Working Paper 95-19
Economics Series 13
April 1995

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THE DISTRIBUTION OF OPPORTUNITIES: A NORMATIVE THEORY

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Abstract

In this paper, we consider the problem of ranking profiles of opportunity sets. First, we take each agent's preferences over (individual) opportunity sets as given. Then, rather than discriminate among possibly competing evaluative criteria, we consider minimal standards for any such ranking. We impose four normative principles, in each case limiting the conditions under which ethical conclusions might be drawn to only those cases that are unambiguous. The first three principles are subrestrictions of the Pareto criterion; they require that Pareto improvements unambiguously enhance social welfare only when they do not conflict with other social objectives. The fourth principle is a minimal equity condition. It requires that if an agent can be identified as being the worst-off, then a necessary condition for social welfare to unambiguously increase when some agents gain is that this agent gains as well, however slightly. We then study the properties of social optima under these restrictions. We show that while optima need not be Pareto efficient, they must be envy-free. Thus, accepting these principles requires commitment to a world in which no agent envies the opportunities available to another.

*Departamento de Economía, Universidad Carlos III de Madrid. I wish to thank Jorge Durán Laguna, Monique Florenzano, Pascal Gourdel, Cuong Le Van, and Hans Peters for helpful conversations. In addition, financial support from the Institute for Policy Reform and the U.S. Agency for International Development is gratefully acknowledged. This publication was prepared under a cooperative agreement between the Institute for Policy Reform (IPR) and the Agency for International Development (AID), Cooperative Agreement No. AEP-5463-A-00-4015-00. Views expressed in this paper are those of the author and do not necessarily reflect those of IPR or AID.

1. Introduction

Clearly, there is a great deal of controversy over the appropriate criteria for evaluating or ranking social outcomes or states. And when confronted with incompatible or inconsistent criteria, one is generally forced to select among them. Here we pursue a different course. Rather than argue for or against one criterion or another, we accept the plurality of such views, and we attempt to find common ground. That is, we propose to consider minimal standards for ranking social states — a common denominator of sorts.

We cast our discussion in terms of the opportunities available to the members of society. Thus, a *social state* is described by a list of sets of the form $\mathbf{O}=(O^1,\ldots,O^n)$, where O^i denotes the opportunities available to agent i.[1] (As will be seen in the sequel, this generalizes the standard economic formulation in which a state corresponds to a resource allocation.) We wish to consider necessary conditions for ranking distributions of opportunity sets.

First, as in standard (economic) choice environments, we do not enquire of the source of preferences but simply take them as given. Here, however, we begin with preferences defined over sets.[2] Thus, any intrinsic value afforded by alternative decision environments is incorporated into the agents' primitive

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[1]The first attempts to rank distributions of opportunity sets in the manner described here were in Kranich (1993a,b). The relationship between the present study and those is discussed in Section 7 below. See also Thomson (1994) for a discussion of equitable opportunities.

[2]There is by now a substantial literature on extending a (preference) relation defined on a set to the power set. [See Fishburn (1993) and Nehring and Puppe (1994) for recent examples and the references cited therein.] This approach presumes that agents' basic preferences are defined over the individual members of the set. However, there is no fundamental reason to presume that such preferences depend only on singletons.

descriptions.[3]

We then posit four normative principles which we consider necessary for a well-behaved social ordering. In each case, we limit the conditions under which ethical conclusions might be drawn to only those cases that are unambiguous.[4]

According to the *Principle of Just Enhancement*, if an agent can be identified as being the worst-off, then, *ceteris paribus*, a rank-preserving enhancement of his or her opportunity set should result in an unambiguous social improvement. Next, the *Principle of Transposition* states that if two agents each prefer the opportunity set of the other, then transposing their sets should unambiguously improve social welfare. The *Restricted Pareto Principle* is a weak form of the Pareto criterion; it requires that at least those Pareto improvements in which each agent prefers his or her own opportunity set should be welfare enhancing. Otherwise, however, Pareto improvements might conflict with other distributional objectives, in which case the effect on social welfare may be ambiguous. Finally, we propose the *Principle of Just Distribution* which again applies only in the event an agent can be identified as being the worst-off, and it requires that in order for social welfare to unambiguously increase when some ("wealthier") agents gain, then it is necessary that the opportunities available to the worst-off agent

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[3]See Sen (1991,1992) for a discussion of the distinction between intrinsic versus instrumental dimensions of choice.

[4]The interpretation is critical. There may be circumstances which nearly everyone agrees are social welfare enhancing. However, if one person disagrees, then we would say the change is "ambiguous." In such circumstances, social welfare may increase or it may not. Indeed, in terms of "unambiguous" changes, the alternative states may be incomparable. Our objective is to describe conditions under which social welfare comparisons are unambiguous, that is, under which we would all agree.

should improve as well, however minimally. Thus, for example, the "poorest" agent should share in the benefits of an increase in aggregate opportunities.[5,6,7]

While these principles are insufficient to ensure a complete ranking of distributions of opportunity sets, they do determine certain characteristics of the maximal set of any such ranking. Specifically, under the aforementioned conditions, a maximal element must be *envy-free*; however, it need <u>not</u> be *Pareto efficient.* Thus, our results show that these principles entail commitment to a world in which no agent envies the opportunities available to another.

The paper is organized as follows. The next section contains preliminary definitions and notation as well as the formal description of the class of problems under investigation. In Section 3, we develop the basic postulates for the two-agent case, where they are most transparent. To evaluate the intuitive appeal of this framework, Section 4 considers the special case in

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[5]This is significantly weaker than the axiom of *resource monotonicity*, which requires that all agents must benefit when aggregate resources increase and all agents must lose when resources decrease. [Cf. Roemer (1988) and Moulin (1990).]

[6]In this paper, we abstract from the source of opportunities, and in particular from the fact that some opportunities may be "deserved," being the result of one's own efforts. At this stage of our analysis, therefore, it is more appropriate to view this work as a generalization of exchange, in which endowments are exogenously specified, rather than generalizing an economy with production. We will address the distinction between endogenous and exogenous opportunities in subsequent work.

[7]Notice, however, that it is consistent with this principle that social welfare might increase in the event the better-off agents improve considerably and the worst-off only minimally; that is, the result is ambiguous and the principle makes no claims in this case. Thus, asymptotically, the principle nearly allows weak Pareto improvements for only the wealthy to be social welfare enhancing.

which opportunity sets are singletons in $\mathbb{R}^{\ell}$. Here the framework reduces to the familiar case of comparing resource allocations. We then consider the general two-agent model in Section 5, and in Section 6 we extend the analysis to include additional agents. Finally, Section 7 contains concluding remarks.

2. Preliminaries

Let $N=\{1,\ldots,n\}$ be a finite set of agents, and let L be a universal set of *opportunities*. For reasons of generality, we do not specify the nature of the elements in L. Let $\mathbb{P}(L)$ denote the set of nonempty subsets of L. We consider a partially ordered set $(\mathcal{L},\leq_o)$, where $\mathcal{L}$ is a nonempty subset of $\mathbb{P}(L)$.[8] $\mathcal{T}$ denotes the order topology on $\mathcal{L}$.

Agent i's *opportunity set* is denoted $O^i\in\mathcal{L}$. We assume each agent has preferences $\mathcal{R}^i$ defined over the elements in $\mathcal{L}$. $\mathcal{I}^i$ and $\mathcal{P}^i$ denote the symmetric and asymmetric components of $\mathcal{R}^i$, respectively.

For $S\subseteq N$, let $\mathcal{L}^S:=\bigtimes_{i\in S}\mathcal{L}$, with generic element $\mathbf{O}^S$. When referring to the entire set of agents, we omit "N" and write $\mathbf{O}$ in place of $\mathbf{O}^N$. We refer to $\mathbf{O}$ as a *profile*. For $\mathbf{O}\in\mathcal{L}^n$, let $\pi_{ij}(\mathbf{O})$ denote the profile obtained by transposing the i^{th} and j^{th} components. Also, we define the *worst-off agents at* $\mathbf{O}$ by $W(\mathbf{O}):=\{j\in N\,|\,O^i\mathcal{R}^iO^j \text{ for all } i\in N\}$.

For $\mathbf{O}\in\mathcal{L}^n$ and for $S\subseteq N$, we will say $\mathbf{O}'^S\in\mathcal{L}^S$ is a *rank-preserving improvement of* $\mathbf{O}^S$ if $O'^i\mathcal{P}^iO^i$ for all $i\in S$, and for all $i,j\in N$, $O^j\mathcal{R}^jO^i$ implies $O^j\mathcal{R}^jO'^i$. Let $RP(\mathbf{O}^S)$ denote the set of rank-preserving improvements of $\mathbf{O}^S$.

A *social evaluation rule* is a binary relation $\succsim$ defined on $\mathcal{L}^n$. We write $\mathbf{O}\succsim\mathbf{O}'$ in place of $(\mathbf{O},\mathbf{O}')\in\succsim$. $\mathbf{O}\succsim\mathbf{O}'$ means social welfare at $\mathbf{O}$ *is unambiguously at*

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[8]We use $\leq_o$ and $\geq_o$ interchangeably.

least as great as at $\mathbf{O}'$. In the sequel, we wish to study the properties of $\succsim$. First, however, we identify the class of problems under consideration.

2.1. Assumptions on $(\mathcal{L},\leq_o)$

We impose the following restrictions on $(\mathcal{L},\leq_o)$:[9]

(A.1) *$\mathcal{L}$ is a continuum with respect to $\leq_o$; i.e., for all $A,B\in\mathcal{L}$, if $A <_o B$, then there exists $C\in\mathcal{L}$ such that $A <_o C <_o B$.*

(A.2) *$\leq_o$ contains the set inclusion relation $\subseteq$; in particular, $<_o$ contains $\subset$.*

The class of partially ordered sets satisfying (A.1) and (A.2) includes "economic" domains in which the elements of $\mathcal{L}$ are singletons consisting of infinitely divisible commodity vectors in $\mathbb{R}^\ell$ and $\leq$ is the standard vector inequality, and in which $\mathcal{L}$ consists of subsets of $\mathbb{R}^\ell$ partially ordered by $\subseteq$.

2.2. Assumptions on $\mathcal{R}^i$

We restrict our attention to $\mathcal{R}^i$ that satisfy the following:

Domain ($\mathcal{R}$-DOM): *$\mathcal{R}^i$ is complete, reflexive and transitive.*

Monotonicity ($\mathcal{R}$-MON): *For all $A,B\in\mathcal{L}$, if $A >_o B$, then $A\mathcal{P}^iB$. Also,*

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[9]Among our notational conventions, we denote vector inequalities in $\mathbb{R}^\ell$ by $\geq$, $>$, and $\gg$ with the usual interpretations. Also, for an arbitrary relation R (with components P and I) defined on a domain $\mathcal{D}$, we define the minimal and maximal sets as $\min(R;\mathcal{D}):=\{d\in\mathcal{D} \mid$ there is no $d'\in\mathcal{D}$ such that $dPd'\}$ and $\max(R;\mathcal{D}):=\{d\in\mathcal{D} \mid$ there is no $d'\in\mathcal{D}$ such that $d'Pd\}$.

$min(\leq_o;\mathcal{L}) \subseteq min(\mathcal{R}^1;\mathcal{L})$.[10,11]

Continuity ($\mathcal{R}$-CONT): *For all $A \in \mathcal{L}$, the sets* $\{A' \in \mathcal{L} \mid A' \mathcal{R}^1 A\}$ and $\{A' \in \mathcal{L} \mid A \mathcal{R}^1 A'\}$ *are closed in $\mathcal{T}$.*

2.3. Assumptions on $\succsim$

We assume $\succsim$ satisfies the following restrictions:

Domain ($\succsim$-DOM): *$\succsim$ is reflexive, transitive, and $\geq_o^n$-complete. That is, it is complete with respect to all pairs $\mathbf{O},\mathbf{O}' \in \mathcal{L}^n$ ordered by $\geq_o^n$, the partial order on $\mathcal{L}^n$ induced by $\geq_o$.*

Continuity ($\succsim$-CONT): *For all $\mathbf{O} \in \mathcal{L}^n$,* $\{\mathbf{O}' \in \mathcal{L}^n \mid \mathbf{O}' \succsim \mathbf{O}\}$ and $\{\mathbf{O}' \in \mathcal{L}^n \mid \mathbf{O} \succsim \mathbf{O}'\}$ *are closed in the product topology on $\mathcal{L}^n$.*

3. A normative theory: the two-agent case

Next, we specify four normative principles for a social evaluation rule. While many would argue for stronger conditions, we consider these to be minimal requirements. For clarity, we begin with the case in which n=2.

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[10] In other words, a $\geq_o$-smallest element of $\mathcal{L}$ is not strictly preferred to any other element.

[11] For example, in economic domains in which preferences over consumption bundles in $\mathbb{R}_+^\ell$ are monotonic with respect to $\leq$, $0 \in \mathbb{R}_+^\ell$ is not strictly preferred to any other bundle. Or if $\mathcal{L}$ consists of subsets of $\mathbb{R}^\ell$ partially ordered by $\subseteq$, then a singleton cannot be strictly preferred to another set *on the basis of size only*. However, if $\leq_o$ is a strict superset of $\subseteq$, then it is consistent with $\mathcal{R}$-MON for $\{x\}\mathcal{P}^1 A$, for some $A \in \mathcal{L}$.

Principle of Just Enhancement (JE): *For all* $\mathbf{O}\in\mathcal{L}^2$, *if* $O^i\mathcal{R}^iO^j$ *and* $O^i\mathcal{R}^jO^j$, for $i\neq j$, then for all $O'^j\in RP(O^j)$, $(O^i,O'^j) > (O^i,O^j)$.[12]

Principle of Transposition (T): For all $\mathbf{O}\in\mathcal{L}^2$, if $O^2\mathcal{R}^1O^1$ and $O^1\mathcal{R}^2O^2$, with strict preference for at least one agent, then $(O^2,O^1) > (O^1,O^2)$.

Restricted Pareto Principle (PP): For all $\mathbf{O}\in\mathcal{L}^2$, if $O^1\mathcal{R}^1O^2$ and $O^2\mathcal{R}^2O^1$, then for all $\mathbf{O}'\in\mathcal{L}^2$ such that (1) $O'^1\mathcal{R}^1O^1$ and $O'^2\mathcal{R}^2O^2$, with strict preference for at least one agent, and (2) $O'^1\mathcal{R}^1O'^2$ and $O'^2\mathcal{R}^2O'^1$, $\mathbf{O}' > \mathbf{O}$.

Principle of Just Distribution (JD): For all $\mathbf{O}\in\mathcal{L}^2$, if $O^i\mathcal{R}^iO^j$ and $O^i\mathcal{P}^jO^j$, for $i\neq j$, then for all $\mathbf{O}'\in\mathcal{L}^2$ in which $\mathbf{O}'\gtrsim\mathbf{O}$, if $O'^i\mathcal{P}^iO^i$, then $O'^j\mathcal{P}^jO^j$.

JE is a subrestriction of the Pareto criterion. It identifies a class of Pareto improvements which would unambiguously increase social welfare without posing a conflict with other evaluative criteria. Specifically, it requires that if one agent's opportunity set is judged unanimously to be better than the other agent's, then a rank-preserving improvement of the latter should increase social welfare.

T is straightforward; moreover, it, too, is a subrestriction of the Pareto criterion.

PP identifies a further class of social welfare enhancing Pareto improvements. It applies when each agent prefers his or her own opportunity set to that of the other agent and when that is the case for the Pareto improvement as well. In this way, it identifies Pareto improvements that are

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[12] Note that if $O^i\mathcal{I}^iO^j$, then there may be no rank-preserving expansion of O^j.

not too skewed in favor of either agent.

Finally, like JE, JD applies when one agent's opportunity set is judged unanimously to be better than the other agent's. It states that a necessary condition for social welfare to unambiguously increase when the welfare of the "rich" agent improves is that the "poor" agent should be better-off as well, however minimally. This rules out social welfare improvements in which the "rich" get richer and the "poor" get poorer, or in which the rich get richer at the expense of the poor.

4. Domain of singletons

To evaluate the intuitive appeal of the above criteria, we consider the case in which $\mathcal{L}$ consists of singleton opportunity sets of the form $\{x\}$, where $x \in \mathbb{R}^{\ell}$, and $\leq_o$ is the standard vector inequality $\leq$ on $\mathbb{R}^{\ell}$. Here, the problem of ranking profiles of opportunity sets reduces to the familiar case of comparing resource allocations.

Thus, consider a pure exchange economy in which fixed quantities of ℓ commodities are to be allocated between two agents. Let $\Omega \in \mathbb{R}^{\ell}_{++}$ denote the aggregate endowment. Agent i, i=1,2, is described by his or her consumption set X^i, and again we denote preferences by $\mathcal{R}^i$. Now, however, we write $x^i \mathcal{R}^i x'^i$ instead of $\{x^i\} \mathcal{R}^i \{x'^i\}$, and we consider $\mathcal{R}^i \subseteq X^i x X^i$. For simplicity, we take $X^i = \mathbb{R}^{\ell}_{+}$.

Let $X = X^1 x X^2$. An allocation $\mathbf{x} = (x^1, x^2) \in \mathbf{X}$ is *feasible* if $x^1 + x^2 = \Omega$,[13] and it is *envy-free* if $x^i \mathcal{R}^i x^j$ for i,j=1,2. Let **Z** denote the feasible set, and let **F** denote the set of envy-free allocations. $\mathbf{x} \in \mathbf{Z}$ is *Pareto efficient* if there is no $\mathbf{x}' = (x'^1, x'^2) \in \mathbf{Z}$ such that $x'^1 \mathcal{R}^1 x^1$ and $x'^2 \mathcal{R}^2 x^2$ with strict preference for at

[13] Note that we consider the case without disposability.

least one agent. We denote the efficient set by **P** and the intersection of **F** and **P** by **FP**.

We are interested in the properties of a social evaluation rule $\succsim_s$ that satisfies our normative principles. Analogous to $\mathcal{R}^i$, we adapt our notation and write $\succsim_s \subseteq X \times X$, and we modify JE, T, PP and JD accordingly. For completeness, we include the modifications.

JE^s: For all $\mathbf{x}=(x^1,x^2)\in\mathbf{X}$, if $x^i\mathcal{R}^i x^j$ and $x^i\mathcal{R}^j x^j$, for i≠j, then for all $x'^j \in RP(x^j)$, $(x^i,x'^j) >_s (x^i,x^j)$.[14]

T^s: For all $\mathbf{x}=(x^1,x^2)\in\mathbf{X}$, if $x^2\mathcal{R}^1 x^1$ and $x^1\mathcal{R}^2 x^2$, with strict preference for at least one agent, then $(x^2,x^1) >_s (x^1,x^2)$.

PP^s: For all $\mathbf{x}=(x^1,x^2)\in\mathbf{X}$, if $x^1\mathcal{R}^1 x^2$ and $x^2\mathcal{R}^2 x^1$, then for all $\mathbf{x}'=(x'^1,x'^2)\in\mathbf{X}$ such that (1) $x'^1\mathcal{R}^1 x^1$ and $x'^2\mathcal{R}^2 x^2$, with strict preference for at least one agent, and (2) $x'^1\mathcal{R}^1 x'^2$ and $x'^2\mathcal{R}^2 x'^1$, $\mathbf{x}' >_s \mathbf{x}$.

JD^s: For all $\mathbf{x}\in\mathbf{X}$, if $x^i\mathcal{R}^i x^j$ and $x^i\mathcal{P}^j x^j$, for i≠j, then for all $\mathbf{x}'\in\mathbf{X}$ in which $\mathbf{x}' \succsim_s \mathbf{x}$, if $x'^i\mathcal{P}^i x^i$, then $x'^j\mathcal{P}^j x^j$.

Remark. As defined, the social evaluation rule $\succsim_s$ and the principles JE^s, T^s, PP^s and JD^s permit comparisons among infeasible allocations. Nevertheless, we are interested in maximal elements in **Z**, as defined below.

First, we note the logical independence of the axioms.

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[14]The modification, $RP(x^j)$, is defined in the obvious fashion.

THEOREM 1. JE^s, T^s, PP^s and JD^s are logically independent.

Proof. Obvious.

LEMMA 2. $\max(\succsim_s;\mathbf{Z})\neq\varnothing$.

Proof. See Border (1985), Theorem 7.12.

THEOREM 2. If $\succsim_s$ satisfies JE^s, T^s and JD^s, then $\max(\succsim_s;\mathbf{Z})\subseteq\mathbf{F}$. That is, a necessary condition for a social optimum is that it be envy-free.[15]

Proof. Let $\succsim_s$ satisfy JE^s, T^s and JD^s, and consider $\mathbf{x}\in\mathbf{Z}$ such that $\mathbf{x}\notin\mathbf{F}$. First, if $x^2\mathcal{R}^1x^1$ and $x^1\mathcal{R}^2x^2$, with strict preference for at least one agent, then by T^S, $(x^2,x^1)\succ_s\mathbf{x}$. Since $(x^2,x^1)\in\mathbf{Z}$, $\mathbf{x}\notin\max(\succsim_s;\mathbf{Z})$.

Next, without loss of generality (wlog), suppose $x^2\mathcal{P}^1x^1$ and $x^2\mathcal{P}^2x^1$. (The case in which $x^2\mathcal{P}^1x^1$ and $x^2\mathcal{I}^2x^1$ is covered above.) Also wlog, suppose $x^2_1>0$.

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[15]Notice that PP^s is not needed for Theorem 2. This calls for a brief discussion of the logical structure of our arguments.

First, in advocating minimal standards for a social evaluation rule, the fewer the restrictions, the more encompassing the result, in particular, for a "positive" result such as Theorem 2, of the form $H_1,\ldots,H_n$ *imply* C. In fact, our generalizations of Theorem 2 in Sections 5 and 6 require even fewer restrictions in that JE is no longer needed. (The precise role of JE is discussed below.)

Conversely, for a "negative" result of the form $H_1,\ldots,H_n$ do not *imply* C, a large number of (logically consistent) hypotheses is a better indicator of the severity, or the full extent of the restrictions necessary to ensure that C does hold.

Subsequently, we discuss such a negative result vis-à-vis Pareto efficiency. Since the Restricted Pareto Principle seems unobjectionable, including it among the restrictions yields a stronger result.

By $\mathcal{R}$-MON and $\mathcal{R}$-CONT, we can find $\varepsilon \in \mathbb{R}_{++}$ such that $x_\varepsilon^2 \mathcal{P}^1 x_\varepsilon^1$ and $x_\varepsilon^2 \mathcal{P}^2 x_\varepsilon^1$, where $x_\varepsilon^1 := (x_1^1+\varepsilon, x_2^1, \ldots, x_\ell^1)$ and $x_\varepsilon^2 := (x_1^2-\varepsilon, x_2^2, \ldots, x_\ell^2)$. And providing $\varepsilon \leq x_1^2$, $\mathbf{x}_\varepsilon = (x_\varepsilon^1, x_\varepsilon^2) \in \mathbf{Z}$. By $\mathcal{R}$-MON, $x_\varepsilon^1 \mathcal{P}^1 x^1$, and since $x_\varepsilon^2 \mathcal{P}^2 x_\varepsilon^1$, $x_\varepsilon^1 \in RP(x^1)$. Therefore, by JE^s, $\mathbf{x}_\varepsilon >_s (x^1, x_\varepsilon^2)$.[16]

Since $\gtrsim_s$ is $\geq$-complete by $\gtrsim$-DOM, and since $\mathbf{x} \geq (x^1, x_\varepsilon^2)$, $\mathbf{x}$ and (x^1, x_ε^2) are $\gtrsim$-comparable. By the transitivity of $\mathcal{R}^1$, $x_\varepsilon^2 \mathcal{P}^1 x^1$ and $x_\varepsilon^2 \mathcal{P}^2 x^1$. And by $\mathcal{R}$-MON, $x^2 \mathcal{P}^2 x_\varepsilon^2$. Hence, by JD^s, it must be the case that $(x^1, x_\varepsilon^2) >_s \mathbf{x}$. It then follows from the transitivity of $\gtrsim_s$ that $\mathbf{x}_\varepsilon >_s \mathbf{x}$. Thus, $\mathbf{x} \notin \max(\gtrsim_s; \mathbf{Z})$. ▮

Next, one might enquire whether JE^s, T^s, JD^s, and even PP^s ensure that a social optimum is necessarily Pareto efficient, i.e., that $\max(\gtrsim_s; \mathbf{Z}) \subseteq \mathbf{P}$. First, however, we note a related consequence of the theorem. In the proof, $(x^1, x_\varepsilon^2) >_s \mathbf{x}$, and yet $\mathbf{x}$ weakly Pareto dominates (x^1, x_ε^2). Hence, we have the following:

Corollary: Under the conditions of Theorem 2, there may exist weak Pareto improvements that reduce social welfare.

Returning to the relationship between $\max(\gtrsim_s; \mathbf{Z})$ and $\mathbf{P}$, in light of Theorem 2, one might ask whether $\max(\gtrsim_s; \mathbf{Z}) \subseteq \mathbf{FP}$?

From Goldman and Sussangkarn (1978), we know that generally there exist envy-free allocations which are inefficient and yet all Pareto improvements introduce envy.[17] Let $\bar{\mathbf{x}} \in \mathbf{F}$ denote such an allocation. Then to conclude that

[16]For future reference, note the role played by JE^s: it (together with the transitivity of $\gtrsim$) serves only to ensure that there is a *feasible* improvement for the case in which commodities are not freely disposable.

[17]Earlier, Kolm (1972) and Feldman and Kirman (1974) showed that there are inefficient envy-free allocations from which a competitive equilibrium

$\max(\succsim_s;\mathbf{Z})\subseteq\mathbf{FP}$ requires that there exist $\mathbf{x}\in\mathbf{Z}$ such that $\mathbf{x}\succ_s\bar{\mathbf{x}}$. Clearly, the conditions JE^s, T^s, PP^s and JD^s do not ensure that such an $\mathbf{x}$ exists.[18] In fact, in comparing any allocation to $\bar{\mathbf{x}}$, the hypotheses of each of the four conditions fail to be satisfied. Thus, the conditions do not apply, and no conclusions whatsoever can be drawn.

5. General two-agent results

In this section we generalize the results of Section 6 to the domain of two-agent opportunity set rankings. We begin by generalizing the appropriate definitions. (For later reference, we write the definitions for the case of n agents.)

First, we will say $\mathbf{O}\in\mathcal{L}^n$ is *envy-free* if $O^i\mathcal{R}^iO^j$ for all $i,j\in N$. Let $\mathcal{F}^n$ denote the set of envy-free profiles. Next, $\mathbf{O}\in\mathcal{L}^n$ is *Pareto efficient*, or simply *efficient*, if there does not exist $\mathbf{O}'\in\mathcal{L}^n$ that Pareto dominates it. Let $\mathcal{P}$ denote the set of efficient profiles. We denote the intersection of $\mathcal{F}$ and $\mathcal{P}$ by $\mathcal{FP}$.

Returning to the two-agent case, we first note that $\succsim$-DOM and $\succsim$-CONT are sufficient to ensure the existence of maximal profiles in the event $\mathcal{L}^2$ is compact.

LEMMA 2. If $\mathcal{L}^2$ is compact, then $\max(\succsim;\mathcal{L}^2)\neq\varnothing$.

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introduces envy.

[18] Although they may not be inconsistent with the existence of such an $\mathbf{x}$. And indeed more extensive restrictions on $\succsim_s$ may be sufficient to ensure existence.

Next, we have the following analogue of Theorem 2:

THEOREM 3. Let $\mathcal{L}$ be compact.[19] If $\succsim$ satisfies T and JD, then $\max(\succsim;\mathcal{L}^2)\subseteq\mathcal{F}$. That is, a necessary condition for a socially optimal distribution of opportunity sets is that it be envy-free.[20]

Proof. Let $\succsim$ satisfy T and JD, and consider $\mathbf{O}\in\mathcal{L}^2$ such that $\mathbf{O}\notin\mathcal{F}$. First, if $O^2\mathcal{R}^1O^1$ and $O^1\mathcal{R}^2O^2$, with strict preference for at least one agent, then by T, $(O^2,O^1)\succ\mathbf{O}$. Since $(O^2,O^1)\in\mathcal{L}^2$, $\mathbf{O}\notin\max(\succsim;\mathcal{L}^2)$.

Next, wlog, suppose $O^2\mathcal{P}^iO^1$, for i=1,2. Then by $\mathcal{R}$-MON, since $O^2\mathcal{P}^1O^1$, $O^2\notin\min(\leq_o;\mathcal{L})$. Therefore, since $\mathcal{L}$ is a continuum and since $\mathcal{R}^1$ is continuous, there exists $O^2_c\in\mathcal{L}$ such that $O^2_c <_o O^2$ and $O^2_c\mathcal{P}^iO^1$, for both i. Since $\succsim$ is $\geq^n_o$-complete, we then have $(O^1,O^2_c)\succ\mathbf{O}$ by JD. Thus, $\mathbf{O}\notin\max(\succsim;\mathcal{L}^2)$. ∎

The following are two examples satisfying the above restrictions:

Example 7.1. Let L be a compact and connected subset of $\mathbb{R}^\ell_+$. Then let $\mathcal{L}$ consist of the Lebesgue measurable subsets of L and let $\leq_o$ be the set inclusion relation $\subseteq$.

Example 7.2. Let $p\in\mathbb{R}^\ell_{++}$ and let $K\in\mathbb{R}$, $K>0$. Then let $\mathcal{L}$ consist of all sets of

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[19] We assume $\mathcal{L}$ is compact only to ensure $\max(\succsim;\mathcal{L}^2)\neq\emptyset$, but it is not needed for the result.

[20] The domain in Theorem 3 differs from that of Theorem 2 in the obvious way that it includes more general profiles of opportunity sets. But also it is rectangular ($\mathcal{L}^2=\mathcal{L}\times\mathcal{L}$). It is for this reason that JE is no longer necessary. Note also that the proof of Theorem 2 can be modified precisely as in the proof of Theorem 3 to allow for the case in which commodities are freely disposable.

the form $\{x\in\mathbb{R}^{\ell}_{+} | p\cdot x\leq k\}$ where $0\leq k\leq K$, and again let $\leq_{o}$ be the set inclusion relation.

6. Multilateral extension

In this section, we extend the results in Section 5 to the fully general case involving an arbitrary number of agents. We begin by generalizing the normative principles.

JE^n: For all $\mathbf{O}\in\mathcal{L}^n$, for all $i\in W(\mathbf{O})$, and for all $\mathbf{O'}^i\in RP(\mathbf{O}^i)$, $(\mathbf{O'}^i,\mathbf{O}^{-i}) > (O^i,\mathbf{O}^{-i})$.[21]

T^n: For all $\mathbf{O}\in\mathcal{L}^n$, if $O^j\mathcal{P}^iO^i$ and $O^i\mathcal{R}^jO^j$, for some $i,j\in N$, then $\pi_{ij}(\mathbf{O}) > \mathbf{O}$.

PP^n: For all $\mathbf{O}\in\mathcal{L}^n$, if $O^i\mathcal{R}^iO^j$ for all $i,j\in N$, then for all $\mathbf{O'}\in\mathcal{L}^n$ such that (1) $O'^i\mathcal{R}^iO^i$ for all $i\in N$, with strict preference for some i, and (2) $O'^i\mathcal{R}^iO'^j$ for all $i,j\in N$, $\mathbf{O'} > \mathbf{O}$.

JD^n: For all $\mathbf{O},\mathbf{O'}\in\mathcal{L}^n$, and for all $j\in W(\mathbf{O})$, if $\mathbf{O'}\succsim\mathbf{O}$ and if $O'^i\mathcal{P}^iO^i$ for some $i\notin W(\mathbf{O})$, then $O'^j\mathcal{P}^jO^j$.

THEOREM 4. Let $\mathcal{L}$ be compact. If $\succsim$ satisfies T^n and JD^n, then $\max(\succsim;\mathcal{L}^n)\cap\mathcal{P}\subseteq\mathcal{F}$. I.e., any socially optimal distribution that is Pareto efficient must be envy-free as well.

Proof. Let $\succsim$ satisfy T^n and JD^n, and consider $\mathbf{O}\in\mathcal{L}^n$ such that $\mathbf{O}\notin\mathcal{F}$. First, if

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[21] $-i$ denotes $N\setminus\{i\}$.

$O^j\mathcal{R}^iO^i$ and $O^i\mathcal{R}^jO^j$, for some $i,j\in N$, with strict preference for at least one of the two, then by T^n, $\pi_{ij}(\mathbf{O})>\mathbf{O}$. Therefore, $\mathbf{O}\notin\max(\succsim;\mathcal{L}^n)$ and, hence, $\mathbf{O}\notin\max(\succsim;\mathcal{L}^n)\cap\mathcal{P}$.

Next, suppose $O^j\mathcal{P}^iO^i$ and $O^j\mathcal{P}^jO^i$ for some $i,j\in N$, and suppose also that $\mathbf{O}\in\mathcal{P}$. We must show that $\mathbf{O}\notin\max(\succsim;\mathcal{L}^n)$.

Wlog let i=1 and j=2 so that $O^2\mathcal{P}^1O^1$ and $O^2\mathcal{P}^2O^1$. Adapting the argument in Varian (1974), Theorem 2.1, since $\mathbf{O}\in\mathcal{P}$, $W(\mathbf{O})\neq\emptyset$. That is, at $\mathbf{O}$, there is at least one agent k such that $O^i\mathcal{R}^iO^k$ for all $i\in N$.[22] Clearly, $2\notin W(\mathbf{O})$. Moreover, by $\mathcal{R}$-MON, $O^2\notin\min(\leq_o;\mathcal{L})$. Since $\mathcal{L}$ is a continuum and $\mathcal{R}^1$ is continuous, there exists $O^2_c\in\mathcal{L}$ such that $O^2_c <_o O^2$ and $O^2_c\mathcal{P}^2O^1$. Therefore, since $\succsim$ is $\geq^n_o$-complete, we have $(O^2_c,\mathbf{O}^{-2})>\mathbf{O}$ by JD. Thus, $\mathbf{O}\notin\max(\succsim;\mathcal{L}^n)$. ∎

7. Conclusion

Our objective in this paper has been two-fold: first, to cast the discussion of social welfare evaluation in terms of the distribution of opportunities, and, second, to search for common ground among possibly competing views of appropriate evaluative criteria. Regarding the first, we have extended our earlier work in several directions.

In Kranich (1993a), we considered finite opportunity sets, and we abstracted from the preferences of the agents. There, our objective was to demonstrate that the axiomatic approach is capable of rendering a <u>complete</u> ranking of profiles of opportunity sets on the basis of fairness. In Kranich (1993b), we again addressed the question of equitable opportunities, and we again required

[22] Otherwise, if for all i there exists j such that $O^i\mathcal{P}^jO^j$, then since N is finite there must exist a cycle of envy among the agents. Transposing the opportunity sets appropriately would yield a Pareto improvement.

that a social evaluation rule be complete, although we extended the analysis to include connected economic domains in $\mathbb{R}^{\ell}$. Moreover, we incorporated some general information about agents' preferences (in particular, that they are monotonic), but we again abstracted from the specific relations.

In the present paper, we no longer address the question of equity *per se*. Indeed, this represents one of the "competing views" we attempt to reconcile. Since our analysis is independent of the overall objective, it is not surprising that we forego the requirement of completeness. In return, the present analysis fully incorporates agents' preferences and it pertains to a larger class of problems.

Regarding our second objective, we have shown that under minimal restrictions, social optima must be envy-free.[23] Thus, whatever stronger conditions one might wish to impose on a social evaluation rule, they must be consistent with the envy-free criterion. In other words, we might limit our search for welfare optima to the envy-free set.

The practical limitation of the present analysis, as discussed in footnote 6, is that it is appropriately viewed as an extension of exchange environments in which opportunity sets are specified exogenously. In our subsequent work, we will extend the analysis to include endogenously determined opportunity sets. Nevertheless, within the framework developed here, our results show that, regardless of one's larger views of social welfare, accepting the aforementioned principles requires commitment to a world in which all agents have equitable opportunities.

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[23]Our use of the term "minimal" was explained in footnote 4.

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