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Dynamic Conditional Score Models with Time-Varying Location, Scale and Shape Parameters

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Abstract

We introduce new dynamic conditional score (DCS) models with time-varying location, scale and shape parameters. For these models, we use the Student's-t, GED (general error distribution), Gen-t (generalized-t), Skew-Gen-t (skewed generalized-t), EGB2 (exponential generalized beta of the second kind) and NIG (normal-inverse Gaussian) distributions. We show that the maximum likelihood (ML) estimates of the new DCS models are consistent and asymptotically Gaussian. As an illustration, we use daily log-return time series data from the S&P 500 index for period 1950 to 2016. We find that, with respect to goodness-of-fit and predictive performance, the DCS models with dynamic shape are superior to the DCS models with constant shape and the benchmark AR-t-GARCH model.

Keywords: dynamic conditional score models, score-driven shape parameters
JEL codes: C22, C52, C58

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1. Introduction

The precise forecasting of the probability distribution of financial returns and, more specifically, the precise forecasting of volatility, are important concerns of practitioners for the effective management of financial portfolios. When probability distributions that include scale and shape parameters are used to model financial returns, then both parameters influence volatility. In standard financial time series models, the scale parameter is dynamic and the shape parameter (if it is specified) is constant over time. We suggest new financial time series models, for which both the scale and shape parameters of financial returns are dynamic. We show that changes in the scale are more related to the normal risk (non-extreme risk) of the investment (i.e., news of low or moderate impacts that frequently updates asset prices), while changes in the shape are more related to the extreme risk of the investment (i.e., news that appears from time to time with significant influence on asset prices). We also show that the normal risk component of dynamic shape is significant, which motivates the use of the new models.

Our models extend the previous financial time series models with constant shape parameters, since: (i) they have a superior likelihood-based statistical performance and forecast performance; (ii) they estimate the dynamics of both scale and shape parameters effectively; (iii) news on asset value updates the distribution of financial return not only through scale, but also through shape; (iv) they use different dynamic tail shape for the left and right tails of the return distribution; (v) they identify extreme events and sudden changes in extreme risk effectively; (vi) they can be used to separate the normal risk and extreme risk components of scale and shape, and to study the influence of those components on volatility.

In the body of literature relevant to this field, different econometric methods are used to investigate dynamic tail shape for financial returns. Quintos et al. (2001) construct tests of tail shape constancy that allow for an unknown breakpoint, and present applications of those tests for stock price data from Thailand, Malaysia and Indonesia. Galbraith and Zernov (2004) present applications of the same tests for the Dow Jones Industrial Average (DJIA) and Standard & Poor's 500 (S&P 500) indexes. More recently, Bollerslev and Todorov (2011) suggest a flexible nonparametric method of dynamic tail shape, which is used by the same authors for high-frequency data from the S&P 500. There are several methods in the body of literature that use options data to estimate dynamic tail shape for financial returns (e.g., Bakshi et al. 2003; Bollerslev et al. 2009; Backus et al. 2011). In relation to options data and dynamic tail shape, we also refer to the recent works of Bollerslev and Todorov (2014) and Bollerslev et al. (2015). Furthermore, by using panel data models, Kelly and Jiang (2014) identify a common variation in the tail shape of United States (US) stock returns. In our paper, (i) we use a new flexible parametric approach to estimate dynamic tail shape; (ii) the proposed econometric models are not only for the dynamic modeling of tail shape, but also for that of the asymmetry and peakedness of the distribution.

The main contribution of this paper is that we introduce new dynamic conditional score (DCS) models (Harvey 2013), for which the shape parameters are dynamic. We introduce those models for the Student's-t, GED (general error distribution), Gen-t (generalized-t), Skew-Gen-t (skewed generalized-t), EGB2 (exponential generalized beta of the second kind) and NIG (normal-inverse Gaussian) distributions. The new models are extensions of the DCS models with constant shape introduced in the works of Harvey (2013), Caivano and Harvey (2014), Harvey and Sucarrat (2014) and Harvey and Lange (2017). In addition, to the best of our knowledge the Skew-Gen-t-DCS and NIG-DCS specifications used in this paper are new, since (i) for Skew-Gen-t-DCS, we use a density function that has not yet been used in the body of DCS literature, and (ii) the NIG distribution has not yet been used in the body of DCS literature.

As an illustration, we use return time series data from the adjusted S&P 500 index for period 1950 to 2016. The analysis of S&P 500 data is useful, for example, for investors of (i) well-diversified US equity portfolios; (ii) S&P 500 futures and options contracts traded at Chicago Mercantile Exchange (CME); (iii) exchange traded funds (ETFs) related to the S&P 500.

We apply the results of Jensen and Rahbek (2004) to argue that the maximum likelihood (ML) estimates of the DCS models with dynamic shape are consistent and asymptotically Gaussian. We compare the statistical performance of the new DCS models with that of the DCS models with constant shape and the standard AR (autoregressive) (Box and Jenkins 1970) plus t-GARCH (generalized autoregressive conditional heteroskedasticity) with leverage effects (Bollerslev 1987; Glosten et al. 1993) model. We find that the score-driven dynamics of shape are significant for the new DCS models, and show that the likelihood-based performance of the new DCS models is superior to that of DCS with constant shape and AR-t-GARCH. We separate the normal risk and extreme risk components of scale and shape, and study the different importances of those components. We find that changes in scale are more related to the normal risk component, and changes in shape are more related to the extreme risk component. Finally, we undertake an out-of-sample exercise, and show that the density forecast performance of EGB2-DCS with dynamic shape is superior to that of t-GARCH with leverage effects.

The remainder of this paper is organized as follows. Section 2 presents the econometric framework. Section 3 presents the model specifications. Section 4 presents the statistical inferences. Section 5 presents the empirical results. Section 6 concludes.

2. Econometric framework

In all econometric specifications of this paper, we model the daily log-return time series $y_t = \ln(p_t/p_{t-1})$ for days $t = 1, \ldots, T$, where p_t is closing price, adjusted for dividends and stock splits, of a financial asset for day t (for p_0, we use pre-sample data).

2.1. Benchmark model

As the benchmark model, we use AR(p) plus t-GARCH(1,1) with leverage effects. For this standard financial time-series model $y_t = \mu_t + v_t = \mu_t + \lambda_t^{1/2}\epsilon_t$, where ϵ_t is the error term with the Student's t-distribution and a constant shape parameter (i.e., the degrees of freedom parameter). The location and squared-scale equations of this model are specified as:

$$\mu_t = c + \sum_{j=1}^{p} \phi_j y_{t-j} \tag{2.1}$$

$$\lambda_t = \omega + \beta\lambda_{t-1} + [\alpha + \alpha^* \mathbb{1}(\epsilon_{t-1} < 0)]v_{t-1}^2 \tag{2.2}$$

respectively, where $\epsilon_t \sim t[\exp(\delta_1) + 2]$ for $t = 1, \ldots, T$ is an i.i.d. sequence, and the degrees of freedom parameter implies a finite conditional variance of y_t. We initialize μ_t by using pre-sample data and λ_t by parameter λ_0. The conditional distribution of y_t is the non-standardized Student's t-distribution $t[\mu_t, \lambda_t^{1/2}, \exp(\delta_1)+2]$. The conditional mean and volatility of y_t are μ_t and $\lambda_t^{1/2}[1+2\exp(-\delta_1)]^{1/2}$, respectively. The log-density of y_t is

$$\ln f(y_t|y_1,\ldots,y_{t-1}) = \ln\Gamma\left[\frac{\exp(\delta_1)+3}{2}\right] - \ln\Gamma\left[\frac{\exp(\delta_1)+2}{2}\right] - \frac{1}{2}\ln(\pi\lambda_t) \tag{2.3}$$

$$-\frac{1}{2}\ln[\exp(\delta_1)+2] - \frac{\exp(\delta_1)+3}{2}\ln\left[1+\frac{\epsilon_t^2}{\exp(\delta_1)+2}\right]$$

2.2. DCS models of location, scale and shape

The general form of DCS models is $y_t = \mu_t + v_t = \mu_t + \exp(\lambda_t)\epsilon_t$, where μ_t and $\exp(\lambda_t)$ are the dynamic location and scale parameters, respectively. For ϵ_t, we use the Student's-t, GED, Gen-t, Skew-Gen-t, EGB2 and NIG distributions. Potentially, there may be more than one shape parameter of ϵ_t, and it is determined by the parameter $\rho_{k,t}$ ($k=1$ if there is one shape parameter, and $k \geq 1$ if the number of shape parameters is greater than one). We consider constant and dynamic alternatives of $\rho_{k,t}$. For the dynamic alternatives, $\rho_{k,t}$ is driven by the conditional score of the log-likelihood (LL) with respect to $\rho_{k,t}$ (hereafter, score function).

We present the DCS models of location, scale and shape, by using the representation of Harvey (2013) that can be related to the unobserved components models (Harvey 1989). A DCS model is obtained from an unobserved components model by replacing its error terms with the score functions. The location, scale and shape equations of the DCS model are given by

$$\mu_t = c + \left(\sum_{j=1}^{p}\phi_j\mu_{t-j}\right) + \theta u_{\mu,t-1} \tag{2.4}$$

$$\lambda_t = \omega + \beta\lambda_{t-1} + \alpha u_{\lambda,t-1} + \alpha^*\mathrm{sgn}(-\epsilon_{t-1})(u_{\lambda,t-1}+1) \tag{2.5}$$

$$\rho_{k,t} = \delta_k + \gamma_k\rho_{k,t-1} + \kappa_k u_{\rho,k,t-1} \tag{2.6}$$

respectively, where $\mathrm{sgn}(x)$ is the signum function. We initialize μ_t by using pre-sample data of

y_t, λ_t by parameter λ_0 and $\rho_{k,t}$ by using $\delta_k/(1-\gamma_k)$. It is worth noting that, as an alternative, we also use parameter $\rho_{k,0}$ to initialize $\rho_{k,t}$. For each DCS model with dynamic shape, we also consider the alternative $\rho_{k,t} = \delta_k$ that is a DCS model with constant shape.

The general notation $\rho_{k,t}$ is specified as ν_t, η_t, τ_t, ξ_t and ζ_t for different shape parameters (Section 3). If a given DCS model includes more than one shape parameter, then we use a different parameter index for each shape parameter. For example, we use δ_1, γ_1 and κ_1 for ν_t and δ_2, γ_2 and κ_2 for η_t for the case of Gen-*t*-DCS (Section 3.3).

The parameters μ_t, λ_t and $\rho_{k,t}$ are updated by lags of the score functions $u_{\mu,t}$, $u_{\lambda,t}$ and $u_{\rho,k,t}$, respectively. For μ_t, we use the DCS-QAR(p) model (Harvey 2013). For λ_t, we use the DCS-EGARCH (exponential GARCH) model with leverage effects (Harvey and Chakravarty 2008; Harvey 2013). In the body of literature, DCS-EGARCH with constant shape parameters that uses the Student's *t*, GED, Gen-*t*, Skew-Gen-*t* and EGB2 distributions for ϵ_t is named as Beta-*t*-EGARCH (Harvey and Chakravarty 2008), GED-EGARCH (Harvey 2013), Beta-Gen-*t*-EGARCH (Harvey and Lange 2017), Beta-Skew-Gen-*t*-EGARCH (Harvey and Lange 2017) and EGB2-EGARCH (Caivano and Harvey 2014), respectively. For $\rho_{k,t}$, we use the DCS-QAR(1) model that is applied for location μ_t in the work of Harvey (2013). To the best of our knowledge, the use of score-driven shape parameters is new in the body of literature.

It is noteworthy that in this paper we use the Skew-Gen-*t* density function from the work of McDonald and Michelfelder (2017) for ϵ_t, which is different from the Skew-Gen-*t* density function used in the work of Harvey and Lange (2017). Furthermore, to the best of our knowledge, the present paper is the first to use the NIG distribution (Barndorff-Nielsen and Halgreen 1977) for DCS models of location, scale and shape. We name DCS-EGARCH with NIG distribution, as the NIG-EGARCH model.

2.3. Error terms of DCS models with time-varying location, scale and shape

In this section, we present the six alternative specifications of ϵ_t. In Fig. 1, for illustration, we present the probability density function of each specification for different shape parameters, and compare each of them with the standard normal distribution.

First, $\epsilon_t \sim t[0, 1, \exp(\nu_t) + 2]$, where ν_t influences the tail-heaviness of ϵ_t. The conditional mean and variance of ϵ_t are

$$E(\epsilon_t | y_1, \ldots, y_{t-1}) = 0 \tag{2.7}$$

$$\text{Var}(\epsilon_t | y_1, \ldots, y_{t-1}) = 1 + 2\exp(-\nu_t) \tag{2.8}$$

respectively. In this specification, the degrees of freedom parameter $[\exp(\nu_t) + 2]$ is greater than two, hence, the conditional variance is finite.

Second, $\epsilon_t \sim \text{GED}[0, 1, \exp(\nu_t)]$, where ν_t influences the peakedness of ϵ_t. The conditional mean and variance of ϵ_t are

$$E(\epsilon_t | y_1, \ldots, y_{t-1}) = 0 \tag{2.9}$$

$$\text{Var}(\epsilon_t | y_1, \ldots, y_{t-1}) = 2^{2\exp(-\nu_t)} \frac{\Gamma[3\exp(-\nu_t)]}{\Gamma[\exp(-\nu_t)]} \tag{2.10}$$

respectively, where $\Gamma(x)$ is the gamma function.

Third, $\epsilon_t \sim \text{Gen-}t[0, 1, \exp(\nu_t) + 2, \exp(\eta_t)]$, where ν_t and η_t influence the tail-heaviness and peakedness of ϵ_t, respectively. The conditional mean and variance of ϵ_t are

$$E(\epsilon_t | y_1, \ldots, y_{t-1}) = 0 \tag{2.11}$$

$$\text{Var}(\epsilon_t | y_1, \ldots, y_{t-1}) = [\exp(\nu_t) + 2]^{2\exp(-\eta_t)} \times \frac{\Gamma\left[3\exp(-\eta_t)\right]\Gamma[\exp(\nu_t - \eta_t)]}{\Gamma\left[\exp(-\eta_t)\right]\Gamma\left[\frac{\exp(\nu_t)+2}{\exp(\eta_t)}\right]} \tag{2.12}$$

respectively. In this specification, the degrees of freedom parameter $[\exp(\nu_t) + 2]$ is greater than two, hence, the conditional variance is finite.

For the Student's-t, GED and Gen-t models, ϵ_t is a martingale difference sequence (MDS), i.e., $E(\epsilon_t | y_1, \ldots, y_{t-1}) = 0$. We estimate the residuals by using $\hat{\epsilon}_t = (y_t - \hat{\mu}_t)\exp(-\hat{\lambda}_t)$. For $\hat{\epsilon}_t$, as suggested by Harvey (2013), we apply the MDS test of Escanciano and Lobato (2009) that involves an automatic procedure for lag selection in the statistical test.

Fourth, $\epsilon_t \sim \text{Skew-Gen-}t[0, 1, \tanh(\tau_t), \exp(\nu_t) + 2, \exp(\eta_t)]$, where $\tanh(x)$ is the hyperbolic tangent function, and τ_t, ν_t and η_t influence asymmetry, tail-heaviness and peakedness, respectively, of ϵ_t. The Skew-Gen-t distribution uses different dynamic tail shape for the left and right tails, similar to the recent works of Bollerslev and Todorov (2014) and Bollerslev et al. (2015). The conditional mean and variance of ϵ_t are

$$E(\epsilon_t|y_1, \ldots, y_{t-1}) = \frac{2\tanh(\tau_t)[\exp(\nu_t) + 2]^{\exp(-\eta_t)} B\left\{\frac{2}{\exp(\eta_t)}, \frac{\exp(\nu_t)+1}{exp(\eta_t)}\right\}}{B\left\{\frac{1}{\exp(\eta_t)}, \frac{\exp(\nu_t)+2}{\exp(\eta_t)}\right\}} \tag{2.13}$$

$$\text{Var}(\epsilon_t|y_1, \ldots, y_{t-1}) = [\exp(\nu_t) + 2]^{2\exp(-\eta_t)} \times \tag{2.14}$$

$$\times \left\{ \frac{[3\tanh^2(\tau_t) + 1] B\left[\frac{3}{\exp(\eta_t)}, \frac{\exp(\nu_t)}{\exp(\eta_t)}\right]}{B\left[\frac{1}{\exp(\eta_t)}, \frac{\exp(\nu_t)+2}{\exp(\eta_t)}\right]} - \frac{4\tanh^2(\tau_t) B^2\left[\frac{2}{\exp(\eta_t)}, \frac{\exp(\nu_t)+1}{\exp(\eta_t)}\right]}{B^2\left[\frac{1}{\exp(\eta_t)}, \frac{\exp(\nu_t)+2}{\exp(\eta_t)}\right]} \right\}$$

respectively, where $B(x, y) = \Gamma(x)\Gamma(y)/\Gamma(x + y)$ is the Beta function. In this specification, the degrees of freedom parameter $[\exp(\nu_t) + 2]$ is greater than two, hence, the conditional variance is finite. Furthermore, the asymmetry parameter $\tanh(\tau_t)$ is in the interval $(-1, 1)$, as required for Skew-Gen-t.

Fifth, $\epsilon_t \sim \text{EGB2}[0, 1, \exp(\xi_t), \exp(\zeta_t)]$, where ξ_t and ζ_t influence both asymmetry and tail-heaviness. EGB2 uses different dynamic tail shape for the left and right tails, similar to the recent works of Bollerslev and Todorov (2014) and Bollerslev et al. (2015). The conditional mean and variance of ϵ_t are

$$E(\epsilon_t|y_1, \ldots, y_{t-1}) = \Psi^{(0)}[\exp(\xi_t)] - \Psi^{(0)}[\exp(\zeta_t)] \tag{2.15}$$

$$\text{Var}(\epsilon_t|y_1, \ldots, y_{t-1}) = \Psi^{(1)}[\exp(\xi_t)] + \Psi^{(1)}[\exp(\zeta_t)] \tag{2.16}$$

respectively, where $\Psi^{(0)}(x)$ and $\Psi^{(1)}(x)$ are polygamma functions of orders 0 and 1, respectively.

Sixth, $\epsilon_t \sim \text{NIG}[0, 1, \exp(\nu_t), \exp(\nu_t)\tanh(\eta_t)]$, where ν_t and η_t influence tail-heaviness and asymmetry, respectively. NIG uses different dynamic tail shape for the left and right tails, similar to the recent works of Bollerslev and Todorov (2014) and Bollerslev et al. (2015). The conditional mean and variance of ϵ_t are

$$E(\epsilon_t|y_1,\ldots,y_{t-1}) = \frac{\tanh(\eta_t)}{[1-\tanh^2(\eta_t)]^{1/2}} \tag{2.17}$$

$$\mathrm{Var}(\epsilon_t|y_1,\ldots,y_{t-1}) = \frac{\exp(-\nu_t)}{[1-\tanh^2(\eta_t)]^{3/2}} \tag{2.18}$$

respectively. In this specification, the absolute value of the asymmetry parameter $|\exp(\nu_t)\tanh(\eta_t)|$ is less than the tail-heaviness parameter $\exp(\nu_t)$ that is required for NIG.

For the Skew-Gen-t, EGB2 and NIG models, $E(\epsilon_t|y_1,\ldots,y_{t-1}) \neq 0$. The conditional mean of y_t for these models is $E(y_t|y_1,\ldots,y_{t-1}) = \mu_t + \exp(\lambda_t)E(\epsilon_t|y_1,\ldots,y_{t-1})$. Given the formula of $E(\epsilon_t|y_1,\ldots,y_{t-1})$ for Skew-Gen-t, EGB2 and NIG, we define the transformed residuals as $\epsilon_t^* = \epsilon_t - E(\epsilon_t|y_1,\ldots,y_{t-1})$. We estimate the transformed residuals by using

$$\hat{\epsilon}_t^* = (y_t - \hat{\mu}_t)\exp(-\hat{\lambda}_t) - \hat{E}(\epsilon_t|y_1,\ldots,y_{t-1}) \tag{2.19}$$

For $\hat{\epsilon}_t^*$, we apply the MDS test of Escanciano and Lobato (2009).

3. DCS specifications of location, scale and shape

In this section, for each error specification, we present the conditional distribution of y_t, the conditional mean and volatility of y_t, the log of the conditional density of y_t, and the score functions with respect to location, scale and shape.

3.1. t-DCS model

The conditional distribution of the log-return y_t is the non-standardized Student's t-distribution $t[\mu_t, \exp(\lambda_t), \exp(\nu_t)+2]$. The conditional mean and volatility of the log-return y_t are μ_t and $\exp(\lambda_t)[1+2\exp(-\nu_t)]^{1/2}$, respectively. The log of the conditional density of y_t is

$$\ln f(y_t|y_1,\ldots,y_{t-1}) = \ln\Gamma\left[\frac{\exp(\nu_t)+3}{2}\right] - \ln\Gamma\left[\frac{\exp(\nu_t)+2}{2}\right] \tag{3.1}$$

$$-\frac{\ln(\pi)+\ln[\exp(\nu_t)+2]}{2}-\lambda_t-\frac{\exp(\nu_t)+3}{2}\ln\left\{1+\frac{\epsilon_t^2}{\exp(\nu_t)+2}\right\}$$

In general, the conditional score of y_t is the partial derivative of $\ln f(y_t|y_1,\ldots,y_{t-1})$ with respect to a time-varying parameter (Harvey 2013). In the t-DCS model, the score functions with respect to μ_t, λ_t and ν_t are as follows. First, the score function with respect to μ_t is

$$\frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1})}{\partial \mu_t}=\frac{\exp(\lambda_t)\epsilon_t}{\epsilon_t^2+\exp(\nu_t)+2}\times\frac{\exp(\nu_t)+3}{\exp(2\lambda_t)}=u_{\mu,t}\times\frac{\exp(\nu_t)+3}{\exp(2\lambda_t)} \tag{3.2}$$

where $u_{\mu,t}$ is the scaled score function. Second, the score function with respect to λ_t is

$$u_{\lambda,t}=\frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1})}{\partial \lambda_t}=\frac{[\exp(\nu_t)+3]\epsilon_t^2}{\exp(\nu_t)+2+\epsilon_t^2}-1 \tag{3.3}$$

Third, the score function with respect to ν_t is

$$u_{\nu,t}=\frac{\exp(\nu_t)}{2}\Psi^{(0)}\left[\frac{\exp(\nu_t)+3}{2}\right]-\frac{\exp(\nu_t)}{2}\Psi^{(0)}\left[\frac{\exp(\nu_t)+2}{2}\right]-\frac{\exp(\nu_t)}{2\exp(\nu_t)+4} \tag{3.4}$$

$$+\frac{\exp(\nu_t)[\exp(\nu_t)+3]\epsilon_t^2}{2[\exp(\nu_t)+2][\epsilon_t^2+\exp(\nu_t)+2]}-\frac{\exp(\nu_t)}{2}\times\ln\left[1+\frac{\epsilon_t^2}{\exp(\nu_t)+2}\right]$$

3.2. GED-DCS model

The conditional distribution of the log-return y_t is the non-standardized GED distribution, denoted as GED$[\mu_t,\exp(\lambda_t),\exp(\nu_t)]$. The conditional mean and volatility of y_t are μ_t and

$$\exp(\lambda_t)2^{\exp(-\nu_t)}\times\left\{\frac{\Gamma[3\exp(-\nu_t)]}{\Gamma[\exp(-\nu_t)]}\right\}^{1/2} \tag{3.5}$$

respectively. The log of the conditional density of y_t is

$$\ln f(y_t|y_1,\ldots,y_{t-1})=-[1+\exp(-\nu_t)]\ln(2)-\lambda_t-\ln\Gamma[1+\exp(-\nu_t)]-\frac{1}{2}|\epsilon_t|^{\exp(\nu_t)} \tag{3.6}$$

The score functions with respect to μ_t, λ_t and ν_t are formulated as follows. First, the score function with respect to μ_t is

$$\frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1})}{\partial \mu_t} = \epsilon_t|\epsilon_t|^{\exp(\nu_t)-2} \times \frac{\exp(\nu_t-\lambda_t)}{2} = u_{\mu,t} \times \frac{\exp(\nu_t-\lambda_t)}{2} \tag{3.7}$$

where $u_{\mu,t}$ is the scaled score function. Second, the score function with respect to λ_t is

$$u_{\lambda,t} = \frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1})}{\partial \lambda_t} = \frac{\exp(\nu_t)}{2}|\epsilon_t|^{\exp(\nu_t)} - 1 \tag{3.8}$$

Third, the score function with respect to ν_t is

$$u_{\nu,t} = \frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1})}{\partial \nu_t} = \exp(-\nu_t)\ln(2) + \exp(-\nu_t)\Psi^{(0)}[1+\exp(-\nu_t)] \tag{3.9}$$

$$-\frac{\exp(\nu_t)}{2}|\epsilon_t|^{\exp(\nu_t)}\ln|\epsilon_t|$$

3.3. Gen-t-DCS model

The conditional distribution of y_t is the non-standardized Gen-t distribution that we denote by Gen-$t[\mu_t, \exp(\lambda_t), \exp(\nu_t)+2, \exp(\eta_t)]$. The conditional mean and volatility of y_t are μ_t and

$$\exp(\lambda_t)[\exp(\nu_t)+2]^{\exp(-\eta_t)} \times \left\{\frac{\Gamma[3\exp(-\eta_t)]\Gamma[\exp(\nu_t-\eta_t)]}{\Gamma[\exp(-\eta_t)]\Gamma\left[\frac{\exp(\nu_t)+2}{\exp(\eta_t)}\right]}\right\}^{1/2} \tag{3.10}$$

respectively. The log of the conditional density of y_t is

$$\ln f(y_t|y_1,\ldots,y_{t-1}) = \eta_t - \lambda_t - \ln(2) - \frac{\ln[\exp(\nu_t)+2]}{\exp(\eta_t)} - \ln\Gamma\{[\exp(\nu_t)+2]\exp(-\eta_t)\} \tag{3.11}$$

$$-\ln\Gamma[\exp(-\eta_t)] + \ln\Gamma\{[\exp(\nu_t)+3]\exp(-\eta_t)\} - \frac{\exp(\nu_t)+3}{\exp(\eta_t)}\ln\left[1+\frac{|\epsilon_t|^{\exp(\eta_t)}}{\exp(\nu_t)+2}\right]$$

The score functions with respect to the time-varying parameters μ_t, λ_t, ν_t and η_t are formulated as follows. First, the score function with respect to μ_t is

$$\frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1})}{\partial \mu_t} = \frac{\exp(\lambda_t)\epsilon_t|\epsilon_t|^{\exp(\eta_t)-2}}{|\epsilon_t|^{\exp(\eta_t)}+\exp(\nu_t)+2} \times \frac{\exp(\nu_t)+3}{\exp(2\lambda_t)} = u_{\mu,t} \times \frac{\exp(\nu_t)+3}{\exp(2\lambda_t)} \tag{3.12}$$

where $u_{\mu,t}$ is the scaled score function. Second, the score function with respect to λ_t is

$$u_{\lambda,t} = \frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1})}{\partial \lambda_t} = \frac{[\exp(\nu_t)+3]|\epsilon_t|^{\exp(\eta_t)}}{|\epsilon_t|^{\exp(\eta_t)}+\exp(\nu_t)+2} - 1 \tag{3.13}$$

Third, the score function with respect to ν_t is

$$u_{\nu,t} = \frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1})}{\partial \nu_t} = -\frac{\exp(\nu_t-\eta_t)}{[\exp(\nu_t)+2]} \tag{3.14}$$

$$-\exp(\nu_t-\eta_t)\Psi^{(0)}\left[\frac{\exp(\nu_t)+2}{\exp(\eta_t)}\right] + \exp(\nu_t-\eta_t)\Psi^{(0)}\left[\frac{\exp(\nu_t)+3}{\exp(\eta_t)}\right]$$

$$-\exp(\nu_t-\eta_t)\ln\left[1+\frac{|\epsilon_t|^{\exp(\eta_t)}}{\exp(\nu_t)+2}\right] + \frac{\exp(\nu_t-\eta_t)[\exp(\nu_t)+3]|\epsilon_t|^{\exp(\eta_t)}}{[\exp(\nu_t)+2][|\epsilon_t|^{\exp(\eta_t)}+\exp(\nu_t)+2]}$$

Fourth, the score function with respect to η_t is

$$u_{\eta,t} = \frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1})}{\partial \eta_t} = 1 + \frac{\ln[\exp(\nu_t)+2]}{\exp(\eta_t)} + \frac{\exp(\nu_t)+2}{\exp(\eta_t)}\Psi^{(0)}\left[\frac{\exp(\nu_t)+2}{\exp(\eta_t)}\right] \tag{3.15}$$

$$+\frac{1}{\exp(\eta_t)}\Psi^{(0)}\left[\frac{1}{\exp(\eta_t)}\right] - \frac{\exp(\nu_t)+3}{\exp(\eta_t)}\Psi^{(0)}\left[\frac{\exp(\nu_t)+3}{\exp(\eta_t)}\right]$$

$$-\frac{[\exp(\nu_t)+3]|\epsilon_t|^{\exp(\eta_t)}\ln|\epsilon_t|}{|\epsilon_t|^{\exp(\eta_t)}+\exp(\nu_t)+2} + \frac{\exp(\nu_t)+3}{\exp(\eta_t)} \times \ln\left[1+\frac{|\epsilon_t|^{\exp(\eta_t)}}{\exp(\nu_t)+2}\right]$$

3.4. Skew-Gen-t-DCS model

The conditional distribution of y_t is

$$y_t|(y_1,\ldots,y_{t-1}) \sim \text{Skew-Gen-}t[\mu_t, \exp(\lambda_t), \tanh(\tau_t), \exp(\nu_t)+2, \exp(\eta_t)] \tag{3.16}$$

The conditional mean of y_t is

$$\mu_t + 2\exp(\lambda_t)\tanh(\tau_t)[\exp(\nu_t)+2]^{\exp(-\eta_t)} \times \frac{B\left\{\frac{2}{\exp(\eta_t)}, \frac{\exp(\nu_t)+1}{\exp(\eta_t)}\right\}}{B\left\{\frac{1}{\exp(\eta_t)}, \frac{\exp(\nu_t)+2}{\exp(\eta_t)}\right\}} \tag{3.17}$$

The conditional volatility of y_t is

$$\exp(\lambda_t)[\exp(\nu_t)+2]^{\exp(-\eta_t)}\times \tag{3.18}$$

$$\times\left\{\frac{[3\tanh^2(\tau_t)+1]B\left[\frac{3}{\exp(\eta_t)}, \frac{\exp(\nu_t)}{\exp(\eta_t)}\right]}{B\left[\frac{1}{\exp(\eta_t)}, \frac{\exp(\nu_t)+2}{\exp(\eta_t)}\right]} - \frac{4\tanh^2(\tau_t)B^2\left[\frac{2}{\exp(\eta_t)}, \frac{\exp(\nu_t)+1}{\exp(\eta_t)}\right]}{B^2\left[\frac{1}{\exp(\eta_t)}, \frac{\exp(\nu_t)+2}{\exp(\eta_t)}\right]}\right\}^{1/2}$$

The log of the conditional density of y_t is

$$\ln f(y_t|y_1,\ldots,y_{t-1}) = \eta_t - \lambda_t - \ln(2) - \frac{\ln[\exp(\nu_t)+2]}{\exp(\eta_t)} - \ln\Gamma\left[\frac{\exp(\nu_t)+2}{\exp(\eta_t)}\right] \tag{3.19}$$

$$-\ln\Gamma[\exp(-\eta_t)] + \ln\Gamma\left[\frac{\exp(\nu_t)+3}{\exp(\eta_t)}\right]$$

$$-\frac{\exp(\nu_t)+3}{\exp(\eta_t)}\ln\left\{1+\frac{|\epsilon_t|^{\exp(\eta_t)}}{[1+\tanh(\tau_t)\mathrm{sgn}(\epsilon_t)]^{\exp(\eta_t)}\times[\exp(\nu_t)+2]}\right\}$$

First, the score function with respect to μ_t is

$$\frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1})}{\partial\mu_t} = \tag{3.20}$$

$$= \frac{\exp(\lambda_t)\epsilon_t|\epsilon_t|^{\exp(\eta_t)-2}}{|\epsilon_t|^{\exp(\eta_t)} + [1+\tanh(\tau_t)\mathrm{sgn}(\epsilon_t)]^{\exp(\eta_t)}[\exp(\nu_t)+2]} \times \frac{\exp(\nu_t)+3}{\exp(2\lambda_t)} =$$

$$= u_{\mu,t} \times \frac{\exp(\nu_t)+3}{\exp(2\lambda_t)}$$

where $u_{\mu,t}$ is the scaled score function. Second, the score function with respect to λ_t is

$$u_{\lambda,t} = \frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1})}{\partial\lambda_t} = \frac{|\epsilon_t|^{\exp(\eta_t)}[\exp(\nu_t)+3]}{|\epsilon_t|^{\exp(\eta_t)} + [1+\tanh(\tau_t)\mathrm{sgn}(\epsilon_t)]^{\exp(\eta_t)}[\exp(\nu_t)+2]} - 1 \tag{3.21}$$

Third, the score function with respect to τ_t is

$$u_{\tau,t} = \frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1})}{\partial \tau_t} = \frac{[\exp(\nu_t)+3]|\epsilon_t|^{\exp(\eta_t)}\text{sgn}(\epsilon_t)\text{sech}(\tau_t)}{[\text{sgn}(\epsilon_t)\text{sinh}(\tau_t)+\cosh(\tau_t)]}\times \tag{3.22}$$

$$\times\left\{|\epsilon_t|^{\exp(\eta_t)}+[1+\tanh(\tau_t)\text{sgn}(\epsilon_t)]^{\exp(\eta_t)}[\exp(\nu_t)+2]\right\}^{-1}$$

Fourth, the score function with respect to ν_t is

$$u_{\nu,t} = \frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1})}{\partial \nu_t} = -\frac{\exp(\nu_t-\eta_t)}{\exp(\nu_t)+2} - \exp(\nu_t-\eta_t)\Psi^{(0)}\left[\frac{\exp(\nu_t)+2}{\exp(\eta_t)}\right] \tag{3.23}$$

$$+\exp(\nu_t-\eta_t)\Psi^{(0)}\left[\frac{\exp(\nu_t)+3}{\exp(\eta_t)}\right]$$

$$+\frac{\exp(\nu_t-\eta_t)[\exp(\nu_t)+3]|\epsilon_t|^{\exp(\eta_t)}}{[\exp(\nu_t)+2]\left\{|\epsilon_t|^{\exp(\eta_t)}+[1+\tanh(\tau_t)\text{sgn}(\epsilon_t)]^{\exp(\eta_t)}[\exp(\nu_t)+2]\right\}}$$

$$-\exp(\nu_t-\eta_t)\ln\left\{1+\frac{|\epsilon_t|^{\exp(\eta_t)}}{[1+\tanh(\tau_t)\text{sgn}(\epsilon_t)]^{\exp(\eta_t)}[\exp(\nu_t)+2]}\right\}$$

Fifth, the score function with respect to η_t is

$$u_{\eta,t} = \frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1})}{\partial \eta_t} = 1+\frac{\ln[\exp(\nu_t)+2]}{\exp(\eta_t)}+\frac{\exp(\nu_t)+2}{\exp(\eta_t)}\Psi^{(0)}\left[\frac{\exp(\nu_t)+2}{\exp(\eta_t)}\right] \tag{3.24}$$

$$+\frac{1}{\exp(\eta_t)}\Psi^{(0)}\left[\frac{1}{\exp(\eta_t)}\right]-\frac{\exp(\nu_t)+3}{\exp(\eta_t)}\Psi^{(0)}\left[\frac{\exp(\nu_t)+3}{\exp(\eta_t)}\right]$$

$$+\frac{\exp(\nu_t)+3}{\exp(\eta_t)}\ln\left\{1+\frac{|\epsilon_t|^{\exp(\eta_t)}[1+\tanh(\tau_t)\text{sgn}(\epsilon_t)]^{-\exp(\eta_t)}}{\exp(\nu_t)+2}\right\}$$

$$+\frac{[\exp(\nu_t)+3]|\epsilon_t|^{\exp(\eta_t)}\ln[1+\tanh(\tau_t)\text{sgn}(\epsilon_t)]}{|\epsilon_t|^{\exp(\eta_t)}+[\exp(\nu_t)+2][1+\tanh(\tau_t)\text{sgn}(\epsilon_t)]^{\exp(\eta_t)}}$$

$$-\frac{[\exp(\nu_t)+3]|\epsilon_t|^{\exp(\eta_t)}\ln(|\epsilon_t|)}{|\epsilon_t|^{\exp(\eta_t)}+[\exp(\nu_t)+2][1+\tanh(\tau_t)\text{sgn}(\epsilon_t)]^{\exp(\eta_t)}}$$

3.5. EGB2-DCS model

The conditional distribution of y_t is EGB2$[\mu_t, \exp(-\lambda_t), \exp(\xi_t), \exp(\zeta_t)]$. The conditional mean and volatility of y_t are $\mu_t+\exp(\lambda_t)\left\{\Psi^{(0)}[\exp(\xi_t)]-\Psi^{(0)}[\exp(\zeta_t)]\right\}$ and $\exp(\lambda_t)\{\Psi^{(1)}[\exp(\xi_t)]+$

$\Psi^{(1)}[\exp(\zeta_t)]\}^{1/2}$, respectively. The log of the conditional density of y_t is

$$\ln f(y_t|y_1,\ldots,y_{t-1}) = \exp(\xi_t)\epsilon_t - \lambda_t - \ln\Gamma[\exp(\xi_t)] - \ln\Gamma[\exp(\zeta_t)] \tag{3.25}$$

$$+\ln\Gamma[\exp(\xi_t)+\exp(\zeta_t)] - [\exp(\xi_t)+\exp(\zeta_t)]\ln[1+\exp(\epsilon_t)]$$

The score functions with respect to μ_t, λ_t, ξ_t and ζ_t are as follows. First, the score function with respect to μ_t is

$$\frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1})}{\partial \mu_t} = u_{\mu,t} \times \{\Psi^{(1)}[\exp(\xi_t)] + \Psi^{(1)}[\exp(\zeta_t)]\}\exp(2\lambda_t) \tag{3.26}$$

where

$$u_{\mu,t} = \{\Psi^{(1)}[\exp(\xi_t)]+\Psi^{(1)}[\exp(\zeta_t)]\}\exp(\lambda_t)\left\{[\exp(\xi_t)+\exp(\zeta_t)]\frac{\exp(\epsilon_t)}{\exp(\epsilon_t)+1} - \exp(\xi_t)\right\} \tag{3.27}$$

is the scaled score function. Second, the score function with respect to λ_t is

$$u_{\lambda,t} = \frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1})}{\partial \lambda_t} = [\exp(\xi_t)+\exp(\zeta_t)]\frac{\epsilon_t\exp(\epsilon_t)}{\exp(\epsilon_t)+1} - \exp(\xi_t)\epsilon_t - 1 \tag{3.28}$$

Third, the score function with respect to ξ_t is

$$u_{\xi,t} = \frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1})}{\partial \xi_t} = \exp(\xi_t)\epsilon_t - \exp(\xi_t)\Psi^{(0)}[\exp(\xi_t)] \tag{3.29}$$

$$+\exp(\xi_t)\Psi^{(0)}[\exp(\xi_t)+\exp(\zeta_t)] - \exp(\xi_t)\ln[1+\exp(\epsilon_t)]$$

Fourth, the score function with respect to ζ_t is

$$u_{\zeta,t} = \frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1})}{\partial \zeta_t} = -\exp(\zeta_t)\Psi^{(0)}[\exp(\zeta_t)] \tag{3.30}$$

$$+\exp(\zeta_t)\Psi^{(0)}[\exp(\xi_t)+\exp(\zeta_t)] - \exp(\zeta_t)\ln[1+\exp(\epsilon_t)]$$

3.6. NIG-DCS model

The conditional distribution of y_t is

$$y_t|(y_1,\ldots,y_{t-1}) \sim \text{NIG}[\mu_t, \exp(\lambda_t), \exp(\nu_t-\lambda_t), \exp(\nu_t-\lambda_t)\tanh(\eta_t)] \tag{3.31}$$

The conditional mean and volatility of y_t are

$$\mu_t + \frac{\exp(\lambda_t)\tanh(\eta_t)}{[1-\tanh^2(\eta_t)]^{1/2}} \tag{3.32}$$

$$\left\{\frac{\exp(2\lambda_t-\nu_t)}{[1-\tanh^2(\eta_t)]^{3/2}}\right\}^{1/2} \tag{3.33}$$

respectively. The log of the conditional density of y_t is

$$\ln f(y_t|y_1,\ldots,y_{t-1}) = \nu_t - \lambda_t - \ln(\pi) + \exp(\nu_t)[1-\tanh^2(\eta_t)]^{1/2} \tag{3.34}$$

$$+\exp(\nu_t)\tanh(\eta_t)\epsilon_t + \ln K^{(1)}\left[\exp(\nu_t)\sqrt{1+\epsilon_t^2}\right] - \frac{1}{2}\ln(1+\epsilon_t^2)$$

where $K^{(1)}(x)$ is the modified Bessel function of the second kind of order 1. The score functions with respect to μ_t, λ_t, ν_t and η_t are as follows. First, the score function with respect to μ_t is

$$\frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1})}{\partial \mu_t} = -\exp(\nu_t-\lambda_t)\tanh(\eta_t) + \frac{\epsilon_t}{\exp(\lambda_t)(1+\epsilon_t^2)} \tag{3.35}$$

$$+\frac{\exp(\nu_t-\lambda_t)\epsilon_t}{\sqrt{1+\epsilon_t^2}} \times \frac{K^{(0)}\left[\exp(\nu_t)\sqrt{1+\epsilon_t^2}\right] + K^{(2)}\left[\exp(\nu_t)\sqrt{1+\epsilon_t^2}\right]}{2K^{(1)}\left[\exp(\nu_t)\sqrt{1+\epsilon_t^2}\right]}$$

where $K^{(0)}(x)$ and $K^{(2)}(x)$ are the modified Bessel functions of the second kind of orders 0 and 2, respectively. We define the scaled score function with respect to μ_t as

$$u_{\mu,t} = \frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1})}{\partial \mu_t} \times \exp(2\lambda_t) \tag{3.36}$$

Second, the score function with respect to λ_t is

$$u_{\lambda,t} = \frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1})}{\partial \lambda_t} = -1 - \exp(\nu_t)\tanh(\eta_t)\epsilon_t + \frac{\epsilon_t^2}{1+\epsilon_t^2} \tag{3.37}$$

$$+\frac{\exp(\nu_t)\epsilon_t^2}{\sqrt{1+\epsilon_t^2}} \times \frac{K^{(0)}\left[\exp(\nu_t)\sqrt{1+\epsilon_t^2}\right] + K^{(2)}\left[\exp(\nu_t)\sqrt{1+\epsilon_t^2}\right]}{2K^{(1)}\left[\exp(\nu_t)\sqrt{1+\epsilon_t^2}\right]}$$

Third, the score function with respect to ν_t is

$$u_{\nu,t} = \frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1})}{\partial \nu_t} = 1 + \exp(\nu_t)[1-\tanh^2(\eta_t)]^{1/2} + \exp(\nu_t)\tanh(\eta_t)\epsilon_t \tag{3.38}$$

$$-\exp(\nu_t)\sqrt{1+\epsilon_t^2} \times \frac{K^{(0)}\left[\exp(\nu_t)\sqrt{1+\epsilon_t^2}\right] + K^{(2)}\left[\exp(\nu_t)\sqrt{1+\epsilon_t^2}\right]}{2K^{(1)}\left[\exp(\nu_t)\sqrt{1+\epsilon_t^2}\right]}$$

Fourth, the score function with respect to η_t is

$$u_{\eta,t} = \frac{\partial \ln f(y_t|y_1,\ldots,y_{t-1})}{\partial \eta_t} = \exp(\nu_t)\mathrm{sech}^2(\eta_t)\epsilon_t - \exp(\nu_t)\tanh(\eta_t)\mathrm{sech}(\eta_t) \tag{3.39}$$

where $\mathrm{sech}(x)$ is the hyperbolic secant function.

4. Statistical inference

We estimate the parameters of all models by using the ML method (Davidson and MacKinnon 2003). We introduce the notation $\Theta = (\Theta_1,\ldots,\Theta_K)'$ for the $K \times 1$ vector of time-constant parameters. The ML estimator of parameters is

$$\hat{\Theta}_{\mathrm{ML}} = \arg\max_{\Theta} \mathrm{LL}(y_1,\ldots,y_T;\Theta) = \arg\max_{\Theta} \frac{1}{T}\sum_{t=1}^{T} \ln f(y_t|y_1,\ldots,y_{t-1};\Theta) \tag{4.1}$$

Our criterion for effective ML estimation is convergence to the maximum LL at interior points of the parameter space, with 10^{-5} convergence tolerance for the gradient. We numerically estimate the standard errors of parameters $\mathrm{SE}_\Theta = (\mathrm{SE}_{\Theta 1},\ldots,\mathrm{SE}_{\Theta K})'$. We use the delta method to estimate the standard errors of transformed parameters. We estimate p-values in order to test

$\mathrm{H}_0 : \Theta_j = 0$, by using the standard normal distribution for $\hat{\Theta}_j/\hat{\mathrm{SE}}_{\Theta j}$. The use of the standard normal distribution for ML is validated in the remainder of this section.

4.1. AR plus t-GARCH with leverage effects

We evaluate two conditions of consistency and asymptotic normality of the ML estimates. First, for the AR(p) equation, we numerically solve $1 - \phi_1 z - \phi_2 z^2 - \ldots - \phi_p z^p = 0$ (Hamilton 1994), and compute the minimum modulus of all roots that we denote as C_μ. This condition requires that $C_\mu > 1$. Second, for the GARCH(1,1) with leverage effects equation, we estimate $C_\lambda = \alpha + 0.5\alpha^* + \beta$ (Glosten et al. 1993). This condition requires that $C_\lambda < 1$.

4.2. DCS models of location, scale and shape

In the first step, we evaluate those conditions of consistency and asymptotic normality of the ML, which are sufficient conditions for the DCS models with constant shape (Harvey 2013). For the QAR(p) location equation, we use Harvey (2013, Chapter 3.5), and denote the maximum modulus of eigenvalues of the matrix A (Harvey 2013, Equation 3.33, Chapters 3.5.2 and 3.5.3) by using C_μ. This condition requires that $C_\mu < 1$. For DCS-EGARCH(1,1) with leverage effects, Harvey (2013, Equation 4.38) defines $C_\lambda = \beta^2 + 2\beta\alpha E(\partial u_{\lambda,t}/\partial\lambda_t) + [\alpha^2 + (\alpha^*)^2]E[(\partial u_{\lambda,t}/\partial\lambda_t)^2]$. Two conditions for DCS-EGARCH(1,1) with leverage effects are $|\beta| < 1$ and $C_\lambda < 1$. For the QAR(1) shape equation, Harvey (2013, Equation 2.35) defines: $C_{\rho,k} = \gamma_k^2 + 2\gamma_k\kappa_k E(\partial u_{\rho,k,t}/\partial\rho_{k,t}) + \kappa_k^2 E[(\partial u_{\rho,k,t}/\partial\rho_{k,t})^2]$. Two conditions for QAR(1) are $|\gamma_k| < 1$ and $C_{\rho,k} < 1$.

In the second step, we use Lemma 1 of Jensen and Rahbek (2004, p. 1206), which provides sufficient conditions for the consistency and asymptotic normality of the ML estimator for DCS models with dynamic shape parameters. The conditions of Lemma 1 are

(A.1) $\mathrm{LL}(y_1, \ldots, y_T; \Theta)$ is three times continuously differentiable in Θ.

(A.2) The true value of parameters Θ_0 is an interior point of the compact parameter space.

(A.3) As $T \to \infty$, $\sqrt{T}\partial\mathrm{LL}(y_1, \ldots, y_T; \Theta_0)/\partial\Theta \to_D N(0, \Omega_S)$, $\Omega_s > 0$.

(A.4) As $T \to \infty$, $-\partial^2\mathrm{LL}(y_1, \ldots, y_T; \Theta_0)/\partial\Theta\partial\Theta' \to_P \Omega_I > 0$.

(A.5) $\max_{h,i,j=1,\ldots,K}\sup_{\Theta\in N(\Theta_0)}|\partial^3\mathrm{LL}(y_1,\ldots,y_T;\Theta_0)/\partial\Theta_h\partial\Theta_i\partial\Theta_j| \leq c_T$, where $N(\Theta_0)$ is a neighborhood of Θ_0, $0 \leq c_T \to_P c$ and $0 < c < \infty$.

Under these conditions, Jensen and Rahbek (2004) demonstrate that

(B.1) With probability tending to one as $T \to \infty$, there exists a unique $\hat{\Theta}_{\mathrm{ML}}$.

(B.2) As $T \to \infty$, $\hat{\Theta}_{\mathrm{ML}} \to_P \Theta_0$.

(B.3) As $T \to \infty$, $\sqrt{T}(\hat{\Theta}_{\mathrm{ML}} - \Theta_0) \to_D N(0, \Omega_I^{-1}\Omega_s\Omega_I^{-1})$.

(A.1) and (A.2) are supported by all models of this paper. (A.3) is supported due to the Lindeberg–Lévy central limit theorem (Harvey 2013, Chapter 2.2.4). (A.4) is supported because the function $\mathrm{LL}(y_1,\ldots,y_T;\Theta)$ is concave and it attains an isolated local maximum for all models estimated in our paper. We verify condition (A.5) in Appendix.

5. Empirical results

5.1. Data

As an illustration, we use daily log-return data from the adjusted S&P 500 index p_t for period 1950 to 2016. Some descriptive statistics of y_t are presented in Table 1. The negative skewness estimate indicates that the mass of the distribution of y_t is concentrated on the right side, and the high excess kurtosis estimate suggests heavy tails of y_t. The negative correlation coefficient $\mathrm{Corr}(y_t^2, y_{t-1})$ suggests that high volatility often follows significant negative returns. We also present the partial autocorrelation function (PACF) (Hamilton 1994) up to 30 lags in Table 1. We find significant serial correlation for the first and second lags, and we also find significant serial correlation for the lags in multiples of around five (this indicates weekly stochastic seasonality effects). Motivated by PACF, we use the lag order 30 for all models of location.

5.2. ML estimation results

In this section, we present the ML results for AR-*t*-GARCH with leverage effects (Table 2), *t*-DCS (Table 2), GED-DCS (Table 3), Gen-*t*-DCS (Table 4), Skew-Gen-*t*-DCS (Table 5), EGB2-DCS (Table 6) and NIG-DCS (Table 7). We compare the LL-based performance of these models in Table 8. We present diagnostics of score functions and residuals in Table 9. We present the evolution of scale parameters, shape parameters and volatility for all DCS models in Figs. 2 to 6.

For t-DCS, GED-DCS and EGB2-DCS, the ML procedure converged effectively for all specifications (i.e., all shape parameters are constant or all shape parameters are dynamic). For Gen-t-DCS, three specifications were identified: (i) ν_t and η_t are constant ($\nu_t = \delta_1$ and $\eta_t = \delta_2$); (ii) ν_t is dynamic ($\nu_t = \delta_1 + \gamma_1 \nu_{t-1} + \kappa_1 u_{\nu,t-1}$) and η_t is constant ($\eta_t = \delta_2$); (iii) ν_t is constant ($\nu_t = \delta_1$) and η_t is dynamic ($\eta_t = \delta_2 + \gamma_2 \eta_{t-1} + \kappa_2 u_{\eta,t-1}$). For Skew-Gen-$t$-DCS, three specifications were identified: (i) τ_t, ν_t and η_t are constant ($\tau_t = \delta_1$, $\nu_t = \delta_2$ and $\eta_t = \delta_3$); (ii) only ν_t is dynamic ($\tau_t = \delta_1$, $\nu_t = \delta_2 + \gamma_2 \nu_{t-1} + \kappa_2 u_{\nu,t-1}$ and $\eta_t = \delta_3$); (iii) only η_t is dynamic ($\tau_t = \delta_1$, $\nu_t = \delta_2$ and $\eta_t = \delta_3 + \gamma_3 \eta_{t-1} + \kappa_3 u_{\eta,t-1}$). For NIG-DCS, two specifications were identified: (i) ν_t and η_t are constant ($\nu_t = \delta_1$ and $\eta_t = \delta_2$); (ii) ν_t is constant ($\nu_t = \delta_1$) and η_t is dynamic ($\eta_t = \delta_2 + \gamma_2 \eta_{t-1} + \kappa_2 u_{\eta,t-1}$). Our estimation results are summarized as follows.

First, for all cases, we find that some of the ϕ_j parameters are significantly different from zero. The scaling parameter of the score function with respect to location θ is positive and significant for all models. For all cases, we find highly significant parameters of conditional volatility. For almost all cases, we find that the dynamic parameters of shape (i.e., γ_1, γ_2 and γ_3) are significant and positive (the only exception is GED-DCS with dynamic ν_t, for which γ_1 is not significant). We also find that the scaling parameter of the score function with respect to the shape (i.e., κ_1, κ_2 and κ_3) is significantly different from zero for all cases (i.e., all DCS specifications with dynamic shape are identified; Harvey 2013).

Second, we use the following model performance metrics: mean LL, mean Akaike information criterion (AIC), mean Bayesian information criterion (BIC) and mean Hannan-Quinn criterion (HQC) (Davidson and MacKinnon 2003). We find that the AIC-, BIC- and HQC-based statistical performances of the DCS model with dynamic shape are superior to the performance of the DCS model with constant shape (Table 2 to 7). We also undertake a likelihood-ratio (LR) test for non-nested models (Vuong 1989). We denote the conditional density functions of y_t of the DCS models with dynamic and constant shape by using $f(y_t|y_1, \ldots, y_{t-1})$ and $g(y_t|y_1, \ldots, y_{t-1})$, respectively. We define $d_t = \ln f(y_t|y_1, \ldots, y_{t-1}) - \ln g(y_t|y_1, \ldots, y_{t-1})$. We test whether LL of DCS with dynamic shape is superior to that of DCS with constant shape by estimating $d_t = c + \epsilon_t$ with OLS-HAC (ordinary least squares heteroskedasticity and autocorrelation consistent; Newey and West 1987). If c is significantly positive then DCS with dynamic shape is superior to DCS with constant shape. For almost all cases, we find that the DCS model with dynamic shape is a superior specification (the only exception is Gen-t-DCS with dynamic ν_t and constant η_t). We rank the LL-based performances of different models in Table 8.

Third, we estimate the Mincer–Zarnowitz (1969) (hereafter, MZ) regression, to rank volatility forecast performances. For each model, we use $\hat{v}_t^2$ (i.e., a conditionally unbiased volatility proxy; Patton 2011) as the dependent variable and the square of the conditional volatility (Section 3) as the explanatory variable. Meddahi (2002) shows that the ranking of models based on the R^2 of the MZ regression is robust to noise for conditionally unbiased volatility proxies. We indicate four models with the highest R^2 by using bold numbers in Table 8. Interestingly, all those models have dynamics in η_t, ξ_t and ζ_t, but not in ν_t. Dynamic ν_t (i.e., dynamic heavy tails) reduces the MZ R^2. The parameters η_t, ξ_t and ζ_t are responsible for dynamic asymmetry and dynamic peakedness of the distribution (Fig. 1).

Fourth, we present the MDS test results for $\hat{\epsilon}_t$ and $\hat{\epsilon}_t^*$ in Table 9. For most of the cases, we find that the null hypothesis of the MDS hypothesis is not rejected at the 10% level of significance (the only exceptions are NIG-DCS with ν_t and η_t constant, and Skew-Gen-t-DCS with τ_t, η_t constant and ν_t variable).

Fifth, the conditions for C_μ, C_λ and $C_{\rho,k}$ are satisfied for all models (Section 4).

Sixth, Figs. 2 to 6 indicate the following: (i) the shape parameters are time-varying for all DCS models; (ii) for the DCS models with dynamic shape, the shape parameters identify the dates of some extreme events; (iii) the scale and shape parameters can be decomposed into a normal risk component influenced by small or moderate changes, and an extreme component influenced by large jumps or falls; (iv) volatility exhibits greater jumps due to extreme events for DCS with dynamic shape than for DCS with constant shape.

5.3. Decomposition of normal risk and extreme risk components

For the DCS models with dynamic shape, news updates volatility through λ_t and $\rho_{k,t}$. The significant parameters β and γ_k (Tables 2 to 7), and the estimates of λ_t and $\rho_{k,t}$ (Figs. 2 to 6) indicate that each series can be decomposed into a normal risk component that can be related to normal events, and an extreme risk component involving significant jumps or falls that can be related to extreme events.

We decompose all $\hat{\lambda}_t$ and $\hat{\rho}_{k,t}$ series by using the equations $\hat{\lambda}_t = c_\lambda + \sum_{j=1}^{30} \phi_{\lambda,j}\hat{\lambda}_{t-j} + e_{\lambda,t}$ and $\hat{\rho}_{k,t} = c_{\rho,k} + \sum_{j=1}^{30} \phi_{\rho,k,j}\hat{\rho}_{k,t-j} + e_{\rho,k,t}$, respectively. We estimate these equations by using OLS (see the parameter estimates in the Separate Appendix). We study the consistency of OLS by undertaking the MDS test (Escanciano and Lobato 2009) for the residuals $\hat{e}_{\lambda,t}$ and $\hat{e}_{\rho,k,t}$. We find that the MDS null hypothesis of the test is never rejected (Table 10). We define the normal risk components of λ_t and $\rho_{k,t}$ by using the fitted values of $\hat{\lambda}_t$ and $\hat{\rho}_{k,t}$, respectively. We define the extreme risk components of λ_t and $\rho_{k,t}$ by using the residuals $\hat{e}_{\lambda,t}$ and $\hat{e}_{\rho,k,t}$, respectively. The use of 30 lags in the AR model is motivated by the PACF results of Section 5.1. The AR model and its OLS estimation are motivated by the work of Hamilton (2017).

For each AR(30) equation, the proportion of the regression sum of squares to total sum of squares (i.e., R^2) is interpreted as the proportion of the dynamic parameter that corresponds to the normal risk component (normal % in Table 10). Furthermore, for each AR(30) equation, the proportion of the residual sum of squares to total sum of squares is interpreted as the proportion of the dynamic parameter that corresponds to extreme risk component (extreme % in Table 10). For the new DCS models, these results suggest that most part of λ_t is associated with the normal risk component, and most part of $\rho_{k,t}$ is associated with the extreme risk component. We find that the normal component of $\rho_{k,t}$ is significant, which motivates the use of the DCS models with dynamic shape rather than the DCS models with constant shape.

The extreme risk component, represented by variables $\hat{e}_{\lambda,t}$ and $\hat{e}_{\rho,k,t}$, updates volatility simultaneously through λ_t and $\rho_{k,t}$, respectively. We study the relation between $\hat{e}_{\lambda,t}$ and $\hat{e}_{\rho,k,t}$ for the DCS models with dynamic shape parameters, by estimating the correlation coefficient of $\hat{e}_{\lambda,t}$

and $\hat{e}_{\rho,k,t}$ for all days of the data window (Table 10). The results suggest that the correlation is negative and significant. Hence, the effects of extreme events are divided between λ_t and $\rho_{k,t}$, and volatility is updated simultaneously by variables $\hat{e}_{\lambda,t}$ and $\hat{e}_{\rho,k,t}$, respectively.

Furthermore, we also estimate the correlation coefficients of $\hat{e}_{\lambda,t}$ and $\hat{e}_{\rho,k,t}$, for risky days and safe days (Table 10). For $\hat{e}_{\lambda,t}$, a risky day is when $\hat{e}_{\lambda,t}$ is above its mean. For $\hat{e}_{\rho,k,t}$, a risky day is when $\hat{e}_{\rho,k,t}$ is below its mean. We use the opposite definition of safe days for $\hat{e}_{\lambda,t}$ and $\hat{e}_{\rho,k,t}$. We find that the correlation coefficient is negative and significant for risky days. For most of the cases, this indicates a stronger negative correlation for risky days than for all days of the data window. We also find that the correlation coefficient is less significant for safe days. Thus, we identify asymmetric relationships of $\hat{e}_{\lambda,t}$ and $\hat{e}_{\rho,k,t}$, with respect to risky days and safe days.

5.4. Out-of-sample density forecast performance

We compare the out-of-sample density forecast performance of EGB2-DCS with dynamic shape parameters and that of t-GARCH with leverage effects. We use EGB2-DCS, since it has the best in-sample volatility forecast performance (Table 8). Furthermore, we use t-GARCH as a benchmark, due to the results in the work of Hansen and Lunde (2005).

We apply the Amisano–Giacomini (2007) out-of-sample density forecast comparison test, for which we use a uniform weighting function. In the body of literature, a more recent test of out-of-sample density forecast comparison is suggested in the work of Gneiting and Ranjan (2011). In this paper we do not use the Gneiting–Ranjan test, since it is not feasible for EGB2-DCS. Moreover, Gneiting and Ranjan (2011) present a simulation-experiment for GARCH, which indicates that the Amisano–Giacomini test with a uniform weighting function takes a correct decision with respect to the out-of-sample density forecast performance. This simulation-experiment provides a further motivation for the use of the Amisano–Giacomini test.

In this paper, we randomly select 1,000 data windows from the full data window (Table 1). Each random data window includes 2,500 observations $(y_1, \ldots, y_{t-1})$. The use of the 1,000 data windows from the total $16,858 - 2,500 = 14,358$ data windows, is motivated by the speed at which the Amisano–Giacomini test procedure performs. For each data window, we estimate the parameters of EGB2-DCS and t-GARCH, and denote them by $\hat{\Theta}_{\text{EGB2}}$ and $\hat{\Theta}_{\text{GARCH}}$, respectively. We forecast the conditional density of $y_t|(y_1,\ldots,y_{t-1})$, and denote the log-densities of EGB2-DCS and t-GARCH by $\ln f(y_t|y_1,\ldots,y_{t-1};\hat{\Theta}_{\text{EGB2}})$ and $\ln g(y_t|y_1,\ldots,y_{t-1};\hat{\Theta}_{\text{GARCH}})$, respectively. For each data window, we define the variable $d_t = \ln f(y_t|y_1,\ldots,y_{t-1};\hat{\Theta}_{\text{EGB2}}) - \ln g(y_t|y_1,\ldots,y_{t-1};\hat{\Theta}_{\text{GARCH}})$. We test whether, on average, d_t for $t = 1,\ldots,1000$ is significantly different from zero by using the linear regression $d_t = c + \epsilon_t$, estimated with OLS-HAC. We find that the robust estimate of c is 0.0314***(0.0090). Therefore, the out-of-sample density forecast performance of EGB2-DCS is superior to that of t-GARCH.

6. Conclusions

In this paper, we have introduced new DCS models with dynamic location, scale and shape parameters. For the new DCS models we have obtained the following results: (i) the statistical and forecast performances of the new DCS models are superior to those of the DCS models with constant shape and AR-t-GARCH with leverage effects; (ii) the dates of extreme events are identified effectively, and all volatility time series exhibit significant jumps due to extreme events; (iii) changes in the scale are more related to the normal risk component, and changes in the shape are more related to the extreme risk component; (iv) the normal risk component of the dynamic shape parameter is significant; (v) we have undertaken an out-of-sample density forecast performance exercise for EGB2-DCS with dynamic shape and t-GARCH with leverage effects, and we have found that EGB2-DCS has a superior forecast performance. These results motivate the practical use of the DCS models with dynamic location, scale and shape parameters.

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Appendix: Third-order derivatives of condition (A.5)

1. Product of bounded functions in absolute value

In the first section of this Appendix, we use the result that the product of bounded functions in absolute value is also bounded in absolute value. We use the general notations m_t for μ_t, λ_t and $\rho_{k,t}$, and $u_{m,t}$ for $u_{\mu,t}$, $u_{\lambda,t}$ and $u_{\rho,k,t}$. For each score-driven parameter m_t and for each time-constant parameter Θ_j, we use the chain rule $\partial \ln f(y_t|y_1,\ldots,y_{t-1};\Theta)/\partial\Theta_j = [\partial \ln f(y_t|y_1,\ldots,y_{t-1};\Theta)/\partial m_t] \times [\partial m_t/\partial\Theta_j]$. To this formula of the first-order derivative, we apply the chain rule twice with respect to Θ, and analyze the terms within the third-order derivatives of condition (A.5). In the following, we focus on those terms of the third-order derivative that may be unbounded in absolute value with respect to Θ.

First, in the formulas of the first-order derivatives of $\ln f(y_t|y_1,\ldots,y_{t-1};\Theta)$ with respect to m_t (Section 3), all denominators include positive values, according to the model specification. Hence, the first- and second-order derivatives of those proportions are bounded in absolute value within the neighborhood $N(\Theta_0)$.

Second, in the formulas of the first-order derivatives of $\ln f(y_t|y_1,\ldots,y_{t-1};\Theta)$ with respect to m_t (Section 3), all $\ln(x)$ functions include positive values, according to the model specification. Hence, the first- and second-order derivatives of those logarithms (i.e., $1/x$ and $-1/x^2$) are bounded in absolute value within the neighborhood $N(\Theta_0)$.

Third, in the formulas of the first-order derivatives of $\ln f(y_t|y_1,\ldots,y_{t-1};\Theta)$ with respect to m_t (Section 3), all $\Psi^{(0)}(x)$, $\Psi^{(1)}(x)$, $K^{(0)}(x)$, $K^{(1)}(x)$ and $K^{(2)}(x)$ functions include positive values, according to the model specification. The derivative of $\Psi^{(j)}(x)$ includes $\Psi^{(j+1)}(x)$, and all $\Psi^{(j+1)}(x)$ are bounded in absolute value for $x > 0$. The derivative of $K^{(j)}(x)$ includes $K^{(j-1)}(x)$ or $K^{(j+1)}(x)$, and all $K^{(j-1)}(x)$ and $K^{(j+1)}(x)$ are bounded in absolute value for $x > 0$.

These suggest that the third-order derivatives of $\ln f(y_t|y_1,\ldots,y_{t-1};\Theta)$ with respect to the μ_t, λ_t and $\rho_{k,t}$ are bounded in absolute value within the neighborhood $N(\Theta_0)$.

Fourth, the formulas of the first-order derivatives of m_t with respect to Θ_j, include 1, m_{t-1}, $u_{m,t-1}$ or $\text{sgn}(\epsilon_{t-1})(u_{m,t-1}+1)$. It is noteworthy that it is enough to study the absolute boundedness of the first-order derivatives with respect to Θ_j, since the second- and third-order derivatives of m_t with respect to Θ_j are zero. If m_t and $u_{m,t}$ are bounded in absolute value, then this will imply that $\partial m_t/\partial\Theta_j$ is bounded in absolute value.

In the first step, we use the augmented Dickey–Fuller (1979) (ADF) test with constant (motivated by the work of Davidson and MacKinnon 2003, Chapter 14) to study the covariance stationarity of $u_{m,t}$. We perform optimal lag order selection in the ADF test by using BIC. If for the ADF test we find that $u_{m,t}$ is covariance stationary, then each $u_{m,t}$ can be written as a sum of a constant parameter and an infinite moving average process of uncorrelated variables according to the Wold representation theorem (Wold 1954). In that case, $u_{m,t}$ is bounded in absolute value within the neighborhood $N(\Theta_0)$.

In the second step, given that $u_{m,t}$ is covariance stationary, we study if the coefficients of the dynamic terms of μ_t, λ_t and $\rho_{k,t}$ support their covariance stationarity. If that is the case, then μ_t, λ_t and $\rho_{k,t}$ can be written as a sum of a constant parameter and an infinite moving average process of uncorrelated variables. Hence, μ_t, λ_t and $\rho_{k,t}$ are bounded in absolute value within the neighborhood $N(\Theta_0)$.

For all models, we find that all score functions $u_{m,t}$ are covariance stationary (Table 9). Furthermore, we also find that the dynamic parameters in μ_t, λ_t and $\rho_{k,t}$ support covariance stationarity (Tables 2 to 7). These results suggest that $\partial m_t/\partial\Theta_j$ is bounded in absolute value.

2. Estimation of the third-order derivatives of LL

In the second section of this Appendix, we investigate condition (A.5) by using two approaches, where each approach involves the estimation of all third-order derivatives of LL.

For the first approach, we study whether all possible third-order derivatives of LL are finite at $\hat{\Theta}_{\mathrm{ML}}$ that is estimated for the S&P 500 dataset. This approach will provide evidence against the consistency and asymptotic normal distribution of the ML estimates if any third-order derivatives estimated at $\hat{\Theta}_{\mathrm{ML}}$ are infinite.

For the second approach, we undertake a Monte Carlo simulation experiment to analyze condition (A.5) in a more robust way. We use 20 alternative sets of true parameter values, and for each set we simulate 2,000 observations of y_t. The first set of parameters Θ_{MC1} coincides with $\hat{\Theta}_{\text{ML}}$, estimated for the S&P 500 dataset. For the remaining 19 sets of parameters $\Theta_{\text{MC2}}, \ldots, \Theta_{\text{MC20}}$, we use alternative covariance stationary dynamics for μ_t, λ_t and $\rho_{k,t}$ (i.e., more or less persistent dynamics with respect to $\hat{\Theta}_{\text{ML}}$), and we also use alternative non-zero values for the score function coefficients (i.e., θ, α, α^*, κ_k) for which DCS is identified (Harvey 2013). The sets of parameters $\Theta_{\text{MC2}}, \ldots, \Theta_{\text{MC20}}$ are obtained by using Monte Carlo simulation around $\Theta_{\text{MC1}} = \hat{\Theta}_{\text{ML}}$. For each set of true parameter values, we evaluate all third-order derivatives of LL, and study their finiteness.

For both approaches, we estimate the third-order derivatives of LL as follows. We use the chain rule $\partial \ln f(y_t|y_1, \ldots, y_{t-1}; \Theta)/\partial \Theta_j = [\partial \ln f(y_t|y_1, \ldots, y_{t-1}; \Theta)/\partial m_t] \times [\partial m_t/\partial \Theta_j]$, to formulate the first-derivative function with respect to each Θ_j. This gives K first-derivative functions with respect to Θ_j for $j = 1, \ldots, K$. The first-derivative function with respect to Θ_j is available in closed form for all models, since: (i) $\partial \ln f(y_t|y_1, \ldots, y_{t-1}; \Theta)/\partial m_t$ is reported in Section 3; (ii) m_t is a simple function of Θ_j for all cases (Section 2.2).

For each first-derivative function corresponding to Θ_j, we numerically estimate the $K \times K$ Hessian matrix with respect to $(\Theta_1, \ldots, \Theta_K)$. We evaluate each Hessian at $\hat{\Theta}_{\text{ML}}$ for the S&P 500 data of the first approach, and we evaluate each Hessian at each of the 20 alternative sets of true parameter values for the simulated data of the second approach. For each Hessian matrix corresponding to Θ_j, we denote the maximum element in absolute value by using $H_{\max,j}$. Furthermore, we introduce the notation $H_{\max} = \max\{H_{\max,1}, \ldots, H_{\max,K}\}$. We study the finiteness of $H_{\max}$ for each DCS specification.

For the first approach, we evaluate $H_{\max}$ at $\hat{\Theta}_{\text{ML}}$ for the S&P 500 dataset, and find that it is finite for all DCS specifications (Separate Appendix). For the second approach, we evaluate $H_{\max}$ at each of the 20 alternative sets of true parameter values (Separate Appendix) in the Monte Carlo simulation experiment, and find that $H_{\max}$ is finite (Separate Appendix). These results encourage the use of the ML method for the estimation of the new DCF models with dynamic location, scale and shape parameters, as suggested in this paper.

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Table 1. Descriptive statistics

Start date	4-Jan-1950	PACF(1)	0.0273***	PACF(11)	−0.0142*	PACF(21)	−0.0181**
End date	30-Dec-2016	PACF(2)	−0.0422***	PACF(12)	0.0280***	PACF(22)	−0.0028
Sample size T	$16,858$	PACF(3)	0.0028	PACF(13)	−0.0005	PACF(23)	−0.0007
Minimum	−0.2290	PACF(4)	−0.0082	PACF(14)	−0.0010	PACF(24)	0.0103
Maximum	0.1096	PACF(5)	−0.0129*	PACF(15)	−0.0130*	PACF(25)	−0.0117
Mean	0.0003	PACF(6)	−0.0055	PACF(16)	0.0325***	PACF(26)	−0.0214***
Standard deviation	0.0097	PACF(7)	−0.0190**	PACF(17)	−0.0045	PACF(27)	0.0192**
Skewness	−1.0110	PACF(8)	0.0105	PACF(18)	−0.0205***	PACF(28)	−0.0049
Excess kurtosis	27.1272	PACF(9)	−0.0108	PACF(19)	0.0016	PACF(29)	0.0236***
Corr(y_t^2,y_{t-1})	−0.0877	PACF(10)	0.0130*	PACF(20)	0.0106	PACF(30)	0.0064

Notes: The lag order of the partial autocorrelation function (PACF) is shown in parentheses. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively. *Source of data*: Yahoo Finance, https://finance.yahoo.com/, accessed 23rd February 2017.

Table 2. Parameter estimates and model diagnostics, AR-*t*-GARCH with leverage effects and *t*-DCS

AR-*t*-GARCH with leverage effects				*t*-DCS constant ν_t				*t*-DCS dynamic ν_t			
c	0.0004***(0.0001)	ϕ_{20}	0.0081(0.0071)	c	0.0001**(0.0000)	ϕ_{20}	−0.1989***(0.0654)	c	0.0001**(0.0000)	ϕ_{20}	−0.2033***(0.0700)
ϕ_1	0.0912***(0.0080)	ϕ_{21}	−0.0091(0.0072)	ϕ_1	−0.1298(0.0790)	ϕ_{21}	−0.1537**(0.0705)	ϕ_1	−0.1640**(0.0793)	ϕ_{21}	−0.1468*(0.0753)
ϕ_2	−0.0274***(0.0079)	ϕ_{22}	−0.0056(0.0071)	ϕ_2	−0.0824(0.0695)	ϕ_{22}	−0.1309*(0.0687)	ϕ_2	−0.0682(0.0706)	ϕ_{22}	−0.1254*(0.0742)
ϕ_3	−0.0008(0.0078)	ϕ_{23}	0.0001(0.0070)	ϕ_3	0.1825**(0.0730)	ϕ_{23}	0.2475***(0.0639)	ϕ_3	0.1397*(0.0743)	ϕ_{23}	0.1752**(0.0686)
ϕ_4	0.0115(0.0077)	ϕ_{24}	0.0166**(0.0069)	ϕ_4	0.0744(0.0781)	ϕ_{24}	−0.0773(0.0643)	ϕ_4	0.0982(0.0793)	ϕ_{24}	−0.0216(0.0653)
ϕ_5	−0.0013(0.0076)	ϕ_{25}	−0.0176**(0.0070)	ϕ_5	0.0023(0.0766)	ϕ_{25}	0.0422(0.0728)	ϕ_5	−0.0175(0.0797)	ϕ_{25}	−0.0392(0.0762)
ϕ_6	−0.0106(0.0076)	ϕ_{26}	−0.0088(0.0070)	ϕ_6	−0.0928(0.0750)	ϕ_{26}	−0.1268*(0.0732)	ϕ_6	−0.0563(0.0781)	ϕ_{26}	−0.0458(0.0757)
ϕ_7	−0.0069(0.0077)	ϕ_{27}	0.0062(0.0068)	ϕ_7	−0.0329(0.0761)	ϕ_{27}	0.1001(0.0748)	ϕ_7	−0.0381(0.0782)	ϕ_{27}	0.0596(0.0751)
ϕ_8	0.0058(0.0075)	ϕ_{28}	0.0040(0.0069)	ϕ_8	−0.0190(0.0702)	ϕ_{28}	0.0688(0.0697)	ϕ_8	−0.0059(0.0766)	ϕ_{28}	0.1098(0.0720)
ϕ_9	0.0003(0.0075)	ϕ_{29}	0.0105(0.0068)	ϕ_9	0.2345***(0.0688)	ϕ_{29}	0.2728***(0.0699)	ϕ_9	0.2024***(0.0745)	ϕ_{29}	0.2624***(0.0712)
ϕ_{10}	0.0159**(0.0075)	ϕ_{30}	0.0076(0.0068)	ϕ_{10}	0.0608(0.0710)	ϕ_{30}	0.0468(0.0699)	ϕ_{10}	0.0701(0.0759)	ϕ_{30}	0.0330(0.0706)
ϕ_{11}	−0.0050(0.0074)	ω	0.0000***(0.0000)	ϕ_{11}	0.2834***(0.0674)	θ	0.9246***(0.0786)	ϕ_{11}	0.2654***(0.0728)	θ	0.9505***(0.0810)
ϕ_{12}	0.0188**(0.0074)	α	0.0562***(0.0031)	ϕ_{12}	0.0464(0.0723)	ω	−0.0622***(0.0066)	ϕ_{12}	0.1078(0.0755)	ω	−0.0534***(0.0061)
ϕ_{13}	0.0066(0.0073)	α^*	0.0389***(0.0048)	ϕ_{13}	0.0014(0.0716)	α	0.0365***(0.0019)	ϕ_{13}	−0.0127(0.0767)	α	0.0339***(0.0019)
ϕ_{14}	−0.0033(0.0073)	β	0.9129***(0.0042)	ϕ_{14}	−0.1163(0.0738)	α^*	0.0267***(0.0015)	ϕ_{14}	−0.0738(0.0797)	α^*	0.0252***(0.0015)
ϕ_{15}	−0.0009(0.0073)	λ_0	0.0000(0.0000)	ϕ_{15}	0.1933***(0.0724)	β	0.9877***(0.0013)	ϕ_{15}	0.1476*(0.0787)	β	0.9895***(0.0012)
ϕ_{16}	0.0158**(0.0072)	δ_1	1.6417***(0.0605)	ϕ_{16}	0.0823(0.0727)	λ_0	−5.3897***(0.3306)	ϕ_{16}	0.0881(0.0778)	λ_0	−5.3884***(0.3192)
ϕ_{17}	−0.0012(0.0072)			ϕ_{17}	0.0933(0.0728)	δ_1	1.6467***(0.0570)	ϕ_{17}	0.0857(0.0793)	δ_1	0.6818***(0.2329)
ϕ_{18}	−0.0052(0.0071)			ϕ_{18}	−0.0520(0.0737)			ϕ_{18}	−0.0485(0.0802)	γ_1	0.5897***(0.1377)
ϕ_{19}	−0.0003(0.0071)			ϕ_{19}	−0.0642(0.0730)			ϕ_{19}	−0.0138(0.0780)	κ_1	0.9099***(0.2772)
LL	3.4433	C_μ	1.1398	LL	3.4465	C_μ	0.9891	LL	3.4473	C_μ	0.9888
AIC	−6.8822	C_λ	0.9886	AIC	−6.8884	C_λ	0.8814	AIC	−6.8898	C_λ	0.9158
BIC	−6.8652			BIC	−6.8710	LR	0.0008*(0.0004)	BIC	−6.8714	C_ν	0.3300
HQC	−6.8766			HQC	−6.8827			HQC	−6.8837		

Notes: Standard errors are reported in parentheses. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively. C_μ, C_λ and C_ν indicate the conditions of consistency and asymptotic normality of ML for the location, scale and shape equations, respectively (Section 4). These are sufficient conditions for DCS models with constant shape parameters. For the LR test, we report the OLS-HAC estimate of $d_t = c + \epsilon_t$ (Section 5.2). Model specification for AR-*t*-GARCH with leverage effects: $y_t = \mu_t + v_t = \mu_t + \lambda_t^{1/2}\epsilon_t$, $\epsilon_t \sim t[\exp(\delta_1) + 2]$, $\mu_t = c + \left(\sum_{j=1}^{30} \phi_j y_{t-j}\right)$ and $\lambda_t = \omega + \beta\lambda_{t-1} + [\alpha + \alpha^* \mathbb{1}(\epsilon_{t-1} < 0)]v_{t-1}^2$. Model specification for *t*-DCS: $y_t = \mu_t + \exp(\lambda_t)\epsilon_t$, $\epsilon_t \sim t[\exp(\nu_t) + 2]$, $\mu_t = c + \left(\sum_{j=1}^{30} \phi_j \mu_{t-j}\right) + \theta u_{\mu,t-1}$ and $\lambda_t = \omega + \beta\lambda_{t-1} + \alpha u_{\lambda,t-1} + \alpha^* \text{sgn}(-\epsilon_{t-1})(u_{\lambda,t-1} + 1)$. For the DCS specification with constant ν_t: $\nu_t = \delta_1$. For the DCS specification with dynamic ν_t: $\nu_t = \delta_1 + \gamma_1 \nu_{t-1} + \kappa_1 u_{\nu,t-1}$.

Table 3. Parameter estimates and model diagnostics, GED-DCS

Constant ν_t				Dynamic ν_t			
c	0.0006**(0.0003)	ϕ_{20}	−0.0447(0.0671)	c	0.0005**(0.0002)	ϕ_{20}	−0.0505(0.0617)
ϕ_1	−0.1473**(0.0665)	ϕ_{21}	−0.0372(0.0669)	ϕ_1	−0.1486**(0.0659)	ϕ_{21}	−0.0314(0.0623)
ϕ_2	−0.0885(0.0662)	ϕ_{22}	−0.0613(0.0672)	ϕ_2	−0.0669(0.0623)	ϕ_{22}	−0.0539(0.0642)
ϕ_3	0.0790(0.0659)	ϕ_{23}	0.1061(0.0659)	ϕ_3	0.1032(0.0659)	ϕ_{23}	0.1369**(0.0649)
ϕ_4	0.0316(0.0664)	ϕ_{24}	−0.0782(0.0606)	ϕ_4	0.0538(0.0644)	ϕ_{24}	−0.0577(0.0613)
ϕ_5	−0.0768(0.0674)	ϕ_{25}	−0.0575(0.0661)	ϕ_5	−0.0682(0.0635)	ϕ_{25}	−0.0561(0.0596)
ϕ_6	−0.0016(0.0642)	ϕ_{26}	−0.0586(0.0620)	ϕ_6	−0.0041(0.0604)	ϕ_{26}	−0.0522(0.0634)
ϕ_7	−0.0896(0.0639)	ϕ_{27}	0.0599(0.0659)	ϕ_7	−0.0716(0.0640)	ϕ_{27}	0.0576(0.0648)
ϕ_8	−0.0447(0.0633)	ϕ_{28}	0.1569**(0.0663)	ϕ_8	−0.0315(0.0641)	ϕ_{28}	0.1654***(0.0629)
ϕ_9	−0.0103(0.0657)	ϕ_{29}	0.0695(0.0653)	ϕ_9	0.0157(0.0633)	ϕ_{29}	0.0829(0.0646)
ϕ_{10}	−0.0467(0.0654)	ϕ_{30}	−0.1014*(0.0611)	ϕ_{10}	−0.0189(0.0648)	ϕ_{30}	−0.1032*(0.0611)
ϕ_{11}	0.1288**(0.0651)	θ	0.0007***(0.0000)	ϕ_{11}	0.1498**(0.0620)	θ	0.0007***(0.0000)
ϕ_{12}	0.0245(0.0671)	ω	−0.0739***(0.0074)	ϕ_{12}	0.0404(0.0662)	ω	−0.0555***(0.0064)
ϕ_{13}	−0.0238(0.0644)	α	0.0383***(0.0018)	ϕ_{13}	−0.0175(0.0632)	α	0.0325***(0.0020)
ϕ_{14}	−0.0973(0.0668)	α^*	0.0204***(0.0013)	ϕ_{14}	−0.0977(0.0652)	α^*	0.0186***(0.0014)
ϕ_{15}	0.1113*(0.0634)	β	0.9863***(0.0014)	ϕ_{15}	0.0988(0.0625)	β	0.9897***(0.0012)
ϕ_{16}	−0.0401(0.0632)	λ_0	−5.8321***(0.3569)	ϕ_{16}	−0.0304(0.0641)	λ_0	−5.7214***(0.3308)
ϕ_{17}	−0.0724(0.0632)	δ_1	0.2751***(0.0095)	ϕ_{17}	−0.0519(0.0637)	δ_1	0.1984***(0.0584)
ϕ_{18}	0.0212(0.0630)			ϕ_{18}	0.0324(0.0576)	γ_1	0.3089(0.1991)
ϕ_{19}	−0.0712(0.0670)			ϕ_{19}	−0.0639(0.0669)	κ_1	0.0091***(0.0012)
LL	3.4407	C_μ	0.9736	LL	3.4419	C_μ	0.9762
AIC	−6.8769	C_λ	0.8792	AIC	−6.8791	C_λ	0.9412
BIC	−6.8594	LR	0.0012*(0.0006)	BIC	−6.8607	C_ν	0.0848
HQC	−6.8711			HQC	−6.8730		

Notes: Standard errors are reported in parentheses. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively. C_μ, C_λ and C_ν indicate the conditions of consistency and asymptotic normality of ML for the location, scale and shape equations, respectively (Section 4). These are sufficient conditions for DCS models with constant shape parameters. For the LR test, we report the OLS-HAC estimate of $d_t = c + \epsilon_t$ (Section 5.2). Model specification: $y_t = \mu_t + \exp(\lambda_t)\epsilon_t$, $\epsilon_t \sim \text{GED}[\exp(\nu_t)]$, $\mu_t = c + \left(\sum_{j=1}^{30} \phi_j \mu_{t-j}\right) + \theta u_{\mu,t-1}$ and $\lambda_t = \omega + \beta\lambda_{t-1} + \alpha u_{\lambda,t-1} + \alpha^* \text{sgn}(-\epsilon_{t-1})(u_{\lambda,t-1} + 1)$. For the specification with constant ν_t: $\nu_t = \delta_1$. For the specification with dynamic ν_t: $\nu_t = \delta_1 + \gamma_1\nu_{t-1} + \kappa_1 u_{\nu,t-1}$.

Table 4. Parameter estimates and model diagnostics, Gen-t-DCS

Constant ν_t and η_t				Dynamic ν_t and constant η_t				Constant ν_t and dynamic η_t			
c	$0.0001^{**}(0.0000)$	ϕ_{21}	$-0.1545^{**}(0.0715)$	c	$0.0001^{**}(0.0000)$	ϕ_{21}	$-0.1596^{**}(0.0774)$	c	$0.0001^{**}(0.0000)$	ϕ_{21}	$-0.1352^{*}(0.0713)$
ϕ_1	$-0.1378^{*}(0.0798)$	ϕ_{22}	$-0.1231^{*}(0.0703)$	ϕ_1	$-0.1771^{**}(0.0809)$	ϕ_{22}	$-0.1278(0.0782)$	ϕ_1	$-0.1515^{**}(0.0768)$	ϕ_{22}	$-0.1221^{*}(0.0690)$
ϕ_2	$-0.0873(0.0710)$	ϕ_{23}	$0.2588^{***}(0.0651)$	ϕ_2	$-0.0756(0.0738)$	ϕ_{23}	$0.1957^{***}(0.0719)$	ϕ_2	$-0.0667(0.0689)$	ϕ_{23}	$0.2068^{***}(0.0645)$
ϕ_3	$0.1837^{**}(0.0743)$	ϕ_{24}	$-0.0688(0.0654)$	ϕ_3	$0.1527^{**}(0.0765)$	ϕ_{24}	$0.0019(0.0676)$	ϕ_3	$0.1484^{**}(0.0726)$	ϕ_{24}	$-0.0691(0.0629)$
ϕ_4	$0.0833(0.0793)$	ϕ_{25}	$0.0565(0.0740)$	ϕ_4	$0.1305(0.0821)$	ϕ_{25}	$0.0027(0.0802)$	ϕ_4	$0.0797(0.0765)$	ϕ_{25}	$-0.0151(0.0717)$
ϕ_5	$0.0050(0.0782)$	ϕ_{26}	$-0.1312^{*}(0.0747)$	ϕ_5	$0.0163(0.0850)$	ϕ_{26}	$-0.0393(0.0799)$	ϕ_5	$-0.0079(0.0754)$	ϕ_{26}	$-0.1057(0.0721)$
ϕ_6	$-0.0996(0.0765)$	ϕ_{27}	$0.0934(0.0759)$	ϕ_6	$-0.0576(0.0831)$	ϕ_{27}	$0.0543(0.0769)$	ϕ_6	$-0.0792(0.0738)$	ϕ_{27}	$0.0601(0.0737)$
ϕ_7	$-0.0413(0.0777)$	ϕ_{28}	$0.0689(0.0708)$	ϕ_7	$-0.0520(0.0826)$	ϕ_{28}	$0.0926(0.0728)$	ϕ_7	$-0.0339(0.0740)$	ϕ_{28}	$0.0754(0.0704)$
ϕ_8	$-0.0280(0.0712)$	ϕ_{29}	$0.2780^{***}(0.0710)$	ϕ_8	$-0.0166(0.0802)$	ϕ_{29}	$0.2645^{***}(0.0723)$	ϕ_8	$0.0025(0.0708)$	ϕ_{29}	$0.2573^{***}(0.0700)$
ϕ_9	$0.2291^{***}(0.0701)$	ϕ_{30}	$0.0535(0.0708)$	ϕ_9	$0.1939^{**}(0.0782)$	ϕ_{30}	$0.0453(0.0715)$	ϕ_9	$0.2278^{***}(0.0690)$	ϕ_{30}	$0.0455(0.0683)$
ϕ_{10}	$0.0560(0.0719)$	θ	$1.0317^{***}(0.1036)$	ϕ_{10}	$0.0608(0.0781)$	θ	$1.0007^{***}(0.0999)$	ϕ_{10}	$0.0810(0.0719)$	θ	$1.1236^{***}(0.1107)$
ϕ_{11}	$0.2844^{***}(0.0681)$	ω	$-0.0633^{***}(0.0067)$	ϕ_{11}	$0.2597^{***}(0.0735)$	ω	$-0.0544^{***}(0.0062)$	ϕ_{11}	$0.2784^{***}(0.0693)$	ω	$-0.0522^{***}(0.0060)$
ϕ_{12}	$0.0474(0.0731)$	α	$0.0370^{***}(0.0020)$	ϕ_{12}	$0.1002(0.0767)$	α	$0.0343^{***}(0.0020)$	ϕ_{12}	$0.0860(0.0736)$	α	$0.0336^{***}(0.0020)$
ϕ_{13}	$-0.0061(0.0726)$	α^*	$0.0268^{***}(0.0015)$	ϕ_{13}	$-0.0195(0.0778)$	α^*	$0.0253^{***}(0.0015)$	ϕ_{13}	$0.0153(0.0743)$	α^*	$0.0252^{***}(0.0015)$
ϕ_{14}	$-0.1259^{*}(0.0744)$	β	$0.9875^{***}(0.0013)$	ϕ_{14}	$-0.0974(0.0801)$	β	$0.9893^{***}(0.0012)$	ϕ_{14}	$-0.0893(0.0771)$	β	$0.9897^{***}(0.0012)$
ϕ_{15}	$0.1904^{***}(0.0731)$	λ_0	$-5.3960^{***}(0.3385)$	ϕ_{15}	$0.1446^{*}(0.0792)$	λ_0	$-5.3964^{***}(0.3246)$	ϕ_{15}	$0.1800^{**}(0.0757)$	λ_0	$-5.3904^{***}(0.3217)$
ϕ_{16}	$0.0912(0.0733)$	δ_1	$1.8418^{***}(0.0914)$	ϕ_{16}	$0.0858(0.0779)$	δ_1	$0.7368^{***}(0.2616)$	ϕ_{16}	$0.0898(0.0756)$	δ_1	$1.8813^{***}(0.0931)$
ϕ_{17}	$0.1011(0.0736)$	δ_2	$0.6287^{***}(0.0277)$	ϕ_{17}	$0.0988(0.0789)$	γ_1	$0.5847^{***}(0.1421)$	ϕ_{17}	$0.0828(0.0767)$	δ_2	$0.2564^{***}(0.0722)$
ϕ_{18}	$-0.0493(0.0747)$			ϕ_{18}	$-0.0539(0.0804)$	κ_1	$0.9249^{***}(0.2855)$	ϕ_{18}	$-0.0427(0.0770)$	γ_2	$0.5889^{***}(0.1152)$
ϕ_{19}	$-0.0716(0.0738)$			ϕ_{19}	$-0.0312(0.0780)$	δ_2	$0.6528^{***}(0.0288)$	ϕ_{19}	$-0.0458(0.0753)$	κ_2	$0.0945^{***}(0.0176)$
ϕ_{20}	$-0.2013^{***}(0.0661)$			ϕ_{20}	$-0.2295^{***}(0.0702)$			ϕ_{20}	$-0.1821^{***}(0.0675)$		
LL	3.4466	C_μ	0.9890	LL	3.4473	C_μ	0.9892	LL	3.4475	C_μ	0.9852
AIC	−6.8886	C_λ	0.8569	AIC	−6.8898	C_λ	0.9161	AIC	−6.8902	C_λ	0.9194
BIC	−6.8707			BIC	−6.8709	C_ν	0.3460	BIC	−6.8714	C_η	0.3316
HQC	−6.8827			HQC	−6.8835	LR	0.0007(0.0005)	HQC	−6.8840	LR	$0.0009^{**}(0.0005)$

Notes: Standard errors are reported in parentheses. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively. C_μ, C_λ indicate the conditions of consistency and asymptotic normality of ML for the location and scale equations, respectively (Section 4). C_ν and C_η indicate the conditions of consistency and asymptotic normality of ML for the shape equations (Section 4). These are sufficient conditions for DCS models with constant shape parameters. For both models, the LR test is performed with respect to the benchmark Gen-t-DCS with constant shape parameters. For the LR test, we report the OLS-HAC estimate of $d_t = c + \epsilon_t$ (Section 5.2). Model specification: $y_t = \mu_t + \exp(\lambda_t)\epsilon_t$, $\epsilon_t \sim \text{Gen-}t[\exp(\nu_t) + 2, \exp(\eta_t)]$, $\mu_t = c + \left(\sum_{j=1}^{30} \phi_j \mu_{t-j}\right) + \theta u_{\mu,t-1}$ and $\lambda_t = \omega + \beta\lambda_{t-1} + \alpha u_{\lambda,t-1} + \alpha^* \text{sgn}(-\epsilon_{t-1})(u_{\lambda,t-1} + 1)$. For the specification with constant ν_t and η_t: $\nu_t = \delta_1$ and $\eta_t = \delta_2$. For the specification with dynamic ν_t and constant η_t: $\nu_t = \delta_1 + \gamma_1\nu_{t-1} + \kappa_1 u_{\nu,t-1}$ and $\eta_t = \delta_2$. For the specification with constant ν_t and dynamic η_t: $\nu_t = \delta_1$ and $\eta_t = \delta_2 + \gamma_2\eta_{t-1} + \kappa_2 u_{\eta,t-1}$.

Table 5. Parameter estimates and model diagnostics, Skew-Gen-t-DCS

Constant τ_t, ν_t and η_t				Dynamic ν_t and constant τ_t, η_t				Constant τ_t, ν_t and dynamic η_t			
c	0.0001**(0.0001)	ϕ_{21}	−0.1855**(0.0788)	c	0.0001**(0.0000)	ϕ_{21}	−0.3491***(0.0883)	c	0.0001**(0.0001)	ϕ_{21}	−0.1516**(0.0730)
ϕ_1	−0.1855**(0.0855)	ϕ_{22}	−0.1596**(0.0801)	ϕ_1	−0.3218***(0.0940)	ϕ_{22}	−0.2706***(0.1014)	ϕ_1	−0.1637**(0.0816)	ϕ_{22}	−0.1507**(0.0711)
ϕ_2	−0.1309(0.0825)	ϕ_{23}	0.2448***(0.0727)	ϕ_2	−0.2473**(0.1049)	ϕ_{23}	0.2499***(0.0928)	ϕ_2	−0.1021(0.0741)	ϕ_{23}	0.2092***(0.0660)
ϕ_3	0.1846**(0.0845)	ϕ_{24}	−0.0518(0.0693)	ϕ_3	0.2149**(0.0981)	ϕ_{24}	0.1188(0.0768)	ϕ_3	0.1552**(0.0784)	ϕ_{24}	−0.1126*(0.0647)
ϕ_4	0.1079(0.0882)	ϕ_{25}	0.0913(0.0823)	ϕ_4	0.2785***(0.0918)	ϕ_{25}	0.3093***(0.0971)	ϕ_4	0.0620(0.0819)	ϕ_{25}	0.0048(0.0752)
ϕ_5	0.0670(0.0889)	ϕ_{26}	−0.1133(0.0852)	ϕ_5	0.2538**(0.1082)	ϕ_{26}	−0.0118(0.1093)	ϕ_5	0.0158(0.0792)	ϕ_{26}	−0.1283*(0.0759)
ϕ_6	−0.0600(0.0885)	ϕ_{27}	0.0853(0.0857)	ϕ_6	0.0678(0.1127)	ϕ_{27}	0.0170(0.1000)	ϕ_6	−0.0683(0.0777)	ϕ_{27}	0.0958(0.0791)
ϕ_7	−0.0098(0.0883)	ϕ_{28}	0.0393(0.0792)	ϕ_7	−0.0176(0.1046)	ϕ_{28}	−0.1363(0.0927)	ϕ_7	0.0000(0.0772)	ϕ_{28}	0.0784(0.0750)
ϕ_8	−0.0166(0.0789)	ϕ_{29}	0.2770***(0.0776)	ϕ_8	−0.0675(0.0855)	ϕ_{29}	0.1367(0.0926)	ϕ_8	0.0101(0.0731)	ϕ_{29}	0.2845***(0.0740)
ϕ_9	0.2107***(0.0764)	ϕ_{30}	0.0497(0.0744)	ϕ_9	0.1424*(0.0811)	ϕ_{30}	0.0234(0.0803)	ϕ_9	0.2336***(0.0701)	ϕ_{30}	0.0483(0.0724)
ϕ_{10}	0.0406(0.0771)	θ	0.9903***(0.1045)	ϕ_{10}	−0.0094(0.0772)	θ	0.8374***(0.0912)	ϕ_{10}	0.0677(0.0729)	θ	1.0586***(0.1103)
ϕ_{11}	0.2840***(0.0715)	ω	−0.0571***(0.0068)	ϕ_{11}	0.2814***(0.0691)	ω	−0.0487***(0.0063)	ϕ_{11}	0.2890***(0.0699)	ω	−0.0475***(0.0062)
ϕ_{12}	0.0735(0.0789)	α	0.0379***(0.0020)	ϕ_{12}	0.1118(0.0829)	α	0.0351***(0.0020)	ϕ_{12}	0.0923(0.0763)	α	0.0347***(0.0020)
ϕ_{13}	0.0416(0.0795)	α^*	0.0274***(0.0015)	ϕ_{13}	0.0928(0.0853)	α^*	0.0262***(0.0015)	ϕ_{13}	0.0686(0.0772)	α^*	0.0260***(0.0015)
ϕ_{14}	−0.1000(0.0805)	β	0.9888***(0.0014)	ϕ_{14}	−0.0835(0.0793)	β	0.9905***(0.0013)	ϕ_{14}	−0.0658(0.0811)	β	0.9908***(0.0012)
ϕ_{15}	0.2171***(0.0772)	λ_0	−5.3697***(0.3377)	ϕ_{15}	0.2161***(0.0721)	λ_0	−5.4050***(0.2738)	ϕ_{15}	0.2072***(0.0793)	λ_0	−5.3215***(0.3188)
ϕ_{16}	0.1114(0.0779)	δ_1	−0.0479***(0.0097)	ϕ_{16}	0.1943***(0.0749)	δ_1	−0.0492***(0.0099)	ϕ_{16}	0.0818(0.0795)	δ_1	−0.0476***(0.0098)
ϕ_{17}	0.0971(0.0797)	δ_2	1.8888***(0.0948)	ϕ_{17}	0.2089**(0.0816)	δ_2	0.7735***(0.2649)	ϕ_{17}	0.0589(0.0806)	δ_2	1.9216***(0.0964)
ϕ_{18}	−0.0593(0.0801)	δ_3	0.6216***(0.0277)	ϕ_{18}	−0.0213(0.0855)	γ_2	0.5790***(0.1383)	ϕ_{18}	−0.0502(0.0799)	δ_3	0.2808***(0.0806)
ϕ_{19}	−0.0985(0.0780)			ϕ_{19}	−0.1606**(0.0790)	κ_2	1.0313***(0.3126)	ϕ_{19}	−0.0586(0.0779)	γ_3	0.5464***(0.1300)
ϕ_{20}	−0.2376***(0.0702)			ϕ_{20}	−0.3821***(0.0701)	δ_3	0.6360***(0.0286)	ϕ_{20}	−0.1878***(0.0688)	κ_3	0.0923***(0.0178)
LL	3.4474	C_μ	0.9924	LL	3.4482	C_μ	0.9952	LL	3.4483	C_μ	0.9909
AIC	−6.8900	C_λ	0.9104	AIC	−6.8914	C_λ	0.9179	AIC	−6.8916	C_λ	0.9197
BIC	−6.8716			BIC	−6.8721	C_ν	0.3381	BIC	−6.8723	C_η	0.2847
HQC	−6.8839			HQC	−6.8850	LR	0.0008*(0.0005)	HQC	−6.8852	LR	0.0009**(0.0005)

Notes: Standard errors are reported in parentheses. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively. C_μ, C_λ indicate the conditions of consistency and asymptotic normality of ML for the location and scale equations, respectively (Section 4). C_ν and C_η indicate the conditions of consistency and asymptotic normality of ML for the shape equations (Section 4). These are sufficient conditions for DCS models with constant shape parameters. For both models, the nested LR test is performed with respect to the benchmark Gen-t-DCS with constant shape parameters. For the LR test, we report the OLS-HAC estimate of $d_t = c + \epsilon_t$ (Section 5.2). Model specification: $y_t = \mu_t + \exp(\lambda_t)\epsilon_t$, $\epsilon_t \sim \text{Skew-Gen-}t[0, 1, \tanh(\tau_t), \exp(\nu_t) + 2, \exp(\eta_t)]$, $\mu_t = c + \left(\sum_{j=1}^{30} \phi_j \mu_{t-j}\right) + \theta u_{\mu,t-1}$ and $\lambda_t = \omega + \beta\lambda_{t-1} + \alpha u_{\lambda,t-1} + \alpha^* \text{sgn}(-\epsilon_{t-1})(u_{\lambda,t-1} + 1)$. For the specification with constant ν_t, τ_t and η_t: $\tau_t = \delta_1$, $\nu_t = \delta_2$ and $\eta_t = \delta_3$. For the specification with dynamic ν_t and constant τ_t, η_t: $\tau_t = \delta_1$, $\nu_t = \delta_2 + \gamma_2\nu_{t-1} + \kappa_2 u_{\nu,t-1}$ and $\eta_t = \delta_3$. For the specification with constant τ_t, ν_t and dynamic η_t: $\tau_t = \delta_1$, $\nu_t = \delta_2$ and $\eta_t = \delta_3 + \gamma_3\eta_{t-1} + \kappa_3 u_{\eta,t-1}$.

Table 6. Parameter estimates and model diagnostics, EGB2-DCS

Constant ξ_t and ζ_t				Dynamic ξ_t and ζ_t			
c	0.0001**(0.0001)	ϕ_{22}	-0.1731**(0.0828)	c	0.0001***(0.0000)	ϕ_{22}	-0.4949***(0.0956)
ϕ_1	-0.2058**(0.0858)	ϕ_{23}	0.2217***(0.0747)	ϕ_1	-0.6459***(0.1081)	ϕ_{23}	0.0922(0.0891)
ϕ_2	-0.1650*(0.0851)	ϕ_{24}	-0.0568(0.0695)	ϕ_2	-0.5789***(0.1243)	ϕ_{24}	0.1597***(0.0585)
ϕ_3	0.1647*(0.0872)	ϕ_{25}	0.1026(0.0832)	ϕ_3	0.0089(0.1052)	ϕ_{25}	0.4287***(0.0830)
ϕ_4	0.1057(0.0894)	ϕ_{26}	-0.1021(0.0868)	ϕ_4	0.3303***(0.0788)	ϕ_{26}	0.0126(0.1050)
ϕ_5	0.0865(0.0905)	ϕ_{27}	0.0887(0.0879)	ϕ_5	0.3652***(0.0999)	ϕ_{27}	-0.2508**(0.1074)
ϕ_6	-0.0469(0.0907)	ϕ_{28}	0.0296(0.0828)	ϕ_6	0.1493(0.1086)	ϕ_{28}	-0.5399***(0.1166)
ϕ_7	-0.0020(0.0901)	ϕ_{29}	0.2806***(0.0797)	ϕ_7	-0.0880(0.1039)	ϕ_{29}	-0.2451**(0.1210)
ϕ_8	-0.0086(0.0817)	ϕ_{30}	0.0624(0.0746)	ϕ_8	-0.2396***(0.0886)	ϕ_{30}	-0.0720(0.0950)
ϕ_9	0.2188***(0.0780)	θ	0.0735***(0.0063)	ϕ_9	0.0269(0.0666)	θ	0.0428***(0.0048)
ϕ_{10}	0.0422(0.0781)	ω	-0.0630***(0.0076)	ϕ_{10}	0.0297(0.0579)	ω	-0.0459***(0.0068)
ϕ_{11}	0.2852***(0.0730)	α	0.0376***(0.0019)	ϕ_{11}	0.4668***(0.0683)	α	0.0359***(0.0019)
ϕ_{12}	0.0898(0.0820)	α^*	0.0255***(0.0015)	ϕ_{12}	0.4655***(0.1059)	α^*	0.0203***(0.0015)
ϕ_{13}	0.0576(0.0833)	β	0.9890***(0.0014)	ϕ_{13}	0.4373***(0.1233)	β	0.9920***(0.0012)
ϕ_{14}	-0.0886(0.0836)	λ_0	-5.8704***(0.3355)	ϕ_{14}	0.2007*(0.1135)	λ_0	-5.8977***(0.2729)
ϕ_{15}	0.2205***(0.0803)	δ_1	-0.2118***(0.0602)	ϕ_{15}	0.4073***(0.0921)	δ_1	-0.0893***(0.0314)
ϕ_{16}	0.1216(0.0809)	δ_2	-0.0900(0.0653)	ϕ_{16}	0.4814***(0.0980)	γ_1	0.5633***(0.0786)
ϕ_{17}	0.1114(0.0835)			ϕ_{17}	0.5552***(0.1169)	κ_1	0.0599***(0.0071)
ϕ_{18}	-0.0551(0.0839)			ϕ_{18}	0.2675**(0.1214)	δ_2	-0.0111(0.0088)
ϕ_{19}	-0.0977(0.0805)			ϕ_{19}	-0.0237(0.1002)	γ_2	0.8868***(0.2365)
ϕ_{20}	-0.2385***(0.0723)			ϕ_{20}	-0.3496***(0.0711)	κ_2	-0.0166***(0.0054)
ϕ_{21}	-0.2015**(0.0810)			ϕ_{21}	-0.4897***(0.0782)		
LL	3.4458	C_μ	0.9931	LL	3.4477	C_μ	0.9979
AIC	-6.8870	C_λ	0.8826	AIC	-6.8904	C_λ	0.8917
BIC	-6.8691	LR	0.0019***(0.0006)	BIC	-6.8706	C_ξ	0.2593
HQC	-6.8811			HQC	-6.8839	C_ζ	0.8140

Notes: Standard errors are reported in parentheses. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively. C_μ, C_λ indicate the conditions of consistency and asymptotic normality of ML for the location and scale equations, respectively (Section 4). C_ξ and C_ζ indicate the conditions of consistency and asymptotic normality of ML for the shape equations (Section 4). These are sufficient conditions for DCS models with constant shape parameters. For the LR test, we report the OLS-HAC estimate of $d_t = c + \epsilon_t$ (Section 5.2). Model specification: $y_t = \mu_t + \exp(\lambda_t)\epsilon_t$, $\epsilon_t \sim \text{EGB2}[0, 1, \exp(\xi_t), \exp(\zeta_t)]$, $\mu_t = c + \left(\sum_{j=1}^{30} \phi_j \mu_{t-j}\right) + \theta u_{\mu,t-1}$ and $\lambda_t = \omega + \beta\lambda_{t-1} + \alpha u_{\lambda,t-1} + \alpha^* \text{sgn}(-\epsilon_{t-1})(u_{\lambda,t-1} + 1)$. For the specification with constant ξ_t and ζ_t: $\xi_t = \delta_1$ and $\zeta_t = \delta_2$. For the specification with dynamic ξ_t and ζ_t: $\xi_t = \delta_1 + \gamma_1 \xi_{t-1} + \kappa_1 u_{\xi,t-1}$ and $\zeta_t = \delta_2 + \gamma_2 \zeta_{t-1} + \kappa_2 u_{\zeta,t-1}$.

Table 7. Parameter estimates and model diagnostics, NIG-DCS

Constant ν_t and η_t				Constant ν_t and dynamic η_t			
c	$0.0014(0.0013)$	ϕ_{20}	$-0.0176(0.0831)$	c	$0.0031(0.0019)$	ϕ_{20}	$-0.0925(0.0915)$
ϕ_1	$-0.5322^{***}(0.0829)$	ϕ_{21}	$-0.0024(0.0780)$	ϕ_1	$-0.6990^{***}(0.0964)$	ϕ_{21}	$-0.1804^{*}(0.0959)$
ϕ_2	$-0.5883^{***}(0.1113)$	ϕ_{22}	$0.0670(0.0855)$	ϕ_2	$-0.8014^{***}(0.1569)$	ϕ_{22}	$-0.0169(0.1191)$
ϕ_3	$-0.2777^{**}(0.1367)$	ϕ_{23}	$0.4544^{***}(0.0791)$	ϕ_3	$-0.4585^{**}(0.2015)$	ϕ_{23}	$0.4086^{***}(0.1113)$
ϕ_4	$-0.1673(0.1320)$	ϕ_{24}	$0.2916^{***}(0.0789)$	ϕ_4	$-0.2203(0.1919)$	ϕ_{24}	$0.3269^{***}(0.0997)$
ϕ_5	$-0.1009(0.1100)$	ϕ_{25}	$0.5055^{***}(0.1020)$	ϕ_5	$-0.1448(0.1460)$	ϕ_{25}	$0.6580^{***}(0.1379)$
ϕ_6	$-0.1476^{*}(0.0879)$	ϕ_{26}	$0.2681^{**}(0.1281)$	ϕ_6	$-0.1086(0.1103)$	ϕ_{26}	$0.3409^{*}(0.1898)$
ϕ_7	$-0.1987^{**}(0.0891)$	ϕ_{27}	$0.3541^{***}(0.1227)$	ϕ_7	$-0.2340^{**}(0.1171)$	ϕ_{27}	$0.3814^{**}(0.1798)$
ϕ_8	$-0.3285^{***}(0.0786)$	ϕ_{28}	$0.1985^{*}(0.1054)$	ϕ_8	$-0.4214^{***}(0.1088)$	ϕ_{28}	$0.1791(0.1391)$
ϕ_9	$-0.2761^{***}(0.0790)$	ϕ_{29}	$0.3475^{***}(0.0790)$	ϕ_9	$-0.4067^{***}(0.0989)$	ϕ_{29}	$0.2722^{***}(0.0847)$
ϕ_{10}	$-0.5345^{***}(0.0893)$	ϕ_{30}	$0.1836^{***}(0.0704)$	ϕ_{10}	$-0.7191^{***}(0.1143)$	ϕ_{30}	$0.2624^{***}(0.0706)$
ϕ_{11}	$-0.2837^{**}(0.1134)$	θ	$0.0285^{***}(0.0029)$	ϕ_{11}	$-0.5053^{***}(0.1593)$	θ	$0.0198^{***}(0.0027)$
ϕ_{12}	$-0.2665^{**}(0.1257)$	ω	$-0.0545^{***}(0.0065)$	ϕ_{12}	$-0.4033^{**}(0.1867)$	ω	$-0.0540^{***}(0.0066)$
ϕ_{13}	$-0.1245(0.1247)$	α	$0.0398^{***}(0.0020)$	ϕ_{13}	$-0.2669(0.1820)$	α	$0.0406^{***}(0.0021)$
ϕ_{14}	$-0.1106(0.1102)$	α^{*}	$0.0245^{***}(0.0015)$	ϕ_{14}	$-0.1763(0.1561)$	α^{*}	$0.0248^{***}(0.0015)$
ϕ_{15}	$0.2301^{**}(0.1005)$	β	$0.9883^{***}(0.0015)$	ϕ_{15}	$0.1784(0.1411)$	β	$0.9883^{***}(0.0015)$
ϕ_{16}	$0.2146^{**}(0.1040)$	λ_0	$-5.0791^{***}(0.3164)$	ϕ_{16}	$0.1956(0.1501)$	λ_0	$-5.3231^{***}(0.3077)$
ϕ_{17}	$0.2512^{**}(0.1153)$	δ_1	$0.7058^{***}(0.0539)$	ϕ_{17}	$0.3047^{*}(0.1666)$	δ_1	$0.7121^{***}(0.0542)$
ϕ_{18}	$0.0690(0.1164)$	δ_2	$-0.0625^{***}(0.0115)$	ϕ_{18}	$0.0244(0.1689)$	δ_2	$-0.0395^{***}(0.0141)$
ϕ_{19}	$0.0320(0.1002)$			ϕ_{19}	$-0.0817(0.1312)$	γ_2	$0.3379^{*}(0.1755)$
						κ_2	$0.0210^{***}(0.0040)$
LL	3.4462	C_μ	0.9986	LL	3.4470	C_μ	0.9993
AIC	-6.8877	C_λ	0.9253	AIC	-6.8891	C_λ	0.9247
BIC	-6.8698	LR	$0.0008^{*}(0.0004)$	BIC	-6.8702	C_η	0.0874
HQC	-6.8818			HQC	-6.8828		

Notes: Standard errors are reported in parentheses. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively. C_μ, C_λ and C_η indicate the conditions of consistency and asymptotic normality of ML for the location, scale and shape equations, respectively (Section 4). These are sufficient conditions for DCS models with constant shape parameters. For the LR test, we report the OLS-HAC estimate of $d_t = c + \epsilon_t$ (Section 5.2). Model specification: $y_t = \mu_t + \exp(\lambda_t)\epsilon_t$, $\epsilon_t \sim \text{NIG}[0, 1, \exp(\nu_t), \exp(\nu_t)\tanh(\eta_t)]$, $\mu_t = c + \left(\sum_{j=1}^{30} \phi_j \mu_{t-j}\right) + \theta u_{\mu,t-1}$ and $\lambda_t = \omega + \beta\lambda_{t-1} + \alpha u_{\lambda,t-1} + \alpha^{*}\text{sgn}(-\epsilon_{t-1})(u_{\lambda,t-1} + 1)$. For the specification with constant ν_t and η_t: $\nu_t = \delta_1$ and $\eta_t = \delta_2$. For the specification with constant ν_t and dynamic η_t: $\nu_t = \delta_1$ and $\eta_t = \delta_2 + \gamma_2\eta_{t-1} + \kappa_2 u_{\eta,t-1}$.

Table 8. Likelihood-based model comparison and R^2 from the Mincer–Zarnowitz regression

Model	LL (rank)	AIC (rank)	BIC (rank)	HQC (rank)	MZ R^2 (rank)
Skew-Gen-t-DCS (τ_t, ν_t constant, η_t dynamic)	3.4483(1)	−6.8916(1)	−6.8723(1)	−6.8852(1)	**10.51%(3)**
Skew-Gen-t-DCS (τ_t, η_t constant, ν_t dynamic)	3.4482(2)	−6.8914(2)	−6.8721(2)	−6.8850(2)	2.77%(13)
EGB2-DCS (ξ_t, ζ_t dynamic)	3.4477(3)	−6.8904(3)	−6.8706(9)	−6.8839(5)	**13.76%(1)**
Gen-t-DCS (ν_t constant, η_t dynamic)	3.4475(4)	−6.8902(4)	−6.8714(5)	−6.8840(3)	**10.23%(4)**
Skew-Gen-t-DCS (τ_t, ν_t, η_t constant)	3.4474(5)	−6.8900(5)	−6.8716(3)	−6.8839(4)	0.38%(14)
Gen-t-DCS (ν_t dynamic, η_t constant)	3.4473(6)	−6.8898(6)	−6.8709(7)	−6.8835(7)	6.24%(11)
t-DCS (ν_t dynamic)	3.4473(7)	−6.8898(7)	−6.8714(4)	−6.8837(6)	7.68%(10)
NIG-DCS (ν_t constant, η_t dynamic)	3.4470(8)	−6.8891(8)	−6.8702(10)	−6.8828(8)	**11.19%(2)**
Gen-t-DCS (ν_t, η_t constant)	3.4466(9)	−6.8886(9)	−6.8707(8)	−6.8827(10)	9.99%(5)
t-DCS (ν_t constant)	3.4465(10)	−6.8884(10)	−6.8710(6)	−6.8827(9)	9.93%(6)
NIG-DCS (ν_t, η_t constant)	3.4462(11)	−6.8877(11)	−6.8698(11)	−6.8818(11)	9.88%(7)
EGB2-DCS (ξ_t, ζ_t constant)	3.4458(12)	−6.8870(12)	−6.8691(12)	−6.8811(12)	9.86%(8)
AR plus t-GARCH with leverage effects	3.4433(13)	−6.8822(13)	−6.8652(13)	−6.8766(13)	8.76%(9)
GED-DCS (ν_t dynamic)	3.4419(14)	−6.8791(14)	−6.8607(14)	−6.8730(14)	0.01%(15)
GED-DCS (ν_t constant)	3.4407(15)	−6.8769(15)	−6.8594(15)	−6.8711(15)	5.95%(12)

Notes: The highest R^2 values from the Mincer–Zarnowitz regression are indicated by bold numbers.

Table 9. ADF test of score functions and MDS test of residuals

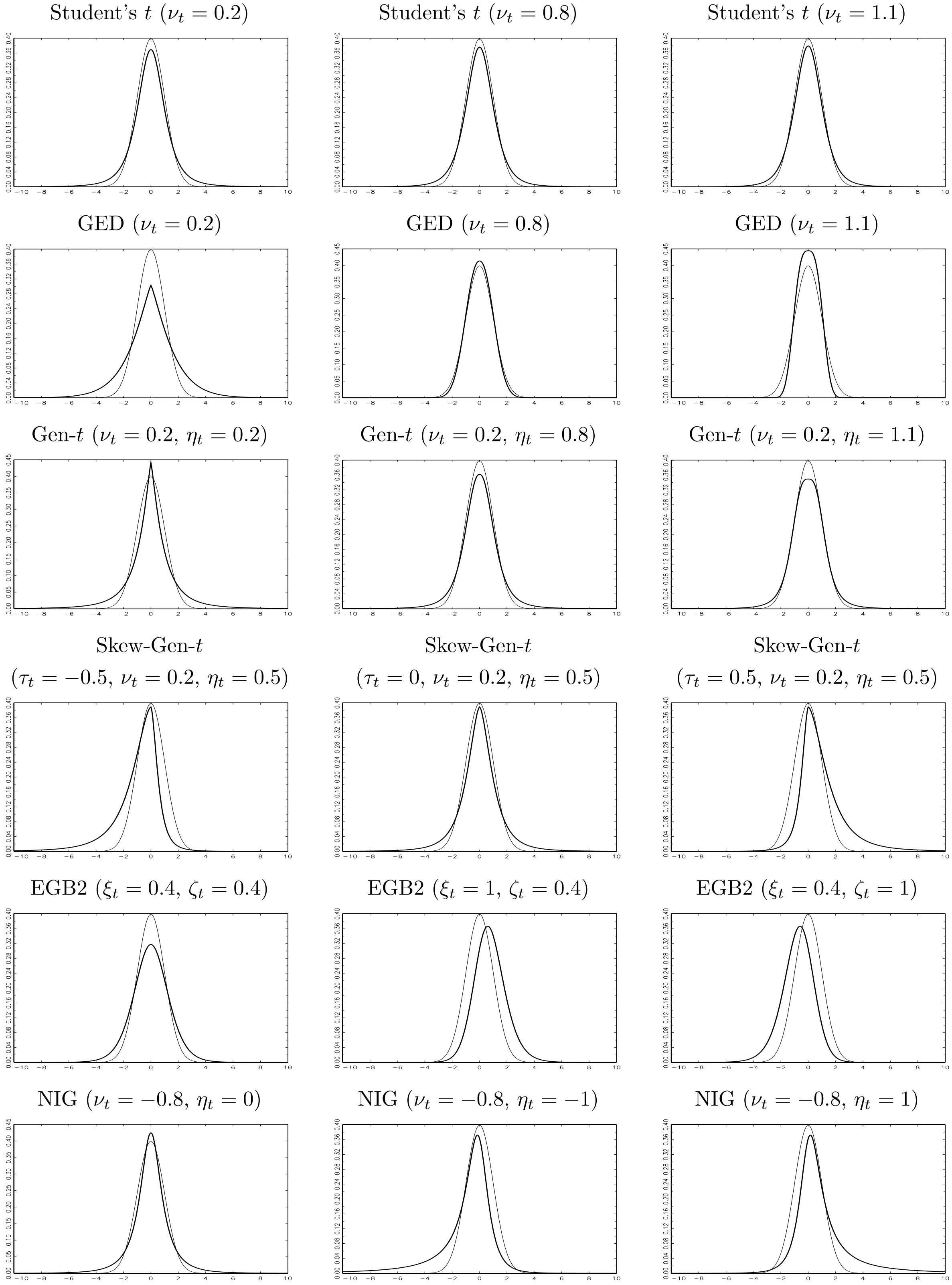
Model	Specification	Score function	ADF test	Residuals	MDS test
t-DCS	ν_t constant	$u_{\mu,t}$	−96.1499***	$\hat{\epsilon}_t$	0.2001
t-DCS	ν_t constant	$u_{\lambda,t}$	−129.9770***		
t-DCS	ν_t dynamic	$u_{\mu,t}$	−96.6024***	$\hat{\epsilon}_t$	0.0345
t-DCS	ν_t dynamic	$u_{\lambda,t}$	−131.4590***		
t-DCS	ν_t dynamic	$u_{\nu,t}$	−128.7600***		
GED-DCS	ν_t constant	$u_{\mu,t}$	−131.0850***	$\hat{\epsilon}_t$	0.8792
GED-DCS	ν_t constant	$u_{\lambda,t}$	−127.9010***		
GED-DCS	ν_t dynamic	$u_{\mu,t}$	−131.1240***	$\hat{\epsilon}_t$	0.3682
GED-DCS	ν_t dynamic	$u_{\lambda,t}$	−131.3500***		
GED-DCS	ν_t dynamic	$u_{\nu,t}$	−130.3980***		
Gen-t-DCS	ν_t, η_t constant	$u_{\mu,t}$	−96.1175***	$\hat{\epsilon}_t$	0.4981
Gen-t-DCS	ν_t, η_t constant	$u_{\lambda,t}$	−129.7930***		
Gen-t-DCS	ν_t dynamic, η_t constant	$u_{\mu,t}$	−137.7060***	$\hat{\epsilon}_t$	0.1888
Gen-t-DCS	ν_t dynamic, η_t constant	$u_{\lambda,t}$	−131.3220***		
Gen-t-DCS	ν_t dynamic, η_t constant	$u_{\nu,t}$	−128.6700***		
Gen-t-DCS	ν_t constant, η_t dynamic	$u_{\mu,t}$	−136.1920***	$\hat{\epsilon}_t$	0.0250
Gen-t-DCS	ν_t constant, η_t dynamic	$u_{\lambda,t}$	−131.5210***		
Gen-t-DCS	ν_t constant, η_t dynamic	$u_{\eta,t}$	−129.2220***		
Skew-Gen-t-DCS	τ_t, ν_t, η_t constant	$u_{\mu,t}$	−95.9761***	$\hat{\epsilon}^*_t$	1.2479
Skew-Gen-t-DCS	τ_t, ν_t, η_t constant	$u_{\lambda,t}$	−129.7120***		
Skew-Gen-t-DCS	τ_t, η_t constant, ν_t dynamic	$u_{\mu,t}$	−135.8840***	$\hat{\epsilon}^*_t$	3.7726*
Skew-Gen-t-DCS	τ_t, η_t constant, ν_t dynamic	$u_{\lambda,t}$	−131.4490***		
Skew-Gen-t-DCS	τ_t, η_t constant, ν_t dynamic	$u_{\nu,t}$	−128.8750***		
Skew-Gen-t-DCS	τ_t, ν_t constant, η_t dynamic	$u_{\mu,t}$	−135.8060***	$\hat{\epsilon}^*_t$	0.4706
Skew-Gen-t-DCS	τ_t, ν_t constant, η_t dynamic	$u_{\lambda,t}$	−131.5260***		
Skew-Gen-t-DCS	τ_t, ν_t constant, η_t dynamic	$u_{\eta,t}$	−129.2150***		
EGB2-DCS	ξ_t, ζ_t constant	$u_{\mu,t}$	−135.2790***	$\hat{\epsilon}^*_t$	2.1107
EGB2-DCS	ξ_t, ζ_t constant	$u_{\lambda,t}$	−128.8170***		
EGB2-DCS	ξ_t, ζ_t dynamic	$u_{\mu,t}$	−69.1279***	$\hat{\epsilon}^*_t$	0.2176
EGB2-DCS	ξ_t, ζ_t dynamic	$u_{\lambda,t}$	−130.0890***		
EGB2-DCS	ξ_t, ζ_t dynamic	$u_{\xi,t}$	−129.3040***		
EGB2-DCS	ξ_t, ζ_t dynamic	$u_{\zeta,t}$	−127.3640***		
NIG-DCS	ν_t, η_t constant	$u_{\mu,t}$	−132.9480***	$\hat{\epsilon}^*_t$	14.2378***
NIG-DCS	ν_t, η_t constant	$u_{\lambda,t}$	−129.4050***		
NIG-DCS	ν_t constant, η_t dynamic	$u_{\mu,t}$	−135.1690***	$\hat{\epsilon}^*_t$	0.0019
NIG-DCS	ν_t constant, η_t dynamic	$u_{\lambda,t}$	−129.4790***		
NIG-DCS	ν_t constant, η_t dynamic	$u_{\eta,t}$	−129.6330***		

Notes: * and *** indicates significance at the 10% and 1% levels, respectively. The definition of residuals and transformed residuals is $\hat{\epsilon}_t = (y_t - \hat{\mu}_t)\exp(-\hat{\lambda}_t)$ and $\hat{\epsilon}^*_t = (y_t - \hat{\mu}_t)\exp(-\hat{\lambda}_t) - \hat{E}(\epsilon_t | y_1, \ldots, y_{t-1})$, respectively (Section 2.3).

Table 10. Decomposition of normal risk and extreme risk components for scale and shape

Model	Specification	Variable	MDS p-value	Normal %	Extreme %	Correlation (all)	Correlation (risky)	Correlation (safe)
t-DCS	ν_t constant	λ_t	0.9840	97.36%	2.64%			
t-DCS	ν_t dynamic	λ_t	0.9802	97.67%	2.33%	−0.5389	−0.5378	−0.2045
t-DCS	ν_t dynamic	ν_t	0.9944	36.15%	63.85%			
GED-DCS	ν_t constant	λ_t	0.9913	97.35%	2.65%			
GED-DCS	ν_t dynamic	λ_t	0.9887	97.94%	2.06%	−0.8375	−0.8310	−0.7323
GED-DCS	ν_t dynamic	ν_t	0.9952	9.49%	90.51%			
Gen-t-DCS	ν_t, η_t constant	λ_t	0.9842	97.34%	2.66%			
Gen-t-DCS	ν_t dynamic, η_t constant	λ_t	0.9807	97.65%	2.35%	−0.5366	−0.5386	−0.1869
Gen-t-DCS	ν_t dynamic, η_t constant	ν_t	0.9950	35.57%	64.43%			
Gen-t-DCS	ν_t constant, η_t dynamic	λ_t	0.9800	97.70%	2.30%	−0.5486	−0.6204	0.2549
Gen-t-DCS	ν_t constant, η_t dynamic	η_t	0.9941	35.43%	64.57%			
Skew-Gen-t-DCS	τ_t, ν_t, η_t constant	λ_t	0.9849	97.30%	2.70%			
Skew-Gen-t-DCS	τ_t, η_t constant, ν_t dynamic	λ_t	0.9822	97.60%	2.40%	−0.5069	−0.5082	−0.1503
Skew-Gen-t-DCS	τ_t, η_t constant, ν_t dynamic	ν_t	0.9939	34.61%	65.39%			
Skew-Gen-t-DCS	τ_t, ν_t constant, η_t dynamic	λ_t	0.9807	97.64%	2.36%	−0.5177	−0.5878	0.2791
Skew-Gen-t-DCS	τ_t, ν_t constant, η_t dynamic	η_t	0.9944	30.58%	69.42%			
EGB2-DCS	ξ_t, ζ_t constant	λ_t	0.9876	97.26%	2.74%			
EGB2-DCS	ξ_t, ζ_t dynamic	λ_t	0.9893	97.87%	2.13%			
EGB2-DCS	ξ_t, ζ_t dynamic	ξ_t	0.9760	31.94%	68.06%	−0.8992	−0.9822	−0.2710
EGB2-DCS	ξ_t, ζ_t dynamic	ζ_t	0.9979	79.26%	20.74%	−0.2105	−0.5404	0.2944
NIG-DCS	ν_t, η_t constant	λ_t	0.9850	97.26%	2.74%			
NIG-DCS	ν_t constant, η_t dynamic	λ_t	0.9830	97.21%	2.79%	−0.6834	−0.9404	0.1165
NIG-DCS	ν_t constant, η_t dynamic	η_t	0.9877	11.51%	88.49%			

Notes: We use the AR(30) model to separate the normal and extreme risk components of each dynamic parameter. The p-value of the MDS test (Escanciano and Lobato 2009) is denoted by MDS p-value. The MDS test is undertaken for the residuals of the AR(30) model to support the consistency of OLS. Normal % is the R^2 of the AR(30) equation, and it is interpreted as the proportion of the dynamic parameter that can be related to the normal risk component. Extreme % is $1 - R^2$ of the AR(30) equation, and it is interpreted as the proportion of the dynamic parameter that can be related to the extreme risk component. For the DCS models with dynamic shape, correlation coefficients are estimated between the AR(30) residuals of λ_t and those of each shape parameter (i.e., ν_t, η_t, ξ_t or ζ_t). We estimate those correlation coefficients for all days, for risky days and for safe days.


Student's t ($\nu_t = 0.2$)

Student's t ($\nu_t = 0.8$)

Student's t ($\nu_t = 1.1$)

GED ($\nu_t = 0.2$)

GED ($\nu_t = 0.8$)

GED ($\nu_t = 1.1$)

Gen-t ($\nu_t = 0.2$, $\eta_t = 0.2$)

Gen-t ($\nu_t = 0.2$, $\eta_t = 0.8$)

Gen-t ($\nu_t = 0.2$, $\eta_t = 1.1$)

Skew-Gen-t ($\tau_t = -0.5$, $\nu_t = 0.2$, $\eta_t = 0.5$)

Skew-Gen-t ($\tau_t = 0$, $\nu_t = 0.2$, $\eta_t = 0.5$)

Skew-Gen-t ($\tau_t = 0.5$, $\nu_t = 0.2$, $\eta_t = 0.5$)

EGB2 ($\xi_t = 0.4$, $\zeta_t = 0.4$)

EGB2 ($\xi_t = 1$, $\zeta_t = 0.4$)

EGB2 ($\xi_t = 0.4$, $\zeta_t = 1$)

NIG ($\nu_t = -0.8$, $\eta_t = 0$)

NIG ($\nu_t = -0.8$, $\eta_t = -1$)

NIG ($\nu_t = -0.8$, $\eta_t = 1$)

Fig. 1. Alternative density functions of ϵ_t (thick lines), compared to N(0,1) (thin lines).

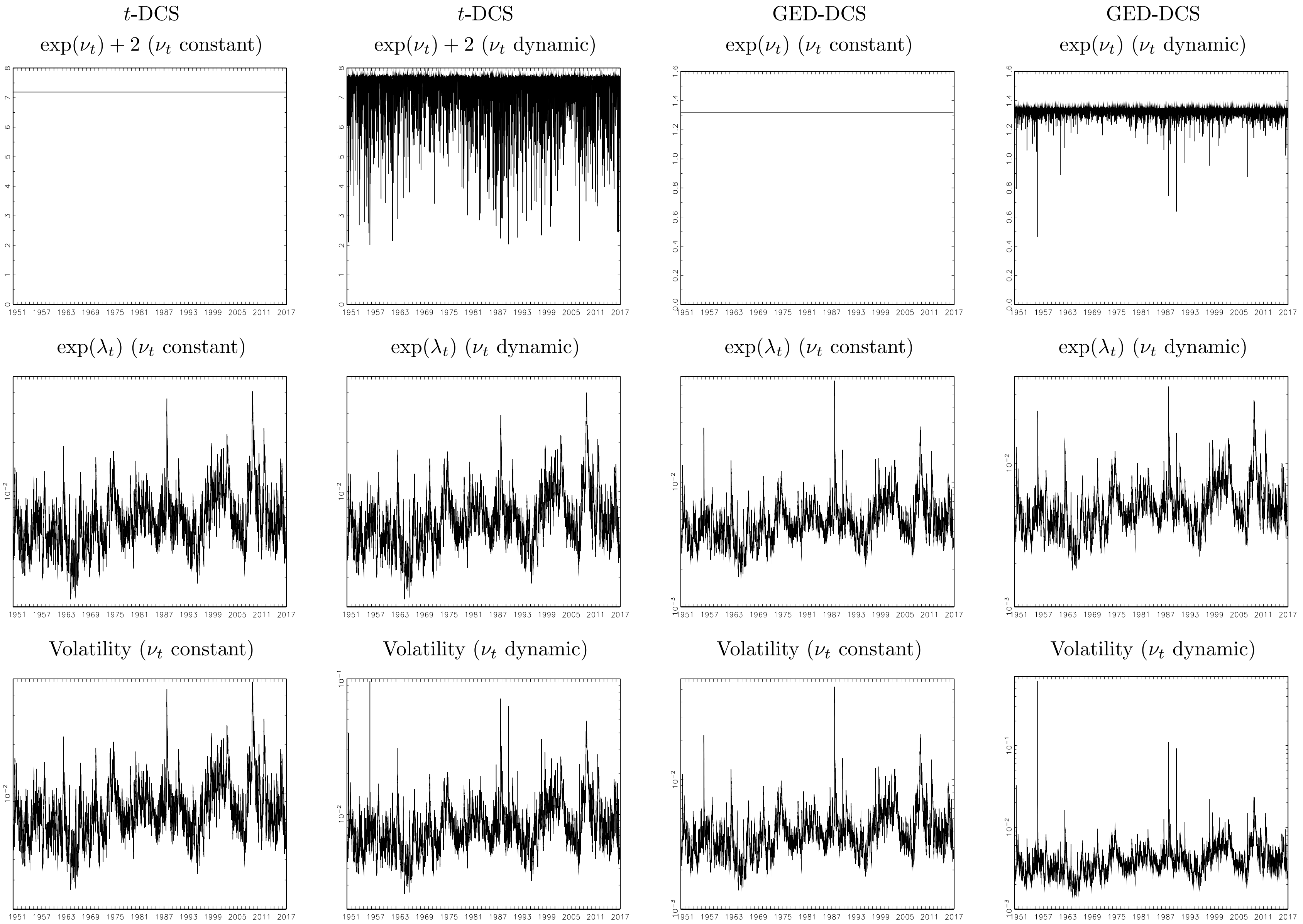


Fig. 2. ***t*-DCS and GED-DCS models (Sections 3.1 and 3.2).**

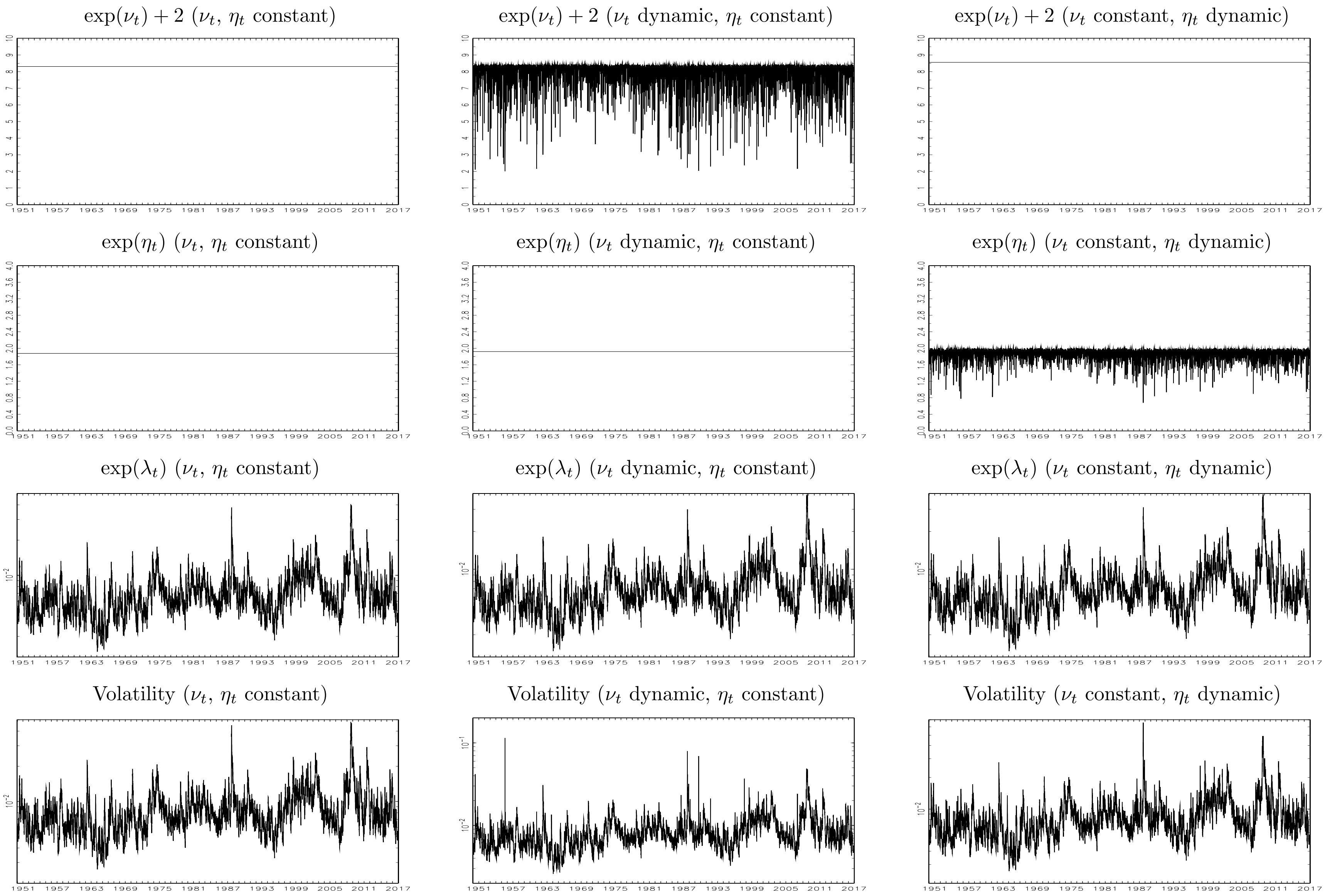
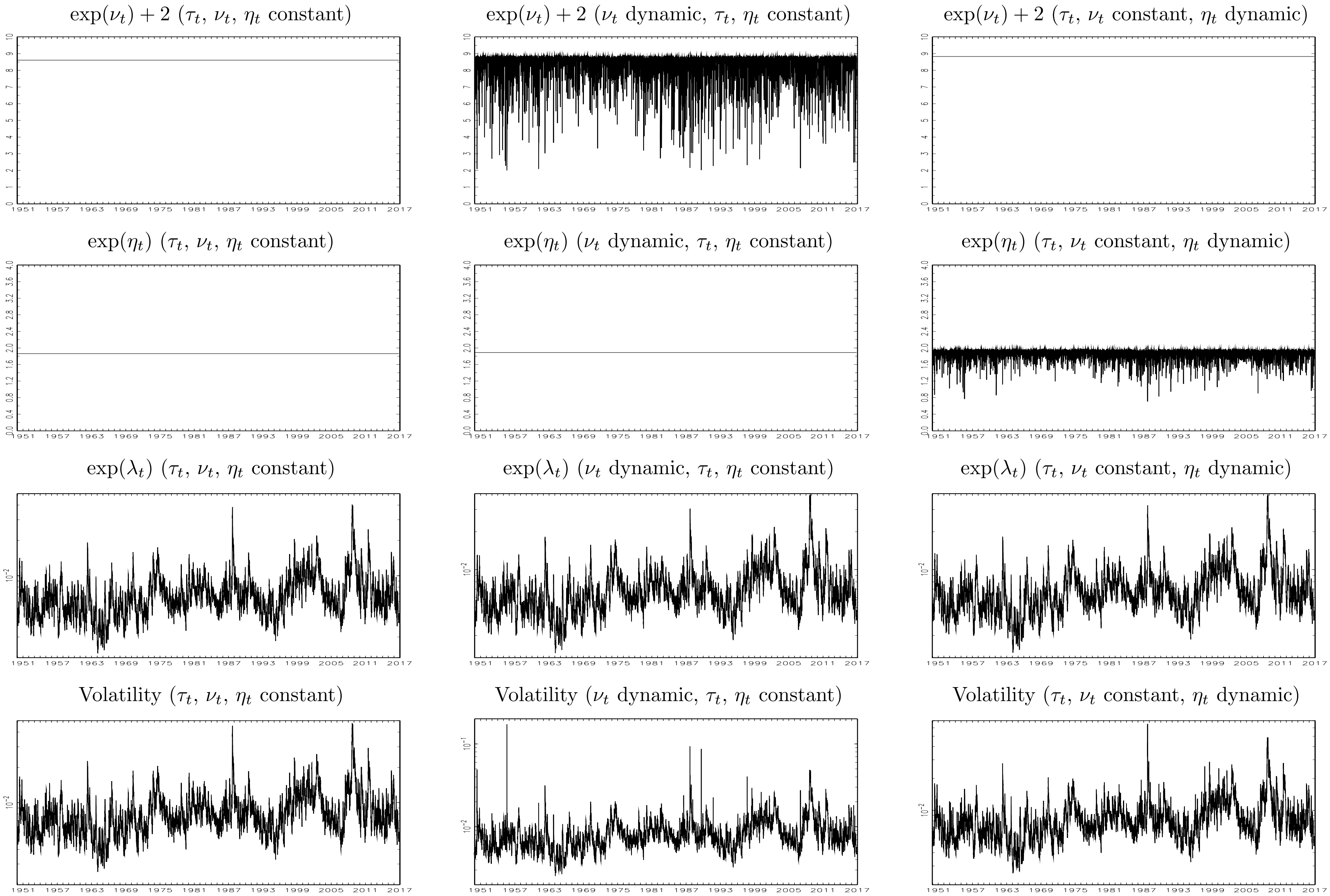


Fig. 3. Gen-t-DCS model (Section 3.3).



$\exp(\nu_t) + 2$ (τ_t, ν_t, η_t constant)

$\exp(\nu_t) + 2$ (ν_t dynamic, τ_t, η_t constant)

$\exp(\nu_t) + 2$ (τ_t, ν_t constant, η_t dynamic)

$\exp(\eta_t)$ (τ_t, ν_t, η_t constant)

$\exp(\eta_t)$ (ν_t dynamic, τ_t, η_t constant)

$\exp(\eta_t)$ (τ_t, ν_t constant, η_t dynamic)

$\exp(\lambda_t)$ (τ_t, ν_t, η_t constant)

$\exp(\lambda_t)$ (ν_t dynamic, τ_t, η_t constant)

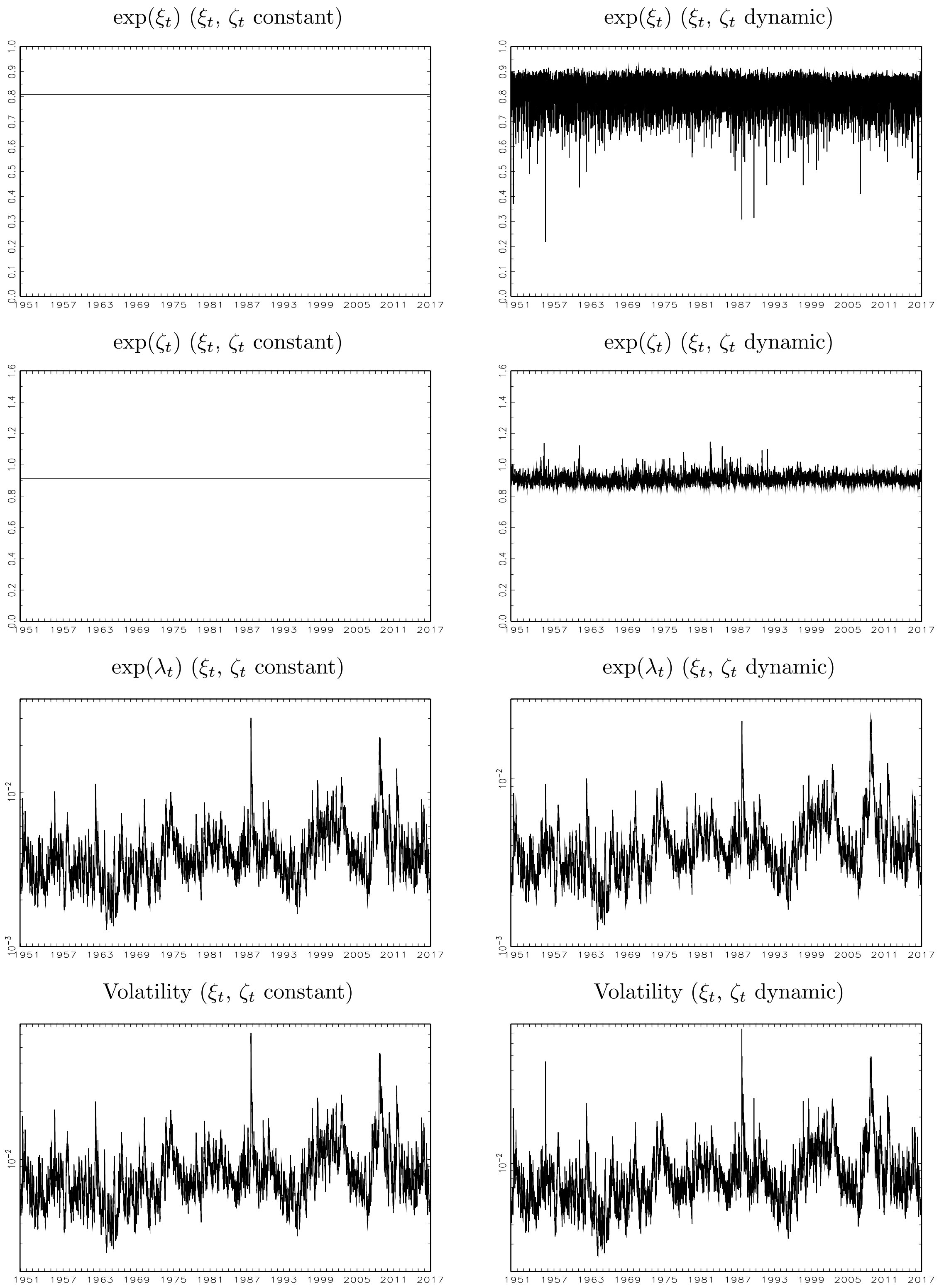
$\exp(\lambda_t)$ (τ_t, ν_t constant, η_t dynamic)

Volatility (τ_t, ν_t, η_t constant)

Volatility (ν_t dynamic, τ_t, η_t constant)

Volatility (τ_t, ν_t constant, η_t dynamic)

Fig. 4. Skew-Gen-t-DCS model (Section 3.4).



$\exp(\xi_t)$ (ξ_t, ζ_t constant)

$\exp(\xi_t)$ (ξ_t, ζ_t dynamic)

$\exp(\zeta_t)$ (ξ_t, ζ_t constant)

$\exp(\zeta_t)$ (ξ_t, ζ_t dynamic)

$\exp(\lambda_t)$ (ξ_t, ζ_t constant)

$\exp(\lambda_t)$ (ξ_t, ζ_t dynamic)

Volatility (ξ_t, ζ_t constant)

Volatility (ξ_t, ζ_t dynamic)

Fig. 5. EGB2-DCS model (Section 3.5).

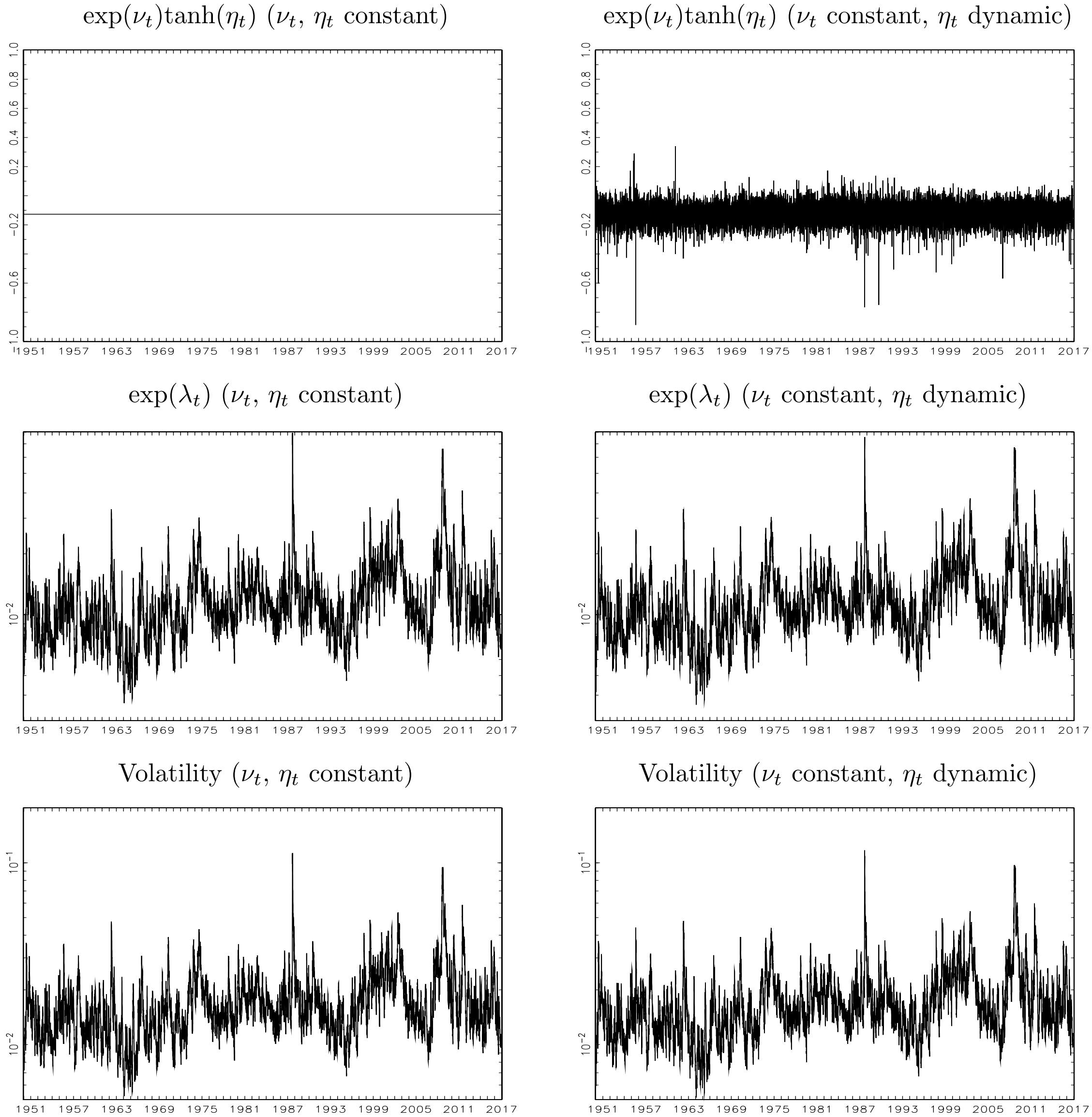


Fig. 6. NIG-DCS model (Section 3.6).

Separate Appendix for "Dynamic Conditional Score Models with Time-Varying Location, Scale and Shape Parameters"

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Table A1. OLS estimates of the AR(30) model for the decomposition of scale and shape

t-DCS ν_t constant		t-DCS ν_t dynamic				GED-DCS ν_t constant		GED-DCS ν_t dynamic			
c_λ	$-0.0637^{***}(0.0071)$	c_λ	$-0.0573^{***}(0.0066)$	c_ν	$0.6604^{***}(0.0276)$	c_λ	$-0.0686^{***}(0.0078)$	c_λ	$-0.0555^{***}(0.0066)$	c_ν	$0.2117^{***}(0.0110)$
$\phi_{\lambda,1}$	$1.0045^{***}(0.0084)$	$\phi_{\lambda,1}$	$1.0025^{***}(0.0078)$	$\phi_{\nu,1}$	$0.5976^{***}(0.0070)$	$\phi_{\lambda,1}$	$1.0092^{***}(0.0103)$	$\phi_{\lambda,1}$	$0.9987^{***}(0.0086)$	$\phi_{\nu,1}$	$0.3035^{***}(0.0105)$
$\phi_{\lambda,2}$	$-0.0032(0.0127)$	$\phi_{\lambda,2}$	$0.0001(0.0118)$	$\phi_{\nu,2}$	$0.0034(0.0076)$	$\phi_{\lambda,2}$	$-0.0056(0.0127)$	$\phi_{\lambda,2}$	$0.0088(0.0114)$	$\phi_{\nu,2}$	$0.0062(0.0073)$
$\phi_{\lambda,3}$	$-0.0066(0.0122)$	$\phi_{\lambda,3}$	$-0.0067(0.0115)$	$\phi_{\nu,3}$	$-0.0027(0.0097)$	$\phi_{\lambda,3}$	$-0.0106(0.0116)$	$\phi_{\lambda,3}$	$-0.0079(0.0114)$	$\phi_{\nu,3}$	$0.0012(0.0071)$
$\phi_{\lambda,4}$	$0.0050(0.0119)$	$\phi_{\lambda,4}$	$0.0063(0.0111)$	$\phi_{\nu,4}$	$-0.0011(0.0076)$	$\phi_{\lambda,4}$	$0.0046(0.0120)$	$\phi_{\lambda,4}$	$0.0045(0.0111)$	$\phi_{\nu,4}$	$0.0011(0.0059)$
$\phi_{\lambda,5}$	$-0.0064(0.0118)$	$\phi_{\lambda,5}$	$-0.0068(0.0113)$	$\phi_{\nu,5}$	$0.0232(0.0143)$	$\phi_{\lambda,5}$	$-0.0090(0.0096)$	$\phi_{\lambda,5}$	$-0.0058(0.0099)$	$\phi_{\nu,5}$	$0.0073(0.0081)$
$\phi_{\lambda,6}$	$-0.0179(0.0110)$	$\phi_{\lambda,6}$	$-0.0157(0.0110)$	$\phi_{\nu,6}$	$-0.0179^{**}(0.0088)$	$\phi_{\lambda,6}$	$-0.0118(0.0085)$	$\phi_{\lambda,6}$	$-0.0148^{*}(0.0088)$	$\phi_{\nu,6}$	$-0.0083(0.0051)$
$\phi_{\lambda,7}$	$0.0050(0.0112)$	$\phi_{\lambda,7}$	$0.0030(0.0113)$	$\phi_{\nu,7}$	$0.0007(0.0071)$	$\phi_{\lambda,7}$	$0.0043(0.0093)$	$\phi_{\lambda,7}$	$0.0036(0.0096)$	$\phi_{\nu,7}$	$-0.0048(0.0052)$
$\phi_{\lambda,8}$	$0.0154(0.0116)$	$\phi_{\lambda,8}$	$0.0170(0.0115)$	$\phi_{\nu,8}$	$0.0066(0.0073)$	$\phi_{\lambda,8}$	$0.0131(0.0106)$	$\phi_{\lambda,8}$	$0.0155(0.0109)$	$\phi_{\nu,8}$	$0.0073(0.0065)$
$\phi_{\lambda,9}$	$-0.0181(0.0110)$	$\phi_{\lambda,9}$	$-0.0194^{*}(0.0111)$	$\phi_{\nu,9}$	$0.0008(0.0087)$	$\phi_{\lambda,9}$	$-0.0063(0.0116)$	$\phi_{\lambda,9}$	$-0.0113(0.0105)$	$\phi_{\nu,9}$	$0.0092(0.0094)$
$\phi_{\lambda,10}$	$0.0154(0.0112)$	$\phi_{\lambda,10}$	$0.0172(0.0113)$	$\phi_{\nu,10}$	$0.0181(0.0123)$	$\phi_{\lambda,10}$	$0.0052(0.0126)$	$\phi_{\lambda,10}$	$0.0100(0.0115)$	$\phi_{\nu,10}$	$-0.0003(0.0068)$
$\phi_{\lambda,11}$	$-0.0065(0.0114)$	$\phi_{\lambda,11}$	$-0.0084(0.0115)$	$\phi_{\nu,11}$	$-0.0194^{**}(0.0088)$	$\phi_{\lambda,11}$	$-0.0084(0.0096)$	$\phi_{\lambda,11}$	$-0.0120(0.0098)$	$\phi_{\nu,11}$	$-0.0042(0.0051)$
$\phi_{\lambda,12}$	$-0.0053(0.0102)$	$\phi_{\lambda,12}$	$-0.0050(0.0103)$	$\phi_{\nu,12}$	$0.0043(0.0064)$	$\phi_{\lambda,12}$	$-0.0037(0.0081)$	$\phi_{\lambda,12}$	$-0.0021(0.0082)$	$\phi_{\nu,12}$	$-0.0047(0.0053)$
$\phi_{\lambda,13}$	$-0.0031(0.0106)$	$\phi_{\lambda,13}$	$-0.0030(0.0107)$	$\phi_{\nu,13}$	$-0.0030(0.0070)$	$\phi_{\lambda,13}$	$-0.0019(0.0086)$	$\phi_{\lambda,13}$	$-0.0030(0.0087)$	$\phi_{\nu,13}$	$-0.0029(0.0052)$
$\phi_{\lambda,14}$	$0.0078(0.0109)$	$\phi_{\lambda,14}$	$0.0085(0.0110)$	$\phi_{\nu,14}$	$0.0119(0.0088)$	$\phi_{\lambda,14}$	$0.0093(0.0095)$	$\phi_{\lambda,14}$	$0.0090(0.0096)$	$\phi_{\nu,14}$	$0.0071(0.0072)$
$\phi_{\lambda,15}$	$-0.0128(0.0110)$	$\phi_{\lambda,15}$	$-0.0142(0.0110)$	$\phi_{\nu,15}$	$-0.0130^{**}(0.0057)$	$\phi_{\lambda,15}$	$-0.0126(0.0097)$	$\phi_{\lambda,15}$	$-0.0120(0.0099)$	$\phi_{\nu,15}$	$-0.0085^{*}(0.0050)$
$\phi_{\lambda,16}$	$0.0035(0.0107)$	$\phi_{\lambda,16}$	$0.0049(0.0108)$	$\phi_{\nu,16}$	$0.0075(0.0071)$	$\phi_{\lambda,16}$	$0.0030(0.0096)$	$\phi_{\lambda,16}$	$0.0028(0.0100)$	$\phi_{\nu,16}$	$-0.0020(0.0053)$
$\phi_{\lambda,17}$	$0.0021(0.0105)$	$\phi_{\lambda,17}$	$0.0011(0.0106)$	$\phi_{\nu,17}$	$-0.0039(0.0064)$	$\phi_{\lambda,17}$	$-0.0018(0.0088)$	$\phi_{\lambda,17}$	$-0.0030(0.0090)$	$\phi_{\nu,17}$	$-0.0063(0.0046)$
$\phi_{\lambda,18}$	$0.0100(0.0104)$	$\phi_{\lambda,18}$	$0.0086(0.0104)$	$\phi_{\nu,18}$	$-0.0015(0.0050)$	$\phi_{\lambda,18}$	$0.0063(0.0082)$	$\phi_{\lambda,18}$	$0.0067(0.0083)$	$\phi_{\nu,18}$	$-0.0014(0.0046)$
$\phi_{\lambda,19}$	$-0.0005(0.0104)$	$\phi_{\lambda,19}$	$0.0012(0.0104)$	$\phi_{\nu,19}$	$0.0099(0.0085)$	$\phi_{\lambda,19}$	$0.0038(0.0087)$	$\phi_{\lambda,19}$	$0.0024(0.0085)$	$\phi_{\nu,19}$	$0.0054(0.0081)$
$\phi_{\lambda,20}$	$-0.0021(0.0109)$	$\phi_{\lambda,20}$	$-0.0066(0.0108)$	$\phi_{\nu,20}$	$-0.0129^{**}(0.0062)$	$\phi_{\lambda,20}$	$-0.0018(0.0092)$	$\phi_{\lambda,20}$	$-0.0014(0.0092)$	$\phi_{\nu,20}$	$-0.0070(0.0048)$
$\phi_{\lambda,21}$	$0.0001(0.0110)$	$\phi_{\lambda,21}$	$0.0053(0.0109)$	$\phi_{\nu,21}$	$0.0146(0.0138)$	$\phi_{\lambda,21}$	$-0.0008(0.0090)$	$\phi_{\lambda,21}$	$-0.0012(0.0089)$	$\phi_{\nu,21}$	$0.0032(0.0087)$
$\phi_{\lambda,22}$	$-0.0180^{*}(0.0108)$	$\phi_{\lambda,22}$	$-0.0219^{**}(0.0108)$	$\phi_{\nu,22}$	$-0.0081(0.0100)$	$\phi_{\lambda,22}$	$-0.0151(0.0093)$	$\phi_{\lambda,22}$	$-0.0149(0.0093)$	$\phi_{\nu,22}$	$-0.0057(0.0059)$
$\phi_{\lambda,23}$	$0.0076(0.0104)$	$\phi_{\lambda,23}$	$0.0110(0.0104)$	$\phi_{\nu,23}$	$-0.0030(0.0054)$	$\phi_{\lambda,23}$	$0.0099(0.0094)$	$\phi_{\lambda,23}$	$0.0108(0.0094)$	$\phi_{\nu,23}$	$-0.0015(0.0058)$
$\phi_{\lambda,24}$	$0.0144(0.0104)$	$\phi_{\lambda,24}$	$0.0125(0.0104)$	$\phi_{\nu,24}$	$0.0078(0.0058)$	$\phi_{\lambda,24}$	$0.0136(0.0091)$	$\phi_{\lambda,24}$	$0.0115(0.0089)$	$\phi_{\nu,24}$	$0.0052(0.0061)$
$\phi_{\lambda,25}$	$0.0014(0.0113)$	$\phi_{\lambda,25}$	$0.0007(0.0111)$	$\phi_{\nu,25}$	$0.0084(0.0103)$	$\phi_{\lambda,25}$	$-0.0073(0.0103)$	$\phi_{\lambda,25}$	$-0.0051(0.0099)$	$\phi_{\nu,25}$	$0.0005(0.0062)$
$\phi_{\lambda,26}$	$-0.0324^{***}(0.0104)$	$\phi_{\lambda,26}$	$-0.0285^{***}(0.0103)$	$\phi_{\nu,26}$	$-0.0260^{***}(0.0075)$	$\phi_{\lambda,26}$	$-0.0260^{***}(0.0091)$	$\phi_{\lambda,26}$	$-0.0261^{***}(0.0090)$	$\phi_{\nu,26}$	$-0.0231^{***}(0.0054)$
$\phi_{\lambda,27}$	$0.0312^{***}(0.0102)$	$\phi_{\lambda,27}$	$0.0276^{***}(0.0102)$	$\phi_{\nu,27}$	$0.0102^{**}(0.0050)$	$\phi_{\lambda,27}$	$0.0289^{***}(0.0091)$	$\phi_{\lambda,27}$	$0.0285^{***}(0.0091)$	$\phi_{\nu,27}$	$0.0054(0.0053)$
$\phi_{\lambda,28}$	$-0.0171(0.0104)$	$\phi_{\lambda,28}$	$-0.0153(0.0105)$	$\phi_{\nu,28}$	$-0.0050(0.0073)$	$\phi_{\lambda,28}$	$-0.0147(0.0093)$	$\phi_{\lambda,28}$	$-0.0142(0.0093)$	$\phi_{\nu,28}$	$-0.0109^{*}(0.0058)$
$\phi_{\lambda,29}$	$0.0143(0.0101)$	$\phi_{\lambda,29}$	$0.0108(0.0099)$	$\phi_{\nu,29}$	$0.0069(0.0085)$	$\phi_{\lambda,29}$	$0.0111(0.0087)$	$\phi_{\lambda,29}$	$0.0081(0.0084)$	$\phi_{\nu,29}$	$0.0029(0.0060)$
$\phi_{\lambda,30}$	$-0.0004(0.0074)$	$\phi_{\lambda,30}$	$0.0017(0.0072)$	$\phi_{\nu,30}$	$-0.0114^{*}(0.0063)$	$\phi_{\lambda,30}$	$0.0024(0.0067)$	$\phi_{\lambda,30}$	$0.0036(0.0062)$	$\phi_{\nu,30}$	$-0.0110^{***}(0.0042)$

Notes: Standard errors are reported in parenthesis. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively.

Table A2. OLS estimates of the AR(30) model for the decomposition of scale and shape

Gen-t-DCS ν_t, η_t constant		Gen-t-DCS ν_t dynamic, η_t constant				Gen-t-DCS ν_t constant, η_t dynamic			
c_λ	−0.0643***(0.0072)	c_λ	−0.0579***(0.0066)	c_ν	0.7179***(0.0297)	c_λ	−0.0569***(0.0065)	c_η	0.2443***(0.0121)
$\phi_{\lambda,1}$	1.0055***(0.0084)	$\phi_{\lambda,1}$	1.0035***(0.0078)	$\phi_{\nu,1}$	0.5933***(0.0071)	$\phi_{\lambda,1}$	1.0021***(0.0077)	$\phi_{\eta,1}$	0.5936***(0.0101)
$\phi_{\lambda,2}$	−0.0044(0.0128)	$\phi_{\lambda,2}$	−0.0006(0.0119)	$\phi_{\nu,2}$	0.0030(0.0076)	$\phi_{\lambda,2}$	−0.0003(0.0116)	$\phi_{\eta,2}$	−0.0001(0.0097)
$\phi_{\lambda,3}$	−0.0070(0.0124)	$\phi_{\lambda,3}$	−0.0074(0.0116)	$\phi_{\nu,3}$	−0.0023(0.0098)	$\phi_{\lambda,3}$	−0.0042(0.0115)	$\phi_{\eta,3}$	−0.0035(0.0108)
$\phi_{\lambda,4}$	0.0049(0.0120)	$\phi_{\lambda,4}$	0.0059(0.0111)	$\phi_{\nu,4}$	−0.0022(0.0073)	$\phi_{\lambda,4}$	0.0046(0.0113)	$\phi_{\eta,4}$	0.0049(0.0097)
$\phi_{\lambda,5}$	−0.0064(0.0118)	$\phi_{\lambda,5}$	−0.0068(0.0113)	$\phi_{\nu,5}$	0.0234*(0.0142)	$\phi_{\lambda,5}$	−0.0053(0.0114)	$\phi_{\eta,5}$	0.0126(0.0113)
$\phi_{\lambda,6}$	−0.0175(0.0110)	$\phi_{\lambda,6}$	−0.0155(0.0110)	$\phi_{\nu,6}$	−0.0180**(0.0086)	$\phi_{\lambda,6}$	−0.0173(0.0109)	$\phi_{\eta,6}$	−0.0166*(0.0095)
$\phi_{\lambda,7}$	0.0051(0.0113)	$\phi_{\lambda,7}$	0.0039(0.0113)	$\phi_{\nu,7}$	0.0004(0.0067)	$\phi_{\lambda,7}$	0.0046(0.0112)	$\phi_{\eta,7}$	0.0093(0.0091)
$\phi_{\lambda,8}$	0.0153(0.0116)	$\phi_{\lambda,8}$	0.0167(0.0115)	$\phi_{\nu,8}$	0.0060(0.0069)	$\phi_{\lambda,8}$	0.0158(0.0115)	$\phi_{\eta,8}$	0.0082(0.0091)
$\phi_{\lambda,9}$	−0.0181(0.0110)	$\phi_{\lambda,9}$	−0.0197*(0.0110)	$\phi_{\nu,9}$	0.0014(0.0086)	$\phi_{\lambda,9}$	−0.0201*(0.0110)	$\phi_{\eta,9}$	−0.0048(0.0089)
$\phi_{\lambda,10}$	0.0159(0.0113)	$\phi_{\lambda,10}$	0.0182(0.0113)	$\phi_{\nu,10}$	0.0169(0.0118)	$\phi_{\lambda,10}$	0.0172(0.0113)	$\phi_{\eta,10}$	0.0163(0.0103)
$\phi_{\lambda,11}$	−0.0073(0.0114)	$\phi_{\lambda,11}$	−0.0088(0.0115)	$\phi_{\nu,11}$	−0.0187**(0.0083)	$\phi_{\lambda,11}$	−0.0079(0.0115)	$\phi_{\eta,11}$	−0.0222**(0.0087)
$\phi_{\lambda,12}$	−0.0052(0.0102)	$\phi_{\lambda,12}$	−0.0051(0.0103)	$\phi_{\nu,12}$	0.0040(0.0060)	$\phi_{\lambda,12}$	−0.0057(0.0102)	$\phi_{\eta,12}$	0.0059(0.0086)
$\phi_{\lambda,13}$	−0.0029(0.0105)	$\phi_{\lambda,13}$	−0.0033(0.0106)	$\phi_{\nu,13}$	−0.0030(0.0066)	$\phi_{\lambda,13}$	−0.0016(0.0106)	$\phi_{\eta,13}$	−0.0031(0.0084)
$\phi_{\lambda,14}$	0.0081(0.0109)	$\phi_{\lambda,14}$	0.0085(0.0110)	$\phi_{\nu,14}$	0.0109(0.0085)	$\phi_{\lambda,14}$	0.0071(0.0110)	$\phi_{\eta,14}$	0.0139(0.0098)
$\phi_{\lambda,15}$	−0.0133(0.0110)	$\phi_{\lambda,15}$	−0.0141(0.0111)	$\phi_{\nu,15}$	−0.0123**(0.0053)	$\phi_{\lambda,15}$	−0.0128(0.0110)	$\phi_{\eta,15}$	−0.0134(0.0083)
$\phi_{\lambda,16}$	0.0039(0.0108)	$\phi_{\lambda,16}$	0.0045(0.0109)	$\phi_{\nu,16}$	0.0065(0.0067)	$\phi_{\lambda,16}$	0.0041(0.0109)	$\phi_{\eta,16}$	0.0112(0.0090)
$\phi_{\lambda,17}$	0.0014(0.0105)	$\phi_{\lambda,17}$	0.0012(0.0106)	$\phi_{\nu,17}$	−0.0037(0.0059)	$\phi_{\lambda,17}$	0.0011(0.0106)	$\phi_{\eta,17}$	−0.0045(0.0090)
$\phi_{\lambda,18}$	0.0101(0.0104)	$\phi_{\lambda,18}$	0.0088(0.0104)	$\phi_{\nu,18}$	−0.0015(0.0046)	$\phi_{\lambda,18}$	0.0088(0.0104)	$\phi_{\eta,18}$	0.0038(0.0079)
$\phi_{\lambda,19}$	0.0000(0.0104)	$\phi_{\lambda,19}$	0.0017(0.0104)	$\phi_{\nu,19}$	0.0099(0.0083)	$\phi_{\lambda,19}$	0.0006(0.0103)	$\phi_{\eta,19}$	0.0099(0.0099)
$\phi_{\lambda,20}$	−0.0026(0.0109)	$\phi_{\lambda,20}$	−0.0071(0.0108)	$\phi_{\nu,20}$	−0.0128**(0.0060)	$\phi_{\lambda,20}$	−0.0049(0.0107)	$\phi_{\eta,20}$	−0.0140*(0.0084)
$\phi_{\lambda,21}$	0.0002(0.0109)	$\phi_{\lambda,21}$	0.0054(0.0109)	$\phi_{\nu,21}$	0.0151(0.0142)	$\phi_{\lambda,21}$	0.0028(0.0108)	$\phi_{\eta,21}$	0.0077(0.0108)
$\phi_{\lambda,22}$	−0.0177(0.0108)	$\phi_{\lambda,22}$	−0.0216**(0.0108)	$\phi_{\nu,22}$	−0.0086(0.0100)	$\phi_{\lambda,22}$	−0.0197*(0.0107)	$\phi_{\eta,22}$	0.0018(0.0093)
$\phi_{\lambda,23}$	0.0077(0.0103)	$\phi_{\lambda,23}$	0.0108(0.0104)	$\phi_{\nu,23}$	−0.0031(0.0051)	$\phi_{\lambda,23}$	0.0102(0.0103)	$\phi_{\eta,23}$	−0.0026(0.0076)
$\phi_{\lambda,24}$	0.0145(0.0104)	$\phi_{\lambda,24}$	0.0124(0.0104)	$\phi_{\nu,24}$	0.0069(0.0055)	$\phi_{\lambda,24}$	0.0122(0.0104)	$\phi_{\eta,24}$	0.0084(0.0084)
$\phi_{\lambda,25}$	0.0009(0.0113)	$\phi_{\lambda,25}$	0.0005(0.0111)	$\phi_{\nu,25}$	0.0084(0.0100)	$\phi_{\lambda,25}$	0.0012(0.0111)	$\phi_{\eta,25}$	0.0117(0.0099)
$\phi_{\lambda,26}$	−0.0330***(0.0104)	$\phi_{\lambda,26}$	−0.0292***(0.0103)	$\phi_{\nu,26}$	−0.0249***(0.0073)	$\phi_{\lambda,26}$	−0.0301***(0.0103)	$\phi_{\eta,26}$	−0.0323***(0.0079)
$\phi_{\lambda,27}$	0.0322***(0.0101)	$\phi_{\lambda,27}$	0.0287***(0.0102)	$\phi_{\nu,27}$	0.0094**(0.0047)	$\phi_{\lambda,27}$	0.0304***(0.0101)	$\phi_{\eta,27}$	0.0200***(0.0076)
$\phi_{\lambda,28}$	−0.0174*(0.0104)	$\phi_{\lambda,28}$	−0.0156(0.0104)	$\phi_{\nu,28}$	−0.0046(0.0070)	$\phi_{\lambda,28}$	−0.0169(0.0104)	$\phi_{\eta,28}$	−0.0147*(0.0076)
$\phi_{\lambda,29}$	0.0142(0.0100)	$\phi_{\lambda,29}$	0.0107(0.0098)	$\phi_{\nu,29}$	0.0067(0.0083)	$\phi_{\lambda,29}$	0.0116(0.0099)	$\phi_{\eta,29}$	0.0094(0.0090)
$\phi_{\lambda,30}$	0.0000(0.0074)	$\phi_{\lambda,30}$	0.0019(0.0072)	$\phi_{\nu,30}$	−0.0111*(0.0061)	$\phi_{\lambda,30}$	0.0010(0.0072)	$\phi_{\eta,30}$	−0.0081(0.0074)

Notes: Standard errors are reported in parenthesis. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively.

Table A3. OLS estimates of the AR(30) model for the decomposition of scale and shape

Skew-Gen-t-DCS τ_t, ν_t, η_t constant		Skew-Gen-t-DCS τ_t, η_t constant, ν_t dynamic				Skew-Gen-t-DCS τ_t, ν_t constant, η_t dynamic			
c_λ	−0.0649***(0.0072)	c_λ	−0.0591***(0.0067)	c_ν	0.7485***(0.0312)	c_λ	−0.0581***(0.0066)	c_η	0.2652***(0.0128)
$\phi_{\lambda,1}$	1.0090***(0.0084)	$\phi_{\lambda,1}$	1.0116***(0.0079)	$\phi_{\nu,1}$	0.5860***(0.0069)	$\phi_{\lambda,1}$	1.0061***(0.0077)	$\phi_{\eta,1}$	0.5510***(0.0100)
$\phi_{\lambda,2}$	−0.0087(0.0127)	$\phi_{\lambda,2}$	−0.0092(0.0121)	$\phi_{\nu,2}$	0.0017(0.0067)	$\phi_{\lambda,2}$	−0.0058(0.0117)	$\phi_{\eta,2}$	0.0007(0.0092)
$\phi_{\lambda,3}$	−0.0064(0.0123)	$\phi_{\lambda,3}$	−0.0062(0.0117)	$\phi_{\nu,3}$	−0.0016(0.0094)	$\phi_{\lambda,3}$	−0.0024(0.0116)	$\phi_{\eta,3}$	−0.0030(0.0107)
$\phi_{\lambda,4}$	0.0041(0.0119)	$\phi_{\lambda,4}$	0.0002(0.0110)	$\phi_{\nu,4}$	−0.0036(0.0069)	$\phi_{\lambda,4}$	0.0032(0.0113)	$\phi_{\eta,4}$	0.0044(0.0095)
$\phi_{\lambda,5}$	−0.0062(0.0118)	$\phi_{\lambda,5}$	−0.0035(0.0112)	$\phi_{\nu,5}$	0.0241*(0.0138)	$\phi_{\lambda,5}$	−0.0040(0.0114)	$\phi_{\eta,5}$	0.0129(0.0112)
$\phi_{\lambda,6}$	−0.0182*(0.0109)	$\phi_{\lambda,6}$	−0.0153(0.0109)	$\phi_{\nu,6}$	−0.0175**(0.0082)	$\phi_{\lambda,6}$	−0.0187*(0.0110)	$\phi_{\eta,6}$	−0.0162*(0.0092)
$\phi_{\lambda,7}$	0.0059(0.0113)	$\phi_{\lambda,7}$	0.0051(0.0114)	$\phi_{\nu,7}$	0.0005(0.0065)	$\phi_{\lambda,7}$	0.0050(0.0112)	$\phi_{\eta,7}$	0.0090(0.0089)
$\phi_{\lambda,8}$	0.0150(0.0115)	$\phi_{\lambda,8}$	0.0158(0.0116)	$\phi_{\nu,8}$	0.0052(0.0068)	$\phi_{\lambda,8}$	0.0152(0.0115)	$\phi_{\eta,8}$	0.0076(0.0088)
$\phi_{\lambda,9}$	−0.0177(0.0110)	$\phi_{\lambda,9}$	−0.0198*(0.0110)	$\phi_{\nu,9}$	0.0019(0.0088)	$\phi_{\lambda,9}$	−0.0196*(0.0109)	$\phi_{\eta,9}$	−0.0036(0.0089)
$\phi_{\lambda,10}$	0.0176(0.0113)	$\phi_{\lambda,10}$	0.0209*(0.0113)	$\phi_{\nu,10}$	0.0148(0.0110)	$\phi_{\lambda,10}$	0.0187*(0.0113)	$\phi_{\eta,10}$	0.0133(0.0100)
$\phi_{\lambda,11}$	−0.0083(0.0114)	$\phi_{\lambda,11}$	−0.0097(0.0115)	$\phi_{\nu,11}$	−0.0169**(0.0079)	$\phi_{\lambda,11}$	−0.0093(0.0115)	$\phi_{\eta,11}$	−0.0194**(0.0085)
$\phi_{\lambda,12}$	−0.0048(0.0101)	$\phi_{\lambda,12}$	−0.0043(0.0103)	$\phi_{\nu,12}$	0.0049(0.0062)	$\phi_{\lambda,12}$	−0.0043(0.0102)	$\phi_{\eta,12}$	0.0064(0.0085)
$\phi_{\lambda,13}$	−0.0038(0.0105)	$\phi_{\lambda,13}$	−0.0051(0.0106)	$\phi_{\nu,13}$	−0.0029(0.0064)	$\phi_{\lambda,13}$	−0.0028(0.0106)	$\phi_{\eta,13}$	−0.0028(0.0082)
$\phi_{\lambda,14}$	0.0073(0.0109)	$\phi_{\lambda,14}$	0.0064(0.0110)	$\phi_{\nu,14}$	0.0096(0.0083)	$\phi_{\lambda,14}$	0.0063(0.0110)	$\phi_{\eta,14}$	0.0122(0.0096)
$\phi_{\lambda,15}$	−0.0133(0.0110)	$\phi_{\lambda,15}$	−0.0135(0.0110)	$\phi_{\nu,15}$	−0.0102*(0.0052)	$\phi_{\lambda,15}$	−0.0129(0.0110)	$\phi_{\eta,15}$	−0.0119(0.0082)
$\phi_{\lambda,16}$	0.0048(0.0108)	$\phi_{\lambda,16}$	0.0050(0.0109)	$\phi_{\nu,16}$	0.0061(0.0068)	$\phi_{\lambda,16}$	0.0059(0.0109)	$\phi_{\eta,16}$	0.0111(0.0089)
$\phi_{\lambda,17}$	0.0008(0.0105)	$\phi_{\lambda,17}$	0.0000(0.0107)	$\phi_{\nu,17}$	−0.0032(0.0060)	$\phi_{\lambda,17}$	0.0001(0.0106)	$\phi_{\eta,17}$	−0.0043(0.0089)
$\phi_{\lambda,18}$	0.0115(0.0103)	$\phi_{\lambda,18}$	0.0130(0.0104)	$\phi_{\nu,18}$	−0.0017(0.0046)	$\phi_{\lambda,18}$	0.0103(0.0104)	$\phi_{\eta,18}$	0.0040(0.0077)
$\phi_{\lambda,19}$	−0.0004(0.0104)	$\phi_{\lambda,19}$	−0.0010(0.0103)	$\phi_{\nu,19}$	0.0099(0.0086)	$\phi_{\lambda,19}$	−0.0011(0.0103)	$\phi_{\eta,19}$	0.0093(0.0098)
$\phi_{\lambda,20}$	−0.0031(0.0108)	$\phi_{\lambda,20}$	−0.0050(0.0108)	$\phi_{\nu,20}$	−0.0124**(0.0062)	$\phi_{\lambda,20}$	−0.0038(0.0107)	$\phi_{\eta,20}$	−0.0123(0.0082)
$\phi_{\lambda,21}$	0.0007(0.0109)	$\phi_{\lambda,21}$	0.0020(0.0109)	$\phi_{\nu,21}$	0.0181(0.0172)	$\phi_{\lambda,21}$	0.0020(0.0108)	$\phi_{\eta,21}$	0.0080(0.0113)
$\phi_{\lambda,22}$	−0.0183*(0.0107)	$\phi_{\lambda,22}$	−0.0174(0.0107)	$\phi_{\nu,22}$	−0.0094(0.0115)	$\phi_{\lambda,22}$	−0.0193*(0.0106)	$\phi_{\eta,22}$	0.0029(0.0094)
$\phi_{\lambda,23}$	0.0083(0.0103)	$\phi_{\lambda,23}$	0.0094(0.0104)	$\phi_{\nu,23}$	−0.0024(0.0054)	$\phi_{\lambda,23}$	0.0101(0.0103)	$\phi_{\eta,23}$	−0.0021(0.0077)
$\phi_{\lambda,24}$	0.0142(0.0104)	$\phi_{\lambda,24}$	0.0118(0.0104)	$\phi_{\nu,24}$	0.0069(0.0055)	$\phi_{\lambda,24}$	0.0122(0.0104)	$\phi_{\eta,24}$	0.0082(0.0083)
$\phi_{\lambda,25}$	0.0000(0.0112)	$\phi_{\lambda,25}$	0.0005(0.0112)	$\phi_{\nu,25}$	0.0076(0.0103)	$\phi_{\lambda,25}$	0.0012(0.0111)	$\phi_{\eta,25}$	0.0116(0.0099)
$\phi_{\lambda,26}$	−0.0325***(0.0104)	$\phi_{\lambda,26}$	−0.0317***(0.0104)	$\phi_{\nu,26}$	−0.0241***(0.0074)	$\phi_{\lambda,26}$	−0.0312***(0.0103)	$\phi_{\eta,26}$	−0.0305***(0.0078)
$\phi_{\lambda,27}$	0.0330***(0.0101)	$\phi_{\lambda,27}$	0.0346***(0.0102)	$\phi_{\nu,27}$	0.0085*(0.0046)	$\phi_{\lambda,27}$	0.0324***(0.0101)	$\phi_{\eta,27}$	0.0176**(0.0075)
$\phi_{\lambda,28}$	−0.0164(0.0104)	$\phi_{\lambda,28}$	−0.0170(0.0105)	$\phi_{\nu,28}$	−0.0030(0.0075)	$\phi_{\lambda,28}$	−0.0171(0.0104)	$\phi_{\eta,28}$	−0.0144*(0.0074)
$\phi_{\lambda,29}$	0.0130(0.0101)	$\phi_{\lambda,29}$	0.0101(0.0100)	$\phi_{\nu,29}$	0.0072(0.0088)	$\phi_{\lambda,29}$	0.0110(0.0099)	$\phi_{\eta,29}$	0.0102(0.0090)
$\phi_{\lambda,30}$	−0.0001(0.0074)	$\phi_{\lambda,30}$	0.0009(0.0072)	$\phi_{\nu,30}$	−0.0116*(0.0063)	$\phi_{\lambda,30}$	0.0011(0.0072)	$\phi_{\eta,30}$	−0.0079(0.0074)

Notes: Standard errors are reported in parenthesis. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively.

Table A4. OLS estimates of the AR(30) model for the decomposition of scale and shape

EGB2-DCS ξ_t, ζ_t constant		EGB2-DCS ξ_t, ζ_t dynamic					
c_λ	$-0.0727^{***}(0.0081)$	c_λ	$-0.0580^{***}(0.0069)$	c_ξ	$-0.1058^{***}(0.0043)$	c_ζ	$-0.0103^{***}(0.0007)$
$\phi_{\lambda,1}$	$1.0106^{***}(0.0087)$	$\phi_{\lambda,1}$	$0.9972^{***}(0.0080)$	$\phi_{\xi,1}$	$0.5656^{***}(0.0078)$	$\phi_{\zeta,1}$	$0.9047^{***}(0.0090)$
$\phi_{\lambda,2}$	$-0.0104(0.0124)$	$\phi_{\lambda,2}$	$-0.0013(0.0113)$	$\phi_{\xi,2}$	$-0.0048(0.0093)$	$\phi_{\zeta,2}$	$-0.0208^{*}(0.0113)$
$\phi_{\lambda,3}$	$-0.0057(0.0121)$	$\phi_{\lambda,3}$	$-0.0035(0.0113)$	$\phi_{\xi,3}$	$-0.0015(0.0091)$	$\phi_{\zeta,3}$	$0.0051(0.0109)$
$\phi_{\lambda,4}$	$0.0016(0.0121)$	$\phi_{\lambda,4}$	$0.0089(0.0116)$	$\phi_{\xi,4}$	$-0.0013(0.0093)$	$\phi_{\zeta,4}$	$-0.0120(0.0103)$
$\phi_{\lambda,5}$	$-0.0052(0.0113)$	$\phi_{\lambda,5}$	$-0.0002(0.0114)$	$\phi_{\xi,5}$	$-0.0004(0.0093)$	$\phi_{\zeta,5}$	$0.0062(0.0107)$
$\phi_{\lambda,6}$	$-0.0188^{*}(0.0101)$	$\phi_{\lambda,6}$	$-0.0201^{**}(0.0102)$	$\phi_{\xi,6}$	$-0.0070(0.0088)$	$\phi_{\zeta,6}$	$0.0041(0.0109)$
$\phi_{\lambda,7}$	$0.0083(0.0107)$	$\phi_{\lambda,7}$	$0.0032(0.0106)$	$\phi_{\xi,7}$	$0.0021(0.0089)$	$\phi_{\zeta,7}$	$0.0002(0.0109)$
$\phi_{\lambda,8}$	$0.0117(0.0114)$	$\phi_{\lambda,8}$	$0.0141(0.0113)$	$\phi_{\xi,8}$	$0.0085(0.0090)$	$\phi_{\zeta,8}$	$0.0021(0.0104)$
$\phi_{\lambda,9}$	$-0.0124(0.0110)$	$\phi_{\lambda,9}$	$-0.0090(0.0109)$	$\phi_{\xi,9}$	$-0.0124(0.0084)$	$\phi_{\zeta,9}$	$-0.0006(0.0106)$
$\phi_{\lambda,10}$	$0.0144(0.0119)$	$\phi_{\lambda,10}$	$0.0140(0.0118)$	$\phi_{\xi,10}$	$0.0205^{**}(0.0094)$	$\phi_{\zeta,10}$	$0.0039(0.0106)$
$\phi_{\lambda,11}$	$-0.0093(0.0110)$	$\phi_{\lambda,11}$	$-0.0096(0.0114)$	$\phi_{\xi,11}$	$-0.0105(0.0085)$	$\phi_{\zeta,11}$	$0.0043(0.0109)$
$\phi_{\lambda,12}$	$-0.0047(0.0096)$	$\phi_{\lambda,12}$	$-0.0079(0.0098)$	$\phi_{\xi,12}$	$-0.0020(0.0080)$	$\phi_{\zeta,12}$	$-0.0101(0.0105)$
$\phi_{\lambda,13}$	$-0.0033(0.0098)$	$\phi_{\lambda,13}$	$-0.0036(0.0100)$	$\phi_{\xi,13}$	$-0.0062(0.0081)$	$\phi_{\zeta,13}$	$0.0140(0.0103)$
$\phi_{\lambda,14}$	$0.0078(0.0104)$	$\phi_{\lambda,14}$	$0.0081(0.0106)$	$\phi_{\xi,14}$	$0.0029(0.0088)$	$\phi_{\zeta,14}$	$-0.0061(0.0104)$
$\phi_{\lambda,15}$	$-0.0129(0.0107)$	$\phi_{\lambda,15}$	$-0.0147(0.0107)$	$\phi_{\xi,15}$	$-0.0147^{*}(0.0089)$	$\phi_{\zeta,15}$	$-0.0011(0.0111)$
$\phi_{\lambda,16}$	$0.0046(0.0106)$	$\phi_{\lambda,16}$	$0.0069(0.0109)$	$\phi_{\xi,16}$	$-0.0009(0.0088)$	$\phi_{\zeta,16}$	$-0.0058(0.0115)$
$\phi_{\lambda,17}$	$-0.0010(0.0101)$	$\phi_{\lambda,17}$	$-0.0024(0.0102)$	$\phi_{\xi,17}$	$-0.0073(0.0082)$	$\phi_{\zeta,17}$	$0.0018(0.0110)$
$\phi_{\lambda,18}$	$0.0107(0.0096)$	$\phi_{\lambda,18}$	$0.0129(0.0096)$	$\phi_{\xi,18}$	$0.0038(0.0080)$	$\phi_{\zeta,18}$	$-0.0017(0.0106)$
$\phi_{\lambda,19}$	$0.0015(0.0097)$	$\phi_{\lambda,19}$	$-0.0027(0.0096)$	$\phi_{\xi,19}$	$0.0021(0.0080)$	$\phi_{\zeta,19}$	$0.0033(0.0111)$
$\phi_{\lambda,20}$	$-0.0042(0.0102)$	$\phi_{\lambda,20}$	$-0.0061(0.0101)$	$\phi_{\xi,20}$	$-0.0060(0.0085)$	$\phi_{\zeta,20}$	$0.0069(0.0115)$
$\phi_{\lambda,21}$	$0.0015(0.0102)$	$\phi_{\lambda,21}$	$0.0027(0.0100)$	$\phi_{\xi,21}$	$-0.0028(0.0086)$	$\phi_{\zeta,21}$	$-0.0104(0.0112)$
$\phi_{\lambda,22}$	$-0.0158(0.0103)$	$\phi_{\lambda,22}$	$-0.0136(0.0102)$	$\phi_{\xi,22}$	$-0.0073(0.0085)$	$\phi_{\zeta,22}$	$0.0094(0.0116)$
$\phi_{\lambda,23}$	$0.0070(0.0101)$	$\phi_{\lambda,23}$	$0.0077(0.0102)$	$\phi_{\xi,23}$	$-0.0033(0.0083)$	$\phi_{\zeta,23}$	$0.0005(0.0104)$
$\phi_{\lambda,24}$	$0.0144(0.0100)$	$\phi_{\lambda,24}$	$0.0141(0.0101)$	$\phi_{\xi,24}$	$-0.0003(0.0085)$	$\phi_{\zeta,24}$	$-0.0044(0.0111)$
$\phi_{\lambda,25}$	$-0.0030(0.0109)$	$\phi_{\lambda,25}$	$-0.0038(0.0109)$	$\phi_{\xi,25}$	$-0.0035(0.0088)$	$\phi_{\zeta,25}$	$-0.0068(0.0110)$
$\phi_{\lambda,26}$	$-0.0300^{***}(0.0099)$	$\phi_{\lambda,26}$	$-0.0297^{***}(0.0099)$	$\phi_{\xi,26}$	$-0.0233^{***}(0.0078)$	$\phi_{\zeta,26}$	$-0.0064(0.0101)$
$\phi_{\lambda,27}$	$0.0321^{***}(0.0098)$	$\phi_{\lambda,27}$	$0.0322^{***}(0.0097)$	$\phi_{\xi,27}$	$0.0159^{*}(0.0084)$	$\phi_{\zeta,27}$	$0.0110(0.0102)$
$\phi_{\lambda,28}$	$-0.0153(0.0101)$	$\phi_{\lambda,28}$	$-0.0157(0.0101)$	$\phi_{\xi,28}$	$-0.0107(0.0081)$	$\phi_{\zeta,28}$	$-0.0065(0.0105)$
$\phi_{\lambda,29}$	$0.0124(0.0098)$	$\phi_{\lambda,29}$	$0.0110(0.0097)$	$\phi_{\xi,29}$	$0.0028(0.0085)$	$\phi_{\zeta,29}$	$0.0178^{*}(0.0106)$
$\phi_{\lambda,30}$	$0.0000(0.0074)$	$\phi_{\lambda,30}$	$0.0004(0.0073)$	$\phi_{\xi,30}$	$-0.0157^{**}(0.0072)$	$\phi_{\zeta,30}$	$-0.0077(0.0083)$

Notes: Standard errors are reported in parenthesis. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively.

Table A5. OLS estimates of the AR(30) model for the decomposition of scale and shape

NIG-DCS ν_t, η_t constant		NIG-DCS ν_t constant, η_t dynamic			
c_λ	−0.0598***(0.0066)	c_λ	−0.0601***(0.0066)	c_η	−0.0430***(0.0016)
$\phi_{\lambda,1}$	1.0148***(0.0087)	$\phi_{\lambda,1}$	1.0089***(0.0084)	$\phi_{\eta,1}$	0.3385***(0.0086)
$\phi_{\lambda,2}$	−0.0121(0.0126)	$\phi_{\lambda,2}$	−0.0099(0.0122)	$\phi_{\eta,2}$	−0.0047(0.0085)
$\phi_{\lambda,3}$	−0.0081(0.0123)	$\phi_{\lambda,3}$	−0.0075(0.0120)	$\phi_{\eta,3}$	0.0054(0.0081)
$\phi_{\lambda,4}$	0.0014(0.0121)	$\phi_{\lambda,4}$	0.0046(0.0119)	$\phi_{\eta,4}$	−0.0079(0.0083)
$\phi_{\lambda,5}$	−0.0050(0.0115)	$\phi_{\lambda,5}$	−0.0059(0.0114)	$\phi_{\eta,5}$	−0.0104(0.0083)
$\phi_{\lambda,6}$	−0.0190*(0.0103)	$\phi_{\lambda,6}$	−0.0171*(0.0102)	$\phi_{\eta,6}$	−0.0032(0.0082)
$\phi_{\lambda,7}$	0.0066(0.0109)	$\phi_{\lambda,7}$	0.0042(0.0108)	$\phi_{\eta,7}$	−0.0010(0.0083)
$\phi_{\lambda,8}$	0.0146(0.0114)	$\phi_{\lambda,8}$	0.0152(0.0113)	$\phi_{\eta,8}$	0.0068(0.0079)
$\phi_{\lambda,9}$	−0.0112(0.0111)	$\phi_{\lambda,9}$	−0.0111(0.0109)	$\phi_{\eta,9}$	−0.0078(0.0080)
$\phi_{\lambda,10}$	0.0148(0.0119)	$\phi_{\lambda,10}$	0.0151(0.0116)	$\phi_{\eta,10}$	0.0154*(0.0081)
$\phi_{\lambda,11}$	−0.0089(0.0113)	$\phi_{\lambda,11}$	−0.0088(0.0113)	$\phi_{\eta,11}$	−0.0019(0.0079)
$\phi_{\lambda,12}$	−0.0076(0.0099)	$\phi_{\lambda,12}$	−0.0079(0.0098)	$\phi_{\eta,12}$	0.0007(0.0078)
$\phi_{\lambda,13}$	−0.0040(0.0101)	$\phi_{\lambda,13}$	−0.0045(0.0100)	$\phi_{\eta,13}$	−0.0036(0.0078)
$\phi_{\lambda,14}$	0.0078(0.0106)	$\phi_{\lambda,14}$	0.0094(0.0106)	$\phi_{\eta,14}$	0.0020(0.0081)
$\phi_{\lambda,15}$	−0.0136(0.0108)	$\phi_{\lambda,15}$	−0.0138(0.0108)	$\phi_{\eta,15}$	−0.0060(0.0082)
$\phi_{\lambda,16}$	0.0056(0.0107)	$\phi_{\lambda,16}$	0.0055(0.0106)	$\phi_{\eta,16}$	−0.0059(0.0083)
$\phi_{\lambda,17}$	−0.0003(0.0102)	$\phi_{\lambda,17}$	0.0007(0.0102)	$\phi_{\eta,17}$	−0.0004(0.0079)
$\phi_{\lambda,18}$	0.0107(0.0098)	$\phi_{\lambda,18}$	0.0083(0.0098)	$\phi_{\eta,18}$	−0.0097(0.0079)
$\phi_{\lambda,19}$	0.0011(0.0098)	$\phi_{\lambda,19}$	0.0021(0.0098)	$\phi_{\eta,19}$	0.0059(0.0081)
$\phi_{\lambda,20}$	−0.0056(0.0104)	$\phi_{\lambda,20}$	−0.0062(0.0103)	$\phi_{\eta,20}$	0.0011(0.0085)
$\phi_{\lambda,21}$	0.0004(0.0104)	$\phi_{\lambda,21}$	−0.0002(0.0103)	$\phi_{\eta,21}$	−0.0140*(0.0084)
$\phi_{\lambda,22}$	−0.0152(0.0103)	$\phi_{\lambda,22}$	−0.0127(0.0103)	$\phi_{\eta,22}$	0.0006(0.0081)
$\phi_{\lambda,23}$	0.0086(0.0102)	$\phi_{\lambda,23}$	0.0068(0.0102)	$\phi_{\eta,23}$	−0.0053(0.0079)
$\phi_{\lambda,24}$	0.0152(0.0101)	$\phi_{\lambda,24}$	0.0161(0.0101)	$\phi_{\eta,24}$	−0.0008(0.0084)
$\phi_{\lambda,25}$	−0.0018(0.0110)	$\phi_{\lambda,25}$	−0.0027(0.0110)	$\phi_{\eta,25}$	−0.0029(0.0081)
$\phi_{\lambda,26}$	−0.0312***(0.0101)	$\phi_{\lambda,26}$	−0.0307***(0.0101)	$\phi_{\eta,26}$	−0.0150**(0.0076)
$\phi_{\lambda,27}$	0.0333***(0.0100)	$\phi_{\lambda,27}$	0.0334***(0.0100)	$\phi_{\eta,27}$	0.0098(0.0082)
$\phi_{\lambda,28}$	−0.0166(0.0103)	$\phi_{\lambda,28}$	−0.0163(0.0103)	$\phi_{\eta,28}$	−0.0069(0.0080)
$\phi_{\lambda,29}$	0.0126(0.0099)	$\phi_{\lambda,29}$	0.0127(0.0099)	$\phi_{\eta,29}$	0.0087(0.0085)
$\phi_{\lambda,30}$	−0.0006(0.0074)	$\phi_{\lambda,30}$	−0.0010(0.0075)	$\phi_{\eta,30}$	−0.0079(0.0076)

Notes: Standard errors are reported in parenthesis. *, ** and *** indicate significance at the 10%, 5% and 1% levels, respectively.

Table A6. $H_{\max}$ **evaluated at** $\hat{\Theta}_{\text{ML}}$ **for the S&P 500 and at** $\Theta_{\text{MC}1}$ **to** $\Theta_{\text{MC}20}$ **for the simulated data**

ϵ_t	DCS specification	$\hat{\Theta}_{\text{ML}}$	Θ_{MC1}	Θ_{MC2}	Θ_{MC3}	Θ_{MC4}	Θ_{MC5}	Θ_{MC6}
t	ν_t constant	1.3E+04	8.2E+07	3.4E+07	5.6E+07	3.9E+08	2.8E+08	1.9E+05
t	ν_t variable	1.3E+04	3.9E+07	3.2E+08	6.2E+07	5.6E+07	8.0E+07	3.4E+07
GED	ν_t constant	4.4E+05	2.1E+09	5.3E+09	1.0E+09	5.2E+07	2.2E+07	7.2E+09
GED	ν_t variable	5.3E+05	5.3E+09	4.8E+08	3.4E+05	8.5E+06	7.9E+06	4.2E+09
Gen-t	ν_t, η_t constant	4.8E+03	2.7E+07	2.0E+07	1.6E+07	1.1E+08	2.6E+07	1.8E+08
Gen-t	ν_t variable, η_t constant	6.7E+03	5.2E+07	3.4E+07	6.2E+07	5.1E+07	1.1E+08	3.4E+07
Gen-t	ν_t constant, η_t variable	1.5E+05	1.6E+08	4.9E+17	2.5E+11	2.6E+08	2.2E+07	5.8E+14
Skew-Gen-t	τ_t, ν_t, η_t constant	8.8E+03	3.9E+07	3.4E+07	8.8E+08	2.9E+07	3.5E+07	1.6E+07
Skew-Gen-t	τ_t, η_t constant, ν_t variable	8.0E+03	1.6E+07	3.8E+05	1.9E+07	2.9E+07	1.2E+07	1.7E+07
Skew-Gen-t	τ_t, ν_t constant, η_t variable	4.8E+03	6.6E+06	6.9E+06	1.2E+06	1.8E+08	7.2E+06	1.2E+07
EGB2	ξ_t, ζ_t constant	1.8E+04	7.6E+07	7.2E+07	7.3E+07	4.9E+07	5.6E+07	7.0E+07
EGB2	ξ_t, ζ_t variable	2.8E+04	1.2E+08	5.1E+08	3.9E+04	3.0E+08	1.5E+07	1.4E+06
NIG	ν_t, η_t constant	3.4E+03	7.3E+09	1.3E+09	6.9E+08	9.5E+09	1.1E+09	1.1E+15
NIG	ν_t constant, η_t variable	4.1E+03	1.7E+09	3.4E+18	2.8E+10	1.8E+09	4.1E+08	3.2E+13

ϵ_t	DCS specification	Θ_{MC7}	Θ_{MC8}	Θ_{MC9}	Θ_{MC10}	Θ_{MC11}	Θ_{MC12}	Θ_{MC13}
t	ν_t constant	2.9E+07	1.7E+13	1.6E+12	5.0E+14	1.1E+21	1.4E+09	1.2E+16
t	ν_t variable	3.8E+07	2.5E+07	1.1E+08	3.2E+08	1.4E+08	1.4E+08	9.8E+07
GED	ν_t constant	1.7E+08	1.2E+16	3.9E+13	9.0E+15	3.3E+07	5.1E+15	1.2E+16
GED	ν_t variable	2.1E+11	3.5E+07	3.8E+07	2.3E+07	1.2E+08	2.1E+09	2.9E+16
Gen-t	ν_t, η_t constant	3.0E+07	3.2E+07	5.6E+07	4.8E+07	1.6E+07	2.7E+08	1.5E+07
Gen-t	ν_t variable, η_t constant	3.1E+07	2.7E+07	1.8E+07	2.3E+07	6.0E+06	5.2E+07	1.5E+08
Gen-t	ν_t constant, η_t variable	5.0E+07	1.1E+08	4.4E+12	1.1E+16	2.7E+18	1.1E+12	3.3E+06
Skew-Gen-t	τ_t, ν_t, η_t constant	6.5E+07	5.5E+06	5.8E+07	1.3E+07	2.0E+07	2.1E+07	2.0E+08
Skew-Gen-t	τ_t, η_t constant, ν_t variable	3.5E+07	1.4E+07	2.2E+06	3.4E+07	2.4E+07	1.7E+07	5.9E+07
Skew-Gen-t	τ_t, ν_t constant, η_t variable	4.0E+06	8.1E+06	1.4E+07	4.5E+06	1.7E+06	3.4E+07	1.2E+07
EGB2	ξ_t, ζ_t constant	7.7E+07	3.3E+07	8.4E+07	8.1E+07	6.3E+07	5.5E+07	1.1E+08
EGB2	ξ_t, ζ_t variable	1.2E+20	2.2E+07	9.9E+07	5.0E+10	6.7E+06	2.2E+06	3.6E+06
NIG	ν_t, η_t constant	1.4E+25	7.1E+15	4.6E+11	1.6E+12	2.2E+16	4.5E+09	3.4E+08
NIG	ν_t constant, η_t variable	1.1E+08	6.6E+15	4.7E+15	1.3E+18	2.2E+19	3.3E+16	1.8E+10

ϵ_t	DCS specification	Θ_{MC14}	Θ_{MC15}	Θ_{MC16}	Θ_{MC17}	Θ_{MC18}	Θ_{MC19}	Θ_{MC20}
t	ν_t constant	6.6E+20	2.0E+09	2.0E+15	1.3E+12	1.2E+20	1.9E+05	6.5E+06
t	ν_t variable	2.7E+07	2.9E+07	5.4E+07	8.2E+07	2.1E+08	5.2E+07	1.0E+08
GED	ν_t constant	6.3E+08	6.5E+15	8.1E+15	2.1E+16	5.8E+09	7.3E+15	3.5E+08
GED	ν_t variable	9.9E+07	4.1E+08	7.1E+08	3.6E+06	2.2E+11	1.5E+16	1.2E+16
Gen-t	ν_t, η_t constant	2.8E+07	1.6E+08	2.5E+07	3.1E+07	1.2E+08	1.6E+09	9.7E+07
Gen-t	ν_t variable, η_t constant	3.9E+07	4.3E+07	3.3E+07	2.5E+07	1.4E+07	4.6E+07	2.8E+07
Gen-t	ν_t constant, η_t variable	1.1E+14	1.9E+09	1.8E+23	4.0E+06	4.2E+20	1.3E+18	3.9E+08
Skew-Gen-t	τ_t, ν_t, η_t constant	6.4E+07	2.0E+08	4.2E+06	1.2E+08	3.6E+07	6.5E+07	1.3E+08
Skew-Gen-t	τ_t, η_t constant, ν_t variable	6.1E+06	8.4E+06	8.7E+07	2.1E+08	1.5E+07	1.8E+07	6.1E+06
Skew-Gen-t	τ_t, ν_t constant, η_t variable	9.2E+06	7.4E+06	1.2E+07	4.5E+06	4.0E+07	9.8E+07	1.3E+07
EGB2	ξ_t, ζ_t constant	2.8E+07	4.9E+07	5.4E+07	4.4E+07	6.9E+07	3.4E+07	3.0E+07
EGB2	ξ_t, ζ_t variable	3.0E+07	5.5E+06	2.6E+06	1.9E+07	2.8E+07	3.8E+07	5.7E+20
NIG	ν_t, η_t constant	2.3E+10	1.3E+10	3.8E+13	7.6E+12	9.9E+10	1.5E+09	1.8E+11
NIG	ν_t constant, η_t variable	9.6E+17	2.9E+16	7.0E+25	2.4E+12	2.6E+20	1.4E+19	4.6E+09

Notes: We use the chain rule $\partial \ln f(y_t|y_1,\ldots,y_{t-1};\Theta)/\partial\Theta_j = [\partial \ln f(y_t|y_1,\ldots,y_{t-1};\Theta)/\partial m_t] \times [\partial m_t/\partial\Theta_j]$, to formulate the first-derivative function with respect to each Θ_j. This gives K first-derivative functions with respect to Θ_j for $j = 1,\ldots,K$. For each first-derivative function corresponding to Θ_j, we numerically estimate the $K \times K$ Hessian matrix with respect to $(\Theta_1,\ldots,\Theta_K)$. For each Hessian matrix corresponding to Θ_j, we denote the maximum element in absolute value by using $H_{\max,j}$. Furthermore, we introduce the notation $H_{\max} = \max\{H_{\max,1},\ldots,H_{\max,K}\}$. We study the finiteness of $H_{\max}$ for each DCS specification.

Table A7. Parameters for Monte Carlo simulation, t-DCS with constant ν_t

	Θ_{MC1}	Θ_{MC2}	Θ_{MC3}	Θ_{MC4}	Θ_{MC5}	Θ_{MC6}	Θ_{MC7}	Θ_{MC8}	Θ_{MC9}	Θ_{MC10}
c	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
ϕ_1	−0.1298	−0.1233	−0.1465	−0.1085	−0.1539	−0.1134	−0.1273	−0.1170	−0.1415	−0.1583
ϕ_2	−0.0824	−0.0774	−0.0890	−0.0924	−0.0857	−0.0854	−0.0797	−0.0901	−0.0801	−0.0867
ϕ_3	0.1825	0.1542	0.1635	0.2245	0.1779	0.2038	0.1726	0.2120	0.1603	0.1785
ϕ_4	0.0744	0.0654	0.0687	0.0765	0.0875	0.0654	0.0646	0.0753	0.0632	0.0845
ϕ_5	0.0023	0.0019	0.0020	0.0024	0.0026	0.0025	0.0021	0.0023	0.0022	0.0026
ϕ_6	−0.0928	−0.1019	−0.0848	−0.0972	−0.0936	−0.0812	−0.0881	−0.0908	−0.1067	−0.0991
ϕ_7	−0.0329	−0.0287	−0.0355	−0.0401	−0.0297	−0.0294	−0.0330	−0.0362	−0.0348	−0.0352
ϕ_8	−0.0190	−0.0234	−0.0169	−0.0214	−0.0171	−0.0181	−0.0150	−0.0186	−0.0208	−0.0183
ϕ_9	0.2345	0.2124	0.2531	0.2026	0.2404	0.2021	0.2213	0.2410	0.2446	0.2145
ϕ_{10}	0.0608	0.0617	0.0618	0.0512	0.0668	0.0633	0.0648	0.0747	0.0637	0.0528
ϕ_{11}	0.2834	0.2683	0.2934	0.3063	0.2621	0.2545	0.2620	0.2912	0.3065	0.3300
ϕ_{12}	0.0464	0.0423	0.0442	0.0457	0.0496	0.0405	0.0355	0.0535	0.0437	0.0402
ϕ_{13}	0.0014	0.0012	0.0012	0.0016	0.0014	0.0013	0.0015	0.0013	0.0015	0.0014
ϕ_{14}	−0.1163	−0.1130	−0.1087	−0.1236	−0.1289	−0.1315	−0.1266	−0.1138	−0.1182	−0.1097
ϕ_{15}	0.1933	0.1936	0.2130	0.1999	0.1966	0.1867	0.2080	0.1885	0.1752	0.2216
ϕ_{16}	0.0823	0.0813	0.0907	0.0806	0.0938	0.0933	0.0813	0.0818	0.0799	0.0836
ϕ_{17}	0.0933	0.0934	0.0950	0.1064	0.0973	0.1092	0.0868	0.0816	0.0937	0.0987
ϕ_{18}	−0.0520	−0.0531	−0.0542	−0.0558	−0.0512	−0.0526	−0.0544	−0.0524	−0.0615	−0.0474
ϕ_{19}	−0.0642	−0.0630	−0.0632	−0.0665	−0.0677	−0.0663	−0.0621	−0.0604	−0.0682	−0.0707
ϕ_{20}	−0.1989	−0.1900	−0.2211	−0.1993	−0.2087	−0.2012	−0.2295	−0.1800	−0.2023	−0.2238
ϕ_{21}	−0.1537	−0.1477	−0.1514	−0.1733	−0.1401	−0.1786	−0.1741	−0.1775	−0.1648	−0.1561
ϕ_{22}	−0.1309	−0.1402	−0.1148	−0.1086	−0.1431	−0.1340	−0.1430	−0.1171	−0.1282	−0.1007
ϕ_{23}	0.2475	0.2262	0.2695	0.2452	0.2455	0.2267	0.2750	0.2532	0.2662	0.1999
ϕ_{24}	−0.0773	−0.0632	−0.0674	−0.0754	−0.0857	−0.0813	−0.0851	−0.0869	−0.0850	−0.0757
ϕ_{25}	0.0422	0.0345	0.0440	0.0532	0.0461	0.0383	0.0489	0.0413	0.0446	0.0399
ϕ_{26}	−0.1268	−0.1370	−0.1273	−0.1388	−0.1148	−0.1301	−0.1354	−0.1187	−0.1101	−0.1426
ϕ_{27}	0.1001	0.0857	0.1002	0.1138	0.0959	0.1117	0.0900	0.0909	0.1277	0.1003
ϕ_{28}	0.0688	0.0606	0.0776	0.0678	0.0719	0.0647	0.0691	0.0624	0.0770	0.0818
ϕ_{29}	0.2728	0.2437	0.2602	0.2359	0.2639	0.2673	0.2364	0.2440	0.3279	0.2550
ϕ_{30}	0.0468	0.0494	0.0431	0.0446	0.0397	0.0472	0.0455	0.0497	0.0493	0.0441
θ	0.9246	0.8863	0.9027	0.9675	0.9329	0.8926	0.9590	0.9338	0.9459	0.9134
ω	−0.0622	−0.0821	−0.0845	−0.0622	−0.0622	−0.0622	−0.0622	−0.0821	−0.0845	−0.0821
α	0.0365	0.0371	0.0400	0.0355	0.0404	0.0408	0.0362	0.0363	0.0351	0.0308
α^*	0.0267	0.0260	0.0235	0.0280	0.0250	0.0273	0.0228	0.0286	0.0300	0.0297
β	0.9877	0.9838	0.9833	0.9891	0.9886	0.9805	0.9876	0.9908	0.9899	0.9921
λ_0	−5.3897	−5.3897	−5.3897	−5.3897	−5.3897	−5.3897	−5.3897	−5.3897	−5.3897	−5.3897
δ_1	1.6467	1.7011	1.6096	1.5771	1.5900	1.4197	1.7980	1.5926	1.6132	1.5907

Table A7 (continued). Parameters for Monte Carlo simulation, t-DCS with constant ν_t

	Θ_{MC11}	Θ_{MC12}	Θ_{MC13}	Θ_{MC14}	Θ_{MC15}	Θ_{MC16}	Θ_{MC17}	Θ_{MC18}	Θ_{MC19}	Θ_{MC20}
c	0.0000	0.0001	0.0001	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
ϕ_1	−0.1061	−0.1470	−0.1549	−0.1170	−0.1238	−0.1411	−0.1209	−0.1531	−0.1170	−0.1240
ϕ_2	−0.0850	−0.0833	−0.0797	−0.1007	−0.0858	−0.0875	−0.0903	−0.0785	−0.0722	−0.0872
ϕ_3	0.1734	0.1681	0.1860	0.1826	0.1952	0.1856	0.1918	0.1929	0.2160	0.1641
ϕ_4	0.0700	0.0636	0.0715	0.0724	0.0724	0.0671	0.0612	0.0810	0.0610	0.0652
ϕ_5	0.0022	0.0024	0.0020	0.0024	0.0021	0.0021	0.0020	0.0024	0.0022	0.0025
ϕ_6	−0.0839	−0.0958	−0.0907	−0.1017	−0.1036	−0.0894	−0.0811	−0.1006	−0.0994	−0.1140
ϕ_7	−0.0271	−0.0300	−0.0263	−0.0327	−0.0342	−0.0338	−0.0301	−0.0307	−0.0283	−0.0382
ϕ_8	−0.0220	−0.0215	−0.0203	−0.0230	−0.0198	−0.0178	−0.0215	−0.0226	−0.0226	−0.0186
ϕ_9	0.2407	0.2614	0.2026	0.2692	0.2394	0.2279	0.2120	0.2799	0.2645	0.2469
ϕ_{10}	0.0548	0.0505	0.0693	0.0534	0.0576	0.0709	0.0704	0.0600	0.0548	0.0527
ϕ_{11}	0.3447	0.2940	0.2709	0.3509	0.2971	0.2392	0.3304	0.2825	0.2869	0.2799
ϕ_{12}	0.0457	0.0451	0.0494	0.0531	0.0485	0.0502	0.0420	0.0422	0.0422	0.0520
ϕ_{13}	0.0015	0.0014	0.0012	0.0014	0.0014	0.0014	0.0013	0.0015	0.0014	0.0014
ϕ_{14}	−0.1185	−0.1043	−0.1153	−0.1241	−0.1203	−0.1172	−0.1235	−0.1452	−0.1046	−0.1163
ϕ_{15}	0.2141	0.2191	0.1648	0.1896	0.1831	0.2242	0.2069	0.2172	0.1936	0.2025
ϕ_{16}	0.0727	0.0870	0.0804	0.0942	0.0880	0.0916	0.0844	0.0905	0.0838	0.0808
ϕ_{17}	0.0871	0.0984	0.1095	0.0942	0.0966	0.1016	0.1058	0.0893	0.0807	0.0802
ϕ_{18}	−0.0546	−0.0584	−0.0468	−0.0523	−0.0555	−0.0498	−0.0527	−0.0496	−0.0561	−0.0613
ϕ_{19}	−0.0760	−0.0654	−0.0740	−0.0601	−0.0679	−0.0655	−0.0589	−0.0656	−0.0621	−0.0605
ϕ_{20}	−0.1864	−0.2031	−0.2368	−0.1832	−0.2007	−0.2063	−0.1926	−0.2080	−0.2009	−0.2003
ϕ_{21}	−0.1604	−0.1669	−0.1660	−0.1558	−0.1563	−0.1397	−0.1700	−0.1447	−0.1760	−0.1424
ϕ_{22}	−0.1374	−0.1469	−0.1284	−0.1165	−0.1277	−0.1341	−0.1688	−0.1316	−0.1453	−0.1006
ϕ_{23}	0.2662	0.2539	0.2349	0.2049	0.2469	0.2818	0.2722	0.2610	0.2446	0.2252
ϕ_{24}	−0.0682	−0.0866	−0.0780	−0.0703	−0.0649	−0.0856	−0.0811	−0.0774	−0.0754	−0.0689
ϕ_{25}	0.0357	0.0505	0.0451	0.0405	0.0396	0.0491	0.0435	0.0392	0.0421	0.0414
ϕ_{26}	−0.1411	−0.1293	−0.1328	−0.1309	−0.1367	−0.1597	−0.0905	−0.1671	−0.1213	−0.1224
ϕ_{27}	0.0881	0.1043	0.1134	0.1010	0.0913	0.1054	0.0789	0.0994	0.0935	0.0869
ϕ_{28}	0.0673	0.0857	0.0578	0.0688	0.0775	0.0721	0.0689	0.0616	0.0582	0.0725
ϕ_{29}	0.3133	0.2955	0.2545	0.3036	0.2907	0.2229	0.2975	0.2567	0.2643	0.3227
ϕ_{30}	0.0465	0.0527	0.0473	0.0487	0.0578	0.0519	0.0482	0.0484	0.0453	0.0391
θ	0.9394	0.9396	0.9250	0.9031	0.9197	0.9478	0.9486	0.9108	0.9385	0.9600
ω	−0.0845	−0.0821	−0.0845	−0.0821	−0.0845	−0.0622	−0.0821	−0.0845	−0.0622	−0.0845
α	0.0326	0.0372	0.0363	0.0309	0.0346	0.0403	0.0429	0.0331	0.0343	0.0362
α^*	0.0231	0.0289	0.0287	0.0235	0.0275	0.0280	0.0280	0.0260	0.0238	0.0255
β	0.9952	0.9864	0.9927	0.9950	0.9859	0.9942	0.9899	0.9944	0.9800	0.9804
λ_0	−5.3897	−5.3897	−5.3897	−5.3897	−5.3897	−5.3897	−5.3897	−5.3897	−5.3897	−5.3897
δ_1	1.7704	1.4448	1.6135	1.7690	1.5496	1.6816	1.6929	1.5108	1.5361	1.6665

Table A8. Parameters for Monte Carlo simulation, t-DCS with dynamic ν_t

	Θ_{MC1}	Θ_{MC2}	Θ_{MC3}	Θ_{MC4}	Θ_{MC5}	Θ_{MC6}	Θ_{MC7}	Θ_{MC8}	Θ_{MC9}	Θ_{MC10}
c	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000
ϕ_1	−0.1640	−0.1505	−0.1439	−0.1710	−0.1466	−0.1694	−0.1725	−0.1835	−0.1414	−0.1631
ϕ_2	−0.0682	−0.0572	−0.0771	−0.0645	−0.0728	−0.0764	−0.0674	−0.0569	−0.0711	−0.0667
ϕ_3	0.1397	0.1372	0.1471	0.1218	0.1403	0.1229	0.1658	0.1219	0.1474	0.1331
ϕ_4	0.0982	0.0950	0.0919	0.0729	0.0826	0.0891	0.0961	0.0929	0.0912	0.0963
ϕ_5	−0.0175	−0.0163	−0.0153	−0.0178	−0.0149	−0.0185	−0.0183	−0.0177	−0.0173	−0.0174
ϕ_6	−0.0563	−0.0463	−0.0570	−0.0572	−0.0596	−0.0555	−0.0583	−0.0581	−0.0476	−0.0512
ϕ_7	−0.0381	−0.0441	−0.0363	−0.0427	−0.0317	−0.0402	−0.0371	−0.0355	−0.0377	−0.0407
ϕ_8	−0.0059	−0.0059	−0.0050	−0.0058	−0.0069	−0.0062	−0.0070	−0.0068	−0.0068	−0.0062
ϕ_9	0.2024	0.2084	0.2002	0.2349	0.1855	0.2373	0.1897	0.1895	0.2215	0.1901
ϕ_{10}	0.0701	0.0615	0.0760	0.0707	0.0535	0.0546	0.0741	0.0671	0.0728	0.0759
ϕ_{11}	0.2654	0.2999	0.2508	0.2566	0.2975	0.2049	0.2373	0.2331	0.2556	0.3256
ϕ_{12}	0.1078	0.1033	0.1015	0.1094	0.0926	0.1065	0.1033	0.1070	0.1164	0.1175
ϕ_{13}	−0.0127	−0.0122	−0.0140	−0.0120	−0.0151	−0.0130	−0.0131	−0.0108	−0.0140	−0.0124
ϕ_{14}	−0.0738	−0.0746	−0.0822	−0.0716	−0.0590	−0.0693	−0.0841	−0.0665	−0.0729	−0.0891
ϕ_{15}	0.1476	0.1476	0.1361	0.1460	0.1476	0.1425	0.1578	0.1498	0.1422	0.1553
ϕ_{16}	0.0881	0.0885	0.0867	0.0880	0.0811	0.0960	0.0963	0.0838	0.0766	0.0856
ϕ_{17}	0.0857	0.0832	0.0815	0.0888	0.0944	0.0632	0.0817	0.0723	0.0825	0.0907
ϕ_{18}	−0.0485	−0.0428	−0.0424	−0.0565	−0.0468	−0.0524	−0.0494	−0.0478	−0.0515	−0.0457
ϕ_{19}	−0.0138	−0.0123	−0.0129	−0.0138	−0.0139	−0.0143	−0.0135	−0.0124	−0.0129	−0.0157
ϕ_{20}	−0.2033	−0.1955	−0.1746	−0.2411	−0.2110	−0.2126	−0.2048	−0.2124	−0.2287	−0.1847
ϕ_{21}	−0.1468	−0.1387	−0.1368	−0.1485	−0.1393	−0.1487	−0.1486	−0.1691	−0.1479	−0.1493
ϕ_{22}	−0.1254	−0.1076	−0.1340	−0.1355	−0.1186	−0.1175	−0.1319	−0.1054	−0.1388	−0.1139
ϕ_{23}	0.1752	0.1923	0.1492	0.1657	0.1725	0.1809	0.1361	0.1531	0.1940	0.2187
ϕ_{24}	−0.0216	−0.0233	−0.0231	−0.0237	−0.0229	−0.0201	−0.0191	−0.0224	−0.0224	−0.0197
ϕ_{25}	−0.0392	−0.0407	−0.0428	−0.0388	−0.0411	−0.0401	−0.0424	−0.0436	−0.0435	−0.0432
ϕ_{26}	−0.0458	−0.0520	−0.0404	−0.0457	−0.0534	−0.0409	−0.0373	−0.0436	−0.0495	−0.0419
ϕ_{27}	0.0596	0.0623	0.0583	0.0481	0.0515	0.0670	0.0642	0.0627	0.0645	0.0580
ϕ_{28}	0.1098	0.1044	0.1220	0.0963	0.0931	0.1292	0.1118	0.1289	0.0897	0.1009
ϕ_{29}	0.2624	0.2421	0.2137	0.2682	0.2945	0.2855	0.2194	0.2639	0.2723	0.2944
ϕ_{30}	0.0330	0.0323	0.0365	0.0314	0.0381	0.0333	0.0300	0.0324	0.0308	0.0296
θ	0.9505	0.9369	0.9590	0.9978	0.9385	0.9467	1.0046	0.9981	0.9602	0.9858
ω	−0.0534	−0.0074	−0.0695	−0.0807	−0.0127	−0.0723	−0.0569	−0.0698	−0.0539	−0.0527
α	0.0339	0.0307	0.0301	0.0268	0.0375	0.0404	0.0314	0.0406	0.0336	0.0260
α^*	0.0252	0.0245	0.0234	0.0273	0.0234	0.0218	0.0245	0.0219	0.0263	0.0257
β	0.9895	0.9985	0.9863	0.9841	0.9975	0.9858	0.9888	0.9863	0.9894	0.9896
λ_0	−5.3884	−5.3884	−5.3884	−5.3884	−5.3884	−5.3884	−5.3884	−5.3884	−5.3884	−5.3884
δ_1	0.6818	0.6499	0.7070	0.7048	0.6559	0.6058	0.6076	0.7260	0.6371	0.7113
γ_1	0.5897	0.6089	0.5746	0.5759	0.6053	0.6355	0.6344	0.5631	0.6166	0.5719
κ_1	0.9099	0.9009	0.8922	0.9537	0.9060	0.9318	0.9246	0.9231	0.9666	0.9382

Table A8 (continued). Parameters for Monte Carlo simulation, t-DCS with dynamic ν_t

	Θ_{MC11}	Θ_{MC12}	Θ_{MC13}	Θ_{MC14}	Θ_{MC15}	Θ_{MC16}	Θ_{MC17}	Θ_{MC18}	Θ_{MC19}	Θ_{MC20}
c	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000
ϕ_1	−0.1623	−0.1752	−0.1556	−0.1781	−0.1948	−0.1841	−0.1631	−0.1617	−0.1771	−0.1609
ϕ_2	−0.0707	−0.0712	−0.0724	−0.0687	−0.0651	−0.0720	−0.0752	−0.0729	−0.0727	−0.0584
ϕ_3	0.1381	0.1427	0.1473	0.1517	0.1522	0.1596	0.1535	0.1335	0.1315	0.1328
ϕ_4	0.1059	0.0723	0.1090	0.0891	0.0939	0.0998	0.1067	0.0896	0.0881	0.0997
ϕ_5	−0.0184	−0.0189	−0.0164	−0.0183	−0.0205	−0.0191	−0.0189	−0.0144	−0.0167	−0.0158
ϕ_6	−0.0456	−0.0570	−0.0578	−0.0614	−0.0591	−0.0523	−0.0597	−0.0602	−0.0580	−0.0760
ϕ_7	−0.0390	−0.0338	−0.0370	−0.0436	−0.0345	−0.0361	−0.0398	−0.0342	−0.0415	−0.0404
ϕ_8	−0.0057	−0.0064	−0.0066	−0.0059	−0.0058	−0.0060	−0.0047	−0.0061	−0.0060	−0.0048
ϕ_9	0.1836	0.2136	0.2050	0.2090	0.1894	0.1970	0.2145	0.2002	0.1695	0.1929
ϕ_{10}	0.0710	0.0733	0.0742	0.0770	0.0622	0.0730	0.0567	0.0672	0.0769	0.0780
ϕ_{11}	0.2703	0.2892	0.2919	0.2435	0.2311	0.2695	0.2616	0.2702	0.2714	0.2690
ϕ_{12}	0.1133	0.1211	0.1294	0.1086	0.1201	0.1022	0.1037	0.0987	0.1076	0.1077
ϕ_{13}	−0.0119	−0.0129	−0.0128	−0.0137	−0.0146	−0.0142	−0.0112	−0.0121	−0.0112	−0.0125
ϕ_{14}	−0.0760	−0.0590	−0.0679	−0.0805	−0.0753	−0.0702	−0.0756	−0.0705	−0.0817	−0.0772
ϕ_{15}	0.1514	0.1518	0.1525	0.1283	0.1474	0.1417	0.1485	0.1358	0.1244	0.1649
ϕ_{16}	0.0866	0.0835	0.0766	0.0779	0.0818	0.0890	0.0831	0.0906	0.0790	0.0940
ϕ_{17}	0.0802	0.0882	0.0814	0.0742	0.0857	0.0819	0.0899	0.0915	0.0845	0.0842
ϕ_{18}	−0.0410	−0.0483	−0.0532	−0.0484	−0.0436	−0.0412	−0.0346	−0.0430	−0.0518	−0.0481
ϕ_{19}	−0.0139	−0.0154	−0.0138	−0.0123	−0.0107	−0.0133	−0.0118	−0.0147	−0.0109	−0.0120
ϕ_{20}	−0.1974	−0.1885	−0.2217	−0.1954	−0.1495	−0.1997	−0.2174	−0.2173	−0.2247	−0.1969
ϕ_{21}	−0.1364	−0.1354	−0.1341	−0.1352	−0.1526	−0.1711	−0.1501	−0.1441	−0.1402	−0.1317
ϕ_{22}	−0.1156	−0.1184	−0.1169	−0.1089	−0.1465	−0.1429	−0.1169	−0.0992	−0.1295	−0.1285
ϕ_{23}	0.1925	0.1748	0.1870	0.1782	0.1549	0.1659	0.2014	0.1827	0.1644	0.1989
ϕ_{24}	−0.0201	−0.0181	−0.0263	−0.0193	−0.0212	−0.0215	−0.0200	−0.0226	−0.0226	−0.0273
ϕ_{25}	−0.0312	−0.0443	−0.0370	−0.0409	−0.0379	−0.0416	−0.0437	−0.0417	−0.0373	−0.0344
ϕ_{26}	−0.0483	−0.0441	−0.0498	−0.0406	−0.0467	−0.0411	−0.0450	−0.0433	−0.0475	−0.0475
ϕ_{27}	0.0586	0.0654	0.0570	0.0528	0.0634	0.0567	0.0562	0.0587	0.0529	0.0611
ϕ_{28}	0.1018	0.1192	0.1176	0.0952	0.1053	0.0802	0.0917	0.0975	0.1240	0.1258
ϕ_{29}	0.2721	0.2418	0.2368	0.2283	0.2515	0.2561	0.2698	0.2471	0.2581	0.2947
ϕ_{30}	0.0272	0.0386	0.0390	0.0329	0.0362	0.0411	0.0292	0.0283	0.0299	0.0314
θ	0.9742	0.9456	0.9351	0.9577	0.9760	0.9977	0.9437	0.9563	0.9618	0.9110
ω	−0.0525	−0.0238	−0.0757	−0.0451	−0.0685	−0.0605	−0.0799	−0.0300	−0.0491	−0.0157
α	0.0343	0.0347	0.0328	0.0367	0.0344	0.0291	0.0335	0.0401	0.0333	0.0372
α^*	0.0227	0.0265	0.0248	0.0249	0.0244	0.0255	0.0317	0.0251	0.0249	0.0264
β	0.9897	0.9953	0.9851	0.9911	0.9865	0.9881	0.9843	0.9941	0.9903	0.9969
λ_0	−5.3884	−5.3884	−5.3884	−5.3884	−5.3884	−5.3884	−5.3884	−5.3884	−5.3884	−5.3884
δ_1	0.6726	0.7232	0.7136	0.7175	0.6662	0.6878	0.6158	0.6874	0.6866	0.6576
γ_1	0.5953	0.5648	0.5706	0.5682	0.5991	0.5861	0.6294	0.5864	0.5869	0.6043
κ_1	0.9711	0.9045	0.8855	0.8665	0.8888	0.9454	0.9239	0.9065	0.8908	0.9561

Table A9. Parameters for Monte Carlo simulation, GED-DCS with constant ν_t

	Θ_{MC1}	Θ_{MC2}	Θ_{MC3}	Θ_{MC4}	Θ_{MC5}	Θ_{MC6}	Θ_{MC7}	Θ_{MC8}	Θ_{MC9}	Θ_{MC10}
c	0.0006	0.0006	0.0006	0.0006	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006
ϕ_1	−0.1473	−0.1369	−0.1274	−0.1643	−0.1653	−0.1480	−0.1263	−0.1315	−0.1432	−0.1459
ϕ_2	−0.0885	−0.0682	−0.0838	−0.1022	−0.0955	−0.0957	−0.0798	−0.0942	−0.0854	−0.0931
ϕ_3	0.0790	0.0851	0.0840	0.0780	0.0776	0.0959	0.0861	0.1014	0.0783	0.0904
ϕ_4	0.0316	0.0329	0.0307	0.0336	0.0280	0.0339	0.0280	0.0344	0.0312	0.0333
ϕ_5	−0.0768	−0.0832	−0.0716	−0.0687	−0.0857	−0.0860	−0.0803	−0.0698	−0.0826	−0.0768
ϕ_6	−0.0016	−0.0017	−0.0014	−0.0018	−0.0016	−0.0015	−0.0019	−0.0016	−0.0016	−0.0016
ϕ_7	−0.0896	−0.1000	−0.0965	−0.0795	−0.0789	−0.0870	−0.0888	−0.0950	−0.0835	−0.0890
ϕ_8	−0.0447	−0.0468	−0.0482	−0.0475	−0.0404	−0.0480	−0.0390	−0.0448	−0.0469	−0.0282
ϕ_9	−0.0103	−0.0098	−0.0103	−0.0097	−0.0112	−0.0085	−0.0081	−0.0094	−0.0104	−0.0100
ϕ_{10}	−0.0467	−0.0511	−0.0468	−0.0431	−0.0449	−0.0463	−0.0484	−0.0520	−0.0493	−0.0442
ϕ_{11}	0.1288	0.1215	0.1447	0.1311	0.1247	0.1302	0.1282	0.1521	0.1297	0.1455
ϕ_{12}	0.0245	0.0259	0.0251	0.0233	0.0269	0.0239	0.0266	0.0238	0.0240	0.0241
ϕ_{13}	−0.0238	−0.0252	−0.0231	−0.0285	−0.0242	−0.0212	−0.0223	−0.0225	−0.0221	−0.0267
ϕ_{14}	−0.0973	−0.0754	−0.1105	−0.1000	−0.0961	−0.1130	−0.0861	−0.0986	−0.0991	−0.0934
ϕ_{15}	0.1113	0.1198	0.0938	0.1082	0.1250	0.1366	0.1037	0.1220	0.1188	0.1105
ϕ_{16}	−0.0401	−0.0442	−0.0478	−0.0350	−0.0420	−0.0415	−0.0416	−0.0398	−0.0434	−0.0336
ϕ_{17}	−0.0724	−0.0753	−0.0593	−0.0708	−0.0753	−0.0813	−0.0717	−0.0713	−0.0827	−0.0769
ϕ_{18}	0.0212	0.0196	0.0172	0.0191	0.0186	0.0197	0.0210	0.0200	0.0205	0.0211
ϕ_{19}	−0.0712	−0.0721	−0.0658	−0.0691	−0.0773	−0.0759	−0.0730	−0.0851	−0.0751	−0.0683
ϕ_{20}	−0.0447	−0.0460	−0.0441	−0.0440	−0.0440	−0.0489	−0.0415	−0.0473	−0.0424	−0.0496
ϕ_{21}	−0.0372	−0.0364	−0.0351	−0.0285	−0.0336	−0.0328	−0.0392	−0.0332	−0.0343	−0.0382
ϕ_{22}	−0.0613	−0.0541	−0.0607	−0.0637	−0.0634	−0.0583	−0.0622	−0.0565	−0.0636	−0.0499
ϕ_{23}	0.1061	0.1161	0.0891	0.0987	0.0937	0.1044	0.1012	0.1120	0.1050	0.1049
ϕ_{24}	−0.0782	−0.0688	−0.0704	−0.0783	−0.0871	−0.0705	−0.0915	−0.0727	−0.0918	−0.0661
ϕ_{25}	−0.0575	−0.0597	−0.0564	−0.0632	−0.0552	−0.0524	−0.0531	−0.0643	−0.0646	−0.0535
ϕ_{26}	−0.0586	−0.0700	−0.0669	−0.0658	−0.0648	−0.0542	−0.0607	−0.0425	−0.0601	−0.0596
ϕ_{27}	0.0599	0.0586	0.0547	0.0657	0.0480	0.0565	0.0579	0.0623	0.0602	0.0599
ϕ_{28}	0.1569	0.1585	0.1491	0.1589	0.1392	0.1406	0.1322	0.1580	0.1614	0.1398
ϕ_{29}	0.0695	0.0761	0.0695	0.0831	0.0565	0.0604	0.0770	0.0839	0.0740	0.0729
ϕ_{30}	−0.1014	−0.1050	−0.0995	−0.1078	−0.1207	−0.1077	−0.0966	−0.1045	−0.1130	−0.0927
θ	0.0007	0.0006	0.0006	0.0007	0.0007	0.0006	0.0007	0.0007	0.0007	0.0007
ω	−0.0739	−0.1107	−0.0722	−0.0739	−0.0739	−0.0739	−0.0739	−0.1107	−0.0722	−0.1107
α	0.0383	0.0422	0.0389	0.0353	0.0393	0.0383	0.0315	0.0399	0.0446	0.0369
α^*	0.0204	0.0228	0.0214	0.0220	0.0213	0.0237	0.0187	0.0184	0.0205	0.0195
β	0.9863	0.9795	0.9866	0.9858	0.9828	0.9890	0.9854	0.9915	0.9897	0.9946
λ_0	−5.8321	−5.8321	−5.8321	−5.8321	−5.8321	−5.8321	−5.8321	−5.8321	−5.8321	−5.8321
δ_1	0.2751	0.2829	0.2670	0.2659	0.2704	0.2784	0.2815	0.2769	0.2779	0.2692

Table A9 (continued). Parameters for Monte Carlo simulation, GED-DCS with constant ν_t

	Θ_{MC11}	Θ_{MC12}	Θ_{MC13}	Θ_{MC14}	Θ_{MC15}	Θ_{MC16}	Θ_{MC17}	Θ_{MC18}	Θ_{MC19}	Θ_{MC20}
c	0.0006	0.0006	0.0007	0.0006	0.0006	0.0006	0.0006	0.0007	0.0006	0.0006
ϕ_1	−0.1285	−0.1592	−0.1428	−0.1554	−0.1359	−0.1590	−0.1317	−0.1704	−0.1458	−0.1495
ϕ_2	−0.0922	−0.0982	−0.1042	−0.0983	−0.0822	−0.0999	−0.0727	−0.0904	−0.0933	−0.0826
ϕ_3	0.0830	0.0658	0.0645	0.0857	0.0769	0.0721	0.0706	0.0791	0.0853	0.0817
ϕ_4	0.0327	0.0290	0.0311	0.0295	0.0288	0.0345	0.0354	0.0338	0.0293	0.0322
ϕ_5	−0.0726	−0.0699	−0.0907	−0.0686	−0.0923	−0.0979	−0.0698	−0.0765	−0.0757	−0.0807
ϕ_6	−0.0015	−0.0014	−0.0015	−0.0017	−0.0015	−0.0013	−0.0018	−0.0014	−0.0018	−0.0015
ϕ_7	−0.0883	−0.1035	−0.1073	−0.0931	−0.0926	−0.0868	−0.1059	−0.0809	−0.1000	−0.0869
ϕ_8	−0.0409	−0.0347	−0.0458	−0.0429	−0.0353	−0.0369	−0.0436	−0.0528	−0.0502	−0.0447
ϕ_9	−0.0110	−0.0109	−0.0116	−0.0104	−0.0119	−0.0099	−0.0099	−0.0105	−0.0099	−0.0095
ϕ_{10}	−0.0562	−0.0435	−0.0552	−0.0453	−0.0433	−0.0462	−0.0407	−0.0484	−0.0460	−0.0482
ϕ_{11}	0.1183	0.1303	0.1232	0.1237	0.1229	0.1598	0.1354	0.1346	0.1336	0.1278
ϕ_{12}	0.0241	0.0237	0.0215	0.0239	0.0185	0.0244	0.0221	0.0262	0.0246	0.0265
ϕ_{13}	−0.0275	−0.0221	−0.0259	−0.0212	−0.0232	−0.0228	−0.0205	−0.0219	−0.0235	−0.0230
ϕ_{14}	−0.0716	−0.0967	−0.0925	−0.0901	−0.0859	−0.0907	−0.0958	−0.1040	−0.1193	−0.0918
ϕ_{15}	0.1051	0.1265	0.1311	0.1295	0.1234	0.0993	0.1054	0.1273	0.1158	0.0844
ϕ_{16}	−0.0446	−0.0442	−0.0446	−0.0363	−0.0433	−0.0328	−0.0410	−0.0435	−0.0390	−0.0406
ϕ_{17}	−0.0866	−0.0551	−0.0751	−0.0739	−0.0686	−0.0749	−0.0832	−0.0809	−0.0605	−0.0736
ϕ_{18}	0.0176	0.0193	0.0195	0.0236	0.0201	0.0231	0.0226	0.0200	0.0235	0.0168
ϕ_{19}	−0.0706	−0.0601	−0.0643	−0.0789	−0.0768	−0.0685	−0.0753	−0.0809	−0.0721	−0.0662
ϕ_{20}	−0.0443	−0.0488	−0.0414	−0.0452	−0.0454	−0.0499	−0.0430	−0.0482	−0.0503	−0.0440
ϕ_{21}	−0.0287	−0.0387	−0.0349	−0.0397	−0.0420	−0.0339	−0.0364	−0.0381	−0.0406	−0.0413
ϕ_{22}	−0.0656	−0.0631	−0.0603	−0.0556	−0.0670	−0.0506	−0.0680	−0.0620	−0.0548	−0.0744
ϕ_{23}	0.0860	0.1098	0.1004	0.1028	0.0983	0.1084	0.1050	0.1021	0.1131	0.1010
ϕ_{24}	−0.0781	−0.0830	−0.0787	−0.0774	−0.0725	−0.0772	−0.0755	−0.0763	−0.0770	−0.0753
ϕ_{25}	−0.0554	−0.0529	−0.0628	−0.0554	−0.0520	−0.0590	−0.0474	−0.0537	−0.0593	−0.0502
ϕ_{26}	−0.0580	−0.0653	−0.0564	−0.0689	−0.0622	−0.0551	−0.0470	−0.0569	−0.0636	−0.0604
ϕ_{27}	0.0608	0.0554	0.0643	0.0645	0.0616	0.0649	0.0497	0.0668	0.0638	0.0558
ϕ_{28}	0.1844	0.1693	0.1523	0.1668	0.1798	0.1268	0.1741	0.1226	0.1563	0.1497
ϕ_{29}	0.0712	0.0717	0.0741	0.0545	0.0660	0.0603	0.0769	0.0699	0.0726	0.0722
ϕ_{30}	−0.1055	−0.0992	−0.1055	−0.1062	−0.0976	−0.1070	−0.0810	−0.0982	−0.0887	−0.0999
θ	0.0007	0.0007	0.0007	0.0007	0.0006	0.0007	0.0006	0.0006	0.0007	0.0006
ω	−0.0722	−0.1107	−0.0722	−0.1107	−0.0722	−0.0739	−0.1107	−0.0722	−0.0739	−0.0722
α	0.0353	0.0380	0.0397	0.0383	0.0368	0.0338	0.0335	0.0370	0.0436	0.0458
α^*	0.0238	0.0236	0.0207	0.0212	0.0197	0.0209	0.0208	0.0227	0.0178	0.0223
β	0.9832	0.9882	0.9900	0.9794	0.9937	0.9952	0.9868	0.9880	0.9946	0.9885
λ_0	−5.8321	−5.8321	−5.8321	−5.8321	−5.8321	−5.8321	−5.8321	−5.8321	−5.8321	−5.8321
δ_1	0.2817	0.2798	0.2875	0.2979	0.2971	0.2923	0.2919	0.2913	0.2884	0.2749

Table A10. Parameters for Monte Carlo simulation, GED-DCS with dynamic ν_t

	Θ_{MC1}	Θ_{MC2}	Θ_{MC3}	Θ_{MC4}	Θ_{MC5}	Θ_{MC6}	Θ_{MC7}	Θ_{MC8}	Θ_{MC9}	Θ_{MC10}
c	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005
ϕ_1	−0.1486	−0.1400	−0.1438	−0.1445	−0.1621	−0.1504	−0.1690	−0.1451	−0.1636	−0.1576
ϕ_2	−0.0669	−0.0652	−0.0663	−0.0677	−0.0656	−0.0685	−0.0659	−0.0697	−0.0584	−0.0681
ϕ_3	0.1032	0.1063	0.1014	0.1146	0.0954	0.0893	0.1285	0.1147	0.1020	0.1057
ϕ_4	0.0538	0.0540	0.0582	0.0475	0.0586	0.0646	0.0592	0.0553	0.0441	0.0650
ϕ_5	−0.0682	−0.0724	−0.0677	−0.0700	−0.0722	−0.0834	−0.0661	−0.0680	−0.0700	−0.0625
ϕ_6	−0.0041	−0.0037	−0.0041	−0.0037	−0.0043	−0.0031	−0.0044	−0.0045	−0.0043	−0.0046
ϕ_7	−0.0716	−0.0688	−0.0732	−0.0700	−0.0642	−0.0609	−0.0674	−0.0664	−0.0731	−0.0708
ϕ_8	−0.0315	−0.0294	−0.0337	−0.0352	−0.0279	−0.0353	−0.0383	−0.0312	−0.0300	−0.0292
ϕ_9	0.0157	0.0164	0.0173	0.0181	0.0182	0.0207	0.0126	0.0163	0.0164	0.0136
ϕ_{10}	−0.0189	−0.0187	−0.0176	−0.0215	−0.0163	−0.0178	−0.0208	−0.0221	−0.0172	−0.0175
ϕ_{11}	0.1498	0.1360	0.1273	0.1613	0.1473	0.1393	0.1711	0.1467	0.1474	0.1565
ϕ_{12}	0.0404	0.0356	0.0431	0.0349	0.0517	0.0406	0.0404	0.0380	0.0389	0.0387
ϕ_{13}	−0.0175	−0.0156	−0.0149	−0.0196	−0.0150	−0.0178	−0.0187	−0.0164	−0.0187	−0.0187
ϕ_{14}	−0.0977	−0.1047	−0.0868	−0.0996	−0.0910	−0.1068	−0.0892	−0.1028	−0.0827	−0.0844
ϕ_{15}	0.0988	0.0933	0.1029	0.0943	0.1038	0.0853	0.0840	0.1077	0.0807	0.1035
ϕ_{16}	−0.0304	−0.0299	−0.0270	−0.0288	−0.0284	−0.0290	−0.0305	−0.0270	−0.0254	−0.0292
ϕ_{17}	−0.0519	−0.0566	−0.0506	−0.0497	−0.0465	−0.0526	−0.0583	−0.0537	−0.0529	−0.0535
ϕ_{18}	0.0324	0.0349	0.0376	0.0358	0.0320	0.0344	0.0324	0.0360	0.0295	0.0313
ϕ_{19}	−0.0639	−0.0633	−0.0774	−0.0702	−0.0691	−0.0627	−0.0721	−0.0637	−0.0653	−0.0666
ϕ_{20}	−0.0505	−0.0575	−0.0460	−0.0484	−0.0510	−0.0481	−0.0448	−0.0501	−0.0433	−0.0519
ϕ_{21}	−0.0314	−0.0324	−0.0311	−0.0274	−0.0313	−0.0339	−0.0303	−0.0316	−0.0276	−0.0326
ϕ_{22}	−0.0539	−0.0477	−0.0573	−0.0556	−0.0507	−0.0507	−0.0584	−0.0533	−0.0565	−0.0519
ϕ_{23}	0.1369	0.1376	0.1530	0.1357	0.1457	0.1360	0.1405	0.1273	0.1319	0.1253
ϕ_{24}	−0.0577	−0.0502	−0.0612	−0.0547	−0.0593	−0.0627	−0.0594	−0.0611	−0.0665	−0.0548
ϕ_{25}	−0.0561	−0.0688	−0.0609	−0.0601	−0.0520	−0.0595	−0.0477	−0.0622	−0.0542	−0.0678
ϕ_{26}	−0.0522	−0.0501	−0.0475	−0.0522	−0.0523	−0.0523	−0.0433	−0.0535	−0.0494	−0.0546
ϕ_{27}	0.0576	0.0518	0.0482	0.0501	0.0672	0.0667	0.0677	0.0567	0.0611	0.0510
ϕ_{28}	0.1654	0.1814	0.1471	0.1593	0.1572	0.1524	0.1667	0.1738	0.1808	0.1549
ϕ_{29}	0.0829	0.0757	0.0769	0.0856	0.0856	0.0812	0.0867	0.0735	0.0830	0.0796
ϕ_{30}	−0.1032	−0.0937	−0.1208	−0.0955	−0.0876	−0.0916	−0.1074	−0.1023	−0.1181	−0.0976
θ	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007	0.0006	0.0007
ω	−0.0555	−0.0555	−0.0555	−0.0555	−0.0555	−0.0555	−0.0555	−0.0555	−0.0555	−0.0555
α	0.0325	0.0289	0.0354	0.0369	0.0306	0.0329	0.0316	0.0315	0.0297	0.0312
α^*	0.0186	0.0212	0.0180	0.0196	0.0184	0.0199	0.0178	0.0180	0.0177	0.0203
β	0.9897	0.9888	0.9825	0.9867	0.9881	0.9912	0.9889	0.9896	0.9904	0.9856
λ_0	−5.7214	−5.7214	−5.7214	−5.7214	−5.7214	−5.7214	−5.7214	−5.7214	−5.7214	−5.7214
δ_1	0.1984	0.1967	0.1812	0.1896	0.2113	0.2043	0.1732	0.2042	0.2068	0.1916
γ_1	0.3089	0.3148	0.3686	0.3394	0.2638	0.2884	0.3965	0.2888	0.2796	0.3324
κ_1	0.0091	0.0080	0.0092	0.0083	0.0105	0.0076	0.0110	0.0085	0.0096	0.0082

Table A10 (continued). Parameters for Monte Carlo simulation, GED-DCS with dynamic ν_t

	Θ_{MC11}	Θ_{MC12}	Θ_{MC13}	Θ_{MC14}	Θ_{MC15}	Θ_{MC16}	Θ_{MC17}	Θ_{MC18}	Θ_{MC19}	Θ_{MC20}
c	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005
ϕ_1	−0.1318	−0.1038	−0.1505	−0.1133	−0.1620	−0.1589	−0.1604	−0.1090	−0.1465	−0.1674
ϕ_2	−0.0750	−0.0603	−0.0710	−0.0708	−0.0747	−0.0703	−0.0717	−0.0651	−0.0656	−0.0727
ϕ_3	0.1086	0.0902	0.0979	0.0978	0.1036	0.1019	0.0977	0.0974	0.1044	0.1012
ϕ_4	0.0498	0.0622	0.0608	0.0601	0.0549	0.0541	0.0575	0.0579	0.0478	0.0513
ϕ_5	−0.0766	−0.0696	−0.0563	−0.0682	−0.0759	−0.0709	−0.0757	−0.0717	−0.0558	−0.0606
ϕ_6	−0.0043	−0.0043	−0.0036	−0.0040	−0.0042	−0.0043	−0.0042	−0.0037	−0.0044	−0.0035
ϕ_7	−0.0594	−0.0709	−0.0744	−0.0797	−0.0708	−0.0659	−0.0856	−0.0728	−0.0803	−0.0813
ϕ_8	−0.0293	−0.0338	−0.0290	−0.0283	−0.0264	−0.0259	−0.0292	−0.0297	−0.0281	−0.0296
ϕ_9	0.0166	0.0134	0.0146	0.0152	0.0161	0.0163	0.0159	0.0140	0.0164	0.0145
ϕ_{10}	−0.0198	−0.0178	−0.0217	−0.0197	−0.0174	−0.0197	−0.0194	−0.0209	−0.0193	−0.0189
ϕ_{11}	0.1615	0.1234	0.1318	0.1609	0.1319	0.1186	0.1452	0.1890	0.1446	0.1466
ϕ_{12}	0.0396	0.0394	0.0447	0.0421	0.0412	0.0476	0.0340	0.0416	0.0418	0.0378
ϕ_{13}	−0.0188	−0.0213	−0.0170	−0.0148	−0.0165	−0.0175	−0.0172	−0.0177	−0.0185	−0.0167
ϕ_{14}	−0.0933	−0.0959	−0.0999	−0.0972	−0.0939	−0.0967	−0.1061	−0.0992	−0.0948	−0.1095
ϕ_{15}	0.1070	0.1069	0.0828	0.1015	0.0800	0.0872	0.1208	0.1063	0.1107	0.1105
ϕ_{16}	−0.0348	−0.0357	−0.0321	−0.0240	−0.0268	−0.0277	−0.0382	−0.0343	−0.0318	−0.0335
ϕ_{17}	−0.0513	−0.0481	−0.0488	−0.0534	−0.0474	−0.0534	−0.0415	−0.0570	−0.0534	−0.0461
ϕ_{18}	0.0271	0.0354	0.0256	0.0358	0.0356	0.0272	0.0341	0.0408	0.0333	0.0332
ϕ_{19}	−0.0623	−0.0636	−0.0632	−0.0657	−0.0659	−0.0589	−0.0561	−0.0570	−0.0502	−0.0627
ϕ_{20}	−0.0449	−0.0527	−0.0466	−0.0582	−0.0413	−0.0485	−0.0493	−0.0447	−0.0544	−0.0409
ϕ_{21}	−0.0325	−0.0348	−0.0310	−0.0309	−0.0306	−0.0338	−0.0347	−0.0282	−0.0290	−0.0322
ϕ_{22}	−0.0530	−0.0545	−0.0509	−0.0503	−0.0565	−0.0582	−0.0630	−0.0534	−0.0482	−0.0443
ϕ_{23}	0.1349	0.1366	0.1449	0.1525	0.1481	0.1109	0.1310	0.1463	0.1279	0.1449
ϕ_{24}	−0.0544	−0.0456	−0.0622	−0.0540	−0.0560	−0.0568	−0.0611	−0.0675	−0.0547	−0.0466
ϕ_{25}	−0.0553	−0.0624	−0.0514	−0.0508	−0.0565	−0.0431	−0.0593	−0.0524	−0.0590	−0.0525
ϕ_{26}	−0.0513	−0.0587	−0.0537	−0.0514	−0.0479	−0.0449	−0.0576	−0.0424	−0.0564	−0.0515
ϕ_{27}	0.0548	0.0543	0.0543	0.0671	0.0496	0.0563	0.0553	0.0457	0.0498	0.0556
ϕ_{28}	0.1498	0.1755	0.1618	0.1696	0.1604	0.1509	0.1864	0.1678	0.1799	0.1853
ϕ_{29}	0.0812	0.0929	0.0902	0.0879	0.0947	0.0906	0.0897	0.0698	0.0859	0.0707
ϕ_{30}	−0.1141	−0.1124	−0.0994	−0.1126	−0.0994	−0.0953	−0.1079	−0.0975	−0.1069	−0.1141
θ	0.0006	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007	0.0007
ω	−0.0555	−0.0555	−0.0555	−0.0555	−0.0555	−0.0555	−0.0555	−0.0555	−0.0555	−0.0555
α	0.0363	0.0330	0.0354	0.0383	0.0374	0.0327	0.0280	0.0313	0.0349	0.0310
α^*	0.0175	0.0170	0.0166	0.0191	0.0177	0.0195	0.0184	0.0224	0.0178	0.0166
β	0.9888	0.9901	0.9924	0.9879	0.9880	0.9902	0.9855	0.9920	0.9940	0.9952
λ_0	−5.7214	−5.7214	−5.7214	−5.7214	−5.7214	−5.7214	−5.7214	−5.7214	−5.7214	−5.7214
δ_1	0.1870	0.1906	0.1921	0.2005	0.2061	0.1976	0.1907	0.1844	0.1944	0.2164
γ_1	0.3487	0.3361	0.3307	0.3014	0.2822	0.3115	0.3358	0.3577	0.3229	0.2462
κ_1	0.0085	0.0086	0.0089	0.0077	0.0082	0.0076	0.0082	0.0091	0.0084	0.0096

Table A11. Parameters for Monte Carlo simulation, Gen-t-DCS with constant ν_t, η_t

	Θ_{MC1}	Θ_{MC2}	Θ_{MC3}	Θ_{MC4}	Θ_{MC5}	Θ_{MC6}	Θ_{MC7}	Θ_{MC8}	Θ_{MC9}	Θ_{MC10}
c	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
ϕ_1	−0.1378	−0.1585	−0.1674	−0.1456	−0.1513	−0.1534	−0.1510	−0.1467	−0.1539	−0.1352
ϕ_2	−0.0873	−0.0749	−0.0868	−0.0918	−0.0933	−0.0913	−0.0833	−0.0960	−0.0794	−0.0798
ϕ_3	0.1837	0.1845	0.1870	0.1942	0.1747	0.1754	0.1772	0.1744	0.2078	0.1568
ϕ_4	0.0833	0.0712	0.0859	0.0741	0.0847	0.0724	0.0695	0.0728	0.0872	0.0841
ϕ_5	0.0050	0.0042	0.0051	0.0043	0.0055	0.0057	0.0054	0.0050	0.0045	0.0054
ϕ_6	−0.0996	−0.0899	−0.0976	−0.0883	−0.0831	−0.0977	−0.0789	−0.0907	−0.1015	−0.0962
ϕ_7	−0.0413	−0.0407	−0.0416	−0.0367	−0.0435	−0.0469	−0.0396	−0.0410	−0.0422	−0.0414
ϕ_8	−0.0280	−0.0306	−0.0287	−0.0293	−0.0255	−0.0255	−0.0253	−0.0293	−0.0264	−0.0286
ϕ_9	0.2291	0.1858	0.2057	0.2316	0.2177	0.2190	0.1802	0.1817	0.2300	0.2185
ϕ_{10}	0.0560	0.0593	0.0488	0.0476	0.0469	0.0636	0.0568	0.0580	0.0496	0.0527
ϕ_{11}	0.2844	0.2624	0.2991	0.3312	0.2463	0.3714	0.2933	0.3021	0.2767	0.2898
ϕ_{12}	0.0474	0.0469	0.0391	0.0516	0.0525	0.0452	0.0425	0.0386	0.0506	0.0492
ϕ_{13}	−0.0061	−0.0065	−0.0058	−0.0062	−0.0063	−0.0065	−0.0065	−0.0071	−0.0062	−0.0058
ϕ_{14}	−0.1259	−0.1128	−0.1276	−0.1362	−0.1287	−0.1355	−0.1469	−0.1391	−0.1278	−0.1353
ϕ_{15}	0.1904	0.1859	0.1907	0.2277	0.2036	0.1943	0.1649	0.2073	0.1771	0.2055
ϕ_{16}	0.0912	0.0838	0.0922	0.0899	0.0872	0.0869	0.1003	0.1017	0.0862	0.0721
ϕ_{17}	0.1011	0.1003	0.0955	0.1131	0.0947	0.1139	0.0997	0.0957	0.0999	0.1072
ϕ_{18}	−0.0493	−0.0524	−0.0536	−0.0511	−0.0504	−0.0569	−0.0470	−0.0487	−0.0536	−0.0468
ϕ_{19}	−0.0716	−0.0745	−0.0892	−0.0817	−0.0785	−0.0685	−0.0705	−0.0828	−0.0719	−0.0824
ϕ_{20}	−0.2013	−0.2066	−0.2583	−0.1965	−0.1922	−0.2329	−0.2242	−0.2148	−0.2451	−0.2025
ϕ_{21}	−0.1545	−0.1426	−0.1506	−0.1553	−0.1580	−0.1536	−0.1461	−0.1443	−0.1603	−0.1579
ϕ_{22}	−0.1231	−0.1237	−0.1197	−0.1161	−0.1206	−0.1340	−0.1262	−0.1153	−0.1345	−0.1252
ϕ_{23}	0.2588	0.2253	0.3161	0.2754	0.2577	0.2362	0.2372	0.2927	0.2500	0.2816
ϕ_{24}	−0.0688	−0.0590	−0.0767	−0.0667	−0.0671	−0.0778	−0.0661	−0.0610	−0.0586	−0.0663
ϕ_{25}	0.0565	0.0574	0.0612	0.0488	0.0550	0.0490	0.0601	0.0557	0.0555	0.0658
ϕ_{26}	−0.1312	−0.1610	−0.1560	−0.1336	−0.1054	−0.1238	−0.1214	−0.1088	−0.1160	−0.1089
ϕ_{27}	0.0934	0.0834	0.1071	0.0804	0.0859	0.0702	0.0800	0.0984	0.0927	0.0874
ϕ_{28}	0.0689	0.0757	0.0584	0.0686	0.0594	0.0700	0.0755	0.0694	0.0687	0.0576
ϕ_{29}	0.2780	0.2622	0.3437	0.2857	0.2677	0.2557	0.2631	0.2592	0.2208	0.2945
ϕ_{30}	0.0535	0.0495	0.0575	0.0485	0.0564	0.0511	0.0568	0.0478	0.0571	0.0599
θ	1.0317	1.0412	1.0443	1.0158	0.9972	1.0368	1.0466	0.9978	1.0325	1.0414
ω	−0.0633	−0.0664	−0.0476	−0.0779	−0.0494	−0.0481	−0.0705	−0.0520	−0.0736	−0.0642
α	0.0370	0.0335	0.0319	0.0350	0.0392	0.0359	0.0374	0.0367	0.0351	0.0387
α^*	0.0268	0.0224	0.0262	0.0284	0.0266	0.0274	0.0248	0.0249	0.0253	0.0308
β	0.9875	0.9869	0.9906	0.9847	0.9903	0.9905	0.9861	0.9898	0.9855	0.9874
λ_0	−5.3960	−5.3960	−5.3960	−5.3960	−5.3960	−5.3960	−5.3960	−5.3960	−5.3960	−5.3960
δ_1	1.8418	1.8770	1.4832	2.0982	1.7272	1.6234	1.8607	1.5721	1.8669	2.0143
δ_2	0.6287	0.6670	0.6631	0.6719	0.7003	0.5311	0.6111	0.7026	0.5483	0.6098

Table A11 (continued). Parameters for Monte Carlo simulation, Gen-t-DCS with constant ν_t, η_t

	Θ_{MC11}	Θ_{MC12}	Θ_{MC13}	Θ_{MC14}	Θ_{MC15}	Θ_{MC16}	Θ_{MC17}	Θ_{MC18}	Θ_{MC19}	Θ_{MC20}
c	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
ϕ_1	−0.1357	−0.1199	−0.1405	−0.1459	−0.1326	−0.1307	−0.1457	−0.1578	−0.1175	−0.1227
ϕ_2	−0.0869	−0.0957	−0.0951	−0.1026	−0.0867	−0.0922	−0.0971	−0.0864	−0.0922	−0.0924
ϕ_3	0.2018	0.1690	0.1496	0.1674	0.1689	0.2061	0.1811	0.1957	0.1637	0.1736
ϕ_4	0.0785	0.0783	0.0768	0.0656	0.0695	0.0952	0.0773	0.0916	0.0882	0.1006
ϕ_5	0.0045	0.0052	0.0052	0.0042	0.0039	0.0048	0.0059	0.0052	0.0060	0.0053
ϕ_6	−0.0750	−0.1033	−0.1071	−0.1147	−0.0972	−0.1216	−0.1057	−0.1020	−0.0841	−0.0979
ϕ_7	−0.0443	−0.0443	−0.0365	−0.0349	−0.0387	−0.0414	−0.0387	−0.0359	−0.0401	−0.0405
ϕ_8	−0.0222	−0.0313	−0.0318	−0.0274	−0.0272	−0.0304	−0.0325	−0.0249	−0.0304	−0.0282
ϕ_9	0.2229	0.2713	0.2221	0.2095	0.2646	0.2110	0.2583	0.2262	0.2477	0.2630
ϕ_{10}	0.0522	0.0575	0.0598	0.0615	0.0527	0.0550	0.0582	0.0449	0.0574	0.0580
ϕ_{11}	0.2567	0.3152	0.2371	0.2760	0.2926	0.2733	0.3012	0.3443	0.2304	0.3506
ϕ_{12}	0.0481	0.0388	0.0441	0.0512	0.0436	0.0554	0.0523	0.0354	0.0507	0.0466
ϕ_{13}	−0.0058	−0.0058	−0.0064	−0.0073	−0.0064	−0.0065	−0.0059	−0.0072	−0.0067	−0.0050
ϕ_{14}	−0.1211	−0.1106	−0.1157	−0.1128	−0.1351	−0.1240	−0.1306	−0.1253	−0.1097	−0.1256
ϕ_{15}	0.2079	0.1814	0.2142	0.2168	0.1849	0.1800	0.2052	0.2109	0.2177	0.1544
ϕ_{16}	0.0902	0.0765	0.0919	0.0946	0.0982	0.0803	0.0899	0.0883	0.0974	0.0920
ϕ_{17}	0.1017	0.1131	0.0966	0.1015	0.1024	0.1060	0.0912	0.1029	0.0905	0.1028
ϕ_{18}	−0.0539	−0.0470	−0.0406	−0.0529	−0.0571	−0.0503	−0.0510	−0.0508	−0.0500	−0.0665
ϕ_{19}	−0.0715	−0.0631	−0.0650	−0.0770	−0.0727	−0.0768	−0.0585	−0.0840	−0.0743	−0.0708
ϕ_{20}	−0.1886	−0.2084	−0.2122	−0.2063	−0.2193	−0.1926	−0.1780	−0.1997	−0.1747	−0.2118
ϕ_{21}	−0.1428	−0.1773	−0.1456	−0.1562	−0.1659	−0.1904	−0.1569	−0.1585	−0.1520	−0.1676
ϕ_{22}	−0.1088	−0.1396	−0.1148	−0.1274	−0.1394	−0.0981	−0.1533	−0.1304	−0.1246	−0.1249
ϕ_{23}	0.2818	0.2502	0.2788	0.2462	0.2560	0.1996	0.2240	0.2786	0.2886	0.2827
ϕ_{24}	−0.0687	−0.0721	−0.0754	−0.0680	−0.0567	−0.0641	−0.0763	−0.0733	−0.0901	−0.0612
ϕ_{25}	0.0651	0.0403	0.0652	0.0620	0.0592	0.0543	0.0562	0.0522	0.0480	0.0430
ϕ_{26}	−0.1443	−0.1319	−0.1352	−0.1302	−0.1463	−0.1425	−0.1456	−0.1461	−0.1169	−0.1332
ϕ_{27}	0.0880	0.0955	0.0954	0.0974	0.0897	0.0737	0.0966	0.1018	0.0952	0.0897
ϕ_{28}	0.0671	0.0651	0.0725	0.0752	0.0774	0.0668	0.0687	0.0648	0.0714	0.0806
ϕ_{29}	0.2165	0.2857	0.2868	0.2879	0.2359	0.2675	0.2422	0.2604	0.2603	0.2548
ϕ_{30}	0.0402	0.0581	0.0604	0.0488	0.0664	0.0483	0.0541	0.0510	0.0534	0.0524
θ	1.0022	1.0404	1.0047	1.0936	1.0297	1.0451	0.9924	1.0577	1.0277	1.0437
ω	−0.0624	−0.0624	−0.0693	−0.0402	−0.0761	−0.0821	−0.0474	−0.0703	−0.0299	−0.0605
α	0.0443	0.0333	0.0365	0.0431	0.0459	0.0353	0.0363	0.0370	0.0388	0.0350
α^*	0.0246	0.0316	0.0313	0.0290	0.0249	0.0294	0.0251	0.0261	0.0261	0.0316
β	0.9877	0.9877	0.9864	0.9921	0.9850	0.9838	0.9907	0.9862	0.9941	0.9881
λ_0	−5.3960	−5.3960	−5.3960	−5.3960	−5.3960	−5.3960	−5.3960	−5.3960	−5.3960	−5.3960
δ_1	1.7966	1.8858	1.7448	1.5722	1.8765	1.8886	1.8607	2.0918	1.6317	2.1115
δ_2	0.6726	0.5780	0.6446	0.7340	0.5886	0.5532	0.5895	0.6053	0.4939	0.6158

Table A12. Parameters for Monte Carlo simulation, Gen-t-DCS with dynamic ν_t, constant η_t

	Θ_{MC1}	Θ_{MC2}	Θ_{MC3}	Θ_{MC4}	Θ_{MC5}	Θ_{MC6}	Θ_{MC7}	Θ_{MC8}	Θ_{MC9}	Θ_{MC10}
c	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
ϕ_1	−0.1771	−0.1810	−0.1930	−0.1867	−0.1912	−0.1614	−0.1665	−0.1707	−0.1720	−0.1947
ϕ_2	−0.0756	−0.0816	−0.0722	−0.0755	−0.0735	−0.0685	−0.0781	−0.0763	−0.0699	−0.0706
ϕ_3	0.1527	0.1677	0.1380	0.1484	0.1587	0.1600	0.1391	0.1307	0.1277	0.1231
ϕ_4	0.1305	0.1445	0.1226	0.1249	0.1389	0.1374	0.1309	0.1464	0.1119	0.1049
ϕ_5	0.0163	0.0137	0.0130	0.0169	0.0155	0.0139	0.0161	0.0201	0.0162	0.0168
ϕ_6	−0.0576	−0.0658	−0.0610	−0.0611	−0.0658	−0.0716	−0.0658	−0.0594	−0.0573	−0.0488
ϕ_7	−0.0520	−0.0463	−0.0523	−0.0543	−0.0534	−0.0586	−0.0626	−0.0501	−0.0581	−0.0519
ϕ_8	−0.0166	−0.0178	−0.0151	−0.0173	−0.0150	−0.0165	−0.0162	−0.0194	−0.0169	−0.0154
ϕ_9	0.1939	0.2028	0.2322	0.2083	0.2072	0.1891	0.2001	0.2055	0.1747	0.1998
ϕ_{10}	0.0608	0.0641	0.0561	0.0643	0.0608	0.0632	0.0666	0.0576	0.0523	0.0553
ϕ_{11}	0.2597	0.2892	0.3098	0.2815	0.2611	0.2579	0.2669	0.1971	0.2262	0.2820
ϕ_{12}	0.1002	0.1190	0.1053	0.0811	0.1035	0.1169	0.1162	0.1038	0.0924	0.0854
ϕ_{13}	−0.0195	−0.0206	−0.0179	−0.0219	−0.0222	−0.0199	−0.0196	−0.0193	−0.0164	−0.0199
ϕ_{14}	−0.0974	−0.0828	−0.1042	−0.0982	−0.0909	−0.0906	−0.0987	−0.0987	−0.1082	−0.0937
ϕ_{15}	0.1446	0.1162	0.1506	0.1788	0.1296	0.1420	0.1376	0.1366	0.1268	0.1057
ϕ_{16}	0.0858	0.0939	0.0731	0.0830	0.0791	0.1004	0.0899	0.0889	0.0872	0.0833
ϕ_{17}	0.0988	0.1037	0.0930	0.1034	0.1084	0.0963	0.0969	0.0950	0.1001	0.1036
ϕ_{18}	−0.0539	−0.0555	−0.0536	−0.0526	−0.0563	−0.0509	−0.0565	−0.0556	−0.0519	−0.0538
ϕ_{19}	−0.0312	−0.0319	−0.0290	−0.0346	−0.0320	−0.0332	−0.0322	−0.0304	−0.0297	−0.0352
ϕ_{20}	−0.2295	−0.2528	−0.1937	−0.2331	−0.2266	−0.2606	−0.2354	−0.2617	−0.2254	−0.2464
ϕ_{21}	−0.1596	−0.1811	−0.1886	−0.1479	−0.1711	−0.1619	−0.1488	−0.1583	−0.1413	−0.1327
ϕ_{22}	−0.1278	−0.1217	−0.1279	−0.1315	−0.1238	−0.1299	−0.1441	−0.1188	−0.1195	−0.1371
ϕ_{23}	0.1957	0.2000	0.1749	0.1851	0.2007	0.1891	0.1902	0.1809	0.2094	0.1973
ϕ_{24}	0.0019	0.0016	0.0025	0.0022	0.0018	0.0018	0.0016	0.0020	0.0017	0.0019
ϕ_{25}	0.0027	0.0027	0.0027	0.0031	0.0024	0.0029	0.0031	0.0026	0.0029	0.0026
ϕ_{26}	−0.0393	−0.0328	−0.0466	−0.0418	−0.0427	−0.0337	−0.0397	−0.0418	−0.0364	−0.0417
ϕ_{27}	0.0543	0.0465	0.0542	0.0452	0.0585	0.0525	0.0570	0.0572	0.0470	0.0571
ϕ_{28}	0.0926	0.1014	0.1087	0.0987	0.0883	0.1051	0.0872	0.0983	0.1114	0.0853
ϕ_{29}	0.2645	0.2192	0.2741	0.2723	0.2667	0.2942	0.2817	0.2166	0.2488	0.2450
ϕ_{30}	0.0453	0.0474	0.0440	0.0409	0.0492	0.0481	0.0465	0.0411	0.0463	0.0445
θ	1.0007	0.9918	1.0155	0.9894	0.9508	1.0009	1.0509	0.9804	0.9239	0.9555
ω	−0.0544	−0.0605	−0.0545	−0.0773	−0.0921	−0.0661	−0.0477	−0.0634	−0.0428	−0.0618
α	0.0343	0.0345	0.0344	0.0368	0.0313	0.0364	0.0387	0.0319	0.0288	0.0307
α^*	0.0253	0.0246	0.0234	0.0279	0.0264	0.0226	0.0273	0.0241	0.0284	0.0280
β	0.9893	0.9881	0.9893	0.9848	0.9819	0.9870	0.9906	0.9875	0.9916	0.9878
λ_0	−5.3964	−5.3964	−5.3964	−5.3964	−5.3964	−5.3964	−5.3964	−5.3964	−5.3964	−5.3964
δ_1	0.7368	0.7445	0.6025	0.5749	0.7881	0.8805	0.8816	0.7519	0.5803	0.9702
γ_1	0.5847	0.5804	0.6605	0.6760	0.5558	0.5038	0.5031	0.5763	0.6730	0.4532
κ_1	0.9249	0.9582	0.8967	0.9753	0.8565	0.8475	1.1018	0.8151	0.8809	0.9928
δ_2	0.6528	0.6618	0.8063	0.7007	0.6563	0.6524	0.6769	0.6233	0.8379	0.6376

Table A12 (continued). Parameters for Monte Carlo simulation, Gen-t-DCS with dynamic ν_t, constant η_t

	Θ_{MC11}	Θ_{MC12}	Θ_{MC13}	Θ_{MC14}	Θ_{MC15}	Θ_{MC16}	Θ_{MC17}	Θ_{MC18}	Θ_{MC19}	Θ_{MC20}
c	0.0001	0.0001	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	0.0001
ϕ_1	−0.1774	−0.1735	−0.1766	−0.1776	−0.1796	−0.1781	−0.1827	−0.1379	−0.1677	−0.1793
ϕ_2	−0.0934	−0.0807	−0.0730	−0.0765	−0.0775	−0.0829	−0.0755	−0.0902	−0.0752	−0.0750
ϕ_3	0.1318	0.1486	0.1680	0.1426	0.1490	0.1501	0.1906	0.1696	0.1326	0.1615
ϕ_4	0.1196	0.1229	0.1468	0.1185	0.1332	0.1226	0.1125	0.1268	0.1597	0.1242
ϕ_5	0.0162	0.0168	0.0177	0.0148	0.0137	0.0170	0.0159	0.0168	0.0167	0.0165
ϕ_6	−0.0582	−0.0478	−0.0668	−0.0631	−0.0520	−0.0588	−0.0597	−0.0584	−0.0560	−0.0532
ϕ_7	−0.0604	−0.0468	−0.0554	−0.0387	−0.0515	−0.0482	−0.0495	−0.0385	−0.0395	−0.0521
ϕ_8	−0.0160	−0.0158	−0.0158	−0.0126	−0.0193	−0.0188	−0.0143	−0.0148	−0.0161	−0.0178
ϕ_9	0.1865	0.2088	0.2213	0.1829	0.2083	0.2150	0.1451	0.1997	0.2025	0.1825
ϕ_{10}	0.0636	0.0540	0.0632	0.0498	0.0614	0.0602	0.0689	0.0561	0.0643	0.0651
ϕ_{11}	0.2893	0.2400	0.2745	0.2594	0.2458	0.2568	0.2528	0.2797	0.3089	0.3024
ϕ_{12}	0.1035	0.1006	0.0999	0.0840	0.1005	0.1002	0.1023	0.1000	0.0965	0.0873
ϕ_{13}	−0.0221	−0.0195	−0.0186	−0.0191	−0.0206	−0.0176	−0.0195	−0.0174	−0.0204	−0.0190
ϕ_{14}	−0.0663	−0.0948	−0.0918	−0.1043	−0.0819	−0.0924	−0.0903	−0.0880	−0.1121	−0.1081
ϕ_{15}	0.1533	0.1626	0.1783	0.1323	0.1477	0.1517	0.1441	0.1396	0.1526	0.1683
ϕ_{16}	0.0881	0.0891	0.0920	0.0874	0.0865	0.0860	0.0844	0.0882	0.0860	0.0964
ϕ_{17}	0.0986	0.1070	0.1083	0.0885	0.0838	0.1111	0.1186	0.1039	0.1121	0.0971
ϕ_{18}	−0.0406	−0.0486	−0.0560	−0.0478	−0.0502	−0.0551	−0.0547	−0.0545	−0.0576	−0.0721
ϕ_{19}	−0.0354	−0.0278	−0.0322	−0.0300	−0.0280	−0.0312	−0.0336	−0.0305	−0.0291	−0.0291
ϕ_{20}	−0.2817	−0.2170	−0.2185	−0.2280	−0.1992	−0.2450	−0.2572	−0.2503	−0.2661	−0.2385
ϕ_{21}	−0.1838	−0.1585	−0.1454	−0.1541	−0.1595	−0.1698	−0.1714	−0.1658	−0.1733	−0.1800
ϕ_{22}	−0.1228	−0.1246	−0.1243	−0.1336	−0.1313	−0.1228	−0.1472	−0.1185	−0.1156	−0.1140
ϕ_{23}	0.2244	0.1869	0.1874	0.2189	0.1851	0.1946	0.2073	0.1746	0.2023	0.1610
ϕ_{24}	0.0018	0.0014	0.0022	0.0016	0.0020	0.0016	0.0015	0.0018	0.0020	0.0019
ϕ_{25}	0.0030	0.0031	0.0028	0.0025	0.0028	0.0027	0.0025	0.0028	0.0030	0.0025
ϕ_{26}	−0.0403	−0.0395	−0.0349	−0.0422	−0.0390	−0.0343	−0.0398	−0.0363	−0.0346	−0.0409
ϕ_{27}	0.0602	0.0587	0.0637	0.0546	0.0482	0.0598	0.0518	0.0447	0.0556	0.0551
ϕ_{28}	0.0968	0.0796	0.0892	0.0917	0.0904	0.1054	0.1011	0.0945	0.0935	0.0876
ϕ_{29}	0.2661	0.2610	0.2961	0.2679	0.2682	0.2704	0.2567	0.2957	0.2691	0.3009
ϕ_{30}	0.0465	0.0429	0.0446	0.0463	0.0406	0.0533	0.0404	0.0437	0.0532	0.0419
θ	1.0000	0.9726	1.0111	0.9738	0.9731	0.9604	0.9905	1.0273	1.0025	1.0297
ω	−0.0763	−0.0609	−0.0398	−0.0623	−0.0560	−0.0308	−0.0542	−0.0306	−0.0700	−0.0535
α	0.0351	0.0341	0.0355	0.0365	0.0388	0.0328	0.0344	0.0361	0.0336	0.0277
α^*	0.0262	0.0216	0.0234	0.0275	0.0280	0.0285	0.0219	0.0288	0.0234	0.0252
β	0.9850	0.9880	0.9922	0.9877	0.9890	0.9940	0.9893	0.9940	0.9862	0.9895
λ_0	−5.3964	−5.3964	−5.3964	−5.3964	−5.3964	−5.3964	−5.3964	−5.3964	−5.3964	−5.3964
δ_1	0.7128	0.8278	0.8667	0.6883	0.6849	0.6747	0.6285	0.7053	0.8937	0.7158
γ_1	0.5983	0.5334	0.5115	0.6121	0.6140	0.6197	0.6458	0.6025	0.4964	0.5966
κ_1	0.8609	0.8753	0.9387	0.9516	1.0460	0.8594	0.8077	0.8924	0.7498	0.8420
δ_2	0.6784	0.8179	0.6523	0.6624	0.6502	0.7704	0.6250	0.6716	0.5570	0.6925

Table A13. Parameters for Monte Carlo simulation, Gen-t-DCS with constant ν_t, dynamic η_t

	Θ_{MC1}	Θ_{MC2}	Θ_{MC3}	Θ_{MC4}	Θ_{MC5}	Θ_{MC6}	Θ_{MC7}	Θ_{MC8}	Θ_{MC9}	Θ_{MC10}
c	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0000	0.0001	0.0001	0.0001
ϕ_1	−0.1515	−0.1488	−0.1479	−0.1418	−0.1447	−0.1610	−0.1567	−0.1479	−0.1409	−0.1361
ϕ_2	−0.0667	−0.0744	−0.0661	−0.0746	−0.0756	−0.0725	−0.0746	−0.0640	−0.0648	−0.0661
ϕ_3	0.1484	0.1318	0.1477	0.1544	0.1668	0.1418	0.1493	0.1408	0.1114	0.1615
ϕ_4	0.0797	0.0832	0.0875	0.0673	0.0902	0.0743	0.0812	0.0828	0.0826	0.0737
ϕ_5	−0.0079	−0.0077	−0.0064	−0.0084	−0.0082	−0.0087	−0.0081	−0.0077	−0.0075	−0.0079
ϕ_6	−0.0792	−0.0767	−0.0926	−0.0829	−0.0794	−0.0784	−0.0737	−0.0795	−0.0778	−0.0853
ϕ_7	−0.0339	−0.0351	−0.0361	−0.0295	−0.0386	−0.0388	−0.0364	−0.0352	−0.0332	−0.0355
ϕ_8	0.0025	0.0024	0.0025	0.0027	0.0021	0.0022	0.0023	0.0021	0.0027	0.0025
ϕ_9	0.2278	0.2468	0.2325	0.2087	0.1998	0.2148	0.2508	0.2129	0.2042	0.2224
ϕ_{10}	0.0810	0.0803	0.0817	0.0771	0.0739	0.0737	0.0711	0.0728	0.0808	0.0749
ϕ_{11}	0.2784	0.2769	0.3043	0.2615	0.3193	0.2965	0.2760	0.2850	0.2877	0.2677
ϕ_{12}	0.0860	0.0838	0.0910	0.0829	0.0927	0.0970	0.0799	0.0749	0.0679	0.1010
ϕ_{13}	0.0153	0.0149	0.0144	0.0172	0.0150	0.0166	0.0143	0.0143	0.0164	0.0162
ϕ_{14}	−0.0893	−0.0809	−0.0902	−0.0812	−0.0979	−0.0918	−0.0830	−0.0912	−0.0826	−0.0916
ϕ_{15}	0.1800	0.1906	0.1940	0.1940	0.1610	0.1871	0.1793	0.1871	0.1620	0.2087
ϕ_{16}	0.0898	0.0965	0.0761	0.0868	0.0986	0.0784	0.1020	0.0982	0.0716	0.0874
ϕ_{17}	0.0828	0.0826	0.0731	0.0859	0.1007	0.0766	0.0961	0.0750	0.0798	0.0937
ϕ_{18}	−0.0427	−0.0409	−0.0490	−0.0428	−0.0407	−0.0377	−0.0495	−0.0433	−0.0419	−0.0431
ϕ_{19}	−0.0458	−0.0465	−0.0448	−0.0458	−0.0416	−0.0374	−0.0509	−0.0430	−0.0411	−0.0551
ϕ_{20}	−0.1821	−0.1709	−0.1713	−0.1714	−0.1826	−0.1776	−0.1427	−0.1747	−0.2241	−0.1909
ϕ_{21}	−0.1352	−0.1468	−0.1434	−0.1434	−0.1329	−0.1348	−0.1239	−0.1223	−0.1530	−0.1293
ϕ_{22}	−0.1221	−0.1472	−0.1214	−0.1208	−0.1014	−0.1380	−0.1330	−0.1295	−0.1184	−0.1302
ϕ_{23}	0.2068	0.1777	0.2040	0.2090	0.2049	0.2082	0.2196	0.2174	0.2038	0.2139
ϕ_{24}	−0.0691	−0.0746	−0.0508	−0.0653	−0.0647	−0.0732	−0.0730	−0.0705	−0.0618	−0.0646
ϕ_{25}	−0.0151	−0.0141	−0.0130	−0.0147	−0.0150	−0.0174	−0.0151	−0.0156	−0.0164	−0.0161
ϕ_{26}	−0.1057	−0.1039	−0.1061	−0.1064	−0.1100	−0.1118	−0.1163	−0.1233	−0.1224	−0.1024
ϕ_{27}	0.0601	0.0605	0.0548	0.0682	0.0588	0.0657	0.0540	0.0601	0.0649	0.0549
ϕ_{28}	0.0754	0.0826	0.0625	0.0905	0.0830	0.0710	0.0802	0.0779	0.0884	0.0759
ϕ_{29}	0.2573	0.2444	0.2168	0.2690	0.2403	0.2835	0.2844	0.2807	0.2408	0.2544
ϕ_{30}	0.0455	0.0471	0.0496	0.0466	0.0508	0.0420	0.0407	0.0477	0.0506	0.0482
θ	1.1236	1.0643	1.1210	1.1648	1.1447	1.1376	1.1432	1.1011	1.1309	1.1058
ω	−0.0522	−0.0913	−0.0424	−0.0073	−0.0388	−0.0469	−0.0619	−0.0711	−0.0583	−0.0296
α	0.0336	0.0326	0.0339	0.0335	0.0319	0.0376	0.0348	0.0347	0.0281	0.0348
α^*	0.0252	0.0260	0.0258	0.0243	0.0268	0.0297	0.0242	0.0236	0.0273	0.0220
β	0.9897	0.9821	0.9917	0.9986	0.9924	0.9908	0.9878	0.9860	0.9885	0.9942
λ_0	−5.3904	−5.3904	−5.3904	−5.3904	−5.3904	−5.3904	−5.3904	−5.3904	−5.3904	−5.3904
δ_1	1.8813	1.7421	1.9430	2.4187	2.1184	1.7932	2.0490	1.7268	1.8689	1.7343
δ_2	0.2564	0.2599	0.2540	0.2999	0.2506	0.2614	0.2283	0.2382	0.2742	0.2591
γ_2	0.5889	0.5834	0.5928	0.5192	0.5983	0.5809	0.6340	0.6182	0.5605	0.5846
κ_2	0.0945	0.0996	0.1008	0.0909	0.0878	0.0915	0.0940	0.0954	0.1026	0.1074

Table A13 (continued). Parameters for Monte Carlo simulation, Gen-t-DCS with constant ν_t, dynamic η_t

	Θ_{MC11}	Θ_{MC12}	Θ_{MC13}	Θ_{MC14}	Θ_{MC15}	Θ_{MC16}	Θ_{MC17}	Θ_{MC18}	Θ_{MC19}	Θ_{MC20}
c	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
ϕ_1	−0.1517	−0.1377	−0.1615	−0.1559	−0.1299	−0.1521	−0.1536	−0.1603	−0.1468	−0.1721
ϕ_2	−0.0784	−0.0629	−0.0726	−0.0551	−0.0665	−0.0718	−0.0622	−0.0636	−0.0811	−0.0595
ϕ_3	0.1497	0.1572	0.1345	0.1388	0.1586	0.1511	0.1294	0.1315	0.1295	0.1565
ϕ_4	0.0804	0.0770	0.0759	0.0737	0.0801	0.0664	0.0810	0.0777	0.0672	0.0859
ϕ_5	−0.0079	−0.0080	−0.0087	−0.0079	−0.0076	−0.0085	−0.0065	−0.0076	−0.0082	−0.0078
ϕ_6	−0.0744	−0.0690	−0.0825	−0.0744	−0.0855	−0.0743	−0.0708	−0.0772	−0.0918	−0.0922
ϕ_7	−0.0353	−0.0289	−0.0352	−0.0331	−0.0361	−0.0315	−0.0315	−0.0326	−0.0348	−0.0359
ϕ_8	0.0027	0.0026	0.0020	0.0023	0.0024	0.0027	0.0026	0.0027	0.0021	0.0024
ϕ_9	0.2196	0.2221	0.2601	0.2158	0.2061	0.2339	0.2396	0.2443	0.2609	0.2244
ϕ_{10}	0.0845	0.0750	0.0823	0.0780	0.0858	0.0729	0.0862	0.0905	0.0781	0.0949
ϕ_{11}	0.2530	0.3224	0.3040	0.2766	0.3017	0.2987	0.2258	0.2723	0.2426	0.2896
ϕ_{12}	0.0719	0.0752	0.1047	0.0963	0.0864	0.0911	0.0976	0.0825	0.0935	0.0830
ϕ_{13}	0.0119	0.0141	0.0164	0.0173	0.0168	0.0150	0.0138	0.0189	0.0154	0.0140
ϕ_{14}	−0.0882	−0.0914	−0.0991	−0.0929	−0.0890	−0.0966	−0.1011	−0.0947	−0.0927	−0.0850
ϕ_{15}	0.1852	0.1421	0.1798	0.2030	0.1925	0.1855	0.1878	0.1979	0.1892	0.1887
ϕ_{16}	0.0820	0.1007	0.0832	0.0819	0.0901	0.0963	0.0869	0.0956	0.0889	0.0928
ϕ_{17}	0.0903	0.0844	0.0836	0.0834	0.0750	0.0692	0.0805	0.1033	0.0816	0.0794
ϕ_{18}	−0.0473	−0.0497	−0.0436	−0.0449	−0.0388	−0.0366	−0.0384	−0.0428	−0.0479	−0.0443
ϕ_{19}	−0.0437	−0.0458	−0.0469	−0.0462	−0.0365	−0.0423	−0.0486	−0.0387	−0.0496	−0.0469
ϕ_{20}	−0.1672	−0.1749	−0.2083	−0.2228	−0.2056	−0.1578	−0.2018	−0.1741	−0.1798	−0.1920
ϕ_{21}	−0.1259	−0.1121	−0.1116	−0.1433	−0.1271	−0.1388	−0.1246	−0.1649	−0.1176	−0.1319
ϕ_{22}	−0.1242	−0.1403	−0.1069	−0.0980	−0.1353	−0.1268	−0.1179	−0.1460	−0.1393	−0.1046
ϕ_{23}	0.1918	0.1715	0.2138	0.1915	0.2004	0.2075	0.2208	0.2055	0.1940	0.1832
ϕ_{24}	−0.0718	−0.0826	−0.0593	−0.0775	−0.0774	−0.0662	−0.0658	−0.0813	−0.0758	−0.0647
ϕ_{25}	−0.0164	−0.0165	−0.0157	−0.0144	−0.0126	−0.0150	−0.0176	−0.0149	−0.0160	−0.0126
ϕ_{26}	−0.1129	−0.0941	−0.1050	−0.0950	−0.0966	−0.1066	−0.0970	−0.1149	−0.1156	−0.1052
ϕ_{27}	0.0684	0.0615	0.0643	0.0674	0.0568	0.0611	0.0540	0.0498	0.0660	0.0651
ϕ_{28}	0.0796	0.0778	0.0756	0.0671	0.0647	0.0827	0.0776	0.0798	0.0740	0.0778
ϕ_{29}	0.2715	0.2166	0.2522	0.2166	0.2252	0.2620	0.2497	0.2457	0.2132	0.2422
ϕ_{30}	0.0476	0.0468	0.0442	0.0499	0.0436	0.0430	0.0485	0.0487	0.0456	0.0439
θ	1.1434	1.1416	1.1740	1.1352	1.1151	1.0949	1.1338	1.1165	1.1208	1.0883
ω	−0.0903	−0.0998	−0.0405	−0.0685	−0.0601	−0.0442	−0.0449	−0.0471	−0.0183	−0.0470
α	0.0328	0.0339	0.0355	0.0380	0.0361	0.0323	0.0319	0.0332	0.0349	0.0281
α^*	0.0224	0.0231	0.0263	0.0240	0.0234	0.0269	0.0252	0.0244	0.0223	0.0239
β	0.9822	0.9804	0.9920	0.9865	0.9882	0.9913	0.9912	0.9907	0.9964	0.9908
λ_0	−5.3904	−5.3904	−5.3904	−5.3904	−5.3904	−5.3904	−5.3904	−5.3904	−5.3904	−5.3904
δ_1	1.8480	1.4580	1.6466	2.2216	1.7666	1.7761	1.8551	1.8209	1.9029	1.7994
δ_2	0.2782	0.2900	0.2624	0.2630	0.2588	0.2460	0.1556	0.3059	0.2881	0.2179
γ_2	0.5540	0.5351	0.5794	0.5784	0.5851	0.6057	0.7506	0.5096	0.5382	0.6506
κ_2	0.0836	0.0865	0.1006	0.0720	0.0761	0.0777	0.0887	0.0975	0.0849	0.0939

Table A14. Parameters for Monte Carlo simulation, Skew-Gen-t-DCS with constant τ_t, ν_t, η_t

	Θ_{MC1}	Θ_{MC2}	Θ_{MC3}	Θ_{MC4}	Θ_{MC5}	Θ_{MC6}	Θ_{MC7}	Θ_{MC8}	Θ_{MC9}	Θ_{MC10}
c	0.0001	0.0001	0.0001	0.0002	0.0002	0.0003	0.0001	0.0002	0.0001	0.0001
ϕ_1	−0.1855	−0.1910	−0.2293	−0.1707	−0.2337	−0.1912	−0.1630	−0.1850	−0.2020	−0.1732
ϕ_2	−0.1309	−0.1044	−0.1321	−0.1568	−0.1161	−0.1529	−0.1143	−0.1149	−0.1517	−0.1411
ϕ_3	0.1846	0.1939	0.1728	0.1400	0.1929	0.1707	0.1645	0.1994	0.1640	0.1847
ϕ_4	0.1079	0.1175	0.1021	0.1230	0.1188	0.1003	0.1019	0.0856	0.1115	0.1095
ϕ_5	0.0670	0.0591	0.0574	0.0690	0.0717	0.0762	0.0519	0.0774	0.0784	0.0763
ϕ_6	−0.0600	−0.0686	−0.0605	−0.0634	−0.0620	−0.0614	−0.0584	−0.0552	−0.0601	−0.0660
ϕ_7	−0.0098	−0.0089	−0.0094	−0.0099	−0.0092	−0.0090	−0.0103	−0.0110	−0.0089	−0.0085
ϕ_8	−0.0166	−0.0159	−0.0154	−0.0192	−0.0172	−0.0193	−0.0144	−0.0175	−0.0162	−0.0178
ϕ_9	0.2107	0.2120	0.1861	0.1685	0.2073	0.2056	0.2166	0.2055	0.2063	0.2020
ϕ_{10}	0.0406	0.0423	0.0364	0.0397	0.0452	0.0406	0.0451	0.0388	0.0390	0.0426
ϕ_{11}	0.2840	0.2851	0.3287	0.3003	0.2798	0.2166	0.3199	0.2397	0.2926	0.2409
ϕ_{12}	0.0735	0.0682	0.0715	0.0777	0.0810	0.0716	0.0696	0.0767	0.0670	0.0653
ϕ_{13}	0.0416	0.0419	0.0363	0.0398	0.0446	0.0401	0.0372	0.0479	0.0407	0.0388
ϕ_{14}	−0.1000	−0.0939	−0.0951	−0.0945	−0.1040	−0.1040	−0.0961	−0.1122	−0.0836	−0.0854
ϕ_{15}	0.2171	0.1981	0.2112	0.2227	0.1892	0.2083	0.2539	0.2158	0.2249	0.2351
ϕ_{16}	0.1114	0.1140	0.1159	0.1304	0.1047	0.0915	0.0997	0.1074	0.1092	0.1227
ϕ_{17}	0.0971	0.0873	0.1111	0.0953	0.0995	0.0981	0.1195	0.0993	0.0973	0.0939
ϕ_{18}	−0.0593	−0.0559	−0.0631	−0.0680	−0.0625	−0.0631	−0.0566	−0.0530	−0.0605	−0.0543
ϕ_{19}	−0.0985	−0.1068	−0.0948	−0.0861	−0.1151	−0.1022	−0.0903	−0.1079	−0.0914	−0.1032
ϕ_{20}	−0.2376	−0.2627	−0.2129	−0.2432	−0.2832	−0.2268	−0.2090	−0.2477	−0.1964	−0.2456
ϕ_{21}	−0.1855	−0.2083	−0.1518	−0.1783	−0.2188	−0.2024	−0.1911	−0.1838	−0.1829	−0.1775
ϕ_{22}	−0.1596	−0.1471	−0.1521	−0.1481	−0.1659	−0.1741	−0.1679	−0.1420	−0.1815	−0.1592
ϕ_{23}	0.2448	0.2684	0.2509	0.2365	0.2625	0.2123	0.2476	0.2256	0.2532	0.2381
ϕ_{24}	−0.0518	−0.0539	−0.0439	−0.0585	−0.0480	−0.0592	−0.0539	−0.0552	−0.0545	−0.0524
ϕ_{25}	0.0913	0.0993	0.0798	0.0747	0.0765	0.0845	0.1021	0.0908	0.1023	0.0910
ϕ_{26}	−0.1133	−0.1184	−0.1317	−0.0957	−0.1017	−0.1098	−0.1240	−0.1052	−0.1227	−0.0964
ϕ_{27}	0.0853	0.0843	0.0822	0.0854	0.0781	0.0775	0.0777	0.0958	0.0897	0.0905
ϕ_{28}	0.0393	0.0395	0.0402	0.0463	0.0442	0.0396	0.0367	0.0421	0.0380	0.0450
ϕ_{29}	0.2770	0.3182	0.3189	0.2583	0.3414	0.2671	0.2767	0.2667	0.2708	0.3080
ϕ_{30}	0.0497	0.0532	0.0538	0.0488	0.0469	0.0467	0.0472	0.0457	0.0476	0.0411
θ	0.9903	1.0030	0.9842	0.9966	1.0293	1.0122	0.9410	1.0063	0.9722	0.9934
ω	−0.0571	−0.0077	−0.0270	−0.0664	−0.0420	−0.0872	−0.0828	−0.0930	−0.0483	−0.0228
α	0.0379	0.0359	0.0417	0.0412	0.0414	0.0288	0.0356	0.0378	0.0371	0.0336
α^*	0.0274	0.0292	0.0259	0.0213	0.0269	0.0315	0.0305	0.0231	0.0284	0.0252
β	0.9888	0.9985	0.9947	0.9870	0.9918	0.9830	0.9838	0.9818	0.9906	0.9955
λ_0	−5.3697	−5.3697	−5.3697	−5.3697	−5.3697	−5.3697	−5.3697	−5.3697	−5.3697	−5.3697
δ_1	−0.0479	−0.0509	−0.0451	−0.0498	−0.0498	−0.0499	−0.0479	−0.0491	−0.0478	−0.0474
δ_2	1.8888	1.8408	1.8426	1.8741	1.7391	1.8317	1.8141	1.8395	1.7077	1.8564
δ_3	0.6216	0.6596	0.5896	0.6395	0.6277	0.6295	0.6428	0.6522	0.6393	0.6013

Table A14 (continued). Parameters for Monte Carlo simulation, Skew-Gen-t-DCS with constant τ_t, ν_t, η_t

	Θ_{MC11}	Θ_{MC12}	Θ_{MC13}	Θ_{MC14}	Θ_{MC15}	Θ_{MC16}	Θ_{MC17}	Θ_{MC18}	Θ_{MC19}	Θ_{MC20}
c	0.0002	0.0001	0.0000	0.0001	0.0000	0.0001	0.0001	0.0002	0.0001	0.0001
ϕ_1	−0.1773	−0.1954	−0.1755	−0.1515	−0.1803	−0.1786	−0.1611	−0.1708	−0.1592	−0.1664
ϕ_2	−0.1206	−0.1465	−0.1343	−0.1408	−0.1401	−0.1278	−0.1316	−0.1429	−0.1255	−0.1330
ϕ_3	0.1907	0.2194	0.1741	0.1751	0.2082	0.1790	0.1656	0.2199	0.1840	0.1660
ϕ_4	0.0884	0.1090	0.1126	0.0956	0.1098	0.1122	0.1126	0.1004	0.1358	0.1137
ϕ_5	0.0698	0.0643	0.0648	0.0601	0.0731	0.0621	0.0676	0.0710	0.0645	0.0615
ϕ_6	−0.0567	−0.0635	−0.0641	−0.0597	−0.0622	−0.0681	−0.0566	−0.0654	−0.0677	−0.0548
ϕ_7	−0.0091	−0.0106	−0.0103	−0.0101	−0.0119	−0.0099	−0.0098	−0.0085	−0.0106	−0.0104
ϕ_8	−0.0145	−0.0186	−0.0181	−0.0156	−0.0167	−0.0142	−0.0153	−0.0144	−0.0183	−0.0159
ϕ_9	0.2329	0.2191	0.2378	0.2320	0.2163	0.2064	0.2207	0.2106	0.1927	0.2297
ϕ_{10}	0.0394	0.0369	0.0345	0.0452	0.0428	0.0431	0.0394	0.0416	0.0381	0.0383
ϕ_{11}	0.2924	0.2754	0.3262	0.3407	0.3072	0.2863	0.2303	0.2849	0.2751	0.2884
ϕ_{12}	0.0773	0.0727	0.0861	0.0684	0.0769	0.0821	0.0784	0.0688	0.0712	0.0754
ϕ_{13}	0.0461	0.0453	0.0331	0.0476	0.0457	0.0419	0.0431	0.0420	0.0449	0.0386
ϕ_{14}	−0.1037	−0.0882	−0.1062	−0.1033	−0.1039	−0.0883	−0.0945	−0.1224	−0.1112	−0.1255
ϕ_{15}	0.1853	0.1956	0.2423	0.1981	0.2020	0.1998	0.2256	0.2300	0.1886	0.1847
ϕ_{16}	0.0908	0.1181	0.1087	0.1032	0.1211	0.1036	0.1006	0.1130	0.1115	0.0996
ϕ_{17}	0.0972	0.1033	0.0999	0.0906	0.0942	0.0975	0.1035	0.0844	0.0832	0.0891
ϕ_{18}	−0.0654	−0.0533	−0.0568	−0.0612	−0.0529	−0.0533	−0.0482	−0.0566	−0.0627	−0.0583
ϕ_{19}	−0.0825	−0.0937	−0.0774	−0.1043	−0.1042	−0.1078	−0.1025	−0.0920	−0.1086	−0.1156
ϕ_{20}	−0.2550	−0.2230	−0.2482	−0.2475	−0.2417	−0.2561	−0.2146	−0.2318	−0.2372	−0.2234
ϕ_{21}	−0.2017	−0.1931	−0.1853	−0.1878	−0.1968	−0.1829	−0.1841	−0.2140	−0.1803	−0.1859
ϕ_{22}	−0.1673	−0.1446	−0.1689	−0.1581	−0.1508	−0.1723	−0.1638	−0.1465	−0.1056	−0.1601
ϕ_{23}	0.2235	0.2173	0.2881	0.2691	0.2666	0.2550	0.2365	0.1871	0.2872	0.2828
ϕ_{24}	−0.0434	−0.0597	−0.0428	−0.0567	−0.0483	−0.0453	−0.0577	−0.0563	−0.0582	−0.0460
ϕ_{25}	0.0754	0.0848	0.0805	0.0914	0.1133	0.1082	0.0781	0.0836	0.1023	0.0941
ϕ_{26}	−0.1033	−0.1292	−0.1073	−0.1062	−0.1100	−0.1153	−0.1109	−0.1004	−0.1196	−0.1243
ϕ_{27}	0.0880	0.0884	0.0938	0.0868	0.0814	0.0693	0.0869	0.0762	0.0908	0.0906
ϕ_{28}	0.0324	0.0432	0.0375	0.0382	0.0377	0.0337	0.0431	0.0359	0.0347	0.0394
ϕ_{29}	0.2938	0.2957	0.3001	0.2998	0.3175	0.2773	0.3208	0.2808	0.2677	0.2764
ϕ_{30}	0.0501	0.0543	0.0548	0.0424	0.0442	0.0529	0.0550	0.0535	0.0489	0.0442
θ	0.9856	0.9719	1.0042	1.0055	0.9875	0.9880	0.9780	0.9985	0.9721	0.9618
ω	−0.0797	−0.0634	−0.0539	−0.0487	−0.0617	−0.0321	−0.0714	−0.0524	−0.0442	−0.0381
α	0.0386	0.0345	0.0315	0.0423	0.0349	0.0474	0.0399	0.0453	0.0378	0.0326
α^*	0.0257	0.0270	0.0321	0.0258	0.0246	0.0263	0.0283	0.0285	0.0249	0.0259
β	0.9844	0.9876	0.9895	0.9905	0.9880	0.9937	0.9860	0.9898	0.9914	0.9926
λ_0	−5.3697	−5.3697	−5.3697	−5.3697	−5.3697	−5.3697	−5.3697	−5.3697	−5.3697	−5.3697
δ_1	−0.0539	−0.0486	−0.0481	−0.0490	−0.0452	−0.0462	−0.0499	−0.0488	−0.0431	−0.0473
δ_2	1.8611	1.7721	1.9438	1.8627	1.8570	1.9964	1.9969	1.9689	1.9806	1.8448
δ_3	0.6302	0.6236	0.6157	0.6317	0.6213	0.6338	0.6304	0.6376	0.6491	0.6285

Table A15. Parameters for Monte Carlo simulation, Skew-Gen-t-DCS with constant τ_t, η_t, dynamic ν_t

	Θ_{MC1}	Θ_{MC2}	Θ_{MC3}	Θ_{MC4}	Θ_{MC5}	Θ_{MC6}	Θ_{MC7}	Θ_{MC8}	Θ_{MC9}	Θ_{MC10}
c	0.0001	0.0001	0.0001	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
ϕ_1	−0.3218	−0.3152	−0.3600	−0.3239	−0.2969	−0.2832	−0.3008	−0.3597	−0.2626	−0.2962
ϕ_2	−0.2473	−0.2427	−0.2439	−0.2585	−0.2547	−0.2308	−0.2365	−0.2320	−0.2349	−0.2547
ϕ_3	0.2149	0.1993	0.2086	0.2028	0.2153	0.2163	0.2251	0.2196	0.2170	0.2022
ϕ_4	0.2785	0.2968	0.2967	0.2742	0.2886	0.2723	0.2856	0.2752	0.2733	0.2834
ϕ_5	0.2538	0.2421	0.2728	0.2475	0.2340	0.2671	0.2469	0.2582	0.2592	0.2671
ϕ_6	0.0678	0.0680	0.0665	0.0709	0.0664	0.0657	0.0648	0.0701	0.0640	0.0666
ϕ_7	−0.0176	−0.0178	−0.0174	−0.0167	−0.0187	−0.0180	−0.0168	−0.0183	−0.0175	−0.0176
ϕ_8	−0.0675	−0.0698	−0.0717	−0.0679	−0.0711	−0.0639	−0.0665	−0.0667	−0.0704	−0.0721
ϕ_9	0.1424	0.1343	0.1401	0.1436	0.1468	0.1344	0.1389	0.1626	0.1411	0.1339
ϕ_{10}	−0.0094	−0.0101	−0.0091	−0.0096	−0.0089	−0.0092	−0.0091	−0.0098	−0.0091	−0.0096
ϕ_{11}	0.2814	0.2854	0.2623	0.2500	0.2654	0.2751	0.2669	0.2920	0.2800	0.2991
ϕ_{12}	0.1118	0.1157	0.1006	0.1099	0.1143	0.1148	0.1123	0.1234	0.1102	0.1166
ϕ_{13}	0.0928	0.0922	0.0904	0.0938	0.0969	0.0860	0.1016	0.0901	0.0956	0.0934
ϕ_{14}	−0.0835	−0.0844	−0.0828	−0.0824	−0.0828	−0.0861	−0.0924	−0.0885	−0.0853	−0.0737
ϕ_{15}	0.2161	0.2153	0.2173	0.2122	0.2185	0.2168	0.2068	0.2237	0.2284	0.2266
ϕ_{16}	0.1943	0.1985	0.1908	0.1926	0.2007	0.1908	0.1859	0.1972	0.1912	0.1808
ϕ_{17}	0.2089	0.2245	0.1932	0.1980	0.2028	0.2202	0.2004	0.2016	0.2071	0.2211
ϕ_{18}	−0.0213	−0.0232	−0.0229	−0.0208	−0.0220	−0.0211	−0.0225	−0.0201	−0.0204	−0.0210
ϕ_{19}	−0.1606	−0.1408	−0.1684	−0.1710	−0.1642	−0.1677	−0.1621	−0.1498	−0.1515	−0.1718
ϕ_{20}	−0.3821	−0.3601	−0.3729	−0.3617	−0.3990	−0.3993	−0.3804	−0.3858	−0.3659	−0.4030
ϕ_{21}	−0.3491	−0.3539	−0.3370	−0.3623	−0.3289	−0.3973	−0.3217	−0.3535	−0.3480	−0.3456
ϕ_{22}	−0.2706	−0.2788	−0.2563	−0.2748	−0.2533	−0.2406	−0.2629	−0.2802	−0.2908	−0.2779
ϕ_{23}	0.2499	0.2526	0.2674	0.2360	0.2425	0.2691	0.2673	0.2443	0.2638	0.2468
ϕ_{24}	0.1188	0.1182	0.1211	0.1242	0.1154	0.1182	0.1172	0.1167	0.1174	0.1267
ϕ_{25}	0.3093	0.3155	0.3345	0.2937	0.3378	0.3273	0.2908	0.3094	0.2824	0.2940
ϕ_{26}	−0.0118	−0.0121	−0.0115	−0.0110	−0.0120	−0.0124	−0.0111	−0.0119	−0.0119	−0.0114
ϕ_{27}	0.0170	0.0184	0.0175	0.0167	0.0169	0.0164	0.0182	0.0173	0.0165	0.0173
ϕ_{28}	−0.1363	−0.1250	−0.1340	−0.1423	−0.1348	−0.1452	−0.1359	−0.1193	−0.1334	−0.1336
ϕ_{29}	0.1367	0.1329	0.1353	0.1311	0.1378	0.1322	0.1467	0.1374	0.1369	0.1392
ϕ_{30}	0.0234	0.0255	0.0238	0.0234	0.0239	0.0206	0.0219	0.0235	0.0253	0.0234
θ	0.8374	0.8180	0.8727	0.8123	0.8309	0.8443	0.8225	0.8343	0.8254	0.8136
ω	−0.0487	−0.0196	−0.0503	−0.0705	−0.0525	−0.0414	−0.0663	−0.0518	−0.0210	−0.0345
α	0.0351	0.0394	0.0331	0.0375	0.0313	0.0390	0.0302	0.0304	0.0358	0.0378
α^*	0.0262	0.0226	0.0242	0.0252	0.0249	0.0260	0.0259	0.0275	0.0230	0.0197
β	0.9905	0.9962	0.9902	0.9863	0.9898	0.9919	0.9871	0.9899	0.9959	0.9933
λ_0	−5.4050	−5.4050	−5.4050	−5.4050	−5.4050	−5.4050	−5.4050	−5.4050	−5.4050	−5.4050
δ_1	−0.0492	−0.0499	−0.0452	−0.0452	−0.0477	−0.0493	−0.0512	−0.0526	−0.0538	−0.0527
δ_2	0.7735	0.7596	0.7468	0.7437	0.7751	0.7896	0.7194	0.7397	0.7834	0.7414
γ_2	0.5790	0.5865	0.5935	0.5952	0.5781	0.5702	0.6084	0.5973	0.5736	0.5964
κ_2	1.0313	1.0149	0.9703	1.0179	1.0423	0.9826	1.1231	1.0358	0.9958	1.0630
δ_3	0.6360	0.6545	0.5973	0.5820	0.6427	0.5584	0.5944	0.6404	0.5903	0.5599

Table A15 (continued). Parameters for Monte Carlo simulation, Skew-Gen-t-DCS with constant τ_t, η_t, dynamic ν_t

	Θ_{MC11}	Θ_{MC12}	Θ_{MC13}	Θ_{MC14}	Θ_{MC15}	Θ_{MC16}	Θ_{MC17}	Θ_{MC18}	Θ_{MC19}	Θ_{MC20}
c	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0002	0.0001	0.0001	0.0001
ϕ_1	−0.3732	−0.3525	−0.2879	−0.3704	−0.3387	−0.2468	−0.3510	−0.3210	−0.2779	−0.3411
ϕ_2	−0.2580	−0.2532	−0.2624	−0.2585	−0.2375	−0.2501	−0.2436	−0.2351	−0.2452	−0.2277
ϕ_3	0.2243	0.2143	0.2108	0.2129	0.1962	0.2015	0.1973	0.2066	0.2043	0.2182
ϕ_4	0.2857	0.2794	0.2795	0.2986	0.2774	0.2851	0.2612	0.2769	0.2580	0.2877
ϕ_5	0.2520	0.2569	0.2423	0.2715	0.2506	0.2552	0.2510	0.2485	0.2384	0.2445
ϕ_6	0.0694	0.0643	0.0719	0.0715	0.0605	0.0669	0.0631	0.0673	0.0730	0.0700
ϕ_7	−0.0186	−0.0179	−0.0186	−0.0188	−0.0176	−0.0170	−0.0179	−0.0182	−0.0177	−0.0198
ϕ_8	−0.0727	−0.0663	−0.0713	−0.0643	−0.0670	−0.0648	−0.0668	−0.0654	−0.0689	−0.0699
ϕ_9	0.1568	0.1378	0.1310	0.1519	0.1346	0.1398	0.1485	0.1501	0.1500	0.1408
ϕ_{10}	−0.0102	−0.0100	−0.0091	−0.0094	−0.0093	−0.0092	−0.0093	−0.0099	−0.0090	−0.0090
ϕ_{11}	0.2862	0.2768	0.2792	0.2995	0.3125	0.2744	0.2869	0.2870	0.2902	0.2818
ϕ_{12}	0.1093	0.1126	0.1113	0.1143	0.1058	0.1117	0.1176	0.1143	0.1079	0.1150
ϕ_{13}	0.0970	0.1005	0.0920	0.0918	0.0993	0.0941	0.0894	0.0941	0.0940	0.0997
ϕ_{14}	−0.0757	−0.0833	−0.0830	−0.0861	−0.0868	−0.0784	−0.0878	−0.0804	−0.0842	−0.0798
ϕ_{15}	0.2306	0.2124	0.2191	0.2363	0.2188	0.2231	0.2201	0.2117	0.2125	0.2101
ϕ_{16}	0.1979	0.2044	0.1898	0.1892	0.1898	0.1909	0.2092	0.2000	0.1985	0.1811
ϕ_{17}	0.2268	0.2310	0.1761	0.2186	0.2249	0.2088	0.2089	0.2194	0.2196	0.1964
ϕ_{18}	−0.0228	−0.0221	−0.0234	−0.0214	−0.0199	−0.0209	−0.0197	−0.0211	−0.0213	−0.0232
ϕ_{19}	−0.1587	−0.1603	−0.1581	−0.1541	−0.1601	−0.1545	−0.1633	−0.1431	−0.1663	−0.1636
ϕ_{20}	−0.3686	−0.3834	−0.3800	−0.4168	−0.3928	−0.3772	−0.4014	−0.3956	−0.3706	−0.3547
ϕ_{21}	−0.3092	−0.3589	−0.3342	−0.3761	−0.3691	−0.3476	−0.3181	−0.3671	−0.3333	−0.3454
ϕ_{22}	−0.2679	−0.2607	−0.2706	−0.2572	−0.2771	−0.2688	−0.2932	−0.2491	−0.2724	−0.2783
ϕ_{23}	0.2459	0.2580	0.2349	0.2433	0.2475	0.2472	0.2480	0.2585	0.2483	0.2556
ϕ_{24}	0.1187	0.1147	0.1171	0.1094	0.1199	0.1135	0.1181	0.1185	0.1108	0.1251
ϕ_{25}	0.3060	0.2981	0.3104	0.2920	0.3187	0.2906	0.2996	0.2878	0.3258	0.3087
ϕ_{26}	−0.0120	−0.0115	−0.0120	−0.0125	−0.0113	−0.0127	−0.0113	−0.0110	−0.0119	−0.0122
ϕ_{27}	0.0171	0.0169	0.0179	0.0167	0.0173	0.0172	0.0154	0.0159	0.0180	0.0172
ϕ_{28}	−0.1382	−0.1324	−0.1308	−0.1280	−0.1299	−0.1315	−0.1409	−0.1414	−0.1181	−0.1260
ϕ_{29}	0.1345	0.1393	0.1338	0.1337	0.1418	0.1449	0.1343	0.1459	0.1310	0.1351
ϕ_{30}	0.0246	0.0237	0.0193	0.0236	0.0235	0.0214	0.0228	0.0216	0.0254	0.0249
θ	0.8227	0.8429	0.8081	0.8315	0.8409	0.8339	0.8533	0.7889	0.8211	0.8512
ω	−0.0596	−0.0765	−0.0467	−0.0403	−0.0519	−0.0618	−0.0551	−0.0600	−0.0351	−0.0186
α	0.0357	0.0307	0.0361	0.0384	0.0381	0.0346	0.0384	0.0335	0.0362	0.0396
α^*	0.0251	0.0273	0.0210	0.0254	0.0242	0.0270	0.0254	0.0277	0.0248	0.0304
β	0.9884	0.9851	0.9909	0.9922	0.9899	0.9880	0.9893	0.9883	0.9932	0.9964
λ_0	−5.4050	−5.4050	−5.4050	−5.4050	−5.4050	−5.4050	−5.4050	−5.4050	−5.4050	−5.4050
δ_1	−0.0535	−0.0467	−0.0524	−0.0480	−0.0442	−0.0501	−0.0489	−0.0483	−0.0496	−0.0496
δ_2	0.7594	0.7189	0.7541	0.7569	0.7568	0.8255	0.6950	0.8154	0.7561	0.8165
γ_2	0.5866	0.6087	0.5895	0.5880	0.5880	0.5506	0.6217	0.5562	0.5884	0.5555
κ_2	0.9901	1.0942	1.0216	1.0011	1.0691	1.0546	0.9735	1.1102	0.8939	1.0971
δ_3	0.7114	0.6011	0.5643	0.6210	0.6156	0.6853	0.5917	0.6472	0.6331	0.6135

Table A16. Parameters for Monte Carlo simulation, Skew-Gen-t-DCS with constant τ_t, ν_t, dynamic η_t

	Θ_{MC1}	Θ_{MC2}	Θ_{MC3}	Θ_{MC4}	Θ_{MC5}	Θ_{MC6}	Θ_{MC7}	Θ_{MC8}	Θ_{MC9}	Θ_{MC10}
c	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
ϕ_1	−0.1637	−0.1591	−0.1540	−0.1643	−0.1681	−0.1639	−0.1618	−0.1678	−0.1708	−0.1616
ϕ_2	−0.1021	−0.1036	−0.1029	−0.1009	−0.1029	−0.1029	−0.1046	−0.1025	−0.1030	−0.1019
ϕ_3	0.1552	0.1556	0.1525	0.1490	0.1542	0.1488	0.1578	0.1566	0.1591	0.1506
ϕ_4	0.0620	0.0624	0.0634	0.0640	0.0614	0.0649	0.0598	0.0606	0.0600	0.0627
ϕ_5	0.0158	0.0150	0.0156	0.0162	0.0152	0.0155	0.0168	0.0151	0.0158	0.0161
ϕ_6	−0.0683	−0.0687	−0.0670	−0.0654	−0.0688	−0.0676	−0.0673	−0.0677	−0.0689	−0.0687
ϕ_7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ϕ_8	0.0101	0.0100	0.0101	0.0099	0.0104	0.0102	0.0099	0.0102	0.0096	0.0103
ϕ_9	0.2336	0.2302	0.2320	0.2276	0.2384	0.2312	0.2357	0.2503	0.2312	0.2327
ϕ_{10}	0.0677	0.0679	0.0702	0.0684	0.0655	0.0655	0.0683	0.0707	0.0642	0.0667
ϕ_{11}	0.2890	0.2913	0.2908	0.2840	0.2929	0.2837	0.2926	0.2974	0.2909	0.3022
ϕ_{12}	0.0923	0.0936	0.0918	0.0917	0.0917	0.0931	0.0917	0.0873	0.0919	0.0947
ϕ_{13}	0.0686	0.0689	0.0687	0.0682	0.0701	0.0699	0.0685	0.0689	0.0705	0.0674
ϕ_{14}	−0.0658	−0.0651	−0.0656	−0.0658	−0.0680	−0.0671	−0.0681	−0.0646	−0.0675	−0.0666
ϕ_{15}	0.2072	0.2046	0.2074	0.2099	0.2016	0.2131	0.2043	0.2116	0.2118	0.2005
ϕ_{16}	0.0818	0.0830	0.0809	0.0779	0.0810	0.0831	0.0824	0.0776	0.0829	0.0832
ϕ_{17}	0.0589	0.0565	0.0593	0.0583	0.0611	0.0596	0.0572	0.0575	0.0587	0.0586
ϕ_{18}	−0.0502	−0.0491	−0.0489	−0.0487	−0.0480	−0.0487	−0.0489	−0.0490	−0.0479	−0.0501
ϕ_{19}	−0.0586	−0.0604	−0.0590	−0.0589	−0.0584	−0.0580	−0.0590	−0.0583	−0.0580	−0.0594
ϕ_{20}	−0.1878	−0.1863	−0.1955	−0.1806	−0.1829	−0.1850	−0.1894	−0.1867	−0.1960	−0.1881
ϕ_{21}	−0.1516	−0.1489	−0.1491	−0.1504	−0.1493	−0.1525	−0.1538	−0.1536	−0.1520	−0.1513
ϕ_{22}	−0.1507	−0.1477	−0.1524	−0.1462	−0.1533	−0.1506	−0.1531	−0.1555	−0.1507	−0.1520
ϕ_{23}	0.2092	0.2117	0.2129	0.2039	0.2116	0.2115	0.2082	0.2098	0.2048	0.2104
ϕ_{24}	−0.1126	−0.1125	−0.1098	−0.1116	−0.1138	−0.1107	−0.1129	−0.1154	−0.1098	−0.1092
ϕ_{25}	0.0048	0.0048	0.0047	0.0049	0.0048	0.0047	0.0047	0.0046	0.0047	0.0050
ϕ_{26}	−0.1283	−0.1294	−0.1284	−0.1341	−0.1283	−0.1307	−0.1304	−0.1303	−0.1315	−0.1241
ϕ_{27}	0.0958	0.0993	0.0970	0.0949	0.0971	0.0960	0.0927	0.0943	0.0926	0.0954
ϕ_{28}	0.0784	0.0773	0.0775	0.0812	0.0820	0.0797	0.0800	0.0809	0.0786	0.0795
ϕ_{29}	0.2845	0.2820	0.2871	0.2804	0.2899	0.2803	0.2773	0.2789	0.2780	0.2874
ϕ_{30}	0.0483	0.0491	0.0480	0.0494	0.0473	0.0478	0.0477	0.0464	0.0481	0.0489
θ	1.0586	1.0556	1.0862	1.0404	1.0673	1.0410	1.0814	1.0650	1.0757	1.0787
ω	−0.0475	−0.0531	−0.0138	−0.0643	−0.0643	−0.0372	−0.0390	−0.0574	−0.0597	−0.0460
α	0.0347	0.0363	0.0320	0.0329	0.0299	0.0304	0.0375	0.0315	0.0346	0.0308
α^*	0.0260	0.0283	0.0310	0.0257	0.0242	0.0288	0.0244	0.0300	0.0253	0.0237
β	0.9908	0.9897	0.9973	0.9875	0.9875	0.9928	0.9924	0.9888	0.9884	0.9911
λ_0	−5.3215	−5.3215	−5.3215	−5.3215	−5.3215	−5.3215	−5.3215	−5.3215	−5.3215	−5.3215
δ_1	−0.0476	−0.0485	−0.0535	−0.0469	−0.0515	−0.0445	−0.0463	−0.0503	−0.0489	−0.0461
δ_2	1.9216	1.8718	2.0209	1.7072	1.9176	2.0058	2.1043	1.9611	2.2358	1.9871
δ_3	0.2808	0.2953	0.2707	0.2573	0.3104	0.2480	0.2544	0.2916	0.2701	0.2831
γ_3	0.5464	0.5229	0.5626	0.5843	0.4985	0.5993	0.5891	0.5289	0.5636	0.5426
κ_3	0.0923	0.0909	0.0889	0.0917	0.0903	0.0930	0.0912	0.0898	0.0870	0.0961

Table A16 (continued). Parameters for Monte Carlo simulation, Skew-Gen-t-DCS with constant τ_t, ν_t, dynamic η_t

	Θ_{MC11}	Θ_{MC12}	Θ_{MC13}	Θ_{MC14}	Θ_{MC15}	Θ_{MC16}	Θ_{MC17}	Θ_{MC18}	Θ_{MC19}	Θ_{MC20}
c	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
ϕ_1	−0.1683	−0.1584	−0.1651	−0.1625	−0.1614	−0.1560	−0.1670	−0.1609	−0.1611	−0.1615
ϕ_2	−0.1059	−0.1037	−0.1027	−0.1058	−0.1022	−0.0966	−0.0991	−0.1010	−0.1044	−0.1024
ϕ_3	0.1491	0.1549	0.1604	0.1602	0.1496	0.1611	0.1451	0.1527	0.1542	0.1549
ϕ_4	0.0624	0.0631	0.0654	0.0584	0.0612	0.0644	0.0620	0.0634	0.0649	0.0616
ϕ_5	0.0161	0.0157	0.0159	0.0160	0.0159	0.0162	0.0158	0.0151	0.0160	0.0155
ϕ_6	−0.0677	−0.0707	−0.0700	−0.0674	−0.0695	−0.0685	−0.0676	−0.0661	−0.0722	−0.0680
ϕ_7	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ϕ_8	0.0100	0.0101	0.0098	0.0101	0.0103	0.0101	0.0104	0.0103	0.0102	0.0100
ϕ_9	0.2357	0.2209	0.2226	0.2400	0.2342	0.2295	0.2423	0.2270	0.2377	0.2276
ϕ_{10}	0.0631	0.0673	0.0678	0.0649	0.0668	0.0655	0.0662	0.0688	0.0666	0.0662
ϕ_{11}	0.2836	0.2945	0.3091	0.2808	0.2801	0.2925	0.2815	0.2794	0.2828	0.2961
ϕ_{12}	0.0966	0.0875	0.0903	0.0922	0.0932	0.0878	0.0944	0.0950	0.0921	0.0940
ϕ_{13}	0.0688	0.0681	0.0681	0.0681	0.0675	0.0680	0.0730	0.0676	0.0678	0.0664
ϕ_{14}	−0.0647	−0.0661	−0.0641	−0.0645	−0.0646	−0.0655	−0.0673	−0.0665	−0.0671	−0.0665
ϕ_{15}	0.2137	0.1994	0.2048	0.2026	0.2075	0.2048	0.2093	0.2083	0.2038	0.2012
ϕ_{16}	0.0817	0.0799	0.0797	0.0799	0.0839	0.0810	0.0842	0.0860	0.0831	0.0777
ϕ_{17}	0.0584	0.0582	0.0581	0.0582	0.0594	0.0575	0.0609	0.0599	0.0582	0.0581
ϕ_{18}	−0.0500	−0.0492	−0.0485	−0.0505	−0.0508	−0.0504	−0.0499	−0.0507	−0.0476	−0.0508
ϕ_{19}	−0.0563	−0.0599	−0.0594	−0.0582	−0.0583	−0.0592	−0.0589	−0.0616	−0.0579	−0.0594
ϕ_{20}	−0.1802	−0.1942	−0.1855	−0.1944	−0.1819	−0.1912	−0.1831	−0.1878	−0.1875	−0.1876
ϕ_{21}	−0.1482	−0.1573	−0.1514	−0.1478	−0.1478	−0.1494	−0.1475	−0.1520	−0.1451	−0.1499
ϕ_{22}	−0.1516	−0.1512	−0.1519	−0.1553	−0.1460	−0.1500	−0.1587	−0.1504	−0.1479	−0.1497
ϕ_{23}	0.2080	0.2068	0.2055	0.2058	0.2077	0.2155	0.2154	0.2123	0.2054	0.2100
ϕ_{24}	−0.1138	−0.1167	−0.1123	−0.1130	−0.1121	−0.1080	−0.1132	−0.1149	−0.1080	−0.1162
ϕ_{25}	0.0047	0.0047	0.0048	0.0049	0.0050	0.0047	0.0047	0.0048	0.0046	0.0048
ϕ_{26}	−0.1310	−0.1338	−0.1283	−0.1290	−0.1270	−0.1277	−0.1272	−0.1276	−0.1352	−0.1229
ϕ_{27}	0.0983	0.0946	0.0968	0.0986	0.0952	0.0982	0.1007	0.0966	0.0959	0.0972
ϕ_{28}	0.0783	0.0750	0.0797	0.0790	0.0816	0.0813	0.0763	0.0793	0.0763	0.0770
ϕ_{29}	0.2786	0.2990	0.2824	0.2953	0.2758	0.2709	0.2856	0.2879	0.2791	0.2811
ϕ_{30}	0.0475	0.0481	0.0460	0.0480	0.0484	0.0484	0.0481	0.0474	0.0500	0.0473
θ	1.0773	1.0730	1.0568	1.0079	1.0516	1.0954	1.0685	1.0292	1.0559	1.0483
ω	−0.0441	−0.0690	−0.0533	−0.0423	−0.0457	−0.0844	−0.0303	−0.0634	−0.0280	−0.0572
α	0.0312	0.0264	0.0363	0.0401	0.0368	0.0332	0.0316	0.0347	0.0346	0.0391
α^*	0.0321	0.0290	0.0272	0.0257	0.0315	0.0227	0.0231	0.0258	0.0279	0.0308
β	0.9914	0.9866	0.9896	0.9918	0.9911	0.9836	0.9941	0.9877	0.9946	0.9889
λ_0	−5.3215	−5.3215	−5.3215	−5.3215	−5.3215	−5.3215	−5.3215	−5.3215	−5.3215	−5.3215
δ_1	−0.0501	−0.0471	−0.0458	−0.0437	−0.0477	−0.0442	−0.0470	−0.0500	−0.0468	−0.0511
δ_2	1.9474	1.8733	1.9930	1.9592	1.7949	1.9543	1.9046	2.1055	1.8988	1.8058
δ_3	0.3020	0.3005	0.2748	0.2703	0.2824	0.2961	0.2755	0.2795	0.2399	0.2701
γ_3	0.5121	0.5145	0.5561	0.5633	0.5438	0.5216	0.5549	0.5484	0.6124	0.5636
κ_3	0.0869	0.0917	0.0884	0.0976	0.0969	0.0954	0.0999	0.0883	0.1011	0.1000

Table A17. Parameters for Monte Carlo simulation, EGB2-DCS with constant ξ_t, ζ_t

	Θ_{MC1}	Θ_{MC2}	Θ_{MC3}	Θ_{MC4}	Θ_{MC5}	Θ_{MC6}	Θ_{MC7}	Θ_{MC8}	Θ_{MC9}	Θ_{MC10}
c	0.0001	0.0001	0.0001	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001
ϕ_1	−0.2058	−0.1980	−0.2018	−0.2107	−0.2133	−0.1955	−0.2060	−0.2042	−0.2025	−0.1988
ϕ_2	−0.1650	−0.1666	−0.1656	−0.1637	−0.1592	−0.1743	−0.1714	−0.1743	−0.1684	−0.1641
ϕ_3	0.1647	0.1582	0.1707	0.1639	0.1646	0.1692	0.1689	0.1745	0.1719	0.1620
ϕ_4	0.1057	0.1083	0.0990	0.1044	0.1087	0.1095	0.1095	0.1087	0.1072	0.1110
ϕ_5	0.0865	0.0909	0.0893	0.0835	0.0940	0.0892	0.0844	0.0857	0.0903	0.0840
ϕ_6	−0.0469	−0.0468	−0.0487	−0.0475	−0.0458	−0.0452	−0.0447	−0.0463	−0.0479	−0.0503
ϕ_7	−0.0020	−0.0020	−0.0020	−0.0020	−0.0019	−0.0020	−0.0021	−0.0019	−0.0022	−0.0021
ϕ_8	−0.0086	−0.0085	−0.0087	−0.0085	−0.0085	−0.0083	−0.0084	−0.0083	−0.0082	−0.0085
ϕ_9	0.2188	0.2160	0.2077	0.2063	0.2081	0.2185	0.2163	0.2231	0.2184	0.2163
ϕ_{10}	0.0422	0.0415	0.0412	0.0435	0.0396	0.0414	0.0434	0.0452	0.0422	0.0396
ϕ_{11}	0.2852	0.2894	0.2856	0.2912	0.2897	0.2855	0.2860	0.2835	0.2790	0.2925
ϕ_{12}	0.0898	0.0923	0.0872	0.0937	0.0896	0.0902	0.0882	0.0910	0.0958	0.0880
ϕ_{13}	0.0576	0.0577	0.0573	0.0584	0.0594	0.0565	0.0548	0.0573	0.0578	0.0593
ϕ_{14}	−0.0886	−0.0832	−0.0845	−0.0877	−0.0878	−0.0868	−0.0883	−0.0864	−0.0885	−0.0807
ϕ_{15}	0.2205	0.2193	0.2345	0.2126	0.2168	0.2156	0.2275	0.2151	0.2216	0.2266
ϕ_{16}	0.1216	0.1184	0.1218	0.1221	0.1253	0.1242	0.1286	0.1219	0.1177	0.1271
ϕ_{17}	0.1114	0.1132	0.1114	0.1112	0.1175	0.1123	0.1083	0.1151	0.1160	0.1086
ϕ_{18}	−0.0551	−0.0528	−0.0536	−0.0531	−0.0545	−0.0588	−0.0548	−0.0548	−0.0601	−0.0546
ϕ_{19}	−0.0977	−0.0974	−0.0973	−0.0985	−0.0982	−0.1012	−0.0982	−0.0996	−0.1037	−0.0994
ϕ_{20}	−0.2385	−0.2349	−0.2444	−0.2429	−0.2399	−0.2400	−0.2408	−0.2353	−0.2351	−0.2481
ϕ_{21}	−0.2015	−0.1814	−0.2123	−0.2004	−0.2062	−0.1899	−0.2013	−0.1993	−0.1842	−0.1984
ϕ_{22}	−0.1731	−0.1824	−0.1738	−0.1704	−0.1817	−0.1778	−0.1700	−0.1755	−0.1690	−0.1731
ϕ_{23}	0.2217	0.2260	0.2334	0.2202	0.2211	0.2260	0.2161	0.2300	0.2233	0.2169
ϕ_{24}	−0.0568	−0.0561	−0.0547	−0.0571	−0.0601	−0.0531	−0.0573	−0.0573	−0.0578	−0.0562
ϕ_{25}	0.1026	0.0977	0.0995	0.0991	0.1043	0.1028	0.1081	0.1086	0.1025	0.0986
ϕ_{26}	−0.1021	−0.1056	−0.1000	−0.1020	−0.1024	−0.0962	−0.1027	−0.0982	−0.0970	−0.1011
ϕ_{27}	0.0887	0.0895	0.0846	0.0855	0.0843	0.0897	0.0892	0.0872	0.0927	0.0877
ϕ_{28}	0.0296	0.0287	0.0300	0.0302	0.0282	0.0312	0.0295	0.0301	0.0293	0.0298
ϕ_{29}	0.2806	0.2679	0.2679	0.2802	0.2752	0.2869	0.2729	0.2819	0.2664	0.2817
ϕ_{30}	0.0624	0.0590	0.0649	0.0643	0.0626	0.0645	0.0608	0.0607	0.0610	0.0630
θ	0.0735	0.0728	0.0727	0.0746	0.0731	0.0760	0.0709	0.0755	0.0735	0.0747
ω	−0.0630	−0.0870	−0.0892	−0.0577	−0.0556	−0.0710	−0.0763	−0.0247	−0.0578	−0.1112
α	0.0376	0.0360	0.0404	0.0334	0.0372	0.0384	0.0345	0.0429	0.0323	0.0364
α^*	0.0255	0.0265	0.0238	0.0211	0.0266	0.0181	0.0251	0.0268	0.0318	0.0242
β	0.9890	0.9848	0.9845	0.9899	0.9903	0.9876	0.9867	0.9957	0.9899	0.9806
λ_0	−5.8704	−5.8704	−5.8704	−5.8704	−5.8704	−5.8704	−5.8704	−5.8704	−5.8704	−5.8704
δ_1	−0.2118	−0.2230	−0.2152	−0.2023	−0.2131	−0.2151	−0.2147	−0.2012	−0.2100	−0.2120
δ_2	−0.0900	−0.0780	−0.0902	−0.0921	−0.0962	−0.0844	−0.0921	−0.0960	−0.0877	−0.0867

Table A17 (continued). Parameters for Monte Carlo simulation, EGB2-DCS with constant ξ_t, ζ_t

	Θ_{MC11}	Θ_{MC12}	Θ_{MC13}	Θ_{MC14}	Θ_{MC15}	Θ_{MC16}	Θ_{MC17}	Θ_{MC18}	Θ_{MC19}	Θ_{MC20}
c	0.0001	0.0002	0.0001	0.0002	0.0002	0.0002	0.0002	0.0001	0.0002	0.0002
ϕ_1	−0.2008	−0.2162	−0.1975	−0.2172	−0.2151	−0.1958	−0.2076	−0.2096	−0.2079	−0.2053
ϕ_2	−0.1717	−0.1597	−0.1637	−0.1647	−0.1746	−0.1749	−0.1679	−0.1681	−0.1616	−0.1675
ϕ_3	0.1664	0.1673	0.1789	0.1575	0.1560	0.1659	0.1706	0.1691	0.1608	0.1643
ϕ_4	0.1034	0.1046	0.1037	0.1036	0.1054	0.1047	0.1052	0.1115	0.1080	0.1069
ϕ_5	0.0863	0.0869	0.0870	0.0863	0.0849	0.0830	0.0878	0.0828	0.0906	0.0855
ϕ_6	−0.0465	−0.0433	−0.0453	−0.0472	−0.0457	−0.0478	−0.0474	−0.0455	−0.0460	−0.0477
ϕ_7	−0.0020	−0.0020	−0.0020	−0.0020	−0.0021	−0.0021	−0.0020	−0.0020	−0.0021	−0.0021
ϕ_8	−0.0083	−0.0082	−0.0089	−0.0081	−0.0085	−0.0083	−0.0086	−0.0084	−0.0088	−0.0091
ϕ_9	0.2358	0.2103	0.2296	0.2283	0.2082	0.2112	0.2224	0.2190	0.2198	0.2257
ϕ_{10}	0.0416	0.0435	0.0424	0.0413	0.0432	0.0412	0.0420	0.0415	0.0434	0.0438
ϕ_{11}	0.2919	0.2725	0.2798	0.2723	0.2687	0.2912	0.2775	0.2847	0.2989	0.2968
ϕ_{12}	0.0892	0.0869	0.0913	0.0827	0.0892	0.0838	0.0929	0.0940	0.0899	0.0928
ϕ_{13}	0.0567	0.0564	0.0581	0.0582	0.0562	0.0588	0.0597	0.0602	0.0575	0.0560
ϕ_{14}	−0.0871	−0.0886	−0.0940	−0.0840	−0.0851	−0.0885	−0.0923	−0.0933	−0.0844	−0.0898
ϕ_{15}	0.2315	0.2344	0.2249	0.2262	0.2372	0.2167	0.2155	0.2179	0.2217	0.2071
ϕ_{16}	0.1212	0.1312	0.1203	0.1299	0.1227	0.1196	0.1201	0.1216	0.1140	0.1273
ϕ_{17}	0.1155	0.1097	0.1070	0.1131	0.1078	0.1141	0.1115	0.1154	0.1105	0.1120
ϕ_{18}	−0.0560	−0.0560	−0.0542	−0.0541	−0.0554	−0.0558	−0.0519	−0.0583	−0.0545	−0.0553
ϕ_{19}	−0.0970	−0.0988	−0.0977	−0.0992	−0.0996	−0.0961	−0.0957	−0.0978	−0.1048	−0.0969
ϕ_{20}	−0.2328	−0.2461	−0.2324	−0.2585	−0.2453	−0.2453	−0.2354	−0.2357	−0.2443	−0.2503
ϕ_{21}	−0.2063	−0.2114	−0.2078	−0.2031	−0.2050	−0.2046	−0.1917	−0.2008	−0.2128	−0.2095
ϕ_{22}	−0.1710	−0.1784	−0.1697	−0.1669	−0.1656	−0.1697	−0.1835	−0.1682	−0.1843	−0.1739
ϕ_{23}	0.2228	0.2235	0.2419	0.2235	0.2327	0.2265	0.2116	0.2224	0.2212	0.2218
ϕ_{24}	−0.0549	−0.0547	−0.0543	−0.0593	−0.0549	−0.0593	−0.0571	−0.0593	−0.0588	−0.0578
ϕ_{25}	0.0982	0.1059	0.1054	0.0984	0.1004	0.1041	0.1059	0.1043	0.1096	0.1029
ϕ_{26}	−0.1033	−0.1050	−0.0996	−0.1030	−0.1019	−0.1015	−0.1046	−0.1034	−0.1053	−0.1011
ϕ_{27}	0.0853	0.0902	0.0907	0.0904	0.0919	0.0938	0.0834	0.0878	0.0878	0.0854
ϕ_{28}	0.0292	0.0288	0.0298	0.0289	0.0297	0.0299	0.0298	0.0290	0.0283	0.0304
ϕ_{29}	0.2866	0.2825	0.2687	0.2760	0.2764	0.2728	0.2684	0.2758	0.2768	0.2758
ϕ_{30}	0.0661	0.0663	0.0649	0.0597	0.0599	0.0622	0.0605	0.0624	0.0650	0.0605
θ	0.0741	0.0734	0.0741	0.0743	0.0730	0.0744	0.0720	0.0727	0.0744	0.0755
ω	−0.0304	−0.0587	−0.0793	−0.0448	−0.0472	−0.0743	−0.0613	−0.0381	−0.0105	−0.0417
α	0.0393	0.0407	0.0327	0.0402	0.0350	0.0349	0.0465	0.0401	0.0386	0.0448
α^*	0.0235	0.0202	0.0248	0.0261	0.0240	0.0224	0.0274	0.0279	0.0249	0.0265
β	0.9947	0.9898	0.9862	0.9922	0.9918	0.9870	0.9893	0.9934	0.9982	0.9927
λ_0	−5.8704	−5.8704	−5.8704	−5.8704	−5.8704	−5.8704	−5.8704	−5.8704	−5.8704	−5.8704
δ_1	−0.2148	−0.2195	−0.2105	−0.1991	−0.2131	−0.2234	−0.2026	−0.2190	−0.1928	−0.2058
δ_2	−0.0961	−0.1024	−0.0907	−0.0897	−0.0937	−0.0923	−0.0960	−0.0879	−0.0923	−0.0871

Table A18. Parameters for Monte Carlo simulation, EGB2-DCS with dynamic ξ_t, ζ_t

	Θ_{MC1}	Θ_{MC2}	Θ_{MC3}	Θ_{MC4}	Θ_{MC5}	Θ_{MC6}	Θ_{MC7}	Θ_{MC8}	Θ_{MC9}	Θ_{MC10}
c	0.0001	0.0002	0.0001	0.0000	0.0002	0.0001	0.0002	0.0001	0.0000	0.0002
ϕ_1	−0.6459	−0.6608	−0.6760	−0.6542	−0.6288	−0.6632	−0.6498	−0.6230	−0.6372	−0.6501
ϕ_2	−0.5789	−0.5932	−0.5846	−0.5945	−0.5849	−0.5888	−0.5584	−0.5905	−0.5624	−0.5845
ϕ_3	0.0089	0.0089	0.0087	0.0087	0.0089	0.0091	0.0094	0.0095	0.0086	0.0090
ϕ_4	0.3303	0.3354	0.3247	0.3183	0.3276	0.3326	0.3283	0.3340	0.3533	0.3295
ϕ_5	0.3652	0.3535	0.3595	0.3725	0.3487	0.3763	0.3676	0.3807	0.3593	0.3489
ϕ_6	0.1493	0.1460	0.1521	0.1471	0.1428	0.1550	0.1538	0.1558	0.1530	0.1500
ϕ_7	−0.0880	−0.0861	−0.0918	−0.0910	−0.0855	−0.0875	−0.0936	−0.0899	−0.0898	−0.0863
ϕ_8	−0.2396	−0.2530	−0.2402	−0.2346	−0.2344	−0.2494	−0.2430	−0.2568	−0.2457	−0.2469
ϕ_9	0.0269	0.0278	0.0275	0.0276	0.0252	0.0267	0.0279	0.0265	0.0279	0.0278
ϕ_{10}	0.0297	0.0286	0.0297	0.0306	0.0295	0.0292	0.0301	0.0288	0.0286	0.0293
ϕ_{11}	0.4668	0.4638	0.4584	0.4801	0.4733	0.4788	0.4459	0.4724	0.4774	0.4499
ϕ_{12}	0.4655	0.4673	0.4898	0.4633	0.4817	0.4598	0.4700	0.4405	0.4347	0.4888
ϕ_{13}	0.4373	0.4461	0.4357	0.4288	0.4030	0.4391	0.4246	0.4193	0.4436	0.4543
ϕ_{14}	0.2007	0.2037	0.2029	0.1987	0.1960	0.2073	0.1979	0.2083	0.2098	0.2013
ϕ_{15}	0.4073	0.4023	0.4043	0.4314	0.4006	0.4202	0.3891	0.3793	0.3986	0.3828
ϕ_{16}	0.4814	0.4802	0.4790	0.4998	0.5064	0.4713	0.4760	0.4977	0.4757	0.4693
ϕ_{17}	0.5552	0.5371	0.5612	0.5518	0.5340	0.5484	0.5433	0.5536	0.5629	0.5354
ϕ_{18}	0.2675	0.2784	0.2761	0.2647	0.2678	0.2592	0.2581	0.2778	0.2810	0.2694
ϕ_{19}	−0.0237	−0.0236	−0.0240	−0.0227	−0.0234	−0.0248	−0.0241	−0.0232	−0.0238	−0.0234
ϕ_{20}	−0.3496	−0.3293	−0.3699	−0.3245	−0.3485	−0.3405	−0.3732	−0.3537	−0.3244	−0.3304
ϕ_{21}	−0.4897	−0.4996	−0.4663	−0.4310	−0.4940	−0.4894	−0.4924	−0.5264	−0.4841	−0.5060
ϕ_{22}	−0.4949	−0.5039	−0.4853	−0.5026	−0.5223	−0.4818	−0.5355	−0.4704	−0.4845	−0.5130
ϕ_{23}	0.0922	0.0911	0.0898	0.0911	0.0944	0.0914	0.0928	0.0907	0.0924	0.0873
ϕ_{24}	0.1597	0.1601	0.1567	0.1674	0.1622	0.1599	0.1626	0.1601	0.1655	0.1604
ϕ_{25}	0.4287	0.4111	0.4506	0.4123	0.4029	0.4237	0.4287	0.4431	0.4308	0.4277
ϕ_{26}	0.0126	0.0123	0.0128	0.0127	0.0128	0.0125	0.0128	0.0121	0.0125	0.0129
ϕ_{27}	−0.2508	−0.2346	−0.2457	−0.2634	−0.2314	−0.2464	−0.2486	−0.2363	−0.2533	−0.2434
ϕ_{28}	−0.5399	−0.5389	−0.5462	−0.5229	−0.5628	−0.5040	−0.5112	−0.5151	−0.5433	−0.5459
ϕ_{29}	−0.2451	−0.2574	−0.2474	−0.2460	−0.2530	−0.2524	−0.2468	−0.2465	−0.2463	−0.2443
ϕ_{30}	−0.0720	−0.0721	−0.0721	−0.0717	−0.0738	−0.0768	−0.0705	−0.0727	−0.0670	−0.0740
θ	0.0428	0.0443	0.0413	0.0436	0.0422	0.0415	0.0456	0.0429	0.0446	0.0425
ω	−0.0459	−0.0459	−0.0459	−0.0459	−0.0459	−0.0459	−0.0459	−0.0459	−0.0459	−0.0459
α	0.0359	0.0463	0.0406	0.0437	0.0380	0.0257	0.0410	0.0370	0.0369	0.0436
α^*	0.0203	0.0201	0.0222	0.0204	0.0205	0.0178	0.0204	0.0234	0.0198	0.0218
β	0.9920	0.9930	0.9836	0.9919	0.9913	0.9882	0.9998	0.9911	0.9913	0.9944
λ_0	−5.8977	−5.8977	−5.8977	−5.8977	−5.8977	−5.8977	−5.8977	−5.8977	−5.8977	−5.8977
δ_1	−0.0893	−0.0944	−0.0898	−0.0940	−0.0808	−0.0900	−0.0949	−0.0861	−0.0771	−0.0792
γ_1	0.5633	0.5380	0.5607	0.5400	0.6047	0.5595	0.5357	0.5788	0.6230	0.6123
κ_1	0.0599	0.0592	0.0610	0.0536	0.0593	0.0598	0.0558	0.0589	0.0584	0.0602
δ_2	−0.0111	−0.0181	−0.0082	−0.0048	−0.0101	−0.0120	−0.0066	−0.0155	−0.0099	−0.0124
γ_2	0.8868	0.8146	0.9158	0.9506	0.8970	0.8770	0.9324	0.8410	0.8989	0.8729
κ_2	−0.0166	−0.0152	−0.0178	−0.0171	−0.0159	−0.0171	−0.0171	−0.0163	−0.0154	−0.0156

Table A18 (continued). Parameters for Monte Carlo simulation, EGB2-DCS with dynamic ξ_t, ζ_t

	Θ_{MC11}	Θ_{MC12}	Θ_{MC13}	Θ_{MC14}	Θ_{MC15}	Θ_{MC16}	Θ_{MC17}	Θ_{MC18}	Θ_{MC19}	Θ_{MC20}
c	0.0001	0.0000	0.0002	0.0001	0.0000	0.0001	0.0001	0.0001	0.0001	0.0001
ϕ_1	−0.6148	−0.6449	−0.6640	−0.6752	−0.6078	−0.6246	−0.6665	−0.6563	−0.6619	−0.6158
ϕ_2	−0.5617	−0.5633	−0.5570	−0.5688	−0.5825	−0.5758	−0.5629	−0.5666	−0.5737	−0.5905
ϕ_3	0.0086	0.0087	0.0093	0.0094	0.0087	0.0092	0.0089	0.0094	0.0089	0.0090
ϕ_4	0.3285	0.3315	0.3333	0.3353	0.3409	0.3454	0.3420	0.3161	0.3260	0.3326
ϕ_5	0.3657	0.3485	0.3783	0.3688	0.3691	0.3630	0.3543	0.3580	0.3672	0.3582
ϕ_6	0.1596	0.1528	0.1612	0.1476	0.1524	0.1510	0.1449	0.1487	0.1442	0.1533
ϕ_7	−0.0826	−0.0921	−0.0912	−0.0906	−0.0841	−0.0967	−0.0930	−0.0840	−0.0854	−0.0890
ϕ_8	−0.2542	−0.2295	−0.2333	−0.2424	−0.2261	−0.2583	−0.2484	−0.2322	−0.2311	−0.2378
ϕ_9	0.0261	0.0265	0.0278	0.0264	0.0279	0.0281	0.0275	0.0291	0.0283	0.0258
ϕ_{10}	0.0289	0.0285	0.0295	0.0297	0.0302	0.0305	0.0299	0.0295	0.0295	0.0295
ϕ_{11}	0.4803	0.4710	0.4362	0.4361	0.4477	0.4790	0.4910	0.4818	0.4641	0.4678
ϕ_{12}	0.4743	0.4826	0.4750	0.4437	0.4715	0.4788	0.4599	0.4671	0.4529	0.4734
ϕ_{13}	0.4473	0.4291	0.4395	0.4479	0.4321	0.4233	0.4187	0.4252	0.4199	0.4449
ϕ_{14}	0.2013	0.1952	0.1948	0.2050	0.2065	0.2122	0.2023	0.1962	0.2041	0.1988
ϕ_{15}	0.4149	0.4195	0.4212	0.4191	0.4303	0.3993	0.4342	0.4121	0.4235	0.4246
ϕ_{16}	0.4881	0.4809	0.4643	0.4524	0.5137	0.4759	0.4828	0.4946	0.5167	0.4910
ϕ_{17}	0.5582	0.5527	0.5464	0.6202	0.5367	0.5638	0.5409	0.5440	0.5663	0.5381
ϕ_{18}	0.2696	0.2635	0.2783	0.2538	0.2707	0.2646	0.2723	0.2647	0.2495	0.2788
ϕ_{19}	−0.0235	−0.0228	−0.0226	−0.0229	−0.0238	−0.0247	−0.0234	−0.0223	−0.0245	−0.0235
ϕ_{20}	−0.3583	−0.3408	−0.3564	−0.3541	−0.3598	−0.3505	−0.3561	−0.3320	−0.3545	−0.3534
ϕ_{21}	−0.4980	−0.4780	−0.5053	−0.5089	−0.4791	−0.4830	−0.4876	−0.4880	−0.5067	−0.4747
ϕ_{22}	−0.4927	−0.4698	−0.5095	−0.4956	−0.4849	−0.4721	−0.4892	−0.5047	−0.5129	−0.5157
ϕ_{23}	0.0892	0.0935	0.0903	0.0946	0.0966	0.0880	0.0891	0.0946	0.0954	0.0834
ϕ_{24}	0.1531	0.1706	0.1723	0.1605	0.1583	0.1539	0.1504	0.1580	0.1628	0.1586
ϕ_{25}	0.4401	0.4586	0.4093	0.4315	0.4293	0.4095	0.4198	0.4348	0.4218	0.4054
ϕ_{26}	0.0136	0.0127	0.0134	0.0127	0.0125	0.0130	0.0128	0.0131	0.0128	0.0123
ϕ_{27}	−0.2659	−0.2594	−0.2570	−0.2732	−0.2409	−0.2508	−0.2507	−0.2643	−0.2530	−0.2502
ϕ_{28}	−0.5468	−0.5677	−0.5545	−0.5162	−0.5617	−0.5618	−0.5500	−0.5607	−0.5370	−0.5545
ϕ_{29}	−0.2439	−0.2367	−0.2528	−0.2483	−0.2448	−0.2334	−0.2532	−0.2305	−0.2490	−0.2401
ϕ_{30}	−0.0738	−0.0737	−0.0705	−0.0723	−0.0693	−0.0677	−0.0711	−0.0737	−0.0746	−0.0712
θ	0.0418	0.0454	0.0426	0.0420	0.0418	0.0433	0.0434	0.0425	0.0411	0.0431
ω	−0.0459	−0.0459	−0.0459	−0.0459	−0.0459	−0.0459	−0.0459	−0.0459	−0.0459	−0.0459
α	0.0345	0.0375	0.0266	0.0351	0.0353	0.0380	0.0373	0.0302	0.0348	0.0356
α^*	0.0209	0.0202	0.0186	0.0222	0.0198	0.0198	0.0164	0.0196	0.0213	0.0182
β	0.9893	0.9883	0.9899	0.9917	0.9888	0.9893	0.9913	0.9911	0.9917	0.9988
λ_0	−5.8977	−5.8977	−5.8977	−5.8977	−5.8977	−5.8977	−5.8977	−5.8977	−5.8977	−5.8977
δ_1	−0.0919	−0.0805	−0.0901	−0.0851	−0.0867	−0.0971	−0.0833	−0.0831	−0.0907	−0.0933
γ_1	0.5506	0.6064	0.5594	0.5835	0.5757	0.5248	0.5923	0.5935	0.5563	0.5438
κ_1	0.0581	0.0629	0.0636	0.0593	0.0608	0.0586	0.0601	0.0567	0.0595	0.0589
δ_2	−0.0089	−0.0135	−0.0073	−0.0122	−0.0144	−0.0200	−0.0095	−0.0182	−0.0132	−0.0135
γ_2	0.9087	0.8620	0.9252	0.8756	0.8522	0.7955	0.9029	0.8141	0.8650	0.8622
κ_2	−0.0178	−0.0170	−0.0169	−0.0154	−0.0165	−0.0153	−0.0157	−0.0167	−0.0171	−0.0177

Table A19. Parameters for Monte Carlo simulation, NIG-DCS with constant ν_t, η_t

	Θ_{MC1}	Θ_{MC2}	Θ_{MC3}	Θ_{MC4}	Θ_{MC5}	Θ_{MC6}	Θ_{MC7}	Θ_{MC8}	Θ_{MC9}	Θ_{MC10}
c	0.0014	0.0013	0.0015	0.0013	0.0013	0.0014	0.0013	0.0013	0.0013	0.0015
ϕ_1	−0.5322	−0.5804	−0.4944	−0.5580	−0.5286	−0.6445	−0.5528	−0.5225	−0.5666	−0.4428
ϕ_2	−0.5883	−0.5502	−0.6515	−0.6212	−0.4357	−0.6057	−0.4745	−0.5255	−0.5440	−0.5521
ϕ_3	−0.2777	−0.3033	−0.2755	−0.2580	−0.2803	−0.2552	−0.2442	−0.2938	−0.2604	−0.3126
ϕ_4	−0.1673	−0.1784	−0.1736	−0.1588	−0.1672	−0.1488	−0.1557	−0.1972	−0.1668	−0.1802
ϕ_5	−0.1009	−0.1036	−0.0947	−0.0929	−0.0926	−0.0885	−0.1065	−0.0890	−0.1016	−0.0890
ϕ_6	−0.1476	−0.1554	−0.1472	−0.1247	−0.1415	−0.1399	−0.1499	−0.1244	−0.1534	−0.1590
ϕ_7	−0.1987	−0.1987	−0.2152	−0.1929	−0.1891	−0.2024	−0.2259	−0.2007	−0.1978	−0.2067
ϕ_8	−0.3285	−0.3111	−0.3308	−0.3273	−0.3359	−0.3361	−0.3565	−0.3360	−0.3255	−0.3226
ϕ_9	−0.2761	−0.3063	−0.2712	−0.3062	−0.2660	−0.2882	−0.2883	−0.2910	−0.3151	−0.3257
ϕ_{10}	−0.5345	−0.5614	−0.5280	−0.4964	−0.5359	−0.5486	−0.5724	−0.6553	−0.5214	−0.5916
ϕ_{11}	−0.2837	−0.2576	−0.2948	−0.2433	−0.2550	−0.3195	−0.3273	−0.2954	−0.3253	−0.2828
ϕ_{12}	−0.2665	−0.2212	−0.2820	−0.2581	−0.2778	−0.2664	−0.2530	−0.2711	−0.2559	−0.2900
ϕ_{13}	−0.1245	−0.1298	−0.1119	−0.1294	−0.1415	−0.1266	−0.1324	−0.1512	−0.1165	−0.1340
ϕ_{14}	−0.1106	−0.0922	−0.1187	−0.0997	−0.1010	−0.1103	−0.1164	−0.1035	−0.1187	−0.1082
ϕ_{15}	0.2301	0.2458	0.2507	0.2382	0.2399	0.2265	0.2721	0.2223	0.2644	0.2717
ϕ_{16}	0.2146	0.2177	0.2168	0.2004	0.2030	0.1932	0.1848	0.2520	0.2040	0.2382
ϕ_{17}	0.2512	0.2744	0.2412	0.2240	0.2203	0.2682	0.2888	0.2715	0.3081	0.2463
ϕ_{18}	0.0690	0.0714	0.0615	0.0698	0.0678	0.0692	0.0649	0.0578	0.0753	0.0679
ϕ_{19}	0.0320	0.0336	0.0314	0.0291	0.0343	0.0304	0.0333	0.0273	0.0291	0.0377
ϕ_{20}	−0.0176	−0.0168	−0.0174	−0.0197	−0.0166	−0.0155	−0.0164	−0.0160	−0.0174	−0.0169
ϕ_{21}	−0.0024	−0.0024	−0.0023	−0.0022	−0.0019	−0.0021	−0.0018	−0.0021	−0.0026	−0.0027
ϕ_{22}	0.0670	0.0725	0.0593	0.0675	0.0664	0.0611	0.0660	0.0670	0.0774	0.0637
ϕ_{23}	0.4544	0.4690	0.4359	0.4493	0.4343	0.5347	0.4400	0.4944	0.4753	0.4273
ϕ_{24}	0.2916	0.3011	0.2588	0.3547	0.2762	0.2650	0.2875	0.2919	0.3313	0.2718
ϕ_{25}	0.5055	0.5161	0.5123	0.5554	0.5080	0.4899	0.5365	0.5445	0.5289	0.5296
ϕ_{26}	0.2681	0.3054	0.2323	0.2369	0.2881	0.2876	0.2430	0.3195	0.2120	0.2190
ϕ_{27}	0.3541	0.3251	0.3773	0.3335	0.3370	0.3175	0.3929	0.3600	0.3100	0.3334
ϕ_{28}	0.1985	0.2329	0.1893	0.1927	0.2178	0.1914	0.1949	0.2044	0.2121	0.2358
ϕ_{29}	0.3475	0.3047	0.3470	0.3739	0.3329	0.4103	0.3428	0.3245	0.3469	0.3050
ϕ_{30}	0.1836	0.1791	0.2070	0.1914	0.1789	0.2120	0.1949	0.2078	0.1708	0.1820
θ	0.0285	0.0282	0.0269	0.0270	0.0279	0.0278	0.0287	0.0281	0.0286	0.0284
ω	−0.0545	−0.0697	−0.0684	−0.0629	−0.0737	−0.0618	−0.0069	−0.0215	−0.0341	−0.0311
α	0.0398	0.0448	0.0300	0.0434	0.0378	0.0397	0.0466	0.0393	0.0356	0.0400
α^*	0.0245	0.0223	0.0296	0.0275	0.0235	0.0247	0.0275	0.0249	0.0257	0.0248
β	0.9883	0.9850	0.9853	0.9865	0.9841	0.9867	0.9985	0.9954	0.9927	0.9933
λ_0	−5.0791	−5.0791	−5.0791	−5.0791	−5.0791	−5.0791	−5.0791	−5.0791	−5.0791	−5.0791
δ_1	0.7058	0.6975	0.6803	0.7945	0.7762	0.7425	0.7471	0.7590	0.7134	0.7348
δ_2	−0.0625	−0.0626	−0.0640	−0.0637	−0.0627	−0.0598	−0.0580	−0.0595	−0.0643	−0.0589

Table A19 (continued). Parameters for Monte Carlo simulation, NIG-DCS with constant ν_t, η_t

	Θ_{MC11}	Θ_{MC12}	Θ_{MC13}	Θ_{MC14}	Θ_{MC15}	Θ_{MC16}	Θ_{MC17}	Θ_{MC18}	Θ_{MC19}	Θ_{MC20}
c	0.0012	0.0012	0.0013	0.0015	0.0014	0.0015	0.0012	0.0015	0.0016	0.0015
ϕ_1	−0.4925	−0.5258	−0.5418	−0.4767	−0.5772	−0.5361	−0.6060	−0.5563	−0.5676	−0.5682
ϕ_2	−0.4847	−0.5291	−0.5776	−0.6715	−0.5805	−0.5883	−0.5550	−0.5373	−0.6072	−0.6321
ϕ_3	−0.2887	−0.2750	−0.2782	−0.2904	−0.2341	−0.2796	−0.2646	−0.2755	−0.3122	−0.2669
ϕ_4	−0.1421	−0.1563	−0.1555	−0.1814	−0.1588	−0.1808	−0.1790	−0.1514	−0.1896	−0.1828
ϕ_5	−0.1076	−0.0954	−0.0996	−0.0961	−0.1086	−0.1201	−0.1051	−0.1013	−0.0913	−0.0902
ϕ_6	−0.1407	−0.1701	−0.1309	−0.1473	−0.1188	−0.1271	−0.1382	−0.1897	−0.1381	−0.1563
ϕ_7	−0.2054	−0.1909	−0.2242	−0.2061	−0.1651	−0.2125	−0.2007	−0.2040	−0.2330	−0.2114
ϕ_8	−0.2829	−0.2681	−0.3383	−0.3290	−0.3556	−0.3284	−0.2839	−0.3161	−0.3328	−0.3586
ϕ_9	−0.2461	−0.3103	−0.2636	−0.3199	−0.2786	−0.2890	−0.2582	−0.3025	−0.2930	−0.2276
ϕ_{10}	−0.5512	−0.5287	−0.5835	−0.5664	−0.4232	−0.6500	−0.5193	−0.5537	−0.5731	−0.4790
ϕ_{11}	−0.2653	−0.2584	−0.2729	−0.2698	−0.2907	−0.2760	−0.3123	−0.3370	−0.2884	−0.2865
ϕ_{12}	−0.2908	−0.2468	−0.2230	−0.2730	−0.2940	−0.2488	−0.2726	−0.2421	−0.2840	−0.2819
ϕ_{13}	−0.1285	−0.1058	−0.1237	−0.1118	−0.1568	−0.1231	−0.1223	−0.1296	−0.1245	−0.1255
ϕ_{14}	−0.1266	−0.1200	−0.1096	−0.1211	−0.0943	−0.1106	−0.0981	−0.1026	−0.1083	−0.1028
ϕ_{15}	0.2424	0.2297	0.2422	0.2319	0.2207	0.2545	0.2399	0.2511	0.2010	0.2464
ϕ_{16}	0.1767	0.2222	0.2501	0.2138	0.2079	0.2119	0.2635	0.2067	0.2049	0.2202
ϕ_{17}	0.2714	0.2842	0.2201	0.2395	0.2650	0.2401	0.2467	0.2504	0.2366	0.2376
ϕ_{18}	0.0754	0.0664	0.0795	0.0638	0.0589	0.0718	0.0696	0.0562	0.0692	0.0655
ϕ_{19}	0.0335	0.0284	0.0318	0.0288	0.0299	0.0323	0.0326	0.0304	0.0276	0.0337
ϕ_{20}	−0.0187	−0.0207	−0.0202	−0.0173	−0.0161	−0.0177	−0.0181	−0.0187	−0.0172	−0.0195
ϕ_{21}	−0.0024	−0.0024	−0.0022	−0.0023	−0.0021	−0.0025	−0.0027	−0.0024	−0.0025	−0.0028
ϕ_{22}	0.0639	0.0750	0.0652	0.0695	0.0679	0.0772	0.0628	0.0743	0.0664	0.0656
ϕ_{23}	0.5768	0.4592	0.4252	0.4530	0.4376	0.5078	0.5703	0.4993	0.4133	0.4035
ϕ_{24}	0.2680	0.2635	0.2666	0.2827	0.2738	0.3308	0.3092	0.2531	0.2558	0.2956
ϕ_{25}	0.3765	0.5480	0.5421	0.5043	0.5127	0.4466	0.5783	0.4668	0.5174	0.4955
ϕ_{26}	0.2948	0.3021	0.2450	0.3248	0.2304	0.2518	0.2638	0.2716	0.2949	0.2903
ϕ_{27}	0.3767	0.3448	0.3324	0.3815	0.3418	0.3210	0.3048	0.3508	0.4062	0.3385
ϕ_{28}	0.1766	0.2068	0.2301	0.1996	0.2127	0.1919	0.1772	0.1863	0.2035	0.1874
ϕ_{29}	0.4237	0.3174	0.3926	0.3212	0.3295	0.3510	0.3816	0.3889	0.3822	0.2968
ϕ_{30}	0.1585	0.1902	0.1859	0.2018	0.1714	0.1500	0.1787	0.1664	0.1619	0.2111
θ	0.0270	0.0276	0.0289	0.0291	0.0284	0.0270	0.0281	0.0305	0.0270	0.0281
ω	−0.0728	−0.0574	−0.0881	−0.0445	−0.0517	−0.0547	−0.1063	−0.0398	−0.0629	−0.0372
α	0.0381	0.0416	0.0420	0.0367	0.0426	0.0351	0.0476	0.0430	0.0440	0.0404
α^*	0.0231	0.0235	0.0237	0.0251	0.0245	0.0291	0.0229	0.0233	0.0241	0.0239
β	0.9843	0.9876	0.9810	0.9904	0.9889	0.9882	0.9771	0.9914	0.9864	0.9920
λ_0	−5.0791	−5.0791	−5.0791	−5.0791	−5.0791	−5.0791	−5.0791	−5.0791	−5.0791	−5.0791
δ_1	0.7317	0.6970	0.6738	0.6972	0.7409	0.6826	0.6966	0.7346	0.6875	0.7704
δ_2	−0.0695	−0.0723	−0.0584	−0.0641	−0.0703	−0.0606	−0.0620	−0.0645	−0.0677	−0.0625

Table A20. Parameters for Monte Carlo simulation, NIG-DCS with constant ν_t, dynamic η_t

	Θ_{MC1}	Θ_{MC2}	Θ_{MC3}	Θ_{MC4}	Θ_{MC5}	Θ_{MC6}	Θ_{MC7}	Θ_{MC8}	Θ_{MC9}	Θ_{MC10}
c	0.0031	0.0032	0.0032	0.0032	0.0032	0.0028	0.0031	0.0033	0.0031	0.0033
ϕ_1	−0.6990	−0.7609	−0.6157	−0.6339	−0.7277	−0.6300	−0.6562	−0.6965	−0.7981	−0.7888
ϕ_2	−0.8014	−0.9423	−0.8024	−0.8307	−0.8341	−0.6458	−0.8610	−0.8381	−0.7812	−0.8508
ϕ_3	−0.4585	−0.4966	−0.4520	−0.4256	−0.4047	−0.3249	−0.5095	−0.4373	−0.4972	−0.3958
ϕ_4	−0.2203	−0.2129	−0.2022	−0.2025	−0.2364	−0.2181	−0.2237	−0.2004	−0.1871	−0.2167
ϕ_5	−0.1448	−0.1414	−0.1409	−0.1466	−0.1300	−0.1359	−0.1584	−0.1531	−0.1371	−0.1482
ϕ_6	−0.1086	−0.1050	−0.0990	−0.1099	−0.0950	−0.1070	−0.1014	−0.1049	−0.0944	−0.1080
ϕ_7	−0.2340	−0.2310	−0.2261	−0.2515	−0.2310	−0.2327	−0.2224	−0.2189	−0.2254	−0.2090
ϕ_8	−0.4214	−0.3854	−0.4886	−0.4078	−0.4596	−0.4302	−0.3884	−0.3599	−0.5037	−0.4141
ϕ_9	−0.4067	−0.4815	−0.4304	−0.4802	−0.4077	−0.2900	−0.4709	−0.4164	−0.3837	−0.4169
ϕ_{10}	−0.7191	−0.7757	−0.6748	−0.8128	−0.7236	−0.5997	−0.7469	−0.8316	−0.7917	−0.8940
ϕ_{11}	−0.5053	−0.4996	−0.4703	−0.5054	−0.5293	−0.5128	−0.5337	−0.5240	−0.4420	−0.5283
ϕ_{12}	−0.4033	−0.4506	−0.4390	−0.4000	−0.4076	−0.4472	−0.4812	−0.4576	−0.3975	−0.4527
ϕ_{13}	−0.2669	−0.2863	−0.2973	−0.2675	−0.2972	−0.2819	−0.2881	−0.2448	−0.2842	−0.2598
ϕ_{14}	−0.1763	−0.1645	−0.1787	−0.1832	−0.1595	−0.1911	−0.1856	−0.1559	−0.1477	−0.1900
ϕ_{15}	0.1784	0.2045	0.1889	0.1512	0.1690	0.1737	0.2028	0.1496	0.1844	0.1793
ϕ_{16}	0.1956	0.1973	0.2047	0.2046	0.1875	0.1728	0.2255	0.1752	0.2040	0.2344
ϕ_{17}	0.3047	0.3319	0.2658	0.3193	0.2886	0.3162	0.3364	0.3136	0.3215	0.2841
ϕ_{18}	0.0244	0.0238	0.0239	0.0228	0.0248	0.0251	0.0274	0.0250	0.0219	0.0231
ϕ_{19}	−0.0817	−0.0753	−0.0897	−0.0733	−0.0741	−0.0784	−0.0957	−0.0902	−0.0803	−0.0868
ϕ_{20}	−0.0925	−0.0981	−0.0898	−0.0927	−0.1038	−0.0971	−0.0957	−0.0972	−0.0971	−0.0973
ϕ_{21}	−0.1804	−0.1657	−0.1844	−0.1702	−0.1611	−0.1706	−0.1954	−0.1958	−0.2269	−0.1828
ϕ_{22}	−0.0169	−0.0201	−0.0173	−0.0176	−0.0199	−0.0176	−0.0171	−0.0172	−0.0149	−0.0150
ϕ_{23}	0.4086	0.4277	0.3368	0.3847	0.4298	0.3807	0.4012	0.4272	0.4042	0.4034
ϕ_{24}	0.3269	0.2930	0.3650	0.3053	0.3466	0.3617	0.3765	0.2960	0.3256	0.3600
ϕ_{25}	0.6580	0.7327	0.7357	0.6401	0.7358	0.6447	0.7235	0.7012	0.6461	0.7580
ϕ_{26}	0.3409	0.3704	0.2980	0.3177	0.3170	0.2849	0.3779	0.3306	0.4106	0.3475
ϕ_{27}	0.3814	0.3818	0.3690	0.3711	0.3214	0.3616	0.3527	0.2682	0.4199	0.3883
ϕ_{28}	0.1791	0.1651	0.1329	0.1969	0.1816	0.1632	0.1890	0.1836	0.1948	0.1744
ϕ_{29}	0.2722	0.3174	0.2712	0.3110	0.2861	0.2272	0.3024	0.2472	0.2753	0.2314
ϕ_{30}	0.2624	0.2780	0.2287	0.2282	0.2222	0.2536	0.2749	0.3034	0.2739	0.2861
θ	0.0198	0.0197	0.0198	0.0197	0.0186	0.0189	0.0203	0.0212	0.0195	0.0201
ω	−0.0540	−0.0753	−0.0446	−0.0547	−0.0714	−0.0779	−0.0862	−0.0321	−0.0991	−0.0517
α	0.0406	0.0396	0.0489	0.0435	0.0338	0.0382	0.0379	0.0445	0.0417	0.0390
α^*	0.0248	0.0258	0.0251	0.0259	0.0293	0.0235	0.0255	0.0212	0.0256	0.0262
β	0.9883	0.9837	0.9904	0.9882	0.9846	0.9832	0.9814	0.9931	0.9786	0.9888
λ_0	−5.3231	−5.3231	−5.3231	−5.3231	−5.3231	−5.3231	−5.3231	−5.3231	−5.3231	−5.3231
δ_1	0.7121	0.7209	0.7176	0.6870	0.6781	0.6400	0.7111	0.7573	0.6858	0.6422
δ_2	−0.0395	−0.0400	−0.0390	−0.0398	−0.0390	−0.0386	−0.0400	−0.0398	−0.0401	−0.0393
γ_2	0.3379	0.3290	0.3464	0.3323	0.3473	0.3535	0.3300	0.3334	0.3288	0.3416
κ_2	0.0210	0.0217	0.0220	0.0209	0.0203	0.0212	0.0194	0.0212	0.0222	0.0200

Table A20 (continued). Parameters for Monte Carlo simulation, NIG-DCS with constant ν_t, dynamic η_t

	Θ_{MC11}	Θ_{MC12}	Θ_{MC13}	Θ_{MC14}	Θ_{MC15}	Θ_{MC16}	Θ_{MC17}	Θ_{MC18}	Θ_{MC19}	Θ_{MC20}
c	0.0031	0.0032	0.0032	0.0034	0.0033	0.0034	0.0030	0.0028	0.0031	0.0031
ϕ_1	−0.6378	−0.7354	−0.7581	−0.6477	−0.7434	−0.7023	−0.6933	−0.7406	−0.6356	−0.7357
ϕ_2	−0.8703	−0.7385	−0.8282	−0.7963	−0.9713	−0.8026	−0.8173	−0.5934	−0.8200	−0.8695
ϕ_3	−0.4718	−0.4782	−0.4849	−0.4777	−0.4948	−0.4024	−0.4457	−0.4534	−0.4685	−0.4909
ϕ_4	−0.2322	−0.1942	−0.2319	−0.2438	−0.2261	−0.2151	−0.2376	−0.2379	−0.2037	−0.2154
ϕ_5	−0.1548	−0.1324	−0.1590	−0.1415	−0.1428	−0.1603	−0.1279	−0.1448	−0.1195	−0.1152
ϕ_6	−0.1288	−0.1205	−0.1275	−0.1185	−0.1134	−0.1233	−0.0832	−0.1142	−0.0932	−0.0919
ϕ_7	−0.2624	−0.2431	−0.2018	−0.2023	−0.2767	−0.2642	−0.2106	−0.2352	−0.2329	−0.1983
ϕ_8	−0.5216	−0.3960	−0.4291	−0.4651	−0.3768	−0.4486	−0.4180	−0.4046	−0.4297	−0.4737
ϕ_9	−0.4275	−0.4500	−0.3731	−0.5040	−0.3768	−0.3889	−0.4899	−0.4652	−0.3865	−0.4292
ϕ_{10}	−0.5549	−0.7046	−0.7102	−0.8094	−0.7798	−0.8237	−0.5280	−0.7138	−0.8354	−0.6956
ϕ_{11}	−0.5773	−0.5127	−0.5434	−0.5628	−0.5027	−0.5568	−0.4592	−0.5615	−0.5181	−0.4681
ϕ_{12}	−0.3775	−0.4129	−0.4653	−0.3915	−0.4266	−0.4194	−0.4499	−0.3378	−0.4204	−0.3371
ϕ_{13}	−0.2453	−0.2796	−0.2887	−0.2797	−0.2313	−0.2868	−0.3084	−0.2655	−0.2962	−0.2331
ϕ_{14}	−0.2075	−0.1987	−0.1751	−0.1998	−0.1560	−0.1839	−0.1731	−0.1790	−0.1663	−0.1852
ϕ_{15}	0.1920	0.1943	0.1775	0.1218	0.1559	0.1815	0.1462	0.1599	0.1832	0.2111
ϕ_{16}	0.2013	0.2047	0.2416	0.1967	0.1959	0.2171	0.2066	0.2204	0.1878	0.2132
ϕ_{17}	0.3427	0.3103	0.3394	0.2737	0.2998	0.2608	0.3332	0.3701	0.3013	0.2654
ϕ_{18}	0.0257	0.0273	0.0258	0.0268	0.0255	0.0238	0.0258	0.0232	0.0219	0.0213
ϕ_{19}	−0.0630	−0.0761	−0.0910	−0.0850	−0.0859	−0.0821	−0.0739	−0.0818	−0.0797	−0.0887
ϕ_{20}	−0.1019	−0.0894	−0.0951	−0.0942	−0.0835	−0.0899	−0.1081	−0.1011	−0.0987	−0.0854
ϕ_{21}	−0.1953	−0.1710	−0.1823	−0.1592	−0.1631	−0.1888	−0.1855	−0.1782	−0.1633	−0.1715
ϕ_{22}	−0.0179	−0.0176	−0.0173	−0.0173	−0.0178	−0.0146	−0.0168	−0.0187	−0.0157	−0.0168
ϕ_{23}	0.4061	0.4447	0.3733	0.3754	0.4444	0.4392	0.4165	0.4979	0.4057	0.3330
ϕ_{24}	0.2919	0.2862	0.3460	0.3472	0.3574	0.3261	0.3434	0.3571	0.3043	0.4003
ϕ_{25}	0.6779	0.6402	0.6933	0.7432	0.6381	0.5295	0.6886	0.6841	0.7268	0.6067
ϕ_{26}	0.3493	0.3166	0.3857	0.3686	0.3890	0.3729	0.2943	0.4058	0.3635	0.3310
ϕ_{27}	0.3802	0.3217	0.3731	0.4050	0.3429	0.4285	0.3840	0.3819	0.2806	0.3977
ϕ_{28}	0.1842	0.1848	0.1930	0.1718	0.1819	0.1597	0.1949	0.1535	0.1983	0.2026
ϕ_{29}	0.2877	0.2441	0.1963	0.2317	0.2549	0.2905	0.2625	0.2866	0.2876	0.2681
ϕ_{30}	0.2752	0.2764	0.2582	0.2209	0.2608	0.2189	0.2746	0.2539	0.2560	0.2621
θ	0.0199	0.0204	0.0201	0.0197	0.0196	0.0207	0.0203	0.0194	0.0197	0.0192
ω	−0.0575	−0.0561	−0.0341	−0.0287	−0.0756	−0.0412	−0.0385	−0.1099	−0.0420	−0.0470
α	0.0413	0.0389	0.0414	0.0474	0.0449	0.0450	0.0370	0.0261	0.0383	0.0409
α^*	0.0201	0.0214	0.0181	0.0269	0.0280	0.0242	0.0281	0.0205	0.0209	0.0261
β	0.9876	0.9879	0.9926	0.9938	0.9837	0.9911	0.9917	0.9763	0.9909	0.9899
λ_0	−5.3231	−5.3231	−5.3231	−5.3231	−5.3231	−5.3231	−5.3231	−5.3231	−5.3231	−5.3231
δ_1	0.6378	0.7702	0.7251	0.7302	0.7377	0.7183	0.7260	0.7089	0.7347	0.6554
δ_2	−0.0387	−0.0390	−0.0388	−0.0398	−0.0391	−0.0385	−0.0384	−0.0400	−0.0384	−0.0375
γ_2	0.3511	0.3473	0.3504	0.3327	0.3447	0.3554	0.3560	0.3291	0.3564	0.3709
κ_2	0.0202	0.0204	0.0223	0.0202	0.0230	0.0199	0.0214	0.0224	0.0202	0.0210