in order that the interfaces lie between two rows of material points as shown in Fig. 2.

In order to approximate second derivatives, central difference formulas were used, while backward and forward difference formulas were employed for points falling on edges. In addition, the following conditions of symmetry were applied:

$$\begin{aligned}\bar{U}_{,\bar{z}}(\bar{y}, 0) &= 0 \\ \bar{V}_{,\bar{z}}(\bar{y}, 0) &= 0 \\ \bar{W}(\bar{y}, 0) &= 0\end{aligned}\tag{13}$$

$$\begin{aligned}\bar{V}(0, \bar{z}) &= 0 \\ \bar{W}_{,\bar{y}}(0, \bar{z}) &= 0\end{aligned}\tag{14}$$

Finally, the following conditions were considered as a means of fixing rigid solid displacements:

$$\begin{aligned}\bar{U}(0, \bar{z}) = \bar{U}(0, 0) &= 0 \\ \bar{V}(0, 0) &= 0 \\ \bar{W}(0, 0) &= 0\end{aligned}\tag{15}$$

By combining all these equations, an algebraic system with the form $\mathbf{A}\mathbf{d} = \mathbf{b}$ can be defined as follows:

- A matrix $\mathbf{A}$ containing the full set of coefficients shown in Eqs. 10-12.
- A vector of variables containing the displacement components $\mathbf{d}$:

$$\mathbf{d} = (\bar{U}_1 \quad \bar{V}_1 \quad \bar{W}_1 \quad \bar{U}_2 \quad \bar{V}_2 \quad \bar{W}_2 \quad \dots \quad \bar{U}_n \quad \bar{V}_n \quad \bar{W}_n)^T \tag{16}$$

- A vector $\mathbf{b}$ containing the independent terms, also shown in Eqs. 10-12.

Alternatively, it should be noted that the structure of the algebraic system can be written per unit value of applied strain:

$$\mathbf{A} \left(\frac{\mathbf{d}}{\varepsilon_0} \right) = \left(\frac{\mathbf{b}}{\varepsilon_0} \right) \tag{17}$$

And given the form of $\mathbf{b}$ vector:

$$\frac{\mathbf{b}}{\varepsilon_0} \neq f(\varepsilon_0) \tag{18}$$

In virtue of the linearity of the operators involved (Eqs. 17 and 18), it is possible to obtain displacement, strain and stress components per unit of axial deformation, so in order to obtain the actual value of the variables it is sufficient to multiply the results by the level of strain imposed.

3. Sensibility analysis and model verification

In this section two symmetric laminates - $[-45/45]_s$ and $[0/-45/90/45]_s$ - with a geometry ratio $R=8$ (see Eq. 9), were used to analyse the dependence of the results on mesh size. Four mesh sizes were selected, with 184, 363, 616 and 864 points respectively. The properties of the graphite/epoxy ply used in this work are shown in Table 1 and were taken from the scientific literature [21].

The interlaminar stress components were analysed both along the width of the laminate at the interface between plies and along the thickness at the free edge. In Figs. 3 and 4 only the results for laminate $[-45/45]_s$ are shown, as the results for the other laminate are very similar. As expected, the out of-plane stresses are zero around 50% of the semi-width of the laminate for the case shown in Fig. 3. This distance is approximately equal to the thickness of the laminate [24, 25]. Stress σ_{xz} grows near the interface between two plies with different orientations (Fig. 4a), while σ_{zz} changes sign as it approaches the interface (Fig. 4b).

Unlike other models, the variation of stresses is independent of the number of points in the mesh, and additional information relating to stress concentration is gathered near the laminate edge when the number of mesh points increases. The results show that it is possible to obtain sufficiently accurate results using a mesh of 616 points, the increase in accuracy achieved using a finer mesh being practically negligible.

In order to verify the methodology used in this work the interlaminar stress components predicted by the model were compared with the results of Lessard et al. [21] for two

laminates $[0/90]_s$ and $[90/0]_s$. The mechanical properties of the material are the same as those shown in Table 1. As an example, only the results of the first of the laminates are presented. The correlation of interlaminar stress components for the laminate $[90/0]_s$ is similar. In this laminate the interlaminar normal stress (σ_{zz}) shows the opposite sign than in the laminate $[0/90]_s$, a result already found by other authors who have employed different methodologies [3,21]. This result also explains the different damage evolution observed experimentally in $[0/90]_s$ and $[90/0]_s$ laminates subjected to in-plane loads, results that Laminate Theory and other 2D theories are not able to predict.

The stress components σ_{zz} and σ_{yz} were analyzed along the semi-width of the laminate at the interface between plies with different orientation (Figs. 5 and 6), and along the thickness at the free edge (Figs. 7 and 8). These stresses were compared with the results of Lessard et al. [21] for an applied strain of 1%. It should be noted that the stress component σ_{yz} along the thickness at the free edge is zero, according to boundary condition, so the curves represented for this stress component along the thickness (Fig. 8) correspond to the values estimated at interior points of the laminate very close to the free edge (specifically in the nodes immediately prior to those located on the free edge). The results obtained from the proposed model reproduce the trends and extreme values of the curves shown in [21]. The variations in interlaminar stress components observed in Figs. 5 and 6 are also similar to those observed by other authors [22, 25]. These results confirm that this model can effectively be used to study the effect of ply orientation on interlaminar stress components.

4. Results and discussion

The longitudinal displacement distribution on the surface of laminates was calculated along the transversal direction (y -direction in Fig.1) and the results correlated with experimental tests in specimens with a $[\theta_7/-\theta_7]$ lay-up and three distinct fibre orientations. In addition, the interlaminar stress components along the transversal direction and along the thickness direction at the laminate edge (z -direction in Fig.1) were estimated in laminates with several lay-ups and the fibre orientation varying between 0° and 90° .

4.1. Displacement gradient analysis

The model predicts the existence of a gradient in the longitudinal displacement (component U of the displacement functions shown in Eq. 5), which is a function of fibre orientation. This gradient is related to the interlaminar stresses near the free edge [5].

The displacement was estimated experimentally near the free edge and the results correlated with the predictions of the model for laminates with different fibre orientations. Laminates made of IM7 carbon fibre and MTM-45-1 epoxy resin were tested. The mechanical properties are shown in Table 2 and are taken from the literature. Tensile tests were carried out on $200 \times 15 \times 3.5$ mm specimens of a laminate with lay-up $[+\theta_7/-\theta_7]_s$ and three fibre orientations: 6° , 12° , and 30° . The tests were performed according to ASTM D3039 standard [31] and both the force-displacement curve until fracture and the longitudinal displacements at the edge were obtained for each test. The tests were recorded on digital camera and the longitudinal displacements measured using optical techniques. Three aligned points were marked on each specimen at the middle section, two close to the edges (points A and B in Fig. 9) and one at the centre (point C in Fig. 9). Using an image processing software the position of each point was

recorded for the same strain for all specimens. The relative longitudinal displacement of the edge points was estimated (function $U(y, z)$ according to Eq. 5.

The longitudinal displacement U was calculated on the surface of the specimen along the width (direction y in Fig. 1) using the model and introducing the properties of Table 2 and the geometry of the experimental tests. Fig. 10 shows that the model is able to accurately predict this variable for the three fibre orientations tested. An applied strain of 0.69% was used for calculating displacements. This strain was selected to avoid initiation of damage in all configurations tested.

The tests showed that this displacement is strongly modified by the fibre orientation angle θ . This behaviour is shown in Fig. 11, where the maximum value of U is drawn as a function of θ , varying the angle between 0° and 90° . For the laminate $[\theta/-\theta]_s$ there is a maximum for an orientation of approximately 14° , decreasing to zero when the orientation angle of the fibre increases.

4.2. Influence of ply orientation

The variation of the interlaminar stresses was studied in the developed model for different lay-ups in which the orientation of the plies was varied between 0° and 90° . The response of four laminate lay-ups was analysed, two with four plies $[\theta/-\theta]_s$ and $[2\theta/-\theta]_s$ and two with six plies $[\theta/\theta/-\theta]_s$ and $[\theta/2\theta/-\theta]_s$. These stacking sequences were defined as follows - for example, for an orientation of $\theta = 30^\circ$, the lay-ups considered were: $[30/-30]$, $[60/-30]$, $[30/30/-30]$ and $[30/60/-30]$.

Employing this model, for each laminate the various components of the interlaminar stresses at points near the free edge were obtained by varying the angle θ for an applied strain of 0.69%. Figs. 12 and 13 show the stress on the interface between the plies with different orientations in the 4-ply laminates ($[\theta/-\theta]_s$ and $[2\theta/-\theta]_s$). And Figs. 14 and 15 show the stress on the interlayer between plies closest to the plane of symmetry in the 6-

ply laminates ($[\theta/\theta/-\theta]_s$ and $[\theta/2\theta/-\theta]_s$). In all the analysed cases, the stress σ_{yz} is zero due to free stress boundary condition.

The most significant component at the free edge in the lay-ups $[\theta/-\theta]_s$ is σ_{xz} for all orientations studied, the maximum value appearing for an angle of approximately 12° . This orientation not being identical to that in which the maximum longitudinal displacement appears in Fig. 11 (about 14°). It must be taken into account that all stresses influence the displacement, and the maximum for the σ_{zz} component appears at about 18° . In addition, the displacements were measured on the surface of the laminate and the stresses shown in Fig. 12 were calculated at the interface. While the component σ_{zz} has positive values of a lower order of magnitude than σ_{xz} , and although it is very small (tensile stress being the subject at hand), it does contribute to the generation of damage by delamination. Both stress components have the same sign for the complete range of orientations.

In contrast, in the lay-up $[2\theta/-\theta]_s$ the component σ_{xz} changes sign for a fibre orientation of 60° (Fig. 13). The component σ_{zz} also changes direction for this angle, above which this stress is compressive, meaning it does not contribute to damage by delamination. The component σ_{xz} is greater than the σ_{zz} both below 30° and above 60° . In this lay-up, the maximum value of the stress component σ_{xz} appears for orientations below 10° , the maximum for the σ_{zz} component appearing with an orientation of 45° .

For the laminate lay-up $[\theta/\theta/-\theta]_s$ the variation of the interlaminar stresses is very similar to that of the lay-up $[\theta/-\theta]_s$, Fig. 14. The component σ_{xz} is maximum for an orientation of roughly 12° , slightly higher than those of the laminate $[\theta/-\theta]_s$. In the other interface, between the two plies with orientation θ , the stress distribution is similar, although the values are smaller because the boundary condition in the upper surface of the laminate implies that the stresses must be zero.

Fig. 15 shows the variation of interlaminar stress components for the laminate lay-up $[\theta/2\theta/-\theta]_s$. As in the previous cases considered, the largest component of interlaminar stress is σ_{xz} , although the component σ_{zz} is greater when compared to that found in the other three laminates lay ups. The maximum for σ_{xz} appears with an orientation slightly lower than 10° , and for σ_{zz} around 42° . Between orientations of 35° and 60° the values of both stresses are very similar, as occurred with the laminate $[2\theta/-\theta]_s$. With an orientation of 60° both interlaminar stresses change sign. Above this orientation the stress σ_{zz} becomes negative, which does not contribute to interlaminar damage.

5. Conclusions

The interlaminar stresses that appear due to the free edge effect in symmetrical laminates subjected to uniaxial loads in their plane were analysed and a number of laminate lay-ups and fibre orientations were studied.

The variation in interlaminar stresses at the free edge - taking into account fibre orientation - was different in the laminates lay-ups studied in this work. Nevertheless, both the fibre orientation at which the maximum shear stress appears and its value were similar in all cases. In addition, this value is higher than the maximum normal stress.

Laminate lay-ups $[2\theta/-\theta]_s$ and $[\theta/2\theta/-\theta]_s$ feature fibre orientations in which normal compression stresses appear and therefore do not contribute to interlaminar damage. In addition, there exist fibre orientations in which the values of normal and shear stresses are similar, though with opposing signs.

It is therefore possible to use the gradient of longitudinal displacements on the surface of the laminate near the free edge as an indicator of the presence of interlaminar stresses, and these displacements can be measured experimentally using digital image correlation techniques.

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Appendix 1

The relations between the constitutive law coefficients expressed in geometric axes $\overline{C}_{ij}$ and the coefficients expressed in orthotropic axes C_{ij} can be written as follows:

$$\begin{aligned}\overline{C}_{11} &= C_{11}\cos^4\theta + C_{22}\sin^4\theta + 2(C_{12} + 2C_{66})\sin^2\theta\cos^2\theta \\ \overline{C}_{22} &= C_{11}\sin^4\theta + C_{22}\cos^4\theta + 2(C_{12} + 2C_{66})\sin^2\theta\cos^2\theta \\ \overline{C}_{12} &= (C_{11} + C_{22} - 4C_{66})\sin^2\theta\cos^2\theta + C_{12}(\sin^4\theta + \cos^4\theta) \\ \overline{C}_{66} &= (C_{11} + C_{22} - 2C_{12} - 2C_{66})\sin^2\theta\cos^2\theta + C_{66}(\cos^4\theta + \sin^4\theta) \\ \overline{C}_{16} &= (C_{11} - C_{12} - 2C_{66})\sin\theta\cos^3\theta + (C_{12} - C_{22} + 2C_{66})\sin^3\theta\cos\theta \\ \overline{C}_{26} &= (C_{11} - C_{12} - 2C_{66})\sin^3\theta\cos\theta + (C_{12} - C_{22} + 2C_{66})\sin\theta\cos^3\theta \\ \overline{C}_{36} &= (C_{13} - C_{23})\sin\theta\cos\theta \\ \overline{C}_{13} &= C_{13}\cos^2\theta + C_{23}\sin^2\theta \\ \overline{C}_{23} &= C_{13}\sin^2\theta + C_{23}\cos^2\theta \\ \overline{C}_{33} &= C_{33} \\ \overline{C}_{44} &= C_{44}\cos^2\theta + C_{55}\sin^2\theta \\ \overline{C}_{55} &= C_{44}\sin^2\theta + C_{55}\cos^2\theta \\ \overline{C}_{45} &= (C_{55} - C_{44})\sin\theta\cos\theta\end{aligned}\tag{A.1}$$

Where θ is the fibre orientation in each ply

The constitutive law in the orthotropic axes can be written as:

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{pmatrix} \begin{pmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{23} \\ \gamma_{13} \\ \gamma_{12} \end{pmatrix} \quad (\text{A.2})$$

In turn, the C_{ij} coefficients are related to the material mechanical properties according to:

$$\begin{aligned} C_{11} &= \frac{1 - \nu_{23}\nu_{32}}{E_{22}E_{33}\Delta} \\ C_{22} &= \frac{1 - \nu_{13}\nu_{31}}{E_{11}E_{33}\Delta} \\ C_{33} &= \frac{1 - \nu_{12}\nu_{21}}{E_{11}E_{22}\Delta} \\ C_{12} &= \frac{\nu_{12} + \nu_{32}\nu_{13}}{E_{11}E_{33}\Delta} \\ C_{13} &= \frac{\nu_{13} + \nu_{12}\nu_{23}}{E_{11}E_{22}\Delta} \\ C_{23} &= \frac{\nu_{23} + \nu_{21}\nu_{13}}{E_{11}E_{22}\Delta} \\ C_{44} &= G_{23} \\ C_{55} &= G_{13} \\ C_{66} &= G_{12} \end{aligned} \quad (\text{A.3})$$

Where:

$$\begin{aligned}
v_{21} &= \frac{E_{22}}{E_{11}} v_{12} \\
v_{31} &= \frac{E_{33}}{E_{11}} v_{13} \\
v_{32} &= \frac{E_{33}}{E_{22}} v_{23} \\
\Delta &= \frac{1 - v_{12}v_{21} - v_{23}v_{32} - v_{31}v_{13} - 2v_{21}v_{32}v_{13}}{E_{11}E_{22}E_{33}}
\end{aligned} \tag{A.4}$$

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FIGURE CAPTION

Fig. 1. Laminate geometry and axes definitions

Fig. 2. Laminate transversal view (yz plane) and discretization points location

Fig. 3. Interlaminar stresses distributions along the interface $-45/45$ in laminate $[-45/45]_s$ for several mesh sizes, a) σ_{xz} , b) σ_{zz}

Fig. 4. Interlaminar stresses distributions along the thickness near free edge in laminate $[-45/45]_s$ for several mesh sizes, a) σ_{xz} , b) σ_{zz}

Fig. 5. σ_{zz} distribution along the interface $0/90$ in laminate $[0/90]_s$ for an applied strain of 1%

Fig. 6. σ_{yz} distribution along the interface $0/90$ in laminate $[0/90]_s$ for an applied strain of 1%

Fig. 7. σ_{zz} distribution along the thickness near free edge in laminate $[0/90]_s$ for an applied strain of 1%

Fig. 8. σ_{yz} distribution along the thickness near free edge in laminate $[0/90]_s$ for an applied strain of 1%

Fig. 9. Location of the point used in the experimental tests to estimate the longitudinal displacement.

Fig. 10. Gradient of longitudinal displacement on the surface of the laminate $[\theta_7/-\theta_7]_s$. Applied strain 0.69%

Fig. 11. Longitudinal displacement at the free edge on the surface of the laminate as a function of fibre orientation. Laminate lay-up $[\theta_7/-\theta_7]_s$. Applied strain 0.69%

Fig.12. Interlaminar stress component at the free edge on the interface $\theta/-\theta$. Laminate lay-up $[\theta/-\theta]_s$. Applied strain 0.69%

Fig. 13. Interlaminar stress component at the free edge on the interface $2\theta/-\theta$. Laminate lay-up $[2\theta/-\theta]_s$. Applied strain 0.69%

Fig. 14. Interlaminar stress component at the free edge on the interface θ/θ . Laminate lay-up $[\theta/\theta/-\theta]_s$. Applied strain 0.69%

Fig. 15. Interlaminar stress component at the free edge on the interface $2\theta/\theta$. Laminate lay-up $[\theta/2\theta/-\theta]_s$. Applied strain 0.69%

TABLE CAPTION

Table 1. Mechanical properties of the material used in the sensibility and verification analysis [21].

Table 2. Mechanical properties of IM7/MTM-45-1 [30].

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure7

Figure 8

Figure 9

Figure 10

Figure 11

Figure 12

Figure 13

Figure 14

Table 1

E_{11} (GPa)	E_{22} (GPa)	E_{33} (GPa)	G_{12} (GPa)	G_{13} G_{23} (GPa)	ν_{12} ν_{13} ν_{23}
137.9	14.5	14.5	5.86	5.86	0.21

Table 2

E_{11} (GPa)	E_{22} (GPa)	E_{33} (GPa)	G_{12} (GPa)	G_{13} G_{23} (GPa)	ν_{12}	ν_{13} ν_{23}
173	7.36	7.36	3.89	2.94	0.32	0.5