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SINR Degradation in MIMO-OFDM Systems with Channel Estimation Errors and Partial Phase Noise Compensation

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Abstract—The phase noise effect in multiple-input-multiple-output systems employing orthogonal frequency division multiplexing is considered in a realistic scenario where the estimated channel matrix is affected by an error. The analytical SINR degradation due to phase noise and channel estimation is obtained for linear receivers (ZF and MMSE).

Index Terms—MIMO-OFDM systems, phase noise, channel estimation, inter-channel interference, linear receivers.

I. INTRODUCTION

ORTHOGONAL Frequency Division Multiplexing (OFDM) is becoming the most frequently used technique for wireless systems, such as Long Term Evolution (LTE), and enhanced standards are contemplating its combination with multiple input-multiple output (MIMO). These systems suffer from inter-channel interference (ICI) introduced by phase noise and channel estimation errors. In most of the works these impairments are treated separately [1]–[4]: with ICI reduction schemes assuming perfect channel knowledge or analyzing the channel estimation error without phase noise. In [1] a scheme is proposed to remove the common phase error (CPE) and in [3] an iterative technique is presented to cancel successively the ICI terms, both assuming ideal channel estimation. In [2] the ICI power is obtained for zero-forcing (ZF) with ideal channel estimation by a first-order approximation of the phase noise term. The channel estimation error is analyzed in [4] for ZF, in fading, without phase noise. In [5]–[7] the effects are considered jointly for SISO-OFDM. The combination of MIMO with OFDM introduces the spatial dimension that causes additional interference. Depending on the chosen receiver scheme, it will have a different impact on the ICI. This is more accentuated if the channel is spatially correlated, as we will show. The degradation of phase noise and channel estimation for MIMO-OFDM is analyzed in [8] where the signal to interference plus noise ratio (SINR) is derived before the application of the receiver. However, phase noise and channel estimation errors are not separable and have different effects depending on the type of receiver. Therefore, it is important to consider them jointly and to characterize the degradation after the equalization of the receiver. Here we

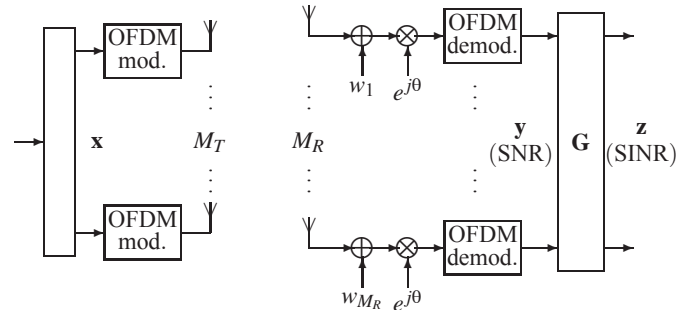


Fig. 1. Scheme of the MIMO-OFDM transmission system.

derive an analytical expression of the SINR degradation after two types of linear receivers, ZF and minimum mean squared error (MMSE). Moreover, we evaluate this degradation in the general case of spatially correlated multipath channel and without any approximation of the phase noise term. Some preliminary results were presented in [9], resorting mainly to simulations. Here we present in detail the analytical derivation of the SINR degradation. In [10] part of these analytical results was used to discuss the system parameters (number of antennas, multipath and phase noise conditions, and estimation errors) only for a spatially white channel.

II. MIMO-OFDM SYSTEM

The spatial multiplexing MIMO-OFDM system [11] is shown in Fig. 1, where M_T independent data streams are OFDM modulated over N sub-carriers and sent to M_T transmit antennas. The receiver has M_R antennas. The vector of transmitted symbols is $\mathbf{x} = [\mathbf{x}_0^T, \dots, \mathbf{x}_{N-1}^T]^T$, where each component $\mathbf{x}_n = [x_{n,1}, \dots, x_{n,M_T}]^T$ groups the symbols transmitted on the n th sub-carrier on all the antennas. For each antenna pair (i, j) , $i = 1, \dots, M_R$, $j = 1, \dots, M_T$ we have a multipath $M_R \times M_T$ impulse response $\mathbf{h}_m[i, j]$, $m = 0, \dots, N_{Ch}$, with length N_{Ch} shorter than the cyclic prefix. The elements of \mathbf{h}_m are randomly distributed with powers determined according to the power delay profile. The spatial correlation is characterized by $E[\mathbf{H}_n \mathbf{H}_n^H] = \mathbf{R}_R$, $E[\mathbf{H}_n^H \mathbf{H}_n] = \mathbf{R}_T$ where $(\cdot)^H$ denotes the conjugate transpose. In a separable channel model, \mathbf{R}_T and \mathbf{R}_R correspond to the antenna correlations at transmitter and receiver, respectively. The phase noise $\theta(t)$ at the receiver, sampled at kT , $\theta_k = \theta(kT)$, coming mainly from the down-conversion by high-frequency oscillators, is assumed to be the same for all the antennas. The received signal after the discrete Fourier transform (DFT), $\mathbf{y} = [\mathbf{y}_0^T, \dots, \mathbf{y}_{N-1}^T]^T$, with $\mathbf{y}_n = [y_{n,1}, \dots, y_{n,M_R}]^T$ grouping all the signals on sub-carrier n , is

$$\mathbf{y} = \mathbf{Q}\mathbf{H}\mathbf{x} + \mathbf{w}. \quad (1)$$

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$\mathbf{H} = \text{diag}[\mathbf{H}_0, \mathbf{H}_1, \dots, \mathbf{H}_{N-1}]$ is the $M_R N \times M_T N$ block diagonal channel frequency response, where each block is the n th sub-carrier component of the channel DFT,

$$\mathbf{H}_n = \sum_{m=0}^{N_{Ch}-1} \mathbf{h}_m e^{-j2\pi \frac{nm}{N}}. \quad (2)$$

The phase noise matrix \mathbf{Q} in (1) is

$$\mathbf{Q} = \begin{bmatrix} \Theta_0 & \Theta_{N-1} & \cdots & \Theta_1 \\ \Theta_1 & \Theta_0 & \cdots & \Theta_2 \\ \vdots & \vdots & \ddots & \vdots \\ \Theta_{N-1} & \Theta_{N-2} & \cdots & \Theta_0 \end{bmatrix} \otimes \mathbf{I}_{M_R}, \quad (3)$$

where \otimes is the Kronecker product and Θ_n is the n th component of the phase noise vector DFT

$$\Theta_n = \sum_{k=0}^{N-1} e^{j\theta_k} e^{-j2\pi \frac{kn}{N}}. \quad (4)$$

The value $E[|\Theta_n|^2]$, required for the ICI power, can be obtained by the phase noise spectral characteristics: $E[|\Theta_n|^2]$ is the power spectral density (PSD) of the sampled phase noise process $\mathcal{P}_0^{(s)}(f)$, evaluated at the n th sub-carrier frequency, $n\Delta f$, where $\Delta f = 1/(NT)$ is the sub-carrier spacing. $\mathcal{P}_0^{(s)}(f)$ is related to the continuous-time phase noise PSD $\mathcal{P}_0(f)$ by periodic repetition

$$E[|\Theta_n|^2] = \mathcal{P}_0^{(s)}(n\Delta f) = \sum_{k=0}^{+\infty} \mathcal{P}_0(n\Delta f + kN\Delta f). \quad (5)$$

The phase noise PSD in PLL-based frequency synthesizers can be expressed as a weighted sum of components $\mathcal{P}_0(f) = a_0 + a_1 \left(\frac{f_1}{f}\right) + a_2 \left(\frac{f_2}{f^2}\right) + a_3 \left(\frac{f_3}{f^3}\right)$, where the characteristic frequencies of each component f_i and the relative weights a_i strongly depend on the actual technology and device [12]. In the case of free running oscillators, they can be accurately characterized by a Wiener phase noise, which corresponds only to the $1/f^2$ PSD component. In this case the amount of phase noise is usually expressed by the 3-dB carrier bandwidth B , normalized to the sub-carrier spacing $B_\theta = BNT$. An alternative approach often employed is the approximation of the exponential $e^{j\theta_k}$ by its first order Taylor series expansion, $1 + j\theta_k$, which holds in the case of small phase noise [2].

Finally \mathbf{w} in (1) is the AWGN contribution, where the phase noise is neglected, due to circular symmetry. We can then introduce an overall equivalent channel matrix $\mathbf{H}_{eq} = \mathbf{Q}\mathbf{H}$, giving $\mathbf{y} = \mathbf{H}_{eq}\mathbf{x} + \mathbf{w}$. We define the reference signal to noise ratio (SNR) as the ratio between the useful component and the noise power, $SNR = E[(\mathbf{H}_{eq}\mathbf{x})^H (\mathbf{H}_{eq}\mathbf{x})] / E[\mathbf{w}^H \mathbf{w}]$.

III. RECEIVER SCHEMES

We will analyze linear receivers (ZF and MMSE), where the recovered signal is obtained by $\mathbf{z} = \mathbf{G}\mathbf{y}$. With ZF the matrix \mathbf{G}_{ZF} , which removes the spatial interference at the expense of enhancing the additive noise, is $\mathbf{G}_{ZF} = \tilde{\mathbf{H}}_{eq}^\dagger$ where $(\cdot)^\dagger$ denotes the Moore-Penrose pseudo-inverse and $\tilde{\mathbf{H}}_{eq} = \tilde{\mathbf{H}}\tilde{\mathbf{Q}}$ is the estimated overall channel matrix. With MMSE the matrix \mathbf{G}_{MMSE} , balancing the spatial interference with the noise, is

$$\mathbf{G}_{MMSE} = \left(\tilde{\mathbf{H}}_{eq}^H \tilde{\mathbf{H}}_{eq} + \frac{1}{SNR} \mathbf{I}_{NM_T} \right)^{-1} \tilde{\mathbf{H}}_{eq}^H. \quad (6)$$

For both receivers we assume that first the CPE is compensated [1] by multiplication of the received signal by the matrix $\tilde{\Theta}_0^{-1} \mathbf{I}_{NM_R}$, in other words \mathbf{Q} is approximated by its diagonal elements $\tilde{\mathbf{Q}} = \tilde{\Theta}_0 \mathbf{I}_{NM_R}$ and $\tilde{\mathbf{H}}_{eq} = \tilde{\Theta}_0 \mathbf{H}$. The CPE can be estimated by means of N_p pilot sub-carriers, inserted in the OFDM symbol at positions p_i , $i = 1, \dots, N_p$ on the antenna streams M_j , $j = 1, \dots, M_p$. It is estimated as the mean phase displacement with respect to the expected symbol [13], by an average

$$\tilde{\Theta}_0 = \frac{1}{M_p} \frac{1}{N_p} \sum_{i=1}^{N_p} \sum_{j=1}^{M_p} \frac{y_{p_i, M_j}}{x_{p_i, M_j} |y_{p_i, M_j}|}, \quad (7)$$

where y_{p_i, M_j} denotes the received value at the positions where the pilot symbols are placed, and x_{p_i, M_j} is the corresponding pilot symbol value. The effect of the estimation process on Θ_0 can be expressed by $\tilde{\Theta}_0 = \Theta_0 + \epsilon_{CPE}$, where ϵ_{CPE} is the error in the CPE estimate, with zero mean and variance σ_{CPE}^2 . The residual error left by CPE estimation will depend on the number of pilots and phase noise characteristics. For example, with the CPE estimation technique reported in [16], 4 pilots out of 64 are enough to obtain the same performance as without phase noise when Wiener phase noise has $B_\theta < 5 \cdot 10^{-2}$ and $SNR = 30$ dB. Channel estimation in MIMO-OFDM uses typically pilot symbols scattered in time and frequency, and has been extensively analyzed (see for example [14] and references therein). Here, we include the final estimation error on \mathbf{H} by an additive term \mathbf{Z} , and the actual channel matrix is $\mathbf{H} = \tilde{\mathbf{H}} + \mathbf{Z}$ where \mathbf{Z} is independent of \mathbf{H} , with zero-mean independent Gaussian elements $z[i, j]$ whose variance σ_{est}^2 is equal to the mean-squared error (MSE) obtained by the channel estimator. The accuracy of the channel estimate obtained by N_p pilots inserted in the OFDM signal depends on the position of the pilots. The best achievable MSE for each SISO-OFDM channel is [17] $MSE = (\sigma_n^2 N_p) / E_p$ where E_p is the pilot energy and σ_n^2 the AWGN variance. Note that CPE and channel estimation are usually performed separately; moreover, from (7) the effect of a channel estimation error on \mathbf{H} has a reduced effect on the CPE estimation, so that we consider \mathbf{Z} and ϵ_{CPE} independent.

IV. SINR DEGRADATION

We define the SINR after the receiver as the ratio between the useful signal power σ_x^2 and the variance of the overall disturbance caused by noise and spatial interference σ_0^2 , that is, $SINR = \sigma_x^2 / \sigma_0^2$. In ideal conditions, that is, without phase noise or estimation error, the SINR for the n th signalling vector at the output of the ZF receiver is [15]

$$SINR_n = \frac{SNR/M_T}{[\mathbf{H}_{eq}^H \mathbf{H}]_{n,n}^{-1}} \quad (8)$$

while for the MMSE receiver, on the n th signalling vector, it has the form

$$SINR_n = \frac{1}{[(SNR/M_T) \mathbf{H}_{eq}^H \mathbf{H} + \mathbf{I}_{NM_T}]_{n,n}^{-1}} - 1 \quad (9)$$

where $[\cdot]_{n,n}$ indicates the (n, n) th entry of a matrix. In the following evaluations the average SINR over all the signalling vectors is considered, $SINR = \frac{1}{M_T} \sum_{n=1}^{M_T} SINR_n$.

The joint effects of phase noise and estimation error will be accounted for by the SINR degradation, defined as the increase to the SINR giving the same error probability as in the ideal case of perfect channel knowledge and no phase noise. In the non-ideal case we should reduce the disturbance power by the overall interference power σ_{ICI}^2 to get the same performance, where

$$\sigma_{ICI}^2 = \frac{1}{N} \sum_{n=0}^{N-1} \sigma_{v_n}^2 \quad (10)$$

with $\sigma_{v_n}^2$ denoting the ICI variance on the n th sub-carrier, averaged over the M_T transmit antennas, obtained in the next section. Then the degradation D is the ratio between the new SINR, with disturbance power reduced by the ICI variance σ_{ICI}^2 , and the original SINR, in dB

$$D = 10 \log_{10} \frac{\sigma_x^2 / (\sigma_0^2 - \sigma_{ICI}^2)}{\sigma_x^2 / \sigma_0^2} = 10 \log_{10} \left(\frac{\sigma_0^2}{\sigma_0^2 - \sigma_{ICI}^2} \right). \quad (11)$$

Clearly, from (11), the degradation goes to infinite if σ_{ICI}^2 gets close to σ_0^2 . This is a floor effect due to the dominance of phase noise and channel imperfections at high SNR.

V. DERIVATION OF THE ICI VARIANCE

The overall interference term \mathbf{v} at the decision point, which includes ICI and imperfect CPE estimation after equalization, is

$$\mathbf{v} = \mathbf{G} [(\mathbf{Q} - \tilde{\Theta}_0 \mathbf{I}_N) \otimes \mathbf{I}_{M_R}] \mathbf{H} \mathbf{x}. \quad (12)$$

If we consider the component on the n th sub-carrier, we have

$$\mathbf{v}_n = \mathbf{G}_n \sum_{i=0}^{N-1} \Theta_{n-i} \mathbf{H}_i \mathbf{x}_i + \varepsilon_{CPE} \mathbf{G}_n \mathbf{H}_n \mathbf{x}_i \quad (13)$$

where \mathbf{G}_n is the n th diagonal block of \mathbf{G} . The ICI variance on the n th sub-carrier, averaged over the transmit antennas M_T , is then

$$\begin{aligned} \sigma_{v_n}^2 &= \frac{\sigma_x^2}{M_T} \sum_{i=0}^{N-1} \sum_{i \neq n} E [|\Theta_{n-i}|^2] \text{Tr} \{ E [\mathbf{H}_i^H \mathbf{G}_n^H \mathbf{G}_n \mathbf{H}_i] \} \\ &+ \sigma_x^2 \frac{\sigma_{CPE}^2}{M_T} \text{Tr} \{ E [\mathbf{H}_n^H \mathbf{G}_n^H \mathbf{G}_n \mathbf{H}_n] \} \end{aligned} \quad (14)$$

where the symbols \mathbf{x}_i have been assumed independent with $E[\mathbf{x}_i \mathbf{x}_i^H] = \sigma_x^2 \mathbf{I}_{M_T}$.

A. ZF receiver

In the expression of the ICI variance (14), for the ZF receiver we have

$$\begin{aligned} \text{Tr} \{ E [\mathbf{H}_i^H \mathbf{G}_n^H \mathbf{G}_n \mathbf{H}_i] \} &= \text{Tr} \left\{ E \left[\mathbf{H}_i^H \left(\mathbf{H}_n^\dagger \right)^H \mathbf{H}_n^\dagger \mathbf{H}_i \right] \right\} \\ &+ \text{Tr} \left\{ E \left[\mathbf{Z}_i^H \left(\mathbf{H}_n^\dagger \right)^H \mathbf{H}_n^\dagger \mathbf{Z}_i \right] \right\}. \end{aligned} \quad (15)$$

Because of the independence of \mathbf{Z} and \mathbf{H} and $E[\mathbf{Z}_i \mathbf{Z}_i^H] = \sigma_{est}^2 \mathbf{I}$, we have

$$\begin{aligned} \text{Tr} \left\{ E \left[\mathbf{Z}_i^H \left(\mathbf{H}_n^\dagger \right)^H \mathbf{H}_n^\dagger \mathbf{Z}_i \right] \right\} \\ = \sigma_{est}^2 \text{Tr} \left\{ E \left[\left(\mathbf{H}_n^H \mathbf{H}_n \right)^{-1} \right] \right\} = \sigma_{est}^2 \frac{\text{Tr} [\mathbf{R}_T^{-1}]}{M_R - M_T}, \end{aligned} \quad (16)$$

for $M_R > M_T$, since the above term represents the expected value of the trace of an inverse Wishart matrix [18]. To evaluate the first right-hand-side term of (15), the joint channel frequency response statistics on sub-carriers i and n are required. The channel frequency response on different sub-carriers can be expressed by a combination of a totally correlated component and an independent component, weighted by the sub-carrier correlation ρ , namely $\mathbf{H}_i = \rho \mathbf{H}_n + \sqrt{1-\rho} \mathbf{H}_I$, where \mathbf{H}_I is independent of \mathbf{H}_n , and the correlation is the Fourier transform of the power delay profile (PDP) evaluated at $(n-i)\Delta f$

$$\rho = E [\mathbf{H}_i(j, k) \mathbf{H}_n(j, k)] = \sum_{m=1}^{N_{CH}} E [|\mathbf{h}_m(j, k)|^2 e^{j2\pi m(n-i)\Delta f}]. \quad (17)$$

For example, for an exponential PDP, $\rho = \frac{1}{1+j2\pi(n-i)\Delta f T_{rms}}$, where T_{rms} is the channel r.m.s. delay spread. Here we assume that the PDP does not depend on the transmit-receive antenna pair, as in [19]. The case of different PDP for different antenna pairs is detailed in the Appendix. Considering the first (correlated) component, $\rho \mathbf{H}_n$, we have

$$\text{Tr} \left\{ E \left[\left(\mathbf{H}_n^\dagger \mathbf{H}_n \right)^H \mathbf{H}_n^\dagger \mathbf{H}_n \right] \right\} = M_T. \quad (18)$$

Considering the second component, $\sqrt{1-|\rho|^2} \mathbf{H}_I$, by applying the trace property $\text{Tr}[AB] = \text{Tr}[BA]$ and the independence between \mathbf{H}_n and \mathbf{H}_I , we have

$$\begin{aligned} \text{Tr} \left\{ E \left[\mathbf{H}_I^H \left(\mathbf{H}_n^\dagger \right)^H \mathbf{H}_n^\dagger \mathbf{H}_I \right] \right\} &= \text{Tr} \left\{ E \left[\left(\mathbf{H}_n^H \mathbf{H}_n \right)^\dagger \right] E \left[\mathbf{H}_I \mathbf{H}_I^H \right] \right\} \\ &= M_R. \end{aligned} \quad (19)$$

For the particular case of Wiener phase noise and spatially uncorrelated channel with exponential PDP, the variance of the overall phase noise interference after ZF can be summarized in

$$\begin{aligned} \sigma_{v_n}^2 &= \sigma_x^2 \sigma_{CPE}^2 + \frac{\sigma_x^2}{M_T} \sum_{i=0}^{N-1} \sum_{i \neq n}^{+\infty} \frac{2\pi B_\theta}{1 + \left(\frac{(n-i)/N+k}{B_\theta} \right)^2} \\ &\times \left[\frac{\sigma_{est}^2 M_T}{M_R - M_T} + \frac{M_T}{1 + 4\pi^2 (n-i)^2 (\Delta f T_{rms})^2} \right. \\ &\left. + \frac{4\pi^2 (n-i)^2 (\Delta f T_{rms})^2}{1 + 4\pi^2 (n-i)^2 (\Delta f T_{rms})^2} M_T M_R \right] \end{aligned} \quad (20)$$

B. MMSE receiver

Also in the case of the MMSE receiver, in the trace of the inner matrix of (14), we can separate the effects of the estimation error and of the sub-carrier correlation,

$$\begin{aligned} \text{Tr} \{ E [\mathbf{H}_i^H \mathbf{G}_n^H \mathbf{G}_n \mathbf{H}_i] \} &= \sigma_{est}^2 E [\text{Tr} \{ \mathbf{G}_n^H \mathbf{G}_n \}] \\ &+ \text{Tr} \left\{ E \left[\mathbf{H}_i^H \mathbf{H}_n \left\{ \left(\mathbf{H}_n^H \mathbf{H}_n + \frac{1}{\text{SNR}} \mathbf{I}_{M_T} \right)^{-1} \right\}^H \right. \right. \\ &\left. \left. \times \left(\mathbf{H}_n^H \mathbf{H}_n + \frac{1}{\text{SNR}} \mathbf{I}_{M_T} \right)^{-1} \mathbf{H}_n^H \mathbf{H}_i \right] \right\}. \end{aligned} \quad (21)$$

Again we can express \mathbf{H}_i as $\mathbf{H}_i = \rho \mathbf{H}_n + \sqrt{1-\rho} \mathbf{H}_I$, where the latter is independent of \mathbf{H}_n . The difference between the MMSE and ZF is relevant in the low SNR region, since for

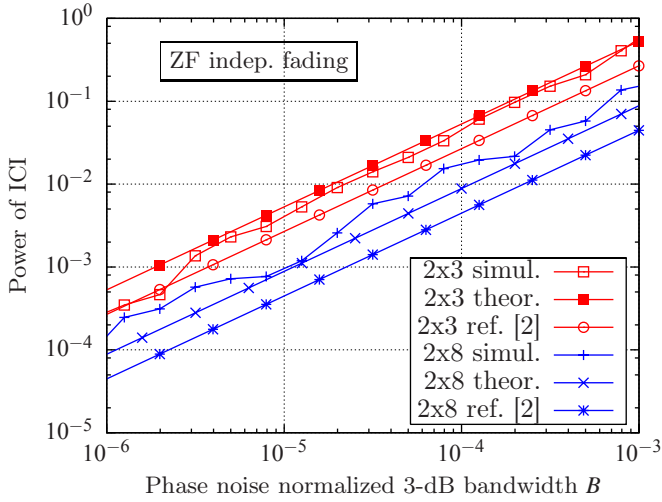


Fig. 2. Power of the ICI contribution for the ZF receiver with independent fading and ideal channel estimation.

high SNR \mathbf{G}_{MMSE} converges to \mathbf{G}_{ZF} . Then, for low SNR, \mathbf{G}_n can be approximated as $\mathbf{G}_n = SNR \mathbf{H}_n^H$, and

$$E[\text{Tr}\{\mathbf{G}_n^H \mathbf{G}_n\}] = SNR^2 E[\text{Tr}\{\mathbf{H}_n^H \mathbf{H}_n\}] = SNR^2 M_R \text{Tr}[\mathbf{R}_T]. \quad (22)$$

In fact the expectation in (22) represents the mean value of the trace of a Wishart matrix [18]. For the correlated component, the expected trace of the square of a Wishart matrix [18] gives

$$\text{Tr}\{E[\mathbf{H}_n \mathbf{H}_n^H \mathbf{H}_n \mathbf{H}_n^H]\} = M_T M_R \left[\text{Tr}(\mathbf{R}_R^2) + \frac{1}{M_T} \text{Tr}^2(\mathbf{R}_R) \right]. \quad (23)$$

In the independent fading term, we have the expected trace of the product of independent matrices [18] with $\text{Tr}\{E[\mathbf{H}_n \mathbf{H}_n^H \mathbf{H}_j \mathbf{H}_j^H]\} = M_T \text{Tr}[\mathbf{R}_R^2]$.

With MMSE at low SNR, the overall phase noise interference variance for the particular case of Wiener phase noise and spatially uncorrelated channel with exponential PDP, is

$$\begin{aligned} \sigma_{v_n}^2 &= \sigma_x^2 \sigma_{CPE}^2 + \frac{\sigma_x^2}{M_T} \sum_{\substack{i=0 \\ i \neq n}}^{N-1} \sum_{k=0}^{+\infty} \frac{2\pi B_\theta}{1 + \left(\frac{(n-i)/N+k}{B_\theta}\right)^2} \\ &\times \left[\frac{\sigma_{est}^2 M_T}{M_R - M_T} + \frac{SNR^2 M_T M_R (M_T + M_R)}{1 + 4\pi^2 (n-i)^2 (\Delta f T_{rms})^2} \right. \\ &\left. + \frac{4\pi^2 (n-i)^2 (\Delta f T_{rms})^2}{1 + 4\pi^2 (n-i)^2 (\Delta f T_{rms})^2} SNR^2 M_T^2 M_R \right]. \quad (24) \end{aligned}$$

VI. PERFORMANCE RESULTS

To validate the above expressions, we show some results with $N = 64$ sub-carriers and, without loss of generality, the same variance of the estimation error on the channel and on the CPE, $\sigma_{est}^2 = \sigma_{CPE}^2$. First, we present the case of independent fading among the sub-carriers and Wiener phase noise, to compare our analytic approach with simulations and with [2]. The curves of the ICI power vs. B_θ of Fig. 2 show a very good matching, although the first order approximation of [2] is slightly optimistic. Note that the assumption of independent fading in sub-carriers is an idealization analyzed here just to compare with [2]. In practical systems, pilot-based channel estimation would not work in such circumstances so that the

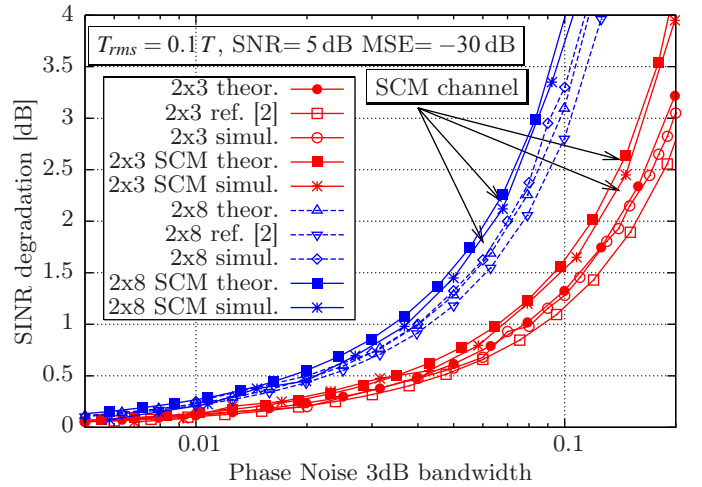


Fig. 3. Degradation for ZF as a function of the phase noise bandwidth, with exponential PDP $T_{rms} = 0.1T$ or SCM channel, estimation error $MSE = -30$ dB and reference $SNR = 5$ dB.

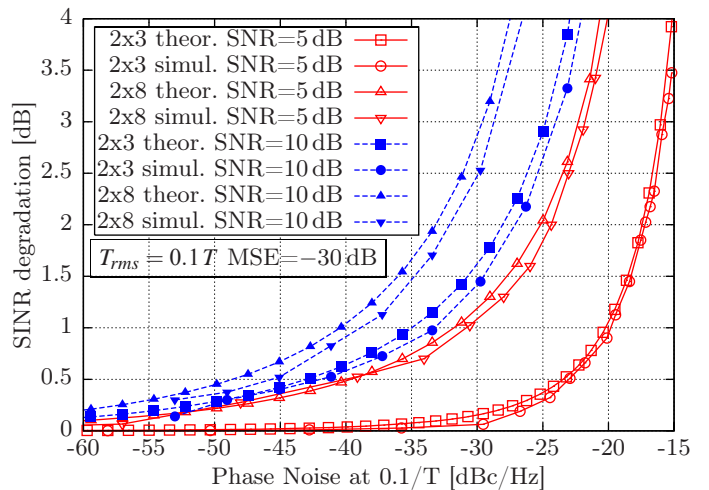


Fig. 4. Degradation for MMSE as a function of the phase noise PSD at $0.1/T$, with exponential PDP $T_{rms} = 0.1T$, estimation error $MSE = -30$ dB and reference $SNR = 5$ dB and $SNR = 10$ dB.

number of sub-carriers is always properly designed to avoid this condition. As examples of more realistic channel models, we use a spatially uncorrelated channel with exponential PDP with $T_{rms} = 0.1$, and on the other hand also a fully realistic spatially correlated channel with the spatial correlation and the PDP specified by 3GPP for the LTE evaluation [19]. Fig. 3 shows the SINR degradation of ZF for $SNR = 5$ dB and estimation error $MSE = -30$ dB, with a good fit between the simulations and the theoretical analysis and again [2] is slightly optimistic. To illustrate the performance of MMSE, a more general phase noise model is adopted [12], with an example of PSD with $1/f^3$ decay up to f_3 , then $1/f^2$ up to $f_2 = 10f_3$, followed by a fixed white noise level of -140 dBc/Hz for $f > f_2$. The value in dBc/Hz of the phase noise PSD at $0.1\Delta f$ is considered as a parameter, thus setting f_3 . Fig. 4 compares the theoretical SINR degradation with the simulation values, again with $MSE = -30$ dB, for reference $SNR = 5$ dB and 10 dB. We note a very good match

between the theoretical and the simulation results, especially for low SNR, while at higher SNR values the theoretical result becomes an upper-bound.

VII. CONCLUSIONS

We derived the expression of the SINR degradation in linear receivers for MIMO-OFDM, considering both the channel estimation error and the phase noise, with partial CPE compensation. The accuracy of the expressions has been compared to previously published works and verified with simulation results.

APPENDIX

CHANNELS WITH DIFFERENT POWER DELAY PROFILES

In the case of different PDP describing the second-order distribution of the channel paths for different antenna pairs, we can use a decomposition similar to $\mathbf{H}_i = \rho \mathbf{H}_n + \sqrt{1-\rho} \mathbf{H}_l$, where instead of a scalar ρ for all the matrix elements, we have different correlations, so that

$$\mathbf{H}_i = \rho \circ \mathbf{H}_n + \rho_C \circ \mathbf{H}_l \quad (25)$$

where \circ represents the Hadamard product, $\rho_C(i, j) = \sqrt{1 - |\rho(i, j)|}$ and the elements of the matrix ρ are given by (17). In this case, when considering the expected trace of products of the kind $E \left[\rho \circ \mathbf{H}_n ((\rho \circ \mathbf{H}_n)^H) \right]$, such as for example in (19) or (22) (similarly for $\rho_C \circ \mathbf{H}_l$), these can be seen as the expected value of the squared Frobenius norm of a Hadamard matrix product. Thus, we can derive an upper bound to the variance, by the property [20], $\|\mathbf{A} \circ \mathbf{B}\|_2 \leq \|\mathbf{A}\|_2 \|\mathbf{B}\|_2$. In practice the results (20) and (24), with $\|\rho\|_2^2$ and $\|\rho_C\|_2^2$ instead of $|\rho|^2$ and $(1 - |\rho|^2)$, would represent an upper bound to the variance of the overall interference.

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