COMPARATIVE STATICS FOR MARKET GAMES: THE STRONG CONCAVITY CASE*

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A B S T R A C T

In this paper we study the effects of a change in some exogenous variable (the number of players or a parameter in the payoff functions) on the strategies played and payoffs obtained in a Nash Equilibrium in the framework of a Market Game (a generalization of the Cournot model). We assume a strong concavity condition which implies that the best reply function of any player is decreasing on sum of the strategies of the remaining players (i.e. strategic substitution). Our results generalize and unify those known in the Cournot model.

I.- INTRODUCTION

In this paper we study the effects of changes in the number of players and shifts in their payoff function on the strategies played and the payoffs obtained in a Nash Equilibrium. We will assume on the class of games under consideration that the payoff function of each player fulfills the following:

I) It can be written as a function of her own strategy (assumed to be one dimensional) and the sum of the strategies of all players. This assumption has been called the "Aggregation Axiom" by Dubey, Mas-Colell and Shubik (1980), p. 346 and the corresponding games are called market games (for a different definition of a market game see Shubik (1984) p. 314). According to M. Shubik (1984) p. 325) "Games with the above property clearly have much more structure than a game selected at random. How this structure influences the equilibrium points has not yet been explored at depth".

II) It satisfies a strong concavity condition slightly stronger than the Strategic Substitutes case studied by Bulow, Geanakoplos and Klemperer (1985). The latter implies that the best reply function of each player (i.e. the mapping selecting the best strategy for a player, given the strategies of the remaining players) is decreasing on the strategies of other players.

Notice that the class of games satisfying I) and II) is large and include

a) Models of strategic competition in quantities (as the Cournot model, competition under rationing schemes -see Romano (1988)-, etc), i.e. oligopoly without or (in some cases) with product differentiation,

b) Models of strategic interaction among firms like technological competition (see Loury (1979)), the problem of the commons (see Dasgupta and Heal (1979) pp. 55-78) and pollution games, and

c) Models focusing on internal organization of firms or the like as contribution and revelation games and principal-many agents models.

In all the above cases uncertainty, taxes and payoff functions different from profit functions (i.e. sales) are allowed.

We first prove that the best reply functions of a game satisfying the aggregation axiom and the strategic substitution condition do not have any structural property beyond that they depend on the sum of strategies of the remaining players and that they are decreasing (Theorem 1). This result can be used to motivate the need of our strong concavity assumption. Assuming the latter we show that:

1) An increase in the number of players, a) decreases the value of the strategy of any incumbent, and increases the sum of all strategies (Proposition 1). b) Decreases the payoff of incumbents (Propositions 2-3).

2) A shift raising the marginal payoff curve of a player, say i, a) increases the sum of strategies and the strategy of player i, and decreases the strategy of any other player (Proposition 4). b) Increases the payoff of player i and decreases the payoff of any other player (Proposition 5).

3) A shift raising the marginal payoff curve of *all* players increases the sum of all strategies (Proposition 6).

4) We provide counterexamples to all Propositions when the strong concavity assumption is not fulfilled. Also two more examples are used to show that in the case considered in 3) above nothing can be said about individual strategies and utilities. Some of these examples are taken from previous work and are included here for the sake of completeness.

Summing up, 1), 2) and 3) above show that under our assumptions, the effects of an increase in the number of players or a shift in their payoff function agrees with our a priori intuition. 1) above has been studied in the Cournot case by Mc Manus (1962), (1964), Frank (1965), Ruffin (1971), Okuguchi (1973), Seade (1980) and Szidarovsky and Yakowitz (1982). It must be noticed that our approach not only generalizes these results but allows for simpler proofs and does not require that the number of players can be treated as a continuous variable. Parts 2)-3) above have been studied in the Cournot case by Dixit (1986) and Quirmbach (1988). Besides the fact that our results apply to a more general class of models, the motivation for our results in an oligopolistic framework is twofold:

i) On the one hand in an imperfectly competitive market *even if a firm cares only about profits*, profit maximization is not, in general, the best policy to be pursued and moreover it does not guarantee survival (see e.g. Vickers (1985)). Therefore the classical hypothesis of profit maximization lacks a convincing foundation in oligopolistic markets and has to be generalized to allow for the maximization of a more complex payoff function. Also, from the classical contribution of Baumol (1959) it is customary to argue that firms might be interested in objectives other than profits.

ii) On the other hand, in contrast with many contributions quoted above, our approach does not rely on dynamics at all. This is not because the author thinks that comparative statics can not profit from stability considerations but because the actual dynamic processes which are used can hardly being justified. Moreover, this stability conditions are usually very strong. For instance in the Cournot model with linear demand and cost functions, the equilibrium is unstable if the number of firms is greater than two. Thus, the aim of the paper is to obtain the best possible results which depend only on the aggregation axiom and the strong concavity condition.

Our results can be compared with those obtained under the (polar) assumption of supermodularity. Roughly speaking, a game is supermodular when for each player her strategy set is the product of compact intervals and the marginal profitability of any action increases with any other action of any player (see Topkis (1979) for a more general definition). When strategy sets are one-dimensional the above definition reduces to that of a game with strategic complementarities (see Bulow, Geanakoplos and Klemperer (1985)). It can be shown that if the marginal profitability of any action is increasing on a parameter, say τ, (this is identical to our assumption 4), the largest and smallest Nash equilibria are increasing functions of τ so if the Nash equilibrium is unique, it is increasing on τ (see Lippman, Mamer and McCardle (1987), Milgrom and Roberts (1990), Milgrom and Shannon (1992))[1]. This is analogous to our Propositions 4 and 6 (but in our case individual strategies are not always increasing on τ, see example 6). Notice that the distinction between idiosyncratic and generalized shocks does not play any role in supermodular games. At the best of my knowledge there are no results in the supermodular games literature on the effect of entry (Propositions 1-3) below) or the effect of a change in τ on payoffs (Proposition 5 and example 6).

The rest of the paper goes as follows. The next Section explains the basic model and the main assumptions. Section 3 studies the effect of an increase in the number of players and Section 4 focuses on shifts of the marginal payoff curve. Finally Section 5 gathers our final comments.

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(1) Other properties of supermodular games are that 1) the existence of a Nash equilibrium does not require quasi-concavity of the payoff functions, and 2) under certain circumstances, if there are several Nash equilibria, they can be Pareto-ranked. Applications of supermodular games include Bayesian games and oligopolistic competition (see Vives (1990)), stability and learning (see Lippman, Mamer and McCardle (1987), Milgrom and Roberts (1990) and Krishna (1993)) and coordination problems in a macroeconomic framework (see Silvestre (1993) for a survey of this literature). For general surveys on supermodular games see Fudenberg and Tirole (1991) and Vives (1993).

II.- THE MODEL

In this Section we will explain the main concepts which will be used in the rest of the paper.

Definition 1.- *A market game* $(U_i(\ ), S_i)_{i\in I}$ *consist of*

a) A set of players (also called agents) $I = 1, 2, \ldots, n$.

b) A collection of strategy sets $S_i = R_+$.

c) A collection of payoff functions $U_i : \underset{i\in I}{X} S_i \longrightarrow R$ *of the form* $U_i(x_i, x)$ *where* $x_i \in S_i$ *and* $x = \sum_{j\in I} x_j$.

In words, in a market game, the so-called "Aggregation Axiom" holds (see Dubey, Mas-Colell and Shubik (1980), p. 346), so the (one dimensional) strategies of the players can be aggregated in an additive way. We remark that all the Propositions below can be proved if $x = f(x_1, \ldots, x_n)$ ($f(\ )$ strictly increasing) introducing suitable concavity assumptions. A market game can be thought of as a generalization of the well-known Cournot model. In this case $U_i = p(x)x_i - C_i(x_i)$, x_i being the output of firm i, x total output, p(x) the inverse demand function and $C_i(x_i)$ the cost function of firm i. This case will be used in most examples below. We remark that our approach can deal with a) payoff functions different from profit (i.e. Welfare-maximizing publicly owned firms, see Fershtman (1990)), b) symmetric uncertainty (for the Cournot case see Horowitz (1987)), c) taxes (for the Cournot case see Dierickx, Matutes and Neven (1988)) and d) in some cases, heterogeneous product (using the trick of Yarrow (1985), p. 517). Other examples of market games (technological competition, the problem of the commons, preference revelation, contribution games, pollution and wage-setting trade unions) are explained in Table 1.

	x_i	x	$U_i(x_i,x)$	$x=f(x_1,...,x_n)$
Trade-Unions	porcentual increase in wage rate	Inflation rate	Utility function of trade union 1	Inflation rate as a function of wage rate increases
Polution	Output of firm i	Amount of Pollution	Profit function	Production of Polution
Contribution Games (Public Goods, Principal Agents)	Private Inputs offered by i	Quantity of the public good / Reward	Utility function of agent i	Production function of the public good / Reward function
Preference Revelation	Preference parameters to be revealed	Social State	Utility function	Social Rule
Problem of the Commons	Inputs used by firm i	An environmental variable	Profit function	Environment as a function of inputs
Oligopoly	Output of firm i	Price	Profit function	Inverse demand function
Technological Competition	Input needed to produce the technology used by firm i	Technological level	Profit function	Technology as a function of inputs

TABLE 1

Now we state our solution concept.

Definition 2: *Given a market game* $(U_i(\ ),S_i)_{i\in I}$, $(x_i^*,x^*)_{i\in I}$ *with* $x^* = \sum_{i\in I}x_i^*$, $x_i^* \in S_i$ $\forall i \in I$ *is said to be a Nash Equilibrium (N.E.) if* $\forall i \in I$

$$U_i(x_i^*, x^*) \geq U_i(x_i, x^* - x_i^* + x_i) \quad \forall x_i \in S_i$$

Now we state and discuss our main assumptions.

Assumption 1: $U_i(\ ) \in \mathscr{C}^1$ $\forall i \in I$.

Notice that under Assumption 1 (A.1 in what follows) if $x_i^* \in$ int. S_i the necessary condition of a N.E. reads as follows:

$$\frac{\partial U_i(x_i^*, x^*)}{\partial x_i} + \frac{\partial U_i(x_i^*, x^*)}{\partial x} = 0 \qquad \forall\ i \in I.$$

Let us define

$$T_i = T_i(x_i, x) \equiv \frac{\partial U_i(x_i,x)}{\partial x_i} + \frac{\partial U_i(x_i,x)}{\partial x} \qquad \forall\ i \in I$$

Let N be the set of active agents (i.e. those for which $x_i^* \in$ int. S_i in a N.E. with n players). N+1 is defined accordingly. We will assume that N $\cap$ N+1 $\neq \emptyset$, i.e. at least one player is active in N.E. with n and n+1 agents respectively.

Assumption 2: $T_i(x_i, x)$ *is strictly decreasing on* x_i *and* x $\forall\ i \in I$.

A sufficient condition for A.2 to hold is that $U_i(\ )$ be strictly concave on x and x_i and(if $U_i \in \mathcal{C}^2$) that $\frac{\partial^2 U_i(\ )}{\partial x_i x} < 0$.

In the homogeneous oligopoly case A.2 is equivalent to a much used condition in the literature on Cournot equilibrium (see e.g. Friedman (1982) p. 496, assumption 3 and the references therein) namely

$$\frac{\partial^2 p(\ )}{\partial x^2} x_i + \frac{\partial p(\ )}{\partial x} < 0 \text{ and } \frac{\partial p(\ )}{\partial x} - \frac{\partial^2 C_i(\ )}{\partial x_i^2} < 0$$

It can be readily seen that A.2 implies that the best reply function is decreasing, i.e. the assumption of Strategic Substitutes in terminology of Bulow, Geanakoplos and Klemperer (1985). Finally, we state our third assumption.

<u>Assumption 3:</u> *$U_i(\ )$ is strictly decreasing on x $\forall\ i \in I$.*

This assumption will be only used in Propositions 2, 3 and 5. If $U_i(\ )$ were strictly *increasing* in x, the reverse conclusions would be true. In the Cournot case A.3 requires a strictly decreasing inverse demand curve.

Notice that A.1 and 2 plus a compactness requirement imply the existence of an unique N.E. and that under A.1 and 3 any interior N.E. can be shown to be inefficient, i.e. there is a strategy vector for which all players are better off (for proofs of these facts see Friedman (1977) pp. 25-6 and 169-71. See Kukushkin (1993) for a more general result on the existence of a N.E.).

The reader may wonder if, under the aggregation axiom, strategic substitution alone may be sufficient to yield well defined answers to our comparative statics questions. The following Theorem looks for structural properties of best reply functions under these two assumptions and finds a negative result. First, let us define $x_{-i} \equiv \sum_{j \neq i} x_j$.

__Theorem 1:__ Let $x_i = f_i(x_{-i})$ $i = 1,\dots, n$ be a collection of $\mathcal{C}^0$ functions defined on a compact set and such that $f_i(\ )$ is strictly decreasing $\forall i$. Then,

a) $\forall i$, $\exists$ $U_i(x_i,x)$, $U_i(\ ) \in \mathcal{C}^1$, concave on x_i such that

$$f_i(x_{-i}) \equiv \text{arg.} \max_{a \in S_i} U_i(a,\ a+x_{-i}),\quad \forall x_{-i}$$

Moreover, $U_i(\ )$ can be taken to be decreasing on x (i.e., fulfilling A.3)

b) $\forall_i$, $\exists$ a $\mathcal{C}^1$ cost function $C_i(x_i)$ and a linear inverse demand function $p = A - x$ such that

$$f_i(x_{-i}) \equiv \text{arg.} \max_{b \in S_i} (A - b - x_{-i})b - C_i(b)\quad \forall x_{-i}$$

__Proof:__ a) First notice that $f_i(\ )$ is invertible. Also, $f_i^{-1}(\ )$ is integrable since $f_i^{-1}(\ ) \in \mathcal{C}^0$ (by the continuity of $f_i(\ )$, see Bartle (1976), p. 156), and it is bounded (see Bartle (1976) p. 427). Let $q_i(x_i)$ be the primitive of $f_i^{-1}(x_i)$. Define $U_i \equiv q_i(x_i) + x_i^2 - x_i x$. Notice that U_i is decreasing on x. Then we have that

$$\frac{\partial U_i}{\partial x_i} = f_i^{-1}(x_i) + 2x_i - x_i - x \equiv f_i^{-1}(x_i) - x_{-i} = 0.$$

and since $f_i^{-1}(\ )$ is strictly decreasing U_i is concave on x_i, so the second order condition of payoff maximization is satisfied and thus, a) holds.

b) Let $p(x) \equiv A - x$ and $C_i(x_i) \equiv Ax_i - x_i^2 - q_i(x_i) + B$, where $q_i(\ )$ is as defined in part a) above. Since x_i is defined on a compact set, B can be taken large enough such that $C(x_i) \geq 0$, $\forall x_i$. Also, taking A large enough, the marginal cost is positive. Then,

$$\Pi_i \equiv p(x)x_i - C(x_i) = (A-x)x_i - Ax_i - x_i^2 - q_i(x_i) - B = q_i(x_i) + x_i^2 - xx_i - B$$

which is identical to the utility function constructed in part a) above.

The main consequence of Theorem 1 is that in games in which both the aggregation axiom and the strategic substitution assumption hold, that the best reply functions depend on the sum of strategies of the other players and that they are decreasing exhaust all the properties of best reply functions. Thus, they are, up to some extent, arbitrary (this result may be regarded as analogous to the lack of structural properties of excess demand functions in General Equilibrium, see Sonnenschein and Shafer (1982) but in our case the root of the problem is not on the aggregation side). Even if payoff functions are restricted to be profit functions, no structural property beyond those quoted above can be found!.

As an easy corollary of Theorem 1 we have that a) the equilibrium set of strategies is arbitrary and b) comparative statics will not yield definitive answers. Both points can be easily seen in the case of two players by constructing best reply mappings which intersect at any given set of points and by considering shifts of these curves and comparing non adjacent equilibria. Thus, we are lead to conclude that in general, we need additional properties to those quoted before in order to tackle comparative statics. As we will see our A.2 will be sufficient for this job.

III.- THE EFFECTS OF ENTRY

In this Section we will study the effects of an increase in the number of players (see Bresnahan and Reiss (1991) and the references there for the empirical evidence in oligopolistic markets). In order to save notation let $y \equiv x_{n+1}(n+1)$. Also, let us denote by $x(n)$, $x_i(n)$ and $U_i(n)$ the equilibrium values of x, x_i and U_i in a game with n players.

Proposition 1: *Under A.1-2 we have that*

a) $x(n) \leq x(n+1)$, $x_i(n) \geq x_i(n+1)$ $\forall$ $i \in N$ *and*

b) if $y > 0$ *the above inequalities are strict.*

Proof: *We first notice that if* $x(n) \geq x(n+1)$ *and* $x_i(n) > 0$, $x_i(n+1) = 0$ *is impossible since* $T_i(x_i(n), x(n)) \geq T_i(0, x(n+1)) \geq T_i(0, x(n))$ *would contradict that* $T_i(\ )$ *is strictly decreasing on* x_i.(2) *Take any* $i \in N \cap N+1$ *(if* $i \notin N+1$, $x_i(n) > x_i(n+1) = 0$*). In both N.E. first order conditions hold so*

$$(1) \quad T_i(x_i(n), x(n)) = T_i(x_i(n+1), x(n+1)).$$

Therefore because A.2 we have only two possibilities:

I.- $x(n+1) \leq x(n)$ *and* $x_i(n+1) \geq x_i(n)$, *with a strict inequality or*

II.- $x(n+1) \geq x(n)$ *and* $x_i(n+1) \leq x_i(n)$.

(2) A similar argument shows that if $x(n) \leq x(n+1)$ and $x(n) = 0$, then $x(n+1) = 0$, so the second inequality in a) in Proposition 1 holds $\forall$ $i \in I$.

If I holds, since all active players at n are active at n+1 and $x = \sum x_i$ *we have a contradiction. Therefore part a) is proved. Part b) is proved noticing that (1) implies that if* $x(n) = x(n+1)$, *then* $x_i(n) = x_i(n+1) \quad \forall\ i \in N \cap N+1$. *But since all active players at n will be active at n+1 and* $y > 0$ *we reach a contradiction. Therefore* $x(n) < x(n+1)$, *A.2. plus (1) show that* $x_i(n) > x_i(n+1) \quad \forall\ i \in N \cap N+1$. *Finally if* $i \notin N+1$ *but* $i \in N$ $x_i(n) > x_i(n+1) = 0$. ■

If A.2 does not hold, Proposition 1 fails as the following examples -which refer to the Cournot model- show.

<u>Example 1.-</u> *(Seade (1980)). Let* $p = x^{-0'8}$, $C_i = x_i$. *Using the first order conditions of profit maximization, it is easily seen that* $x_i(1) < x_i(2)$.

<u>Example 2.-</u> $p = a - bx$, $C_i = cx_i + d/2\ x_i^2$ *with* $a > c$, $d < 0$, $d + 2b > 0$ *and* $d + b < 0$. *(Total costs will be negative for* x_i *large enough, but this can be fixed). Then* $x = \dfrac{(a - c)n}{b+d+nb}$ *so x is decreasing with n if* $b + d < 0$. *On the other hand second order conditions are fulfilled if* $d+2b > 0$. *A graphical argument similar to Example 2 can be found in Mc Manus (1964).*

We now turn to study how payoffs change with entry.

<u>Proposition 2:</u> *Under A.1, A.2 and A.3*

a) $U_i(n) \geq U_i(n+1)$.

b) If $y > 0$ *the above inequalities are strict.*

Proof: In order to save notation let us write $x_{-i}(n)$ as the strategies of all players except i in a N.E. with n players, i.e. $x_{-i}(n) = x(n) - x_i(n)$. Also define $V_i(\ ) \equiv U_i(x_i, x_{-i} + x_i) \equiv V_i(x_i, x_{-i})$. Then, if Proposition 2 a) were not true, $V_i(x_i(n+1), x_{-i}(n+1)) > V_i(x_i(n), x_{-i}(n)) \geq V_i(x_i(n+1), x_{-i}(n))$. Thus, $x_{-i}(n) > x_{-i}(n+1)$ which contradicts that $x_{-i}(n)$ is non-decreasing in n by Proposition 1 a). In order to show b) let us assume that $U_i(n) = U_i(n+1)$. Then, reasoning as above we get $x_{-i}(n) \geq x_{-i}(n+1)$ contradicting that if $y > 0$, $x_{-i}(n)$ is strictly increasing in n (by Proposition 1 b)).■

If A.2 holds but $U_i(\ )$ is increasing on x we have the reverse conclusion. The following example shows that if A.2 does not hold, Proposition 2 may fail.

Example 3.- Let us assume 2 agents with identical payoff functions (see Figure 1). Because A.3, payoffs increase in the direction of the arrows. Point A is a symmetrical N.E. with 2 players since any player can only change unilaterally x and x_i on the 45^0 line (x and x_i change in the same amount since the strategies of other players are given). By the same token B is a symmetrical N.E. with 3 players and such that the payoff of 1 and 2 is now greater (notice that if n = 1 A'A and OA would be identical and the example does not work).

Notice than in Example 3 we have that n > 1. If this is not the case, i.e. there is a unique incumbent player, the entry of a new player will always decrease the payoff of the incumbent, i. e. her payoff is bigger under monopoly than under duopoly as shown by the next Proposition (notice that Assumptions 1-2 are not required and that if U( ) is increasing in x, it is easy to show that entry increases the payoff of the incumbent.

Proposition 3.- *Under A.3 we have that*

a) $U_i(1) \geq U_i(2)$ *and*

b) if $x_2(2) > 0$ *then the above inequality is strict.*

Proof: *Suppose it is not. Defining* $V_i(\ )$ *as before we have that*

$$V_1(x_1(2),\ x_2(2)) \geq V_1(x_1(1),\ 0) \geq V_1(x_1(2),\ 0)$$

And since $V_i(\ )$ *is decreasing on* x_{-i} *we get a contradiction* .■

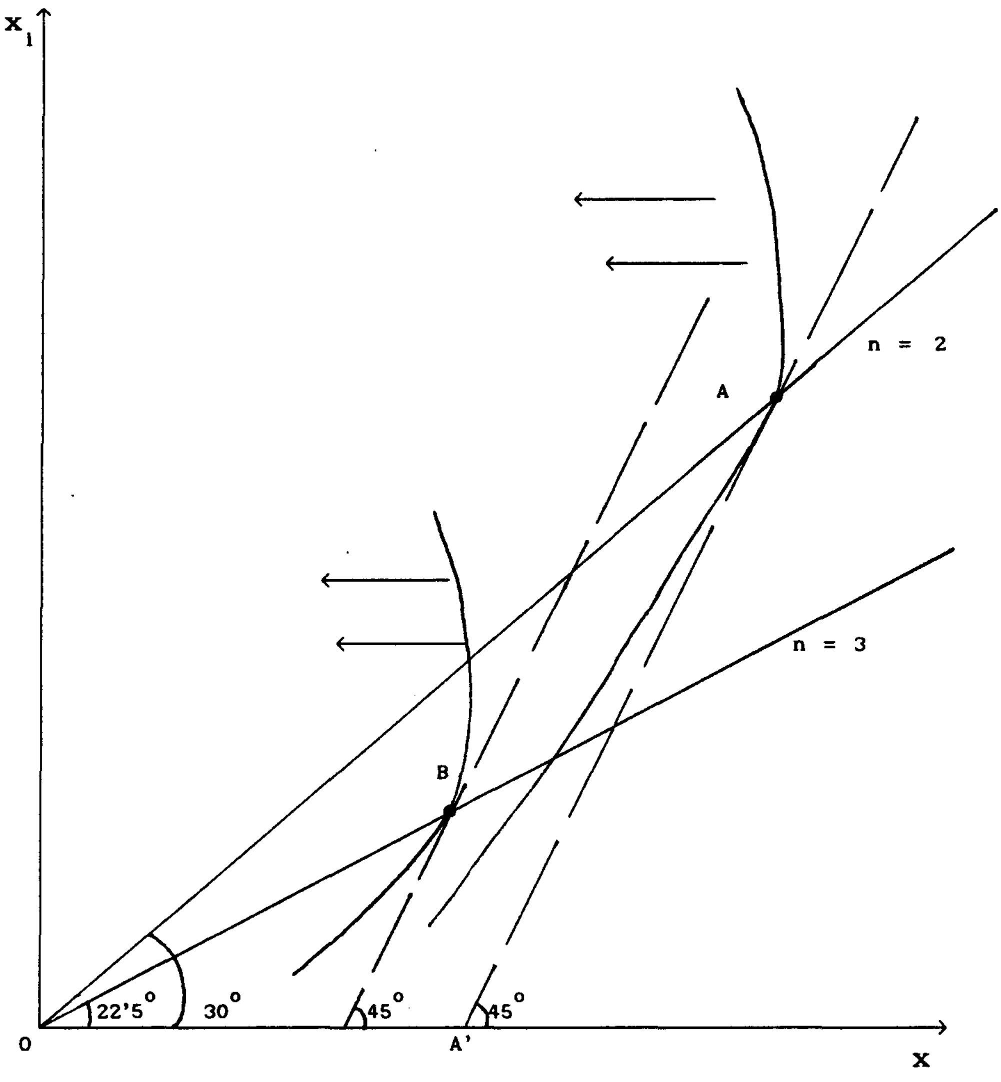

FIGURE 1

IV.- THE EFFECTS OF SHOCKS

In this Section we will study the effect of an exogenous shift in the payoff function on the relevant variables. We will assume that the payoff function of player i can be written as $U_i = U_i(x_i, x, t_i)$ where t_i is a one dimensional parameter which is possibly different for different players (in the Cournot model t_i may represent either the factors behind the demand side or the cost side or as in Farrell and Shapiro (1990) the quantity of capital own by firm i). In this Section, in order to simplify the proofs we will assume that Nash Equilibria are interior. Then, the first order condition reads $T_i(x_i, x, t_i) = 0$. Finally the values of the strategies and payoffs in a Nash Equilibrium will be denoted by x^*_i, x^*, and U^*_i.

<u>*Assumption 4:*</u> *$T_i(\ )$ is strictly increasing in t_i.*

This assumption allows us to interpret increases in t_i as shifts to the right of the marginal payoff curve, i.e. t_i can be regarded as a measure of the impact of a shock on the marginal payoff of player i.

We will distinguish two types of shocks: idiosyncratic and generalized. In the first we will study the impact on the market of a variation in a single t_i (i.e. an increase in the price of the factors or the taxes payed by player i). In the second we consider a simultaneous variation in all t_i, i = 1,...,n. This corresponds, for instance, to a shift in the common demand function or the price of a factor used by all players in the industry. In this case, without loss of generality we will write the first order condition as a function of a single t, i.e. $T_i(x_i, x, t) = 0$.

Intuition suggests that in the case of an idiosyncratic shock an increase in t_i will increase the strategy of player i and it will decrease those of its competitors. This intuition is formalized in the next Proposition:

Proposition 4: Under A. 1, A. 2 and A. 4 an increase in t_i, a) increases the sum of strategies b) increases the strategy of player i and c) decreases the strategy of any other player in the market.

Proof: Since the proof is fairly analogous to the proof of Proposition 1 we will indicate only the guidelines. First it is proven that the sum of strategies can not be constant. Second, if the sum of strategies decreases, the strategy of all players must increase in order to maintain first order conditions and this is a contradiction. Thus, the sum of strategies increases. Again first order conditions of all players except i imply that the strategy of these players must fall. Therefore the strategy of i must increase.■

Of course if the inequality in A.4 is reversed so are the conclusions of Proposition 4. An implication of this Proposition is -in contrast with supermodular games- the absence of multiplier effects i.e. $dx/dt_i < dx_i/dt_i$ (see Fudenberg and Tirole (1991) p. 498). The next example -which again refers to the Cournot model- will show that A.2 is needed for the result to hold.

Example 4.- *Suppose that there are three firms and that in a (sufficiently large) neighborhood of a N.E. the relevant functions read* $p = a' - x$, $C_1 = cx_1 - d/2\, x_1^2 - t_1 x_1$ *with* $a' > c$, $d > 0$, $d - 2 < 0$ *(so the second order condition holds),* $d - 1 > 0$, *and* $C_i = c'x_i$, *with* $a' > c'$, $i = 2, 3$. *Let* $\bar{a} \equiv a' - c$ *and let* $a \equiv a' - c'$. *Profit maximization implies that* $x_1 = (x - \bar{a} - t_1)/(d - 1)$ *and* $x_i = a - x$, $i = 2, 3$. *Solving the system we get* $x = (2a\,(d - 1) - \bar{a} - t_1) / (3(d - 1) - 1)$. *If, for instance,* $a = 10$, $d = 1.5$, $t_1 = 5$ *and* $\bar{a} = 1$ *we have that* $x^* = 8$, $x_1^* = 4$, $x_i^* = 2$. *But if* $t_1 = 5.5$, $x^* = 7$, $x_1^* = 1$, $x_i^* = 3$.

For the next Proposition we will need an additional assumption. This assumption plus A.4 implies that a variation in t_i affects both marginal and total payoff in the same direction.

Assumption 5: $U_i(\ )$ *is increasing on* t_i.

Proposition 5: *If all payoff functions are* $\mathscr{C}^2$ *and A. 2-5 hold, an increase in* t_i, *a) increases the payoff of i and b) decreases the payoff of any other player*

Proof: *First, it is easy -but tedious- to show that all variables are continuously differentiable functions of* t_i *in a neighborhood of equilibrium, since assumption 2 implies that the Jacobian matrix of* $T_i(\ )$ *has a non vanishing determinant. Then, taking into account the first order conditions for player* $j \neq i$, *we have that*

$$dU_j/dt_i = \partial U_j(\ )/\partial x \circ (dx/dt_i - dx_j/dt_i)$$

and Proposition 4 and A. 3 imply b) above. In the case of player i we have that

$$dU_i/dt_i = \partial U_i(\ )/\partial x \circ (dx/dt_i - dx_i/dt_i) + \partial U_i(\ )/\partial t_i$$

and since the strategy of all competitors has decreased and A. 5 we obtain a) above.■

The next example shows the necessity of A.2 for Proposition 5 to hold

Example 5.- *Suppose that the market is as in example 4. Then it is easily calculated that if* $t_1 = 5$, $U^*_1 = 4$ *and* $U^*_i = 4$, $i = 2, 3$. *But if* $t_1 = 5.5$, $U^*_1 = 0$ *and* $U^*_i = 9$, $i = 2, 3$.

We will end this Section by studying the effects of a generalized shock.

Proposition 6: Under A. 1, 2 and 4 an increase in t increases x

Proof: First, by analogous reasoning to Proposition 1 it can be shown that x can not be constant. And if x decreases all x_i *must increase. Contradiction.*■

The effect of *t* on individual strategies and payoffs in equilibrium depends on how payoff functions are affected (see Dixit (1986) and Quirmbach (1988)). This means that, in the Cournot model, a technological improvement in costs might decrease the output and profits *of the most efficient firm* (see example 6). Finally without A. 2 Proposition 6 does not hold (see example 7).

Example 6.- Let $p = a - x$, $n = 2$, $C_1 = c_1 x_1$, $C_2 = \alpha\, c_1 x_2$. Take $t = - c_1$, so A. 2 and 4 hold. It can be easily shown that in the N.E. $x^*_1 = (a + \alpha\, c_1 - 2c_1)/3$ and $U^*_1 = ((a - t_1(\alpha - 2)) / 3)^2$. Thus if $\alpha > 2$, the output and profits of firm 1 (which is the most efficient firm) decreases with t.

Example 7.- Let $p = x + t - A$, $C_i = 2.5\, x_i^2/2$, $n = 2$, with $A > t$ (this implies that for x small p is negative but since p is positive in equilibrium the inverse demand function can be substituted by $p = \max(0, x + t - A)$). Thus, $T_i = x + t - A - 1.5x_1$ so A.4 and the second order condition are fulfilled. Then, $x = 4(A - t)$, i.e. x is decreasing on t.

V.- CONCLUSIONS.

In this paper I have tried to integrate several models -some of them very much used in Industrial Organization and Welfare Economics- and to show that the qualitative properties of comparative statics of these models conform with our intuition as long as i) the game is a market game and ii) a strong concavity condition, which implies strategic substitution, is met. This is because in our case the combination of i) and ii) above implies that the best reply function is a contraction mapping: uniqueness and "right" comparative statics properties follow from that.

It would be very nice if it could be shown that the qualitative properties of models of strategic substitutes and strategic complements are similar, i.e. that a raise in taxes always decreases total output and increases prices. If this were the case we would not need to worry about which model is the right one, since both would yield the same qualitative predictions. This would alleviate the long-standing polemic between supporters of quantity-setting models (Cournot) and price-setting models (Bertrand). However, the case of strategic complements presents greater difficulties and might require different methods. First, an additional assumption is needed in order to guarantee that the best reply function is a contraction (see e.g. Friedman (1982) p. 504, assumption 6). Second, in the case of entry, it is not clear how to model the price of a firm which is not in the market. And third, unless additional assumptions are made, the game is not a market game.

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