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Simulations on Distribution of Phase Retardation through Liquid Crystal Lenses

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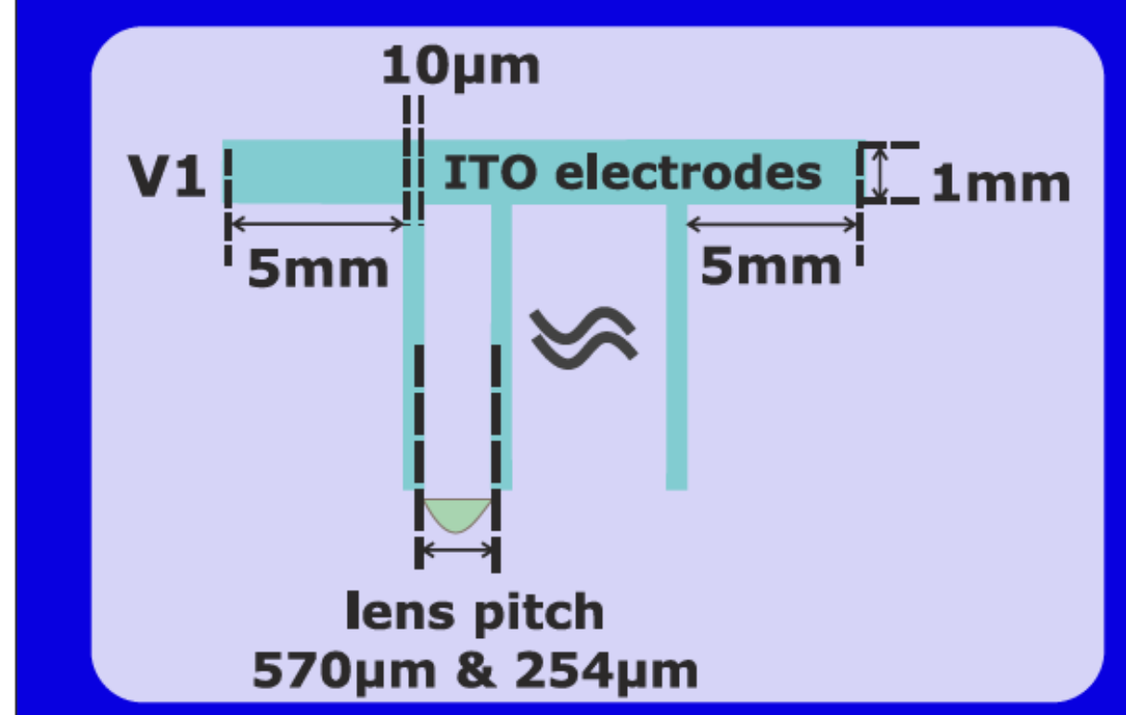
ABSTRACT

For this research work, a simulation program for simulate liquid crystal lens electrooptical behaviour, based on Frank-Oseen's free energy formulation, has been developed and the results have been compared to previous experimental data. This simulation program has been done using MATLAB software and can be used for reproducing different kind of tunable liquid crystal lenses, according to their specific technology, modal lenses, or lenses with hole pattern electrodes, among others.

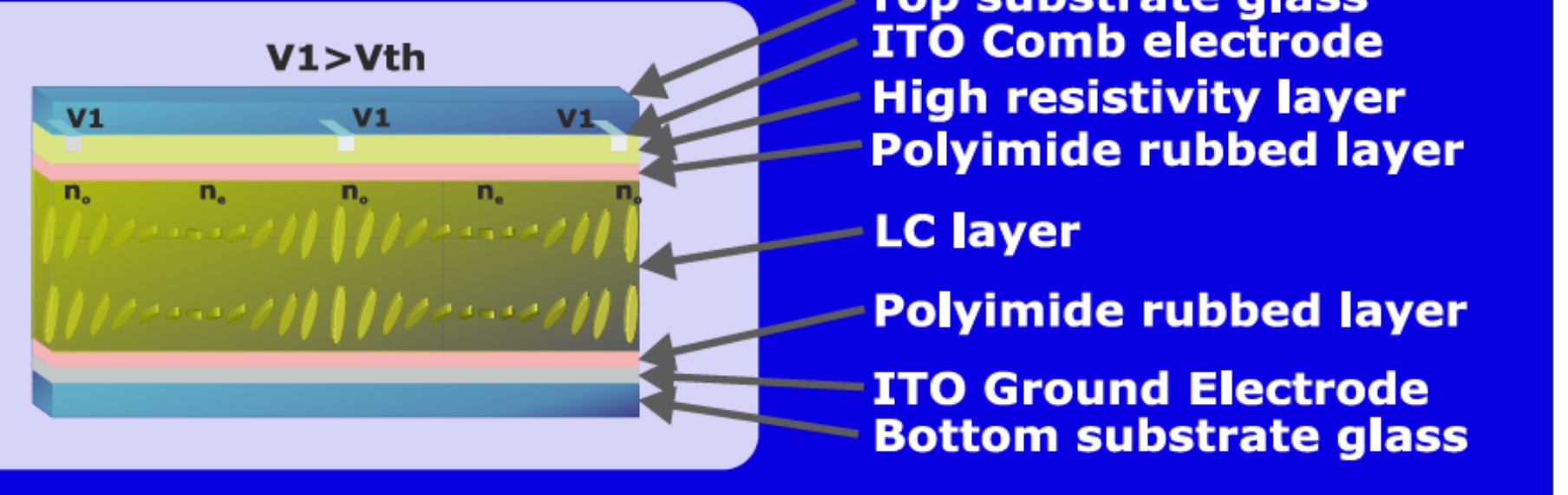
The electrical field in the liquid crystal surface has been estimated by Partial Differential Equations (PDE). By knowing director angles, the application shows the liquid crystal molecular distribution inside the cell, in 2D images of transversal sections of the sandwich from the top to the bottom electrodes. Then, the effective refractive index and the phase retardation profiles are calculated

3 LC LENSES EXPERIMENTAL & SIMULATED RESPONSE

DESIGN

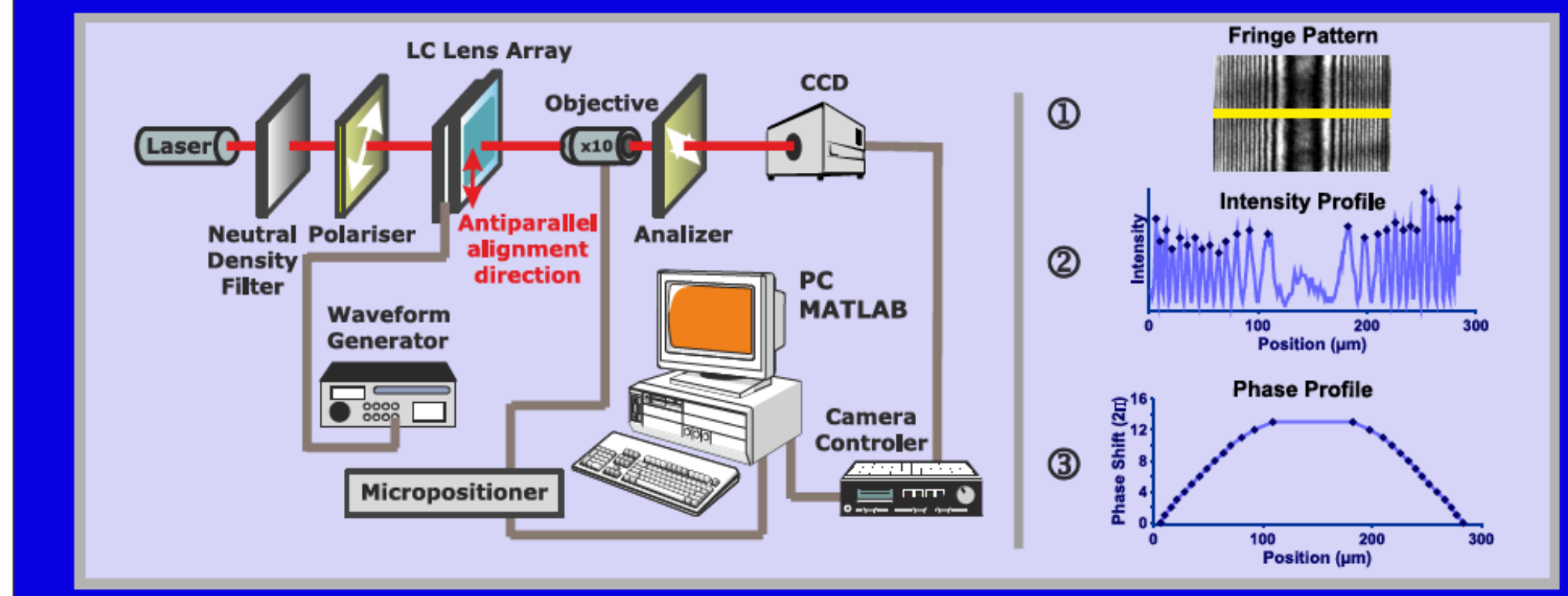


ARRAY CELLS
Resistivity Layer: 2-3 GΩ/sq
NLC: MDA-98-1602
Birefringence: 0.266
Dielectric Anisotropy: 12
K1: 13.6 pN
K2: 30.9 pN
K3: 15.7 pN



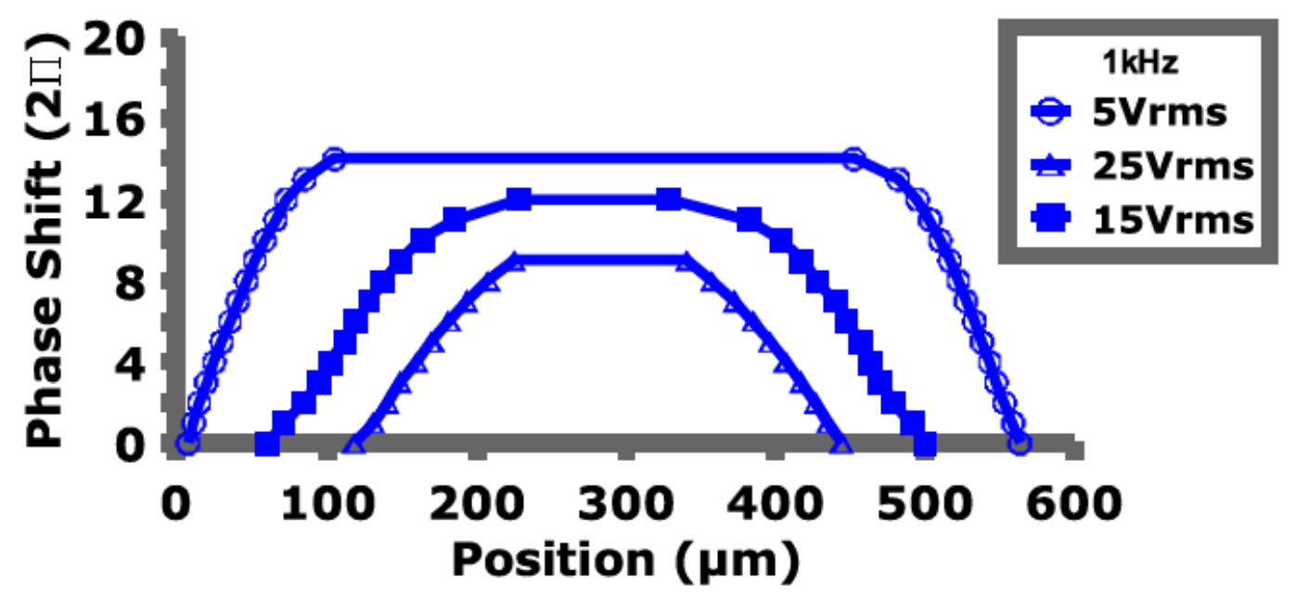
- **Patterns consisted on one-dimensional arrays of cylindrical lenses.**
- **Patterns were designed with comb interdigitated electrodes.**
- **A High resistivity layer is deposited between electrodes.**

EXPERIMENTAL RESULTS

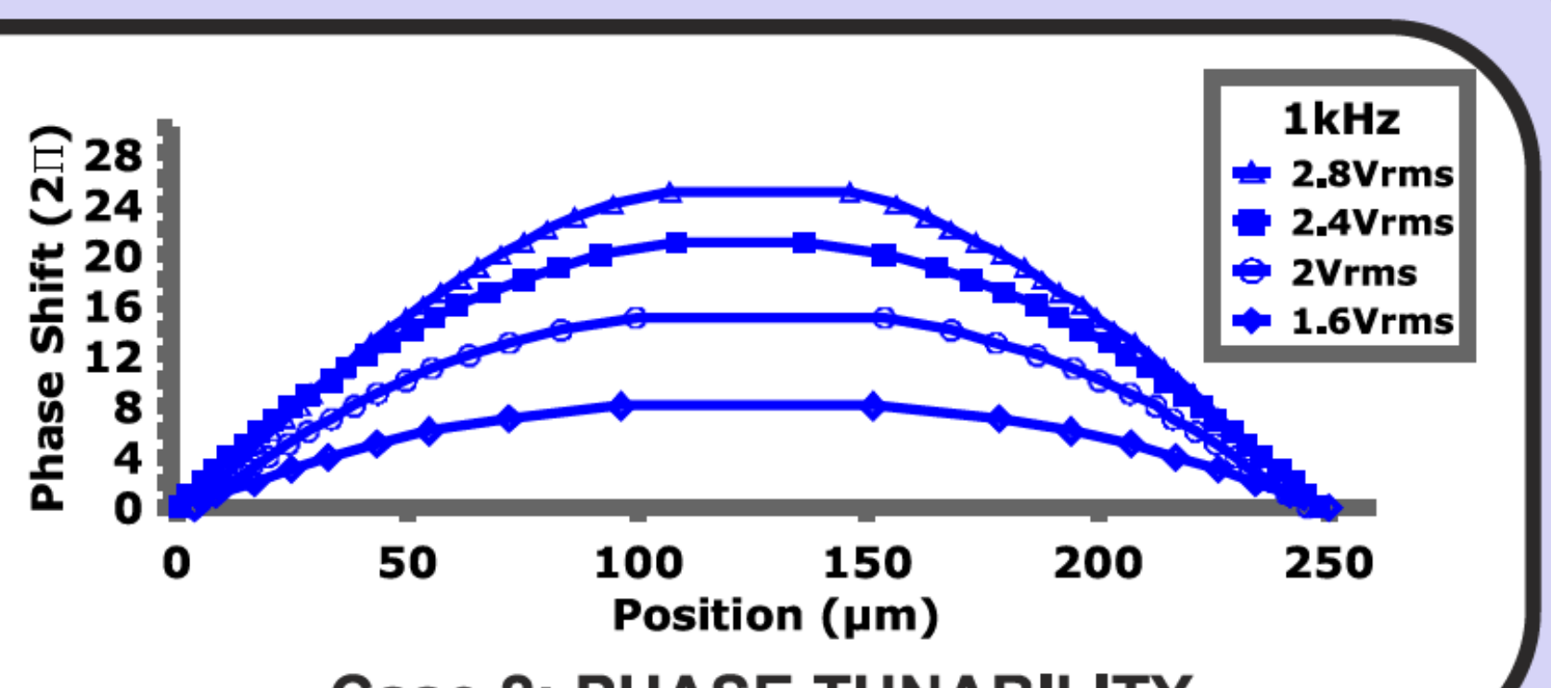


- **Nematic liquid crystal lenses have been driven with different AC square signals at 1kHz.**
- **Lenticular arrays have been placed between crossed polarizers with the alignment direction at p/4 from the linear polarization at the input. Interference fringes have given the *phase retardation*.**

Case 1:
Cell gap: 50µm
Surface area: 6.64cm²
Lens Pitch: 570µm
Case 2:
Cell gap: 120µm
Surface area: 3.18cm²
Lens Pitch: 254µm



Case 1: PHASE DEFECTS

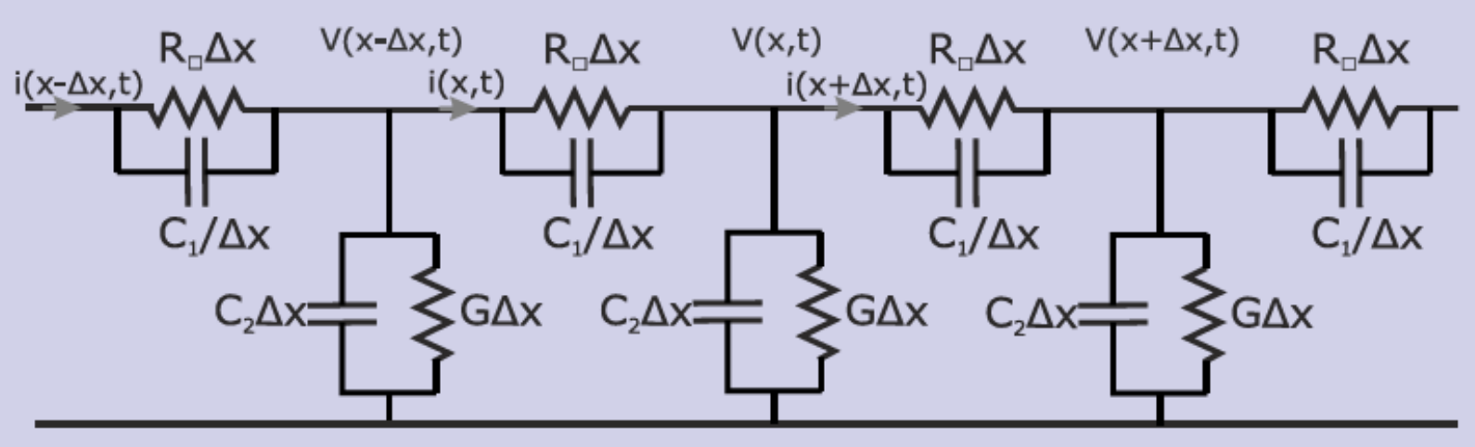


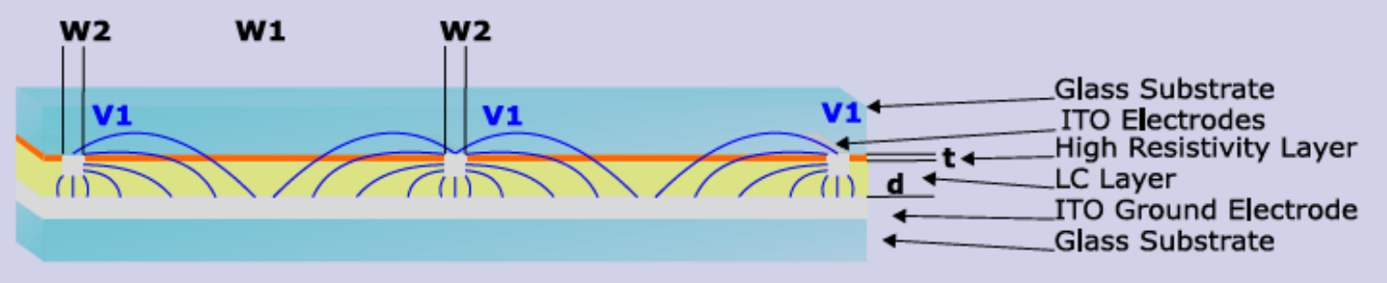
Case 2: PHASE TUNABILITY

- **Too high resistivity has given rise to a different effects. (Case 1)**
- **If the diameter and the thickness has a specific relation the LC produces the voltage gradient. (Case 2)**

SIMULATED RESULTS

ELECTRICAL MODEL

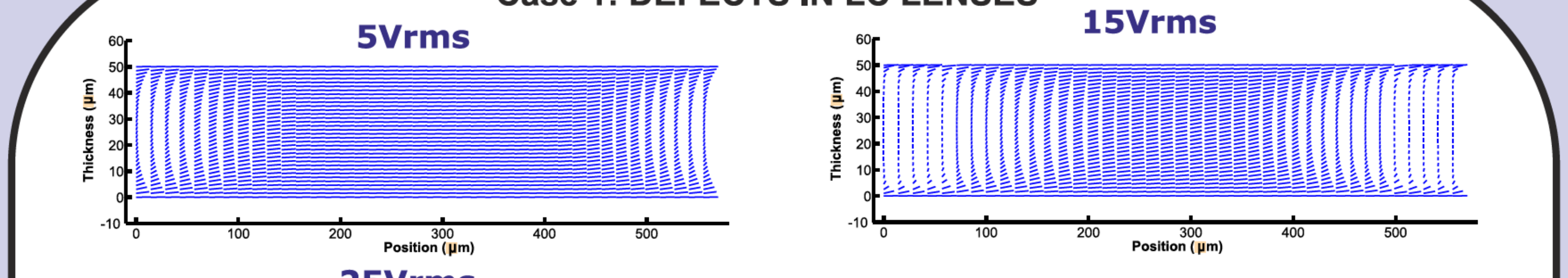


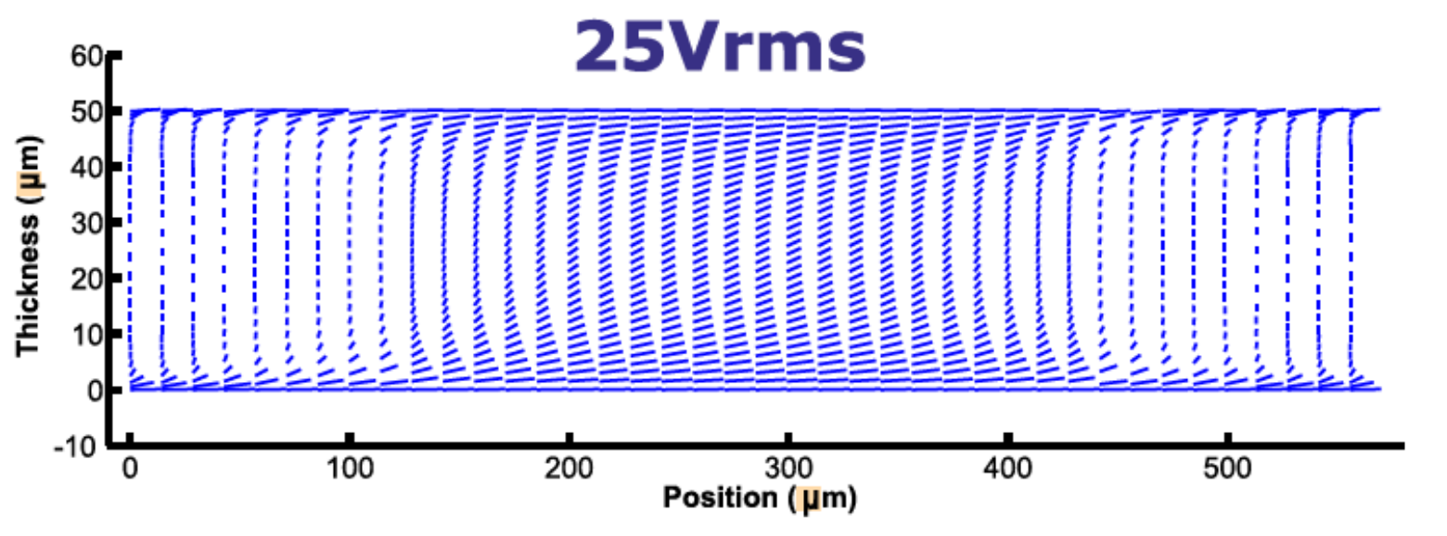


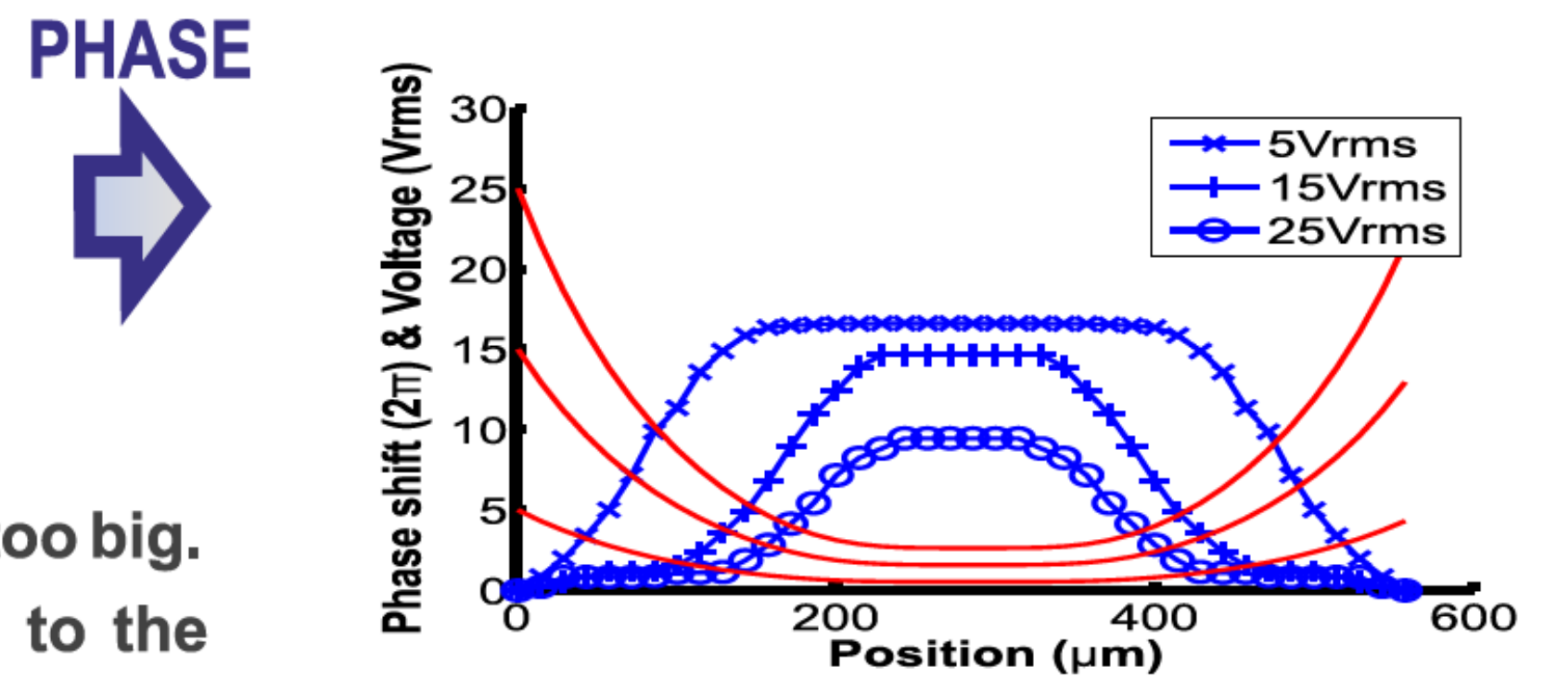
$$\frac{\partial V^2(x)}{\partial x^2} = \frac{\omega i C_2 + G}{\frac{1}{R_\square} + \omega i C_1} \cdot V(x)$$

- Case 1: If Z_{C1} is bigger than high resistivity layer, modal lens.
- Case 2: If Z_{C1} is smaller than the high resistivity layer, hole patterned lens.

Case 1: DEFECTS IN LC LENSES

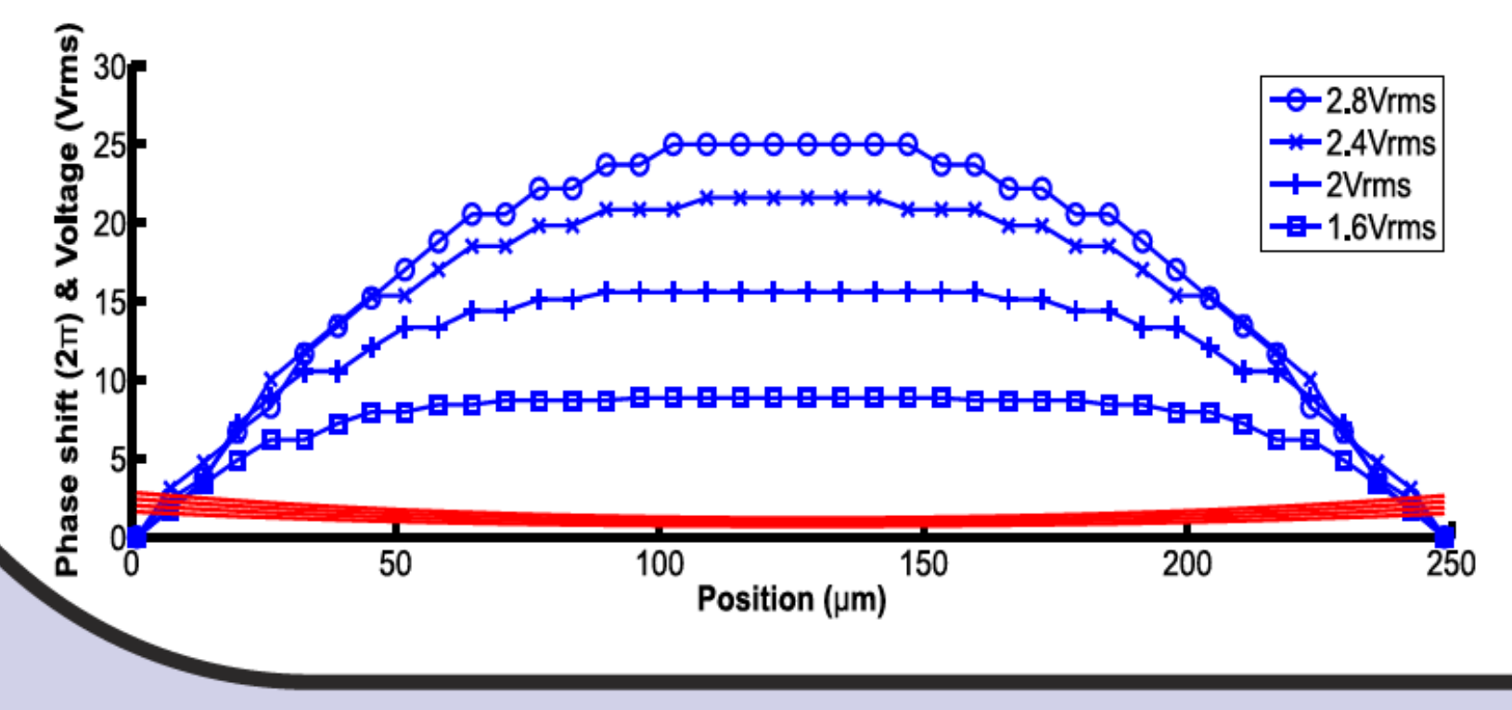


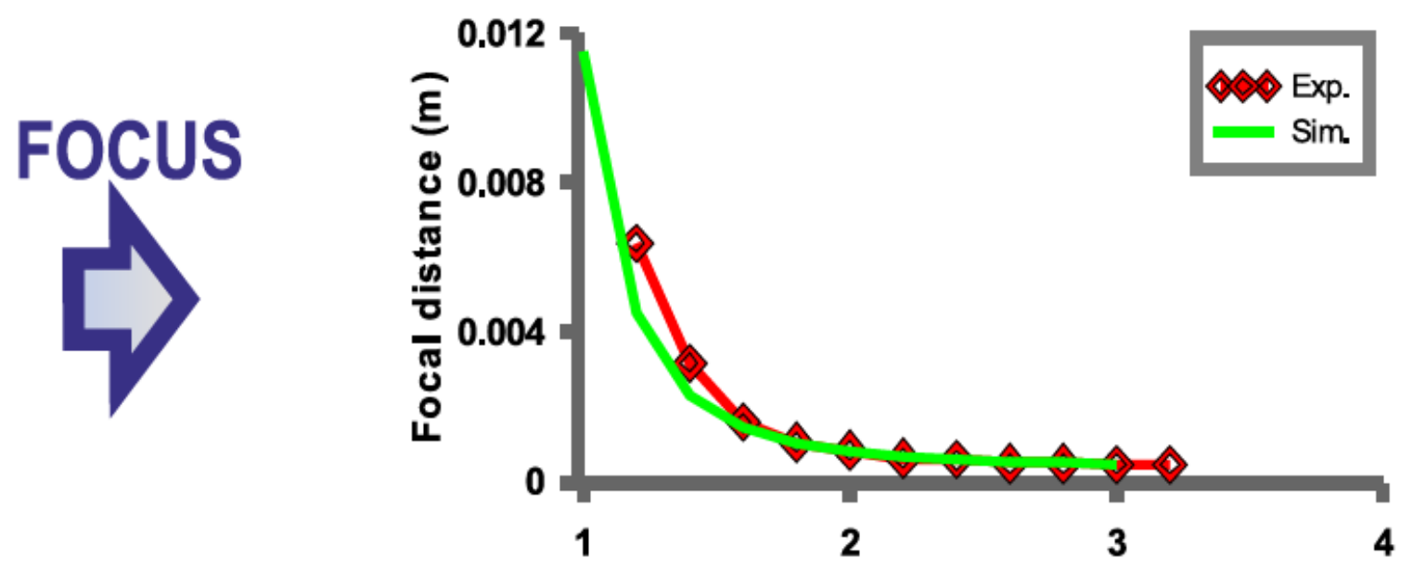




- In case 1 the diameter and the resistivity layer are too big.
- In case 2 the resistivity layer doesn't affect due to the small diameter and high thickness.

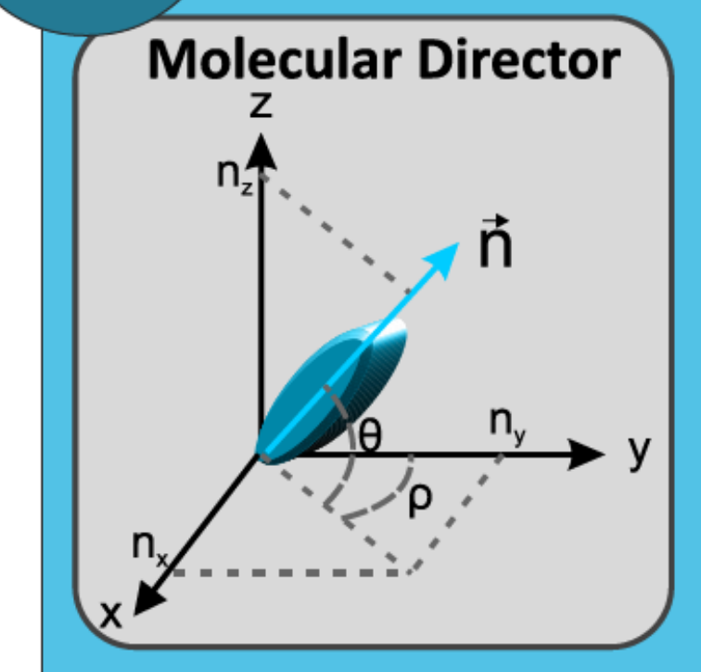
Case 2: TUNABILITY IN LC LENSES





1 SOLVING CONTINUUM THEORY

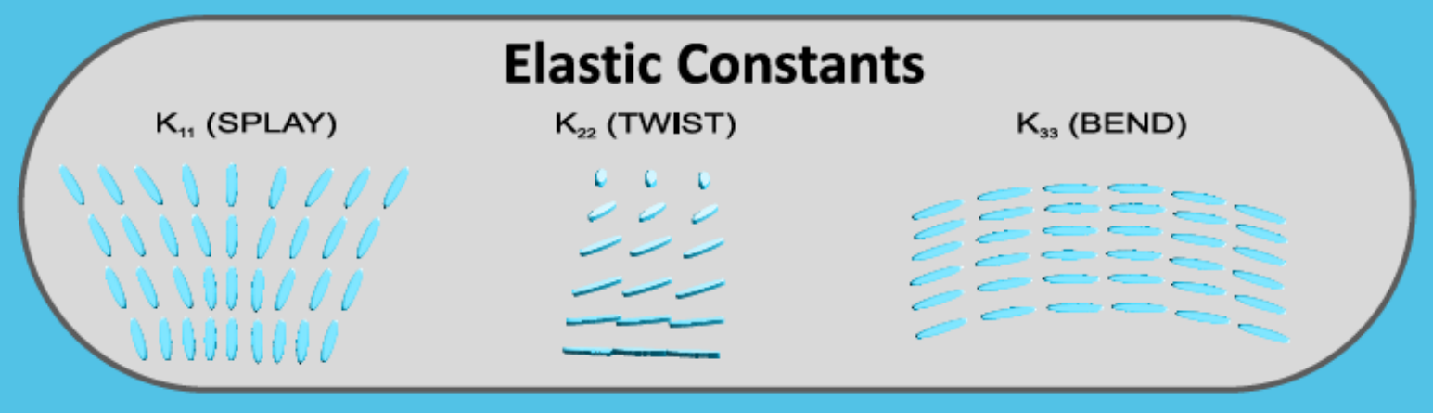
Molecular Director



Gibbs Energy (Frank Oseen - Electrical Force) → **Solutions to Euler-Lagrange Equations**

$$\frac{1}{2}\int_\Omega\left\{K_{11}\left(\nabla\vec{n}\right)^2+K_{22}\left(\vec{n}\cdot\nabla\times\vec{n}+\frac{2\pi}{\xi}\right)^2+K_{33}\left|\vec{n}\times\nabla\times\vec{n}\right|^2-\varepsilon_0\left(\Delta\varepsilon(\vec{n}\cdot\vec{E})^2+\varepsilon_\perp\cdot\vec{E}\cdot\vec{E}\right)\right\}$$

Elastic Constants



$$\frac{\partial F_G}{\partial\theta}-\frac{d}{dz}\left(\frac{\partial F_G}{\partial\theta'}\right)=0$$

$$\frac{\partial F_G}{\partial\varphi}-\frac{d}{dz}\left(\frac{\partial F_G}{\partial\varphi'}\right)=0$$

Euler-Lagrange Equations: Maple functions

```
Q := remove(has, EulerLagrange(F_g, z, [theta(z), phi(z)]), K)
(Thetadif, Phidif) := selectremove(has, Q, diff(diff(theta(z), z), z))
```

Extract Second Derivatives: Maple functions

```
> solve(Thetadif, diff(diff(theta(z), z), z))
> solve(Phidif, diff(diff(phi(z), z), z))
> solve(divergence(e0·ez·E, [z]), diff(diff(V(z), z), z))
```

- Polar coordinates makes solving it easier, but despite this, analytically, is still very tedious. With a few statements Maple software finds the Euler-Lagrange solutions and gives the correct terms.
- For Shooting Method we need the second derivative functions of each parameter (θ´´,φ´´,V´´).

SHOOTING METHOD

BVP (Boundary Value Problem)

- As the voltage at electrodes is on the top and bottom of the cell, Continuum Theory is a BVP. For example with voltage parameter:

$$y''=f(x,y,y'),\quad y(a)=V_1,\quad y(b)=V_2$$

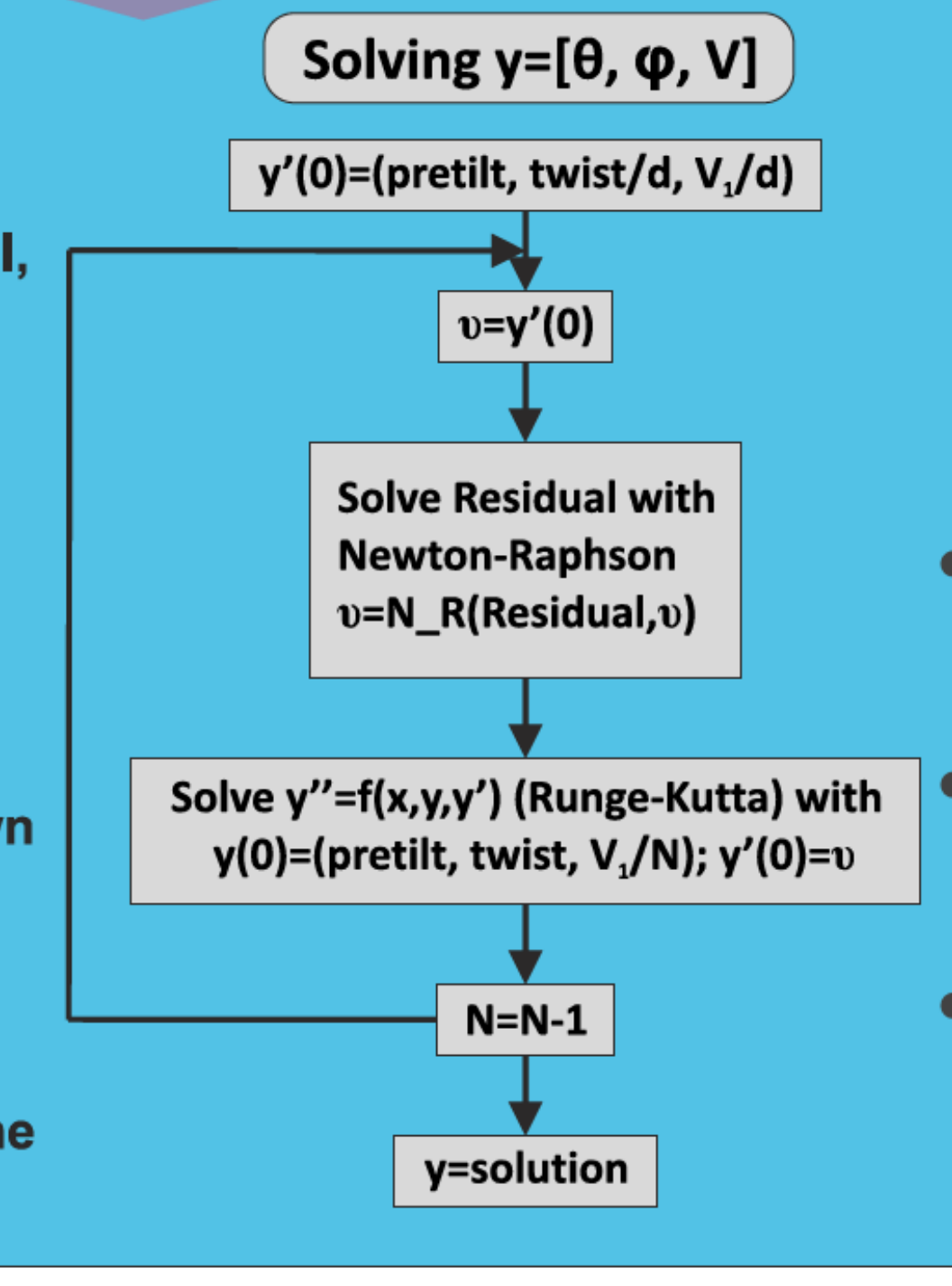
- This can be converted to a classic "initial value problem".

$$y''=f(x,y,y'),\quad y(a)=V_1,\quad y'(a)=\upsilon$$

- The key to success is finding the correct value of the unknown condition υ.

$$r(\upsilon)=g(\upsilon)-V_2=0$$

- Where g(υ) is the solution to f(x,y,y') with υ as initial condition.
- If the BVP has a solution, then r has a root, and that root is just the value of y'(a)=υ which yields a solution y(t) of the BVP.



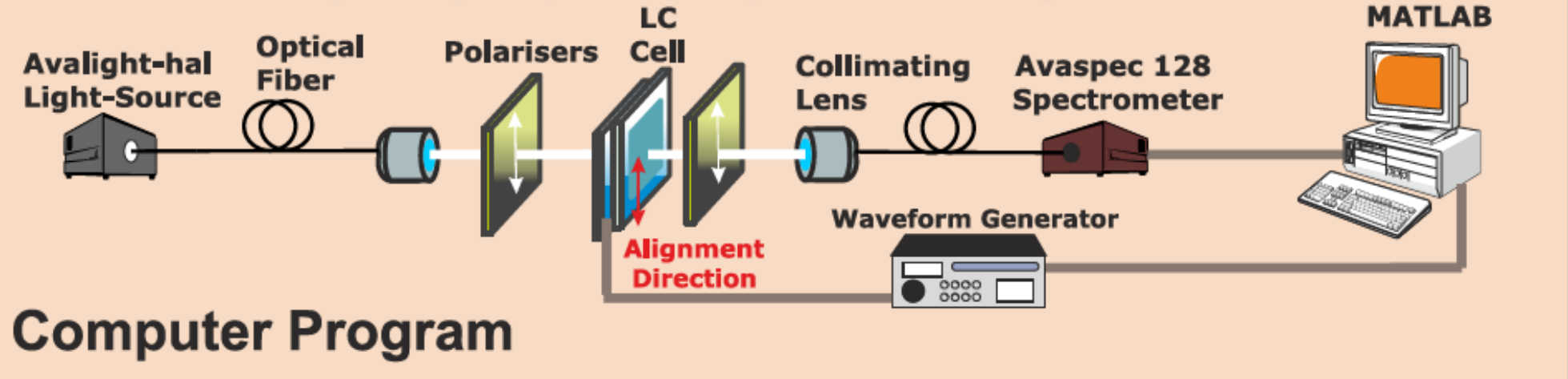
ALGORITHM

Residual
1. Solve $y''=f(x,y,y')$ (Runge-Kutta) with $y(0)=(\text{pretilt, twist}, V_1/N)$; $y'(0)=\upsilon$
2. $r(\upsilon)=y(d)-\theta_2$
   $r(\upsilon)=y(d)-\varphi_2$
   $r(\upsilon)=y(d)-V_2$

- The key to success is finding the correct value of the unknown condition υ.
- For this, the Newton-Raphson Method solves the residual equation with a low tolerance.
- This method is simple and fast, giving good results.

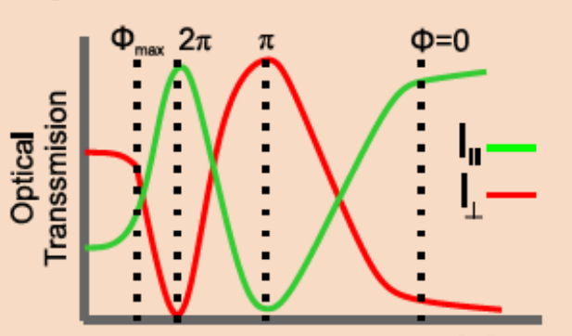
2 MONOPIXEL NLC CELLS EXPERIMENTAL & SIMULATED RESPONSE

BIREFRINGENCE CHARACTERIZATION



Computer Program

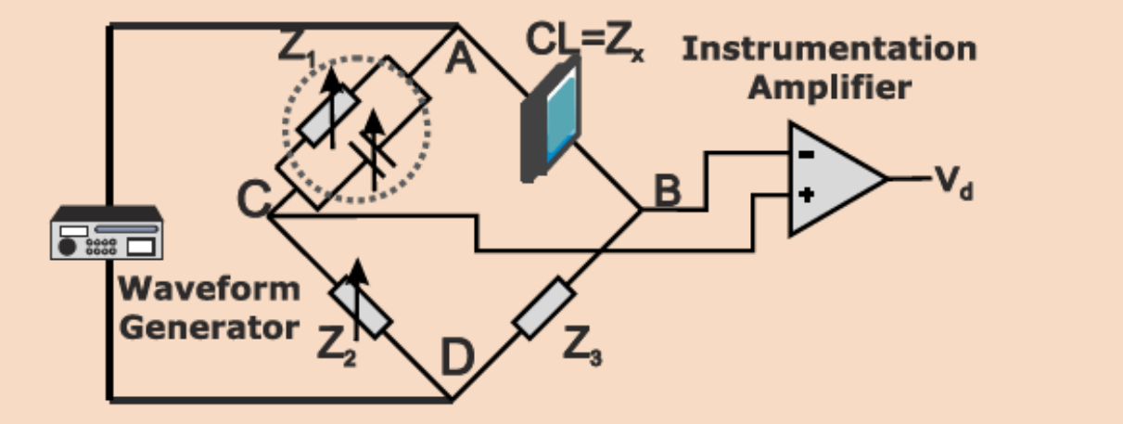
The process to obtain the birefringence is based on transmitance between parallel and crossed polarisers.



$$\Phi=\begin{cases}N\pi+\arccos\left(\frac{I_\parallel-I_\perp}{I_\parallel+I_\perp}\right) & N=0,2,4,6,\ldots\\(N+1)\pi+\arccos\left(\frac{I_\parallel-I_\perp}{I_\parallel+I_\perp}\right) & N=1,3,5,7,\ldots\end{cases}$$

DIELECTRIC PERMITTIVITY CHARACTERIZATION

Wheatstone Bridge

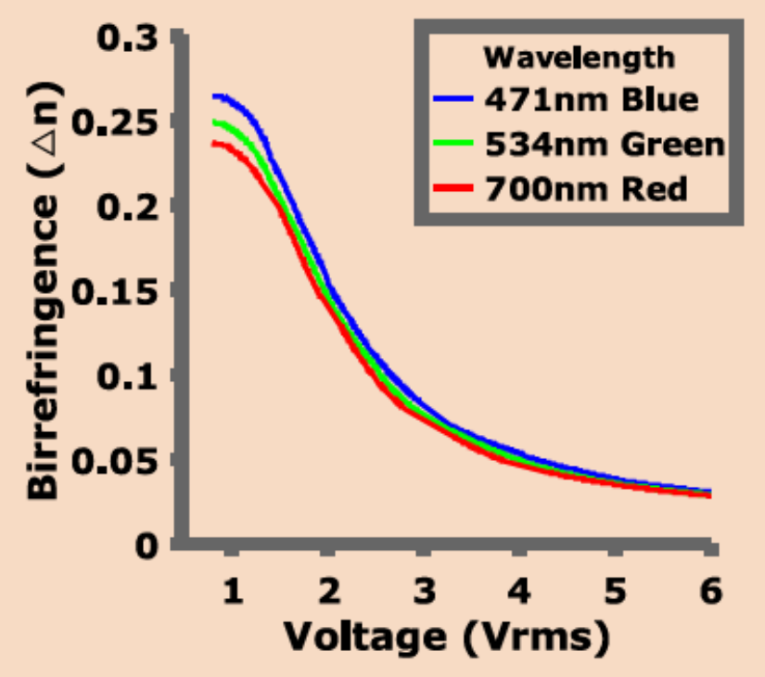


MONOPIXEL CELL:
NLC: MDA-98-1602
Cell gap: 6.5 µm
Surface area: 5cm²

$$Z_x=\frac{Z_1\cdot Z_3}{Z_2}$$

EXPERIMENTAL RESULTS

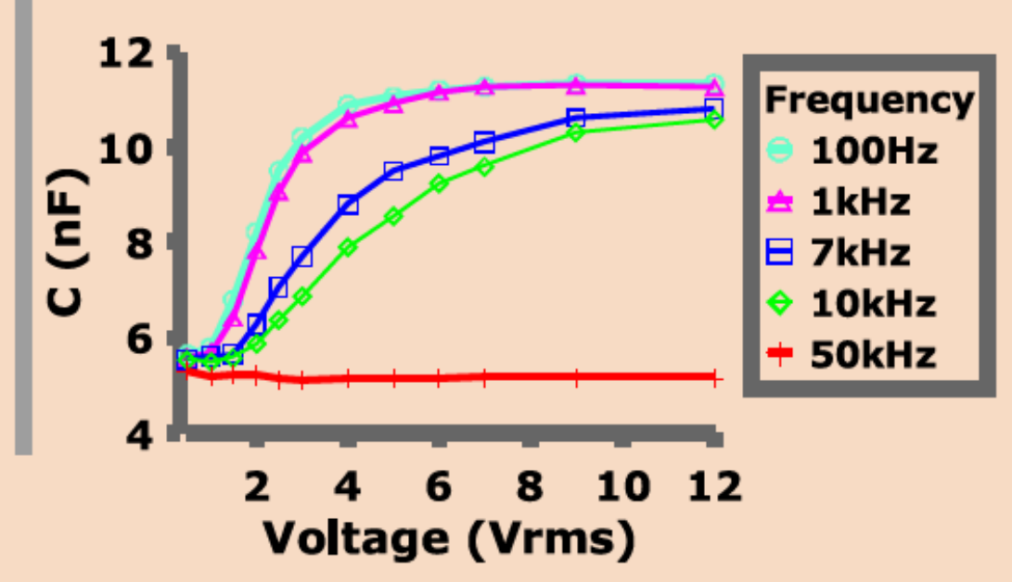
BIREFRINGENCE

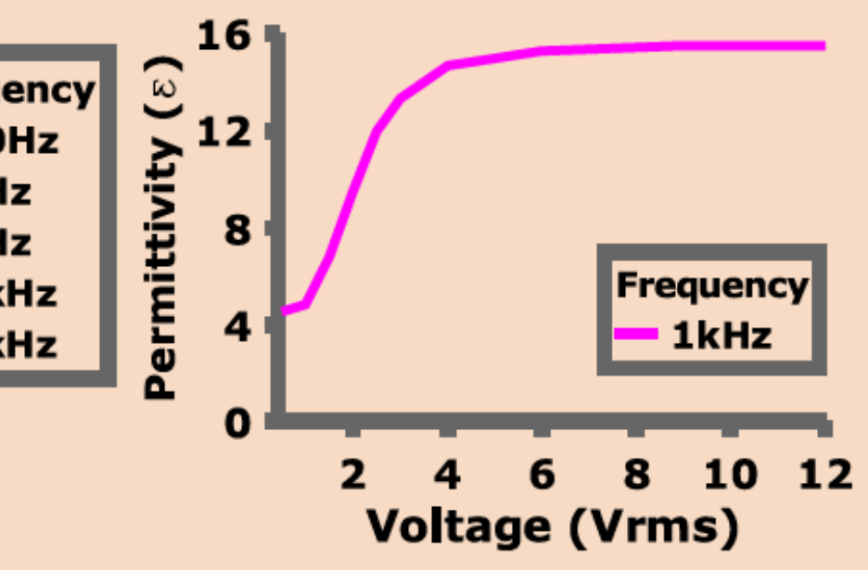


- The measurements at different wavelengths shows dispersion for different colours.
- This fact has to be considered in LC lens for optical applications.
- This dispersion can be modeled after Cauchy theory.

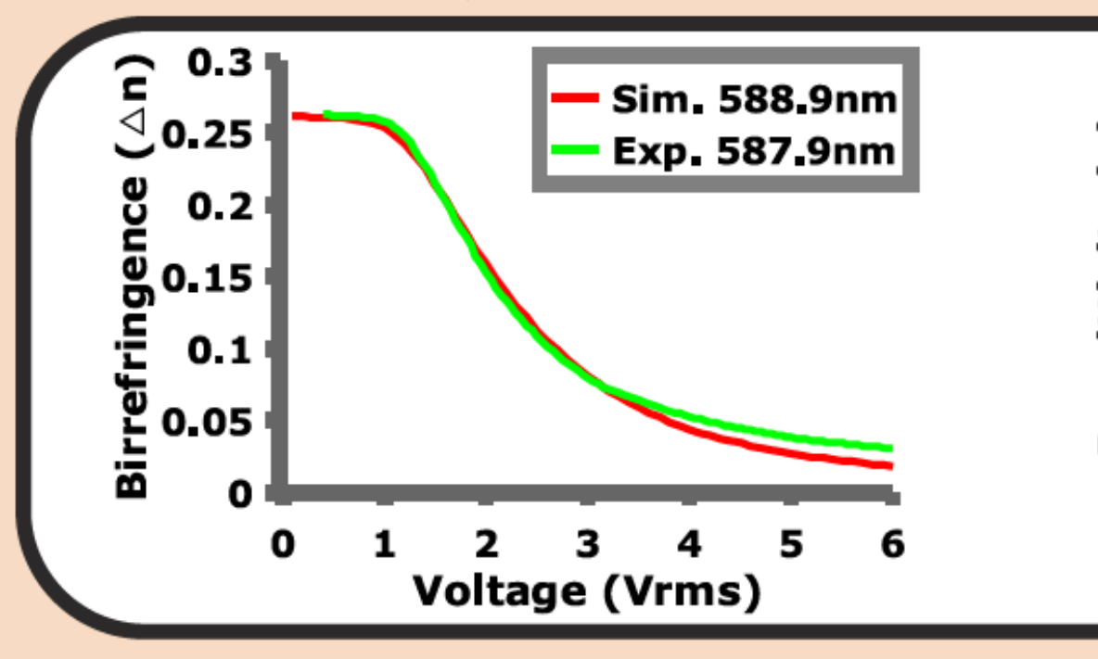
DIELECTRIC PERMITTIVITY

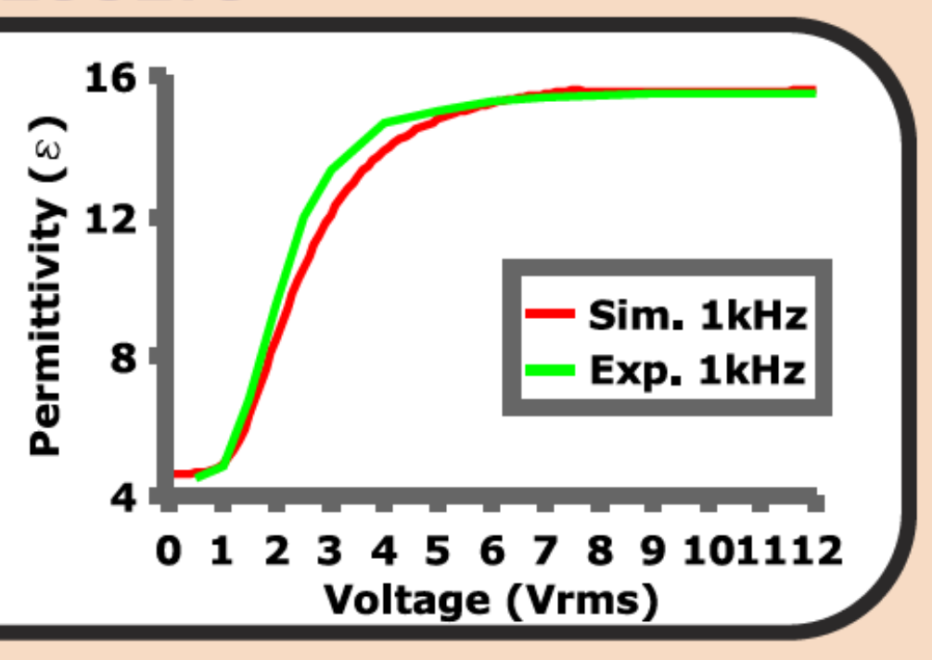
- This experiment permits the observance of dispersion and anisotropy.
- Knowing the cell dimensions, the dielectric permittivity can be extracted from capacitance.





SIMULATED & EXPERIMENTAL RESULTS





- The molecular position is estimated for different voltages.
- With the angular position the birefringence and dielectric permittivity for different voltages is calculated:

$$\varepsilon=\varepsilon_o\cos^2\theta+\varepsilon_e\sin^2\theta;\qquad n_{eff}=\frac{n_o n_e}{\sqrt{\frac{\sin^2\theta}{n_e^2}+\frac{\cos^2\theta}{n_o^2}}}$$

- Experimental and stimated results are fairly in agreement.

CONCLUSIONS

- **A numerical method, simple and fast, for solve the Continuum Theory has been developed.**
- **The birefringence and dielectric permittivity has been measured and validates the simulation results.**
- **Two LC lenses based on modal control have been fabricated. In the first case different defects are caused by a too high resistivity layer. In the second case we achieve a tunability based on a high thickness and small diameter.**
- **In the first case the simulation of molecular distributions shows the causes of these defects. The phase diagrams are fairly in agreement with experimental data.**
- **For the second case this simulation program can predict with a high precision the tunability of LC lenses.**

Acknowledgements: This work was supported in part by the Ministerio de Ciencia e Innovación of Spain (grant no.TEC2009-13991-C02-01) and Comunidad de Madrid (grant no. S2009/ESP-1781).

24th International Liquid Crystal Conference, ILCC 2012

Simulations on Distribution of Phase Retardation through Liquid Crystal Lenses

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Liquid crystal lenses have been subject to research for over 30 years. During that time, great developments have occurred, due mainly to the numerous applications these lenses have, just as gradual index lenses do (beam steering, miniature cameras, spectacles, optics instrumentation in telecommunications, 3D display systems, etc.), with the added advantages of weight reduction, volume reduction and a tunable focal length. In addition, liquid crystal lenses have been considered as promising technology for endoscope devices. In recent years, new techniques have improved the manufacturing process and simplified the voltage control. Among them it is worth noting modal control [1], polymer gel stabilization, curved surface, carbon nanotubes, concentration, etc. All of these technologies share the same kind of liquid crystal mixture, the nematic liquid crystal. Due to the extended use of nematic liquid crystals, they are one of the most extensively researched types. For a proper study of it, crystallographic theory formalisms, hydrodynamic theory, and even an understanding of the electronic and molecular structure, will be necessary. It is a known fact that nematic liquid crystals possess an elongated molecular structure, essential for generating optical anisotropic property (birefringence). The average local orientation of molecules is mathematically represented by a vector called director, n. The whole of the directors that form a liquid crystal cell, determines the anisotropy inside it. Also, in these mixtures there are three principal deformation modes (splay, twist, and bend). Their mathematical description is defined in terms of the free energy deformation (Frank-Oseen's free energy equation) that has a minimum in the equilibrium state.

For this research work, a simulation program for emulating liquid crystal lens electrooptical behavior [2], based on Frank-Oseen's free energy formulation, has been developed and the results have been compared to previous experimental data. This simulation program has been done using MATLAB software and can be used for reproducing different kind of tunable liquid crystal lenses, according to their specific technology, modal lenses or lenses with hole pattern electrodes, among others. The program requires information about characteristic parameters of the liquid crystal cell, such as, elastic constants, birefringence, dielectric constant and thickness. In modal control case, resistivity of the control electrode layer and root mean square voltages are also input parameters. The electrical field in the liquid crystal surface has been estimated by Partial Differential Equations (PDE). By knowing director angles, the application shows the liquid crystal molecular distribution inside the cell, in 2D images of transversal sections of the sandwich from the top to the bottom electrodes. Then, the effective refractive index and the phase retardation profiles are calculated. Some other output simulation results are the voltage gradient electrically induced in the lens and the focal length. The latter parameter, focal length, can be extracted from phase profiles, as has been demonstrated through our previous work experience. To finish, the simulation results have been compared to experimental data from lenticular lenses manufactured with this purpose in view. The validation of the application has been probed and both results are fairly in agreement.

References:

[1] N. Fraval and J.L. de Bougrenet de la Tocnaye, *Low Aberrations Symmetrical Adaptive Modal Liquid Crystal Lens with Short Focal Lengths*, Appl. Opt. **49** (15), 2778 (2010).

[2] S.L. Subota, V.Y. Reshetnyak, S.P. Pavliuchenko, and T.J. Sluckin, *Numerical Modeling of Tunable Liquid-Crystal-Polymer-Network Lens*, Mol. Cryst. Liq. Cryst. **489**, 40 (2008).

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