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THE THEORY OF IMPLEMENTATION: WHAT DID WE LEARN?*

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ARTICLE OUTLINE

GLOSSARY

Type of an agent. All the information possessed by this agent. It may refer to the preferences of this agent and/or to the knowlege of this agent of the preferences of other agents,

State of the World. Description of all information possessed by all agents.

Social Choice Rule. A correspondence mapping the set of states of the world in the set of allocations. It represents the social objectives that the society or its representatives want to achieve.

Mechanism. A list of message spaces and an outcome function mapping messages into allocations. It represents the communication and decision aspects of the organization.

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Equilibrium Concept. A mapping (or a collection of them) from the set of states of the world into allocations yielded by equilibrium messages. This equilibrium is a game-theoretical notion of how agents behave, e.g. Nash Equilibrium, Bayesian Equilibrium, Dominant Strategies, etc.

Implementable Social Choice Rule in an Equilibrium Concept (e.g. Nash Equilibrium). A Social Choice Rule is implementable in an equilibrium concept (e. g. Nash Equilibrium) if there is a mechanism such that for each state of the world the allocations prescribed by the Social Choice Rule and those yielded by the equilibrium concept coincide.

DEFINITION

Implementation theory studies which social objectives (i.e. Social Choice Rules) are compatible with the incentives of the agents (i.e. are implementable). In other words it is the systematic study of the social goals that can be achived when agents behave strategically.

1 Introduction

Dear colleague;

I wrote this survey with you in mind. You are an economist doing research who would like to know why implementation is important. And by this I do not mean why some people won the Nobel Prize working in this area. I mean, what are the deep insights found by implementation theory and what applications are delivered by these tools. I propose a simple game: try to answer the following questions. If you cannot answer them, but you think they are important, read the survey. At the end of this survey, I will give you the answers. I will also tell you why I like implementation theory so much!

1. Why are agents price-takers? Is price-taking possible in economies with a finite number of agents?
2. Suppose two firms wish to merge. They claim that the merger will bring large cost reductions but some people fear that the firms just want to avoid competition. What would be your advice?
3. How should a monopoly be regulated when regulators do not know the cost function or the demand function of the monopolist?
4. How should it be determined whether or not a public facility -a road, a bridge, a stadium- should be constructed and who should pay for it?
5. Is justice possible in this world? Can we reconcile justice and self-interest?
6. Can an uninformed planner achieve better allocations than those produced by completely-informed agents in an unregulated market?
7. In competitive ice skating, the highest and lowest marks awarded by judges are discarded and the remaining are averaged. Do you think that this procedure eliminates incentives to manipulate votes?
8. What kind of policies would you advocate to fight Global Warming?

The answers to these questions are found in Section 6. The rest of this paper goes as follows. Section 2 is a historical introduction that can be skipped. Section 3 explains the basic model. Section 4 explains the main results. Section 5 offers some thoughts about the future direction of the topic.

2 Brief History of Implementation Theory

From, at least Adam Smith on, we have assumed that agents are motivated by self-interest. We also assumed that agents interact in a market economy where prices match supply and demand. This tradition crystallized in the Arrow-Debreu-McKenzie model of General equilibrium in the 1950s. But it was quickly discovered that this model had important pitfalls other than focussing on a narrow class of economic systems: On the one hand, an extra agent was needed to set prices, the auctioneer. On the other hand agents follow rules, i.e. to take prices as given, which are not necessarily consistent with self-interest. An identical question had arisen earlier when Taylor (1929) and Lange (1936-7), following Barone (1908), proposed a market socialism, where socialist managers maximize profits: Why would socialist managers choose output in the way prescribed to them (or who will provide and preserve capital in a system where the private property of such items is forbidden?)? Samuelson (1954) voiced identical concern about the Lindahl solution to allocate public goods: "It is in the selfish interest of each person to give false signals". This concern gave rise later on to the golden rule of incentives -as stated by Roger Myerson (1985): "An organization must give its members the correct incentives to share information and act appropriately". Earlier, it had aroused the interest of Leonid Hurwicz, the father of Implementation theory, in economic systems other than the market. In any case it was clear that an important ingredient was missing in the theory of economic systems. This element was that not all information needed for resource allocation was transmitted by prices: Some vital items have to be transmitted by agents.

Several proposals arose to fill the gap: On the one hand, models of markets under asymmetric information, Vickrey (1961), Akerlof (1970), Spence (1973) and Rothchild and Stiglitz (1976). On the other hand models of public intervention, like optimal taxation, Mirless (1971), and mechanisms for allocating public goods, Clarke (1971) and Groves (1973), with the so-called Principal-Agent models somewhere in the middle. The key word was "Truthful Revelation" or "Incentive Compatibility": Truthful revelation of information must be an equilibrium strategy, either a dominant strategy, as in Clarke and Groves, or a Bayesian equilibrium as in Arrow (1977) and D'Aspremont and Gerard-Varet (1979). A motivation for this procedure was provided by the "Revelation

Principle", Gibbard (1973), Myerson (1979), Dasgupta, Hammond and Maskin (1979) and Harris and Townsend (1981): If a mechanism yields certain allocations in equilibrium, telling the truth about one's characteristics must be an equilibrium as well (however, telling the truth may not be an equilibrium in the original mechanism you might have to use an equivalent direct mechanism). This result is of utmost importance and it will be thoroughly considered in Section 3. However, it was somehow misread as "there is no loss of generality in focussing on incentive compatibility". But what the revelation principle asserts is that truthful revelation is *one* of the, possibly, many equilibria. It does not say that truthful revelation is *the only* equilibrium. As we will see in some cases it is a particularly unsatisfactory way of selecting equilibria.

The paper by Hurwicz (1959), popularized by Reiter (1977), presented a formal structure for the study of economic mechanisms which has been followed by all subsequent papers. Maskin (1999), whose first version circulated in 1977, is credited as the first paper where the problem of multiple equilibria was addressed as a part of the model and not as an afterthought, see the report of the Nobel Prize Committee (2007). Maskin studied implementation in Nash equilibrium (see Glossary). Later his results were generalized to Bayesian Equilibrium by Postlewaite and Schmeidler (1986) and Palfrey and Srivastava (1987), (1989).

Finally, Moulin (1979) studied Dominance Solvability and Moore and Repullo (1988) Subgame Perfect Equilibrium. The century closed with several characterizations on what can be implemented in other equilibrium concepts: Moore and Repullo (1990) in Nash Equilibrium, Palfrey and Srivastava (1991) in Undominated Nash Equilibrium, Jackson (1991) in Bayesian Equilibrium, Dutta and Sen (1991a) in Strong Equilibrium and Sjöström (1993) in Trembling Hand Equilibria. With all these papers in mind, the basic aspects of implementation theory are now well understood.

The interested reader may complement the previous account with the surveys by Maskin and Sjöström (2002) and Serrano (2004) which cover the basic results and by Baliga and Sjöström (2007) for new developments including experiments. See also Maskin (1985), Moore (1992), Corchón (1996), Jackson (2001) and Palfrey (2002). Several important applications of Implementation Theory are not surveyed here: Auctions, see Krishna (2002), Contract theory, see Laffont and Martimort (2001), Matching, see Roth (forthcoming) and Moral Hazard see Ma, Moore and Turnbull (1988).

3 The Main Concepts

We divide this section into four subsections: The first describes the environment, the second deals with social objectives, the third revolves around the notion of a mechanism and the last defines the equilibrium concepts that we will use here.

3.1 The Environment

Let $I = \{1, ..., n\}$ be the set of agents. Let θ_i be the type of i. This includes all the information in the hands of i. Let Θ_i be agent i's type set. The set $\Theta \subset \prod_{i=1}^{n} \Theta_i$ is the set of states of the world. For each $\theta \in \Theta$ we have a feasible set $A(\theta)$ and a preference profile $R(\theta) = (R_1(\theta), ..., R_n(\theta))$. $R_i(\theta)$ is a complete, reflexive and transitive binary relation on $A(\theta)$. $I_i(\theta)$ denotes the corresponding indifference relation. Set $A \equiv \bigcup_{\theta \in \Theta} A(\theta)$. Let $a = (a_1, a_2, ..., a_n) \in A$ be an allocation, also written (a_i, a_{-i}), where $a_{-i} \equiv (a_1, a_2, ..., a_{i-1}, a_{i+1}, ..., a_n)$.

The standard model of an exchange economy is a special case of this model: θ is an economy. $X_i(\theta) \subset \Re^k$ is the consumption set of i. $w_i(\theta) \in intX_i(\theta)$ are the endowments in the hands of i. The preferences of i are defined on $X_i(\theta)$. The set of allocations $A(\theta)$ is defined as

$$A(\theta) = \{a \mid \sum_{i=1}^{n} (a_{ij} + w_{ij}(\theta)) \leq 0,\ j = 1, 2, ..., k,\ (a_{i1}, a_{i2}, ..., a_{ik}) \in X_i(\theta),\ \forall i \in I\}.$$

A special case of an exchange economy is bilateral trading: Here there are two agents, the seller and the buyer. The seller has a unit of an indivisible good and both agents are endowed with an infinitely divisible good ("money"). Preferences are representable by linear utility functions. The type of each agent, also called her valuation, is the marginal rate of substitution between both goods. Finally, the set of types is a closed interval of the real line.

Another example is the social choice model where the set of states of the world is the Cartesian product of individual type sets, $\Theta = \prod_{i=1}^{n} \Theta_i$. The set of feasible allocations is constant. The preferences of each agent only depend on her type, for all $\theta \in \Theta$, $R_i(\theta) = R_i(\theta_i)$ all $i \in I$.

The model of public goods is a hybrid of the social choice and the exchange economy models. For a subset of goods, say $1, 2, ..., l$, agents receive the same bundle (these are the public goods). For goods $l + 1, ..., k$, agents can consume possibly different bundles.

3.2 Social objectives

Implementation begins by asking what allocations *we* want to achieve. In this sense, implementation theory reverses the usual procedure, namely, fix a mechanism and see what the outcomes are. The theory is rather agnostic as to who is behind *we*: It could be a democratic society, it could be a dictator, a benevolent planner, etc. Formally, a correspondence $F : \Theta \twoheadrightarrow A$ such that $F(\theta) \subseteq A(\theta)$ for all $\theta \in \Theta$ will be called a *Social Choice Rule* (*SCR*). Under risk or uncertainty, allocations are state-dependent (recall the concept of contingent commodities in General Equilibrium). Thus an allocation is a single-valued function $f : \Theta \rightarrow A$. The notion of a SCR is replaced by that of a *Social Choice Set* (SCS) defined as a collection of functions mapping Θ into A. Examples of SCR are the Pareto rule, which maps every state into the set of Pareto efficient allocations for this state, the Walrasian SCR which maps every economy in the set of allocations that are a Walrasian Equilibrium for this economy, etc.

If states of the world were contractible, i.e. if they could be written in an enforceable contract specifying the allocations in each state, SCR or SCS would be directly achieved, assuming that those not complying could be punished harshly. Unfortunately, states of the world are a description of preferences and productive capabilities, being those difficult to describe and therefore easy to manipulate. Thus, we have to find another method to reach the desired allocations.

3.3 Mechanisms

If the information necessary to judge the desirability of allocations is in the hands of agents, it seems that the only way of retrieving this information is by asking them. But, of course, agents cannot be trusted to reveal truthfully their information because they might lose by doing so. Thus the owner of a defective car will think twice about revealing the true state of the car if the price of defective cars is less than the price of reliable cars. But perhaps we may design ways in which the messages sent by different agents are checked one against the other. We may also design ways in which agents send information by indirect means, say by raising flags, making gestures, and so on and so forth. This is the idea behind the concept of a mechanism (also called a game form).

Formally, a *mechanism* is a pair (M, g) where $M \equiv \prod_1^n M_i$ is the *message space* and $g : M \rightarrow A$ is the outcome function. M_i denotes agent i's *message space with typical element* m_i. In some cases, i.e. when goods are indivisible, the outcome function maps M into the set of lotteries on A, denoted by $\mathcal{L}A$. In this case the outcome function yields the probability of obtaining an object. Let $m = (m_1, .., m_n) \in M$, be a list of messages, also written (m_i, m_{-i}) where m_{-i} is a list of all messages except those sent by i.

Another interpretation of a mechanism, more in tune with decentralized systems, is that messages describe contracts among agents and the outcome function is a legal system that converts contracts into allocations.

If feasible sets are state dependent we have a problem: Suppose that at θ we want to achieve allocation $a \in A(\theta)$. So there must be a message, say m such that $g(m) = a$. But what if there is another state, say θ' for which $a \notin A(\theta')$? In this case $g(m) \notin A(\theta')$. In other words, since mechanisms are not state dependent they may yield unfeasible allocations. We will postpone the discussion of this problem until Section 5. For the time being, let us assume that feasible sets are not state dependent.

3.4 Equilibrium

Since the messages sent by agents are tied to their incentives, it is clear that we have to use an equilibrium concept borrowed from game theory. Thus, given $\theta \in \Theta$, a mechanism (M, g) induces a game in normal form (M, g, θ). There are many "solutions" to what would constitute an equilibrium. Let us begin by considering the notion of a Nash equilibrium

Definition 1: A message profile $m^* \in M$ is a Nash equilibrium for (M, g, θ) if, for all $i \in I$ $g(m^*)R_i(\theta)g(m_i, m^*_{-i})$ for all $m_i \in M_i$.

Let $NE(M, g, \theta)$ be the set of allocations yielded by all Nash equilibria of (M, g, θ). We now ask, given a SCR, what mechanism, if any, would produce outcomes identical to the SCR. In this sense, the mechanism is the variable of our analysis i.e. the mechanism "solves" the equation $NE(M, g, \theta) = F(\theta)$, for all $\theta \in \Theta$. Formally,

Definition 2: The SCR F is implementable in Nash equilibrium if there is a mechanism (M, g) such that, for all $\theta \in \Theta$, $NE(M, g, \theta) \neq \emptyset$ and:

1: $F(\theta) \subseteq NE(M, g, \theta)$.

2: $NE(M,g,\theta) \subseteq F(\theta)$.

The previous concept can be easily generalized. Given a mechanism (M,g) an equilibrium concept is a mapping, say $E_{(M,g)} : \Theta \twoheadrightarrow A$ such that $E_{(M,g)}(\theta) \subseteq A(\theta)$ for all $\theta \in \Theta$. For instance $E_{(M,g)}(\theta)$ may be the set of allocations arising from dominant strategy profiles in θ when the mechanism (M,g) is in place. The notion of implementation in an equilibrium concept easily follows. See Thomson (1996) for a discussion of other concepts of implementation.

The problem is that some equilibrium concepts can not be written in the way we just described because the actions to be taken in state, say θ', depend on preferences in states other than θ'. To see this, suppose that agents attach a vector of probabilities to each possible type of the other agents, Harsanyi (1967/68). Denote by $q(\theta_{-i}/\theta_i)$ the vector of probabilities attached by i that other agents have types θ_{-i} given that she is of type θ_i. For simplicity assume that it is a strictly positive vector. Suppose that preferences are representable by a von Neumann-Morgenstern utility index $V_i(a,\theta)$. In this framework, (as first noticed by Vickrey [1961]) a strategy for i, denoted by s_i, is no longer a message but a function from the set of types of i in the set of messages of i, namely, $s_i : \Theta_i \to M_i$. A strategy profile, s, is a collection of strategies, one for each agent, $s = (s_1, ..., s_n)$ also written as (s_i, s_{-i}). For simplicity, the next definition assumes that type sets are finite.

Definition 3: A Bayesian Equilibrium (BE) for $(M,g,R(\cdot))$ is a s^* such that for all i, $\theta \in \Theta$, and $m_i \in M_i$,

$$\sum_{\theta_{-i} \in \Theta_{-i}} q(\theta_{-i} \mid \theta_i) V_i(g(s^*(\theta)), \theta) \geq \sum_{\theta_{-i} \in \Theta_{-i}} q(\theta_{-i} \mid \theta_i) V_i(g(m_i, s^*_{-i}(\theta_{-i})), \theta)$$

Thus, an equilibrium concept -given a mechanism- is a collection of functions, denoted by $H_{(M,g)}$, such that for all $h_{(M,g)} \in H_{(M,g)}$ $h_{(M,g)} : \Theta \to A$. Finally, the definition of implementable SCS in BE follows.

Definition 4: The mechanism (M,g) implements a SCS F in BE if:

1: For any BE s there exists $x \in F(\theta)$, such that $g(s(\theta)) = x(\theta)$ for all $\theta \in \Theta$.

2: For any $x \in F$, there is a BE s such that $g(s(\theta)) = x(\theta)$ for all $\theta \in \Theta$.

Looking at our definitions of an implementable SCR or SCS we see that the first requirement is that all equilibria yield "good" allocations. The second requirement is that given an allocation to be implemented, there is an equilibrium "sustaining" this allocation. These two requirements bear some resemblance to the two fundamental theorems of welfare economics, namely that competitive equilibrium is efficient and that any efficient allocation can be achieved as a competitive equilibrium with the appropriate endowment redistribution. Notice that endowment redistribution is not used in the definition of implementation.

4 The Main Insights

We group our results here under three headings: The Revelation Principle and its consequences, Monotonicity and how to avoid it and the limits of design. We will discuss them each in turn.

4.1 The Revelation Principle and its Consequences

The definition of a mechanism is extremely abstract. No conditions have been imposed on what might constitute a message space or an outcome function. And since implementation theory considers the mechanism the variable to be found, this is an unhappy situation: we are asked to find something whose characteristics we do not know! Fortunately the revelation principle comes to the rescue by stating a necessary condition for implementation: If a single valued SCR, which we will call a Social Choice Function (SCF) is implementable, there is a *revelation mechanism* for which telling the truth is an equilibrium. A revelation mechanism (associated with a SCF) is a mechanism in which the message space for each agent is her set of types and the outcome function is the SCF. We say that a SCF is truthfully implementable or incentive compatible if truth-telling is a Bayesian equilibrium (or a dominant strategy) of the direct mechanism associated with it. The following result formally states the revelation principle:

Theorem 1. If f is a Bayesian (resp. dominant strategy) implementable SCF, f is incentive compatible.

Proof. Let f be Bayesian implementable. Therefore, there exists a mechanism (M, g) and a Bayesian equilibrium s^* such that $g(s^*(\theta)) = f(\theta)$ for every $\theta \in \Theta$. Since $s^*(\cdot)$ is a Bayesian equilibrium, $\forall \theta \in \Theta, \forall m_i \in M$

$$\sum_{\theta_{-i} \in \Theta_{-i}} q(\theta_{-i} \mid \theta_i) V_i(g(s^*(\theta)), \theta) \geq \sum_{\theta_{-i} \in \Theta_{-i}} q(\theta_{-i} \mid \theta_i) V_i(g(m_i, s^*_{-i}(\theta_{-i})), \theta).$$

Which implies that $\forall\theta'_i \in \Theta_i, \forall\theta_{-i} \in \Theta_{-i}$,

$$\sum_{\theta_{-i}\in\Theta_{-i}} q(\theta_{-i} \mid \theta_i)V_i(f(\theta), \theta) \geq \sum_{\theta_{-i}\in\Theta_{-i}} q(\theta_{-i} \mid \theta_i)V_i(f(\theta'_i, \theta_{-i}), \theta).$$

The proof for the case of dominant strategies is identical. ■

Theorem 1 (T.1 in the sequel) can be explained in terms of a mediator, i.e. somebody to whom you say "who you are" and who chooses the strategy that maximizes your payoffs on your behalf. Would you try to fool such a person? If you do so, you are fooling yourself because the mediator would choose a strategy that is not the best for you. Thus, the best thing for you to do is to tell the truth (providing an unexpected backing to the aphorism "honesty is the best policy"!).

Consider now the following results, due to Hurwicz (1972) (who proved it for the case of $n = 2$) and to Gibbard (1973) and Satterthwaite (1975) respectively.

Theorem 2. In exchange economies environments there is no SCF such that:

1) It is truthfully implementable in dominant strategies.

2) It selects individually rational allocations.

3) It selects efficient allocations.

4) Its domain includes all economies with convex and continuous preferences.

Theorem 2'. In social choice environments there is no SCF such that:

1) It is truthfully implementable in dominant strategies.

2) It is non-dictatorial.

3) Its range is A, with $\#A > 2$.

4) Its domain includes all possible preference profiles.

It is clear that there are trivial SCF in which any three conditions in T. 2-2' are compatible. But T.2-2' are very robust in the sense that they hold for small domains of economies (Barberá and Peleg, 1990, Barberá, Sonnenschein and Zhou, 1991), for weaker notions of individual rationality (Saijo, 1991 and Serizawa and Weymark 2003) and in public goods domains (Ledyard and Roberts, 1974). Moreover, assuming quasi-linear utility functions, Hurwicz and Walker (1990), building on a previous paper by Walker, proved that the set of economies for which conditions 1-3 in T.2 are incompatible is open and dense. Beviá and Corchón (1995) show that these conditions are incompatible for *any* economy where utility functions are quasi-linear, strictly concave, differentiable and fulfill a very mild regularity condition. These results show that

Vickrey-Clarke-Groves mechanisms fail to achieve efficient allocations in general (Vickrey-Clarke-Groves mechanisms are revelation mechanisms that work in public good economies where utility functions are quasi-linear in "money". The outcome function selects the level of public good that maximizes the sum of utilities announced by agents and the money received by each individual is the sum of the utility functions announced by all other agents. For an exposition of these mechanisms, see Green and Laffont, 1979).

A proof of T. 2 can be found in Serizawa (2002). Simple proofs of T. 2 ' can be found in Barberá (1983), Benoit (2000) and Sen (2001).

T. 1 and 2-2' imply that there is no mechanism implementing an efficient and individually rational (resp. non-dictatorial) SCF in dominant strategies when the domain of the SCF is large enough. In other words, the revelation principle implies that the restriction to mechanisms where agents announce their own characteristic is not important when considering negative results. Thus, the Revelation principle is an appropriate tool for producing negative results. But we will see that to rely entirely on this principle when trying to implement a SCF may yield disastrous results.

A natural question to ask is what happens with the above impossibility results when we weaken the requirement of implementation in dominant strategies to that of implementation in Bayesian equilibrium. The following result, due to Myerson and Satterthwaite (1983), answers this question.

Theorem 2". In the bilateral trading environment there is no SCF such that:

1) It is truthfully implementable in Bayesian Equilibrium.

2) It selects individually rational allocations once agents learn their types.

3) It selects ex-post efficient allocations.

4) Its domain includes all linear utility functions with independent types distributed with positive density and the sets of types have a nonempty intersection.

Proof. (Sketch, see Krishna and Perry [1997] for details) By the revenue equivalence theorem (see Klemperer, 1999, Appendix A), all mechanisms fulfilling conditions 2) and 3) above raise identical revenue. So it is sufficient to consider the Vickrey-Clarke-Groves which, as we remarked before, is not efficient. ■

Again the weakening of any condition in T.2" may produce positive results (Williams (1999), Table 1, presents an illuminating discussion of this issue). For instance, suppose seller valuations are 1 or 3, and buyer valuations are 0 or 2. The mechanism fixes the price at 1.5 and a sale occurs when the valuation of the buyer is larger than the valuation of the seller. This mechanism implements truthfully a SCF satisfying 2) and 3) above. Unfortunately, it does not work when valuations are drawn from a common interval with positive densities.

But unlike T.2-2', there are robust examples of SCF truthfully implementable in Bayesian Equilibrium when conditions 2) or 4) are relaxed. Also, inefficiency converges to zero very quickly when the number of agents increases (see Gresik and Satterthwaite, 1989). This is because the equilibrium concept is now weaker and we are approaching a land where incentive compatibility has no bite, as we will see in T. 3 below.

First, d'Aspremont and Gerard-Varet (1975), (1979), and Arrow (1979) showed that conditions 1)-3)-4) are compatible with individual rationality *before* agents learn their types in the domain of public goods with quasi-linear utility functions. They proposed the "expected externality mechanism" in which each agent is charged the expected externality she creates on the remaining players. Later on, Myerson (1981) and Makowski and Mezzetti (1993) presented incentive compatible SCF yielding ex-post efficient and individually rational allocations in the domain of exchange economies with quasi-linear preferences and more than two buyers. In Myerson (1981), agents have correlated valuations. Buyers are charged even if they do not obtain the object or they may receive money and no object or even receive the object plus some money. Makoswki and Mezzetti (1993) assume no correlation and that the highest possible valuation for a buyer is larger than the seller's highest possible valuation. They consider a family of mechanisms, called Second Price Auction With Seller (SPAWS), in which the highest bidder obtains the object, the seller receives the first bid and the winning buyer pays the second price. These mechanisms not only induce truthful behavior and yield ex-post efficient and individually rational allocations: For any other mechanism with these properties we can find a SPAWS mechanism yielding the same allocation.

Suppose now that information is *Non-Exclusive* in the sense that the type of each player can be inferred from the knowledge of all the other players' type. Intuition suggests that in this case, incentive compatibility has no bite whatsoever (i.e. T2" does not apply) since the behavior of each player can be "policed" by the remaining players. In order to prove this, we will concentrate on an extreme, but illuminating, case of non-exclusive information, namely Nash equilibrium. In this framework, since information is complete, a direct mechanism is one where each agent announces a state of the world.

Consider the following assumption:

- (W) $\exists z \in A$ such that $\forall \theta \in \Theta, \forall a \in A$, $aR_i(\theta)z$, $\forall i \in I$.

This assumption will be called "universally worst outcome" because it postulates the existence of an allocation which is unanimously deemed as the worst. In an exchange economy this allocation would be zero consumption for everybody. Now we have the following result (Repullo [1986], Matsushima [1988]):

Theorem 3. If $n = 2$ *and W holds, any SCF is truthfully implementable in Nash Equilibrium. If* $n > 2$, *any SCF is truthfully implementable in Nash Equilibrium.*

Proof. When $n = 2$ consider the following outcome function: $g(\theta', \theta') = f(\theta')$ $\forall \theta' \in \Theta$, $g(\theta', \theta'') = z$ for all $\theta' \neq \theta''$. Clearly, truth is an equilibrium. When $n > 2$, consider the following outcome function: If m is such that $n-1$ agents announce state θ', then $g(m) = f(\theta')$. Otherwise, $g(\cdot)$ is arbitrary. Clearly truth is an equilibrium as well in this case ■.

The first thing to notice is the difference between the cases of two and more than two individuals. We will have more to say about this in the next section. The second is that the construction in Theorem 3 produces a large number of equilibria, and that there seems to be no good reason for individuals to coordinate in the truthful equilibria.

For instance, suppose workers can be either fit or unfit. When a profit-maximizing firm asks its employees about their characteristics, and all workers are fit, a unanimous announcement such as "we are all unfit" is an equilibrium. If fit workers are required hard work and unfit workers are asked to light work, do you think it is reasonable that workers coordinate in the truthful equilibrium? A more elaborate example was produced by Postlewaite and Schmeidler (1986): There are three agents. The first agent has no information and agents 2 and 3 are perfectly informed. The ranking of agent 1 over alternatives is the opposite of agents 2 and 3 who share the same preferences. The SCF is the top alternative of agent 1 in each state. It is intuitively clear that besides the truthful equilibria, there is another untruthful equilibrium where both informed agents lie and they are strictly better off than under truthful behavior. Again, coordination in the truthful equilibrium seems very unlikely. Thus, we have to recognize that we have a problem here. The next section will tell you how we can solve it.

Summing up, what do we learn from the results in this section?

1. When looking for an implementable SCF, a useful first test is whether this SCF yields incentives for the agents to tell the truth, see T.1. But this test is incomplete because of the existence of equilibria other than the truthful one, see T.3. These untruthful equilibria sometimes sound more plausible than the truthful one.

2. All impossibility theorems -T. 2-2'-2"- have the same structure: Truthful implementation, individual rationality/non-dictatorship, efficiency/large range of the SCR and large domain. Usually in social choice environments conditions 2 and 3 are weaker than in economic environments but the condition on the domain is stronger.

3. The classic story of the market making possible efficient allocation of resources under private information has to be revised. Private information in many cases precludes the existence of *any* mechanism achieving efficient and individually rational allocations under informational decentralization, see T. 2-2'-2".

4. The same remarks apply to naive applications of the Coase theorem where agents are supposed to achieve Pareto efficient allocations just because they have contractual freedom (ditto about Bargaining Theory). In the parlance of Coase, private information is an important transaction cost.

5. When mechanisms with adequate properties exist, like those proposed by Arrow, d'Aspremont and Gerard-Varet, Myerson and Makowski and Mezzetti, they are not of the kind that we see in the streets. Careful design is needed. These mechanisms are tailored to specific assumptions on valuations, thus their range of applicability may be limited.

4.2 Monotonicity and how to avoid it

We have seen that equilibria other than the truthful one are likely to arise. We have also seen that these equilibria cannot be disregarded a priori. So we have to find a way of getting rid of equilibria that do not yield desirable allocations. Under Dominant Strategies, clearly, if all preference orderings are strict, implementation and truthful implementation becomes identical, see Dasgupta, Hammond and Maskin (1979), Corollary 4.1.4 (Laffont and Maskin [1982] presents other conditions under which this result holds. See Repullo (1985) for the case where implementation and truthful implementation in dominant strategies do not coincide). For the ease of exposition we consider next Nash equilibria.

It turns out that the key to this issue in the case of Nash Equilibrium is the following monotonicity property, sometimes called Maskin monotonicity because Maskin (1977) established its central relevance to implementation.

- (M) A SCR F is Monotonic if

$$\{a \in F(\theta),\ aR_i(\theta)b \rightarrow aR_i(\theta')b\ \forall i \in I\} \rightarrow a \in F(\theta')$$

Monotonicity says that if an allocation is chosen in state θ and this allocation doesn't fall in anybody's ranking in state θ', this allocation must also be chosen in θ'. We will also speak of a "monotonic transformation of preferences at θ" when the requirement $aR_i(\theta)b \rightarrow aR_i(\theta')b\ \forall i \in I$ is satisfied. This requirement simply says that the set of preferred allocations shrinks when we go from θ to θ'.

Monotonicity looks like a not unreasonable property, even though as we will see in a moment, there are cases in which it is incompatible with other very desirable properties. In any case the importance of monotonicity comes from the fact that it is a necessary condition for implementation in Nash Equilibrium, as proved by Maskin (1977).

Theorem 4. If a SCR is implementable in Nash Equilibrium it is Monotonic.

Proof. If F is Nash implementable, there must be a mechanism (M, g) such that $\forall a \in F(\theta)$, there is a Nash equilibrium m^*, such that $g(m^*) = a$. Since $aR_i(\theta)b \rightarrow aR_i(\theta')b\ \forall i \in I$, m^* is also a Nash Equilibrium at θ'. Since F is implementable, $a \in F(\theta')$ ■.

Let us now discuss the concept of monotonicity. First, the bad news. Popular concepts in voting, like Plurality, Borda Scoring and Majority Rule are not monotonic, neither is the Pareto correspondence, see Palfrey and Srivastava (1991), p. 484. Even the venerable Walrasian correspondence is not monotonic! The failure of the Pareto and the Walrasian SCR to be monotonic can be amended: If preferences are strictly increasing in all goods, the Pareto SCR is monotonic in economic environments. The *Constrained Walrasian* SCR - in which consumers maximize with respect to the budget constraint *and* the availability of resources- is also monotonic. More serious is a result due to Hurwicz (1979) that uses two weak conditions on a SCR defined in the domain of exchange economies.

- (L) The domain of F contains all preferences representable by linear utility functions.
- (ND) If $a \in F(\theta)$ and $aI_i(\theta)b \; \forall i \in I$, then $b \in F(\theta)$.

The first condition is a rather modest requirement on the richness of the domain of F. The second is a non-discrimination property which says that if everybody considers two allocations to be indifferent and one allocation belongs to the SCR then it must be the other. Now we have the following:

Theorem 5. Let F be a SCR satisfying L and ND and such that:
1) It is Nash implementable.
2) It selects individually rational allocations.
Then, if x is a Walrasian allocation at θ, $x \in F(\theta)$.

Proof. (Sketch, see Thomson (1985) for details) Take an economy θ. Let x be a Walrasian allocation for θ. Consider a new economy, called θ^L, where the marginal rates of substitution among goods are constant and equal to a vector of Walrasian prices. By individual rationality, F must select an allocation which is indifferent to x. By ND, $x \in F(\theta^L)$. Since F is Nash implementable, it satisfies M. Now since $xR_i(\theta^L)b \rightarrow xR_i(\theta)b \; \forall i \in I$, by M, $x \in F(\theta)$. ∎.

Thus under weak conditions, Walrasian allocation are always in the set of those selected by a Monotonic SCR. And these allocations may fail to satisfy properties of fairness or justice as pointed out by the critics of the market. Under stronger assumptions, the converse is also true, i.e. only Walrasian allocations can be selected by a Nash implementable SCR, Hurwicz (1979). Also, T. 5 has the following unpleasant implication.

Theorem 6. There is no SCF in exchange economies such that:
1) It is Nash implementable.
2) It selects individually rational allocations.
3) ND holds.

4) It is defined on all exchange economies.

Proof. T. 5 implies that any Walrasian allocation belongs to the allocations selected by F. Since Walrasian equilibrium is not unique for some economies in the domain, hence the result. ■.

T. 6 has a counterpart in social choice domains, Muller and Satterthwaite (1977).

Theorem 6'. There is no SCF in a social choice domain such that:

1) It is monotonic.

2) It is not dictatorial.

3) Its range is A with $\#A > 2$.

4) It is defined on all possible preferences.

An implication of T. 6-6' is that single valued SCR are still problematic. But the consideration of multivalued SCR, brings a new problem: The existence of several Nash equilibria. For instance, if $a, b \in F(\theta)$ with a and b being efficient allocations, agents play a kind of "Battle of the Sexes" game with no clear results. Moreover the Nash Equilibrium in mixed strategies may yield allocations outside $F(\theta)$ (the concern about mixed-strategy equilibria was first raised by Jackson [1992]).

Now let us come to the good news. Firstly, the ND condition, which is essential for T. 5 to hold, is not as harmless as it appears to be. For instance, it is not satisfied by the Envy-Free SCR, see Thomson (1987) for a discussion. Secondly, there are perfectly reasonable SCR which are monotonic: we have already encountered the Constrained Walrasian SCR. Also any SCR selecting interior allocations in $\mathcal{L}A$ when preferences are von Neumann-Morgenstern is monotonic. In the domain of exchange economies with strictly increasing preferences, the Core and the Envy-Free SCR are also monotonic. In domains where indifference curves only cross once -the single-crossing condition- monotonicity vacuously holds. So Monotonicity, restrictive as it is, is worth a try. But before this, let us introduce a new assumption

- (NVP) A SCR f satisfies No Veto Power if $\forall \theta \in \Theta$,

$$\{aR_i(\theta)b, \ \forall b \in A, \text{ for at least } n-1 \text{ agents}\} \rightarrow a \in F(\theta)$$

In other words, if there is an allocation which is top-ranked by, at least, $n-1$ agents, NVP demands that this allocation belongs to the SCR. This sounds like a reasonable property for large n. Also in exchange economies with strictly increasing preferences and more than two agents, NVP is vacuously satisfied because there is no top allocation for $n-1$ agents.

The following positive result, a relief after so many negative results, was stated and proved by Maskin (1977), although his proof was incomplete.

Theorem 7. If a SCR satisfies M and NVP is Nash implementable when $n > 2$.

Proof. (Sketch) Consider the following mechanism. $M_i = \Theta \times A \times \mathbb{N}$ where $\mathbb{N}$ is the set of natural numbers. The outcome function has three parts.

Rule 1 (Unanimity). If m is such that all agents announce the same state of the world, θ, the same allocation a with $a \in F(\theta)$ and the same integer, then $g(m) = a$.

Rule 2 (One Dissident). If there is only one agent whose message is different from the rest, this agent can choose any allocation that leaves her worse off, according to her preference as announced by others.

Rule 3 (Any other case). $a \in g(m)$ iff a was announced by the agent who announced the highest integer (ties are broken by an arbitrary rule).

Let us show that such a mechanism implements any SCR with the required conditions. Clearly if the true state is $\tilde{\theta}$, $m_i = (\tilde{\theta}, a, 1)$ with $a \in F(\tilde{\theta})$ is a Nash Equilibrium since no agent can gain by saying otherwise, so Condition 1 in the definition of Nash implementation holds. Let us now prove that Condition 2 there also holds. Suppose we have a Nash Equilibrium in Rule 1. Could it be an "untruthful" equilibrium? If so we have two cases. Either the announced preferences are a monotonic transformation of preferences at $\tilde{\theta}$, in which case, M implies that the announced allocation is also optimal at $\tilde{\theta}$. If they are not, there is an agent who can profitably deviate. Clearly, if equilibrium occurs in Rule 2, with, say, agent i as the dissident, any agent other than i can drive the mechanism to Rule 3, so it must be that all these agents are obtaining their most preferred allocation, which by NVP belongs to $F(\tilde{\theta})$. An equilibrium in Rule 3 implies that all agents are obtaining their most preferred allocation which, again by NVP belongs to $F(\tilde{\theta})$. $\blacksquare$.

The interpretation of the mechanism given in the proof of T. 7 is that if everybody agrees on the state and the allocation is what the planner wants, this allocation is selected. If there is a dissident (a term due to Danilov [1992]) she can make her case by choosing an allocation (a "test allocation") in her lower contour set, as announced by others. Finally, with more than one dissident, it's the jungle! Any agent can obtain her most preferred allocation by the choice of an integer. Typically, there is no equilibrium in this part of the mechanism. Notice that (M) is just used to eliminate unwanted equilibria.

The mechanism is an "augmented" revelation mechanism (a term due to Mookherjee and Reichelstein [1990]), where the announcement of the state is complemented with the announcement of an allocation -this can be avoided if the SCR is single valued- and an integer. The final proof of T. 7 was done independently by Williams (1986), Repullo (1987), Saijo (1988) and McKelvey (1989).

The case of two agent is more complicated because when an agent deviates from a common announcement and becomes a dissident, she converts the other agent into another dissident! As in T. 3, W does the job, i.e. any SCR satisfying M, NVP and W is Nash implementable, see Moore and Repullo (1990) and Dutta and Sen (1991b) for a full characterization. Again, the cases of two agents and more than two agents are different. In some areas of mathematics, such as statistics and differential equations the cases of two dimensions and more than two dimensions are also different. The relationship of these with the findings of implementation is not yet fully explored, see Saari (1987).

Under asymmetric information M is substituted by a -rather ugly- *Bayesian Monotonicity* (BM) condition which is a generalization of M to these environments. BM is again necessary and in conjunction with some technical conditions plus incentive compatibility, sufficient for implementation in BE. The interested reader can do no better than to read the account of these matters in Palfrey (2002). It must be remarked that many well-known SCR -including Arrow-Debreu contingent commodities and some efficient SCR- do not satisfy BM and thus cannot be implemented in BE. However, the Rational Expectations Equilibria and the (interim) Envy-Free SCR satisfy BM, see Palfrey and Srivastava (1987).

T. 7 was the first positive finding of implementation theory. And it prompted researchers to be more ambitious: Can we implement without Monotonicity? An interesting observation, due to Matsushima (1988) and Abreu and Sen (1991), is that if agents have preferences representable by von Neumann-Morgenstern utility functions, *any* SCR can be "virtually implemented" in the sense that the set of allocations yielded by Nash equilibria is arbitrarily close to the set of desired allocations. This is because, as we saw before, any SCR mapping in the interior of $\mathcal{L}A$ is Monotonic. Thus allocations in the boundary can be arbitrarily approximated by allocations in the interior.

A more satisfying approach was introduced by Moore and Repullo (1988) by introducing subgame perfection as the solution concept. It is not possible to explain fully this approach here because it would take us too far; in particular the notion of a mechanism must be generalized to "stage mechanism". Instead, we give a result that conveys the force of subgame perfect implementation. It refers to public good economies with quasi-linear utility functions -where under dominant strategies the set of economies with inefficient outcomes is large- and with two individuals -where Nash implementability is harder to obtain.

Suppose that utility functions read $U_i = V(y, \theta_i) + m_i$ where $y \in Y \subseteq \Re$, $\theta_i \in \Theta_i$ with $\#\Theta_i < \infty$ and $m_i \in \Re$, $i = 1, 2$. The set of allocations $A = \{(y, m_1, m_2) \in Y \times \Re^2 / m_1 + m_2 \leq \omega\}$ where ω are the endowments of "money". Moore and Repullo (1988) proved the following:

Theorem 8. Any SCF is implementable in Subgame Perfect Equilibrium in the domain of economies explained above.

Moore and Repullo proved that many SCR which could not be implemented in Nash Equilibrium can be implemented in Subgame Perfect Equilibrium. This is because subgames can be designed to kill unwanted equilibria without using monotonicity. Their result was improved upon by Abreu and Sen (1991). The problem with this approach is that the concept of subgame perfection is problematic because it requires that, no matter what has happened in past, in the remaining subgame, players are rational, even if this subgame was attained because some players made irrational choices.

The Moore-Repullo result was not only important by itself but it opened the way to the consideration of other equilibrium concepts that allow very permissive results. For instance, Palfrey and Srivastava (1991) proved the following result

Theorem 8'. Any SCR satisfying NVP is implementable in Undominated Nash Equilibrium.

At this point, it seemed that by invoking the adequate refinement of Nash equilibrium, any SCR could be implemented. But the implementing mechanisms were getting weird and some people were beginning to get suspicious. Why and how is discussed in the next section.

Summing up the results obtained here, we have the following:

1. (Maskin) Monotonicity is a necessary and in many cases sufficient condition for implementation in Nash Equilibrium, see T. 4 and 7. Similar results are obtained with Bayesian Monotonicity in Bayesian Equilibrium.

2. The Monotonicity requirements are not harmless. Many solution concepts do not satisfy it. Even worse, Monotonicity has some unpalatable consequences, see T. 5-6.

3. Monotonicity can be avoided by considering stage games or refinements of Nash Equilibrium. Practically, any reasonable SCR can be implemented in this way, see T. 8-8'.

4.3 The Limits of Design

So far we have assumed that there are no limits to what the designer can do. She can pick up any mechanism with no restrictions on its shape. This procedure, indeed, pushes the possibilities of design to the limit. But by doing this, we have learnt a good deal about the limitations of the theory of implementation. It is fair to say that today the consensus is that there are *some extra properties* which should be considered when designing an implementing mechanism. We review here five approaches to this question.

1: Game-Theoretical Concerns. Jackson (1992) was the first to point out that some mechanisms had unusual features from the point of view of game theory: Some subgames have no Nash equilibrium. Message spaces, which in the corresponding game become strategy spaces, are unbounded or open. Thus, in the integer game considered in T. 7, if agents eliminate dominated strategies, each integer is dominated by the next highest one and no integer is undominated: Those agents who eliminate dominated strategies are unable to make a choice. These constructions eliminate unwanted equilibria, which as we saw before, is the problem with Nash implementation. Jackson illustrates his point by showing that under no restrictions on mechanisms, any SCR can be implemented in undominated strategies, a weak solution concept. Then he requires that the mechanism be *Bounded* in the following sense: whenever a strategy m_i is dominated, there is another strategy dominating m_i and which is undominated. He shows that implementation in undominated strategies with bounded mechanisms result many times in incentive compatibility, which as we saw in Section

3 is a hard requirement. This shows the bite of the boundedness assumption. However, in the case of implementation with undominated Nash equilibrium, the boundedness assumption has little impact, see Jackson, Palfrey, and Srivastava (1994) and Sjöström (1994). The first of these papers introduced a related requirement, the *Best Response Property*: for every strategy played by the other agents, each agent has a best response.

2: Natural Mechanisms. Given that we have run so far from the kind of mechanism we are used to, it seems reasonable to ask what can be implemented by mechanisms that resemble real-life mechanisms. These mechanisms must be simple too because simplicity is an important characteristic in practice. Let us call them *Natural Mechanisms.* Dutta, Sen and Vohra (1995) consider mechanisms in which messages are prices and quantities and thus resemble market mechanisms. Their approach was refined by Saijo, Tatamitani and Yamato (1996) who demanded the best response property as well. They showed that several well-known SCR such as the (constrained) Walrasian, are implementable in Nash equilibrium. Beviá, Corchón and Wilkie (2003) showed that in Bertrand-like market games, the Walrasian SCR is implementable in Nash and Strong equilibrium, showing that the fear of coalitions destabilizing market outcomes is, at least, partially unwarranted. Sjöström (1996) considered quantity mechanisms, reminiscent of those used by Soviet planners, with negative results about what these mechanisms can achieve. In public good economies, Corchón and Wilkie (1996) and Peleg (1996) introduced a market mechanism implementing Lindahl allocations in Nash and Strong equilibrium. The mechanism works because Lindahl prices have to add up to the marginal cost. If an agent pretends to free ride she decreases the quantity of the public good. Here, contrary to Samuelson's dictum it is in the selfish interest of each person to give true signals. Pérez-Castrillo and Wettstein (2002) offered a bidding mechanism that implements efficient allocations when choosing between a finite number of public projects. They also applied these ideas to back up Shapley value.

3: Credibility. Another implicit assumption is that once the mechanism is in place, there is no way to stop it. Thus, if for some m, $g(m)$ is a "universally worst outcome", the planner has to deliver this allocation even if she is trying to implement a Pareto Efficient allocation. Is this a credible procedure? In many cases, if the planner is a real person it seems that she would do her best to avoid $g(m)$! Here we have two possibilities: Either we identify additional constraints on the planner that look reasonable or we jump to model the planner as a full-fledged player. The first road leads us to identify a subset of allocations of A, say X, which can never be used by the mechanism. For instance, in Chakravorty, Corchón and Wilkie (2006) X is the set of allocations that are never selected by the SCR for some state of the world, i.e. $X = \{a \in A / \nexists \theta \in \Theta,\, a \in F(\theta)\}$. The motivation for this definition is that it hardly seems credible that the planner can choose an allocation that is never intended to be implemented. Redefining the allocation set as $\mathbf{A} \equiv A \backslash X$ the definitions of a mechanism and an implementable SCR can be easily translated in this framework. However, depending on the domain, SCR that are monotonic when defined on A are no longer monotonic when defined on $\mathbf{A}$: For instance, the (constrained) Walrasian SCR. Thus, these SCR can not be implemented when the planner can only use allocations in $\mathbf{A}$. A weakness of this approach is that the list of reasonable constraints on allocations may be large. The second possibility drives us to model implementation as a signalling game where the planner receives signals -messages- from the agents, updates her beliefs and then chooses an allocation which maximizes her expected utility (Baliga, Corchón and Sjöström [1997]). Again, some SCR that are Nash implementable, are not implementable in this framework. However, in this case there are SCR that are not Nash implementable but are implementable in this framework. This is because the model takes a basic assumption of game theory to the limit, namely, that agents know the strategies of other players. In this case, the planner knows if a report on agents' types is truthful or not before the allocation is delivered!

4: Renegotiation. Another strong assumption is that the mechanism prescribes actions that can not be changed by agents. This contradicts experiences such as black markets where agents trade on the existing goods (Hammond [1987]). A way of modelling this is to assume that agents are able to renegotiate some allocations (Maskin and Moore [1999]. Renegotiation in a different context was considered by Rubinstein and Wolinsky [1992]). Assuming that agents have complete information, this is formalized by means of the concept of a reversion function. This function, say r, maps each allocation and each state of the world into a new allocation, i.e. $r : A \times \Theta \rightarrow A$. The reversion function induces new preferences, called *reverted preferences* (this is the "translation principle" in Maskin and Moore [1999]). Notice that reverted preferences are state dependent even if preferences are not. Formally, given a reversion function r, the *reversion of* $R(\theta)$, denoted by $R^r(\theta)$ is defined as $aR_i^r(\theta)b \Leftrightarrow r(a,\theta)R_i(\theta)r(b,\theta), \forall a,b \in A, \forall i \in I$. Given a reversion function r, we can interpret that agents' preferences are the reverted preferences. Then, all definitions given before can be adapted to this case. Again, SCR that were monotonic there, are not so in this framework and viceversa. See Jackson and Palfrey (2001) for applications. An extension to the case where there are several renegotiation functions is given by Amoros (2004). A weakness of this approach is that models renegotiation as a "black box".

5: Multiple Implementation. Maskin (1985) was the first to realize that the notion of implementation requires the planner to know the solution concept used by the agents to analyze the game. He proposed the notion of "Double Implementation" where a SCR was implemented at the same time in Nash and Strong equilibria. He showed that many Nash implementable SCR indeed are doubly implementable. We have seen in Point 2 above that the (constrained) Walrasian and Lindahl SCR are doubly implementable by natural mechanisms. They are also doubly implementable by abstract mechanisms, Schmeidler (1980). Double implementation also occurs with several solutions to the problem of the commons, Shin and Suh (1997) and Pigouvian Taxes, Alcalde, Corchón and Moreno (1999). Yamato (1993) introduced another type of double implementation by requiring implementation in Nash and Undominated Nash Equilibria (1993). He showed that in a large class of exchange economies with at least three agents, monotonicity is necessary and sufficient for double implementation. Saijo, Sjöström and Yamato (2007) considered implementation in Dominant Strategies and Nash Equilibrium. Clearly, other variations of the idea of Double Implementation are possible, see Point 4 in Section 5 below.

Summing up, it is now clear that implementing mechanisms can not be just "anything". Their features matter. Demanding that mechanisms satisfy the best response property, be simple, not use extreme allocations, be robust to the possibility of renegotiation and implement in several equilibrium concepts makes our lives more difficult but makes our models a great deal better.

5 Unsolved Issues and Further Research

1. Implementation with state dependent feasible sets. A motivation of implementation theory was to study the possibility of socialism. However, *all* the results presented in this survey refer to environments where the feasible set is given, a far cry from any kind of planning procedure. In fact, there are only a handful of papers dealing with implementation when the feasible set is unknown: Postlewaite (1979) and Sertel and Sanver (1999) studied manipulation of endowments. Hurwicz, Maskin and Postlewaite (1995) studied implementation assuming that endowments/production possibilities can be hidden or destroyed but never exaggerated. Instead of a mechanism we have a collection of state dependent mechanisms each meant for an economy. After the mechanism is played, production capabilities are shown, e.g. endowments are put on the table. This idea was worked out in a series of papers by Hong on private good economies, Hong (1998), and by Tian on public good economies, Tian and Li (1995). Serrano and Vohra (1997) worked out implementation of the core and Dagan, Serrano and Volij (1999) of taxation methods. And that's all folks! Why has such an important issue been almost neglected? My explanation is that the proposed mechanisms are difficult to understand. Another approach has been tried by Corchón and Triossi (2005) where a reversion function takes care of restoring feasibility when messages lead to unfeasible allocations. The approach is tractable and simpler but relies on the black box of the renegotiation function.

2. Sociological factors/Bounded Rationality. So far, all the solution concepts describing the behavior of agents are game-theoretical. In recent years, we have seen a host of equilibrium concepts based on "irrational" agents. It would be interesting to see what SCR can be implemented with these forms of behavior. Eliaz (2002) considers "Fault Tolerant" implementation where a subset of players ("faulty players") fail to achieve their optimal strategies. Under complete information, No Veto Power and a strong form of Monotonicity are sufficient for implementation when the number of faulty players is less than $n/2-1$, $n>2$. Matsushima (2008) shows that a small preference for honesty is sufficient to knock down unwanted equilibria.

3. Dynamic Implementation. The theory presented here is static but there are some papers dealing with implementation in dynamic set-ups. We mention a few: Freixas, Guesnerie and Tirole (1985) studied the "Ratchet Effect", where firms underproduce for fear of being asked to do too much in the future. Kalai and Ledyard (1988) showed that if the planner is sufficiently patient, every SCR is dominant-strategy implementable. Burguet (1990/94) showed that the revelation principle does not hold when outcomes are chosen in several periods. Candel (2004) proved a revelation principle in a model where a public good is produced in two periods. Finally, Cabrales (1999) and Sandholm (2007) studied implementation in an evolutionary setting. A related topic is that of complexity, see Conitzer and Sandholm (2002). It seems likely that a dynamic theory of incentives will bring new insights and will need new analytical tools.

4. Robustness Under Incomplete Information. When designing a mechanism, sometimes the planner does not know the structure of information. In this case a mechanism must implement regardless the structure of information, i.e. priors of agents, type spaces, etc. Corchón and Ortuño-Ortín (1995) approached the problem by assuming that the economy is composed of "islands" and that there is complete information inside each island. A mechanism robustly implements a SCR if it does it in BE for every possible prior (compatible with the island assumption) and in Uniform Nash Equilibrium. The latter requires that an equilibrium strategy for an agent must be the best reply to what other agents in the island play and to any possible message sent by agents outside the island when they follow their equilibrium strategies (D'Aspremont and Gerard-Varet [1979]). They showed that any SCR satisfying M and NVP is robustly implementable (a later contribution by Yamato (1994) showed that Robust and Nash Implementation coincide in this framework). The same concern has been approached in a series of papers by Bergemann and Morris (see e.g. [2005]) where they ask SCR to be implemented whatever the players' beliefs and higher order beliefs about other player' types. Artemov, Kunimoto and Serrano (2007) require implementation for the payoff type space and the space of first-order beliefs about other agents' payoff types. They obtain very permissive results.

In a different vein, Koray (2005), has argued that, since priors are not contractible, the regulator needs to be regulated in order to stop her from manipulating the priors. He shows that the outcomes of this game vary over a wide spectrum. Again the need of prior-free implementation is clear.

6 Answers to the Questions

1. Yes. We already saw in 4.3, Point 2, that "Bertrand-like" mechanisms implement the Constrained Walrasian SCR in Nash and Strong equilibrium. But this is not all: Schmeidler (1980) exploited the connection between price taking

-which underlies Walrasian equilibrium- and "strategy taking", which underlies Nash and Strong equilibrium and obtained double implementation by a mechanism which does not resemble the market. Implementation of the Lindahl SCR by an abstract mechanism was obtained by Walker (1981) building on previous papers by Groves and Ledyard and Hurwicz. Unfortunately, these positive results turn negative when we consider Arrow-Debreu contingent commodities, Chattopadhyay, Corchón and Naeve (2000) and Serrano and Vohra (2001).

2. A merger affects social welfare in two ways: Positively, from cost savings and negatively, from restricting competition. The first effect is uncertain and, by now, I do not have to convince you that we should take with utmost caution all announcements made by firms concerning cost savings. Corchón and Faulí-Oller (2004) show that under a condition that is fulfilled in several standard IO models, the SCR that maximize social surplus can be implemented by a dominance solvable mechanism with budget balance.

3. There is a very simple mechanism which attains maximum surplus, Loeb and Magath (1979). But in this mechanism the monopolist receives all the surplus and the demand function must be known by the planner. These points were worked out by subsequent contributions from Baron and Myerson, Lewis and Sappington, Sibley and others.

4. By now the reader should know the difficulties of implementing efficient public decisions. When information is exclusive this is impossible, even though an approximate efficient decision can be obtained when the number of agents is large. When information is complete, we have seen several examples of mechanisms implementing efficient outcomes.

5. There is no difference between implementing market or fair outcomes. Both have to pass the same tests, i.e. incentive compatibility, monotonicity and simplicity/credibility of design. In exchange economies, Thomson (2005) presents a simple and elegant mechanism that implements envy-free allocations in Nash Equilibrium. In cooperative production, Corchón and Puy (2002) presented a family of mechanisms that implement in Nash Equilibrium any efficient SCR where the distribution of rewards is a continuous function of efforts.

6. Yes! An uninformed planner can set up a mechanism that yields efficient outcomes in circumstances where the market yields inefficient allocations, i.e. under externalities or public goods see Point 5 in 4.3 above. All we need is non-exclusive information and that the SCR be Monotonic, the latter requirement

can be skipped under refinements of Nash Equilibrium.

7. Not completely. Suppose complete information among three or more judges and that they all perceive the same quality of a given performance. Clearly, truth is an equilibrium, because if all judges minus you tell the truth you cannot change the outcome by saying something different. Unfortunately, any unanimous announcement is also an equilibrium by the same reason. Thus we are in a situation akin to T. 3. Fortunately, if preferences of judges fulfill certain restrictions, full implementation of the true ranking of ice skaters is possible, because Monotonicity and No Veto Power hold so T. 7 applies, Amoros, Corchón and Moreno (2002). If judges have differential information, the truth is no longer implementable as suggested by T. 2". See Gerardi, McLean and Postlewaite (2005) for further insights and references on this problem.

8. ????? Do you think that we have all answers? This is just economics!!

Finally I will tell you why I like implementation theory so much. Firstly, the implementation model solves the problems of the General Equilibrium model mentioned in Section 2, namely: 1: It models a general economic system. 2: All variables are endogenously determined by the interaction of agents. 3: Agents incentives are carefully modeled and are taken fully into account. Secondly, the theory is not based on assumptions like convexity or continuity/differentiability which, no matter how much we are used to them, are very stringent. By the way, a beautiful paper by Laffont and Maskin (referenced in their 1982 survey) developed incentive compatibility in a differentiable framework.

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