Three-Dimensional Method for Simulation of Multimode Interference Couplers

Carmen Vázquez, Francisco José Mustieles, and F. Hernández-Gil

Abstract—A three-dimensional (3-D) method for modeling multimode interference devices based on a finite element formulation is presented as an alternative to models having one-dimensional cross-sections only. The method is tested for couplers with two different strongly confined waveguides structures. The results show that full treatment of two-dimensional cross-section is of special importance for design and simulation of waveguide devices for which the effective index approximation is no longer valid. For deep rib waveguide geometries, excess loss greater than 15 dB can be obtained if the 3-D method is not used in the design of the couplers.

I. INTRODUCTION

OPTICAL couplers are basic elements in photonic integrated circuits, where they can implement signal routing and signal processing functions. Multimode interference (MMI) couplers are reaching a growing interest nowadays due to their low excess loss, high extinction ratio, small size, and low sensitivity to fabrication tolerances and polarization [1], [2].

Modeling of the MMI couplers employing one-dimensional (1-D) cross-section calculations has been previously reported [2].

In this paper, we present a method for modeling MMI couplers based on the multimode propagation in z -direction combined with a three-dimensional (3-D) description based on a finite element formulation (FEM).

After a description of the numerical method in Section II, it is illustrated by the design of MMI couplers with two different waveguide structures. The results obtained with our method and the ones obtained with a two-dimensional (2-D) analysis based on the effective index (EI) approximation are compared in Section III. The high sensitivity of the MMI-section length for a minimum excess loss device to the propagating constants of the modes of the MMI-section is derived after this comparison. Tolerance ranges of the couplers are analyzed to discuss the results.

Special attention is paid to rib waveguides with large rib height, where the adoption of the effective index approximation [3], [4] can introduce large errors in the calculated propagation constants of the modes of the MMI-section.

Manuscript received February 27, 1995; revised July 18, 1995. This work was supported by a Grant from the Spanish Ministry of Education and Science. C. Vázquez and F. Hernández-Gil are with Telefónica Investigación y Desarrollo, División de Optoelectrónica, Madrid 28043, Spain.

F. J. Mustieles is with Universidad Alfonso X el Sabio, 28691 Villanueva de la Cañada, Spain.

IEEE Log Number 9414867.

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II. METHOD DESCRIPTION

The modes in an optical waveguide can be obtained solving the following Maxwell equation in electric field:

$$\nabla \times (\nabla \times \bar{E}) - k_0^2 [n^2] \bar{E} = 0$$

or in magnetic field

$$\nabla \times ([n^2]^{-1} \nabla \times \bar{H}) - k_0^2 \bar{H} = 0. \quad (1)$$

If quasi TE or TM modes and diagonal, isotropic permittivity tensors are considered, the vectorial equations can be reduced to a scalar equation in the main component ($\Phi(x, y)$)

$$\begin{aligned} & \frac{\partial}{\partial x} \left[\frac{1}{p(x, y)} \frac{\partial \Phi}{\partial x} \right] + \frac{\partial}{\partial y} \left[\frac{1}{p(x, y)} \frac{\partial \Phi}{\partial y} \right] \\ & + \frac{1}{p(x, y)} [n^2(x, y) k_0^2 - \beta^2] \Phi = 0 \end{aligned} \quad (2)$$

if TE mode $p(x, y) = 1$ $\Phi = E_y$
if TM mode $p(x, y) = n^2(x, y)$ $\Phi = H_y$

This reduces the computation time required to solve the problem, while the resolution is only slightly altered. We have checked it in our implementation of the vectorial finite element method [5]. Second-order rectangular elements of the serendipity type are used in the finite element formulation [6] employed to solve (2). This formulation leads to an eigenvalue problem where the propagation constants β are related to the eigenvalues and the eigenvectors define the field distribution E_y or H_y .

The normalization of the modes is chosen so they have unity power. For the n th mode, it will have the form [7]

$$P_n = \frac{\lambda \beta_n}{\pi c} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{ct(x, y)} \Phi_n^2(x, y) dx dy = 1 \quad (3)$$

if TE mode $ct = \mu_0$ $\Phi_n = E_{y_n}$
if TM mode $ct = \epsilon_0 n^2(x, y)$ $\Phi_n = H_{y_n}$

where ϵ_0 , μ_0 , λ , c are the permittivity, permeability, wavelength, and speed of light in vacuum, respectively. β_n is the propagation constant of the n th mode, and $n(x, y)$ is the refractive index of the medium. Each modal field of the input waveguide $\Phi'_m(x, y)$ is expanded in a set of orthonormal basic functions $\Phi_n(x, y)$:

$$\Phi'_m(x, y) = \sum_{n=1}^N a_{nm} \Phi_n(x, y) \quad (4)$$

Fig. 1. (a) Layout of the MMI-coupler. (b) Cross-section—Coupler 1: Buried structure. Coupler 2: Deep rib structure. (c) 1-D cross-section of coupler 2 for the effective index method.

These $\Phi_n(x, y)$ correspond to the modal fields of the MMI-section, which supports N guided modes, after being normalized following (3). So, the coefficients of this expansion will have the form

$$a_{nm} = \frac{\lambda \beta_n}{\pi c} \int \int_{-\infty}^{\infty} \frac{1}{ct(x, y)} \Phi'_m \Phi_n^* dx dy \quad (5)$$

if TE mode $ct = \mu_0$ $\Phi_n = E_{y_n}$ $\Phi'_m = E_{y_m}$
 if TM mode $ct = \epsilon_0 n_{MMI}^2(x, y)$ $\Phi_n = H_{y_n}$ $\Phi'_m = H_{y_m}$

where $n_{MMI}(x, y)$ is the refractive index of the medium at the MMI-section. If the M modes of the input waveguide are considered, the excitation coefficient of each mode of the MMI-section is given by

$$b_n = \sum_{m=1}^M a_{nm} c_m; \quad n = 1, \dots, N \quad (6)$$

with c_m the percentage of each modal field at the input waveguide.

Now, in the MMI-section, each mode propagates independently of the presence of the other modes. So, at the end of

Fig. 2. Output intensities P_3 and P_4 versus the length of the MMI-section calculated by the FEM. 3-dB state (L_{3dB}) and cross state (L_{cross}) are marked. (See second row of Table I for the coupler parameters.)

this section the field is given by

$$\Omega(x, y, L_{MMI}) = \sum_{n=1}^N b_n e^{i\beta_n L_{MMI}} \Phi_n(x, y). \quad (7)$$

This field is expanded independently in the modal field (Φ_l'') of each output waveguide with coefficients a_l'

$$a_l' = \frac{\lambda \beta_l}{\pi c} \int \int_{-\infty}^{\infty} \frac{1}{ct(x, y)} \Omega(x, y, L_{MMI}) \Phi_l''^* dx dy \quad (8)$$

if TE mode $ct = \mu_0$ $\Phi_l'' = E_{y_l}$ $\Omega = E_y$
 if TM mode $ct = \epsilon_0 n_0^2(x, y)$ $\Phi_l'' = H_{y_l}$ $\Omega = H_y$

where β_l is the propagation constant of the l th mode of each output waveguide and $n_0(x, y)$ is the refractive index of the medium at the output waveguide. $\Phi_l''(x, y)$ are normalized following (3).

The optical power in each output branch will be $\sum_{l=1}^L |a_l'|^2$, where L is the total number of modes at each output waveguide.

III. APPLICATIONS

In order to show the validity and usefulness of the 3-D method and to emphasize the importance of the 2-D cross-section treatment, we present the analysis of different couplers structures.

MMI couplers operating at $1.55 \mu m$ in an InGaAsP buried-type waveguide on InP substrate (coupler 1) and deep rib waveguides with larger lateral confinement (coupler 2), as shown in Fig. 1(b), are simulated. Two characteristics are defined for this analysis—the excess loss $\gamma = 10 \log(P_3 + P_4)/P_1$ and the splitting ratio $S = 10 \log \frac{P_3}{P_4}$ [the labels P_3, P_4 can be seen in Fig. 1(a)].

In Fig. 2 we show the intensities at the output waveguides for the structure described in the second row of Table I. The relevant lengths L_{3dB} for equal power splitting, and L_{cross} for cross coupling are marked. In our following discussion we will consider the 3 dB coupler behavior unless another objective was stated.

TABLE I
COMPARISON OF THE CALCULATED LENGTH OF THE MMI
SECTION (L_{MMI}) FOR COUPLER 1 AND COUPLER 2
COMPUTED BY THE FEM AND THE EI APPROXIMATION

coupler type	W_{MMI} (μm)	h_{MMI} (μm)	L_{MMI} (μm)		M number of modes
			2-D EI	3-D FEM	
1	12 ¹	.27	230(+2.2%)	225	10 (7) ²
2	18.5	.30	1422(-4.7%)	1492	16

¹Input waveguides positioned at $\frac{W_{MMI}}{3}$; ²only seven modes are really excited.

Fig. 3. Excess loss of the MMI coupler as a function of the length of the MMI-section (L_{MMI}) calculated by 2-D applying the effective index (EI) approximation and by 3-D using the FEM. (See second row of Table I for the coupler parameters.)

The optimum design length for the MMI-section (L_{MMI}), which means a minimum excess loss, has been calculated with our 3-D method and with a 2-D one using the effective index approximation. For the structure with a larger confinement the 1-D cross-sections are depicted in Fig 1(c), and the radiation modes that are not considered in our analysis will be those that follow $n_n < n_s$, with n_n the index of the n th mode of the MMI section and n_s the index of the substrate (InP). The MMI-section width (W_{MMI}) and thickness of the core (h_{MMI}) of the simulated couplers are set down in Table I. Input/output waveguides present the same cross-section and a width of 2 μm .

For the MMI coupler with the buried structure, the two values of L_{MMI} are almost equal (see Table I). The EI approximation overestimates L_{MMI} by only 2.2%. The discrepancy implies deviations in the excess loss <0.5 dB.

But in the case of the deep rib structure, the optimum design length found by the 3-D FEM is 4.7% larger than that derived from the 2-D EI calculations. Now, this deviation is related to an excess loss as large as 15 dB. This can be observed in Fig. 3, where the excess loss of coupler 2 is plotted versus the length of the MMI section. Solid lines correspond to the 3-D FEM while dotted lines represent the 2-D EI calculations.

Concerning computation times, the 2-D-EI method is obviously quicker; it takes only 20 seconds to calculate the excess loss of an MMI coupler with 10 guided modes, while the 3-D-FEM takes 25 minutes for the same coupler. If a sweep

Fig. 4. Excess loss of the MMI coupler as a function of the length of the MMI-section (L_{MMI}) calculated by 2-D with the propagating constants of the MMI-section evaluated by the finite element method (FEM), by 2-D applying the effective index (EI) approximation, and by 3-D using the FEM. (See second row of Table I for the coupler parameters.)

in L_{MMI} is considered, every step takes <2 seconds with the 2D-EI method against the 2 min. 25 seconds when using the 3-D-FEM. Although the computation times of the 3-D-FEM are higher, they are still tolerable.

If we simulate the coupler 2 with the 2-D effective index approximation—but using the propagation constants of the MMI-section computed with the finite element formulation—the same L_{MMI} is obtained as if the 3-D analysis was completely carried out (see Fig. 4). However, a higher excess loss (1.06 dB) is predicted. So, in a case where only the precised value of L_{MMI} is required, this would be a quicker method (you can save almost the 2 min. 25 seconds of each sweep in L_{MMI}). These results confirm how sensitive the optimum length is to the propagation constants of the MMI-section.

The errors in the calculated indexes with the effective index approximation for buried structures are fairly low, so the deviation when calculating L_{MMI} was almost insignificant. When calculating the propagation constants for deep rib structures, the effective index approximation gives less accurate results [3], [4]. That is the reason for the larger deviations in L_{MMI} for coupler 2.

When evaluating the dependence on fabrication tolerances we confirm that the width of the MMI-section is the most critical parameter [8]. Relaxed tolerances for wider devices should be expected from the quadratic dependence of the L_{MMI} on the W_{MMI} [9], but that is not the case.

In Table II, the width tolerances to obtain an excess loss <1.5 dB (and S better than 0.043) is shown for various widths of the MMI-section. All data have been calculated for the structure of coupler 1. It can be seen how the optimum width is around 13 μm . The excess loss limits the tolerance range unless small widths are considered and better results are obtained when injecting at $W_{MMI}/3$.

If the same analysis is carried out for the structure of coupler 2, we see that the tolerance ranges are smaller. For a W_{MMI} of 18.5 μm , when applying the FEM, a factor of 2 appears. So to stay under an excess loss of 1.5 dB, the width of a coupler with

TABLE II
WIDTH TOLERANCE RANGE ($\gamma < 1.5$ dB) FOR COUPLERS WITH THE
CROSS-SECTION OF COUPLER 1 CALCULATED BY THE EI APPROXIMATION.

W_{MMI} (μm)	h_{MMI} (μm)	L_{MMI} (μm)	δW_{MMI} (μm)	M number of modes
6.	.30	188	± 0.4	5
8.	.27	326	± 0.3	7
12. ¹	.27	230	± 0.4	10(7) ²
12.	.27	695	± 0.24	10
13. ¹	.27	262	± 0.5	11(8) ²
15. ¹	.27	348	± 0.4	12(8) ²
18. ¹	.27	492	± 0.34	15(10) ²
18.5	.27	1563	± 0.12	15

^{1,2} As in Table I.

the structure of coupler 1 must be $18.5 \pm 0.12 \mu m$, whereas for coupler 2 is must be $18.5 \pm 0.06 \mu m$.

All the previous considerations reveal the smaller tolerance ranges of coupler 2. That explains the large excess loss (15 dB) obtained for a certain deviation in L_{MMI} (4.7%). A 2.2% change in L_{MMI} results in additional loss > 6 dB when analyzing the deep rib structure; while for coupler 1 this only implies a loss of around 0.5 dB.

It is interesting to notice that these MMI couplers show large fabrication tolerances if the parameter to optimize is the splitting ratio (from Fig. 2 it can be derived that S is better than 0.17 dB for that 2.2% change in L_{MMI})

IV. CONCLUSION

A 3-D method for modeling MMI couplers using a finite element formulation has been developed. The method is illustrated by the analysis of 3-dB couplers with different cross-sections. The comparison of the results obtained with our method and a 2-D analysis based on the effective index approximation shows the dependence of the optimum design length (minimum loss) on the propagation constants of the MMI-section.

The use of a 1-D cross-section analysis is valid for those structures in which variations around 5% in the length of the coupler does not imply large additional loss as in buried-type structures. But for deep rib waveguides where the effective index approximation is not as accurate, the use of a 2-D cross-section analysis is important, otherwise excess loss greater than 15 dB can be obtained in the design of the coupler.

ACKNOWLEDGMENT

The contribution of K. Khan to the preparation of this article and the encouragement from López-Amo are gratefully acknowledged.

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Carmen Vázquez was born in Madrid, Spain, in 1968. She received the M.S. degree in applied physics (electronics) in 1991 from the Complutense University of Madrid (UCM), and the Ph.D. degree in 1995 from the Polytechnic University of Madrid (UPM).

In 1992, she joined the Optoelectronics Division of "Telefónica Investigación y Desarrollo" in Madrid. Her interests include modeling, characterization techniques, and applications of optoelectronic integrated circuits as well as fiber-optic devices with

optical amplifiers.

Francisco José Mustieles was born in Madrid, Spain, in 1961. He received both the M.S. degree in 1985 and the Ph.D. degree in 1989 in mining engineering from the Polytechnic University of Madrid (UPM). In 1990, he received the Ph.D. degree in Applied Mathematics from "Ecole Polytechnique" in Paris.

In 1990, he joined the Optoelectronics Division of "Telefónica Investigación y Desarrollo" in Madrid. In 1994, he joined the University Alfonso X el Sabio in Madrid, where he is currently leading the Applied

Mathematics Department. His current research work includes design and modeling of optoelectronics devices, numerical simulation of semiconductor devices, electromagnetic phenomena simulations, and finite elements and particle methods.

F. Hernández-Gil was born in Salamanca, Spain, in 1957. He received the B.Sc. degree in telecommunication engineering in 1981 and the Ph.D. degree in 1987, both from the Polytechnic University of Madrid (UPM).

In 1985, he joined the Research and Development Center of Telefónica, leading the Optoelectronic Department. His current research work includes design, modeling and fabrication of optoelectronics devices, and multimedia services and applications.