

This is a postprint version of the following published document:

Pernas-Sánchez, J., Artero-Guerrero, J. A., López-Puente, J., & Varas, D. (2018). Numerical methodology to analyze the ice impact threat: Application to composite structures. *Materials & Design*, 141, 350-360.

doi:<https://doi.org/10.1016/j.matdes.2017.12.044>

© Elsevier, 2017



This work is licensed under a [Creative Commons Attribution-NonCommercialNoDerivatives 4.0 International License](https://creativecommons.org/licenses/by-nc-nd/4.0/).

Numerical methodology to analyze the ice impact threat: application to composite structures.

J. Pernas-Sanchez, J.A. Artero-Guerrero, J. López-Puente, D. Varas*

Department of Continuum Mechanics and Structural Analysis. University Carlos III of Madrid. Avda. de la Universidad, 30. 28911 Leganés, Madrid, Spain

Abstract

Impacts on composites produce interlaminar failure (delamination) which is difficult to detect in common maintenance tasks, and affects the structural integrity. Therefore it is critical, for safety, to improve prediction tools in order to perform tolerant damage designs. The numerical modelling of impacts of deformable objects (such as ice) on composite panels is still a challenge. Not only the modelling of the laminate should be appropriate to reproduce its behaviour and failures, but the modelling of the deformable projectile should be capable to induce the corresponding response and damages. In this work a two-step numerical methodology is proposed for the study of ice impact. First, the deformable impactor is analysed, studying the impact process on a rigid target (a steel plate attached to a load cell). Once the deformable projectile behaviour is fully captured, the ice impact on a deformable target (a carbon/epoxy laminate) is studied. The composite material model takes into account intralaminar and interlaminar failure in order to reproduce the laminate behaviour. Different ice sphere diameters (30, 40 and 50mm) and impact velocities (50 – 250 m/s) are considered in this study. All the results from the numerical simulations have been compared with the experimental results.

Keywords: Composite laminate; failure mechanism; delamination; Numerical

*corresponding author

Email address: dvaras@ing.uc3m.es; *Fax/Phone:*+34916248460 (D. Varas)

modeling; Ice behavior; impact.

1. Introduction

Fiber reinforced composites are, at present, one of the most common materials in aerospace and aeronautic industries. Its excellent stiffness/weight and strength/weight ratios make this type of materials one of the best choices to safe weight in structural parts; for example the main commercial aircraft constructor uses them in more than 50 % (in terms of weight) in his recent designs. Moreover these materials are able to safe manufacturing and maintenance times due to the ability to reproduce large pieces without joints. There are some drawbacks to these excellent properties: the brittle-like behavior of the composites increases the risk of sudden failure, and in addition they may present some damages (which affect the structural integrity) that are difficult to detect in common maintenance task. One of these damages could be the delamination produced by impacts, therefore the authorities have reported that kind of menace as a key factor in the design of composite structures: literally from an EASA 2011 report “*A critical safety issue for the design of primary aircraft structures is vulnerability and damage tolerance due to foreign object impact from bird strike, hail, tyre rubber and metal fragments*” [1].

Different authors have studied such type of events, from an experimental and a numerical point of view, using different kind of impactors. Hard projectile impacts (such as metal fragments) on composites have been studied experimentally since early 80's [2, 3, 4]; regarding the numerical approach, the development of reliable material models has allowed to analyse and to achieve a better understanding of these kind of events, examples of it can be found in [5, 6, 7, 8, 9, 10, 11, 12]. The literature about impact events using hard impactors are extensive due to the simplicity of testing and modeling a non-deformable projectile. The study and testing of deformable impactors is inherently more difficult, nevertheless it is possible to find studies regarding bird

29 impacts [13, 14, 15], rubber impacts [16, 17, 18] or ice impacts [19, 20, 21],
30 mainly from an experimental point of view.

31

32 Focusing on the experimental study of ice impacts, two different kind of
33 works can be found in literature: those that study the ice impact against a rigid
34 target and others that analyse the ice impact event onto a deformable target.
35 The first group of works performs impacts of different ice geometries to obtain
36 the impact force time history by means of a load cell or a Hopkinson tube. An
37 example of these studies is the one carried out by Kim et al. [22], where a steel
38 plate is attached to a load cell in order to measure the induced force produced
39 by an ice sphere. The present authors published a previous work [23] in which
40 impact tests on a load cell are performed to obtain the contact force, using a
41 spring-mass model to reconstruct it, from the data measured by the load cell.
42 Later Tippman et al. [24] used a Hopkinson tube to measure the impact force,
43 by means of the unidimensional wave propagation theory, and hence avoiding
44 the use of load cells. A different ice impactor geometry was studied by Pereira et
45 al. [25] who performed tests to measure the impact force exerted by the impact
46 of ice cylinders. The other mentioned group of works, which analyse the ice
47 impact event onto a deformable target, can be subdivided attending the kind
48 of target to be impacted: a metal (mainly aluminum) [21, 26] or a composite
49 target [22, 27, 28, 29]. These kind of works analyse the deflection and damage
50 appearance due to the ice impact, focusing in the interlaminar damage in the
51 case of composites because it is the most important damage that appears. The
52 present authors also studied in a previous experimental work [20] the influence
53 of different ice sphere diameters impacting CFRP panels with different thickness.

54

55 Literature describing numerical models for the simulation of ice impact is
56 likewise scarce. The first attempts to describe the constitutive behaviour of
57 ice consisted on the use J2 elastoplastic models with different modes of failure
58 [21, 30], which are mainly standard models available in commercial codes. De-
59 spite the simplicity of the mentioned models, the works showed its ability to

60 capture some of the effects produced by ice impacts on different material plates.
61 Nevertheless the models fail in the overall description of the ice behaviour dur-
62 ing the impact. Later Carney et al. [31] developed a constitutive behaviour for
63 the ice taking into account its strain rate sensitivity and the hydrostatic pres-
64 sure dependence of ice, validating the model with experimental results of high
65 velocity impacts of ice cylinders against a rigid plate [25]. Chuzel [32] used a
66 constitutive equation based on the damage model of Mazars [33] and modified
67 it to allow degradation in the strengths; the validation of the model was carried
68 out with both impacts against a rigid plate and a deformable plate (aluminium
69 panel). Recently, Tippmann et al. [24] proposed a J2 elastoplastic model with
70 pressure cut-off and rate dependent yield strength, validating it with experi-
71 mental test performed against a rigid plate.

72
73 Taking into account the different behaviour of a 'soft' or deformable pro-
74 jectile when impact onto a rigid or a deformable target, the differences of the
75 impact process as well as the consequences in both cases, it is reasonable to
76 think that any model proposed to describe the ice impact behaviour should
77 require a wide validation to be on the security side. In this work a two-step
78 numerical methodology is proposed for the study of the ice impact. First, the
79 deformable impactor is analysed studying the impact process on a rigid target,
80 thus the CFRP behaviour is not taken into account. A Drucker-Prager cons-
81 titutive model with strain rate dependence for the ice is validated for impacts
82 against a "rigid" target (a steel plate attached to a load cell). Once the de-
83 formable projectile behaviour is fully captured and the model validated, the
84 study of ice impacts on a deformable target (a carbon/epoxy laminate) can
85 be performed. The composite material model implemented takes into account
86 both intralaminar and interlaminar failures in order to reproduce the laminate
87 behaviour. Different ice sphere diameters and impact velocities are considered
88 to study the force time histories induced on the load cell and the damage in
89 composite laminates of different thickness.

90

91 In this work the authors propose a methodology with the aim of having a nu-
92 merical model capable of reproducing the impact of a deformable projectile, on
93 different kind of structures, and hence study its effects which could contribute
94 in future design developments. The methodology consists firstly on analyze the
95 ice impact behaviour on a “rigid” plate to validate the numerical model of the
96 ice (a deformable projectile) and then study the ice impacts on a carbon/epoxy
97 laminate (a deformable structure). It has been considered spherical ice projec-
98 tiles to assure that the ice model is valid when different stress states appear
99 on the projectile (compression and indirect tension) as well as a wide range of
100 velocities in order to demonstrate its validity on different circumstances. In
101 addition it is proposed a simplified CFRP model, in order to achieve a low
102 computational cost, that predicts the damages that appear on the laminate and
103 allows to analyze how they are produced during the impact process.

104

105 The first section of the article presents the material models for the composite
106 laminate and the ice. Then, the experimental tests used for the validation are
107 briefly explained (impacts on “rigid” plate and on composite laminate) as well
108 as the numerical modeling developed. Finally, the validation of both types of
109 impact and the results obtained in the numerical simulations are showed. The
110 last section summarizes the main conclusions of the research.

111

112 **2. Material modeling**

113 The numerical methodology proposed in this work pursues to study the
114 ice sphere impacts at different velocities onto different targets: steel plate and
115 carbon/epoxy laminate. In this section the material models implemented for
116 composite and ice are explained. The numerical methodology has been imple-
117 mented using the commercial explicit finite element software LS-Dyna v.971 [34].

118

119 *2.1. Ice material model*

120 The ice material definition used in the numerical simulations consists on
 121 an hypoelastic, until failure, with strain rate dependence model. In order to
 122 capture the pressure dependence of the ice, a yield function based on a Drucker-
 123 Prager criterion [35], including strain rate dependence in the yield definition is
 124 considered. The model has been implemented in the commercial finite element
 125 code LS-Dyna v.971 [34] using an user subroutine.

126

127 The elastic behaviour is provided by the following expression of Hooke's law:

$$\dot{\sigma} = C : \dot{\varepsilon}^e = C : (\dot{\varepsilon} - \dot{\varepsilon}^p) \quad (1)$$

128 where $\dot{\sigma}$ is the objective rate of the Cauchy stress tensor, $\dot{\varepsilon}$ is the strain rate
 129 tensor, $\dot{\varepsilon}^e$ and $\dot{\varepsilon}^p$ the corresponding elastic and plastic the strain rate tensors,
 130 and C is the Hooke stress-strain tensor defined by the elastic constants.

131

132 The elastic regime defined is limited by a yield function, to this end the model
 133 assumes a Drucker-Prager yield function which includes the pressure dependence
 134 of the ice [36, 37] and the increasing trend of the compressive strength with strain
 135 rate. The Drucker-Prager yield function (f_{DP}), defined in terms of equivalent
 136 stress ($\bar{\sigma}$) and the pressure (p) can be expressed as:

$$f_{DP} = \bar{\sigma} - (\sigma_{0y} + 3\alpha p) \quad (2)$$

137 where σ_{0y} is the material cohesion and α is a parameter related to the internal
 138 friction angle of the material. Both parameters may be related to the uniaxial
 139 stress limits in compression (σ_C) and in tension (σ_T):

$$\sigma_{0y} = \frac{2\sigma_C\sigma_T}{\sigma_C + \sigma_T} \quad \alpha = \frac{\sigma_C - \sigma_T}{\sigma_C + \sigma_T} \quad (3)$$

140 so that the Drucker-Prager yield surface is completely defined once σ_T and σ_C
 141 are known. According to the experimental findings of several authors [19, 36, 38],
 142 the tensile strength is constant but the compressive strength is dependent on

143 strain rate; hence the authors suggest a power law with strain rate sensitivity
 144 [39]:

$$\sigma_C(\dot{\epsilon}^p) = \sigma_{C0} \left(\frac{\dot{\epsilon}^p}{\dot{\epsilon}_0^p} \right)^m \quad (4)$$

145 being $\dot{\epsilon}^p$ the equivalent plastic strain rate, σ_{C0} the initial compressive strength
 146 and $m, \dot{\epsilon}_0^p$ material model's constant parameters.

147

148 To describe the inelastic flow, a non-associated plastic flow is proposed and
 149 integrated following the Kuhn-Tucker complementary condition and the con-
 150 sistency condition. Finally, two different pressure cut-offs were proposed for
 151 tension (P_T^{lim}) and compression (P_C^{lim}):

$$\begin{cases} p > P_T^{lim} = -\frac{\sigma_T}{3} \\ p < P_C^{lim} = \frac{\sigma_C}{3} \end{cases} \quad (5)$$

152

153

154 Once any of the pressure cut-off is reached, the deviatoric part of the stress is
 155 set to zero and the material only can withstand compressive hydrostatic stress.
 156 The parameters needed for the ice are obtained from the literature (Table 1).

157

| Property | Symbol | Magnitude |
|-------------------------------------|---------------|-------------------------|
| Density | ρ | 897.6 kg/m ³ |
| Young's modulus | E | 9.31 GPa |
| Poisson rate | ν | 0.33 |
| Initial compressive strength | σ_{C0} | 10.976 MPa |
| Compressive strain rate sensitivity | m | 0.0093783 |
| Tensile strength | σ_{T0} | 1.72 MPa |

Table 1: Model parameters for ice. [39]

158 It is worth to mention that the ice model, briefly explained in this section,
 159 was proposed by the authors in a previous work to study ice cylinder impacts

160 [39], where further details about the model can be found. On that work, the
161 model was validated using experimental results obtained from the work of Car-
162 ney et al. [31]. In order to perform a wider validation of the ice numerical model,
163 it has been considered necessary to validate it with a different ice geometry (an
164 sphere), a wider range of impact velocities and impacting against a deformable
165 target.

166

167 2.2. Carbon/epoxy laminate material model

168 In order to model the behaviour of the unidirectional carbon/epoxy lami-
169 nate, different approaches are used to take into account both interlaminar and
170 intralaminar failures. An orthotropic elastic material until failure is used to
171 reproduce the intralaminar failure, whereas the interlaminar damage or delam-
172 ination is modeled by means of the use of cohesive interactions. Both, material
173 and cohesive interaction are available in the commercial code. A similar ap-
174 proach for high velocity impacts, obtaining good results, was adopted by the
175 authors of the current work in previous articles [7, 40].

176

177 The intralaminar damage model considers different type of damages based
178 in the Chang-Chang model [41]. Distinguishing between fiber and matrix fail-
179 ure for tension and compression, different variables e_i (based on stresses) are
180 defined for each failure mechanism. When the value of any of these variables
181 reaches the value of 0 ($e_i^2 \geq 0$), the material is unable to withstand more stress
182 in this direction, instant in which the stiffness of the material involved in this
183 failure mechanism is set to zero [42]. This sudden decrease of stiffness could pro-
184 mote excessive distortion, and thus numerical instabilities, which are mitigated
185 removing the elements using a maximum strain criteria ($\varepsilon < 0.05$). The failure
186 mechanisms are defined as a function of the stress tensor component (σ_{ij}) and
187 the material strengths (X_i, Y_i, S_{12}).

| Property | Symbol | Magnitude |
|--------------------------|------------|------------------------|
| Density | ρ | 1580 kg/m ³ |
| Young modulus 0° | E_1 | 139 GPa |
| Young modulus 90° | E_2 | 9.4 GPa |
| In-plane shear modulus | G_{12} | 4.5 GPa |
| Poisson coefficient 12 | ν_{12} | 0.3089 |
| Compressive strength 0° | X_c | 1656 MPa |
| Tensile strength 0° | X_t | 2105 MPa |
| Compressive strength 90° | Y_c | 175 MPa |
| Tensile strength 90° | Y_t | 79 MPa |
| Shear strength | S_c | 114 MPa |

Table 2: Carbon epoxy AS4/8552 properties from the manufacturer Hexcel

- Fiber failure:

$$e_f^2 = \begin{cases} \left(\frac{\sigma_{11}}{X_t}\right)^2 - 1 & \text{if } \sigma_{11} > 0 \\ \left(\frac{\sigma_{11}}{X_c}\right)^2 - 1 & \text{if } \sigma_{11} \leq 0 \end{cases} \quad (6)$$

- Matrix failure:

$$e_m^2 = \begin{cases} \left(\frac{\sigma_{22}}{Y_t}\right)^2 + \left(\frac{\sigma_{12}}{S_{12}}\right)^2 - 1 & \text{if } \sigma_{11} > 0 \\ \left(\frac{\sigma_{22}}{2S_{12}}\right)^2 + \frac{\sigma_{22}}{Y_c} \left[\left(\frac{Y_c}{2S_{12}}\right)^2 - 1\right] + \left(\frac{\sigma_{12}}{S_{12}}\right)^2 - 1 & \text{if } \sigma_{11} \leq 0 \end{cases} \quad (7)$$

188 The values of stiffness and strength are taken from the manufacturer Hexcel
189 Composites and are listed in Table 2.

190 The interlaminar damage is taking into account through a cohesive inter-
191 action based on a traction-separation law, in which it is necessary to define a
192 damage onset and a damage evolution law. Both define a relative displacement
193 (δ) between the surfaces. This relative displacement is given as a function of
194 the displacement between the surfaces in mode I ($\delta_I = \delta_3$) and in mode II
195 ($\delta_{II} = \sqrt{\delta_1^2 + \delta_2^2}$), following the equation:

$$\delta_m = \sqrt{\delta_I^2 + \delta_{II}^2} \quad (8)$$

| Property | Symbol | Magnitude |
|---------------------------------|----------|----------------|
| Maximum onset stress in mode I | T | 35 <i>MPa</i> |
| Maximum onset stress in mode II | S | 45 <i>MPa</i> |
| Coefficient in mixed mode | μ | 1.45 |
| Energy release rate in mode I | G_I | 250 <i>J/m</i> |
| Energy release rate in mode II | G_{II} | 750 <i>J/m</i> |

Table 3: Cohesive interaction parameters.

196 The initiation criteria is function of the displacement of the softening onset
197 (δ^0):

$$\delta^0 = \delta_I^0 \delta_{II}^0 \sqrt{\frac{1 + \beta^2}{(\delta_{II}^0)^2 + (\beta \delta_I^0)^2}} \quad (9)$$

198 where $\delta_I^0 = T/E_N$ y $\delta_{II}^0 = S/E_T$ are the onset of softening in mode I and II, T
199 and S are the peak stresses onset of softening in each mode; and E_N and E_T
200 the stiffness of the different modes. Finally, $\beta = \delta_{II}/\delta_I$ is the ratio between the
201 modes.

202

203 The ultimate displacement (δ^F) is defined as function of G_{IC} and G_{IIC} (the
204 energy release rate for mode I and II respectively), and the power law coefficient
205 (μ), Eq. 10. The properties used for the cohesive interaction can be seen in
206 Table 3.

207

$$\delta^F = \frac{2(1 + \beta)^2}{\delta^0} \left[\left(\frac{E_N}{G_{IC}} \right)^\mu + \left(\frac{E_T \cdot \beta^2}{G_{IIC}} \right)^\mu \right]^{-\frac{1}{\mu}} \quad (10)$$

208 3. Experimental tests for validation and numerical models

209 This section briefly describes the experimental tests used to validate the
210 numerical methodology proposed, as well as the numerical models developed.
211 The numerical modeling implemented pursues to reproduce the impacts of ice

212 spheres onto a “rigid” and a deformable target. In order to validate and study
213 the capacity of damage prediction of the model, different ice sphere diameters
214 (30, 40 and 50 *mm*) and a wide range of impact velocities (from 50 to 250 *m/s*)
215 were considered.

216

217 *3.1. Ice sphere impact on steel plate with a load cell*

218 *3.1.1. Experimental tests*

219 In order to observe how ice behaves under impact conditions, ice spheres of
220 three different diameters (30, 40 and 50 *mm*) were launched against a load cell,
221 measuring the force induced during the impact. The ice spheres were acceler-
222 ated using a one stage light gas gun, which uses pressurized air to impel the
223 projectile through a 5 meters long, 60 *mm* calibre barrel (Fig. 1). The impact
224 velocity was varied from 50 to 250 *m/s* modifying the canon air pressure. A
225 laser sensor placed between the barrel muzzle and the target allows to measure
226 the ice velocity before the impact. In order to launch ice spheres of different
227 diameter using a canon of 60 *mm* calibre, a foam sabot was designed to adjust
228 the mismatch diameter, isolating and protecting the ice from the friction during
229 the acceleration, avoiding melting. The aerodynamic design of the sabot al-
230 lowed its separation from the ice projectile during the flight (using drag forces)
231 before the impact. The ice sphere impacts on a cylindrical 200 *mm* diameter
232 steel plate (Fig. 2) which is screwed to the load cell (model PB-2 manufactured
233 by Microtest, designed to measure load under impact conditions) in order to
234 measure the induced impact force. Finally, the steel plate-load cell group was
235 held to a back structure as it is shown in Fig. 2. The data acquisition system
236 employed to register the data measured by the load cell was a DEWETRON
237 DEWE-800 system specifically designed for dynamic applications, capable to
238 capture data up to 1MHz. In addition a Photron Ultima APX digital high-
239 speed camera was used to record the impact, and to verify that the ice reached
240 the load cell without losing its integrity; different frame rate was selected to
241 record the impacts ranging from 18000 to 100000 *fps*. The lighting, in order to

242 assure clear high-speed images, was provided by an Arrisun 12 Plus lamp head
243 with a 1200 W Hydrargyrum Medium-arc Iodide (HMI) lamp.

244

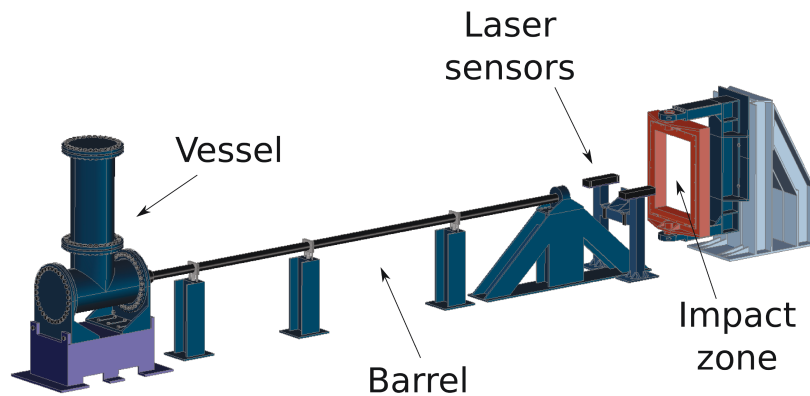


Figure 1: One stage light gas gun.

245 3.1.2. Numerical model

246 The numerical model reproduces the perpendicular impact of ice spheres
247 against a steel plate attached to a load cell. The ice spheres are meshed with
248 reduced integration hexahedral solid elements. After a mesh convergence study
249 the number of element used for the 30, 40 and 50 *mm* ice diameter are 23625,
250 56000 and 109375 respectively, using a similar element size in all the cases. In
251 a previous work [39], the authors explored the possibility of using an arbitrary
252 lagrangian-eulerian mesh and a smooth particle hydrodynamic approximation
253 to model the ice, but the results obtained by these techniques were similar to
254 the results obtained by the lagrangian one, moreover this last technique was
255 chosen because of its reduced computational cost. In order to avoid numerical
256 instabilities due to large distortion suffered by the elements, a deletion criteria
257 is implemented. This criteria is based on the equivalent strain; the maximum
258 value has been chosen in order to not interfere with the simulations results

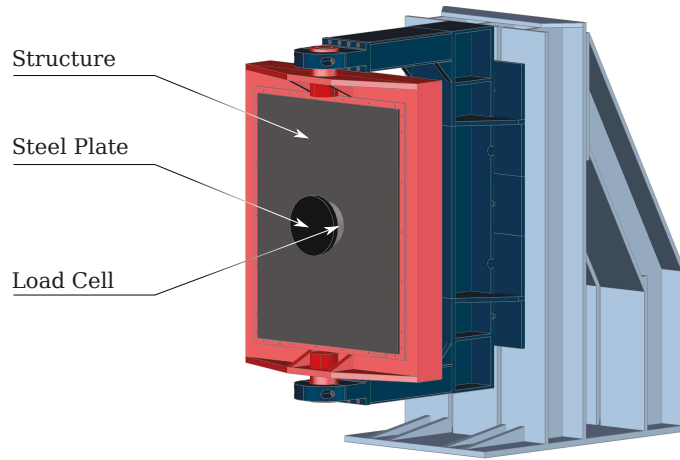


Figure 2: Steel plate, load cell and impact structure used in experimental tests.

259 ($\bar{\epsilon} = 1.5$).

260

261 Taking into account the experimental structure (steel plate-load cell-back
 262 structure) in which the ice spheres impact (Fig. 2), a spring-mass system with
 263 a steel plate has been used in the numerical simulations to reproduce the ex-
 264 perimental set-up (Fig. 3). The steel plate is modeled, using hexahedral solid
 265 elements (3772 elements), as an elastic material ($\rho = 7850 \text{ kg/m}$, $E = 210 \text{ GPa}$
 266 and $\nu = 0.3$) because no plastic deformation was observed. The other compo-
 267 nents of the load cell are modeled using unidimensional elements (springs) and
 268 masses with the values detailed in Table 4. These values have been obtained
 269 from the manufacturer specification and by modal analysis of the experimental
 270 facility [23].

271

272 Regarding the boundary conditions of the model, the end of spring k_1 is
 273 fixed and all the displacements, out of impact direction, in m_1 , m_2 and the steel
 274 plate are impeded. In order to reproduce the impact velocity, an initial velocity
 275 was applied to the ice spheres. Finally, a penalty stiffness contact between the
 276 nodes of the spheres and the surface of the steel plate was defined.

| Parameter | Value |
|-------------|-----------------------|
| m_1 | 16.2 Kg |
| m_2 | 17 Kg |
| Steel plate | 5.545 Kg |
| k_1 | $1.61 \cdot 10^8$ N/m |
| k_2 | $1 \cdot 10^8$ N/m |
| k_3 | $3.7 \cdot 10^8$ N/m |

Table 4: Masses and springs to model the load cell system.

277

278 3.2. Ice sphere impact on CFRP panel

279 3.2.1. Experimental tests

280 A campaign of ice impact tests on CFRP panels was carried out in order to
 281 study how this kind of impacts may affect these composite structures, widely
 282 used in the aeronautic industry. Two different ice diameters (40 and 50 mm)
 283 were launched against laminates at velocities ranging from 50 to 250 m/s . In
 284 order to perform the tests, the launcher previously described was used (Fig. 1).

285

286 In this case, the ice impacts on square plates of 300 mm side with two dif-
 287 ferent thickness: 4 and 6 mm approximately (21 and 32 plies respectively) with
 288 the ply sequence presented in Table 5. The laminates were clamped to the rig
 289 structure located in the impact zone (Fig. 1) using a screwed steel frame, leav-
 290 ing a free span of 280×280 mm . The impact event was recorded by means of
 291 a high speed camera (Photron Ultima APX); the lighting, as in the case of the
 292 ice impacts on a steel plate, was provided by an HMI lamp. A set-up scheme of
 293 the experiments can be observed in Fig. 4. In addition, some of the composite
 294 plates were monitored during the impact using strain gauges. The strain gauges
 295 were located on the back face at 30 mm away from the impact point; each gauge
 296 measures the strain in the direction parallel to the side of the plate, as it is
 297 showed in Fig. 4. Once the impact was performed, the composite laminate

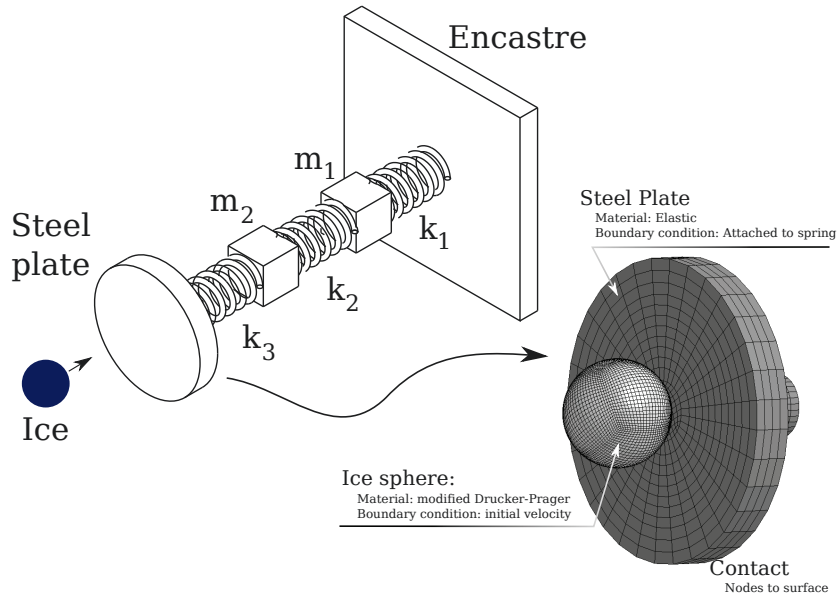


Figure 3: Steel plate and load cell system used in the numerical simulations

298 was inspected using ultrasonic techniques in order to measure the interlaminar
 299 damage extension.

300

| Laminate | Ply sequence | Thickness |
|----------|--|-----------|
| 21 Plies | (45/ - 45/90/0/90/ - 45/45/90/0/90/0)S' | 3.9 mm |
| 32 Plies | (45/ - 45/90/0/90/ - 45/45/90/0/90/45/ - 45/90/90/ - 45/45)S | 6.0 mm |

Table 5: Ply sequence of laminates used.

301 3.2.2. Numerical model

302 In this case, the numerical model reproduces the experimental tests in which
 303 ice spheres impact perpendicularly against a composite panel. The model pur-
 304 sues to study the capability of the numerical method to predict the effects of an
 305 ice impact on a CFRP structure. The ice sphere model is identical to the one
 306 previously detailed.

307

308 The two different thickness (4 and 6 mm) square composite plates afore-

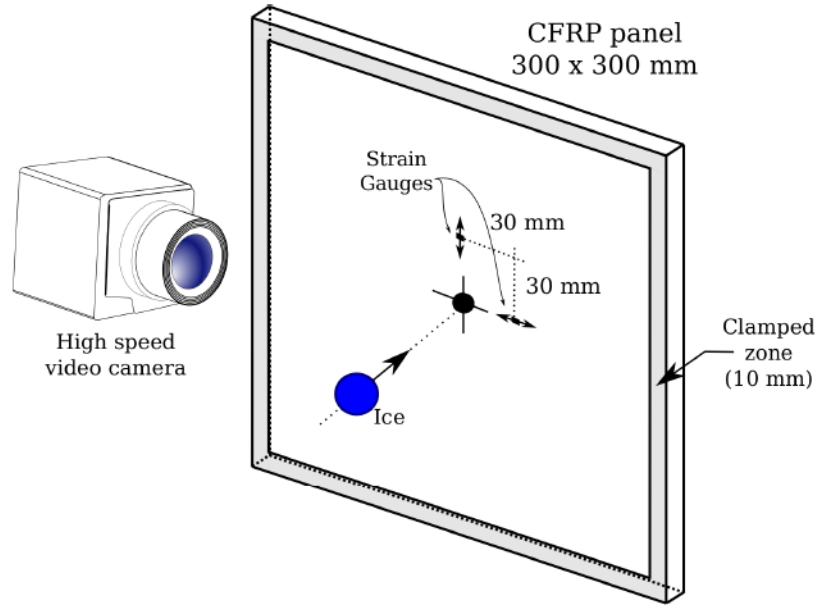


Figure 4: Experimental scheme.

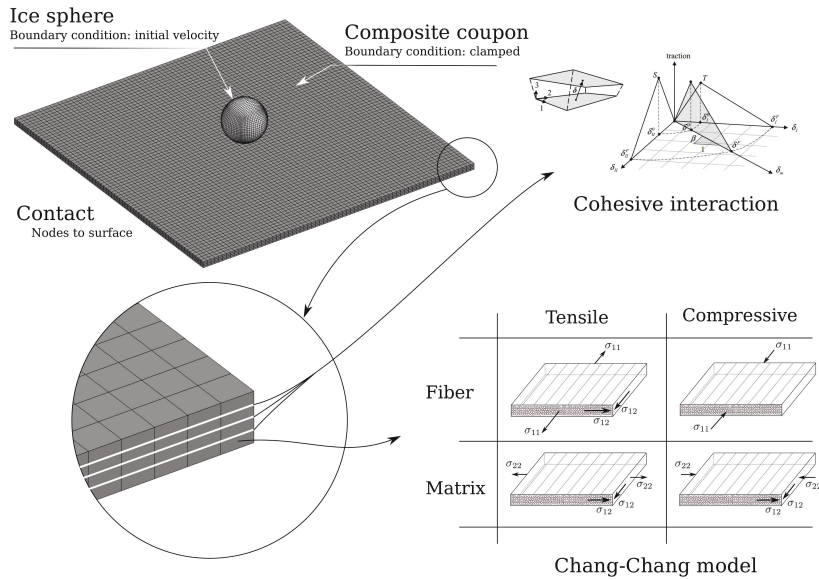
309 mentioned are modeled, as it has been already explained, taking into account
 310 intralaminar failures by means of the material model considered and the inter-
 311 laminar failures by means of cohesive interactions (Fig. 5). In order to reduce
 312 the high computational cost that induces to have a cohesive interaction between
 313 each ply of the laminate, clusters of plies were used, including a cohesive inter-
 314 action between these groups of plies. This approach was used by the authors of
 315 the current article on previous works where composite laminates were subjected
 316 to high velocity impacts, obtaining good results [7, 40]. The configuration of
 317 clusters, including the cohesive interaction ($/ \leftrightarrow /$) between the plies with mis-
 318 match angle orientation and in clusters of 7 plies for the 21 plies laminate and
 319 clusters of 8 plies in the case of 32 plies laminate, is the following:

- 320 • 21 plies: $\underbrace{(45/ - 45/90/0/90/ - 45/45/}_{/ \leftrightarrow /} \underbrace{90/0/90/0)}_{S'}$
- 321 • 32 plies: $\underbrace{(45/ - 45/90/0/90/ - 45/45/90/}_{/ \leftrightarrow /} \underbrace{0/90/45/ - 45/90/90/ - 45/45)}_{S} \leftrightarrow$

322 The discretization has been done using hexahedral shell elements (thick shell

323 elements in LS-Dyna notation), each cluster was meshed with 4900 thick shell
 324 elements (and a Gauss point for each ply), resulting in 14700 (102900 Gauss
 325 points) and 19600 (156800 Gauss points) elements for the 21 and 32 plies lam-
 326 inates respectively, determined after a mesh convergence study. Regarding the
 327 boundary conditions of the model, the laminate sides were clamped (The 6 DOF
 328 were restricted along the 280x280 mm plates sides). Finally, as in the case of
 329 impacts on a steel plate a penalty stiffness contact between the nodes of the
 330 spheres and the laminate surface was defined.

331



332 4. Validation and discussion

333 This section shows the validation of the numerical methodology developed
 334 in order to study the ability of the numerical simulation to reproduce the be-
 335 haviour and effects of an ice impact, as well as its capacity of damage prediction
 336 on composite panels. The results of experimental ice impacts on a steel plate

337 (with a load cell) are mainly used to validate the ice model proposed. Once
338 validated the ice modeling, it can be used to study the ice impact effects on a
339 composite laminate, in particular the appearance of damage. The experimental
340 and numerical results are compared and analysed.

341

342 *4.1. Ice sphere impacts on steel plate with load cell*

343 The model of ice used in this work was previously validated [39], as it was
344 already mentioned, using experimental results of ice cylinders impacting against
345 a load cell [31]. The stress state promoted by the impact in a cylinder is com-
346 pletely different to the one that appears in an sphere. In a slender cylinder
347 the stress state is close to uniaxial compression whereas in a sphere, both com-
348 pression and indirect tension appears. Therefore in order to assure that the
349 proposed model can reproduce the ice behaviour on different circumstances, it
350 has been considered appropriate to extend the validation to a different ice ge-
351 ometry (spheres) and in a wide range of impact velocities.

352

353 The force time history, obtained experimental and numerically (Force in k_3
354 in Fig. 3), of impacts with two different ice diameter and impact velocities are
355 shown in figures 6(a) and 6(b). The ice diameter and impact velocities were
356 selected in order to show extreme values of kinetic energies (low velocity-small
357 diameter & high velocity-large diameter) and hence observe the differences or
358 similarities on the whole range of energies considered. The experimental and
359 numerical curves show a good agreement between them: a sudden increase of
360 the force at the beginning of impact followed by a gentle small plateau, and
361 finally a slope to reach the maximum force value. Not only the trend, but also
362 the maximum value is well predicted. In addition, the pulse force duration pro-
363 duced by the ice impact is predicted correctly by the numerical simulations and
364 hence the final impulse is well reproduced.

365

366 The maximum force values, experimental and numerically obtained, for all

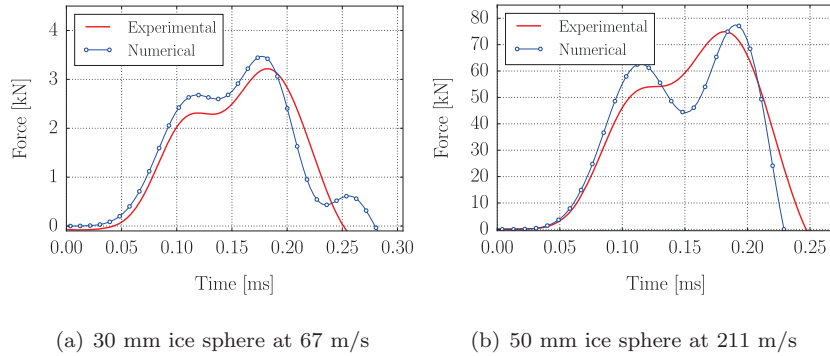


Figure 6: Experimental and numerical force time histories for impacts of different ice spheres at different velocities.

367 the ice diameters and impact velocities are shown in Fig. 7. It can be observed
 368 that the maximum force varies from less than 5 kN to more than 85 kN , for the
 369 cases with a higher kinetic energy. Fig. 7 also shows how the maximum force
 370 increases as the impact velocity raises and how the slope of the force curve raises
 371 when the diameter is bigger. As it can be seen, the numerical model not only
 372 predicts adequately the maximum force values, but capture the aforementioned
 373 trends. According to this fact, it could be said that the model is capable of
 374 reproducing the physical phenomena that appears in the impact process of an
 375 ice for complex stress fields.

376

377 In order to study the ice impact phenomenon, not only the force induced by
 378 the ice projectile should be predicted by the numerical simulations, but also the
 379 general behaviour of the material should be represented. Ice impactors flow over
 380 the structure, spreading the impact load; thus for a reliable damage prediction
 381 of impact it is necessary that the numerical model predicts faithfully how the
 382 ice deforms during the impact. Fig. 8 shows different frames of the high-speed
 383 video recorded during the impact (18000 fps), at a velocity of 115 m/s, of an
 384 ice sphere with a diameter of 40 mm and the corresponding numerical frames.
 385 It can be observed that the numerical model is capable of reproducing, qualita-
 386 tively, the spreading of the ice along the steel plate as the impact develops.

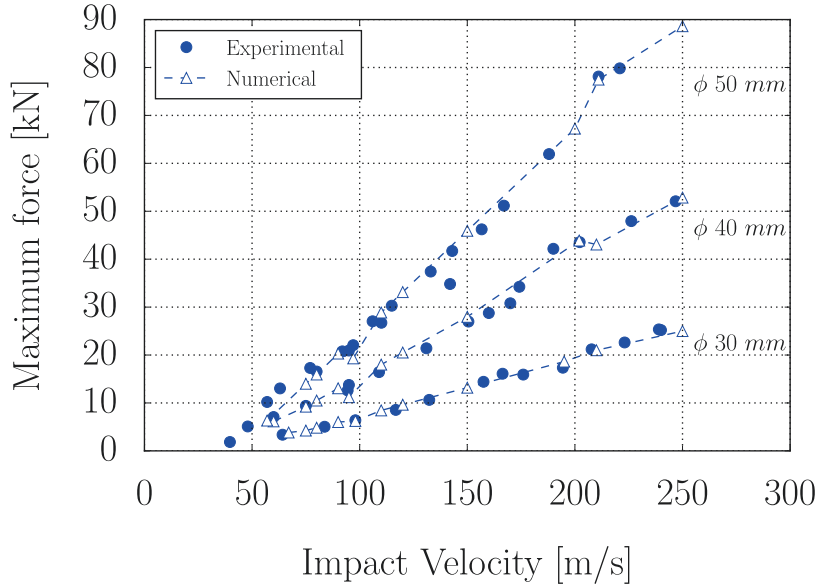


Figure 7: Experimental and numerical maximum forces against impact velocity

387

388 The images of the impact recorded by the high speed camera show how the
 389 ice becomes white at the beginning of the impact ($t = 55 \mu s$). This is due
 390 to the appearance of cracks inside the projectile [23]. In order to estimate the
 391 failure or fragmentation process given by the numerical simulations, an internal
 392 variable of failure (which indicates when the behaviour of the ice is out of the
 393 elastic regime) is defined to evaluate if the numerical model represents faithfully
 394 the ice behaviour. Fig. 9 compares the numerical and experimental images of
 395 the impact process (recorded at 100000 fps), focusing on the failure process
 396 evolution. The upper part of the images represents the contour plot of the
 397 defined failure internal variable, whereas the bottom part of the images shows
 398 how the experimental fragmentation evolves, according to the whiteness of the
 399 fragmented zone [23]. Although the numerical simulation does not fragment as
 400 the ice does, it can be said that the numerical model is able to reproduce the
 401 failure development during the impact.

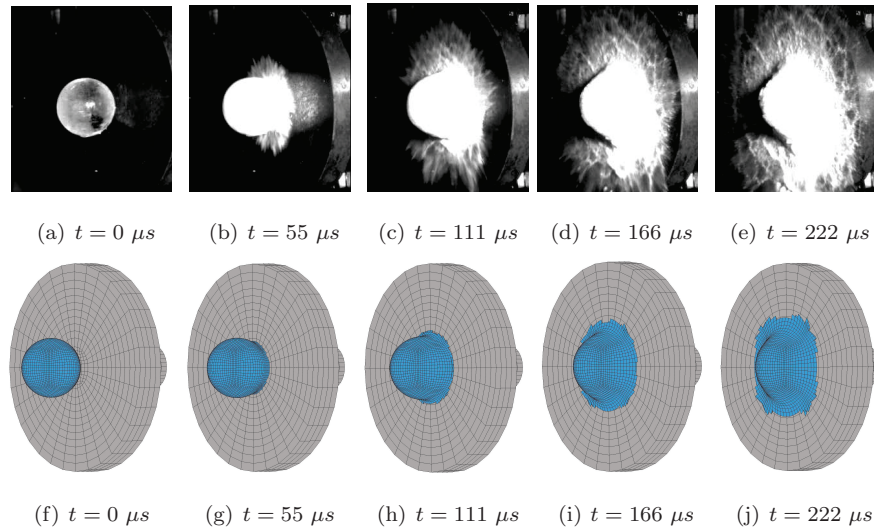


Figure 8: Comparison between numerical and experimental images of a 40 mm ice diameter projectile impacting at 115 m/s.

402

403 According to the observed result it is possible to state that the strain suf-
 404 fered by the ice during the impact is small before the aforementioned failure
 405 variable is activated, therefore it can be concluded that defining properly the
 406 elastic regime of the ice model becomes capital to obtain good numerical results.
 407 The overall predicted behaviour of the ice is in accordance with the statement
 408 of other authors that propose an embrittlement with the strain rate of the ice
 409 [37, 43, 44]. Taking into account the results obtained it is possible to state that
 410 the proposed ice model, based on a Drucker-Prager criterion with strain rate
 411 dependence, predicts faithfully this region of the failure process.

412

413 According to the aforementioned results, it can be concluded that the ice
 414 material model is validated, not only for ice cylinders [39] but, also for ice
 415 sphere impacts. In addition, the numerical results show the capability of the
 416 lagrangian approach to reproduce the impact phenomena. The validation per-
 417 formed enable the use of the model proposed to analyse more complex problems.

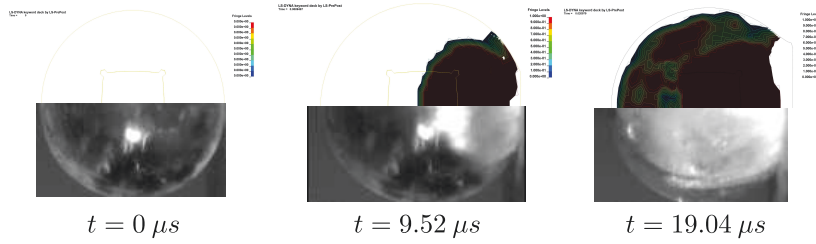


Figure 9: Failure development during the impact experimental and numerical comparison.

418

419 4.2. Ice sphere impacts on CFRP panel

420 Once the ice model is validated, it can be used to validate and study ice
 421 impacts on composite panels. The goal of this second group of numerical simu-
 422 lations is to demonstrate that the numerical model developed is able to capture
 423 the appearance of the damage in composite laminates subjected to ice impact
 424 and that it can be a useful tool to analyse and design structures under this kind
 425 of events.

426

427 Prior to study numerically the damages induced in the laminates by the ice
 428 impact, the overall behaviour of the plates is studied by means of the strain
 429 gauges measurements. Fig. 10 shows the strain experimentally measured and
 430 predicted by the numerical simulations, for a 21 plies laminate impacted by a 40
 431 mm ice sphere at 157 m/s. At the beginning of the impact, the ice promotes a
 432 compression wave followed by a plate bending; this behaviour can be measured
 433 by the strain gauges. The numerical simulations reproduce the overall trends
 434 and the values measured. It is observed that the magnitude of the compres-
 435 sion wave is underestimated, probably because of the use of a continuum shell
 436 approach. However, the tension magnitude and decay as well as the intervals
 437 of change between compression and tension are well captured by the numerical
 438 model proposed.

439

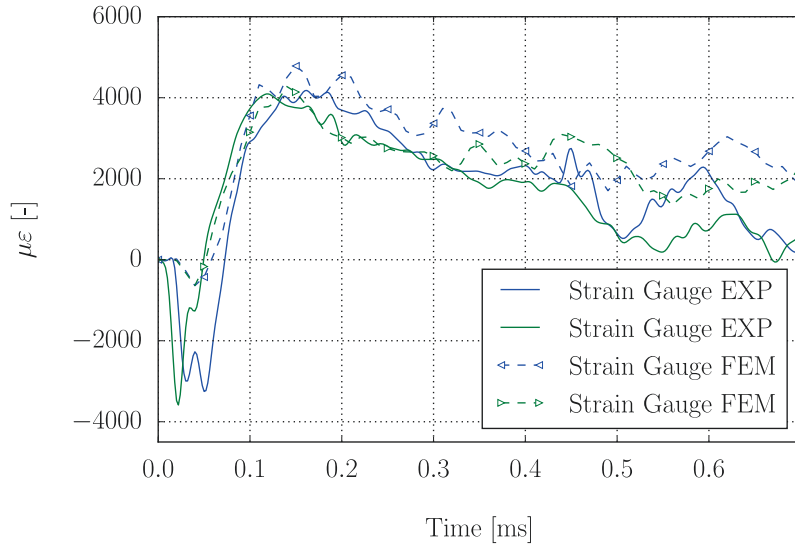


Figure 10: Experimental and numerical measurements of the strain gauges for an impact of 40 mm diameter ice sphere against 21 plies laminate at 157 m/s

440 In order to verify if the composite material model is reproducing accordingly
 441 the behaviour of the impacted laminate panel, the different kind of failures that
 442 may appear are studied, comparing the numerical and experimental results.
 443 The model implemented is able to capture intralaminar damages, such as fiber
 444 breakage or matrix cracking, and interlaminar failures (delaminations) through
 445 the presence of cohesive interactions. The intralaminar damages barely can be
 446 observed, making difficult the comparison between numerical and experimental
 447 results: in the simulations hardly progress from the impact point, and exper-
 448 imentally only can be visualized in the cases impacted at high kinetic energy.
 449 Nevertheless, the interlaminar damage is spread out over the laminate from
 450 the impact point and can be perfectly quantified. Figs. 11 and 12 show the
 451 experimental (C-Scan) and numerical results of delamination in two laminates
 452 impacted at different velocities; the areas colored in red denote interlaminar
 453 damages. Qualitatively comparing the results, it can be observed that in both
 454 cases the numerical model predicts adequately the extension and shape of the

455 delamination. At low impact velocity (Fig. 11) the delamination spreads around
 456 the impact zone, which is well reproduced by the numerical model although it
 457 has to be said that, in this case, the numerical results underestimate slightly
 458 the delaminated area. When the panel is impacted at higher velocity (215 m/s)
 459 (Fig. 12) the interlaminar damage spreads out from the impact point over al-
 460 most the whole free area, only the zones near the clamped boundaries do not
 461 show delamination; this phenomenon is well reproduced by the numerical model.
 462

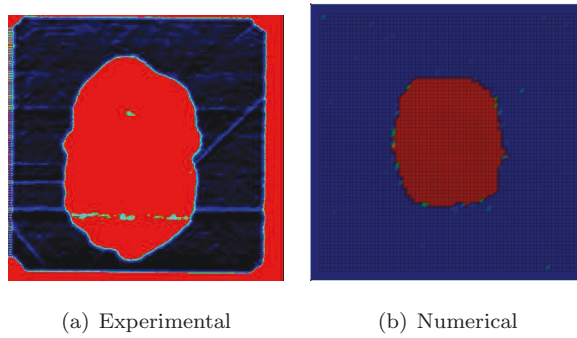


Figure 11: Experimental and numerical delamination for an impact of 40 mm ice diameter impacting against a 21 plies laminate at 157 m/s . (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

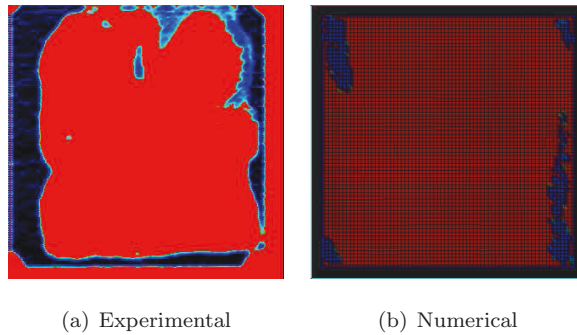


Figure 12: Experimental and numerical delamination for an impact of 40 mm ice diameter impacting against a 21 plies laminate at 215 m/s . (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

463 In order to compare quantitatively the interlaminar failure, Fig. 13 de-

464 picts the percentage of delaminated area versus the impact velocity for all the
465 ice sphere diameters (40 and 50 *mm*) and laminate thicknesses (4 and 6 *mm*)
466 tested. In all cases it is observed a sudden transition between zero delamina-
467 tion to full delamination, which can be considered when the delaminated area
468 is greater than 75% (red lines in Fig. 13). This transition occurs at a different
469 impact velocity, which will be called critical velocity from now on, for each ice
470 sphere diameter-laminate thickness combination. It can be seen how the nu-
471 merical model is able to capture this behaviour for each combination, although
472 the experimental results show a sharper transition than the numerical results.
473 The influence of the ice sphere diameter and panel thickness, in the critical ve-
474 locity, is also well reproduced by the numerical model. It is observed that, for
475 the same panel thickness, it is necessary a higher impact velocity to reach full
476 delamination with a smaller ice diameter. For a given ice diameter, the impact
477 velocity that produces the full delamination is higher on the thicker panel.

478

479 Taking into account the validation and the study carried out for different
480 kind of ice impacts, it can be said that the numerical method proposed is ca-
481 pable of reproducing faithfully the behaviour of the ice subjected to an impact,
482 and the effects that it can produce onto a “rigid” and flexible target for different
483 ice diameters and panel thickness. In addition, it has been demonstrated that
484 the numerical model can predict the delaminations caused by an ice impact on
485 a composite panel, being a useful tool to analyse this kind of events.

486

487 Once validated, the numerical methodology proposed could be used as a de-
488 sign tool previous to the manufacturing phase, or even to analyse some aspects
489 of these impact events that experimentally could be very difficult, or unable to
490 perform. Fig. 14 shows numerical results of the delamination and displacement
491 evolution in a thin laminate (4 *mm*) impacted by an ice sphere of 40 *mm* at
492 175 *m/s*. The displacement of the laminate centre increases suddenly at the
493 beginning of the impact, as delamination does. Then, the displacement reaches
494 a small plateau, while delamination continues growing, followed by the maxi-

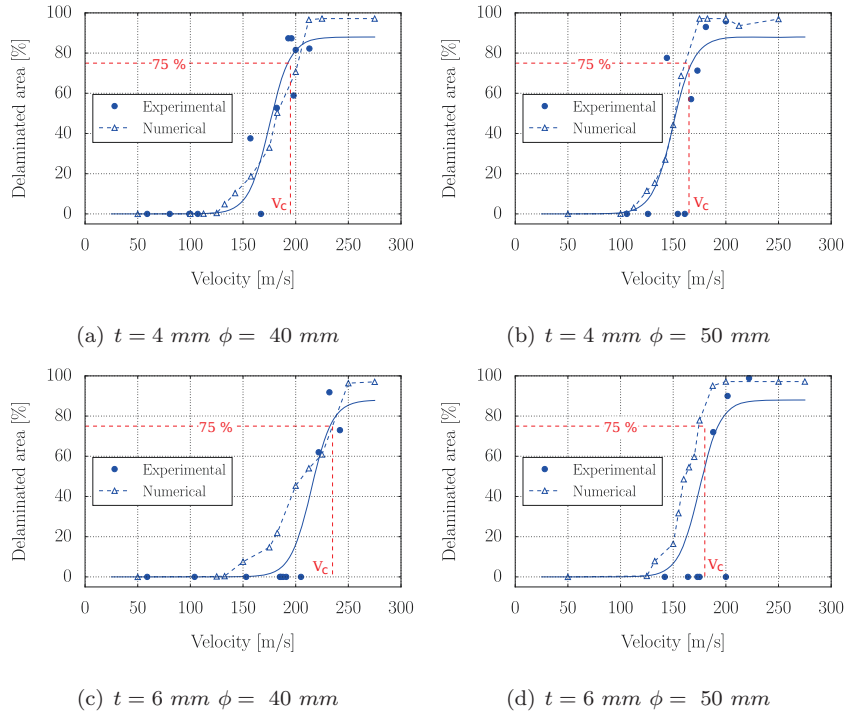


Figure 13: Experimental and numerical percentage of delaminated area against impact velocity for the two plate thickness and different projectile diameter.

495 mum displacement. It is reasonable to think that the peak force induced by
 496 the ice impact is reached at that instant. From that point, the displacement
 497 diminishes while delamination continues spreading out slowly. Therefore it can
 498 be said that the timing of delamination and displacement are directly related
 499 and that the main part of the delaminated area is produced at the beginning of
 500 the impact.

501

502 5. Conclusions

503 In this work a numerical methodology has been proposed to predict the be-
 504 havior and threat of ice sphere projectiles impacting at high velocity. To this
 505 end a two step validation approach has been performed. Firstly, simulations of

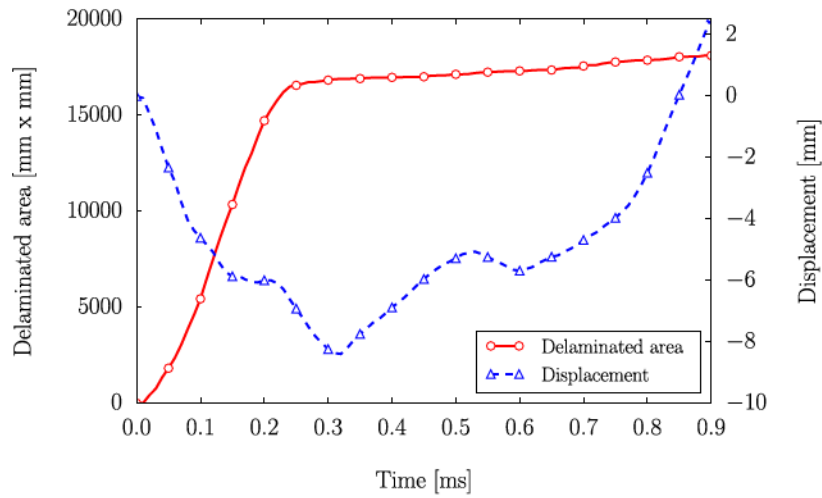


Figure 14: Center panel displacement and delamination evolution for an impact of 40 mm ice diameter impacting against a 21 plies laminate at 175 m/s .

506 high velocity impacts have been carried out in a wide range of impact velocities,
 507 for different ice diameters against a steel plate with a cell load, measuring and
 508 comparing the impact force produced by the ice spheres. Moreover, the pro-
 509 gression of failure inside the ice was also validated. Once the material model
 510 has been proved reliable, the ice model has been used to simulate an impact
 511 event on carbon/epoxy laminates. To this end, different ice diameters have
 512 been impacted against carbon/epoxy laminates, studying the damages induced
 513 and comparing it with the experimental results. The main conclusions are as
 514 follows:

- 515 • The ice model implemented reproduces faithfully the behaviour of the ice
 516 subjected to an impact, and the effects that it can produce, for multiple
 517 conditions and test set-ups (“rigid” and flexible target). The input to the
 518 model is based on clearly defined mechanical properties.
- 519 • Regarding the numerical modeling of the laminate panel; the strategy of
 520 clustering the plies, reducing the interfaces in which delamination may
 521 appear, proves to be feasible to reproduce the delaminated area.

- 522 • The numerical model is able to reproduce the behaviour of composite pan-
523 els impacted by an ice sphere, for all the combinations ice spheres-laminate
524 thickness studied. The numerical model provides, faithfully, the critical
525 velocity in which the laminate suffers an intense interlaminar failure.
- 526 • The validated numerical model has allowed to study the displacement
527 and delamination evolution. The results show a direct relation between
528 them and that the main part of the delaminated area is produced at the
529 beginning of the impact.
- 530 • It has been proved that the numerical methodology proposed can be a
531 useful tool to study or design composite structures that may be subjected
532 to this kind of impacts.

533 Acknowledgements

534 This research was done with the financial support of the Spanish Ministry of
535 Economy and Competitiveness under Project reference DPI2013-41094-R, and
536 the Vicerrectorado de Política Científica UC3M (Projects 2014/00006/002 and
537 2013/00413/002).

538 References

- 539 [1] J. A. Toso N, LIBCOS-Load upon impact behaviour of composite Struc-
540 ture Research Project EASA.2009/3, Tech. rep., European Aviation Safety
541 Agency (2011).
- 542 [2] W. Cantwell, J. Morton, Detection of impact damage in CFRP laminates,
543 Compos. Struct. 8223 (3) (1985) 241–257.
- 544 [3] W. Cantwell, J. Morton, Geometrical effects in the low velocity impact
545 response of CFRP, Compos. Struct. 8223 (89).
- 546 [4] W. Cantwell, J. Morton, Comparison of the low and high velocity impact
547 response of cfrp, Composites 20 (1989) 545–51.

- 548 [5] J. López-Puente, R. Zaera, C. Navarro, Experimental and numerical anal-
549 ysis of normal and oblique ballistic impacts on thin carbon/epoxy woven
550 laminates, *Composites Part A* 39 (2) (2008) 374–387.
- 551 [6] A. F. Johnson, A. K. Pickett, P. Rozycki, Computational methods for pre-
552 dicting impact damage in composite structures., *Compos. Sci. Technol.* 61
553 (2001) 2183–92.
- 554 [7] J. Pernas-Sánchez, J. Artero-Guerrero, J.Zahr Viñuela, D. Varas, J. López-
555 Puente, Numerical analysis of high velocity impacts on unidirectional lam-
556 inates, *Compos. Struct.* 107 (2014) 629 – 634.
- 557 [8] V. Lopresto, V. Melito, C. Leone, G. Caprino, Effect of stitches on the im-
558 pact behaviour of graphite/epoxy composites, *Compos. Sci. Technol.* 66 (2)
559 (2006) 206 – 214.
- 560 [9] J. López-Puente, R. Zaera, C. Navarro, The effect of low temperatures on
561 the intermediate and high velocity impact response of CFRPs., *Composites*
562 *Part B* 33 (2002) 559–66.
- 563 [10] W. B., J. Xiong, X. Wang, L. Ma, G. Zhang, L. Wu, J. Feng, Energy
564 absorption efficiency of carbon fiber reinforced polymer laminates under
565 high velocity impact, *Materials & Design* 50 (Supplement C) (2013) 140 –
566 148. doi:<https://doi.org/10.1016/j.matdes.2013.01.046>.
- 567 [11] Q. Ang, F. Kunkun, L. Wei, Z. Chengbi, T. Youhong, Mod-
568 elling low-speed drop-weight impact on composite laminates,
569 *Materials & Design* 60 (Supplement C) (2014) 520 – 531.
570 doi:<https://doi.org/10.1016/j.matdes.2014.04.041>.
- 571 [12] Shaktivesh, N. Nair, C. Sessa Kumar, N. Naik, Ballistic impact perfor-
572 mance of composite targets, *Materials & Design* 51 (Supplement C) (2013)
573 833 – 846. doi:<https://doi.org/10.1016/j.matdes.2013.04.093>.
- 574 [13] J. Wilbeck, Impact behavior of low strength projectiles, *Tech. Rep. AFML-*
575 *TR-77-134*, Air Force Materials Laboratory (1978).

- 576 [14] L. Nizampatnam, Models and methods for bird strike load predictions,
577 Ph.D. thesis, Wichita State University (2007).
- 578 [15] T. Baughn, L. Graham, Simulation of a birdstrike impact on aircraft canopy
579 material, *Journal of Aircraft* 25 (1988) 659–664.
- 580 [16] D. Karagiozova, R. Mines, Impact of aircraft rubber tyre fragments on
581 aluminium alloy plates: II - Numerical simulation using LS-DYNA, *Int. J.*
582 *Impact Eng.* 34 (2007) 647–667.
- 583 [17] R. Mines, S. McKown, R. Birch, Impact of aircraft rubber tyre fragments
584 on aluminium alloy plates: I-experimental, *Int. J. Impact Eng.* 34 (2007)
585 627–646.
- 586 [18] P. Guégan, R. Othman, D. LeBreton, F. Pasco, N. Swiergiel, P. Thevenet,
587 Experimental investigation of rubber ball impacts on aluminium plates,
588 *Int. J. Crashworthiness* 15 (2010) 391–399.
- 589 [19] H. Kim, J. N. Keune, Compressive strength of ice at impact strain rates,
590 *Journal of Materials Science (full set) September 2007* (2007) 2802–2806.
- 591 [20] J. Pernas-Sánchez, J. A. Artero-Guerrero, D. Varas, López-Puente, Ex-
592 perimental analysis of ice sphere impacts on unidirectional carbon/epoxy
593 laminates, *Int. J. Impact Eng.* 96 (2016) 1 – 10.
- 594 [21] M. Anghileri, F. Invernizzi, M. Mascheroni, A survey of numerical models
595 for hail impact analysis using explicit finite element codes, *Int. J. Impact*
596 *Eng.* 31 (2005) 929–944.
- 597 [22] H. Kim, D. A. Welch, K. T. Kedward, Experimental investigation of high
598 velocity ice impacts on woven carbon/epoxy composite panels, *Composites*
599 *Part A* 34 (2003) 25–41.
- 600 [23] J. Pernas-Sánchez, J. A. Artero-Guerrero, D. Varas, J. López-Puente, Anal-
601 ysis of ice impact process at high velocity, *Exp. Mech.* 55 (9) (2015) 1669–
602 1679.

- 603 [24] J. Tippmann, H. Kim, J. Rhymer, Experimentally validated strain rate de-
604 pendent material model for spherical ice impact simulation, *Int. J. Impact*
605 *Eng.* 57 (2013) 43–54.
- 606 [25] J. M. Pereira, S. A. Padula, D. M. Revilock, M. E. Melis, Forces Generated
607 by High Velocity Impact of Ice on a Rigid Structure, Tech. Rep. July, NASA
608 (2006).
- 609 [26] A. Combescure, Y. Chuzel-Marmot, J. Fabis, Experimental study of high-
610 velocity impact and fracture of ice, *Int. J. Solids Struct.* 48 (20) (2011)
611 2779–2790.
- 612 [27] G. J. Appleby-Thomas, P. J. Hazell, G. Dahini, On the response of two
613 commercially-important CFRP structures to multiple ice impacts, *Compos.*
614 *Struct.* 93 (10) (2011) 2619–2627.
- 615 [28] L. Asp, R. Juntikka, High velocity impact on NCF reinforced composites,
616 *Compos. Sci. Technol.* 43 (2009) 1478–82.
- 617 [29] J. Rhymer, H. Kim, D. Roach, The damage resistance of quasi-isotropic car-
618 bon/epoxy composite tape laminates impacted by high velocity ice, *Com-*
619 *posites Part A* 43 (7) (2012) 1134–1144.
- 620 [30] H. Kim, K. T. Kedward, Modeling Hail Ice Impacts and Predicting Impact
621 Damage Initiation in Composite Structures, *AIAA Journal* 38 (7) (2000)
622 1278–1288.
- 623 [31] K. Carney, D. Benson, P. Dubois, R. Lee, A phenomenological high strain
624 rate model with failure for ice, *Int. J. Solids Struct.* 43 (25-26) (2006) 7820–
625 7839.
- 626 [32] Y. Chuzel, Caractérisation expérimentale et simulation numérique d im-
627 pacts de glace a haute vitesse., phdthesis (2009).
- 628 [33] J. Mazars, G. Pijaudier Cabot, Continuum damage theory application to
629 concrete, *Journal of Engineering Mechanics* 115 (2) (1989) 345–365.

- 630 [34] L. S. TECHNOLOGY, LS-Dyna KEYWORD USER ' S MANUAL v.971,
631 Livermore, California, rev 5 Edition (2010).
- 632 [35] D. C. Drucker, W. Prager, Soil mechanics and plastic analysis or limit
633 design, Q J Applied Mathematics X (2) (1952) 157–165.
- 634 [36] M. Shazly, V. Prakash, B. A. Lerch, High strain-rate behavior of ice under
635 uniaxial compression, Int. J. Solids Struct. 46 (6) (2009) 1499–1515.
- 636 [37] E. Schulson, Brittle failure of ice, Eng. Fract. Mech. 68 (2001) 1839–87.
- 637 [38] S. J. Jones, High Strain-Rate Compression Tests on Ice, The Journal of
638 Physical Chemistry B 101 (32) (1997) 6099–6101.
- 639 [39] J. Pernas-Sánchez, D. Pedroche, D. Varas, J. López-Puente, R. Zaera, Nu-
640 merical modeling of ice behavior under high velocity impacts, Int. J. Solids
641 Struct. 49 (14) (2012) 1919–1927.
- 642 [40] D. Varas, J. Artero-Guerrero, J. Pernas-Sánchez, J. López-Puente, Analysis
643 of high velocity impacts of steel cylinders on thin carbon/epoxy woven
644 laminates, Compos. Struct. 95 (2013) 623–629.
- 645 [41] F. K. Chang, K. Y. Chang, A Progressive Damage Model for Laminated
646 Composites Containing Stress Concentrations, J. Compos. Mater. 21 (834).
- 647 [42] H. Hahn, S. Tsai, On the Behavior of Composite Laminates After Initial
648 Failures, J. Compos. Mater. 8 (3) (1974) 288–305.
- 649 [43] R. Juntikka, R. Olsson, Experimental and modelling study of hail impact on
650 composite plates, in: 17th INTERNATIONAL COMMITTEE ON COM-
651 POSITE MATERIALS, 2009.
- 652 [44] E. Schulson, The brittle compressive fracture of ice, Acta Metall. Mater.
653 38 (10) (1990) 1963 – 1976.