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# Optimization of passive vibration absorbers to reduce chatter in boring

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## ABSTRACT

This paper is focused on the optimal selection of the parameters of a passive dynamic vibration absorber (DVA) attached to a boring bar. The boring bar was modeled as an Euler Bernoulli cantilever beam and the stability of the system was analyzed in terms of the bar and the absorber characteristics. To obtain the optimum parameters of the absorber, a classical method for unconstrained optimization problems has been used. The selection criterion consisted of the maximization of the minimum values of the stability lobe diagram. Empirically fitted expressions for the frequency and damping ratio of the DVA (which permit to obtain its stiffness and damping) are proposed. These expressions are fully applicable when the damping ratio of the boring bar is non null as it is in practical operations. The computed results show a clear improvement in the stability performance regarding other methodologies previously used.

## 1. Introduction

Chatter or self excited vibration is the most significant type of vibration in machining operations. Chatter occurrence involves several negative effects as poor surface quality, decreased removal rate, accelerated tool wear, high noise level, environmental consequences in terms of materials and energy among others. Two categories of chatter are generally recognized: primary chatter that can be produced by the cutting process itself (friction, thermo mechanical or mode coupling effects) and secondary chatter caused by the regeneration of waviness of the work piece surface, the latter being more detrimental for machining operations.

Although Taylor [1] had identified the problem of chatter for machining productivity at the beginning of the 20th century, and the earliest study of chatter theory in simple machine tool systems was stated by Arnold [2] in the 1940s, chatter is still a very important topic in manufacturing research as it can be seen in the very recent reviews by Quintana and Ciurana [3] and Siddhpura and Paurobally [4].

This paper is focused in boring processes in which, the problem of vibration becomes more significant because of the flexibility of the tool. Boring operations need long and slender bars to machine the internal zones of the workpiece (see Fig. 1a).

The interest of these processes in the industry and special geometry of the tool has motivated the development of numerous research works on the subject. Parker [5] analyzed the stability behavior of a slender boring bar modeled as a two degree of freedom mass spring damper system. The mode coupling was experimentally studied for a range of cutting parameters. Zhang and Kapoor [6] developed a two degree of freedom model of a clamped boring bar with four cutting force components. Despite

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Nomenclature			
$A$	boring bar cross sectional area	$V_D$	<i>DVA</i> displacement
$a$	non dimensional <i>DVA</i> position	$V_{D0}$	arbitrary constant
$b$	non dimensional cutting force position	$\tilde{V}_D$	non dimensional <i>DVA</i> displacement
$c$	boring bar damping coefficient	$v$	transverse displacement of the boring bar
$C_D$	<i>DVA</i> damping constant	$W$	auxiliary variable
$C_f$	frequency correction factor	$w_c$	chip width
$C_\xi$	damping correction factor	$w_{c\lim}$	minimum value of $w_c$ in the stability diagram
$D$	boring bar diameter	$x$	spatial axial coordinate
$E$	boring bar Young's modulus	$\tilde{x}$	non dimensional spatial axial coordinate
$F_y$	radial component of the cutting force	$x_a$	<i>DVA</i> position from the clamped end
$f_{optimal}$	optimized tuning frequency	$x_b$	cutting force position from the clamped end
$f_1, f_2, f_3, f_4$	tuning frequencies	$Y$	non dimensional factor
$G$	objective function	$y$	spatial radial coordinate
$g_{ij}^f$	fitted coefficients for the frequency correction factor	$Z$	vector containing variables in the optimized problem
$g_{ij}^r$	fitted coefficients for the damping correction factor	$z$	spatial tangential coordinate
$I$	moment of inertia of the boring bar	$\alpha$	real part of the complex eigenvalues
$K_D$	<i>DVA</i> equivalent spring constant	$\beta$	imaginary part of the complex eigenvalues
$k_c$	specific cutting force	$\delta$	Dirac delta function
$k_r$	non dimensional chip width	$\lambda_D$	<i>DVA</i> non dimensional spring constant
$(k_r)_{\lim}$	minimum value of $k_r$ in the stability diagram	$\lambda_E$	non dimensional equivalent spring constant
$(k_r)_{\lim}^*$	$(k_r)_{\lim}$ normalized	$\lambda_1$	first eigenvalue of the boring bar without <i>DVA</i>
$L$	boring bar length	$\mu$	mass ratio
$M$	mass of the boring bar	$\mu^*$	effective mass ratio
$M_D$	mass of the <i>DVA</i>	$\Omega$	spindle rotational speed
$N$	mode of vibration	$\omega_0$	reference frequency
$p$	non dimensional rotational speed	$\omega_1$	fundamental frequency
$q_{10}$	arbitrary constant	$\phi_j(x)$	orthogonal eigenfunctions of the beam with out <i>DVA</i>
$q_j(t)$	time dependent generalized coordinates	$\phi_1(\tilde{x})$	fundamental vibration mode
$s$	complex eigenvalue	$\rho$	boring bar mass density
$T$	time delay	$\tau$	auxiliary variable
$\tilde{T}$	non dimensional time delay	$\xi_1, \xi_2, \xi_3, \xi_4$	tuning damping ratios
$t$	time	$\xi_E$	non dimensional equivalent damping constant
$\tilde{t}$	non dimensional time	$\xi_D$	non dimensional <i>DVA</i> damping constant
$u$	vector containing fixed parameters in objective function	$\zeta_1$	boring bar damping ratio

the influence of clamping conditions of the bar on its dynamic properties [7], most authors have simplified the analysis of boring vibrations accounting just for the lower order bending modes [8]. Andren et al. [9] studied boring bar chatter comparing an analytical Euler Bernoulli model with a time series approach. Both cutting force and dynamic behavior of the boring bar depend on the geometry of the cutting insert, as has been demonstrated by different authors (see for instance Rao et al. [10], Kuster and Gyga [11], Lazaglu et al. [12], Ozlu and Budak [13,14] and Moetakef Imani and Yussefian [15]).

Different strategies have been developed to avoid or diminish vibrations in boring operations. Improved tool holder and clamping design [7] has shown the ability to improve the dynamic behavior of the system in the chatter control of boring, including sophisticated methods, such as the use of electro rheological [16] and magneto rheological fluids [17] and active dynamic vibration absorbers [18–23]. Another possibility for active chatter suppression is varying the spindle speed to interrupt regenerative chatter effects. Several methodologies related with this concept have been presented in several research papers [24–28], among others.

Although the use of active techniques to avoid or mitigate chatter is increasing [4], passive *DVA* [29–37] is a simple solution and it is still a promising field of research for chatter suppression, not only in boring operations.

The common procedure to analyze the boring bar stability with a passive vibration absorber is modeling the system as a two degree of freedom system. The first degree of freedom corresponds to the first vibration mode of the beam, modeled as an Euler Bernoulli cantilever beam. The vertical displacement of the *DVA* corresponds to the second degree of freedom. The absorber design implies the identification of optimal parameters (mass, stiffness and damping) leading to the desired response of the system.

Rivin and Kang [29] proposed an analytical approach to design an absorber. By means of experimental results, they showed significant improvements in using their procedure. Targ et al. [30] achieved the modification of the frequency response function, *FRF*, of the cutting tool, tuning a vibration absorber. The enhancement of cutting stability was demonstrated with both experimental and analytical results. The potential of impact dampers for improving damping capability of boring bars and chatter suppression was investigated by Ema and Marui [31] through the development of bending, impact, and cutting tests. The improvement of the dynamic response of the cutting tool due to the presence of a passive absorber was demonstrated by Lee [32]. In this case the absorber had a large damping ratio and its natural frequency was similar to the natural frequency of the cutting tool. The influence on vibration response of the location of the absorber along the boring bar was analyzed by Moradi et al. [34]. The criterion for absorber parameters selection was minimizing the deflection of the free end of the boring bar. The damping of the absorber was assumed to be negligible in the analysis. The use of viscoelastic bars has been recently proposed by Saffury and Altus [38]. Sortino et al. [39] compared conventional and high damping boring bars, analyzing different geometries (bar diameter and aspect ratios length over diameter) and bar materials (alloy steels sintered and carbide materials). Experimental damping values were considerably higher in the case of high damping boring bars, due to the higher intrinsic damping properties of sintered carbide materials with respect to alloy steels.

Sims [33] proposed an analytical method relevant for a wide range of machining chatter problems. In turning and boring operations, passive *DVA* can be tuned using the analytical technique developed in [33]. Recently, Miguélez et al. [35] presented an improvement in the passive absorber design. Based on Sims results, they proposed simple analytical expressions for the tuning frequency improving the behavior of the system against chatter.

The aim of this paper is the optimal selection of the parameters of a passive *DVA* attached at a generic section of a boring bar. The boring bar was modeled as an Euler Bernoulli cantilever beam and only its first mode of vibration was taken into account. The stability of the two degree of freedom model was analyzed in terms of the stability diagram dependent on the bar characteristics and also on the absorber parameters (mass, stiffness, damping, and position). The classical Nelder Mead [40] method for unconstrained optimization problems has been used and the selection criterion consisted of the maximization of the minimum values of the stability lobe diagram. From the analysis of wide intervals of mass of the *DVA* and the damping ratio of the boring bar, empirically fitted expressions for the frequency and damping ratio of the *DVA* (which permit to obtain its stiffness and damping) are proposed. The obtained results show a clear improvement in the stability performance regarding other methodologies previously used.

## 2. Stability analysis of a boring bar with a dynamic vibration absorber

Boring operation is schematically illustrated in Fig. 1a. The boring bar is modeled as a uniform Euler Bernoulli beam with one end clamped and the other free, vibrating in  $x$   $y$  plane (see Fig. 1a), with length  $L$ , cross sectional area  $A$ , second moment of inertia  $I$ , Young's modulus  $E$ , density  $\rho$ , and damping  $c$ .

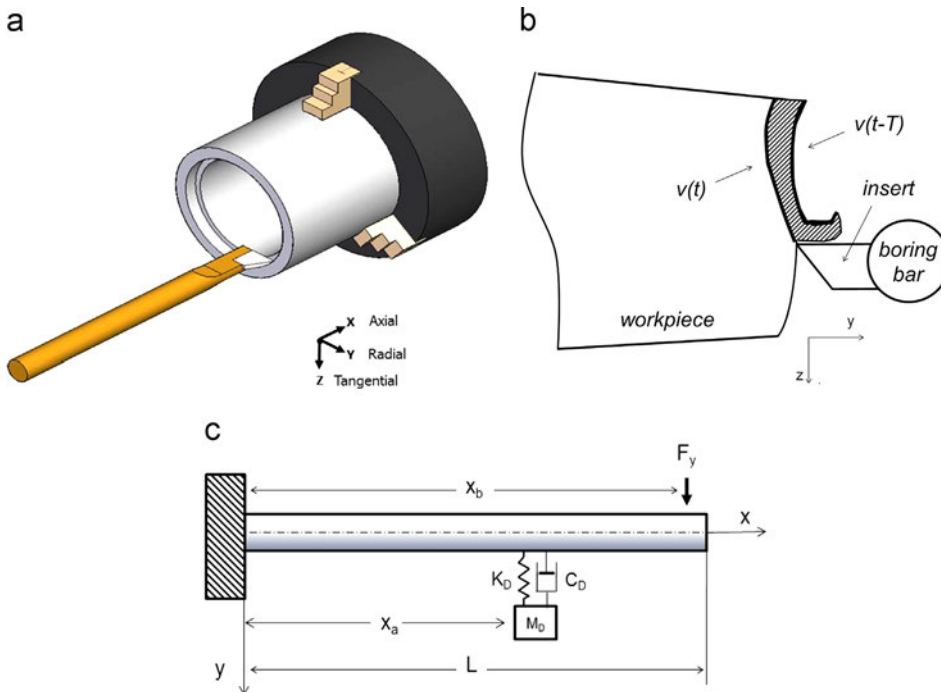


Fig. 1. Schematic view of a boring operation and its model.

The rigidity of the bar is much higher along the feed or axial direction than in the radial and tangential bending directions. On the other hand the bar exhibits higher stiffness in torsion than in bending solicitations. Thus, bending vibrations in both radial and tangential directions should be accounted in the analysis. However, the deflections in tangential direction ( $z$  axis in Fig. 1a) have negligible influence on chip thickness variation [12]. Therefore, just the vibrations of boring bar in radial direction ( $y$  axis in Fig. 1a) are considered to analyze the regenerative chatter phenomenon.

The cutting force, which is proportional to the chip cross sectional area, is applied at a certain distance  $x_b$  from the clamped end, near the free end of the bar. Then, in accordance with the classical regenerative chatter theory (see [41], for instance), the radial component of cutting force,  $F_y$ , relevant to analyze the regenerative process is given by

$$F_y = k_c w_c (v(x, t - T) - v(x, t)) \delta(x - x_b) \quad (1)$$

where  $k_c$  is the specific cutting force (a constant parameter depending on workpiece material and the cutting angles of the tool),  $w_c$  is the chip width,  $v(x, t)$  is the dynamic transverse displacement of the boring bar in radial direction, and  $T = 2\pi/\Omega$  is the delay between the current time and the previous one at which the tool has passed the point under consideration,  $\Omega$  being the spindle rotational speed, and  $\delta$  represents the Dirac delta function.

The chip section is calculated with a simple approach. Only the variation of the depth of cut (caused by surface undulations due to the previous cutting pass of the tool) was considered for the component of cutting force in the radial direction,  $F_y$  (see Fig. 1b). Thus, geometrical details of the tool, such as the insert nose radius, were not accounted in the model. Despite its simplicity, this approach has been widely used to analyze the dynamic behavior of the boring bar [17,20,34].

The beam has a passive DVA attached at a section located at a distance  $x_a$  from the clamped end (see Fig. 1c). The dynamic absorber is characterized by a mass  $M_D$ , an equivalent spring constant  $K_D$  and a damping constant  $C_D$ .

The assumptions previously described allow calculating the transverse displacement of the boring bar  $v(x, t)$  and the displacement of the mass absorber,  $V_D$ , through the resolution of the following equations:

$$\begin{aligned} \rho A \frac{\partial^2 v(x, t)}{\partial t^2} + c \frac{\partial v(x, t)}{\partial t} + EI \frac{\partial^4 v(x, t)}{\partial x^4} + \left[ C_D \left( \frac{\partial v(x, t)}{\partial t} - \frac{dV_D(t)}{dt} \right) + K_D (v(x, t) - V_D(t)) \right] \delta(x - x_a) \\ + k_c w_c [v(x, t) - v(x, t - T)] \delta(x - x_b) = 0 \end{aligned}$$

$$M_D \frac{d^2 V_D(t)}{dt^2} + C_D \left[ \frac{dV_D(t)}{dt} - \frac{\partial v(x, t)}{\partial t} \right]_{x=x_a} + K_D [V_D(t) - v(x, t)] = 0 \quad (3)$$

The analysis of the problem has already been presented in [35], but we resume here the main details for completeness. In the following, it is assumed that the boring bar deflection  $v(x, t)$  at any point  $x$  can be expressed as

$$v(x, t) = L \sum_{j=1}^N q_j(t) \phi_j(x) \quad (4)$$

where  $q_j(t)$  represent the unknown time dependent generalized coordinates and  $\phi_j(x)$  are the well known orthogonal eigenfunctions of the clamped free beam without DVA (see [42], for instance).

Considering that the dynamics of the beam is well represented by the first mode of vibration,  $N=1$  in Eq. (4), and using the new variables

$$\tilde{x} = \frac{x}{L}; \quad a = \frac{x_a}{L}; \quad b = \frac{x_b}{L}; \quad \tilde{V}_D = \frac{V_D}{L}; \quad \tilde{t} = \omega_0 t; \quad \tilde{T} = \omega_0 T; \quad \omega_0 = \sqrt{\frac{EI}{\rho AL^4}}$$

it is possible to reduce the problem by solving the following system of two second order equations that can be written as [35]

$$\begin{aligned} \begin{pmatrix} 1 & 0 \\ 0 & \mu \end{pmatrix} \begin{Bmatrix} q_1(t) \\ V_D(t) \end{Bmatrix} + \begin{pmatrix} 2\xi_E \lambda_E^2 & 2\xi_D \phi_1(a) \\ 2\xi_D \phi_1(a) & 2\xi_D \end{pmatrix} \begin{Bmatrix} \dot{q}_1(t) \\ \dot{V}_D(t) \end{Bmatrix} + \begin{pmatrix} \lambda_E^4 (1 + Y) & \lambda_D^4 \phi_1(a) \\ \lambda_D^4 \phi_1(a) & \lambda_D^4 \end{pmatrix} \begin{Bmatrix} q_1(t) \\ V_D(t) \end{Bmatrix} \\ + \begin{pmatrix} \lambda_E^4 Y & 0 \\ 0 & 0 \end{pmatrix} \begin{Bmatrix} q_1(t - T) \\ V_D(t - T) \end{Bmatrix} = 0 \end{aligned} \quad (5)$$

Eq. (5) corresponds to the motion of the two degree of freedom system representing the boring bar with an attached dynamic absorber.

In the above equation,  $(\dot{\bullet})$  indicates temporal derivatives, and the coefficients have the following significance:

$$\mu = \frac{M_D}{\rho AL}; \quad \xi_D = \frac{C_D}{2\omega_0 \rho AL}; \quad \lambda_D^4 = \frac{K_D}{\omega_0^2 \rho AL}; \quad k_r = \frac{k_c w_c}{\omega_0^2 \rho AL}; \quad \omega_1 = \lambda_1^2 \omega_0; \quad \zeta_1 = \frac{c}{2\rho A \omega_1}$$

with

$$\lambda_E^4 = \lambda_1^4 + \lambda_D^4 \phi_1^2(a); \quad \xi_E = \frac{\zeta_1 \lambda_1^2 + \xi_D \phi_1^2(a)}{\lambda_E^2}$$

and

$$Y = \frac{k_r \phi_1^2(b)}{\lambda_E^4}$$

$\lambda_1$  being the first eigenvalue ( $\lambda_1 = 1.8751$ ) associated with the fundamental mode of vibration of the boring bar given by [42]

$$\phi_1(\bar{x}) = \cosh \lambda_1 \bar{x} - \cos \lambda_1 \bar{x} \frac{\cosh \lambda_1 + \cos \lambda_1}{\sinh \lambda_1 + \sin \lambda_1} (\sinh \lambda_1 \bar{x} - \sin \lambda_1 \bar{x}) \quad (6)$$

To analyze the stability of the solution of Eq. (5), we assume that

$$q_1(t) = q_{10} e^{st}; \quad V_D(t) = V_{D0} e^{st}; \quad (7)$$

where  $q_{10}$  and  $V_{D0}$  are arbitrary constants and  $s = \alpha + i\beta$  is the complex eigenvalue. Positive values of  $\alpha$  lead to unstable behavior of the system.

Substituting Eq. (7) in Eq. (5), we get a homogeneous system for the unknown constants  $q_{10}$  and  $V_{D0}$ . This system has non trivial solutions with purely imaginary eigenvalues, i.e.  $s = i\beta$ , if the following two conditions are satisfied [35]:

$$\lambda_E^4 ((\lambda_D^4 - \mu \lambda_E^4 W^2)(1 - W^2 + Y(1 - \cos(W\tau))) + 4W^2 \xi_D (\phi_1^2(a) \xi_D - \lambda_E^2 \xi_E)) - \phi_1^2(a) \lambda_D^8 - 2W \lambda_E^6 \xi_D \sin(W\tau) = 0 \quad (8)$$

$$W \lambda_E^2 \xi_D (2\lambda_E^4 (1 + Y - W^2) - 4\phi_1^2(a)) - 2WY \lambda_E^6 \xi_D \cos(W\tau) + \lambda_E^4 (\lambda_D^4 - W^2 \lambda_E^4 \mu) (2W \xi_E + Y \sin(W\tau)) = 0 \quad (9)$$

in which, the auxiliary variables  $W$  and  $\tau$  have been introduced:

$$W = \frac{\beta}{\lambda_E^2}, \quad \tau = \frac{T}{\lambda_E^2} \quad (10)$$

Note that once the characteristics of boring bar and DVA (including its location) are selected, the variables  $\lambda_D$ ,  $\lambda_E$ ,  $\mu$ ,  $\xi_E$ ,  $\xi_D$  and  $\phi_1(a)$  are known.

Thus, for each value of the auxiliary variable  $W$  it is possible to solve Eqs. (8) and (9) obtaining the values of variables  $Y$  and  $\tau$ , and from these values we calculate

$$k_r = \frac{k_c W_c L^3}{EI} = \frac{Y \lambda_E^4}{\phi_1^2(b)} \quad (11)$$

$$p = \frac{2\pi \lambda_E^2}{\tau} = \frac{\Omega}{\omega_0} \quad (12)$$

The curves  $k_r$  versus  $p$  constitute the stability boundaries of the system. Fig. 2 shows a schematic view of a typical stability lobe diagram. The minimum values of these curves,  $(k_r)_{lim}$ , are related to the maximum value of chip width that can be removed irrespective of the turning speed. Then, for values of  $k_r$  lower than  $(k_r)_{lim}$  the system is unconditionally stable.

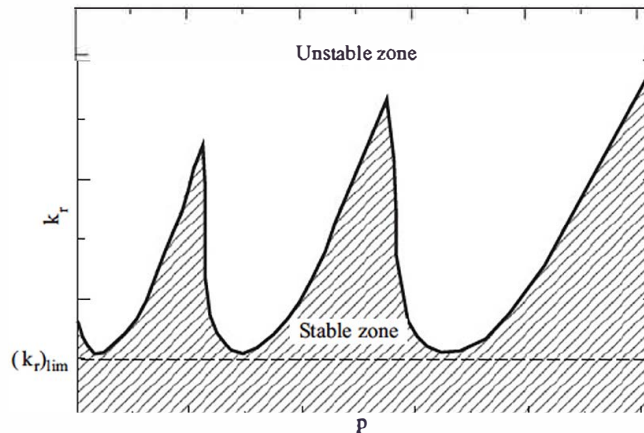


Fig. 2. Stability boundaries of the system.

**Table 1**

Parameters of the boring bar and attached DVA.

Boring bar (main structure)		Dynamic vibration absorber	
Length, $L$	0.3 m	Mass, $M_D$	0.05 kg
Diameter, $D$	0.020 m	Distance to clamping, $x_a$	0.195 m
Section, $A$	$3.142 \times 10^{-4} \text{ m}^2$	Non-dimensional position, $a = x_a/L$	0.65
Inertia, $I$	$7.8540 \times 10^{-9} \text{ m}^4$	Mass ratio, $\mu$	0.068
Density, $\rho$	$7800 \text{ kg/m}^3$	Effective mass ratio, $\mu^*$	0.075
Mass of the beam, $M$	0.735 kg	Equivalent spring constant, $K_D$	52 265 N/m
Young modulus, $E$	$2 \times 10^{11} \text{ Pa}$	Damping constant, $C_D$	16.00 N s/m
Cutting force position, $x_b$	0.294 m		
Non-dimensional cutting force position, $b = x_b/L$	0.98		
Specific cutting force, $k_c$	$9 \times 10^8 \text{ N/m}^2$		
Damping ratio, $\zeta_1$	0.025		

To illustrate this methodology, a representative example corresponding to a typical boring bar with a DVA attached at a certain location is presented below. Table 1 summarizes the characteristics of both boring bar and vibration absorber. From these data, the variables  $\lambda_D$ ,  $\lambda_E$ ,  $\xi_E$ ,  $\xi_D$  and  $\phi_1(a)$  can be calculated, and Eqs. (8) and (9) solved.

From this, a value of  $(k_r)_{\text{lim}} = 0.913$  is obtained that corresponds to a chip width  $(w_c)_{\text{lim}} = 0.059 \text{ mm}$ .

### 3. Absorber parameter selection with previous methodologies

Previous methodologies used to design the DVA for chatter mitigation rely on the analysis of the Frequency Response Function, *FRF*, of a system composed by a mass attached to the undamped main structure (boring bar in this case) through an elastic spring and a viscous damper. Den Hartog [43] proposed a tuning frequency from the analysis of the modulus of *FRF*:

$$f_1 = \frac{\omega_a}{\omega_1} = \frac{1}{1 + \mu^*} \quad (13)$$

where  $\omega_a$  is the absorber frequency given by  $\omega_a = \sqrt{K_D/M_D}$ ,  $\omega_1$  is the natural frequency of the main structure (in this case,  $\omega_1$  corresponds to the fundamental frequency of the boring bar, given by  $\omega_1 = \lambda_1^4 \omega_0$ ), and  $\mu^*$  is the effective mass ratio given by

$$\mu^* = \mu \phi_1^2(a) \quad (14)$$

$\mu$  being the mass ratio written as  $\mu = M_D/M_S$ . Here,  $M_S = \rho AL$  is the mass of the beam.

Notice that the effective mass ratio must be considered in order to take into account the absorber position along the boring bar (see Appendix in [35]).

Accordingly, Den Hartog [43] proposed a damping ratio as

$$\xi_1 = \sqrt{\frac{3\mu^*}{8(1 + \mu^*)}} \quad (15)$$

Sims [33] proposed different values for the tuning frequencies and damping ratios analyzing the real part of the *FRF*:

$$f_2 = \sqrt{\frac{\mu^* + 2 + \sqrt{2\mu^* + \mu^{*2}}}{2(1 + \mu^*)^2}} \quad (16)$$

$$f_3 = \sqrt{\frac{\mu^* + 2 - \sqrt{2\mu^* + \mu^{*2}}}{2(1 + \mu^*)^2}} \quad (17)$$

and

$$\xi_2 = \sqrt{\frac{\mu^*(\mu^* + 3 + \sqrt{2\mu^* + \mu^{*2}})}{4(1 + \mu^*)(\mu^* + 2 + \sqrt{2\mu^* + \mu^{*2}})}} \quad (18)$$

$$\xi_3 = \sqrt{\frac{\mu^*(\mu^* + 3 - \sqrt{2\mu^* + \mu^{*2}})}{4(1 + \mu^*)(\mu^* + 2 - \sqrt{2\mu^* + \mu^{*2}})}} \quad (19)$$



As it was demonstrated in [35], the best stability performance is achieved using the tuning frequency given by (16) ( $f_2$  of Sims model) combined with the damping ratio given by Eq. (19) ( $\xi_3$  of Sims model). The combination of frequency  $f_2$  and the damping ratio ( $\xi_2$  of Sims model) leads to similar results, but the minimum of the stability lobe diagram is slightly lower.

Moreover, Miguélez et al. [35] performed an analysis varying the frequency in the proximity of the value given by Eq. (16),  $f_2$ , with a fixed damping ratio, and proposed new expressions for tuning frequency given by

$$f_4 = \left(1 + \frac{\mu^*}{2}\right) \sqrt{\frac{\mu^* + 2 + \sqrt{2\mu^* + \mu^{*2}}}{2(1 + \mu^*)^2}} \quad (20)$$

together with the damping ratio given by Eq. (18) or

$$f_5 = \left(1 + \frac{\mu^*}{4}\right) \sqrt{\frac{\mu^* + 2 + \sqrt{2\mu^* + \mu^{*2}}}{2(1 + \mu^*)^2}} \quad (21)$$

combined with the damping ratio given by Eq. (19).

These results were obtained for undamped main structure (boring bar). Note that the models of Den Hartog [43] and Sims [33] are strictly applicable only to this case of the undamped main structure.

From a generic value of frequency  $f$ , given by Eq. (13), (16), (17), (20), or (21) and  $\xi$ , given by Eq. (15), (18), or (19), the corresponding dimensionless parameters characterizing the absorber used can be written as

$$\lambda_D^4 = \mu \lambda_1^4 f^2 \quad (22)$$

$$\xi_D = \xi \lambda_1^2 \sqrt{\mu} = \xi \lambda_1^2 \mu f \quad (23)$$

and, finally, the dimensional stiffness and damping of the absorber are given by

$$K_D = \lambda_D^4 \frac{EI}{L^3} \quad (24)$$

$$C_D = 2\xi_D \sqrt{\frac{\rho AEI}{L^2}} \quad (25)$$

#### 4. Optimization strategy for absorber parameters selection

The methodologies previously revised based the selection of the absorber parameter on the analysis of the behavior of the main structure with the dynamic passive absorber attached, enforcing certain conditions in its *FRF*. In this work, a new procedure is proposed to treat the design process as an optimization problem. Thus, given an effective mass ratio  $\mu^*$  and a damping ratio of the main structure  $\zeta_1$ , the corresponding values of non dimensional spring constant and non dimensional damping constant for the dynamic absorber ( $\lambda_D$  and  $\xi_D$ , respectively) that maximize the value of the function  $(k_r)_{\lim}$  need to be obtained. Then an optimization problem can be formulated maximizing the following objective function:

$$(k_r)_{\lim} = G(Z; u) \quad (26)$$

where  $Z = \{\lambda_D, \xi_D\}$  and  $u = \{\mu^*, \zeta_1\}$  are vectors containing the variables to be calculated in the optimization process and parameters fixed in each analysis, respectively. The objective function  $G$  is obtained solving the corresponding stability problem described in Section 2.

The well known Nelder Mead algorithm [40], a simplex method for finding a local minimum of a scalar function of several variables, has been used. Since the method gives the minimum of a function, the negative value of  $(k_r)_{\lim}$  is taken as the objective function to be minimized. The procedure is implemented as a standard routine in MATLAB<sup>®</sup> environment. The values of frequency and damping ratios given by the Sims method [33], Eqs. (16) and (18), respectively, are used as starting points of the optimization process. Although strictly speaking, Nelder Mead is not a true global optimization algorithm, as the starting points are values taken from Sims' formulation based on a *FRF*, it is expected that the method leads to global optimized values.

#### 5. Fitted optimal parameters

Following the procedure mentioned in the previous section, given an effective mass and a damping of the main structure, which act as the fixed parameters in the objective function (see Eq. (26)), it is possible to calculate the characteristic parameters of the passive vibration absorber. This analysis has been done for 36 values of effective mass, from 0.025 to 0.20 with increments of 0.005. For each value of  $\mu^*$ , 11 values of the damping of the main structure, from 0.0 to 0.1 with increments of 0.01, were used. Then a total of 396 ( $36 \times 11$ ) cases were analyzed.



The obtained values of the frequency and damping ratios were normalized by the corresponding values given by Sims [33] (Eqs. (16) and (18), respectively). Thus, the optimal values for the frequency ratio and damping ratio can be written, respectively, as

$$\begin{aligned} f_{optimal} &= C_f f_2 \\ \xi_{optimal} &= C_\xi \xi_2 \end{aligned} \quad (27)$$

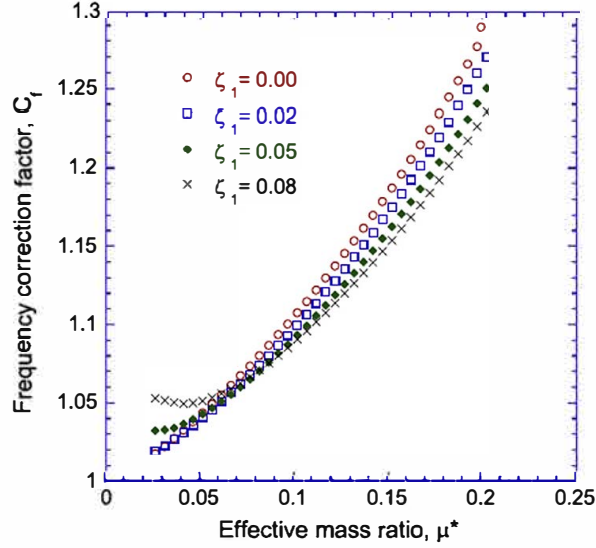


Fig. 3. Correction factor to obtain the optimal frequency ratio.

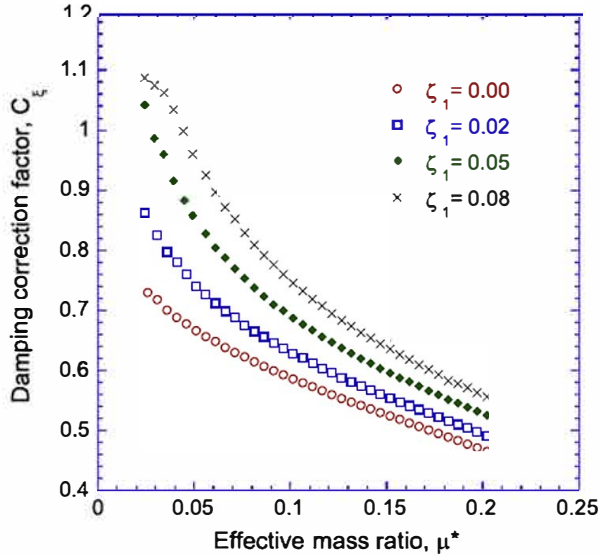


Fig. 4. Correction factor to obtain the optimal damping ratio.

**Table 2**  
Coefficients  $g_{ij}^f$  for the frequency correction factor.

$i$	$j$			
	0	1	2	3
0	0.9954	0.9170	1.7198	5.0762
1	0.3067	-13.19	83.53	-242.2
2	6.36	-75.65	428.4	-645.5

where  $C_f$  and  $C_\xi$  are correction factors to be applied to the values proposed by Sims [33] for the frequency ratio Eq. (16) and damping ratio Eq. (16), respectively.

Fig. 3 shows the correction factor  $C_f$  as a function of the effective mass for four different values of the damping of the boring bar.

**Table 3**

Coefficients  $g_{ij}^c$  for the damping correction factor.

$i$	$j$			
	0	1	2	3
0	0.8090	-3.2698	14.62	-33.058
1	7.947	-110.5	645.7	-1306
2	-20.86	468	-3494	8204

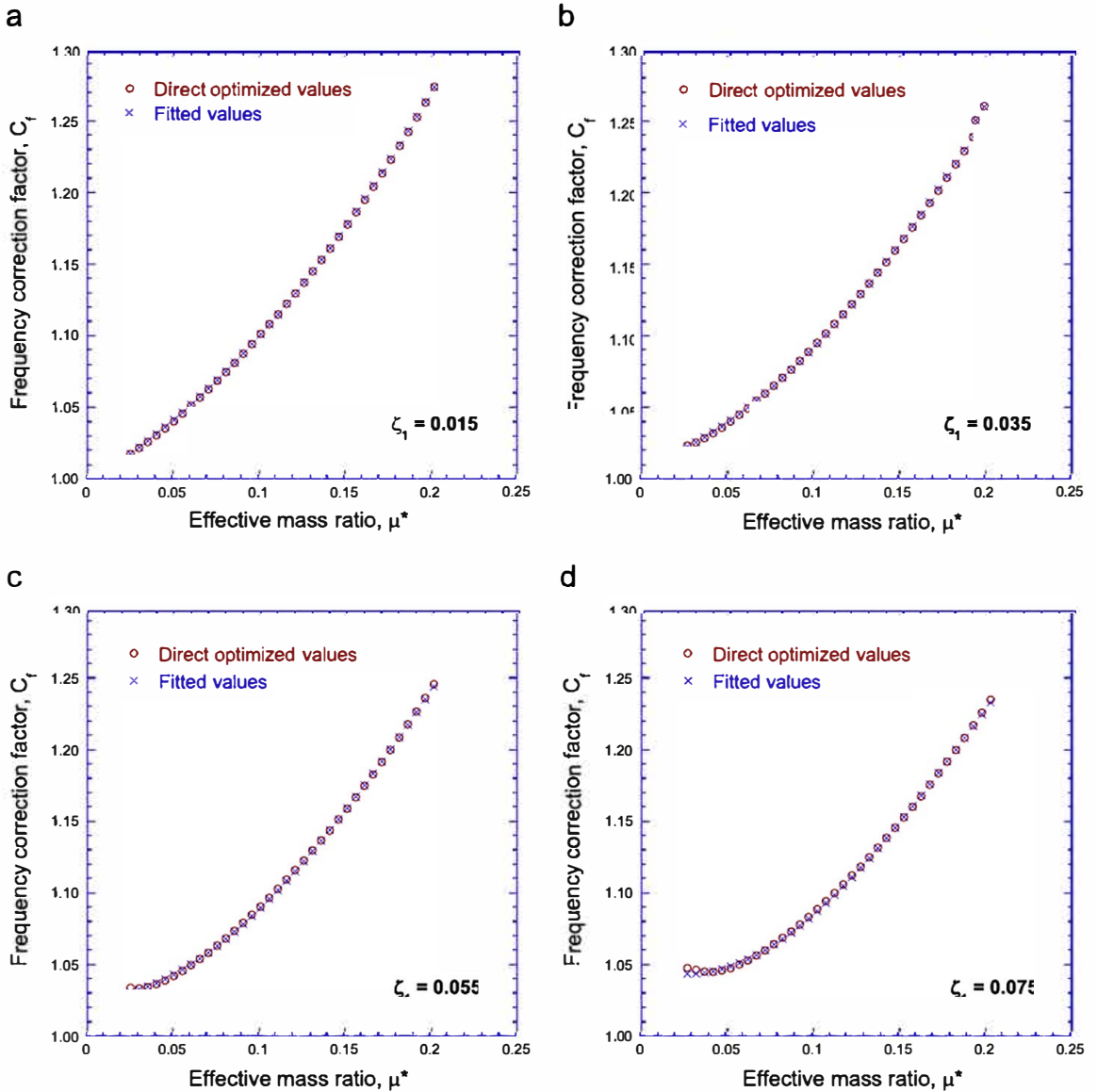


Fig. 5. Fitted frequency correction factors for different values of main structure damping: (a)  $\zeta_1 = 0.015$ , (b)  $\zeta_1 = 0.035$ , (c)  $\zeta_1 = 0.055$  and (d)  $\zeta_1 = 0.075$ .

Fig. 4 shows the correction factor  $C_\xi$  as a function of the effective mass for four different values of the damping of the boring bar.

In view of these results, expressions for the fitted values of the corresponding corrections factors  $C_f$  and  $C_\xi$  were intended. The following expressions are proposed:

$$C_f = \sum_{i=0}^2 \zeta_1^i \sum_{j=0}^3 g_{ij}^f \mu^* \quad (28)$$

and

$$C_\xi = \sum_{i=0}^2 \zeta_1^i \sum_{j=0}^3 g_{ij}^\xi \mu^* \quad (29)$$

Tables 2 and 3 give the fitted coefficients  $g_{ij}^f$  and  $g_{ij}^\xi$ , respectively.

To demonstrate the accuracy of the above proposed fitted expressions, Fig. 5 shows a comparison between the direct optimized values of the frequency correction, obtained by the Nelder Mead algorithm, factor and the fitted ones as a function of the effective mass ratio,  $\mu^*$ , for four values of the damping of the main structure  $\zeta_1 = 0.015, 0.035, 0.055, 0.075$ . In Fig. 6, the same comparison for the damping correction factor is represented.

As it can be seen, the fitted values for both the correction factors are virtually the same as those calculated with the direct optimization method.

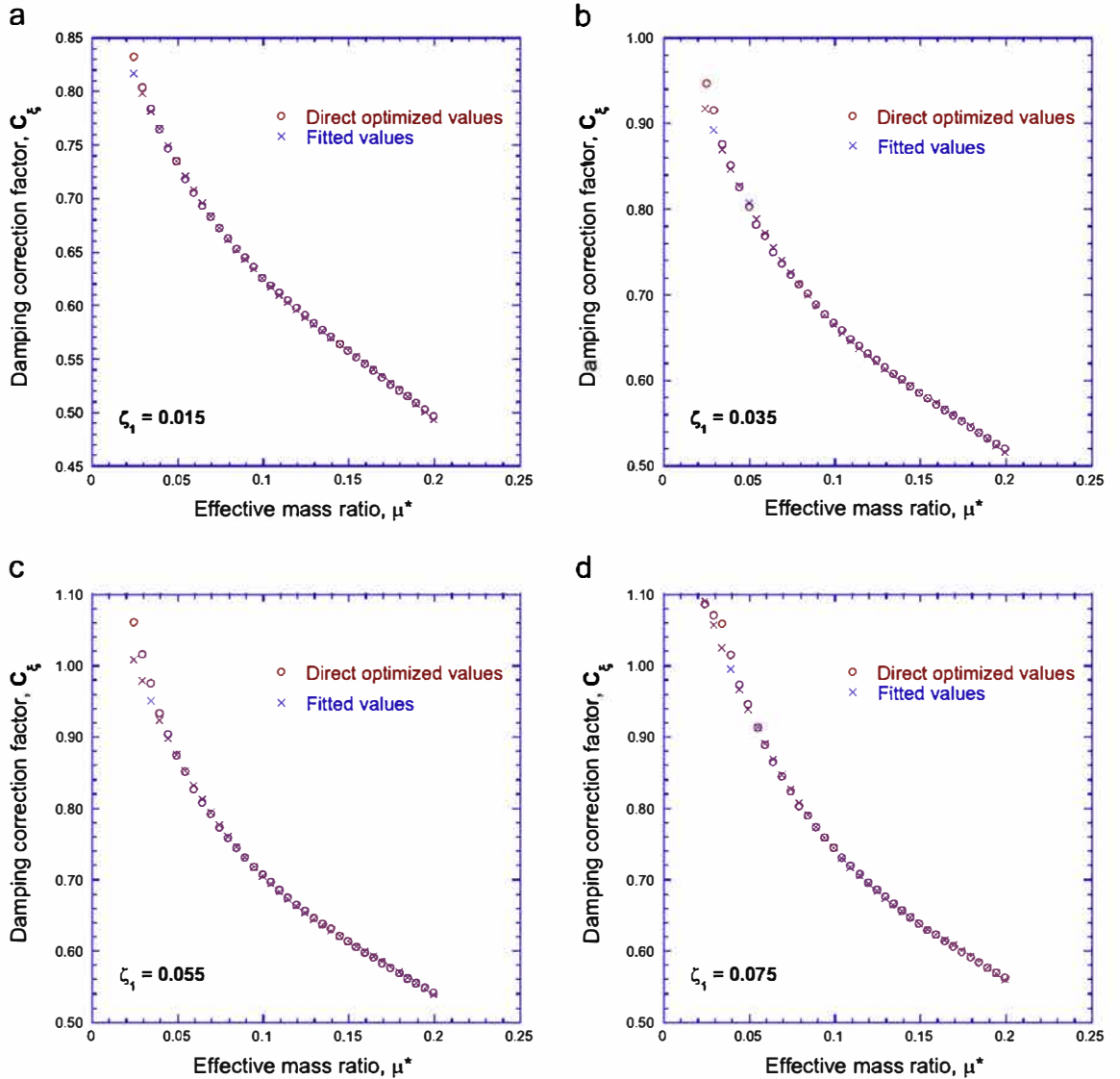


Fig. 6. Fitted damping correction factors for different values of main structure damping: (a)  $\zeta_1 = 0.015$ , (b)  $\zeta_1 = 0.035$ , (c)  $\zeta_1 = 0.055$  and (d)  $\zeta_1 = 0.075$ .

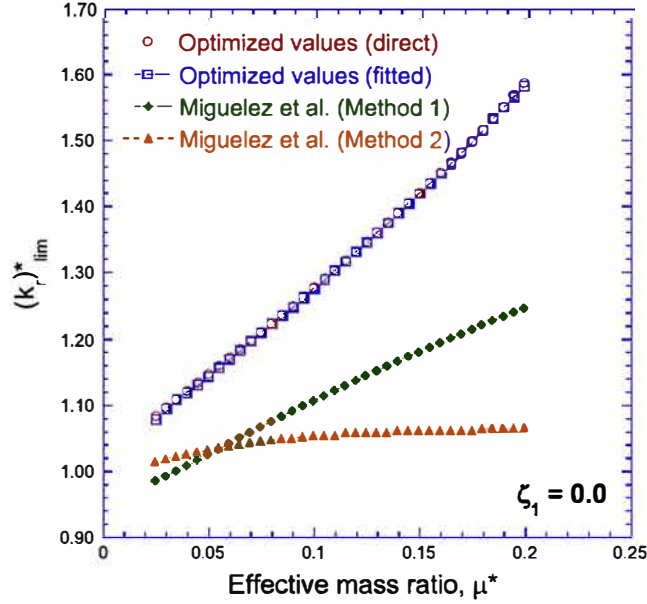


Fig. 7. Comparison of different procedures. Case of null damping ratio of the main structure.

## 6. Comparison with other methodologies

In the previous section we presented different methodologies to select the parameter of a passive vibration absorber with a fixed effective mass ratio  $\mu^*$ . In particular, we remind the following procedures:

- (a) Sims method [33], corresponding to the use of frequency ratio  $f_2$ , Eq. (16) and damping ratio  $\xi_2$ , Eq. (18).
- (b) Method 1 of Miguelé et al. [35] (frequency ratio  $f_4$ , Eq. (20) and damping ratio  $\xi_2$ , Eq. (18)).
- (c) Method 2 of Miguelé et al. [35] (frequency ratio  $f_5$ , Eq. (21) and damping ratio  $\xi_3$ , Eq. (19)).
- (d) Direct optimization method using the Nelder Mead procedure, according to the methodology presented in Section 4.
- (e) Fitted values for the frequency and damping ratios using empirical expressions Eq. (28) and (29), respectively.

Note that the classical tuning frequency method of Den Hartog has not been considered in view of results given by Miguelé et al. [35], showing that other procedures lead to better stability performance.

It is possible to define  $(k_r)_{lim}^*$  as the ratio of the  $(k_r)_{lim}$  value obtained with procedures (b) (e) normalized with the obtained method (a), taken as reference. Thus, this value represents the improvement over the Sims method, method (a), achieved in chatter stability using the other procedures.

Fig. 7 shows the variation of  $(k_r)_{lim}^*$  as a function of mass effective ratio  $\mu^*$  for null damping ratio of main structure ( $\zeta_1 = 0$ ). The same information is given in Fig. 8 for four different values of  $\zeta_1$ .

From the results showed in the above figures, two main conclusions arise. In all cases the parameters selected with the Nelder Mead optimization method give better stability performance than the other ones. In fact, the ratio  $(k_r)_{lim}^*$  continuously increases with the effective mass ratio, reaching minimum values of the stability lobe diagram that are 40% higher than those obtained with the procedure of Sims for  $\mu^* = 0.2$ . On the other hand, for small values of effective mass ratio, the improvement is smaller but not less than 10%.

The second conclusion is that these optimal parameters can be fitted from the expressions in Eqs. (28) and (29) (frequency and damping ratio for the absorber, respectively), leading virtually to identical results with the direct optimization method.

## 7. Numerical example

The proposed methodology to select optimized parameters for a DVA attached to a boring bar was applied to solve a representative example. The characteristics of the boring bar were previously presented in Table 1.

For comparison, three tuning methodologies were considered (Sims method [33], method 1 of Miguelé et al. [35], and the proposed method). Using the Sims method [33], the tuning frequency and damping ratio are calculated from Eqs. (16) and (18), respectively. The method 1 of Miguelé et al. [35] corrects the tuning frequency according to Eq. (20) and takes the same damping ratio as that in the previous method by Sims. Finally, the tuning frequency and damping ratio here proposed are obtained from Eq. (27), in which the correction factors  $C_f$  and  $C_\xi$  are the corresponding fitted values given by Eqs. (28) and (29) respectively, for values of  $\mu^* = 0.075$  and  $\zeta_1 = 0.025$ .

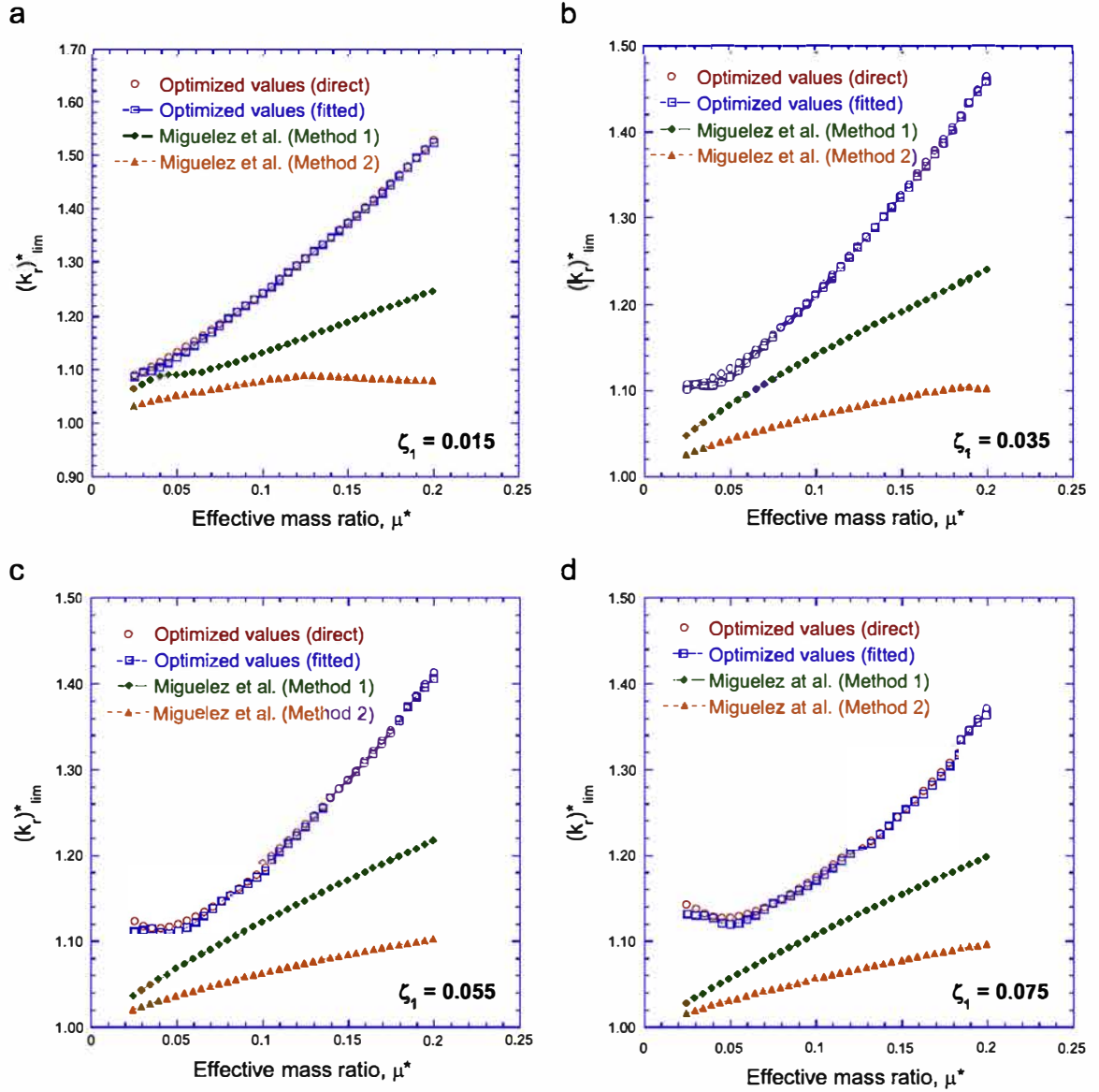


Fig. 8. Comparison of different procedures for several values of main structure damping: (a)  $\zeta_1 = 0.015$ , (b)  $\zeta_1 = 0.035$ , (c)  $\zeta_1 = 0.055$  and (d)  $\zeta_1 = 0.075$ .

Table 4  
Selected parameters of the DVA for the example considered.

Method	$f$	$\xi$	$\lambda_D$	$\xi_D$	$(k_r)_{lim}$	$(w_c)_{lim}$ (mm)	$K_D$ (N/m)	$C_D$ (N s/m)
Sims	1.034	0.157	0.974	0.039	0.913	0.059	52265	16.00
Miguelé et al.	1.072	0.157	0.992	0.040	1.029	0.066	56261	16.61
Proposed	1.106	0.109	1.007	0.029	1.055	0.068	59826	11.98

The obtained values for the spring constant and damping constant of the DVA are given in Table 4, and the complete stability lobe diagrams in Fig. 9. As can be seen, the proposed method gives the highest value of  $(w_c)_{lim}$ .

It is important to note that an increase of the effective mass ratio,  $\mu^*$ , would lead to higher values of  $(w_c)_{lim}$ , as can be seen in Figs. 7 and 8. For higher  $\mu^*$  values, a DVA with higher spring constant but smaller damping constant is required, as can be appreciated in Figs. 3 and 4.

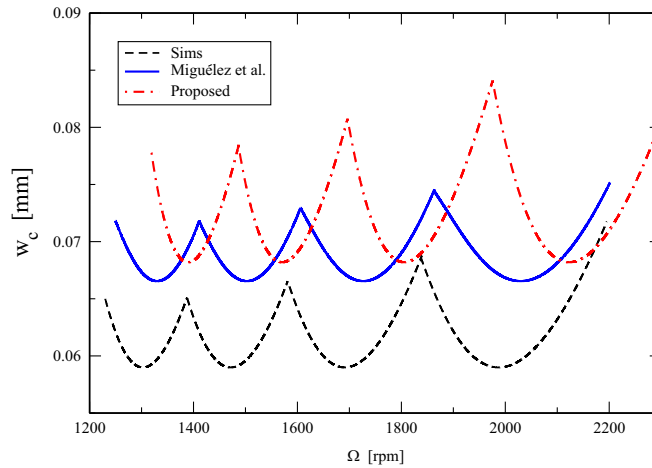


Fig. 9. Stability boundaries for the example considered.

## 8. Conclusions

This paper deals with the chatter stability of a boring bar with an attached passive dynamic vibration absorber. The boring bar is modeled as an Euler Bernoulli cantilever beam, and the absorber is considered attached by a spring and a damper at a certain section of the beam. The stability problem has been properly solved and the stability lobe diagram constructed. To determine the optimum values of the absorber parameters, the criterion was to maximize the minimum values of the stability lobe diagram. To this end, the classical Nelder Mead [40] method for unconstrained optimization problems has been used. Analyzing of wide intervals of mass of the DVA and the damping ratio of the boring bar, empirically fitted expressions for the optimal values of the frequency and damping ratios of the absorber are proposed. The obtained results show a clear improvement in the stability performance over other methodologies previously presented by Sims [33] and Miguélez et al. [35].

The method could be easily implemented in the design procedure of passive absorbers in boring operations, and note that it is fully applicable when the damping ratio of the boring bar is non null as it occurs in practical operations.

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