UNIVERSIDAD CARLOS III DE MADRID
ESCUELA POLITÉCNICA SUPERIOR

PARAMETRIC SIZING OF HTP FLIGHT LOADS

Bachelor Thesis

Author
Marta Girón Carrero

Leganés, September 2015

Department of Aerospace Engineering
ESCUELA POLITÉCNICA SUPERIOR

PARAMETRIC SIZING OF HTP FLIGHT LOADS

Bachelor in Aerospace Engineering

Author
Marta Girón Carrero

Supervisor
Javier Hilario Montes

Leganés, September 2015

Acknowledgment

I would love to thank my advisor, Javier Hilario Montes for his constant guidance along this process. I value his knowledge and professionalism as a great example for future experiences. I also would like to help the department of Aerospace Engineering of Carlos III University for encouraging me along these past years and in the future performance of the master.

I would like to give a special thanks to my parents for all the support and love that they have gave me. I am also fortunate to say that a have so many great friends, classmates and teachers.

Thank you all

Abstract

Structural load analysis relates the calculation and determination of the loads that act in the aircraft for its manoeuvres, landing, turbulence flight...

This project has the aim to study and synthesize the parameters and manoeuvres that size the horizontal tailplane for a preliminary design. This is performed with a combination of structural equations as well as analytical calculation reaching a graphical representation of the dependency of the parameters for different situations in the flighting envelope of the aircraft.

The analysis performed and discussed might be understood as a general method that may need the inclusion of significant parameters for a particular configuration that have been neglected in the equations of the analysis in order to simplified the formulations.

Contents

Acknowledgment **i**

Abstract **iii**

List of Figures **vii**

List of Tables **ix**

1 INTRODUCTION **1**
1.1 State of Art 1
1.2 Objectives 2

2 MANOEUVRES ANALYZED TO OBTAIN THE PARAMETRIC SIZING **5**
2.1 Manoeuvring Balanced Conditions 5
2.2 Specified Control Displacement 7
2.3 Maximum Control Displacement at V_A 8
2.4 Discrete Gust Design Criteria 9

3 FORMULATION **11**
3.1 General Formulation 11
3.1.1 F_{z_H} due to load factor 14
3.1.2 F_{z_H} due to zero-effects 15
3.1.3 F_{z_H} due to pitch rate 15
3.1.4 F_{z_H} due to pitch acceleration 15
3.1.5 F_{z_H} due to gusts 15
3.2 Reduced Formulation 18

4 SIZING PARAMETERS STUDY **19**
4.1 Balanced Conditions Manoeuvre 19
4.2 Specified Control Displacement Manoeuvre 28
4.3 Maximum Displacement at V_A Manoeuvre 32
4.4 Discrete Gust Design Criteria 33
4.5 Possible Sizing Parameters 38

5 CONCLUSIONS **41**

6 FURTHER ANALYSIS **45**

A Budget **47**

B Bibliography **49**

List of Figures

2.1 Manoeuvering envelope. Ref: [1] 5
2.2 Fowler Flap 6
2.3 Example of wing with flaps and ailerons 6
2.4 Cockpit control deflection vs. time . Ref: [2] 7

3.1 Reference axis of the aircraft . Ref: [4] 11
3.2 Wing airfoil 12
3.3 Function of the gust vs. x 16
3.4 Representation of a step gust 16

4.1 $\bar{F}_{z_{H_1}}$ vs. $\Delta\bar{x}$ for balanced conditions manoeuvre where Flaps-up $n =$ 2.5: (—), Flaps-up $n = -1$: (—) and Flaps-down $n = 2$: (—) . 20
4.2 Nose-down and nose-up movement of an aircraft 21
4.3 $\bar{F}_{z_{H_2}}$ vs. $C_{m_\alpha}/C_{L_\alpha}$ for balanced conditions manoeuvre with Flaps-up configuration and $n = 2.5$. 22
4.4 $\bar{F}_{z_{H_2}}$ vs. $C_{m_\alpha}/C_{L_\alpha}$ for balanced conditions manoeuvre with Flaps-down configuration and $n = 2$. 22
4.5 $\bar{F}_{z_{H_2}}$ vs. V for balanced conditions manoeuvre 23
4.6 $\bar{F}_{z_{H_3}}$ vs. h for balanced conditions manoeuvre with Flaps-up configuration and $n = 2.5$ where pull-up : (—), push-over : (—) and steady turn : (—) 25
4.7 $\bar{F}_{z_{H_3}}$ vs. h for balanced conditions manoeuvre comparing push-over movement for different values of m/s 26
4.8 $\bar{F}_{z_{H_3}}$ vs. h for balanced conditions manoeuvre comparing steady turn movement for different values of C_{m_q} 26
4.9 $\bar{F}_{z_{H_3}}$ vs. c for balanced conditions manoeuvre with Flaps-up configuration and $n = 2.5$ where pull-up : (—), push-over : (—) and steady turn : (—) 27
4.10 $\bar{F}_{z_{H_1}}$ vs. $\Delta\bar{x}$ for specified control displacement manoeuvre where $n =$ 2.5: (—) and $n = 1$: (—) 28
4.11 $\bar{F}_{z_{H_2}}$ vs. $C_{m_\alpha}/C_{L_\alpha}$ for specified control displacement manoeuvre comparing different configuration of velocities and wing loading 29
4.12 $\bar{F}_{z_{H_2}}$ vs. V for specified control displacement manoeuvre for different configuration of wing loading where $m/s = 300kg/m^2$: (—), $m/s =$ $500kg/m^2$: (—) and $m/s = 700kg/m^2$: : (—) 30

4.13 $\bar{F}_{z_{H_4}}$vs. $\dot{q}$ for specified control displacement manoeuvre where $n = 2.5$: (—) and $n = 1$: (—) 31
4.14 $\bar{F}_{z_{H_4}}$vs.c for specified control displacement manoeuvre for $n = 1$ where $V = 150KEAS$: (—) and $V = 350KEAS$: (—) 32
4.15 $\bar{F}_{z_{H_4}}$vs. $\dot{q}$ for maximum displacement manoeuvre for different values of the chord where $c = 3m$: (—), $c = 5m$: (—) and $c = 7m$: (—) 33
4.16 $\bar{F}_{z_{H_2}}$vs.v for discrete gust manoeuvre for different configuration of flaps and wing loading 34
4.17 $\bar{F}_{z_{H_{gust}}}$vs. $V_H/(m/s)$ for discrete gust manoeuvre for Flaps-up and $n = 1$ with different values of the velocity where $V = 150KEAS$: (—), $V = 250KEAS$: (—) and $V = 350KEAS$: (—) 35
4.18 $\bar{F}_{z_{H_{gust}}}$vs. $V_H/(m/s)$ for discrete gust manoeuvre for Flaps-down and $n = 1$ with different values of the velocity where $V = 75KEAS$: (—), $V = 125KEAS$: (—) and $V = 200KEAS$: (—) 36
4.19 $\bar{F}_{z_{H_{gust}}}$vs.v for discrete gust manoeuvre for different values of the aerodynamic coefficients 37
4.20 $\bar{F}_{z_{H_{gust}}}$vs. C_1 for discrete gust manoeuvre for different increment in n 38
4.21 $\bar{F}_{z_H}$vs.v for critical manoeuvres for upwards forces 39
4.22 $\bar{F}_{z_H}$vs.v for critical manoeuvres for downwards forces 40

List of Tables

4.1 Values for C_{m_0} and C_{L_0} depending on the flap configuration 21
4.2 Range of equivalent speed depending on the flap configuration 21
4.3 Values of the analysed gusts 36

CHAPTER
ONE

INTRODUCTION

Load calculation is an activity that relate the aircraft design (aerodynamics and flight mechanics and other related areas) with structural design. Basically, it consists on the calculation of the maximum loads in different parts of the aircraft (wing, fuselage, stabilizer, ...) in every manoeuvre and gust encounter that the aircraft must bear. Later on, considering those loads as an data input for the design, structural design and justification.

In the particular case of the horizontal tailplane, the conditions that usually result in maximum loads are:

- Manoeuvres due to longitudinal cockpit control actions that produce angular acceleration ($\dot{q}$).
- Steady level flight with flaps-up and flaps-down configuration encountering a vertical gust.
- Manoeuvring balanced conditions at max/min load factor.

The final loads depend on many parameters of the aircraft design (mass, aerodynamic characteristics, position of the centre of gravity, etc).

Many studies and technical books offer analytical expressions involving the parameters that drive the maximum loads of the wing-body or only the wing. On the other hand, it exists limited information regarding the achievement of compact expressions and figures regarding the topic that this project is based on.

The preliminary structural sizing of an aircraft can be done with little information about the final product. It is enough for the wing-body but nowadays, there is not a simple way to obtain the sizing loads of the HTP.

1.1 State of Art

In the detailed design phase, the loads of the major structural components are obtained by sum of distributed aerodynamic, inertia and propulsive loads. Usually the aerodynamic loads are based on panel pressure data coming from complex CFD calculations and inertia loads are based on refined mass discretization. Aeroelastic modelling is also needed for flexible aircrafts flying at high dynamic pressure. These tasks might need the resolution of mathematical problems with many degrees of freedom.

In general, there are many load conditions and point of the flight envelope that must be investigated. Mass distributions of the aircraft must be accounted for, inside the limits of the envelope of total aircraft mass versus centre of gravity, and considering all the possible combinations of payload and fuel.

To perform these methods, many variables and parameters related to the aircraft design must be established and/or determined. It is usual to manage computer programs that solve the aircraft response with 6 degrees of freedom (6DOF), incorporating adequate models of the Flight Control System (FCS) and aeroelastic models instead of rigid aerodynamic ones.

Before these advanced technologies can be used for a new aircraft, the primary analysis can be performed with simpler equations that allow the engineers to complete a preliminary design knowing only a few fundamental parameters of the final product. These type of analysis are easy to apply for some components of the aircraft such as the wing, due to the abundant formulas existing in technical books and reports for this component.

Before the advanced technologies that have been used, the analysis were performed with simpler equation that allowed the engineers to complete a preliminary design with few parameters of the final product. These type of analysis are still been used for some components of the aircraft such as the wing due to the numerous analysis and books dedicated to the sizing of this component.

On the other hand, there are not as much studies dedicated to a simpler resolution of the structural loads of the HTP which drive the necessity to create a model that might be useful to develop the preliminary design of this component.

1.2 Objectives

This project has the aim to push forward in the knowledge of the dependency of the parameters involved in the calculation of the sizing of the horizontal tailplane. As well as the obtention of practical design rules that are useful for the preliminary design.

Firstly, it is essential the studying and understanding of the different manoeuvres that are going to be analysed in this project as well as the flight envelope that appears in the international regulation.

The structural formulation for the aircraft have to be defined. Also, typical values of the main parameters that appears in the formulation may be found to perform the desired analysis.

Lastly, the manoeuvres at their critical points in the flight envelope must be represented to obtain a final conclusion.

Complex calculation are not required for these preliminary analysis. The objective is the combination of analytical calculation and simple structural equation to determine and synthesize the final results with the aim of graphical representation of the results.

The criteria for the conditions and manoeuvres that might size the horizontal tailplane are for large aircraft under European regulation CS-25 as well as USA regulation FAR-25.

CHAPTER

TWO

MANOEUVRES ANALYZED TO OBTAIN THE PARAMETRIC SIZING

As explained in the previous chapter, some manoeuvres that appear in the international Airworthiness Standards will be analyzed in order to obtain the required information for this thesis.

Each of them will be explained individually in the next sections.

2.1 Manoeuvring Balanced Conditions

The manoeuvre is described in *CS 25.331 (b)* of the European Aviation Safety Agency. The manoeuvring flight conditions must be investigated along the manoeuvring envelope from A to I with wing-flaps up and wing-flaps down respectively.

The envelope that is referred to is the one in *CS 25.333 (b)*.

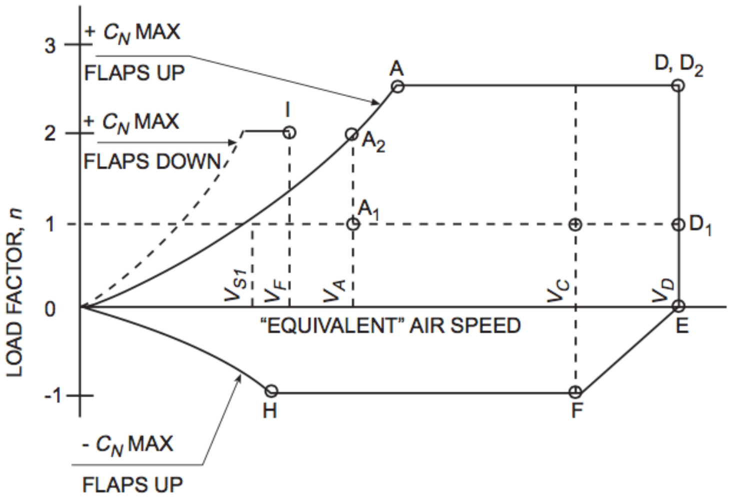

Figure 2.1: Manoeuvering envelope. Ref: [1]

The previous figure shows the boundaries where the strength requirements must be met. It is a V *vs.* n diagram where V is the equivalent air speed of the aircraft and n is the correspondent load factor which is the vertical aerodynamic force to aircraft weigh ratio.

It can be observed that the shape of the envelope is not the same for the different flaps configuration. This difference can be explained taking into consideration the use of the flaps.

In *fig. 2.2*, the two possible configurations are depicted. The flight envelope that covers positive and negative values of the load factor needs to be fulfilled when flaps are retracted or as it appears in *fig. 2.1*, wing-flaps up configuration.

On the other hand, wing-flaps down configuration or deployed position required a lower value for positive load factor. This is due the fact that this configuration is only used in some phases of the flight and it has been considered enough based on experience.

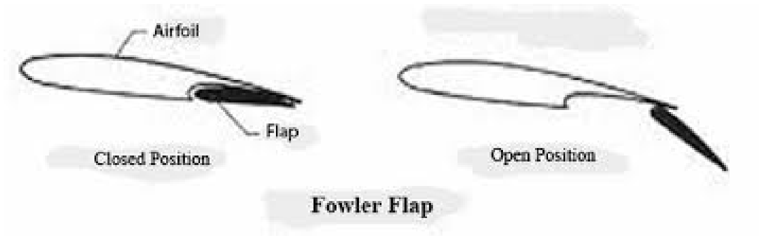

Figure 2.2: Fowler Flap

An example of a real wing with deployed flaps configuration is depicted in *fig. 2.3*.

Figure 2.3: Example of wing with flaps and ailerons

This manoeuvre is investigated with the two possible configurations along the flight envelope as previously mentioned. In addition, it is assumed that the aircraft is in equilibrium with zero pitching acceleration. Therefore, the next equation must be fulfilled.

$$\dot{q} = 0 \tag{2.1}$$

2.2 Specified Control Displacement

It consists on a checked manoeuvre based on pitching motion. As well as the the previous manoeuvre, it must be established between the limit of the flight envelope. The following manoeuvres will also have this same characteristic. It can be found in *CS 25.331 (c)(2)* of the European Aviation Safety Agency.

This restrictions as well as the requirements for the manoeuvre have been changed along the past years. In the 1990s for the CS-25 and 2015 for FAR-25, the calculation is made based on the movement of the cockpit control deflection along the time. A graph similar to *fig. 2.4* would be obtained in relation to the short period natural frequency. The shape of the movement depends on the characteristics of the aircraft Flight Control System, taking into account if it is a reversible system or a fly-by-wire based on n demand, etc.

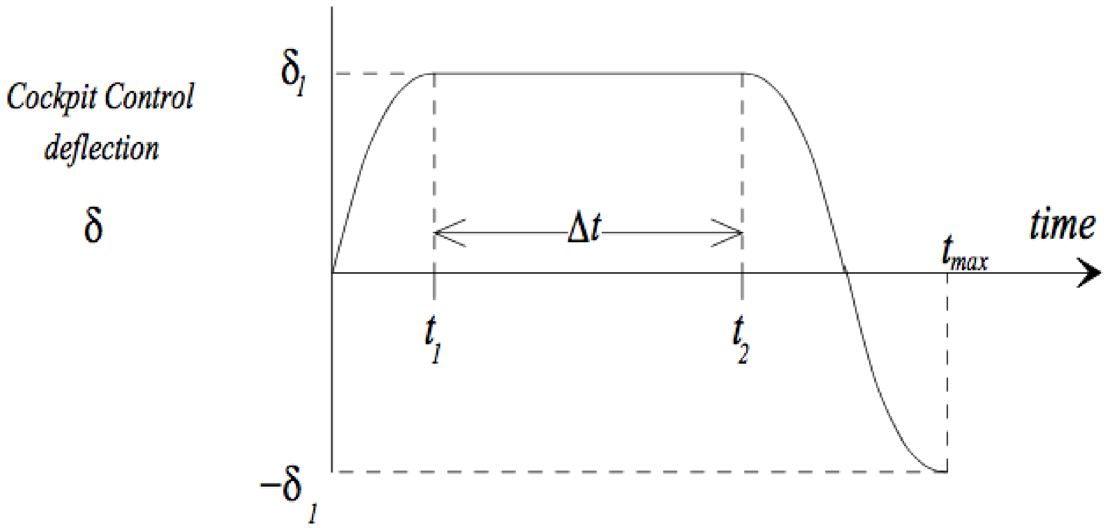

Figure 2.4: Cockpit control deflection vs. time . Ref: [2]

Later on, a relation between deflection of cockpit control displacement and deflection of the elevator is used to obtain the final result that would be the relation between the increment in deflection of the elevator in order to reach the increment in load factor that is required by the regulation.

For the calculation, several variables and their relations must be taken into account. The equations that need to be solved are complex. In order to facilitate it, the formulation that appears in the norm before the last changes will be used in this thesis.

Two equations for the pitching acceleration could be found in *old FAR 25.331 (Amdt 25-91)*. A positive pitching acceleration must be reached with load factor equal to 1 between the points $A1$ and $D1$ of *fig. 2.1* . Where V_A is the design manoeuvring speed and V_D is the design dive speed. The positive acceleration must be equal to at least:

$$\dot{q} = \frac{39n}{V}(n - 1.5) \tag{2.2}$$

Where n is the positive load factor and V is the equivalent airspeed.

A different equation is need for the case of negative pitching acceleration that is reached between the points $A2$ and $D2$ of the same figure. In this case, the negative acceleration must be equal to at least:

$$\dot{q} = \frac{-26n}{V}(n - 1.5) \tag{2.3}$$

In *eq. 2.2* and *eq. 2.3*, $\dot{q}$ is in rad/s^2

2.3 Maximum Control Displacement at V_A

This manoeuvre is also known as unchecked manoeuvre because as the name indicates, it is a non-realistic manoeuvre where the maximum elevator deflection is maintained without checking the effect on the aircraft load factor. It is a conservative manoeuvre that cover the maximum down load that could be applied on the HTP. It appears described in *CS 25.331 (c)(1)* of the European Aviation Safety Agency.

The aircraft is assumed to be flight at point $A1$, steady level flight and the pitch control is moved suddenly in order to obtain maximum pitching acceleration.

In this case, the pitching acceleration can not be obtained as previously, with a simple equation. The maximum deflection of the elevator must be consider in this case.

The perfect scenario would be taking into account the rotation that the aircraft will experience when the pitch control is moved suddenly. It would required another additional variable to the equation and therefore, the rotation is not going to appear in the analysis. The calculation are going to be performed with the value of $\dot{q}$ reached at maximum deflection of the elevator that could lead to a similar formulation that for the previous section.

2.4 Discrete Gust Design Criteria

This section can be found in *CS 25.341 (a)* of the European Aviation Safety Agency. It explains that aircraft may be subjected to vertical gust in level flight. The limit gust loads are determined with a series of equations that are depicted in the previously mentioned section of the international norms. These formulas will be explained in detail in the following chapters of the paper because the aim of this chapter is the understanding of the manoeuvres that are going to be used later on for the sizing of HTP. The shape of the vertical gusts used in the analysis will also be explained in the following chapters.

Continuous turbulences conditions are not considered in the analysis due to the complexity of the numerous variables that are needed in the calculations.

CHAPTER
THREE

FORMULATION

In previous chapter, the manoeuvres that can be found in this part are already explained. The process to get the required formulation will be depicted in the following sections explaining all the considerations and assumptions that need to be made in order to reach the results.

3.1 General Formulation

The aircraft is going to be consider as if it was just wing-body and the horizontal tail was placed in order to counteract the motion of the wing body.

The sign convention is shown in *fig. 3.1.*

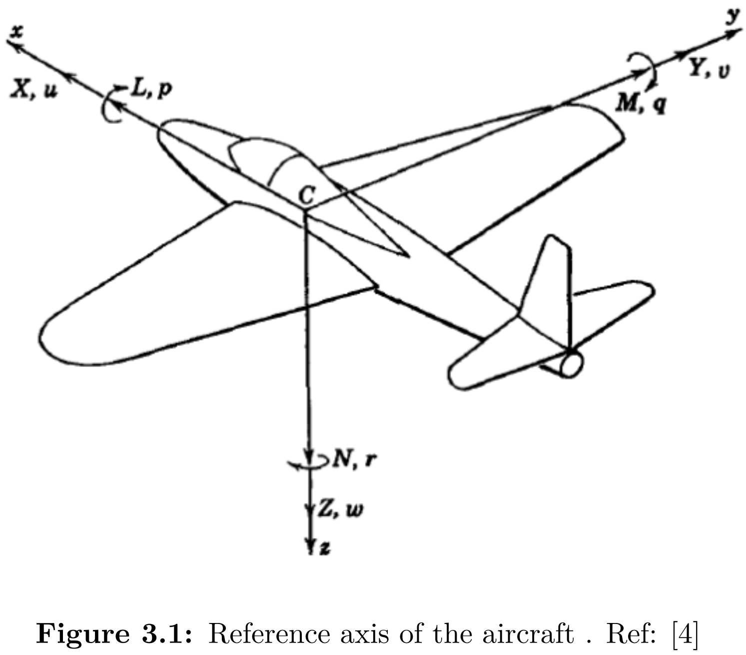

Figure 3.1: Reference axis of the aircraft . Ref: [4]

Balanced conditions manoeuvre as well as both pitch manoeuvres have similar formulation with some differences depending of the point in the envelope of *fig. 2.1* that the aircraft is at.

To begin with the general formulation, the center of gravity and aerodynamic point of reference of the wing-body were placed as it can be appreciated in the *fig. 3.2*. This aerodynamic point is usually located at 25% of the mean aerodynamic chord.

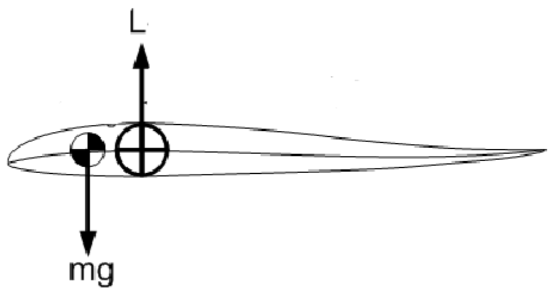

Figure 3.2: Wing airfoil

The wing airfoil is represented with the aircraft inertial force that acts at the centre of gravity and the lift that appears at the reference point. The aircraft inertial force must be multiplied by the load factor in case it is not equal to unity.

A set of equations, related to vertical force and pitching moment, are required in order to reach the desired formulation. Firstly, the vertical forces will be added.

$$L = nmg \tag{3.1}$$

n this approximation it is assumed that the aerodynamic force F_z is equal to the lift. It had to appear a cosine of α because the lift is in wind-axis and the force in body-axis, that is approximated to 1.

The lift coefficient is:

$$C_L = \frac{nmg}{QS} \tag{3.2}$$

Where Q is the dynamic pressure and is equal to:

$$Q = \frac{1}{2}\rho V^2 \tag{3.3}$$

It is convenient to separate the lift coefficient into HTP contribution and wing-body contribution, and the latter into 0-effect, alpha effect and pitch rate effect have to be separated into the different terms that composed it. In *eq. 3.4*, the superindex *wb* appears in the coefficients that are referred to wing-body.

$$C_L = C_{L_0}^{wb} + C_{L_\alpha}^{wb}\alpha + C_{L_{\bar{q}}}^{wb}\bar{q} + C_{L_H} \tag{3.4}$$

Some terms of *eq. 3.4* also need to be decomposed and explained. $\bar{q}$ is the non-dimensional term of the angular velocity of the aircraft and it is equal to:

$$\bar{q} = \frac{qc}{2V} \tag{3.5}$$

In *eq. 3.5*, the value of q depends of the type of movement that the aircraft is performing. It can be three different types: pull-up manoeuvre, push-over manoeuvre and steady turn manoeuvre. The values of the angular velocities of each of them are formulated in *eq. 3.6*, *eq. 3.7* and *eq. 3.8*, respectively.

$$q = (n-1)\frac{g}{V} \tag{3.6}$$

$$q = (n+1)\frac{g}{V} \tag{3.7}$$

$$q = (n-\frac{1}{n})\frac{g}{V} \tag{3.8}$$

The lift coefficient of the horizontal tailplane appears as a single term in *eq. 3.4* but it may be as well divided into parts as it is shown in *eq. 3.4*.

$$C_{L_H} = C_{L_{H_0}} + C_{L_{H_\alpha}}\alpha + C_{L_{\delta e}}\delta e + C_{L_{H_{\bar{q}}}}\bar{q} \tag{3.9}$$

The second equation that is used in order to obtain the motion analysis is the sum of moment around the center of pressure of the wing-body. In *eq. 3.10*, the moment coefficient of the wing-body is divided into its different terms plus the moments created by the weight and the lift created by the horizontal tailplane.

$$C_{m_0}^{wb} + C_{m_\alpha}^{wb} + C_{m_{\bar{q}}}\bar{q}^{wb} - \frac{C_{L_H} l_H QS}{QSc} + \frac{nmg}{QSc}(\bar{x}_{cg} - 0.25)c = \frac{I_{yy}\dot{q}}{QSc} \tag{3.10}$$

Where l_H is the distance between the aerodynamic centers of the wing-body and the HTP.

The lift created by the horizontal tailplane is located upwards (negative z-axis) for this calculation. Therefore, the term in *eq. 3.10* is negative.

Some unknowns appear in *eq. 3.4* and *eq. 3.10* which are α and C_{L_H}. The rest are known values of the system. The easiest method to solve the system of equations is obtain the equation of α from *eq. 3.4* and substitutes it *eq. 3.10*. Isolating α, the equation obtained is:

$$\alpha = \frac{\frac{nmg}{QS} - C_{L_0}^{wb} - C_{L_{\bar{q}}}^{wb}\bar{q} - C_{L_H}}{C_{L_\alpha}^{wb}} \tag{3.11}$$

Then, substituting *eq. 3.11* in *eq. 3.10*, the equation that results is the following:

$$\frac{nmg}{QS}(\Delta\bar{x}_{cg}+\frac{C^{wb}_{m_\alpha}}{C^{wb}_{L_\alpha}})+(C^{wb}_{m_0}-C^{wb}_{L_0}\frac{C^{wb}_{m_\alpha}}{C^{wb}_{L_\alpha}})-\frac{I_{yy}\dot{q}}{QSc}+(C^{wb}_{m_{\bar{q}}}\bar{q}-C^{wb}_{L_{\bar{q}}}\bar{q}\frac{C^{wb}_{m_\alpha}}{C^{wb}_{L_\alpha}}) = C_{L_H}\frac{l_H}{c}(1+\frac{c}{l_H}\frac{C^{wb}_{m_\alpha}}{C^{wb}_{L_\alpha}}) \tag{3.12}$$

Where $\Delta\bar{x}_{cg}$ is $\bar{x}_{cg} - 0.25$.

Eq. 3.12 is complex and introduce parameters to C_{L_H} of different natures and to understand the physical meaning of the different terms, the formula is going to be separated in $F_{z_{H_1}}$, $F_{z_{H_2}}$, $F_{z_{H_3}}$ and $F_{z_{H_4}}$.

The required term is the force created in z-axis by the horizontal tailplane. This force is equal to:

$$F_{z_H} = QSC_{L_H} \tag{3.13}$$

Note that each semi-tailplane, i.e, right and left HTP support one half of F_{z_h}.

Along the process, it is possible to evaluate some terms with the typical orders of magnitude to simplify the formulas.

$$\frac{c}{l_H} \simeq \frac{1}{4} \tag{3.14}$$

$$\frac{C^{wb}_{m_\alpha}}{C^{wb}_{L_\alpha}} \simeq 0.1 \tag{3.15}$$

$$1 + \frac{c}{l_H}\frac{C^{wb}_{m_\alpha}}{C^{wb}_{L_\alpha}} \simeq 1 \tag{3.16}$$

Combining *eq. 3.12* and *eq. 3.13*, the force that is desired can be obtained. It is going to be divided into four different terms as it was explained before.

3.1.1 F_{z_H} due to load factor

The first one includes the load factor as well as the weight.

$$F_{z_{H_1}} = \frac{c}{l_H}nmg(\Delta\bar{x}_{cg} + \frac{C^{wb}_{m_\alpha}}{C^{wb}_{L_\alpha}}) \tag{3.17}$$

Knowing that:

$$C^{wb}_{m_\alpha} = C^{wb}_{L_\alpha}(-\bar{x}_{ca} + 0.25) \tag{3.18}$$

Eq. 3.17 can be reduced to:

$$F_{z_{H_1}} = \frac{c}{l_H}nmg(\bar{x}_{cg} - \bar{x}_{ca}) \tag{3.19}$$

3.1.2 F_{z_H} due to zero-effects

On the other hand, the second term of the force includes various moment and lift coefficients of wing-body.

$$F_{z_{H_2}} = \frac{c}{l_H} QSC^{wb}_{m_0}(1 - \frac{C^{wb}_{m_\alpha}}{C^{wb}_{L_\alpha}} \frac{C^{wb}_{L_0}}{C^{wb}_{m_0}}) \tag{3.20}$$

3.1.3 F_{z_H} due to pitch rate

The third term of the force can be reduced taking into account that:

$$C^{wb}_{L_\alpha} \gg C^{wb}_{L_{\bar{q}}} \tag{3.21}$$

As the angular velocity appears in this term, there will be three different equation substituting the values from *eq. 3.6*, *eq. 3.7* and *eq. 3.8*.

$$F_{z_{H_3}} = \frac{c^2}{4l_H} \rho S g C^{wb}_{m_{\bar{q}}}(n-1) \tag{3.22}$$

$$F_{z_{H_3}} = \frac{c^2}{4l_H} \rho S g C^{wb}_{m_{\bar{q}}}(n+1) \tag{3.23}$$

$$F_{z_{H_3}} = \frac{c^2}{l_H} \frac{\rho S g}{4} C^{wb}_{m_{\bar{q}}}(n - \frac{1}{n}) \tag{3.24}$$

3.1.4 F_{z_H} due to pitch acceleration

The moment of inertia can be found in the last term of the force. The moment of inertia is equal to:

$$I_{yy} = m r_y^2 \tag{3.25}$$

Substituting *eq. 3.25* into the equation, the last force will be:

$$F_{z_{H_4}} = -m \frac{r_y^2}{l_H} \dot{q} \tag{3.26}$$

Where r_y is the radius of curvature of the movement of the aircraft.

3.1.5 F_{z_H} due to gusts

When the aircraft is subjected to a vertical gust, an additional term will be added to the general formulation.

Until the 1980s, a gust that must be analyzed would have the shape of the following function.

$$f = (1 - cos\frac{\pi x}{H}) \tag{3.27}$$

Where H is the distance between the beginning of the gust and the highest point of it and x is the distance of the aircraft inside the gust. The value of H had the value of:

$$H = 12.5\bar{c} \tag{3.28}$$

Where c was the mean aerodynamic chord for each aircraft analysed. Nowadays, the value of H varies between $30fts$ and $350fts$ for every aircraft.

The function of the gust versus x is depicted in *fig. 3.3* when H is equal to $1m$.

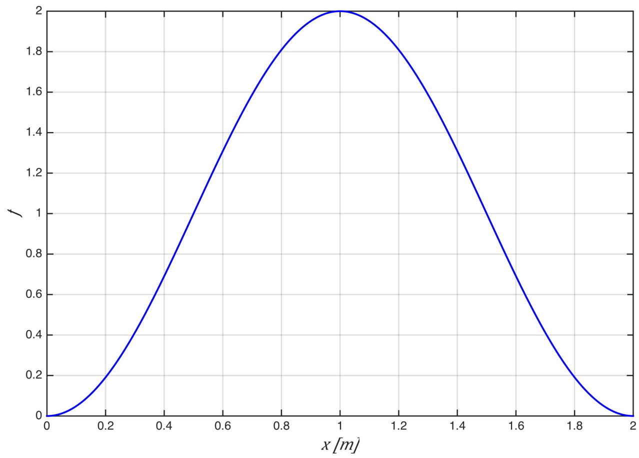

Figure 3.3: Function of the gust vs. x

To complete the analysis, the vertical gusts may be approximated to a step gust as the one depicted in *fig. 3.4*.

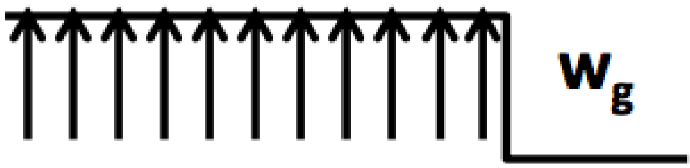

Figure 3.4: Representation of a step gust

W_g is the value of the vertical velocity of the gust. It may be multiplied by an allevation factor k_g.

$$k_g = \frac{0.88\mu}{5.3 + \mu} \tag{3.29}$$

Where μ depends on altitude, geometric and aerodynamic characteristics being maximum when aircraft loading is high at low C_{L_α}.

$$\mu = \frac{2m}{\rho S c C_{L_\alpha}} \tag{3.30}$$

The increment of the angle of attack experienced by the wing will be:

$$\Delta\alpha = arctan\frac{k_g W_g}{V} \simeq \frac{k_g W_g}{V} \tag{3.31}$$

The aircraft is going to be exposed to a change in the load factor as a consequence of the increment in the angle of attack.

$$\Delta n = \frac{\Delta L}{mg} = \frac{\frac{1}{2}\rho v^2 S C_{L_\alpha}\Delta\alpha}{mg} \tag{3.32}$$

Combining the previous equations, the expression for the increment in the load factor is *eq. 3.33*.

$$\Delta n = \frac{1}{\mu c g} k_g V W_g \tag{3.33}$$

The relation between α_H and α must be studied.

$$\alpha_H = \alpha - \varepsilon \tag{3.34}$$

ε (downwash angle) also has a dependence on α.

$$\varepsilon = \varepsilon_0 + \frac{d\varepsilon}{d\alpha} \tag{3.35}$$

Finally, the expression of the angle of attack experienced by the tail follows the expression of *eq. 3.36*.

$$\alpha_H = -\varepsilon_0 + (1 - \frac{d\varepsilon}{d\alpha})\alpha \tag{3.36}$$

Using *eq. 3.36* and *eq. 3.31*, the increment of α_H is:

$$\Delta\alpha_H \simeq (1 - \frac{d\varepsilon}{d\alpha})\frac{k_g U_{ds}}{V} \tag{3.37}$$

Finally the term of the vertical force that appears due to the gusts has the following definition.

$$F_{z_{H_{gust}}} = \frac{1}{2}\rho_0 V^2 S_H C_{L_{\alpha_H}} \Delta\alpha_H \tag{3.38}$$

Substituting *eq. 3.37*, the final formula would be *eq. 3.26*.

$$F_{z_{H_{gust}}} = \frac{1}{2}\rho_0 V S_H C_{L_{\alpha_H}} k_g U_{ds}(1 - \frac{d\varepsilon}{d\alpha}) \tag{3.39}$$

U_{ds} is the same that W_g. This change is made to use the same formulation that can be found in the regulation.

3.2 Reduced Formulation

In order to clarify the calculations and the results, a dimensionless formulation is reached.

The total force would be divided by the total mass of the aircraft, gravity as well as the term $\frac{c}{l_H}$. Those terms are chosen to eliminate some of the parameters in the resultant formulation. Therefore, each separated term of the force will not depend of geometric characteristics that vary for each aircraft.

The resultant term of the force will be shown in *eq. 3.40*, *eq. 3.41*, *eq. 3.42*, *eq. 3.43*, *eq. 3.44*, *eq. 3.45* and *eq. 3.46*.

$$\bar{F}_{z_{H_1}} = n(\bar{x}_{cg} - \bar{x}_{cp}) \tag{3.40}$$

$$\bar{F}_{z_{H_2}} = \frac{QS}{mg} C^{wb}_{m_0}(1 - \frac{C^{wb}_{m_\alpha}}{C^{wb}_{L_\alpha}} \frac{C^{wb}_{L_0}}{C^{wb}_{m_0}}) \tag{3.41}$$

$$\bar{F}_{z_{H_3}} = \frac{c\rho}{4\frac{m}{S}} C^{wb}_{m_{\bar{q}}}(n - 1) \tag{3.42}$$

$$\bar{F}_{z_{H_3}} = \frac{c\rho}{4\frac{m}{S}} C^{wb}_{m_{\bar{q}}}(n + 1) \tag{3.43}$$

$$\bar{F}_{z_{H_3}} = \frac{c\rho}{4\frac{m}{S}} C^{wb}_{m_{\bar{q}}}(n - \frac{1}{n}) \tag{3.44}$$

$$\bar{F}_{z_{H_4}} = -\frac{r_y^2}{c} \frac{\dot{q}}{g} \tag{3.45}$$

$$\bar{F}_{z_{H_{gust}}} = \frac{(\frac{l_H}{c} \frac{S_H}{S})(\frac{1}{2}\rho_0 V U_{ds}}{\frac{m}{S} g} C_{L_{\alpha_H}} k_g (1 - \frac{d\varepsilon}{d\alpha}) \tag{3.46}$$

Eq. 3.42, *eq. 3.43* and *eq. 3.44* correspond to pull-up, push-over and steady turn manoeuvre.

CHAPTER
FOUR

SIZING PARAMETERS STUDY

In previous chapters, the formulation needed to study the manoeuvres have been introduced. Every term of the formulation have been separated in order to analyse the effect of the parameters in the force created by the horizontal tailplane to performed the desired manoeuvre. Therefore, the bigger forces obtained in this analysis will correspond to the sizing manoeuvre for the aircraft.

4.1 Balanced Conditions Manoeuvre

The first manoeuvre to be represented is the balanced conditions manoeuvre. As explained in previous chapters, the aircraft is assumed to be flighting at zero pitching acceleration which means that *eq. 3.45* is equal to zero.

In this manoeuvre, there are not vertical gust. Therefore, there are only three terms of the force that may be analysed.

Three situations of the envelope represented in *fig. 2.1* must be taken into account to determine the importance of this manoeuvre in the sizing of the desired element. The range of velocities between points A and D when the load factor is maximum ($n = 2.5$) at Flaps-up configuration must be analysed. As well as the same configuration for minimum load factor ($n = -1$) between the points H and F of the envelope.

Flaps-down configuration must also be included when the load factor is maximum for this configuration ($n = 2$). In the case of minimum load factor for this configuration, $\bar{F}_{z_{H_1}}$ will be equal to zero.

Eq. 3.40 is the first term represented for these cases.

In *fig. 4.1*, the three different possible sizing points are represented. The only parameters in this term are the load factor that is constant for each one of the cases and the distance between the centre of gravity and the aerodynamic centre. The range of values for this distance that is represented can vary between -15% and 15%. For positive values of the distance, the centre of gravity is forward from the aerodynamic centre. On the contrary, the centre of gravity will be afterwards for negative value of the distance.

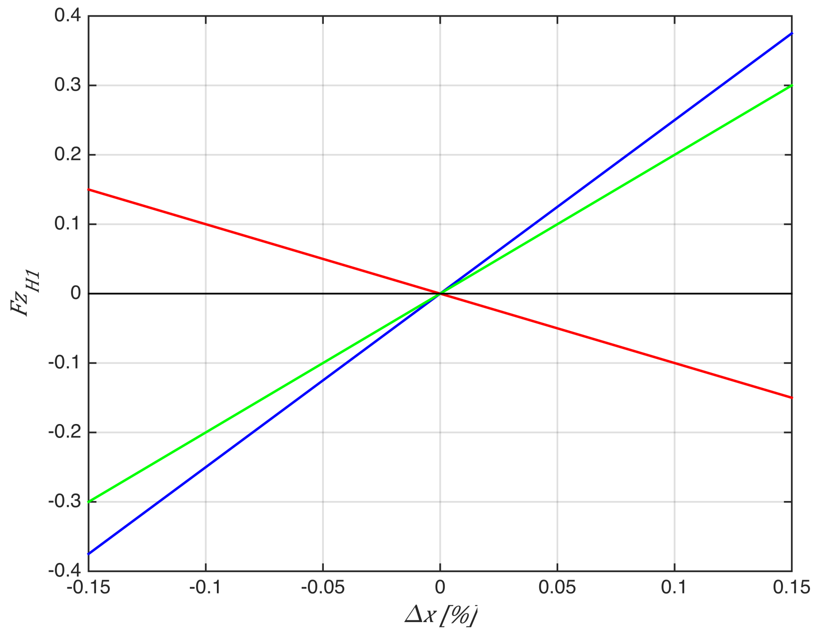

Figure 4.1: $\bar{F}_{z_{H_1}}$ vs. $\Delta\bar{x}$ for balanced conditions manoeuvre where Flaps-up $n = 2.5$: (blue line), Flaps-up $n = -1$: (red line) and Flaps-down $n = 2$: (green line)

At it may be deduced, the case with lower absolute value of the load factor is the one with lower force which is the case of flaps-up configuration with a negative value of the load factor as it can be observed in *fig. 4.1*.

The sign of the forces is not the same for every case. A positive value of the force corresponds to forces that are placed upwards which creates a negative moment in the centre of pressure in order to performed a movement of nose-down. The opposite occurs for negative values of the force. It generates a positive moment around the centre of gravity that creates a movement nose-up.

Therefore, the upwards forces will appear at maximum values of the load factor when a nose-down movement is need.

In *fig. 4.2*, the aircraft has positive load factor when it is climbing. At the highest point, a negative n is needed in order to perform a nose-down movement.

The first term is easy to analyse because it just depends on two parameters. The second term has many parameters that change with velocity, wing loading (m/s) and aerodynamic characteristics.

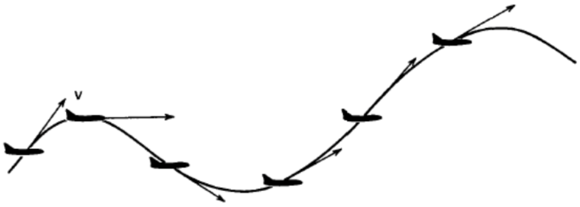

Figure 4.2: Nose-down and nose-up movement of an aircraft

The first parameters that are going to be studied for this term are $C_{m_\alpha}/C_{L_\alpha}$. The value of this fraction is going to vary depending of the specific aerodynamic characteristics of the aircraft to study. The values of C_{m_0} and C_{L_0} is going to remain constant depending on the flap configuration.

Table 4.1: Values for C_{m_0} and C_{L_0} depending on the flap configuration

	Flaps-up	Flaps-down
C_{m_0}	-0.1	-0.4
C_{L_0}	0.3	1

The values were chosen based on experience and typical values for transport aircrafts. The analysis will be made with this values but for an specific aircraft, the values might be changed to its aerodynamic characteristics.

The term of the force that is been analysed does not depend on altitude because the dynamic pressure of *eq. 3.41* is calculated with equivalent speed. Therefore, the density that might be used is ρ_0.

The range of velocities used in the analysis for each case of the envelope can be found in *table 4.2*.

Table 4.2: Range of equivalent speed depending on the flap configuration

	Flaps-up	Flaps-down
n maximum	$150KEAS - 350KEAS$	$75KEAS - 200KEAS$
n minimum	$100KEAS - 350KEAS$	$75KEAS - 200KEAS$

Firstly, *fig. 4.3* shows the case of flaps-up configuration and maximum load factor and *fig. 4.4* ilustrates the case of flaps-down configuration when the load factor is also maximum.

In *fig. 4.3* for $V = 150KEAS$, two wing loading were represented: $m/s = 300kg/m^2$: (—) and $m/s = 700kg/m^2$: (—). The same values were analyzed for $V = 350KEAS$, $m/s = 300kg/m^2$: (—) and $m/s = 700kg/m^2$: (—).

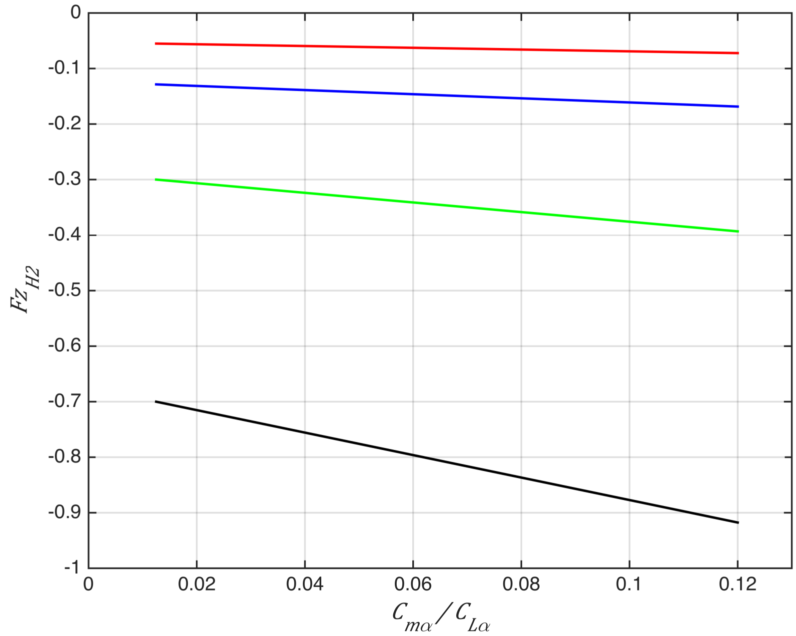

Figure 4.3: $\bar{F}_{z_{H_2}}$ vs. $C_{m_\alpha}/C_{L_\alpha}$ for balanced conditions manoeuvre with Flaps-up configuration and $n = 2.5$.

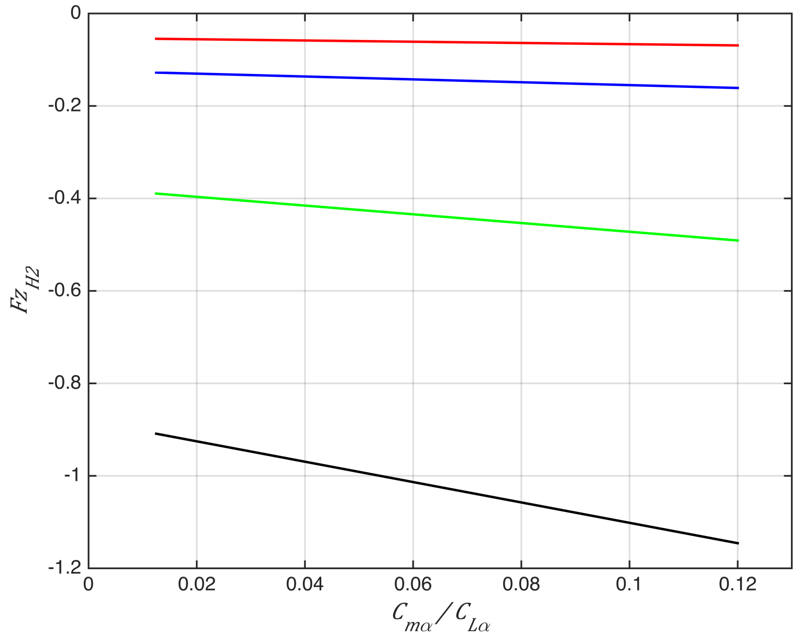

Figure 4.4: $\bar{F}_{z_{H_2}}$ vs. $C_{m_\alpha}/C_{L_\alpha}$ for balanced conditions manoeuvre with Flaps-down configuration and $n = 2$.

In *fig. 4.4* for $V = 75KEAS$, two wing loading were represented: $m/s = 300kg/m^2$: (—) and $m/s = 700kg/m^2$: (—). The same values were analyzed for $V = 200KEAS$, $m/s = 300kg/m^2$: (—) and $m/s = 700kg/m^2$: (—).

As it can be observed in *fig. 4.3* and *fig. 4.3*, the influence of the relation of these two parameters (C_{m_α} and C_{L_α}) does not vary a lot the force created. For instance, the cases with a high value of the wing loading at a low velocity maintain almost constant along the variation of these aerodynamic variables.

On the other hand, the variation in velocity and in wing loading result in a significant change in the force. The higher and therefore, most critical for the sizing of the horizontal tailplane is the situation with low wing loading at high equivalent speed.

It must be clarified that in this term, the values of the forces are downwards even through for cases of maximum n the force needed must be upwards. This is due to the sign of C_{m_0} and that the parenthesis in *eq. 3.41* is always going to be positive because *eq. 4.1* is fulfilled.

$$\frac{C^{wb}_{m_\alpha}}{C^{wb}_{L_\alpha}} \frac{C^{wb}_{L_0}}{C^{wb}_{m_0}} < 1 \tag{4.1}$$

In *fig. 4.5*, the influence of the velocity and wing loading is illustrated.

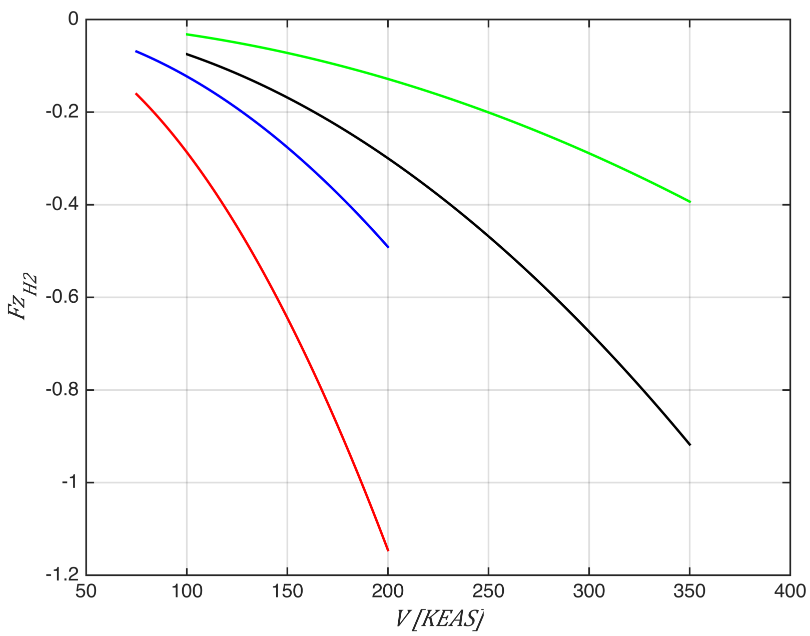

Figure 4.5: $\bar{F}_{z_{H_2}}$ vs. V for balanced conditions manoeuvre

Two wing loading configuration were analyzed in *fig. 4.5* for each point in the envelope explained before.

For $m/s = 300kg/m^2$, Flaps-up : (—) and Flaps-down : (—).

For $m/s = 700kg/m^2$, Flaps-up : (—) and Flaps-down : (—).

The both flaps configuration lines are the same for both maximum and minimum load factor because the velocity at which the manoeuvres are performed is the same and the value of the load factor does not appear in the formula represented.

As it was illustrated in *fig. 4.3* and *fig. 4.4*, the critical case for the different cases is the one with low wing loading and high equivalent airspeed. It might be anticipated taking into account *eq. 3.41*.

Also, the value of C_{m_0} and C_{L_0} have relevancy. As the values of these aerodynamic variables are higher for flaps-down configuration, the forces created when this configuration is deployed are more critical.

The analysis performed for the third term must take into account other parameter dependance. The variation with altitude as well as the value of the chord appear in *eq. 3.42*, *eq. 3.43* and *eq. 3.44*.

Three movements are possible depending on the load factor for each situation. For values of maximum load factor, the three movements are going to be analysed. Therefore, *fig. 4.6* show the variation of the force for flaps-up configuration at maximum n.

The calculations in *fig. 4.6* were performed with $m/s = 300kg/m^2$ and $C_{m_q} = 0.75$.

As it can be deduced from *eq. 3.42*, *eq. 3.43* and *eq. 3.44*, the higher force will be created by push-over manoeuvre for a case where the wing loading and C_{m_q} are fixed. Also, the value of the chord for the representation will be adjusted to an average value of the aircraft or the international regulations ($5m$).

At sea level, the force that must be created to fulfill the movement is going to be higher due to the influence of the density.

The most critical movement when the load factor is maximum is push-over due to the fact that a nose-down movement must be created in order to counteract the high load factor and start the decrease in altitude.

As *fig. 4.6* shows that the most critical manoeuvre is push-over, the next figure compares this type of manoeuvre for the two types of flap configuration at their maximum load factor possible with different wing loading configurations.

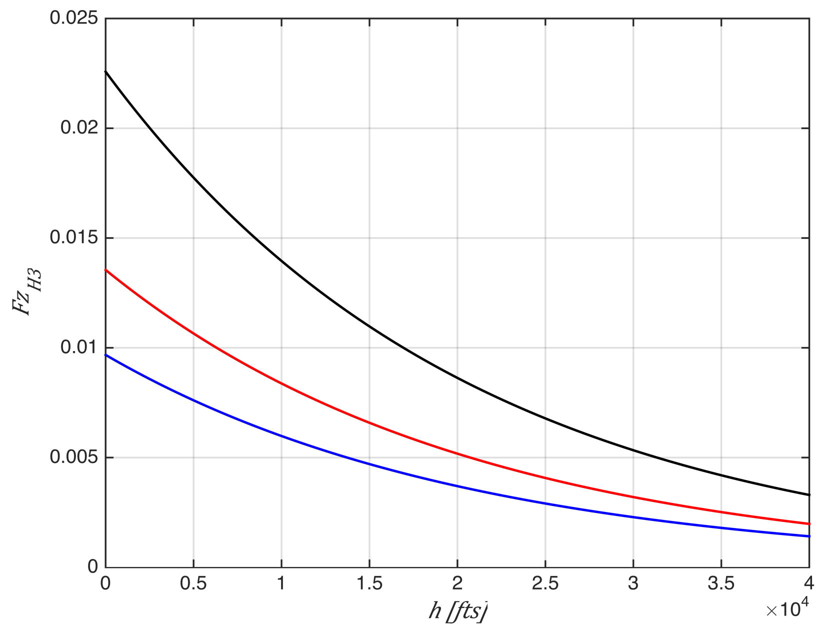

Figure 4.6: $\bar{F}_{z_{H_3}}$ vs. h for balanced conditions manoeuvre with Flaps-up configuration and $n = 2.5$ where pull-up : (—), push-over : (—) and steady turn : (—)

In *fig. 4.7*, push-over manoeuvre is compared between two flaps configurations using $C_{m_q} = 0.75$. Flaps-up configuration and $n = 2.5$ where $m/s = 300kg/m^2$: (—) and $m/s = 700kg/m^2$: (—). Flaps-down configuration and $n = 2$ where $m/s = 300kg/m^2$: (—) and $m/s = 700kg/m^2$: (—)

The value of the chord was chosen as a representation of the mean values for transport aircrafts for both of the cases. In the formulation for this manoeuvres, the parameters that vary from one case to another is the wing loading and the load factor. Therefore, it might be deduced from the formulation that the higher load factor with the lowest wing loading will be the case with higher force that must be created to perform the manoeuvre.

Until this point, the value of C_{m_q} has been chosen equal to 0.75 in order to represent the importance of other parameters.

The influence of this parameter must also be studied. In *fig. 4.8*, the push-over manoeuvre is pictured varying the value of this parameter as so far, have been the most critical manoeuvre for this term of the force.

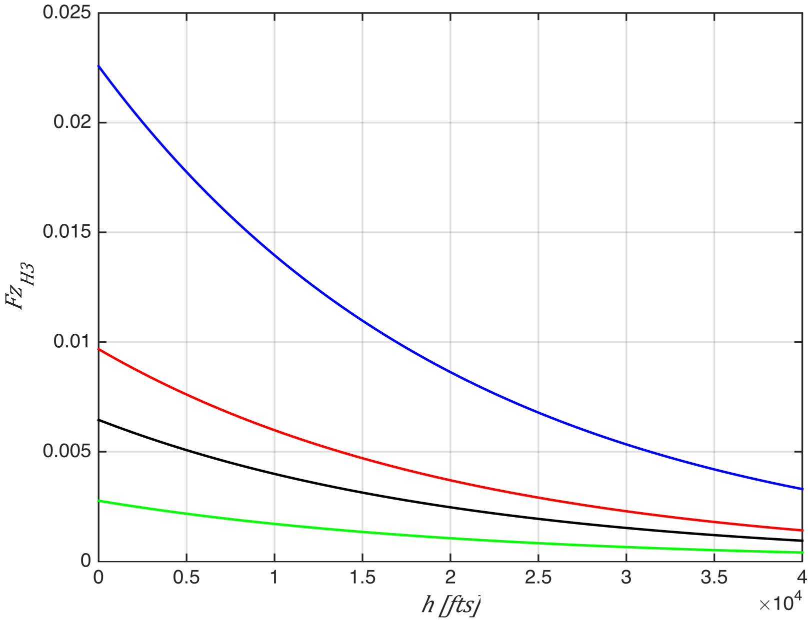

Figure 4.7: $\bar{F}_{z_{H_3}}$ vs. h for balanced conditions manoeuvre comparing push-over movement for different values of m/s

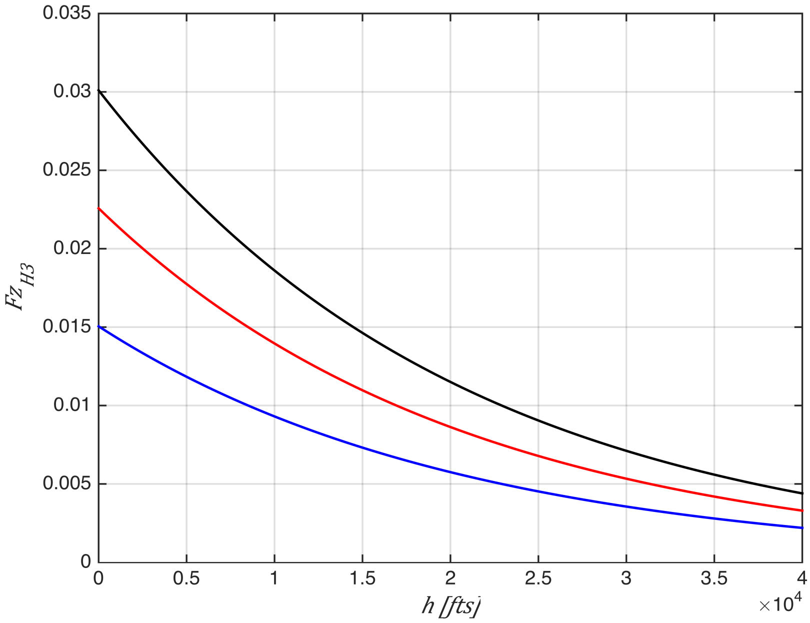

Figure 4.8: $\bar{F}_{z_{H_3}}$ vs. h for balanced conditions manoeuvre comparing steady turn movement for different values of C_{m_q}

The calculation in *fig. 4.8* is performed using $m/s = 300kg/m^2$ and Flaps-up configuration with $n = 2.5$. The values considered for C_{m_q} are $C_{m_q} = 0.5$: (—), $C_{m_q} = 0.75$: (—) and $C_{m_q} = 1$: (—)

It is demonstrates as it should be expected, that the highest value must be generated for the highest value of C_{m_q}. The increase in force is directly proportional to the change in this parameter.

The influence of this parameter is actually really relevant for the variation of this term of the force.

The last parameter that was analysed for this term was the variation of the chord. Until this moment, the value of the chord used has been an average value of $5m$. But there exits other aircraft with smaller and bigger geometric characteristics. Therefore, this parameters cannot be discarded.

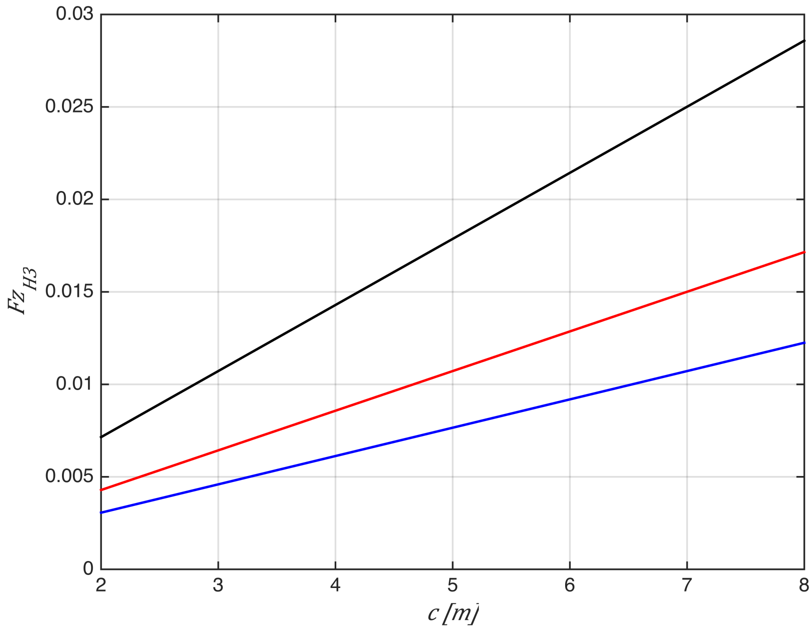

Figure 4.9: $\bar{F}_{z_{H_3}}$ vs. c for balanced conditions manoeuvre with Flaps-up configuration and $n = 2.5$ where pull-up : (—), push-over : (—) and steady turn : (—)

The calculations in *fig. 4.9* were performed with $m/s = 300kg/m^2$ and $C_{m_q} = 1$.

The study of this parameter has been executed with the lowest value of wing loading, case of highest load factor and maximum C_{m_q} for the three different manoeuvre in order to obtain the most critical case. The highest value is as expected when the chord is bigger which means a bigger aircraft and therefore, it would need a more force in order to perform the manoeuvre.

Comparing the terms of the force, some conclusions can be noticed. The distance between the centre of gravity and the centre of pressure of the aircraft is a relevant parameter as well as the chord. But their influence in the total force is not as much as the influence of other parameters. The altitude is a significant parameter for the third term of the force but as this term is not the predominant one in the total force, the parameter is not as critical as the velocity.

The velocity as well as the load factor have the most critical influence in this manoeuvre. The point in the envelope of *fig. 2.1* at which the aircraft is performing the manoeuvre is crucial to determine if it can be a sizing manoeuvre for the aircraft.

4.2 Specified Control Displacement Manoeuvre

The specified control displacement manoeuvre must be analysed in two conditions: for a positive pitching acceleration (nose-up) and for a negative pitching acceleration (nose-down). The positive pitching acceleration is performed at flaps-up configuration when $n = 1$ and a nose-up movement is needed. On the other hand, the same flap configuration at $n = 2.5$ would demand a negative acceleration to obtain a nose-down movement.

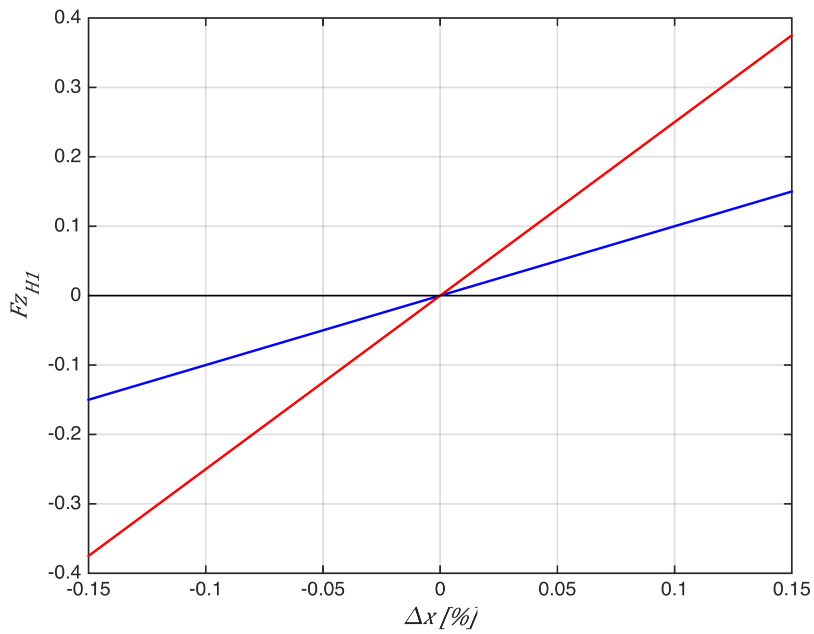

Figure 4.10: $\bar{F}_{z_{H_1}}$ vs. $\Delta\bar{x}$ for specified control displacement manoeuvre where $n = 2.5$: ( ) and $n = 1$: ( )

As previously, the first term of the force is going to be illustrated in order to show the evolution of the force as $\Delta\bar{x}$ varies. It is expected that the manoeuvre performed at higher load factor must need a higher forces to be executed as it was explained in the previous section.

The same parameters for the second term of the force are going to be analysed. The values of C_{m_0} and C_{L_0} that might be used for the study are the ones that correspond to flap-up configuration. In this term of the formulation does not appear the load factor which means that *fig. 4.11* might be valid for both of the cases.

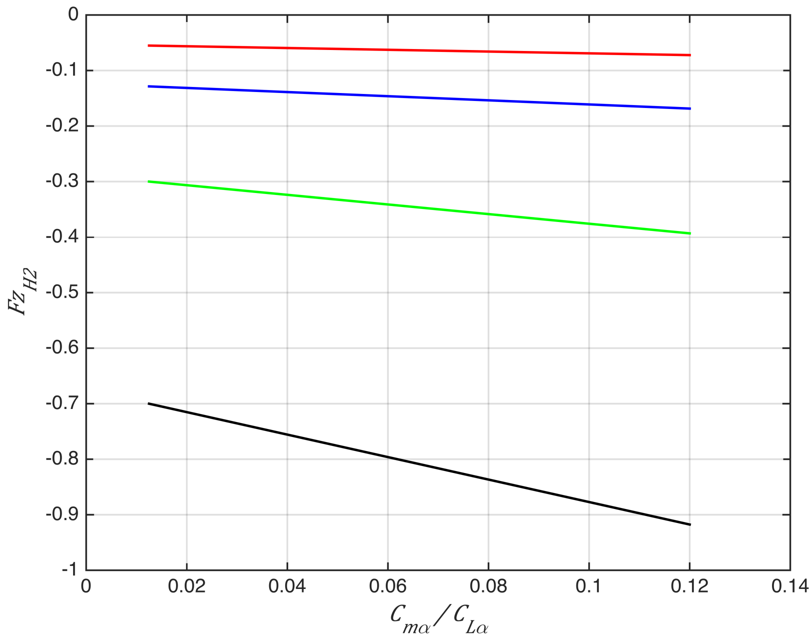

Figure 4.11: $\bar{F}_{z_{H_2}}$vs. $C_{m_\alpha}/C_{L_\alpha}$ for specified control displacement manoeuvre comparing different configuration of velocities and wing loading

In *fig. 4.11* for $V = 150KEAS$, two wing loading were represented: $m/s = 300kg/m^2$: (—) and $m/s = 700kg/m^2$: (—). The same values were analyzed for $V = 350KEAS$, $m/s = 300kg/m^2$: (—) and $m/s = 700kg/m^2$: (—).

As it was shown in the previous section and in *fig. 4.11*, the highest force needed to perform the manoeuvre is reached for the situation of high equivalent speed and low aircraft loading for each of the cases.

The variation of the parameters C_{m_0} and C_{L_0} does not affect in a significant manner $\bar{F}_{z_{H_2}}$ as was seen before.

In the balanced conditions manoeuvre, the most significant parameter was the velocity. Therefore, the variation of $\bar{F}_{z_{H_2}}$ must be illustrated varying the velocity.

Different wing loading configurations were represented. The results obtained had the same behaviour has the ones obtained for the balanced conditions manoeuvre. The influence of the equivalent airspeed is much more relevant that the variation of the aerodynamic characteristics.

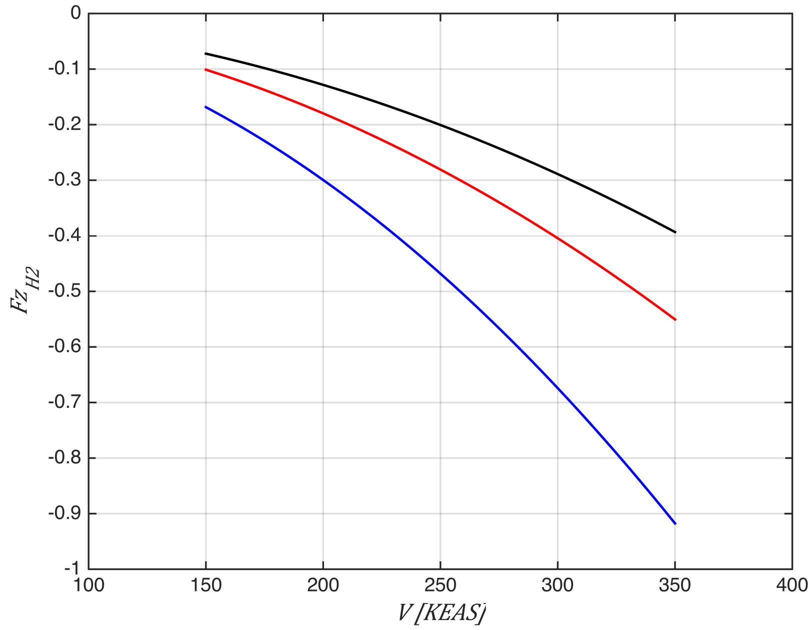

Figure 4.12: $\bar{F}_{z_{H_2}}$ vs. V for specified control displacement manoeuvre for different configuration of wing loading where $m/s = 300kg/m^2$: (—), $m/s = 500kg/m^2$: (—) and $m/s = 700kg/m^2$: : (—)

The term $\bar{F}_{z_{H_3}}$ for this manoeuvre has the same representation as in the previous section. It is not going to be shown again because the conclusion obtained before can be used for this case.

The term that includes the pitching acceleration is observed in *fig. 4.13*.

The calculations in *fig. 4.13* were performed at a constant average value of the chord equal to 5.

The equations to obtain the pitching acceleration for nose-up and nose-down movements are *eq. 2.2* and *eq. 2.3*, respectively.

The results are obtained maintaining the relation $\frac{r_y^2}{c}$ constant along the manoeuvre. It is consider as an average value that:

$$\frac{r_y}{c} \simeq 2 \tag{4.2}$$

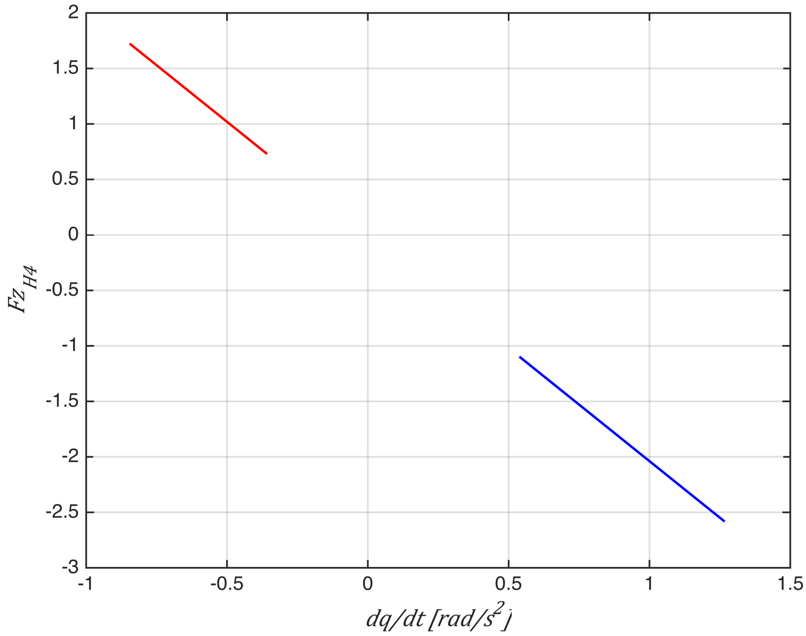

Figure 4.13: $\bar{F}_{z_{H_4}}$ vs. $\dot{q}$ for specified control displacement manoeuvre where $n = 2.5$: (——) and $n = 1$: (——)

This assumption has been made due to experience.

A simple analysis was also made comparing the aircraft with a bar which total distance from the most forward point to the bottom is $2l$.

The moment of inertia would be:

$$I = \frac{1}{12} m(2l)^2 = mr_y^2 \tag{4.3}$$

Therefore, the relation between r_y and c is:

$$r_y = \frac{l_H}{\sqrt{3}} \simeq \frac{4c}{\sqrt{3}} \simeq 2c \tag{4.4}$$

Fig. 4.13 shows that the higher absolute values of $\dot{q}$ must be overcame with higher values of the force. Being the most critical for flaps-up configuration when $n = 1$ (positive pitching acceleration).

Different geometric characteristics must be consider. So $\bar{F}_{z_{H_4}}$ is illustrated with the variation of the chord in *fig. 4.14*.

The variation of the value of the chord also have a significant influence in $\bar{F}_{z_{H_4}}$. In conclusion, the velocity parameter is the one with the highest influence in the results. The point in the flight envelope is fundamental for the analysis.

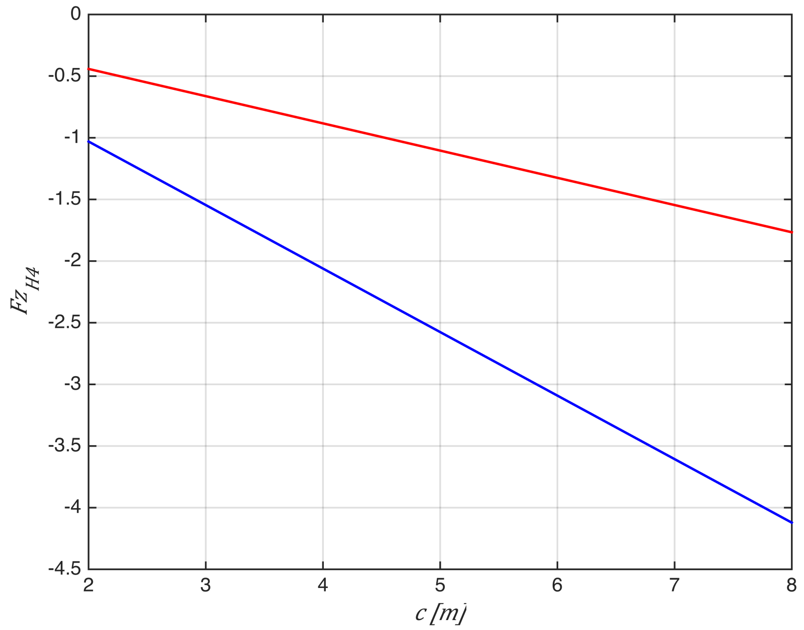

Figure 4.14: $\bar{F}_{z_{H_4}}$vs.c for specified control displacement manoeuvre for $n = 1$ where $V = 150KEAS$: (—) and $V = 350KEAS$: (—)

In general, a specified control manoeuvre demands more force to be performed than the balanced condition manoeuvre. Therefore, it might be consider for the sizing of the horizontal tailplane for positive pitching acceleration as well as negative acceleration. Later on, they might be compared to another sizing manoeuvre.

4.3 Maximum Displacement at V_A Manoeuvre

Maximum displacement manoeuvre consists on an aircraft flight in steady level flight at point $A1$ on the envelope ($n = 1$) and suddenly, the cockpit pitch control is moved to obtain the maximum pitching acceleration. A nose-up movement is desired for this purpose.

Therefore, this manoeuvre is similar to the previous one. Terms $\bar{F}_{z_{H_1}}$ and $\bar{F}_{z_{H_2}}$ have been already analysed for the case of load factor equal to 1 with flaps-up configuration to obtain a positive pitching acceleration.

The term that is slightly different is $\bar{F}_{z_{H_4}}$. In this manoeuvre, $\dot{q}$ does not have a simple formula that represents its relation with the velocity and the load factor. For a modern commercial aircraft, the pitching acceleration have values around

$-0.5rad/s^2$ and $0.5rad/s^2$.

A positive pitching acceleration is need in order to obtained the desired nose-up movement. The analysis was perform with various chord values in order to estimate its influence.

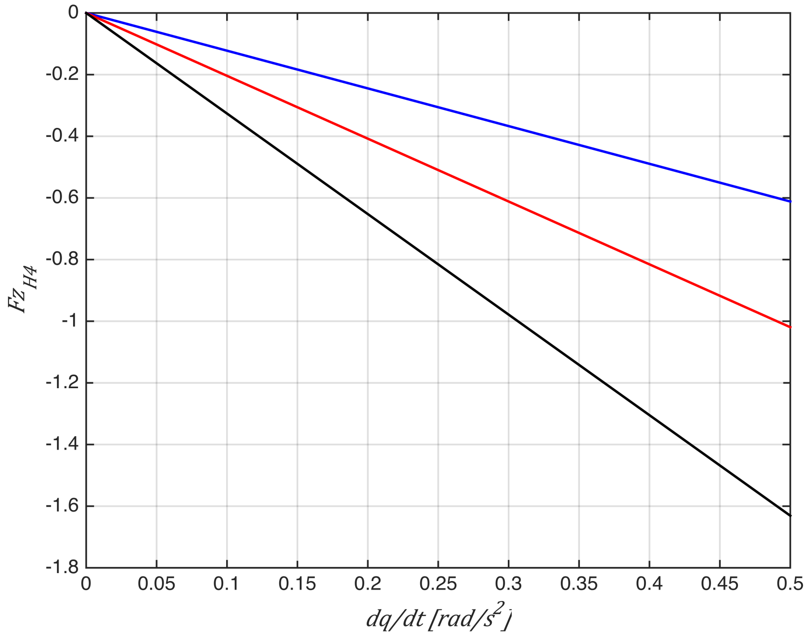

Figure 4.15: $\bar{F}_{z_{H_4}}$ vs. $\dot{q}$ for maximum displacement manoeuvre for different values of the chord where $c = 3m$: (▬), $c = 5m$: (▬) and $c = 7m$: (▬)

As it was expected, the values from bigger geometric characteristics are the ones that required a higher force to perform the desired manoeuvre. This fact was also observed in the previous manoeuvre when the chord parameter was being analysed.

The parameter c has influence in the determination of the total $\bar{F}_{z_H}$ but as previously illustrated, the parameter with highest influence is still the velocity at which the manoeuvre is performed.

4.4 Discrete Gust Design Criteria

The last manoeuvre that was depicted is the one with the appearance of a vertical gust when the aircraft is flight at steady level flight ($n = 1$).

The analysis was performed for two situations: flaps-up and flaps-down configuration. For the flaps-up configuration, the most critical case would be when the force created in the HTP is upwards and it appears a gust going up which can counteract the motion that the aircraft is trying to perform.

On the contrary, the critical case for the flaps-down configuration would be when the force in the HTP is downwards and it encounter a gust down.

$\bar{F}_{z_{H_1}}$ is not represented in this section because it can be found already in *fig. 4.10* where the value of the force can be positive or negative depending of the position of the centre of gravity.

Therefore, *fig. 4.16* shows the situation of both flaps configuration for different values of wing loading .

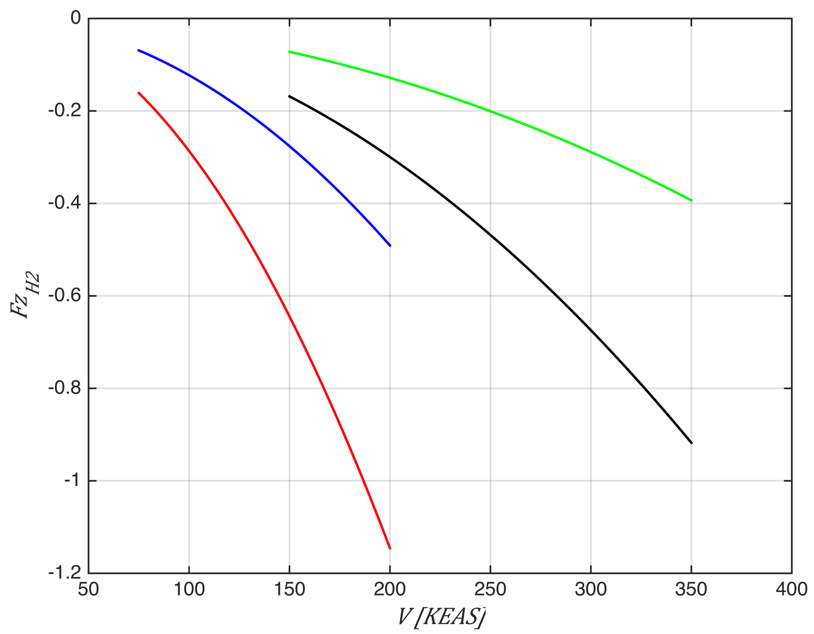

Figure 4.16: $\bar{F}_{z_{H_2}}$vs.v for discrete gust manoeuvre for different configuration of flaps and wing loading

In *fig. 4.16* for Flaps-up and $n = 1$, two wing loading were represented: $m/s = 300kg/m^2$: (—) and $m/s = 700kg/m^2$: (—). The same values were analyzed for Flaps-down and $n = 1$: $m/s = 300kg/m^2$: (—) and $m/s = 700kg/m^2$: (—).

The velocities for each of the cases are not the same. The manoeuvre for flaps-up configuration would take place at V_C. On the other hand, V_F woul be the velocity at the envelope at which the analysed manoeuvre would be studied.

As it was explained before, due to *eq. 3.41* the critical case for both configuration would be the one with higher velocity and lower wing loading. The influence of this parameter is significant for the total value of $\bar{F}_{z_H}$. Being the velocity, the parameter with higher relevancy.

It must be taken in consideration that the aircraft is at steady level and therefore, the terms of the forces $\bar{F}_{z_{H_3}}$ and $\bar{F}_{z_{H_4}}$ are equal to zero. There is not angular velocity nor angular acceleration for this case.

Therefore, the last term to be represented is $\bar{F}_{z_{H_{gust}}}$. Firstly, it is represented against the geometric characteristics of the aicraft for different values of velocity with flaps-up configuration.

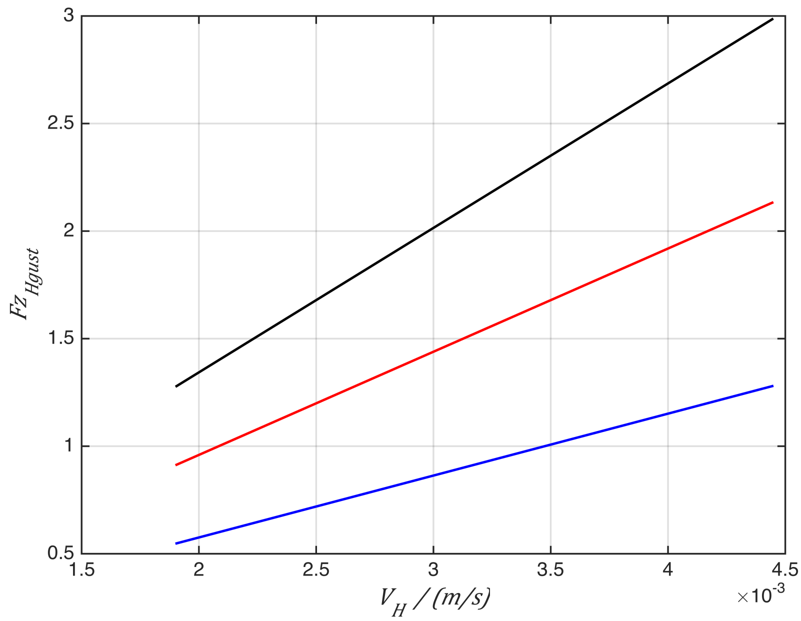

Figure 4.17: $\bar{F}_{z_{H_{gust}}}$ vs. $V_H/(m/s)$ for discrete gust manoeuvre for Flaps-up and $n = 1$ with different values of the velocity where $V = 150KEAS$: (—), $V = 250KEAS$: (—) and $V = 350KEAS$: (—)

Where V_H is the horizontal tail volume coefficient.

$$V_H = \frac{l_H}{c}\frac{S_H}{S} \tag{4.5}$$

The aerodynamic coefficients ($K_g C_{L_{\alpha_H}}(1 - \frac{d\varepsilon}{d\alpha})$) were set to an average value of 3.5.

Also, the other case to study was depicted using the same procedure. The velocities in this case were adjusted to V_F.

As previously, the aerodynamic coefficients were set to an average value of 3.5.

To fulfill the requirement that appear in the international norms, the values for the velocity of the gust that must be analysed for the two cases are:

Table 4.3: Values of the analysed gusts

	U_{ds} [ft/s]
V_F	25
V_C	56

Fig. 4.17 and *fig. 4.18* show some difference between the cases. The values of the force for the flaps-up configuration are higher than the case with flaps-down configuration. This is due to the different values of velocity at which the manoeuvre is taking place as well as the velocity of the gust.

As both values of the velocities are higher for the case of the flaps-up configuration, its was expected that the case required more force to be performed.

Previously, the value of the aerodynamic coefficients ($K_g C_{L_{\alpha_H}}(1-\frac{d\varepsilon}{d\alpha})$) was set to a fixed value. In *fig. 4.19*, different value for this coefficients were represented along the velocity for both cases.

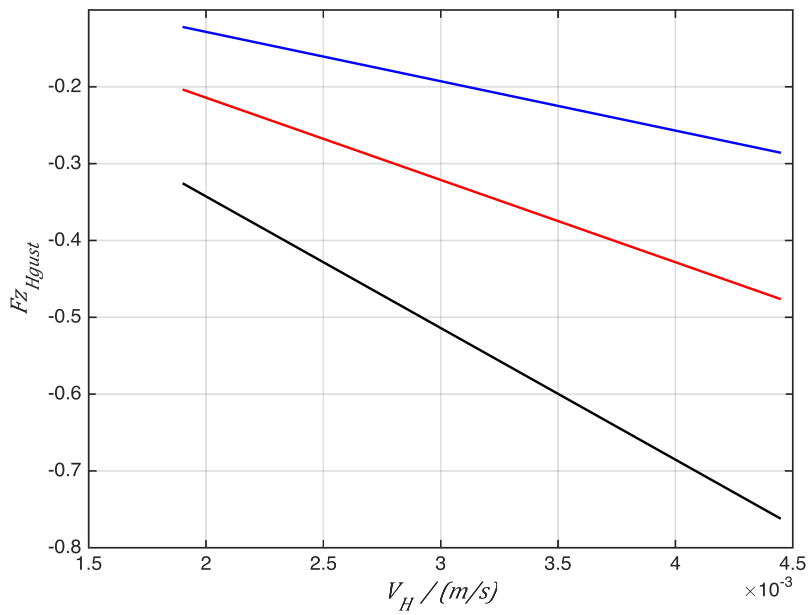

Figure 4.18: $\bar{F}_{z_{H_{gust}}}$ vs. $V_H/(m/s)$ for discrete gust manoeuvre for Flaps-down and $n = 1$ with different values of the velocity where $V = 75KEAS$: (—), $V = 125KEAS$: (—) and $V = 200KEAS$: (—)

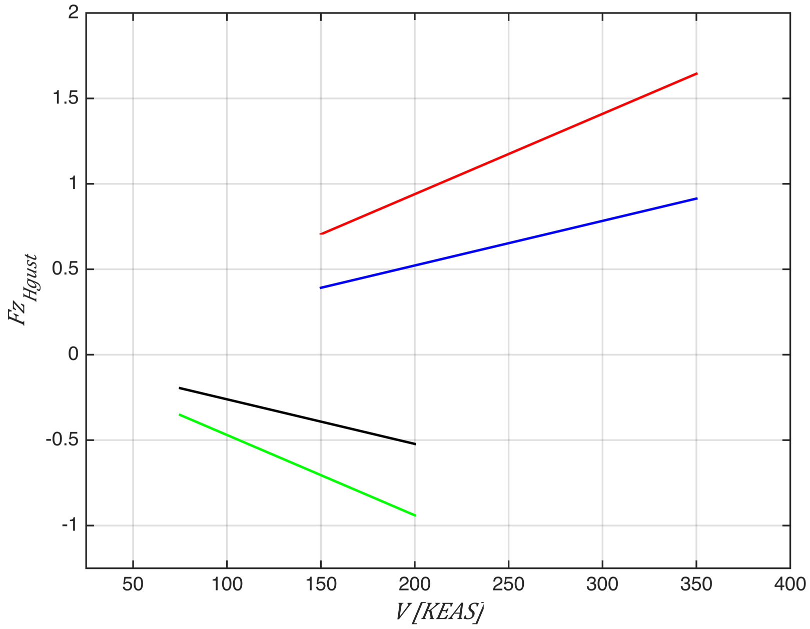

Figure 4.19: $\bar{F}_{z_{H_{gust}}}$ vs.v for discrete gust manoeuvre for different values of the aerodynamic coefficients

In *fig. 4.19* for Flaps-up and $n = 1$, two different values were associated to the aerodynamic coefficient: $C = 2.5$: (—) and $C = 4.5$: (—). The same values were analyzed for Flaps-down and $n = 1$: $C = 2.5$: (—) and $C = 4.5$: (—).

In this case, V_H was set to an average value of 4/3 and the wing loading to a maximum value of $700 kg/m^2$.

The effect of the variation of these parameter was as expected looking at *eq. 3.46*. A higher value of the aerodynamic coefficients would make the force higher to perform the manoeuvre.

Another method can be used in order to analysed the variation of $\bar{F}_{z_{H_{gust}}}$.

It is known that Δn_{gust} follow the formulation of *eq. 4.6*.

$$\Delta n_{gust} = \frac{\rho_0 V C_{L_\alpha} k_g U_{ds}}{2 \frac{m}{s} g} \tag{4.6}$$

Comparing *eq. 3.46* and *eq. 4.6*, a new dimiensionlesss formulation for $\bar{F}_{z_{H_{gust}}}$ can be created.

$$\bar{F}_{z_{H_{gust}}} = \Delta n_{gust} \frac{C_{L_{\alpha_H}}}{C_{L_\alpha}} (1 - \frac{d\varepsilon}{d\alpha}) V_H \tag{4.7}$$

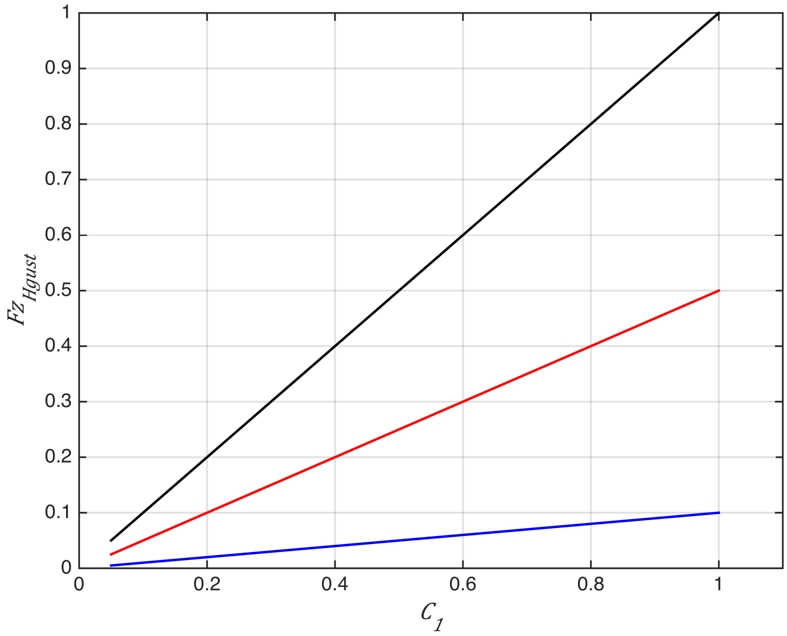

Figure 4.20: $\bar{F}_{z_{H_{gust}}}$ vs. C_1 for discrete gust manoeuvre for different increment in n

Where C_1 is:

$$C_1 = \frac{C_{L_{\alpha_H}}}{C_{L_\alpha}}(1 - \frac{d\varepsilon}{d\alpha})V_H \tag{4.8}$$

In *fig. 4.20* for Flaps-up and $n = 1$, different values for the increment in n were analyzed: $\Delta n_{gust} = 0.1$: ( ), $\Delta n_{gust} = 0.5$ ( ) and $\Delta n_{gust} = 1$ ( ).

Both case can be consider critical in the determination of the manoeuvre that size the horizontal tailplane of the aircraft.

Every single manoeuvre for different points in the envelope have been analysed individually. Therefore, there is time to compare the more critical one. In the next section, two manoeuvres will be study for the cases of upwards force and downwatds force.

4.5 Possible Sizing Parameters

The manoeuvres for upwards forces will be analysed first. Both of them are at flaps-up configuration. When there is a negative angular acceleration (nose-down motion) at V_A and the load factor is maximum, the values of $\bar{F}_{z_H}$ reach high values in order to perform the desired motion.

The other critical manoeuvre is when it encounters an vertical gust up when it is flighting at V_C. The load factor for this case is $n = 1$.

Fig. 4.21 shows both manoeuvre at their critical velocity for two different wing loading values.

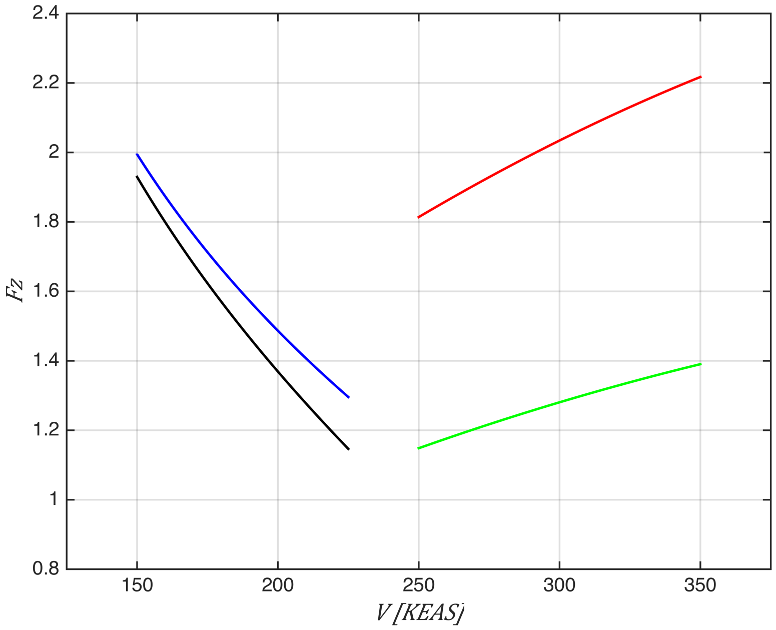

Figure 4.21: $\bar{F}_{z_H}$vs.v for critical manoeuvres for upwards forces

In *fig. 4.21* for maximum load factor and negative pitching acceleration manoeuvre, two different values were associated to the wing loading: $m/s = 300kg/m^2$: (—) and $m/s = 500kg/m^2$: (—). The same values were analyzed for vertical gust up manoeuvre and $n = 1$: $m/s = 300kg/m^2$: (—) and $m/s = 500kg/m^2$: (—).

The sizing situation for upwards $\bar{F}_{z_H}$ would be the on of the appearance of a vertical gust up when the aircraft is flighting at V_C and the wing loading is low. Therefore, when the aircraft is light in weight, the influence is of the impact with the gust is really high.

A different situation happens when the wing loading is higher. The aircraft weights more which means that is more difficult to perform a nose-down motion when it is flighting at a lower velocity at maximum load factor.

The case of downwards force must be analysed separately because the manoeuvre that can size the HTP are different.

When the aircraft is flight at steady level flight and suddenly perform a nose-up movement because of a positive angular acceleration , the values of $\bar{F}_{z_H}$ reach high values in order to perform the desired motion at flaps-up configuration.

The other critical manoeuvre is when it encounters an vertical gust down when it is flighting at V_F with flaps-down configuration. The load factor for this case is $n = 1$.

Both manoeuvres are represented in *fig. 4.22*.

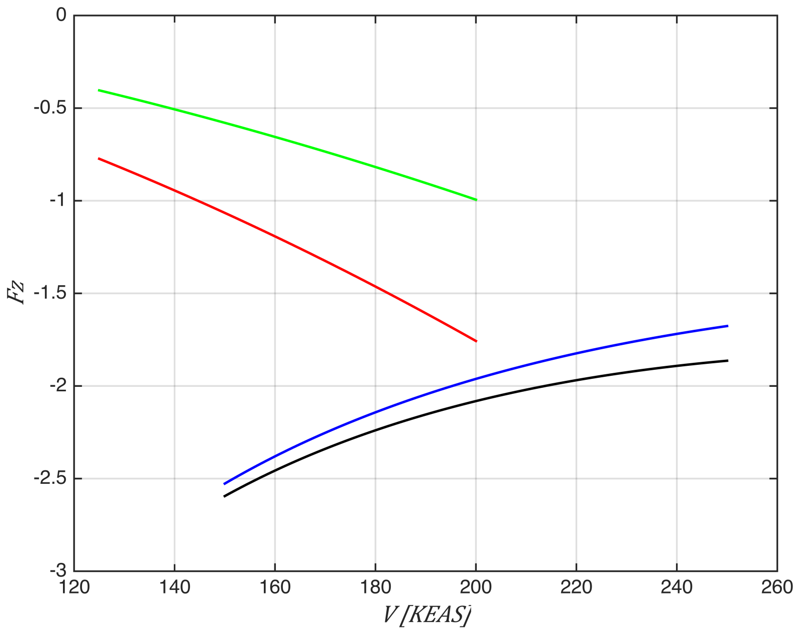

Figure 4.22: $\bar{F}_{z_H}$vs.v for critical manoeuvres for downwards forces

In *fig. 4.22* for positive pitching acceleration manoeuvre, two different values were associated to the wing loading: $m/s = 300kg/m^2$: ( ) and $m/s = 500kg/m^2$: ( ). The same values were analyzed for vertical gust down manoeuvre: $m/s = 300kg/m^2$: ( ) and $m/s = 500kg/m^2$: ( ).

In this case, the values of downward $\bar{F}_{z_H}$ are higher for any wing loading configuration at positive pitching acceleration manoeuvre.

CHAPTER
FIVE

CONCLUSIONS

The objectives of this project was the study of the dependency of the parameters that are involved on the sizing of the horizontal tailplane. These parameters have been analysed for each term of F_{z_H} to determine the importance and influence of each one of them in the sizing.

As it was observed, the wing loading as well as the velocity at which each manoeuvre was studied, were fundamental in order to determine if the aircraft would be structural sized by an encounter with a gust in steady level flight or by any symmetrical manoeuvre that involve pitching acceleration.

F_{z_H} has been made dimensionless dividing by mgc/l_H. The previous figures that have been represented have the order of magnitude of unity. As a first approximation, the order of magnitude of F_{z_H} is mainly given by the weight of the aircraft and the geometric relation c/l_H which is approximately around $20-25\%$.

A brief summary of the results is presented for each manoeuvre.

For the balanced conditions manoeuvre, the maximum values for each term of the force have been:

- $F_{z_{H_1}}$: ± 0.4
- $F_{z_{H_2}}$: -1
- $F_{z_{H_3}}$: 0.03

$F_{z_{H_3}}$ can be neglected compared to the other terms. The total order of the force for this manoeuvre is ± 1.5. Taking into account that it is dimensionless and the values of the geometric characteristics, F_{z_H} would be around $30-40\%$ of the weight of the aircraft.

For the checked and unchecked pitch manoeuvre, the maximum values are:

- $F_{z_{H_1}}$: ± 0.4 for $n = 2.5$ and ± 0.15 for $n = 1$

- $F_{z_{H_2}}$: -1
- $F_{z_{H_3}}$: 0.03 (can be neglected)
- $F_{z_{H_4}}$: 1.7 for $n = 2.5$ and 2.5 for $n = 1$

The order of the values of F_{z_H} for these manoeuvres is:

- Upwards F_{z_H} : 2 ($40 - 50\%$ of the weight of the aircraft) for pitch-down
- Downwards F_{z_H} : -3.5 ($70 - 90\%$ of the weight of the aircraft) for pitch-up

In the case of vertical gusts, the maximum values for each term of the force have been:

- $F_{z_{H_1}}$: ± 0.15 for $n = 1$
- $F_{z_{H_2}}$: -1
- $F_{z_{H_{gust}}}$: ± 2 for flaps-up and ± 1 for flaps-down configuration

Both of the terms are highly dependent of the velocity which make more difficult the estimation.

- Upwards F_{z_H} : $25 - 60\%$ of the weight of the aircraft depending of the value of V_C for flaps-up configuration and gust up
- Downwards F_{z_H}: for moderate values of V_C, the higher values would be obtained for gust down and flaps-down configuration

It must be consider that the estimation of gust with flaps-up configuration was very conservative covering very unfavorable parameters and conditions.

With the combination of structural equations and the analytical results for each case separately, it was possible to determine and analyse the possibilities of the structural sizing manoeuvres for each combination of velocity, wing loading, aerodynamic and geometric characteristics...

For upwards loads in the HTP, light aircraft (low wing loading value) will be more affected by the encounter for a vertical gust with upwards velocity. This case would size the preliminary design of the horizontal tailplane. Negative pitching acceleration manoevre will size heavy aircraft that are not as easy to be affected by the already mention gust.

On the contrary, the case of positive pitching for downwards force will be higher for different types of wing loading configuration mainly due to the effect of $\dot{q}$. The case of flaps-down configuration with gusts may be higher because of the case when

C_{m_0} is quite negative.

These analysis will be an useful start for the engineers to complete a preliminary design of this component of the aircraft with the knowledge of few parameters that might be included in the final result and the requirement that need to fulfill to meet the criteria of the European regulation as well as the United States regulation.

CHAPTER

SIX

FURTHER ANALYSIS

The analysis for this project has been made using F_{z_H}. Some considerations can be added in order to obtain more accurate results.

The formulation was performed considering the mass of the wing-body. The addition in the formulas of the mass of the HTP would sum another term to the general formulation for the manoeuvres.

The weight of the HTP will be multiplied by the load factor of the horizontal tailplane that has the following value:

$$n_H = n - \frac{\dot{q} l_H}{g} \tag{6.1}$$

Knowing the typical values of the relation between the mass of the wing-body and the mass of the HTP, the term that will be added due to this mass is negligible but can be consider to obtain accuracy in the results.

$$\frac{m_H}{m} \simeq 0.02 \tag{6.2}$$

Instead of performing the analysis with F_{z_H}, it could be completed with M_x or M_y.

The case of the momentum around x-axis is comparable and equivalent of the one made for this project because of their relation that can be found in *eq. 6.3*.

$$M_x = M_{x_A} + M_{x_I} \simeq F_{x_A} y_{ac} + F_{x_I} y_{cg} \simeq (F_{x_A} + F_{x_I})\bar{y} \tag{6.3}$$

Where the sum between F_{x_A} and F_{x_I} is equal to F_{z_H}. And y_{cg} and y_{ac} are the distance in the y-axis form the reference axis to the centre of gravity and the aerodynamic centre, respectively.

For the case of M_y, it is more complex because the cases of fixed and variable HTP must be analysed separately.

When the HTP is fixed, the value of i_H (tailplane incidence) is a geometric parameter that is known and the deflection of the elevator must be the necessary to perform the motion of the aircraft.

On the other hand, there will be two cases for variable HTP. The value of the deflection of the elevator would be zero for steady level flight and the incidence will be the one needed for $n = 1$. When the load factor is not equal to one and/or there is pitching acceleration, the tailplane incidence will be set to the value of steady level flight and the deflection of the elevator would be needed to perform the manoeuvre.

It is case the relation of F_{z_H} and the momentum is not a simple equation as a function of a distance. A lot of cases for the different options have to be studied and represented using computer simulations.

This case of analysis is out is scope of this project but may be a reliable for future studies.

APPENDIX

A

Budget

- Salary of a junior aerospace engineer that perform the project in 400 hours with a wage of 10 euros per hour : **4000 euros**.
- Salary of a senior aerospace engineer that supervised the project a total of 25 hours with a wage of 20 euros per hour : **500 euros**.
- Academic license of MATLAB : **500 euros**.
- Computer used to perform the calculation and the analysis of the project (Macbook Air) : **1000 euros**.

Total budget for this project : **6000 euros**.

APPENDIX

B

Bibliography

[1] Anon. *Part 25 - Certification Specification for Large Aeroplanes.* European Aviation Safety Agency, 2007.

[2] Anon. *Part 25 - Airworthiness Standards: Transport Category Airplanes.* Federal Aviation Regulations. U.S. Dept. of Transportation, Jan. 1994.

[3] Anon. *MIL-F-8785C - Military Specification.* U.S. Dept. of Defense, 1980

[4] T. Lomax. *Structural Load Analysis for Commercial Transport Aircraft:Theory and Practice.* AIAA Education Series. Air Force Institute of Technology, 1996.

[5] Ajoy Kumar Kundu. *Aircraft Design.* Cambridge Aerospace Series, 2010.

[6] B. Etkin and Ll. Duff Reid. *Dynamics of Flight: Stability and Control.* Wiley and Sons, 1996.

[7] F. M. Hoblit. *Gust Loads on Aircraft: Concepts and Applications*, AIAA Education Series, AIAA, 1989.

[8] H. A. Pearson, W. A. McGowan and J. J. Donegan, *Horizontal Tail Loads in Maneuvering Flight* NACA Rept. 1007, Feb. 1950.

[9] L. Nicolai. *Fundamentals of Aircraft and Airship Design.* AIAA Education Series, AIAA, 2010.

[10] C. Perkins and R.Hage. *Airplane Performance, Stability and Control.* Wiley, 1949.

[11] A.Silverstein and S. Katzoff. *Design Charts for Predicting Downwash Angles and Wake Characteristics Behind Plain and Flapped Wings.* NACA TR-648, 1939.