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ESSAYS IN INDUSTRIAL AND ORGANIZATIONAL ECONOMICS

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Getafe, de de

a mi hermosa familia y mis grandes amigos

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Resumen en Castellano

El trabajo de investigación realizado para la obtención del grado de Doctor en Economía se compone de tres trabajos en el área de Organización Industrial y Diseño Organizacional de las Empresas. El primer capítulo analiza la delegación óptima de la toma de decisiones dentro de una empresa monopólica que vende varios productos en varios mercados con el objetivo de balancear los incentivos a adquirir y transmitir la información de mercado dentro de la organización. El segundo capítulo responde a la pregunta si la introducción de mecanismos que ayuden a los consumidores a realizar reclamaciones cuando reciben productos defectuosos puede motivar a las empresas a proveer productos de calidad inferior. El tercer capítulo es un trabajo conjunto con Daniel García González. En este capítulo estudiamos los efectos de la provisión de información pública y privada en los incentivos a competir en un duopolio con productos diferenciados y costes heterogéneos, cuando hay información imperfecta sobre las preferencias en la demanda.

Capítulo 1: Organizational Design of Multi-Product Multi-Market Firms.

Este capítulo busca entender los determinantes del diseño organizacional y del esquema de compensación en una empresa multi-mercado y multi-producto cuando la rentabilidad está condicionada por los incentivos a transmitir la información del mercado dentro de la firma. Modificando la estructura organizacional- centralizando o decentralizando la toma de decisiones- y cambiando los incentivos a través de las compensaciones de los managers, la empresa puede afectar los flujos y la veracidad de la información, y, por ende, la rentabilidad de la empresa. Por medio de un modelo matemático formal, en este trabajo encontramos que, al ser multi-producto (lo que implica tener que compartir un recurso escaso en diferentes mercados), la oficina central de una empresa vincula la toma de decisiones de diferentes productos. La empresa decentraliza la toma de decisiones sobre productos con mayor retorno a la diferenciación, mientras que centraliza la toma de decisiones de productos con menor retorno a la diferenciación. Como la centralización es complementaria con la estandarización del producto y la decentralización es complementaria con la diferenciación de productos, el diseño organizacional condiciona la estrategia que la empresa sigue en el mercado. La relación entre los derechos en la toma de decisiones para diferentes productos se mantiene aún cuando la oficina central de la empresa no puede controlar la asignación de recursos dentro de cada división subsidiaria. La contribución de este trabajo es extender la literatura económica de diseño interno de la firma para el caso de empresas multi-producto, lo que implica introducir un problema de multi-tarea en el problema del diseño organizacional.

Capítulo 2: Product Reliability, Consumers' Complaints and Market Performance: The case of Consumers' Associations.

Este trabajo analiza la relación entre la calidad de un producto, medida como la probabilidad de que el producto sea defectuoso, ofrecida por un monopolista y la disposición a reclamar de los consumidores que reciben productos defectuosos. Existe amplia evidencia que algunos consumidores no realizan reclamos para cambiar, reparar o pedir alguna compensación cuando reciben un producto defectuoso. Esta evidencia sugiere que existe heterogeneidad en el coste de realizar reclamos entre los consumidores. Por otro lado, la empresa determina la probabilidad con la que un consumidor recibe un producto defectuoso. Obviamente que la empresa anticipa el comportamiento de los consumidores en el momento de elegir la probabilidad con la que un producto falla. A partir de esta relación, se estudia el efecto de una reducción en el coste de realizar reclamos de los consumidores sobre la elección que la empresa realiza sobre la probabilidad de que el producto sea defectuoso. La reducción en el coste de reclamar puede estar generada, por ejemplo, por la introducción de asociaciones de consumidores.

El resultado principal del trabajo consiste en demostrar que la empresa puede elegir subir la probabilidad con la que el producto resulta defectuoso cuando se reduce el coste de reclamar de los consumidores. A través de un modelo matemático identifiamos dos efectos. Por un lado, incrementar dicha probabilidad reduce el coste de producción de la empresa aunque incrementa el coste de gestionar los reclamos de los consumidores (producción y gestión de los reclamos). Por otro lado, aumentar la probabilidad de que el producto sea defectuoso reduce el valor que los consumidores asignan al producto. El efecto neto dependerá de la circunstancias del mercado. Si los consumidores tienen un menor coste de reclamar realizan más reclamos lo cual motiva una reducción en la probabilidad de que el producto sea defectuoso. Sin embargo, la disponibilidad a pagar de los consumidores se vuelve menos sensible a la probabilidad de recibir un producto defectuoso cuando los consumidores tienen un menor coste de hacer reclamos: una vez que los consumidores tienen un mecanismo barato que les garantiza un producto defectuoso, ya sea porque el producto no falla o porque lo cambian a bajo coste, el valor que se le asigna a la fiabilidad del producto cae.

La principal implicación empírica de este trabajo es que las asociaciones de consumidores pueden tener objectivos en conflicto. Sus objetivos de reducir el coste de hacer reclamos de los consumidores y garantizar la provisión de productos de alta fiabilidad pueden no estar alineados.

Capítulo 3: Hotelling Competition for a Consumer with Unknown Taste.

En este capítulo (un trabajo en conjunto con Daniel García-González) analizamos un mercado donde ni el consumidor ni el productor tienen información precisa sobre cuáles son las preferencias del consumidor. Las empresas deben primero diseñar los productos y luego competir en precios por la demanda. Sin embargo, el consumidor puede proveer información adicional a las empresas sobre sus propias preferencias que ayudarían a las empresas en el diseño de sus productos. Formalmente, utilizamos un modelo de Hotelling de diferenciación horizontal donde hay dos empresas que eligen localización-luego-competencia-en-precios con costes de transporte cuadráticos y heterogeneidad en los costes de producción. En este trabajo mostramos que la existencia de heterogeneidad en costes y la provisión de señales informativas puede fomentar la competencia entre empresas, incrementando el excedente del consumidor a pesar de reducir beneficios y el bienestar. Finalmente, probamos que la existencia de señales privadas puede fomentar la competencia, mientras que la existencia de señales públicas puede desincentivar la competencia, generando importantes implicaciones de política para el diseño de mercados y de subastas para adquirir productos.

Contents

Chapter 1

Introduction

This thesis dissertation consists on three independent research papers in Industrial and Organizational Economics. The first chapter, in Organizational Economics, studies the organizational design of a multi-product multi-market firm. The second chapter, in Industrial Economics, analyzes the interaction of consumers complaining behavior and the firm's choice of product reliability. The third chapter (with Daniel García-González), in Industrial Economics, seeks to understand the incentives of a consumer to provide information about her own preferences to the suppliers before firms design their products.

In the first chapter we seek to understand how a multi-product multi-market firm (for example, a multinational firm) designs its organizational structure and compensation scheme when its profitability is conditioned by how market information flows within the company. By modifying its organizational structure–centralizing or decentralizing decision making–and changing the weights of its compensation scheme, the firm can shape how information flows and is represented, changing the firm's profitability. We find that, when being multi-product (having to allocate a scarce resource between markets), the headquarters links the organizational design of decision rights between different product markets. The headquarters decentralizes decision rights in products with higher returns to product differentiation while it centralizes decision rights in products with lower returns to product differentiation. As centralization is complementary with product standardization and decentralization is complementary with product differentiation, the organizational design conditions the firm's market policy. The relation among product's decision rights remains even when the headquarters cannot control how local managers allocate resources in their own local divisions. Our results are robust to different generalizations. Our paper therefore, contributes to the literature on organizational design by analyzing the case of multi-product multi-market firms.

In the second chapter we analyze the relation between consumers' claiming behavior and product's reliability. In their dealings with retailers and suppliers, regulations and warranties ensure that consumers can seek a repair, a replacement or a refund if the good they have purchased is faulty. The evidence, however, indicates that few consumers pursue any form of compensation, suggesting that consumers have heterogeneous claiming costs. Moreover, product's reliability, which determines how likely it is that a consumer receives a defected product, is endogenous to the problem of the firm. The fact that some consumers have high claiming cost and that firms choose the product's reliability provides a rationale for the role that consumers' associations play.

In this paper, we analyze the monopolist's pricing and product reliability problem when consumers are entitled to product replacement and assess the implications of a decrease in consumers' transaction costs. Our results suggest that the appearance of the consumers' associations could, instead, lower product reliability. A reduction in consumers' claiming cost increases the cost associated with replacements but also reduces the sensitivity to product reliability of consumers' willingness to pay. Alone, the manufacturing cost of the replacement units would make the firm increase the level of its product reliability, but the presence of the expected utility effect may yield opposite results.

The main empirical implication of the paper is to show that consumer associations may have conflicting goals: reducing consumers' claiming cost and increasing product reliability may not be aligned. The firm may decide to produce a less reliable product if more consumers request a replacement of a defected product. We draw empirical evidence from the pattern of recalls and complaints in the U.S. car market around 1995 (the year in which the National Highway Traffic Safety Administration (NHTSA) incorporated on-line filings) and find that it appears consistent with this prediction.

In the third chapter, a joint work with Daniel García-González, we study a market where neither the consumer nor the suppliers have precise information about the consumer's taste. Firms first design their products and then compete in prices for consumer's demand. However, the consumer can provide some additional information about her own preferences to the suppliers that can be used for designing their products. This information can be provided through private meetings or public announcements. Formally, we analyze a Hotelling's duopoly game of location-then-price-competition choice with quadratic transportation cost and firms with heterogeneous costs, under the assumption that firms are uncertain about consumer's taste. We show that firms' heterogeneity and the existence of informative signals can foster competition, thereby increasing consumer surplus in spite of decreasing profits and welfare. Finally, we show that private signals can foster competition while public signals can discourage competition, having important policy implications for markets and procurement design.

Chapter 2

Organizational Design of Multi-Product Multi-Market Firms

2.1 Introduction

In this paper, we seek to understand how a multi-product multi-market firm (for example, a multinational firm) designs its organizational structure and compensation scheme when its profitability is conditioned by how market information flows within the company. By modifying its organizational structure–centralizing or decentralizing decision making–and changing the weights of its compensation scheme, the firm can shape how information flows and is used, changing the firm's profitability. Our paper contributes to the literature on organizational design by analyzing the case of multi-product multi-market firms highlighting a relation on how decision rights are allocated within a firm.

In the model, a monopolist manufactures two different products to sell in two (country) markets. The products have independent demands (they are, for example, coffee and candy bars) and can be customized to meet the country demand's specificities. In each country, the firm has a manager who oversees demand information and who may have product-design decision rights if decision making has been decentralized.

The two country managers are under the umbrella of a headquarters' office (hereafter, HQ) that has the same objective function as the firm. Country demands are heterogeneous (for example, they have different demand elasticities) and thus country-level profits are maximized when the product's characteristics are tailored to the specificities of the country's demand. The country manager, however, only imperfectly observes his own country's demand heterogeneity, but can improve the quality of his signal by devoting more time and effort to information acquisition.

Managerial time, however, is a scarce resource that must be split between the two product markets the manager oversees. Time and effort are complementary. Effort determines the quality of the signal, while time lowers the cost of exerting it—that is, exerting the same amount of effort over a longer period of time is less costly to the manager. Nonetheless, the decisions in the two markets are not independent: as time is a limited resource, the additional time devoted to one market is not devoted to the other and this results in an increase in the latter market's cost of effort.

The firm must decide whether, and how much, to customize the products to the countries it sells to, adding hazelnut flavoring to its coffee or mixing crunchy rice puffs in its chocolate bars. This decision can be delegated to the country's manager, resulting in a decentralized structure, or be centralized in the HQ's office. Country managers are self-interested and their pays are endogenous to the firm. If their pay is simply a share of the firm's aggregate profits, we say their interests are aligned, while, instead, if they are an unequal average of the two countries' profits, we say their incentives are misaligned.

After observing their private signals, each manager acts upon his information. With centralized decision-making, the managers simultaneously send reports to the HQ's office. It is this office that, after receiving the two reports, decides the specificities of the products to manufacture. Instead, with decentralized decision-making, reports are exchanged between country managers, who then unilaterally determine the characteristics of their two products.

Obviously, when sending reports, the manager may be strategic, aiming to bias the characteristics of the products chosen by either the HQ or the other country's manager. Managers may misreport their observed signals to strategically bias product decisions. If the chosen products are identical across country markets, there are economies of scale in production and thus, cost savings. If, instead, the products are heterogeneous, the firm raises revenues as it is implementing third-degree price discrimination. Therefore, when misreporting, the manager seeks to have the two countries sell the same product, yet he wants this product be the ideal product in his own country. From the firm's point of view, having the manager misreport information is costly since it lowers the accuracy of the information transmitted, upon which other agents (HQ or other country) make product decisions. To align the manager's incentives, the firm can modify the manager's compensation, making it more or less aligned with the firm's profit, and/or decentralize decision-making.

To understand the workings of the model and gauge intuition for the results obtained in the described environment, it is useful to start with the existing literature. As our model builds upon this literature, it inherits some of its workings. Alonso, Dessein, and Matouschek (2008)[1] analyze the problem of a multi-market (two countries) single-product firm when the manager of each country perfectly, but privately observes information about true demand characteristics. As in our paper, there are economies of scale in homogenizing products and there are also gains from price discrimination when customizing the products to the respective country markets. Different from our model, however, are that information is exogenous and does not arise from exerting effort or allocating a scarce resource and that the compensation scheme is exogenously given.

Alonso, Dessein, and Matouschek (2008) show that, for compensation schemes that align incentives, the firm prefers to decentralize decisions, while for those that misalign them, the firm prefers

[1]See Rantakari (2008) for the case of asymmetric organizational design in the environment of two country single-product firm with private (perfectly observed) information.

to centralize them. To see this, consider the case of perfect alignment (country managers and HQ have the same objective function). Since there is perfect alignment, the maximization problems of all agents are the same and thus, regardless of who decides, the same decisions are made. As the misalignment increases, the weight on the manager's own country profit does too, the manager, then, starts having incentives to misrepresent information. In his report, the manager seeks to implement product characteristics that are close to his ideal product, as this makes his country's division profit grow. If the misalignment is small, the policy the country manager chooses with decentralization is similar to the one the HQ would choose, albeit the manager makes his choice with better information, and reduces the mismatch with the country's true ideal product. The manager bases his choice on his own observed information and the report that the other sends. Since the two managers have similar objective functions (the misalignment is small), there is little incentive to lie about the reports to each other and thus there is only a small bias in their choices. As the misalignment of incentives increases, however, decisions' biases increase under decentralization as objective functions now differ. Moreover communication under decentralization becomes too poor as they seek to effectively bias the other country's product implementation. As a result, the HQ prefers to centralize decisions and pursue lower production costs through product standardization. Although the managers also misreport information in their communication with the HQ the bias is smaller than it would be if the information were sent to the other manager. The HQ's objective function, being the sum of the two country profits, is closer to the country's objective function than the objective function of the other country and thus there are fewer incentives to lie.

Rantakari (forthcoming) can be viewed as adding a new trade-off to this single-product two-country environment. Now the manager does not perfectly observe the country's characteristics, instead, like in our paper, he obtains a signal that can be made more precise by exerting costly effort. As in Alonso, Dessein, and Matouschek (2008), if the objective functions are aligned, the manager has no incentive to lie when reporting the market's characteristic. However, because the cost of effort is only incurred by the manager and thus in his aligned objective function it carries more weight than the country's profit, he exerts less effort than optimal for the firm. With less effort, the manager obtains lower quality information, by choosing a product that does not coincide with the country's ideal which lowers profits. To give incentives to the manager to exert more effort and obtain higher quality information, the firm must compensate him by misaligning the objective function, but then the manager has incentives to misreport the signal, which, in turn, lowers profits. When the misalignment needed to induce effort is too large, the communication between countries is too imperfect, as their diverging objective functions lead them to strongly bias the reports, and this makes the firm prefer centralized decision-making.

In our paper, we extend this environment to make the firm multi-product and multi-market. Being multi-product may have multiple effects in the problem of the firm, here we focus on one: the country manager allocates a scarce resource—managerial time—to acquire information in the two markets he oversees. The product markets have independent demand yet differ in their returns to product differentiation, which makes the quality of information more valuable in one market than in the other.

Being multi-product and endogenously determining the allocation of the scarce resource qualitatively modify the findings. If time allocation between product markets were, instead, exogenous, the problem of the firm would be equivalent to the problem of two multi-market single-product firms and thus the results in Rantakari (forthcoming) would apply separately to each market. But, if the allocation of time is endogenous to the manager, as it is in our paper, the results are no longer an immediate generalization of Rantakari (forthcoming). The asymmetry of the returns to differentiation makes the firm want to shift resources (time and effort) to the market with higher returns to differentiation. To provide incentives for such a shift, the firm misaligns incentives, decentralizing decision-making in the high-return market, and aligning incentives and centralizing decision marking in the market with lower returns to differentiation. That is, to induce the correct shift of resources, the firm jointly modifies the return to effort and time in the two markets.

Nonetheless, this negatively correlated allocation of decision rights may not be optimal as it misses out on one effect in the market with lower return. If the difference in return between the two markets exists but is small, the cost of shifting resources away from the market with low return and of obtaining bad quality information in this market, is large. Low quality information means that product decisions with centralization are "incorrect" and yield low profitability although initially this market was almost as profitable as the other. To offset this effect, the firm modifies its decision to also decentralize the market with lower returns to differentiation so that more resources are devoted to it. That is, if the differences in returns to differentiation between markets are small, the firm prefers to decentralize the two markets. If, instead, the differences are large, the market with higher returns to differentiation is decentralized while the other is not.

It is worth noting that whether the firm prefers to decentralize the two markets depends critically on the manager's allocation of time not being verifiable. If the HQ was to monitor the allocation of time, the firm would not need to decentralize the market with lower returns to provide incentives. Instead, the firm could simply force the manager to devote more time to it. Then, with more time being devoted, the manager would obtain, and transmit, better information and this would, in turn, raise the profitability of product standardization and increase the returns from centralizing the market with lower returns.

Lastly, there are other strains of literature that our work relates to. Athey and Roberts (2001), Friebel and Raith (2010), and Dessein, Garicano, and Gertner (forthcoming) explore other environments but share with Rantakari (forthcoming) and our paper the trade-off between the choice of effort and the quality of decision making. Our paper also relates to the organizational design literature that uses a mechanism design approach, which is summarized in Mookherjee (2006). This literature analyzes the trade-offs between performance and incentives (not the allocation of decision rights) by assuming that contracts are complete, which implies that the revelation principle applies. In our framework, and in the one of Alonso, Dessein, and Matouschek (2008) and Rantakari (forthcoming), imposing completeness of contracts would imply that decision rights are always centralized, making the framework unsuitable to understand the problem of organizational design: contract incompleteness (i.e., pays that are contingent on realized profits) and decision-making allocation are inherent to the same problem.

The paper is organized as follows. In Section 2.2, we describe the model and solve it in Section 4.3. In Section 2.4 we extend the model to consider endogenous coordination needs and externalities in information acquisition, showing that our results are robust. In Section 3.6, we conclude.

2.2 Setup of the Model

A multi-division firm produces two products, A and B, which it sells in two different regional markets, 1 and 2. Each regional division is controlled by a local division manager who is in charge of obtaining information about demands' characteristics. We denote by θ_{ji} the demand characteristic of product j in region i, for $j \in \{A,B\}$ and $i \in \{1,2\}$.[2] Product demands are unrelated both by region and by product, which implies that demands' characteristics are independently distributed. We assume that θ_{ji} is uniformly distributed over the interval $[-\overline{\theta}_j, \overline{\theta}_j]$, where $\overline{\theta}_A$ and $\overline{\theta}_B$ are bounded and, without loss of generality, $\overline{\theta}_A \geq \overline{\theta}_B$.

We define a_{1A} as the type of product A the firm offers in region 1, where $a_{1A} \in \mathbb{R}$. We define a firm's product strategy for market A as a pair of actions (a_{1A}, a_{2A}).[3] For instance, given a strategy for product A, (a_{1A}, a_{2A}), and a taste for product A in region 1, θ_{1A}, profits derived from product A in region 1 are,

$$\Pi_{1A} = K - (a_{1A} - \theta_{1A})^2 - \beta(a_{1A} - a_{2A})^2.$$

The term K captures the maximum potential profits that the firm can obtain from product A.[4] The potential profits K and the actual profits Π_{1A} can differ for two reasons: 1) the firm does not achieve a good fit between product strategy and local demand characteristics in region 1, represented by the term $(a_{1A} - \theta_{1A})^2$; 2) the firm's strategy does not accomplish a good coordination in product A across regions, represented by $(a_{1A} - a_{2A})^2$. The more standard the product strategy, the lower the term $(a_{1A} - a_{2A})^2$, and the better the coordination across regions. Similarly, we define the profit for each regional division in each product as Π_{1B}, Π_{2A} and Π_{2B}.

The payoff of each local manager depends on the compensation scheme designed by the headquarters. For each product j, we denote by s_j the share of the division 2's profit in market of product j

[2] The parameter θ represents demand characteristics that can be used by the firm to increase its profits. For example, suppose a market with horizontal product differentiation and installed capacity, where preferences are single-peaked at θ, such that firm's profits increase as its product is closer to the bliss point θ.

[3] In this paper we use the concept of strategy to represent two different ideas. "Firm's strategy" is intended to represent the set of decisions that the firm as a whole takes and the objectives it pursues. "Players' strategies", in the sense of the game-theoretic literature, will refer to the mappings from histories into actions that every agent is entitled to choose.

[4] In an extension in Section 2.4.1, the term K is assumed to be an increasing function of the level of divisional integration. The level of integration represents the losses for not having a good coordination between product strategies. Some papers require that this level of divisional integration is a firm's choice variable and some papers do not. See Section 2.4 for a discussion.

that is awarded to division 1 and $(1-s_j)$ the share of division 1's profits in market of product j that remains in division 1. The value of s_j $(\in[0,0.5])$ defines how much aligned the incentives of local managers are in terms of product j. For example, in one extreme case when $s_j=0$, a local manager only cares about his own profits in market of product j; in the other extreme case when $s_j=0.5$, a local manager cares equally about his own profits and the other manager's profits in market of product j, i.e., the manager receives half of the total firm's profits. The payoff of the local manager of division 1 in market j is

$$U_{j1}=(1-s_j)\Pi_{j1}+s_j\Pi_{j2}-C(e,t), \quad j\in\{A,B\},$$

where $C(e,t)$ is the cost of acquiring information of precision e when the manager allocates the amount t of resources, e.g., managerial time.

The headquarters chooses the organizational design of the firm, which is defined as an allocation of the decision rights, g, and as a compensation scheme, s for each product market, to maximize the expected sum of division payoffs. Decision rights can be centralized by the headquarters or decentralized to the local managers, $g\in\{C;D\}$, where C stands for centralization and D for decentralization. Under centralization of decision rights, each manager sends reports about demands' characteristics to the headquarters before it makes a decision of the product strategy. Under decentralization, local managers may communicate between themselves before taking a decision about the product to be sold in their own regions. This means that under decentralization the firm's product strategy results from the addition of two separate decisions.[5]

The problem of the headquarters choosing the optimal organization design would be simple if there were no agency problems. But, as communication is soft and non-verifiable, local managers act strategically to exaggerate their local information in their own interest. Following the literature starting from Alonso et al (2008), we model informal communication as a one-round cheap talk model (Crawford and Sobel, 1982).[6]

Moreover, information about demand characteristics is not perfectly observed by local managers. Instead, local managers observe an imperfect signal of demand characteristics with the following technology: the realization of the signal $\hat{\theta}$ equals the true value θ with probability $\sqrt{e}$ and equals a random draw from the distribution of θ with probability $1-\sqrt{e}$. The quality of this signal reflects the effort and resources local managers apply to learning about demand characteristics, and it is observable but non-verifiable to the organizational participants.

[5] Our framework is symmetric and we follow the literature to concentrate on symmetric structures for each market j ($j\in\{A,B\}$) which implies centralizing or decentralizing decision making. Defining s_{ji} the share of product j in region i the symmetric structure implies $s_{j1}=s_{j2}=s_j$. Similarly if g_{ji} is the allocation of decision right of product j in region i the symmetric structure implies $g_{j1}=g_{j2}=g_j$. For these results in asymmetric structures see Alonso et al (2008) and Rantakari (2008).

[6] Also see Geanakoplos and Milgrom (1991) for a reflection over information transmission within organizations: "we assume that only the surface content of a message like "produce 100 widgets" can be grasped costlessly; the subtler content, which depends on drawing an inference from the message using knowledge of the sender's decision rule, can be inferred only at a cost."

The cost of acquiring a signal of quality $\sqrt{e}$ exerting a level e of effort and allocating t resources is $C(e,t) \equiv \mu(t)C(e)\sigma^2$. Each local division has a budget of 1 of resources, i.e., $0 \leq t \leq 1$ and $t_A + t_B \leq 1$ per local division, and effort is a free choice in $e \in [0,1]$. We assume $C(e) > 0$, $C'(e) > 0$, $C''(e) > 0$, $\mu(t) > 0$, $\mu'(t) < 0$, $\mu''(t) < 0$, and $\lim_{t\to 0}\mu(t) = +\infty$. We normalize the cost function to be proportional to product demand variability, σ^2, i.e., it is more difficult to find the information when it is disperse in a bigger interval. The function $C(e)$ provides the convexity of the cost of effort in learning about market characteristics. The function $\mu(t)$ scales the marginal cost of that effort. Exerting effort and allocating resources in collecting information in one market are complementary. The effort determines the precision of the signal, while resources assigned reduce the cost of this effort. For example, it is less costly for a manager to acquire an amount of information if this is exerted over a longer period of time.[7] We assume that $C(e) = -(e + \log(1-e))$ and $\mu(t) = \overline{\mu}\left(\frac{0.5}{t}\right)^b$ with $\overline{\mu} \in \mathbb{R}_+$ and $b \in [0,1]$.[8] We analyze two different cases concerning the allocation of resources. In our benchmark case, the headquarters controls how resources are allocated. In the second case, local managers are in charge of resource allocation.[9]

Finally, the timing of the model for the benchmark case where the headquarters controls resource allocation is as follows (Figure 2.1): first, the headquarters chooses the firm's structure (decision making and compensation scheme) for each product and an allocation of resources in each division; second, local managers simultaneously and independently choose how much effort, e, to devote to collect local information about each product taste; third, signals $\hat{\theta}_{ij}$ with precisions e_{ij} about the true values of θ_{ij} are observed; fourth, strategic communication takes place; fifth, products a_{ij} are chosen and, finally, payoffs are delivered. Figure 2.2.a shows a mix organizational design where the firm decentralizes the decision right of product A and centralizes the decision rights of product B. Manager 1 chooses a_{1A} and manager 2 chooses a_{2A} after they communicate with each other. The headquarters chooses a_{1B} and a_{2B} after communicating with country managers. Figure 2.2.b shows an organizational design where the firm decentralizes the decision rights of both products.

For the case where managers can decide resource allocation (Figure 2.1), the timing changes in that resource allocation is made by local managers simultaneously with their choice of effort.

We focus on Pareto Efficient and Perfect Bayesian Nash equilibria. We derive the equilibria that

[7] Resources and effort in one activity are complementary. Then, the efforts exerted to collect information about different products are substitutes. If managers allocate resources, our model can be interpreted as the multi-tasking model of Holmstron and Milgrom (1991) for substitute effort in the organizational design environment.

[8] These expressions capture all the general properties and contribute with simplicity in solving the model.

[9] We motivate the analysis of these two cases as follows: the headquarters may have calculated and allocated which is the optimal amount of resources in each local division for operational functioning (motivating our benchmark case). However, it could be argued that within each division, local managers administer how to distribute these resources for learning about market characteristics (motivating our second case). See Geanakoplos and Milgrom (1991): "To take advantage of the information processing potential of a group of managers, it is necessary to have the managers attend to different things. But these differences are themselves the major cause of failure of coordination among the several managers." In their model, "a chief executive allocates production targets, capital and other resources to division managers who in turn reallocate the budgeted items to their subordinates, etc. until the resources and targets reach the shops where production takes place." Nevertheless, delegating resource allocation may be based upon positive externalities in market learning. We adapt our model to this case in an extension.

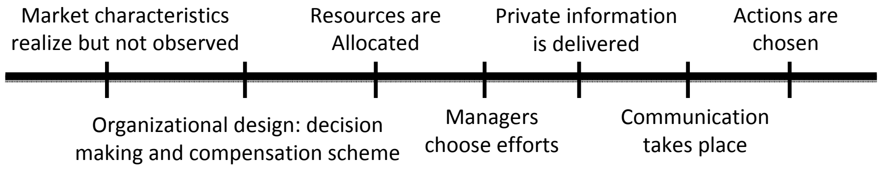
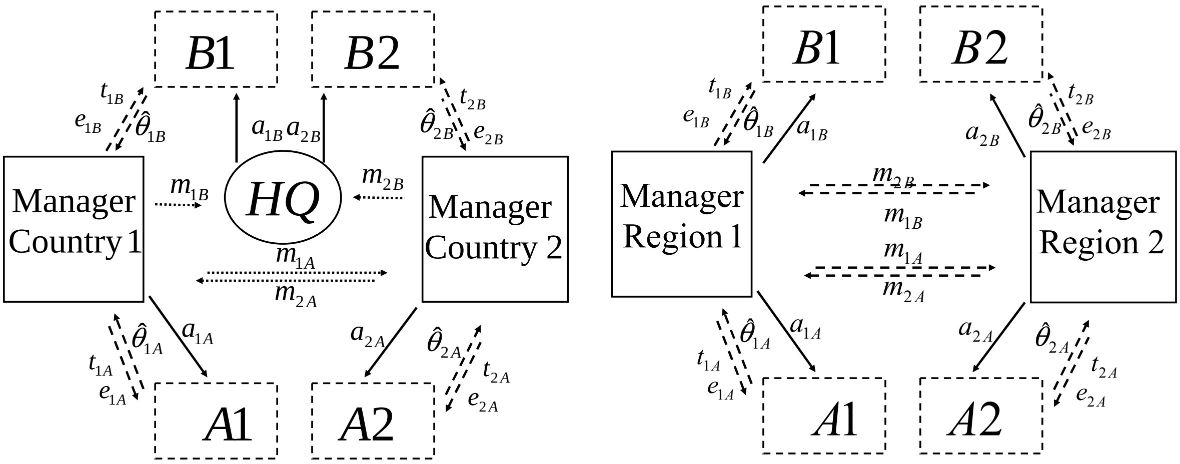


Figure 2.1: Timing.



(a) Mixed centralization and decentralization (b) Decentralization in both products' markets

Figure 2.2: Organizational Structure depending on decision making.

maximize total expected profits by backward induction. Given the signals and the communication outcome, we find the best response functions. Then, anticipating these best response functions, local managers engage in optimal strategic communication. Anticipating the optimal strategic communication and actions, each local manager allocates resources and exerts effort to collect accurate local information. Finally, given optimal behavior, the headquarters chooses the optimal organizational design of the firm.

2.3 Equilibrium

To find the optimal structure we calculate the incentive scheme that maximizes total expected profits of the organization for the four possible allocation of decision rights, taking as given the best response of local managers in collecting, transmitting, and using information. Then, we compare which structure provides higher expected profits.

First we analyze how information is transmitted and used (Section 2.3.1). Second, we study how local managers acquire local information (Section 2.3.2). Third, we describe the resource allocation problem (Section 2.3.3). Finally, we characterize the organization structure that maximizes total profits (Section 2.3.4).

We focus on one product in sections 2.3.1 and 2.3.2, omitting product subindex j until section 2.3.3. We can omit the subindex because product A and B have unrelated demands and profit functions.

They are related only because they share a common input, i.e., resource allocation.
With respect to the equilibrium concept, we focus on Pareto Efficient and Perfect Bayesian Nash equilibrium. In some stages, there are multiple equilibria that can be ranked from a pareto optimality perspective; therefore we concentrate on those equilibria that leave the agents with the maximum expected payoffs.[10] We assume that agents can coordinate over those equilibria when it is mutually beneficial.

2.3.1 Actions and Communication: Transmitting and Using Information

We proceed by backward induction and we analyze the communication and decision making stages for a given organizational design (allocation of decisions rights and a compensation scheme), and the amount of information acquired by local managers. Each local manager i has private information about market characteristic in his own region, θ_i. We solve how information is transmitted by managers and used by decision makers for one product. The solution is similar for both products, and follows from Alonso, Dessein and Matouscheck (2008).
Under centralization, the headquarters communicates with each local manager and forms beliefs about local demand characteristics, i.e., $E[\theta_1]$ and $E[\theta_2]$, and chooses the firm's product strategy solving

$$\max_{a_1,a_2} E[U_1+U_2] = E[\pi_1+\pi_2] = E[K(\beta)-(a_1-\theta_1)^2-(a_2-\theta_2)^2-2\beta(a_1-a_2)^2].$$

Under decentralization, local managers communicate between them, form beliefs about the other manager's action, i.e., manager 1 forms beliefs $E[a_2]$, and chooses his action solving

$$\max_{a_1} E[U_1] = E[(1-s)\pi_1+s\pi_2] = E[K(\beta)-(1-s)(a_1-\theta_1)^2-s(a_2-\theta_2)^2-\beta(a_1-a_2)^2].$$

The following proposition characterizes the optimal actions under centralization and decentralization.

Proposition 1. *1.a Conditioned on beliefs, the optimal actions under centralization are*

$$\begin{aligned} a_1^C(m_1,m_2) &= \frac{1+2\beta}{1+4\beta}E[\theta_1|m_1]+\frac{2\beta}{1+4\beta}E[\theta_2|m_2], \text{ and} \\ a_2^C(m_1,m_2) &= \frac{2\beta}{1+4\beta}E[\theta_1|m_1]+\frac{1+2\beta}{1+4\beta}E[\theta_2|m_2]. \end{aligned}$$

1.b Conditioned on beliefs defined by $E_2[\theta_1]\equiv E_2[\theta_1|m_1]$ and $E_1[\theta_2]\equiv E_1[\theta_2|m_2]$, the optimal

[10] This criteria for selecting equilibria satisfies also the NITS condition of **?**.

actions under decentralization are

$$\begin{aligned} a_1^D(m_1,m_2,\theta_1) &= \frac{(1-s)}{(1-s+\beta)}\theta_1 + \frac{\beta}{(1-s+\beta)}\left[\frac{\beta}{1-s+2\beta}E_2[\theta_1] + \frac{1-s+\beta}{1-s+2\beta}E_1[\theta_2]\right], \text{ and} \\ a_2^D(m_1,m_2,\theta_2) &= \frac{(1-s)}{(1-s+\beta)}\theta_2 + \frac{\beta}{(1-s+\beta)}\left[\frac{\beta}{1-s+2\beta}E_1[\theta_2] + \frac{1-s+\beta}{1-s+2\beta}E_2[\theta_1]\right]. \end{aligned}$$

Proposition 1 is based on Alonso, Dessein, and Matouschek (2008) and Rantakari (forthcoming). Optimal decision making reveals that local managers cannot truthfully transmit the information acquired. Let $\overline{m}_i \equiv E_j[\theta_i|m_i]$ be the receiver j's expectation of θ_i after receiving the message m_i.[11] Each local manager has incentives to lie in order to improve the profits of his own division. The intrinsic incentives to lie under decentralization are $\overline{m}_1 - \theta_1 = \frac{(1-2s)(1-s+\beta)}{s(1-s)+\beta}\theta_1 \equiv \omega_D\theta_1$ and under centralization are $\overline{m}_1 - \theta_1 = \frac{(1-2s)\beta}{1-s+\beta}\theta_1 \equiv \omega_C\theta_1$. Given these incentives to lie, the only incentive compatible communication for the sender is, as described in Crawford and Sobel (1982), a partition of the state space. Recall that we have assumed that demand's characteristic of product j is uniformly distributed in $[-\overline{\theta}_j, \overline{\theta}_j]$. Then, we characterize the truthfully revealing partitions and communication equilibria.

Proposition 2. *Fix two positive integers N_1 and N_2, there exists at least one Perfect Bayesian Nash equilibrium characterized by the functions $(\upsilon_1(m_1|\theta_1), \upsilon_2(m_2|\theta_2), a_1(m_2,m_1,\theta_1), a_2(m_1,m_2,\theta_2), g_1(\theta_2|m_2), g_2(\theta_1|m_1))$. The communication rule $\upsilon_i(m_i|\theta_i)$, decision rule $a_i(m_i,m_{-i},\theta_i)$, and beliefs $g_i(\theta_{-i}|m_{-i})$ satisfy:*

- 2 .a *$\upsilon_i(m_i|\theta_i)$ is uniform, with support on $[d_{i,h-1}, d_{i,h}]$ if $\theta_i \in [d_{i,h-1}, d_{i,h}]$.*
- 2 .b *$g_i(\theta_{-i}|m_{-i})$ is uniform, with support on $[d_{i,h-1}, d_{i,h}]$ if $m_{-i} \in [d_{i,h-1}, d_{i,h}]$.*
- 2 .c *The boundaries are defined by: i) $d_{i,h+1} - d_{i,h} = d_{i,h} - d_{i,h-1} + 4\frac{(1-2s)(1-s+\beta)}{s(1-s)+\beta}d_{i,h}$ for $h = 1,\ldots,N_i - 1$ under decentralization; and ii) $d_{i,h+1} - d_{i,h} = d_{i,h} - d_{i,h-1} + 4\frac{(1-2s)\beta}{1-s+\beta}d_{i,h}$ for $h = 1,\ldots,N_i - 1$ under centralization.*
- 2 .d *The decision of each worker is defined by part 1.b of Proposition 1 under decentralization and by part 1.a of Proposition 1 under centralization.*

Taking the boundaries $d_0 = -\overline{\theta}$ and $d_N = \overline{\theta}$ of the space, the solution is defined by

$$d_h = \overline{\theta}\frac{x^h(1+y^N) - y^h(1+x^N)}{x^N - y^N} \quad 0 \le h \le N,$$

with $x = (1+2\omega) + \sqrt{(1+2\omega)^2 - 1}$ and $y = (1+2\omega) - \sqrt{(1+2\omega)^2 - 1}$, $\omega = \omega_D \equiv \frac{(1-2s)(1-s+\beta)}{s(1-s)+\beta}$ under decentralization, and $\omega = \omega_C \equiv \frac{(1-2s)\beta}{1-s+\beta}$ under centralization. Note that $xy = 1$, $x > 1$ and $y < 1$.

[11] After sending a message m_1, the receiver forms a posterior of $\overline{m}_1$ about θ_1. Communication is not perfect when the posterior $\overline{m}_1$ and the real value θ_1 differ.

Proposition 2 is based on Alonso, Dessein, and Matouschek (2008), and Rantakari (forthcoming). Proposition 2 describes the communication equilibria with N-partition of the space $[-\overline{\theta}_j, \overline{\theta}_j]$. After communication takes place, the successful rate of communication is $E[\overline{m}_i^2] = V_i \sigma_j^2$, where V_i represents the proportion of the local information variance ($\sigma_j^2 \equiv \frac{\overline{\theta}_j^2}{3}$) that is communicated. The rate V_i is increasing in the number of partitions N. The equilibria with $+\infty$-partition are the ones that achieve the maximum expected payoffs for both managers. Onwards, we concentrate on these $+\infty$-partitions equilibria, which deliver the rate of transmission equal to $V_j = \frac{3+3\omega}{3+4\omega}$, with $\omega \in \{\omega_C, \omega_D\}$.[12]

After information is acquired there are two costs in the communication and action stages. One cost is related with how well information is used, and is characterized by Λ in equation (2.1). Under centralization, the headquarters achieves the minimum cost, given that it maximizes total expected profits with the information available. Under decentralization, however, each local manager has a bias to the profit of his own division and does not internalize the externality that the decision about product strategy has on the other division.

The second cost is associated with information transmission and is characterized by $\Gamma(1-V)$ in equation (2.1). The factor V represents how well information is communicated between the ones who have the information and the ones who make decisions, i.e., how accurate communication is. The factor Γ represents how important this communication is for the expected profits, i.e., the value of communication accuracy. Under centralization there is more accurate communication than under decentralization because under the former the conflict between each manager and headquarters is lower, i.e., there is less incentive to exaggerate the private information. However, the value of that communication is also higher under centralization. The information that is not communicated under centralization is lost, in the sense that nobody can use that information for making decisions. Under decentralization, however, local managers use, at least, their own information for making their own decisions.

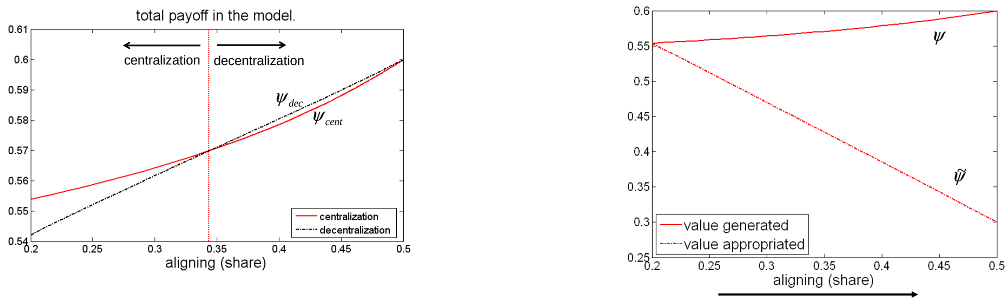
We identify these costs for each local manager under both centralization and decentralization. The expected profits of each division in each product are described in the following lemma. To avoid awkward notation we focus on local manager 1.

Lemma 1. *Under both structures the expected profit function for each local manager is characterized by*

$$E[\Pi_1] = K - \left[\underbrace{\Lambda_1 + \Gamma_{11}(1-V_1)}_{\textit{due to } 1 \textit{ information}} + \underbrace{\Lambda_1 + \Gamma_{21}(1-V_2)}_{\textit{due to } 2 \textit{ information}} \right] \sigma_\theta^2. \tag{2.1}$$

with the following expressions for centralization $V_C = \frac{3(1-s)(1+2\beta)}{(1-2s)\beta+3(1-s)(1+2\beta)}$, $\Lambda_{1C} = \frac{\beta}{1+4\beta}$, $\Gamma_{11C} = 1-\Lambda_{1C}$, *and* $\Gamma_{12C} = -\Lambda_{1C}$. *For decentralization the values are* $V_D = \frac{3(1-s)(1-s+2\beta)}{(1-2s)(1-s+\beta)+3(1-s)(1-s+2\beta)}$,

[12] These equilibria satisfy the Non Incentives To Separate (NITS) criteria of **?** when the number of partitions is infinite. For finite partitions, it satisfies NITS if the number of partitions is odd.



(a) Centralization versus decentralization (b) Value generated versus appropriated

Figure 2.3: Comparing payoffs under centralization and decentralization (a) and comparing the value generated and the value appropriated (b) by each manager .

$\Lambda_{1D} = \frac{\beta[(1-s)^2+\beta]}{(1-s+2\beta)^2}$, $\Gamma_{11D} = \frac{\beta[(1-s)^2+\beta]}{(1-s+\beta)^2} - \Lambda_{1D}$, *and* $\Gamma_{12D} = \beta\frac{(1-s)^2}{(1-s+\beta)^2} - \Lambda_{1D}$.

Lemma 1 describes the situation under perfect information (the algebra for constructing these expressions are in Appendix 2.7). In brace we identify the value generated in region 1 by the information of manager 1 and manager 2.

Summarizing the results of this section, we have characterized the value of information under both centralization and decentralization. After information is acquired, it is transmitted and used through local managers to decision makers. Under both centralization and decentralization, the value of information increases in incentive alignment, s. Misaligning incentives (reducing s) reduces the value of information under both structures. This effect is higher under decentralization if $s \leq \underline{s}$. Hence, the value of information is higher under centralization if incentives are sufficiently misaligned, i.e., s low.[13] In Figure 2.3.a, we show the relation of the value generated in the communication and action stages under centralization (continuous line) and decentralization (dashed line) as a function of the compensation scheme s.

2.3.2 Acquiring Information

Let us assume now that information is imperfect and costly. Each manager invests an effort e in acquiring an imperfect signal $\hat{\theta}$ of the true value θ. The realization of the signal $\hat{\theta}$ equals the true value θ with probability $\sqrt{e}$ and a random draw from the same distribution of θ with probability $1-\sqrt{e}$. The higher the effort, the more accurate the signal. Following Rantakari (2010) we use e as a measure of the *quality of primary information* which has a cost $\mu(t)C(e)\sigma^2$ with $e \in [0,1]$ and $C(e) = -(e+\log(1-e))$.

[13]For extension 2.4.1 it is worth noting that, under both centralization and decentralization, the value of information is decreasing in local divisions' integration, β. The cutoff $\underline{s}(\beta)$ is increasing in β. Increasing local divisions' integration (increasing β) reduces the value of information under both structures but more under decentralization.

We first describe the objective functions with imperfect signals, and then we point out the private incentives of managers and the headquarters to acquire information. Finally, we characterize how information is acquired.

Acquiring information of quality e_i by local manager i, with $i \in \{1,2\}$, with the corresponding cost of acquiring that information means that now the expected profits of division i in equation (2.1) becomes, (see Appendix 2.7)

$$E[\Pi_i] = K - \left[1 - e_i \underbrace{[1 - \Lambda_i - \Gamma_{ii}(1-V_i)]}_{\text{due to } i \text{ information}} + e_k \underbrace{[\Lambda_i + \Gamma_{ki}(1-V_k)]}_{\text{due to } k \text{ information}} + \mu C(e_i) \right] \sigma_\theta^2, i = \{1,2\} \text{ and } k \neq i. \quad (2.2)$$

Since local managers do not internalize the externality that their own information generates on the other manager, we distinguish between the profit captured by each local manager, $E[\Pi_i]$, and the profit generated by each local manager, $E[\pi_i]$. Despite the fact that these values are not the same $\pi_i \lesseqgtr \Pi_i$, the aggregate profit captured equals the aggregate profit generated, i.e., $\sum_{i=1,2} \pi_i = \sum_{i=1,2} \Pi_i$. The profit captured by a local manager is represented in equation (2.2); however, the profit generated by a local manager is

$$E[\pi_i] = K - \left[1 - e_i \underbrace{[1 - (\Lambda_i + \Lambda_k) - (\Gamma_{ii} + \Gamma_{ik})(1-V_i)]}_{=\psi_{ii} \text{ due to information of manager } i} + \mu C(e_i) \right] \sigma_\theta^2. \quad (2.3)$$

where ψ_{ii} represents the value generated by manager i with a signal of precision e_i. A local manager acquires information considering the effect that his own information has on his own payoff and not on the value that the information generates. For this reason there is an inefficiency in information acquisition. Given g and s the expected utility of each manager over a product is $E[U_i] = (1-s)E[\Pi_i] + sE[\Pi_k]$ or,

$$E[U_i] \quad = \quad K + \left[-1 + e_i \tilde{\psi}_{ii} + e_k \tilde{\psi}_{ki} - \mu C(e_i) \right] \sigma_\theta^2, \quad (2.4)$$

where $\tilde{\psi}_{ii}$ represents the value appropriated by manager i with a signal of precision e_i, $\tilde{\psi}_{ki}$ is the externality to manager i generated by manager k who has acquired a signal of precision e_k. The expressions are defined by $\tilde{\psi}_{ii} \equiv (1-s)[1 - \Lambda_i - \Gamma_{ii}(1-V_i)] - s[\Lambda_k + \Gamma_{ik}(1-V_i)]$, $\tilde{\psi}_{ki} \equiv s[1 - \Lambda_k - \Gamma_{kk}(1-V_k)] - (1-s)[\Lambda_i + \Gamma_{ki}(1-V_k)]$, $\tilde{\psi}_{kk} \equiv (1-s)[1 - \Lambda_k - \Gamma_{kk}(1-V_k)] - s[\Lambda_i + \Gamma_{ki}(1-V_k)]$ and $\tilde{\psi}_{ik} \equiv s[1 - \Lambda_i - \Gamma_{ii}(1-V_i)] - (1-s)[\Lambda_k + \Gamma_{ik}(1-V_i)]$. It is important to notice (Figure 2.3.b) that the value of the information captured by a local manager, $\tilde{\psi}(s,g)$, is decreasing in s under both centralization and decentralization. However, the value of information generated $\psi(s,g)$ is increasing in s under both centralization and decentralization. Because $\psi_i \neq \tilde{\psi}_{ii}$, the information acquired is not optimal. The following lemma describes the effort choice

Lemma 2. *Under both decentralization and centralization the effort choice is characterized as* *follows:*

2.a *The optimal level of effort is given by* $\psi = \mu C'(e^*)$.

2.b *Each local manager chooses effort according to* $\tilde{\psi} = \mu C'(\hat{e})$.

The comparative static implies that $\frac{\partial e}{\partial \mu} = -\frac{C'(e)}{\mu C''(e)} < 0$, $\frac{\partial e^*}{\partial s} > 0$ *and* $\frac{\partial \hat{e}}{\partial s} < 0$ *under both centralization and decentralization, since* $\frac{\partial \psi}{\partial s} > 0$ *and* $\frac{\partial \tilde{\psi}}{\partial s} < 0$. *For the function* $C(e) = -(e + \log(1-e))$, $\hat{e} = \frac{\tilde{\psi}}{\mu + \tilde{\psi}}$ *and* $e^* = \frac{\psi}{\mu + \psi}$.

The proof of Lemma 2 consists in solving the first order condition of the objective function respect to e. The objective function is defined by equation 2.3 in part 2.a of Lemma 2 and by equation 2.4 in part 2.b of Lemma 2. The information acquired by a local manager $\hat{e}$ is decreasing in his incentive alignment s because it depends on the perceived value of that information, $\tilde{\psi}$, while the optimal amount of information is increasing in incentive alignment, s, because it depends on the real value of information, ψ.

So far, we have described the incentives to exert effort for acquiring information, e, and how much value that information generates, $\psi(s)$. Under both structures, centralization and decentralization, local managers face similar trade-offs. A manager effort increases but the value generated decreases when local managers incentives are narrowed to their own division's profit. Under decentralization, however, the perceived value of information tends to be greater than under centralization.[14]

In this paper efforts are neither strategic complement nor strategic substitutes, since strategic effects cancel out due to the following assumptions: 1) independence of θ_{j1} and θ_{j2} (for $j \in \{A, B\}$); 2) messages $\overline{m}_1$ and $\overline{m}_2$ are unrelated in the communication game; and 3) the functional form of the profit function.[15] Relaxing these assumptions to capture the strategic interaction of the efforts appears a promising avenue for future research.

2.3.3 Resource Allocation

Again subindex $j \in \{A, B\}$ stands for product A and product B respectively. Local managers have resources equal to 1 which are allocated to acquire information about different demands' tastes. The allocation of these resources determines the marginal cost of information, i.e., $\mu_A(t_A)$ and $\mu_B(t_B)$, where $t_A + t_B \leq 1$. The objective function of the headquarters is $\sum_{i=1,2} E[\pi_{iA}] + E[\pi_{iB}]$,

$$\sum_{i=1,2} \left\{ K - [1 - e_{iA}\psi_A + \mu_A C(e_{iA})]\, \sigma_A^2 \right\} + \left\{ K - [1 - e_{iB}\psi_B + \mu_B C(e_{iB})]\, \sigma_B^2 \right\}. \tag{2.5}$$

[14] $\tilde{\psi}$ is greater under decentralization except when β is excessively high and s excessively low. The advantage of decentralization is that a local manager uses his own information to adapt his product to his local market. Managers value more the information under decentralization except in some extreme circumstances where standardization is very important and compensation schemes are narrowed to local divisions' profits.

[15] Appendix 2.7 shows the form of the expected profit function under both structures.

Note that $E[\pi_{iA}]+E[\pi_{iB}]$ is the value generated by local manager i within the firm. The objective function of a local manager in division i is $E[U_{iA}]+E[U_{iB}]$

$$\{K-\sigma_A^2[1-e_{iA}\tilde{\psi}_{iiA}-e_{kA}\tilde{\psi}_{kiA}+\mu_A C(e_{iA})]\}+\{K-\sigma_B^2[1-e_{iB}\tilde{\psi}_{iiB}-e_{kB}\tilde{\psi}_{kiB}+\mu_B C(e_{iB})]\}. \quad (2.6)$$

We analyze two situations. As a benchmark case we analyze the optimal resource allocation, i.e., how the headquarters allocates resources within each division. The headquarters chooses t_{Ai} for local manager i maximizing the expected profit in equation (2.34) subject to optimal choice of effort described in part 2.b of Lemma 2. In the second case, local managers allocate resources. Each manager chooses t_A and t_B maximizing his expected payoff defined in equation (2.38). In the following lemma we summarize how resource allocation is chosen.

Lemma 3. *The resource allocation t_A within each division is determined as follows:*

3.a The headquarters chooses t_A according to

$$-\frac{\partial \mu_A}{\partial t_A}\sigma_A^2\left[C(e_A)-\frac{\partial e_A}{\partial \mu_A}[\psi_A-\mu_A C'(e_A)]\right]=-\frac{\partial \mu_B}{\partial t_B}\sigma_B^2\left[C(e_B)-\frac{\partial e_B}{\partial \mu_B}[\psi_B-\mu_B C'(e_B)]\right]. \quad (2.7)$$

Replacing $\tilde{\psi}=\mu C'(e)$ we have $[\psi-\tilde{\psi}]$ as a measure of the moral hazard problem in each product market.

3.b A local manager chooses t_A according to[16]

$$-\frac{\partial \mu_A}{\partial t_A}C(e_{iA})\sigma_A^2 \quad = \quad -\frac{\partial \mu_B}{\partial t_B}C(e_{iB})\sigma_B^2. \quad (2.8)$$

We assume that $\mu(t)$ is sufficiently convex for an interior solution to exist.[17] A comparative static shows that more resources are allocated to learn about a product when more information is acquired, i.e., higher e, and when the product has higher returns to differentiation, i.e., higher σ^2. In effect, information acquisition and resource allocation are complementary. There are incentives to allocate more resources in markets where managers acquire more information, e.g., the higher e_A the higher t_A. Also more information is acquired in markets that receive more resources, the higher t_A the higher e_A.

As noted in the Section 2.3.2, the effort in information acquisition of local managers is not efficient, and thus their allocation of resources will also be distorted. Indeed, if the information is acquired efficiently, the term $\frac{\partial e}{\partial \mu}[\psi-\mu C'(e)]$ disappears due to part 2.a of Lemma 2 and the resource allocation is always characterized by part 2.b of Lemma 3. The headquarters corrects this inefficiency in information acquisition through resource allocation as described in part 3.a of Lemma 3, or through organizational design, as is described below in the following section.

[16] We apply envelope theorem to get this condition.

[17] The convexity of resource allocation outweighs the effort convexity problem and incentive alignment convexity problem.

Whenever local managers allocate resources, they choose t_A considering the opportunity cost of those resources on their own expected utility. This is consistent with Geanakoplos and Milgrom (1991) who conclude "that managers at each level optimally focus attention only on those variables that determine the marginal productivity of resources and the marginal costs of production in the units under their command". As described above, the choice of t_A is only affected by the returns to differentiation σ^2 and the function $C(e)$. Hence, to reallocate resources in favor of market A, the headquarters not only can promote a more considerable effort in market A, increasing $C(e_A)$, but also can discourage effort in market B, reducing $C(e_B)$.

If $\sigma_A^2 = \sigma_B^2$, there is a symmetric allocation of resources and identical effort. Note from Lemma 3.b that the allocation of resources is symmetric only if $e_A = e_B$, and from Lemma 2.a we can have identical effort if resource allocation is symmetric.

The function $C(e) = -(e + \log(1-e))$ has a slope $C'(e) = \frac{e}{1-e}$. For the function $\mu = \overline{\mu}\left(\frac{0.5}{t}\right)^b$, with $b \in (0,1)$ and $\overline{\mu} \in \mathbb{R}_{++}$, the resource allocation choice is $t_A = \frac{1}{1+H^{\frac{1}{1+b}}}$ with[18]

$$H_1 \equiv \frac{C(e_{iB})}{C(e_{iA})}\frac{\sigma_B^2}{\sigma_A^2} \quad \text{and} \quad H_0 \equiv \frac{C(e_{iB}) - \frac{\partial e_B}{\partial \mu_B}[\psi_B - \mu_B C'(e_B)]}{C(e_{iA}) - \frac{\partial e_A}{\partial \mu_A}[\psi_A - \mu_A C'(e_A)]}\frac{\sigma_B^2}{\sigma_A^2}.$$

The term is H_1 when resource allocation is decided by each local manager and H_0 when resource allocation is chosen by the headquarters. The cost of acquiring information is $\mu_A(t_A) = \overline{\mu}0.5^b\left(1 + H^{\frac{1}{1+b}}\right)^b$. The following Lemma is crucial for our results.

Lemma 4. *Define t^* as the value that maximizes $[-\mu(t) - \mu(1-t)H]$. Assume $\mu(t,b) = \overline{\mu}\left(\frac{0.5}{t}\right)^b$ and $0 < H \leq 1$. Given $b_0 \in [0,1]$, the set of all functions with $b_1 \in [b_0, 1]$ satisfies the property that $\mu(t_0^*, b_0) \geq \mu(t_1^*, b_1)$.*

The proof of the Lemma 4 is in Section 2.6.1. This Lemma describes that if resource allocation is more important, then the marginal cost of acquiring information is lower in the market with higher variance. In other words, a higher b implies lower μ_A and higher μ_B if $\sigma_A^2 > \sigma_B^2$. Consequently, b represents the importance of resource allocation.

2.3.4 Optimal Structure

To find the optimal structure of the firm, we must evaluate the best response of local managers for each possible structure and compare the total expected payoff obtained under each possible combination of decision rights, i.e., centralization and decentralization in each product market. The problem is as follows

$$\max_{g,s} \sum_{i=1,2} \left\{K - [1 - e_{iA}\psi_A + \mu_A C(e_{iA})]\,\sigma_A^2\right\} + \left\{K - [1 - e_{iB}\psi_B + \mu_B C(e_{iB})]\,\sigma_B^2\right\}, \tag{2.9}$$

[18] With the properties $\mu'(t) = -b\overline{\mu}\left(\frac{0.5^b}{t^{1+b}}\right) < 0$, $\mu''(t) = b(1+b)\overline{\mu}\left(\frac{0.5^b}{t^{2+b}}\right) > 0$, $\frac{\partial \mu'(t)}{\partial b} < 0$, and $\frac{\partial \mu''(t)}{\partial b} > 0$.

subject to the optimal effort choice described in part 2.b of Lemma 2 and resource allocation choice described in part 3.a of Lemma 3, in the benchmark case where the headquarters chooses t, or part 3.b in Lemma 3, when local managers choose t. For simplicity, we assume $\sigma_B^2 \equiv 1$, such that the ratio of market variances is equal to the variance of product A, i.e., $\frac{\sigma_A^2}{\sigma_B^2} \equiv \sigma_A^2$, with $\sigma_A^2 \geq 1$ since $\overline{\theta}_A \geq \overline{\theta}_B$.[19] The first order conditions for the propositions of this section are in Appendix 2.7.
We first analyze the situation when resource allocation is not important, i.e., $b \to 0$, as an starting point to understand our benchmark case, where the headquarters allocates resources, and our first extension, where managers do. For this case, the optimal design for market A is independent of the optimal design for market B. In the following proposition we summarize the result.

Proposition 3. *Assume $\mu = \overline{\mu}\left(\frac{0.5}{t}\right)^b$ with $b \to 0$. There exists a threshold $\tilde{\mu}$ above which centralization outperforms decentralization and the choice of decision rights is independent of market of product A and market of product B.*

In Appendix 2.6.2 we develop a sketch of the formal proof in Rantakari (2010). We provide here some useful intuition for our next results. As explained in Section 2.3.1 and represented in Figure 2.3.a, centralization performs better than decentralization when the incentives of each local manager and the headquarters are sufficiently misaligned, i.e., s is low. The main trade off in designing the compensation scheme, i.e., in choosing s, is clearly observed in the first order condition respect to s,

$$\frac{\partial e}{\partial s}[\psi - \tilde{\psi}] + e\frac{\partial \psi}{\partial s} \quad = \quad 0. \tag{2.10}$$

The first term in the left hand side is negative, since $\frac{\partial e}{\partial s}$ is negative, and the second term is positive. When the manager's payoff depends more on total firm's profit, i.e., higher s, there is an increase in the value of the information acquired, $e\frac{\partial \psi}{\partial s}$, but also an increment in the value of acquiring further information $\frac{\partial e}{\partial s}[\psi - \tilde{\psi}]$, because ψ increases in s while $\tilde{\psi}$ decreases in s. When the cost of information is low, there is a lot of information acquisition, and the headquarters aligns managers' incentives with the firm's profits to increase the value of that information. In this case, decentralization performs better than centralization. When the cost of information is high, the headquarters prioritizes information acquisition, narrowing local managers incentives to their own division profit, and, eventually, the firm performs better under centralization.
The structure of the firm balances the moral hazard problem of suboptimal information acquisition with the suboptimal value generated by this information in the decision making process. When informational cost is low, the headquarters follows a strategy of product differentiation through decentralization, but when informational cost is high, it follows a strategy of product standardization through centralization.

[19] The absolute value of market variances matters if we endogenize β. For robustness, we describe the results in the case that the headquarters also chooses β in Section 2.4.1.

Although it is not directly stated, we can infer by Proposition 3 that when centralization performs as well as decentralization, the compensation scheme under both structures differs, i.e., $s^C < s^D$. Assume that the informational cost is just the threshold $\tilde{\mu}$, and the headquarters is indifferent to centralizing or decentralizing decision making. This indifference between centralization and decentralization requires that $s \leq \underline{s}$.[20] Figure 2.3.a (Section 2.3.1) shows that when $s \leq \underline{s}$ the value of information is more sensitive to s under decentralization than under centralization. Hence, at the same informational cost, the optimal profit sharing s under centralization must be lower than the profit sharing under decentralization, i.e., $s^C < s^D$. This result arises because, once the headquarters provides incentives for information acquisition, centralization handles better, at the margin, the trade-off between acquiring and transmitting information. Then, it can foster more information acquisition through a lower s under centralization.

Summing up the result of Proposition 3, when $b \to 0$, resource allocation is not important, and the marginal cost of information is given by $\overline{\mu}$ in each market. The headquarters decentralizes decision making in both markets if $\overline{\mu} \leq \tilde{\mu}$, and centralizes them otherwise. Let us now see how the optimal structure changes as resource allocation becomes important.

Proposition 4. *Assume $\mu = \overline{\mu}\left(\frac{0.5}{t}\right)^b$ with $b \in (0,1)$ and $\overline{\mu} \in \mathbb{R}_{++}$. If resources are allocated efficiently, that is by the Headquarters, there exists a threshold in the ratio of returns to product differentiation $\tilde{\sigma}_{AB}$ above which the optimal design requires a mix structure with centralization in market of product B and decentralization in market of product A. Moreover, $\frac{\partial \tilde{\sigma}_{AB}}{\partial b} < 0$.*

Proof. The headquarters chooses t_A and t_B according to condition (2.7) in Lemma 3. We guarantee an interior solution in resource allocation when the function $\mu(t)$ is sufficiently convex. The allocation of t_A is increasing in the ratio σ_A^2, implying that $\mu_B(1-t_A)$ is increasing in σ_A^2, and $\mu_A(t_A)$ is decreasing in σ_A^2. The resource allocation, that is t_A, defines the costs' values μ_A and μ_B, and Proposition 3 applies in each market: if $\mu_j > \tilde{\mu}$, centralization outperforms decentralization, but if $\mu_j < \tilde{\mu}$, decentralization outperforms centralization in market of product j. The proof of $\frac{\partial \tilde{\sigma}_{AB}}{\partial b} < 0$ is in appendix 4. □

Proposition 4 has several implications for the optimal internal design of the firm and, consequently, the optimal firm's product strategy. First, the headquarters recognizes that the value of information is higher in markets with higher returns to differentiation than in markets with lower returns to differentiation. The reason is straightforward: the expected losses of a wrong product strategy are higher when consumers' tastes are more uncertain. Hence, the optimal resource allocation concentrates more resources in markets with higher returns to differentiation. This is shown in Lemma 3.a: the higher the ratio σ_A^2, the more resources the firm allocates to learn about product A and the fewer resources the firm allocates to learn about product B demand characteristics. Thus,

[20]Since for $s > \underline{s}$, not only decentralization generates more value for a given effort e, i.e., $\psi^D > \psi^C$, but also local managers exert a higher effort, i.e., $e^D > e^C$ since $\tilde{\psi}^D > \tilde{\psi}^C$.

there is higher informational cost and less information acquisition in market of product B, and lower informational cost and more information acquisition in market of product A.

Second, the headquarters recognizes that the information acquired by local managers is suboptimal in both markets. The suboptimal level of effort, however, is more important in a market with higher returns to differentiation. Through resource allocation, the headquarters reduces the effort cost and encourages more learning in the market with higher returns to differentiation.

As the ratio of the returns to differentiation between markets increases, more resources are allocated to market A and fewer to market B. The headquarters will, eventually, find it optimal to follow a strategy of product differentiation in market A and a strategy of product standardization in market B. To implement these strategies, the headquarters decentralizes decision making in market A, providing a compensation scheme that aligns local managers' incentives with the firm's profits. In market B, however, since the aim is to pursue standardization, it is better to centralize decision making with a compensation scheme that narrows local managers incentives to their own divisions' profit.

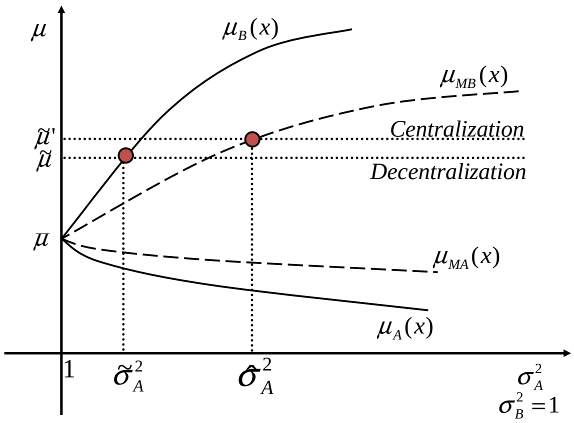


Figure 2.4: Optimal decision making as a function of the ratio in returns (we assume $\sigma_B^2 = 1$) and informational cost.

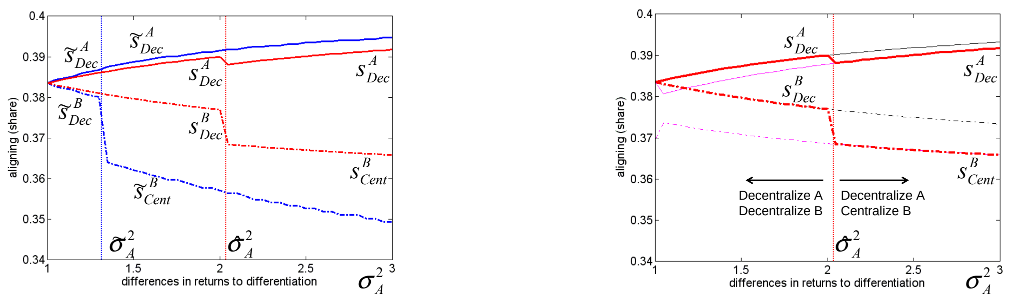
Figure 2.4 shows the relation between informational cost μ and returns to differentiation for each market when resources are allocated by the headquarters, for the particular case that $\overline{\mu} < \tilde{\mu}$. If $\overline{\mu} < \tilde{\mu}$, the headquarters decentralizes decision rights in both markets if $\sigma_A^2 < \tilde{\sigma}_A^2$ and chooses a mix decentralizing decisions about product A and centralizing decisions about product B otherwise. If $\overline{\mu} > \tilde{\mu}$, however, there exists $\breve{\sigma}_A^2$ such that the headquarters centralizes decision rights in both markets if $\sigma_A^2 > \breve{\sigma}_A^2$ and chooses a mix decentralizing decisions about product A and centralizing decisions about product B otherwise. Figure 2.5.a shows in blue the optimal compensation scheme as a function of the returns to product differentiation for a numerical example.[21]

Rantakari (forthcoming) describes a positive causal relation between returns to product differentia-

[21]For this example $\beta = 4$, $\overline{\mu} = 0.65$, $b = 0.5$.

tion (that he calls volatility) and decentralization. If decision rights are centralized in our model, an increment in product A's returns to differentiation can motivate the headquarters to decentralize the decision right of product A. Rantakari (forthcoming) drives this causality through a change in the needs for coordination β, while we drive this causality through a reallocation of resources.

There is an ongoing discussion on the literature about how exogenous is the coordination need.[22] In any case, most authors agree that not all elements of the structure (e.g., contracts, decision rights, divisional integration) can be modified with the same ease and speed. An organization can revise the compensation scheme or its allocation of decision rights more often than its degree of integration which may require updating the equipment, logistic, and information technologies.[23] Our mechanism is more direct than Rantakari (forthcoming)'s one, and we can account for transitory decentralization. Nevertheless, our results are robust to endogenize the needs for coordination that we develop in an extension.



(a) Comparing contracts (b) Contract when managers allocate resources

Figure 2.5: Optimal contract when the headquarters allocates resources (blue) and when local managers allocate resources (red).

In the following proposition we point out that our previous result is robust to the case where managers can decide or affect the allocation of resources. We show that, despite the headquarters can not control the allocation of resources, there exists a cutoff in the ratio of returns to differentiation above which the optimal structure combines decentralization and centralization. When the head-

[22]Some authors argue that the headquarters is free to decide the degree of integration between the two different units (See Rantakari (2010)). Some other authors, however, believe that the need for coordination is an exogenous constraint given by technology, legal environment, culture, etc. (See Alonso, et al (2008) and Dessein, Garicano and Gertner (2010)).

[23]Eccles and Holland (1989) describe the case of Suchard when European Union merges most western european markets as a unique market. It took several years and lots of resources for the company to adapt the company to the new situation. Thomas (forthcoming) also mentions how the market structure of western european markets modifies the needs for standardization. Procter & Gamble and Unilever reorganized their production after the pass of European Regulations in 1992. Firms have spent a lot of time and resources to launch programs to reduce the number of products and to centralize production in fewer plants, e.g., "path to growth" (Unilever in 2000), "Unilever 2010" (Unilever in 2004) and "Organization 2005" (Procter & Gamble in 1999). The program of Unilever included "a more streamlined brand portfolio, moving from 1600 brands in 1999 to a target number of 400 by the end of 2004". "P&G aimed to improve supply chain management of the proliferation of product, pricing, labeling, and packaging variations".

quarters has no control over resources, she modifies the strategy of the firm to capture as much benefit as possible from resource allocation.

Proposition 5. *Assume $\mu = \overline{\mu}\left(\frac{0.5}{t}\right)^b$ with $b \in (0,1)$ and $\overline{\mu} \in \mathbb{R}_{++}$. When local managers allocate resources, there exists a threshold for the ratio of market returns to product differentiation $\tilde{\sigma}_A^2(\tilde{\mu})$ above which the optimal design requires a mixed structure with centralization in market of product B and decentralization in market of product A. Moreover, $\tilde{\sigma}_A^2 \leq \hat{\sigma}_A^2$.*

Proposition 5 claims that our result even though managers control the allocation of resources. However, there are two effects that make the relation less intensive, generating a less intensive relation between returns to differentiation and the mixed structure, i.e., $\tilde{\sigma}_A^2 \leq \hat{\sigma}_A^2$. Let's assume $\sigma_A^2 = \tilde{\sigma}_A^2$ and $\overline{\mu} < \tilde{\mu}$, which means that decentralization is optimal if markets have similar returns to differentiation, i.e., if ratio $\sigma_A^2 \sim 1$. Compared with Proposition 4 both effects favour decentralization in the market with low dispersion, i.e., they favour a strategy of product differentiation in market B.
First, the headquarters would allocate more resources in the market with high returns to differentiation than local managers in fact do. Local managers allocate more resources to learn about the resources according to the condition in Lemma 3.b. The cost of acquiring information changes with the ratio of markets' return to product differentiation, i.e., t_A and μ_B increase with ratio σ_A^2, and μ_A decreases with σ_A^2. However, as local managers do not internalize the effect that resources can have in reducing the inefficiency in information acquisition, they allocate more resources in markets with low return to product differentiation. This favors decentralization in markets with low return to product differentiation.
Second, to correct the manager's misallocation of resources in favor of markets with high returns to differentiation, the headquarters changes the organizational design of the firm. Lemma 3.b describes how local managers allocate resources. The headquarters takes advantage of the complementarity between information acquisition and resource allocation to promote not only more information acquisition in those markets with high return to differentiation but also less information acquisition in markets with low return to differentiation. The headquarters implements this change through incentives alignment in markets with low return to differentiation and incentives misalignment in markets with high return to differentiation. These changes in structure, however, may also affect the firm's product strategy.
These two channels increase the likelihood that the headquarters chooses decentralization in market B, as a way to overcome the inefficiency in manager's resource allocation. The ratio of returns σ_A^2 must be higher for the mixed structure of decentralization and centralization to be optimal.
Summarizing, the externalities in information sharing generate suboptimal information acquisition. The headquarters alleviates this inefficiency allocating more resources to the markets where the problem is more serious, i.e., the markets with high return to product differentiation. However, when lacking control over resources, she uses the internal design of the firm, allocating decisions rights and modifying the compensation scheme to correct the effect of resource allocation. Hence,

resource allocation and decision rights are imperfect substitutes in the sense that controlling resources leads to a mix strategy of centralization and decentralization, but lacking control derives in a decentralized organization.

By symmetry, we can analyze the implications of Proposition 5 for the case with $\overline{\mu} > \tilde{\mu}$, in which case the optimal organizational design when the ratio σ_A^2 is small requires centralizing decision making and a mix structure arises when the ratio σ_A^2 of the returns to differentiation is high.

Figure 2.5.b shows how the optimal contract depends on the ratio of returns to product differentiation in a numerical example. The aligning of incentives is higher in the market with higher returns. There are more returns to differentiation than in the other market. The difference in alignment increases with the ratio of returns to product differentiation. There is a threshold above which the organizational design changes, centralizing decisions about the product with lower returns. This change also affects the contract. The objectives functions depend more on the local division profits to motivate more information acquisition in both markets, however they are more misaligned in the centralized product.

Our results can be empirically identified in at least two ways. First, comparing multi-product multi-market firms with single-product multi-market firms and observing that there may be differences in their organizational design. This identification strategy is relevant since there are differences in organizational structure of firms operating in the same market that cannot be explained either by demand differences or by supply conditions or retailers environment. In this paper we offer a framework for differences that are born within internal organization.

A second way to identify our result is comparing multi-product multi-market firms along time. If there is a shock affecting the returns to product differentiation of some particular product, a firm can modify its organizational design to increase its profits. However, it may be difficult to observe this case because firms might modify informally their organizational design without changing formals procedures.[24]

2.4 Extensions

2.4.1 Coordination

In the previous section we discuss that in some papers the level of coordination among divisions is endogenous, while in some others is exogenous. Our results hold if we endogenize this choice. We extend results of Propositions 4 and 5 for the case that the headquarters chooses β. Figure 2.6.a shows the extension of Proposition 3, Figure 2.6.b of Proposition 4, and Figure 2.7 of Proposition 5.

[24]For instance, suppose a centralized organization with a formal procedure to introduce new products through the following procedure: a country manager writes a project suggesting a product which requires Headquarters' approval; however, the headquarters can commit (through reputation) to relax its approval requirements over those projects related to some particular products' lines or segments. This is interpreted as informal decentralization. I have been informally told by managers in international companies that this is a common proceeding.

The choice of β does not modify our previous results qualitatively, even when it does quantitatively. If the headquarters decides the optimal β for the structure of the organization, there is a negative relation between β and σ^2. There is a high risk of requiring high levels of coordination between local managers, as long as there is high uncertainty about demand characteristics, i.e., high σ^2. For high values of σ^2 the headquarters prefers to provide autonomy to local managers about what products to be offered in their markets and to follow a strategy of product differentiation, i.e., the higher σ^2 the lower β.

Analogously, the headquarters follows a standard product strategy in those markets with low returns to product differentiation. The effect of losing control over resources would also affect the decision of β. For details see Appendix 2.8.2. For the function $\mu = \overline{\mu}\left(\frac{0.5}{t}\right)^b$ with $b \in (0,1)$, there exists a cutoff $\tilde{\sigma}^2(\overline{\mu})$ and increasing relations $\tilde{\sigma}^2_{AM}(\sigma^2_B, \overline{\mu})$, $\tilde{\sigma}^2_A(\sigma^2_B, \overline{\mu})$, $\tilde{\sigma}^2_{BM}(\sigma^2_A, \overline{\mu})$ and $\tilde{\sigma}^2_B(\sigma^2_A, \overline{\mu})$ such that:

a- If resources are allocated efficiently (headquarters), decision rights over market j are centralized if $\sigma^2_j < \tilde{\sigma}^2_j(\sigma^2_{-j}, \overline{\mu})$, for $j \in \{A, B\}$.

b- If resources are allocated by local managers, decision rights over market j are centralized if $\sigma^2_j < \tilde{\sigma}^2_{jM}(\sigma^2_{-j}, \overline{\mu})$, for $j \in \{A, B\}$.

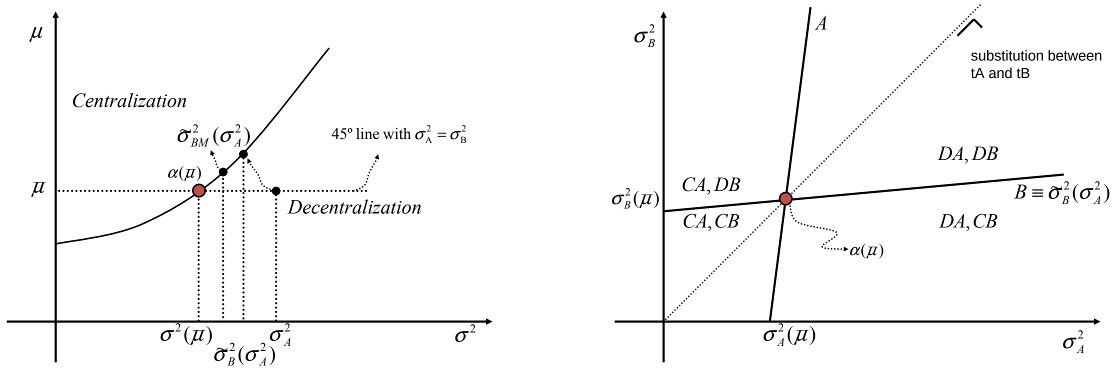
c- $\tilde{\sigma}^2_A(\sigma^2_B, \overline{\mu}) < \tilde{\sigma}^2_{AM}(\sigma^2_B, \overline{\mu})$ and $\tilde{\sigma}^2_B(\sigma^2_A, \overline{\mu}) < \tilde{\sigma}^2_{BM}(\sigma^2_A, \overline{\mu})$.

Centralization is chosen as an optimal structure if the return to product differentiation is sufficiently low, which is more likely for higher informational cost μ. This relation is shown in Figure 2.6.a. We remark now the effect of resource allocation on the headquarters decision over centralization and decentralization. When resource allocation becomes important, the headquarters promotes a lower cost in markets with high returns to differentiation. Not only the ratio of the returns to differentiation but also the level of the returns to differentiation matter in all markets to determine whether to centralize or decentralize decision rights.

A low β fits better in a market with high return to differentiation, which also makes decentralization more attractive. On markets with high returns to differentiation, the cutoff ratio that makes the headquarters to centralize decision making in some markets increases. These thresholds are represented by increasing relations in Figure 2.6.b. As in our baseline analysis, these thresholds are higher when the headquarters lacks control over resources, which is shown in Figure 2.7.a. Relations are steeper when the resource allocation is more important in determining the information cost μ, as shown in Figure 2.7.b.

2.4.2 Delegation

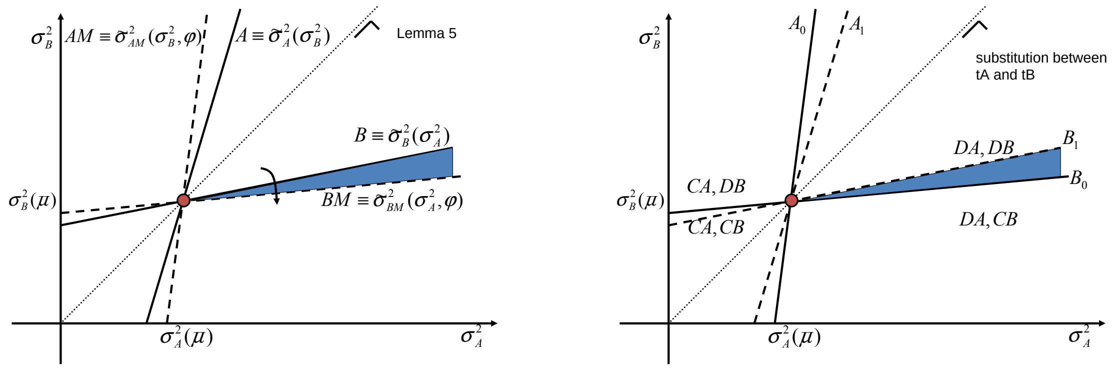
We assume that the headquarters delegates resource allocation on local division managers. We can extend our model to see that delegation may arise as the optimal's choice of the headquarters if there are externalities when learning about regional demands. For example, if learning about demand's taste of product B in region 1 can provide some additional information when learning



(a) Comparing μ and σ^2

(b) Comparing σ^2_A and σ^2_B

Figure 2.6: Optimal structure depending on marginal informational cost, μ, and market returns to differentiation, σ^2.



(a) Firm's and Managers' cutoff

(b) Change in resource allocation importance

Figure 2.7: Optimal structure for change on the importance of resource allocation

about demand's taste of product A in region 1, the firm can prefer to merge two subdivisions in region 1. In this way, the firm designs the organization by regional divisions, as many international firms do. Let us call subdivisions $A1$ and $B1$ for the rest of this section.

Assume the following technology of the signal $\hat{\theta}_{1A}$: the signal equals the true value θ_{1A} with probability $\sqrt{e_{1A} + \alpha e_{1B}}$ and a random draw from the same distribution of θ_{1A} with probability $1 - \sqrt{e_{1A} + \alpha e_{1B}}$, with $0 < \alpha < 1$, and $e \in [0, \frac{1}{1+\alpha}]$.[25] This technology of the signal generates an externality in information acquisition that is described as follows:

1- If subdivisions $A1$ and $B1$ are separated, local manager in division $A1$ collects information according to: $\tilde{\psi}_A(\beta, s, g) = \mu_A C'(\hat{e}_A)$.

2- If subdivisions $A1$ and $B1$ are integrated, local manager in division $A1$ collects information

[25] Introducing this change in the model does not modify the communication problem.

according to: $\tilde{\psi}_A(\beta,s,g)+\frac{\sigma_B^2}{\sigma_A^2}\alpha\tilde{\psi}_B(\beta,s,g)=\mu_A C'(\hat{e}_A)$.

3- If effort were contractible, the headquarters would ask the local manager to collect information according to: $\psi_A+\frac{\sigma_B^2}{\sigma_A^2}\alpha\psi_B=\mu_A C'(e_A^*)$.

The net effect of this learning externality about market characteristics is intuitive. For the function $\mu=\overline{\mu}\left(\frac{0.5}{t}\right)^b$, the parameter b represents the discretion of local managers to reallocate resources, the higher the b the higher the managers' discretion. For a given value of α, integration is profitable if b is sufficiently low. There will be a cutoff on b such that delegation is optimal as long as b is below that value. As α increases, learning from products A and B becomes more complementary, which means that products are less rival. The inefficiency in resource allocation is reduced. Although there are some new insights, the main implications of this paper over the firm's optimal structure remain. To see a comparison of this problem with our previous model see Appendix 2.8.2.

2.5 Conclusion

In this paper, we analyze the internal design of a multi-product multi-division firm. We model the economic interactions of being multi-product that arise in an organization. We focus on the effect that the opportunity cost of resources has on decision making and compensation schemes.

Implementing a product differentiation policy on some products is more profitable than on other products. This relative profitability generates an opportunity cost of allocating scarce resources in each market. We show that the firm may prefer to centralize decision making in markets with lower returns to differentiation and decentralize decision making in markets with high returns to differentiation.

Empirically our result can be observed comparing the organizational design of multi-product multi-market firms with single-product multi-market firms. Alternatively, it can also be observed following dynamic organizational changes of a multi-product multi-market firm that suffers shocks which modify the relative returns to product differentiation.

In this paper we have made a first move to extend the economic literature of organizational design into the analysis of multi-product firms. Much of this literature focuses on internal economic problems that frequently appear in international multi-product firms. Henceforth, an analysis of the multi-product multi-division firm is not only important but also necessary. We show that the allocation of decision rights and firm's strategy is not independent among different products. This is in line with Roberts (2004) who points out that "The structure [of the firm] does not follow strategy any more than strategy follows structure".

We have focused on the impact of the opportunity cost of resources on the internal organizational design. It is still necessary to carry out further work on analyzing other economic interactions that multinational firms face for being multi-product, e.g., interactions that arise on the demand side.

2.6 Appendix A: Proofs

2.6.1 Proof of Lemma 4

Proof. Without loss of generality, assume $\overline{\mu} = 1$, $\mu(t) = \left(\frac{0.5}{t}\right)^b$. Looking for t^* that maximizes $[-\mu(t) - \mu(1-t)H]$ we find that $t^* = \frac{1}{1+(H)^{\frac{1}{1+b}}}$ and $\mu(t^*) = 0.5^b\left[1+(H)^{\frac{1}{1+b}}\right]^b$. Since $0 < H \leq 1$ then $t^* \geq 0.5$.

We need to prove that $\mu(t^*)$ is decreasing in b for $0 < b < 1$, i.e., $\frac{\partial \mu(t^*)}{\partial b} = \mu(t^*)\frac{\partial \log \mu(t^*)}{\partial b} < 0$. Taking logarithm and derivating respect to b we have $\log \mu(t^*) = b \log 0.5 + b \log\left[1+(H)^{\frac{1}{1+b}}\right]$, and

$$\frac{d \log \mu(t^*)}{db} = \log 0.5 + \log\left[1+(H)^{\frac{1}{1+b}}\right] - \frac{(H)^{\frac{1}{1+b}}}{\left[1+(H)^{\frac{1}{1+b}}\right]}\frac{b}{(1+b)^2}\log(H). \tag{2.11}$$

Note in the last term of equation 2.11 that $1-t^* \equiv \frac{(H)^{\frac{1}{1+b}}}{\left[1+(H)^{\frac{1}{1+b}}\right]}$ and that $1-t^* < 0.5$. Working the last two terms, equation 2.11 can be expressed as $\log 0.5 + \log[H2]$, with $H2 \equiv \left(\frac{1}{H}\right)^{\frac{(1-t^*)b}{(1+b)^2}} + (H)^{\frac{1+bt^*}{(1+b)^2}}$. I need to prove that $H2 < 2$.[26] For the extreme case that $t^* = 0.5$ and $b = 1$ we have that $\left(\frac{1}{H}\right)^{\frac{1}{8}} + (H)^{\frac{3}{8}} \leq 2$ if $0.008 < H \leq 1$. Note that the expression $H2$ decreases as b decreases or t^* increases. For completing the proof, we check these derivatives respect to b and t^*,

$$\frac{\partial H2}{\partial b} = \left(\frac{1}{H}\right)^{\frac{(1-t^*)b}{(1+b)^2}} \log(\frac{1}{H})(1-t^*)\frac{1-b}{(1+b)^3} - \log(H)\frac{2-t^*+bt^*}{(1+b)^3}(H)^{\frac{1+bt^*}{(1+b)^2}} > 0.$$

Since $H \leq 1$, note that $\log(H) < 0$ and $\log(\frac{1}{H}) > 0$. And the derivative respect to t^* is,

$$\frac{\partial H2}{\partial t^*} = -\left(\frac{1}{H}\right)^{\frac{(1-t^*)b}{(1+b)^2}} \log(\frac{1}{H})\frac{b}{(1+b)^2} + \log(H)\frac{b}{(1+b)^2}(H)^{\frac{1+bt^*}{(1+b)^2}} < 0.$$

Finally, we check that the derivative $\frac{\partial \mu(t^*)}{\partial H} = \frac{b}{1+b}0.5^b\left[1+(H)^{\frac{1}{1+b}}\right]^{b-1}(H)^{\frac{-b}{1+b}} > 0$ which implies that if H decreases, then the cost $\mu(t^*)$ decreases and the cost $\mu(1-t^*)$ increases. The proof is complete. □

2.6.2 Proof of Proposition 3

Sketch of the proof. (Based on Proposition 5 of Rantakari (2010)) Consider the optimal structure design for market A. We proceed in two steps: first determining the contract s under decentralization and under centralization, and second, comparing which structure generates higher profits. The first order condition respect to s shows a balance between less information which has more value and more information which has less value: $e_A\frac{\partial \psi_A}{\partial s_A} + \frac{\partial e_A}{\partial s_A}[\psi_A - \tilde{\psi}_A] = 0$. Since local managers

[26] Recall that $\log 0.5 + \log 2 = \log 1 = 0$.

appropriate the amount $\tilde{\psi}$ of the value generated, ψ, they acquire a suboptimal amount of information, e. However, this inefficiency can be partially corrected with the compensation scheme. A comparative static shows that incentives are narrowed to local manager's payoff when information is more expensive, i.e., $\frac{\partial s}{\partial \mu} < 0$. To have a good balance between the amount of information acquired and the value generated with this information, e and ψ respectively, the headquarters fosters information acquisition when it is expensive, even when this reduces the value generated by the information acquireds.

Evaluated in the optimal compensation scheme, decentralization outperforms centralization if the surplus value is greater, i.e., if $e_A\psi_A - \mu_A C(e_A)$ is higher under decentralization than under centralization. Recalling that: 1) e decreases and ψ increases when s increases; 2) s decreases when μ increases; 3) for high values of s, ψ and $\tilde{\psi}$ are higher under decentralization, and for low values of s, ψ is higher under centralization but $\tilde{\psi}$ is higher under decentralization; and finally 4) the information acquired increases with the value appropriated by local managers, i.e., the higher the $\tilde{\psi}$ the higher the e.

Low μ generates high e, which allows the firm to increase s and to decentralize. In words, low cost of information generates high amount of information, allowing the firm to increase the degree of incentives alignment and to decentralize decision rights. This result follows directly from section 2.3.1 (based on Alonso et al, 2008) and from section 2.3.2. □

2.6.3 Proof of Proposition 4

Proof. For the last part of Proposition 4, we prove that the cutoff $\tilde{\sigma}_{AB}$ decreases with b. For a given σ_{AB}, the higher the b the lower the μ_A and the higher the μ_B. Recall that t^* and μ_B depend on H and b and from Lemma 4 we have

$$\frac{\partial \mu_B(1-t^*)}{\partial b} = \mu_B(1-t^*)\frac{\partial \ln[\mu_B(1-t^*)]}{\partial b} > 0. \tag{2.12}$$

A higher b implies a higher μ_B. At $\tilde{\sigma}_{AB}$ the headquarters is indifferent between centralizing and decentralizing decision rights about product characteristics in market B. If b increases, μ_B increases and now the headquarters strictly prefers to centralize decision rights in market of product B. The cutoff $\tilde{\sigma}_{AB}$ decreases when b increases. □

2.6.4 Proof of Proposition 5

Proof. When local managers allocate resources, headquarters internalizes an additional effect of modifying the contract. Increasing s_A not only reduces the amount of effort e_A but also reduces the amount of resources that a local managers allocate to acquire information about product A, which indirectly also reduces the effort e_A. Moreover, a reduction in t_A fosters more information acquisition about product B.

If the headquarters allocates resources, t_A^*, the optimal s_A is defined by $\frac{\partial \psi_A}{\partial s_A}e_A + \frac{\partial e_A}{\partial s_A}(\psi_A - \tilde{\psi}_A) = 0$.

However, if local managers allocate resources the first order condition becomes:

$$\begin{aligned}
\frac{\partial \psi_A}{\partial s_A} e_A + \frac{\partial e_A}{\partial s_A}(\psi_A - \tilde{\psi}_A) &= -\frac{\partial t_A}{\partial s_A}\frac{\sigma_B^2}{\sigma_A^2}\Big[\frac{\sigma_A^2}{\sigma_B^2}\frac{\partial e_A}{\partial t_A}[\psi_A - \tilde{\psi}_A] + \frac{\partial e_B}{\partial t_A}[\psi_B - \tilde{\psi}_B]\Big],\\
\frac{\partial \psi_B}{\partial s_B} e_B + \frac{\partial e_B}{\partial s_B}(\psi_B - \tilde{\psi}_B) &= -\frac{\partial t_A}{\partial s_B}\Big[\frac{\sigma_A^2}{\sigma_B^2}\frac{\partial e_A}{\partial t_A}[\psi_A - \tilde{\psi}_A] + \frac{\partial e_B}{\partial t_A}[\psi_B - \tilde{\psi}_B]\Big].
\end{aligned}$$

With $\frac{\partial t_A}{\partial s_A} < 0$ and $\frac{\partial t_A}{\partial s_B} > 0$, and calling $\varphi \equiv \frac{\sigma_A^2}{\sigma_B^2}\frac{\partial e_A}{\partial t_A}[\psi_A - \tilde{\psi}_A] + \frac{\partial e_B}{\partial t_A}[\psi_B - \tilde{\psi}_B]$. These expressions separate the indirect effect of s_A on t_A from the direct effect of s_A on e_A (similarly for s_B). In Section 2.7.7.1 we describe that each manager reaction to an increase in σ_A^2 are $\frac{\partial e_A}{\partial \sigma_A^2} > 0$, $\frac{\partial e_B}{\partial \sigma_A^2} < 0$ and $\frac{\partial t_A}{\partial \sigma_A^2} > 0$. As σ_A^2 increases, more resources are allocated to product A; the HQ increases s_A to increase ψ_A and it reduces s_B to increase $\tilde{\psi}_B$ fostering higher effort in market B, i.e., higher e_B. Eventually, the headquarters chooses to centralize decision making in product B and decentralize decision making in product A.

However, comparing with the situation where headquarters allocates resources, the indirect effects make the cutoff $\hat{\sigma}_{AB}$ to be higher than $\tilde{\sigma}_{AB}$. To show that $\tilde{\sigma}_{AB} < \hat{\sigma}_{AB}$ we identify the two indirect effects that foster the headquarters to decentralize product B when σ_{AB} is around $\tilde{\sigma}_{AB}$. Let us assume that $\sigma_{AB} = \tilde{\sigma}_{AB}$. If this is the case and local managers allocate resources, the optimal organizational design, for the case where $\underline{\mu} < \tilde{\mu}$, requires to decentralize decision making in both products.

First, since local managers ignore the inefficiency in exerting effort, represented by $[\psi - \tilde{\psi}]$, they allocate more resources to market B than the headquarters would, i.e., $\hat{t}_A < t_A^*$. Given $\hat{t}_A < t_A^*$, we have that $\mu_A(\hat{t}_A) > \mu_A(t_A^*)$ and $\mu_B(1-\hat{t}_A) < \mu_B(1-t_A^*)$. Formally, in section 2.3.3 we show that the term H differs depending on whether headquarters or local managers allocate resources, being higher in the latter case. This difference in H comes from the fact that local managers ignore the inefficiency $[\psi - \tilde{\psi}]$, which affects directly the cost μ_B.

$$\frac{\partial \mu_B(1-t^*)}{\partial H} = \frac{\partial}{\partial H}\left[\overline{\mu} 0.5^b \left(1 + H^{\frac{-1}{1+b}}\right)^b\right] < 0.$$

There is lower effort cost μ_B, when managers allocate resources. If information is cheaper in market B, the headquarters increases s_B in market B to increase the value generated by this information, which, at the same time, favors decentralization of decisions rights in market B.

Second, the headquarters recognizes the inefficient allocation of resources of local manager, and changes the organizational design to correct it. With this purpose, the headquarters increases s_B which also favors decentralization of decision rights in market B.

Both effects favor to decentralize market of product B. At $\sigma_{AB} = \tilde{\sigma}_{AB}$ the headquarters strictly prefers to decentralize decision rights in market B. Then, it requires a higher value of σ_{AB} to centralize decisions rights in the market of product B. □

2.7 Appendix B: Solving the Problem

2.7.1 Close Forms

In this appendix we construct the close form solutions of the expressions of ψ and $\tilde{\psi}$ under centralization and decentralization that are the following:

$$\psi_c = \frac{(3(1-s))(1+2\beta)^2}{(1+4\beta)(\beta(1-2s)+(3(1-s))(2\beta+1))},$$
$$\psi_d = -(1-s)\frac{5s^3(1-2\beta)+s^2(5\beta+14(\beta^2-1))+s(16\beta+2\beta^2+13)-(4+6\beta^3+11\beta+12\beta^2)}{((1-s)+2\beta)(1-s+\beta)((1-s+\beta)(1-2s)+3(1-s)(1-s+2\beta))},$$
$$\tilde{\psi}_c = (3(1-s))\frac{(1+2\beta)((1-s)+\beta(3-4s)}{((1+4\beta)(\beta(1-2s)+(3(1-s))(2\beta+1))}, \text{ and}$$
$$\tilde{\psi}_d = (1-s)\frac{9\beta^2+11\beta+13s^2\beta+4-13s+14s^2-5s^3-12s\beta(2+\beta)}{((1-s)+2\beta)((1-s+\beta)(1-2s)+3(1-s)(1-s+2\beta)))}.$$

And its derivatives under centralization are:

$$\frac{\partial \psi^c}{\partial s} = \frac{3\beta(2\beta+1)^2}{(1+4\beta)(8\beta(1-s)-\beta+3(1-s))^2} \geq 0,$$
$$\frac{\partial \tilde{\psi}^c}{\partial s} = \frac{3(2\beta+1)(3(1-s)^2+\beta^2+2(1-s)\beta(10(1-s)-1)+8\beta^2(1-s)(4(1-s)-1))}{(1+4\beta)^2(8\beta(1-s)-\beta+3(1-s))^2} < 0 \quad (2.13)$$

And under decentralization are:

$$\frac{\partial \psi_d}{\partial s} = \frac{\beta^2((1-s)^3H_1+\beta(1-s)^2H_2+\beta^2(1-s)H_3+\beta^3H_4+12\beta^4)}{((1-s)+2\beta)^2(1-s+\beta)^2((1-s+\beta)(1-2s)+3(1-s)(1-s+2\beta))^2} \geq 0, \quad (2.14)$$

with $H_1 \equiv 160(1-s)^3-120(1-s)^2+27(1-s)-2$, $H_2 \equiv 680(1-s)^3-516(1-s)^2+1266(1-s)+10$, $H_3 \equiv 956(1-s)^3-744(1-s)^2+204(1-s)-16$ and $H_4 \equiv 448(1-s)^3-360(1-s)^2+102(1-s)-8$.

$$\frac{\partial \tilde{\psi}_d}{\partial s} = \frac{5(1-s)^5(5(1-s)-2)+6\beta(1-s)^4(30(1-s)-11)+(1-s)^4+6\beta^2(1-s)^3(69(1-s)-20)}{((1-s)+2\beta)^2(5(1-s)^2-(1-s)+8\beta(1-s)-\beta)^2}+$$
$$\frac{6(1-s)^3\beta+4\beta^3(1-s)^2(104(1-s)-23)+9(1-s)^2\beta^2+8(1-s)\beta^3+6\beta^4(32(1-s)^2-8(1-s)+1)}{((1-s)+2\beta)^2(5(1-s)^2-(1-s)+8\beta(1-s)-\beta)^2} < 0.$$

2.7.2 Decision Making: Using Information

In this section we build the expression in equation (2.1) for Centralization and Decentralization before communication outcome is introduced. To have the same terms we must replace the expression $E[\overline{m}^2] = [1-(1-V)]E[\theta^2]$.

We build the indirect function for $E[\Pi|m]$ given the equilibrium beliefs for $m \equiv E[\theta|m]$ and $E[\theta^2|m] \equiv m^2$. Remember that $\Pi_{1A} = K(\beta)-(a_{1A}-\theta_{1A})^2-\beta(a_{1A}-a_{2A})^2$.

Given the optimal policy for decision making and the information transmission process, the objective function of each division can be expressed as a function of σ_1^2, σ_2^2, $E[\theta_1^2|m]$ and $E[\theta_2^2|m]$. We show how to arrive to the optimal expressions.

2.7.2.1 Centralization

The optimal decision making under centralization are:

$$\begin{aligned} a_1^C(m_1,m_2) &= \frac{1+2\beta}{1+4\beta}E[\theta_1|m_1]+\frac{2\beta}{1+4\beta}E[\theta_2|m_2], \text{ and} \\ a_2^C(m_1,m_2) &= \frac{2\beta}{1+4\beta}E[\theta_1|m_1]+\frac{1+2\beta}{1+4\beta}E[\theta_2|m_2]. \end{aligned}$$

Replacing them in the expected profit of division 1 we get the following terms:

$$\begin{aligned} (a_1^C-\theta_1)^2 &= \left(E[\theta_1|m_1]-\theta_1+\frac{2\beta\left(E[\theta_2|m_2]-E[\theta_1|m_1]\right)}{1+4\beta}\right)^2, \\ &= \left(\overline{m}_1-\theta_1\right)^2+\frac{4\beta^2\left(\overline{m}_2-\overline{m}_1\right)^2}{\left(1+4\beta\right)^2}+2\beta\left(\overline{m}_1-\theta_1\right)\frac{\left(\overline{m}_2-\overline{m}_1\right)}{\left(1+4\beta\right)}, \\ &= \left(\overline{m}_1^2-2\overline{m}_1\theta_1+\theta_1^2\right)+\frac{4\beta^2\left(\overline{m}_2^2+\overline{m}_1^2-2\overline{m}_2\overline{m}_1\right)}{\left(1+4\beta\right)^2}+2\beta\left(\overline{m}_1-\theta_1\right)\frac{\left(\overline{m}_2-\overline{m}_1\right)}{\left(1+4\beta\right)} \quad (2.15) \\ (a_1^C-a_2^C)^2 &= \left(\frac{E[\theta_1|m_1,m_2]-E[\theta_2|m_1,m_2]}{1+4\beta}\right)^2=\frac{\overline{m}_1^2-2\overline{m}_1\overline{m}_2+\overline{m}_2^2}{(1+4\beta)^2}. \quad (2.16) \end{aligned}$$

Taking expectations we get that

$$\begin{aligned} E[(a_1^C-\theta_1)^2] &= \left(E[\theta_1^2]-E[\overline{m}_1^2]\right)+\frac{4\beta^2\left(E[\overline{m}_2^2]+E[\overline{m}_1^2]\right)}{\left(1+4\beta\right)^2}, \quad (2.17) \\ E[(a_1^C-a_2^C)^2] &= \frac{E[\overline{m}_1^2]+E[\overline{m}_2^2]}{(1+4\beta)^2}. \quad (2.18) \end{aligned}$$

Now, lets build the expected profits $E[\Pi]$ which add both terms weighted by 1 and β respectively. Notice that taking ex-ante expectation the following terms are null: $E[\overline{m}_1\overline{m}_2]=0$, $E[\left(\overline{m}_1-\theta_1\right)\left(\overline{m}_2-\overline{m}_1\right)]=0$. Also note that $E[\overline{m}_1\theta_1]=E[\overline{m}_1^2]$

$$E[\Pi_1] = \left(K(\beta)-\left[E[\theta_1^2]-\frac{1+3\beta}{(1+4\beta)}E[\overline{m}_1^2]+\frac{\beta}{(1+4\beta)}E[\overline{m}_2^2]\right]\right). \quad (2.19)$$

REMARK: Since $E[\overline{m}_1]=0$ and $E[\theta_1]=0$, then $E[\overline{m}_1^2]=VE[\theta_1^2]=E[\theta_1^2]-(1-V)E[\theta_1^2]$ The division payoff is $E[U_1]=(1-s)\Pi_1+s\Pi_2$.

2.7.2.2 Decentralization

The optimal decision making under decentralization are:

$$\begin{aligned} a_1^D(m_1,m_2,\theta_1) &= \frac{(1-s)}{(1-s+\beta)}\theta_1 + \frac{\beta}{(1-s+\beta)}\left[\frac{\beta}{1-s+2\beta}\overline{m}_1 + \frac{1-s+\beta}{1-s+2\beta}\overline{m}_2\right], \text{ and} \\ a_2^D(m_1,m_2,\theta_2) &= \frac{(1-s)}{(1-s+\beta)}\theta_2 + \frac{\beta}{(1-s+\beta)}\left[\frac{\beta}{1-s+2\beta}\overline{m}_2 + \frac{1-s+\beta}{1-s+2\beta}\overline{m}_1\right]. \end{aligned}$$

Replacing it in the expected profit of division 1 we get the following terms:

$$\begin{aligned} (a_1^D-\theta_1)^2 &= \left(-\frac{\beta}{(1-s+\beta)}\theta_1 + \frac{\beta}{(1-s+\beta)}\overline{m}_1 + \frac{\beta}{1-s+2\beta}(\overline{m}_2-\overline{m}_1)\right)^2, \\ &= \frac{\beta^2}{(1-s+\beta)^2}(\overline{m}_1-\theta_1)^2 + \frac{\beta^2}{(1-s+2\beta)^2}(\overline{m}_2-\overline{m}_1)^2 + \frac{2\beta^2(\overline{m}_1-\theta_1)(\overline{m}_2-\overline{m}_1)}{(1-s+\beta)(1-s+2\beta)}, \\ &= \frac{\beta^2\left(\overline{m}_1^2-2\overline{m}_1\theta_1+\theta_1^2\right)}{(1-s+\beta)^2} + \frac{\beta^2\left(\overline{m}_2^2+\overline{m}_1^2-2\overline{m}_2\overline{m}_1\right)}{(1-s+2\beta)^2} + \frac{2\beta^2(\overline{m}_1-\theta_1)(\overline{m}_2-\overline{m}_1)}{(1-s+\beta)(1-s+2\beta)} \end{aligned} \quad (2.20)$$

$$\begin{aligned} (a_1^D-a_2^D)^2 &= \left(\frac{(1-s)(\theta_1-\theta_2)}{(1-s+\beta)} + \frac{\beta}{(1-s+\beta)}\frac{(1-s)(\overline{m}_2-\overline{m}_1)}{1-s+2\beta}\right)^2, \\ &= \frac{(1-s)^2}{(1-s+\beta)^2}(\theta_1^2+\theta_2^2-2\theta_1\theta_2) + \frac{\beta^2}{(1-s+\beta)^2}\frac{(1-s)^2(\overline{m}_2^2+\overline{m}_1^2-2\overline{m}_1\overline{m}_2)}{(1-s+2\beta)^2}, \\ &\quad + \frac{2\beta}{(1-s+\beta)^2}\frac{(1-s)^2}{1-s+2\beta}(\overline{m}_2\theta_1+\overline{m}_1\theta_2-\overline{m}_2\theta_2-\overline{m}_1\theta_1). \end{aligned} \quad (2.21)$$

Now, lets build the expected profits $E[\Pi]$ which add both terms weighted by 1 and β respectively. Notice that taking ex-ante expectation the following terms are null: $E[\overline{m}_1\overline{m}_2]=0$,$E[\overline{m}_1\theta_2]=0$, $E[\overline{m}_2\theta_1]=0$,$E[\theta_1\theta_2]=0$, $E[(\overline{m}_1-\theta_1)(\overline{m}_2-\overline{m}_1)]=0$. Also note that $E[\overline{m}_1\theta_1]=E[\overline{m}_1^2]$ By parts[27]

$$\begin{aligned} E[(a_1^D-\theta_1)^2] &= \frac{\beta^2 E[\theta_1^2]}{(1-s+\beta)^2} + \frac{(1-s+\beta)^2-(1-s+2\beta)^2}{(1-s+\beta)^2(1-s+2\beta)^2}\beta^2 E[\overline{m}_1^2] + \frac{\beta^2}{(1-s+2\beta)^2}E[\overline{m}_2^2], \\ &= \frac{\beta^2 E[\theta_1^2]}{(1-s+\beta)^2} - \frac{2(1-s)+3\beta}{(1-s+\beta)^2(1-s+2\beta)^2}\beta^3 E[\overline{m}_1^2] + \frac{\beta^2}{(1-s+2\beta)^2}E[\overline{m}_2^2]. \end{aligned}$$

$$E[(a_1^D-a_2^D)^2] = \frac{(1-s)^2}{(1-s+\beta)^2}(E[\theta_1^2]+E[\theta_2^2]) - \frac{[2(1-s)+3\beta]}{(1-s+\beta)^2}\frac{\beta(1-s)^2}{(1-s+2\beta)^2}(E[\overline{m}_2^2]+E[\overline{m}_1^2]).$$

The expected profits $E[\Pi_1]$ are

$$\left(K(\beta) - \left[\frac{\beta(1-s)^2}{(1-s+\beta)^2}E[\theta_2^2] + \beta\frac{\beta+(1-s)^2}{(1-s+\beta)^2}E[\theta_1^2] - \beta^2\frac{[2(1-s)+3\beta]}{(1-s+\beta)^2}\frac{\beta+(1-s)^2}{(1-s+2\beta)^2}E[\overline{m}_1^2] + \left(\beta\frac{\beta+(1-s)^2}{(1-s+2\beta)^2} - \frac{\beta(1-s)^2}{(1-s+\beta)^2}\right)E[\overline{m}_2^2]\right]\right). \quad (2.22)$$

[27] In the profit function we have $E[(a_1^D-\theta_1)^2]+\beta E[(a_1^D-a_2^D)^2]$. Note that $\frac{(1-s+\beta)^2-(1-s+2\beta)^2}{(1-s+\beta)^2(1-s+2\beta)^2} = -\beta\frac{2(1-s)+3\beta}{(1-s+\beta)^2(1-s+2\beta)^2}$.

Since $E[\theta]=0$, $E[\theta^2]=V(\theta)=\frac{\overline{\theta}^2}{3}=\sigma^2$.

2.7.3 Strategic Communication: Transmission

2.7.3.1 Building $E[\overline{m}^2]$ for any Finite Partition.

I must find the payoff for any finite partition. The general formula for a N_j partition is:[28]

$$E[\overline{m}^2]=\frac{1}{2\overline{\theta}}\sum_{j\in N_j}\int_{d_{j-1}}^{d_j}\left(\frac{d_j+d_{j-1}}{2}\right)^2 d\theta. \tag{2.23}$$

By uniform distribution we get that $\int_{d_{j-1}}^{d_j}\left(\frac{d_j+d_{j-1}}{2}\right)^2 d\theta=(d_j-d_{j-1})\left(\frac{d_j+d_{j-1}}{2}\right)^2$.
From Proposition 2 we have $d_{j,i+1}-d_{j,i}=d_{j,i}-d_{j,i-1}+4bd_{j,i}$ with $b\equiv\frac{(1-2s)(1-s+\beta)}{s(1-s)+\beta}$ under decentralization and $b\equiv\frac{(1-2s)\beta}{1-s+\beta}$ under centralization. Also $d_i=\overline{\theta}\frac{x^i(1+y^N)-y^i(1+x^N)}{x^N-y^N}\quad 0\leq i\leq N$ with $x=(1+2b)+\sqrt{(1+2b)^2-1}$ and $y=(1+2b)-\sqrt{(1+2b)^2-1}$. Property $xy=1$ and $x>1$ applies to both cases and are used all along the algebra. Replacing it in the expression above we have (Equation 27 in Alonso, Dessein and Matouscheck 2008).

$$\begin{aligned}E[\overline{m}^2] &= \frac{\overline{\theta}^2}{3}\left[\frac{(x^{3N_j}-1)(x-1)^2}{(x^{N_j}-1)^3(x^2+x+1)}-\frac{(x^{N_j}+1)^2(x+1)(1+y)}{x^{N_j}(x^{N_j}-y^{N_j})^2}\right],\\ &= \frac{\overline{\theta}^2}{3}\left[\frac{(x^{3N_j}-1)(x-1)^2}{(x^{N_j}-1)^3(x^2+x+1)}-\frac{x^{N_j}(x+1)^2}{x(x^{N_j}-1)^2}\right].\end{aligned} \tag{2.24}$$

For the case with infinite partitions we have

$$\lim_{N_j\to+\infty}E[\overline{m}^2]=\frac{\overline{\theta}^2}{4}\frac{(x+1)^2}{(x^2+x+1)}=\overline{\theta}^2\frac{1+b}{3+4b}=V\sigma^2=[1-(1-V)]\sigma^2. \tag{2.25}$$

If the sender is truly believed, he will report exaggerating the signal. For example, if $s=0$, a manager has incentives to misreport $\theta^R-\theta=\frac{\beta}{1+\beta}\theta$ under centralization and $\theta^R-\theta=\frac{1+\beta}{\beta}\theta$ under decentralization. For this case, his report is: $\theta^R\equiv\frac{(1+4\beta)(1+2\beta)}{(1+2\beta)^2+\beta}\theta=\frac{1+2\beta}{1+\beta}\theta$ under centralization and $\theta^R\equiv\frac{1+2\beta}{\beta}\theta$ under decentralization.

2.7.4 Imperfect Signals

Given the information acquired, specified by the effort e, each manager have a posterior about the true value. That is, given the realization of the signal $\hat{\theta}$ (that coincide with the true value of θ with probability $\sqrt{e}$), the manager's posterior is $\tilde{\theta}=E[\theta|\hat{\theta}]=\sqrt{e}\hat{\theta}$.[29] This posterior is the best guess

[28]Note that $\overline{m}^2=\left(\frac{d_j+d_{j-1}}{2}\right)^2$ is the expected value given a particular signal. Then, $E[\overline{m}^2]$ is the ex-ante expected value in the expression.

[29]Note that we care about the mean of the posterior and not the posterior distribution. The posterior mean distribution is uniform in $[-\sqrt{e}\overline{\theta},\sqrt{e}\overline{\theta}]$.

that a manager has over the true value θ, and he prefers to design a product that is closest to this estimation of consumers' tastes.

With the posterior, the headquarters has also inferences after she receives managers' reports that are $E[\tilde{\theta}_1^2|m]$ and $E[\tilde{\theta}_2^2|m]$ The optimal decision making under centralization are:

$$\begin{aligned} a_1^C(m_1,m_2) &= \frac{1+2\beta}{1+4\beta}E[\tilde{\theta}_1|m_1] + \frac{2\beta}{1+4\beta}E[\tilde{\theta}_2|m_2], \text{ and} \\ a_2^C(m_1,m_2) &= \frac{2\beta}{1+4\beta}E[\tilde{\theta}_1|m_1] + \frac{1+2\beta}{1+4\beta}E[\tilde{\theta}_2|m_2]. \end{aligned}$$

Then, replacing it in the expected profit of manager in country 1 we get the following terms:

$$\begin{aligned} (a_1^C-\theta_1)^2 &= \left(E[\tilde{\theta}_1|m_1]-\theta_1+\frac{2\beta\left(E[\tilde{\theta}_2|m_2]-E[\tilde{\theta}_1|m_1]\right)}{1+4\beta}\right)^2, \\ &= \left(\overline{m}_1-\theta_1\right)^2+\frac{4\beta^2\left(\overline{m}_2-\overline{m}_1\right)^2}{(1+4\beta)^2}+2\beta\left(\overline{m}_1-\theta_1\right)\frac{\left(\overline{m}_2-\overline{m}_1\right)}{(1+4\beta)}, \\ &= \left(\overline{m}_1-\theta_1\right)^2+\frac{4\beta^2\left(\overline{m}_2^2+\overline{m}_1^2-2\overline{m}_2\overline{m}_1\right)}{(1+4\beta)^2}+2\beta\left(\overline{m}_1-\theta_1\right)\frac{\left(\overline{m}_2-\overline{m}_1\right)}{(1+4\beta)}. \quad (2.26) \\ (a_1^C-a_2^C)^2 &= \left(\frac{E[\tilde{\theta}_1|m_1,m_2]-E[\tilde{\theta}_2|m_1,m_2]}{1+4\beta}\right)^2=\frac{\overline{m}_1^2-2\overline{m}_1\overline{m}_2+\overline{m}_2^2}{(1+4\beta)^2}. \quad (2.27) \end{aligned}$$

Notice that taking ex-ante expectation the following terms are null $E[\overline{m}_1\overline{m}_2]=0$, $E[(\overline{m}_1-\theta_1)(\overline{m}_2-\overline{m}_1)]=0$. Also note that $E[\overline{m}_1\theta_1]=E[\overline{m}_1^2]$. Also, note that $\overline{m}^2=E[\tilde{\theta}_1]^2=e_1\hat{\theta}_1^2$ and then $E[\overline{m}_1^2]=e_1E[\hat{\theta}_1^2]=e_1V_1\frac{\overline{\theta}_1^2}{3}$. Taking expectations

$$\begin{aligned} E[(a_1^C-\theta_1)^2] &= E[(\overline{m}_1-\theta_1)^2]+\frac{4\beta^2\left(E[\overline{m}_2^2]+E[\overline{m}_1^2]\right)}{(1+4\beta)^2}, \quad (2.28) \\ E[(a_1^C-a_2^C)^2] &= \frac{E[\overline{m}_1^2]+E[\overline{m}_2^2]}{(1+4\beta)^2}. \quad (2.29) \end{aligned}$$

Building the expected profits $E[\Pi]$ which add both terms weighted by 1 and β respectively.

$$E[\Pi_1] = \left(K(\beta)-E[(\overline{m}_1-\theta_1)^2]+\frac{\beta}{(1+4\beta)}(E[\overline{m}_1^2]+E[\overline{m}_2^2])\right). \quad (2.30)$$

where the term $E[(\overline{m}_1-\theta_1)^2]$ is the difference between the real realization of θ and what the HQ believes of $\tilde{\theta}$ after receiving the report. Note that the message of the posterior is equivalent as a message of the signal, then $\overline{m}=\sqrt{e}E[\hat{\theta}|m]$.

$$\begin{aligned} E[(\overline{m}-\theta)^2] &= E[\left(\overline{m}-\sqrt{e}\hat{\theta}+\sqrt{e}\hat{\theta}-\theta\right)^2], \\ &= E[\left(\sqrt{e}E[\hat{\theta}|m]-\sqrt{e}\hat{\theta}\right)^2]+E[\left(\sqrt{e}\hat{\theta}-\theta\right)^2]. \end{aligned}$$

with the first term being the communication accuracy of the signal, i.e.,$E[\left(\sqrt{e}E[\hat{\theta}|m]-\sqrt{e}\hat{\theta}\right)^2]=$

$eVE[\theta^2]$. The second term is the loss due to the lack of precision in the signal, i.e., $E[\left(\sqrt{e}\hat{\theta}-\theta\right)^2] = (1-e)E[\theta^2]$.[30] Recall that V is the proportion of the variance communicated, i.e., how accurate the communication is. We prove now that the omitted term is equal to zero, i.e., $E[\left(\overline{m}-\sqrt{e}\hat{\theta}\right)\left(\sqrt{e}\hat{\theta}-\theta\right)] = 0$.

Proof.

$$\begin{aligned} E[\left(\overline{m}-\sqrt{e}\hat{\theta}\right)\left(\sqrt{e}\hat{\theta}-\theta\right)] &= E[\overline{m}\left(\sqrt{e}\hat{\theta}-\theta\right)] - E[\sqrt{e}\hat{\theta}\left(\sqrt{e}\hat{\theta}-\theta\right)], \\ &= E[\overline{m}\left(\sqrt{e}\hat{\theta}-\theta\right)] - E[\sqrt{e}\hat{\theta}\left(\sqrt{e}\hat{\theta}-\theta\right)]. \end{aligned}$$

The first term is not problematic $E[\overline{m}\left(\sqrt{e}\hat{\theta}-\theta\right)] = 0$, but the second term deserves a little more of attention to notice that $E[\sqrt{e}\hat{\theta}\left(\sqrt{e}\hat{\theta}-\theta\right)] = E[e\hat{\theta}^2] - E[\sqrt{e}\hat{\theta}\theta] = 0$. I prove that $E[e\hat{\theta}^2] = eE[\theta^2] = E[\sqrt{e}\hat{\theta}\theta]$:

$$\begin{aligned} E[\sqrt{e}\hat{\theta}\theta] = E[\sqrt{e}\sqrt{e}\theta\theta + (1-\sqrt{e})\sqrt{e}x\theta] = E[e\theta^2 + (1-\sqrt{e})\sqrt{e}x\theta] =, \\ eE[\theta^2] + (1-\sqrt{e})\sqrt{e} \underbrace{E[x\theta]}_{=0 \text{ independent}} = eE[\theta^2]. \end{aligned} \tag{2.31}$$

Now, we show the other part.

$$E[e\hat{\theta}^2] = E[\sqrt{e}e\theta^2 + (1-\sqrt{e})ex^2] = \sqrt{e}eE[\theta^2] + (1-\sqrt{e})eE[x^2] = eE[\theta^2]. \tag{2.32}$$

The proof is complete. □

REMARK: Since $E[\overline{m}_1] = 0$ and $E[\theta_1] = 0$, then $E[\overline{m}_1^2] = e_1 E[\hat{\theta}_1^2] = e_1 V_1 \frac{\overline{\theta}_1^2}{3}$

$$E[\Pi_1] = \left(K(\beta) - \frac{\overline{\theta}^2}{3}\left[1 - \left(1 - V_1\frac{1+3\beta}{(1+4\beta)}\right)e_1 + e_2 V_2 \frac{\beta}{(1+4\beta)}\right]\right).$$

And the division payoff is $E[U_1] = (1-s)\Pi_1 + s\Pi_2$. Adding up for both divisions

$$\begin{aligned} E[\Pi_1] + E[\Pi_2] &= \left(2K(\beta) - \frac{\overline{\theta}^2}{3}\left[1 - \left(1 - V_1\frac{1+2\beta}{(1+4\beta)}\right)e_1 - \left(1 - V_2\frac{1+2\beta}{(1+4\beta)}\right)e_2\right]\right), \\ &= \left(2K(\beta) - \frac{\overline{\theta}^2}{3}\left[1 - e_1\psi_1 - e_2\psi_2\right]\right). \end{aligned}$$

A similarly analysis accounts for the case under decentralization. For further details you can see Proposition 5 in Rantakari (2010).

[30] $E[\left(\sqrt{e}\hat{\theta}-\theta\right)^2] = E[e\hat{\theta}^2] + E[\theta^2] - 2E[\sqrt{e}\hat{\theta}\theta] = (1-e)E[\theta^2]$, which equation (2.31) proves that $E[e\hat{\theta}^2] = eE[\theta^2]$ and $E[\sqrt{e}\hat{\theta}\theta] = eE[\theta^2]$.

2.7.5 Indirect Profit Function before Communication and Decisions

Value created and value captured:

$$\begin{aligned}\psi_1(s) &= 1-\phi_{11}(s)-\phi_{12}(s),\\ \tilde{\psi}_{11}(s) &= (1-s)(1-\phi_{11}(s))-s\phi_{12}(s),\\ \tilde{\psi}_{11}(s) &= (1-s)\psi_1(s)+(1-2s)\phi_{12}(s).\end{aligned}$$

It is worth noting that from communication and decision making both $\phi_{11}(s)$ and $\phi_{12}(s)$ are decreasing in s. Under centralization the reason is that communication is becoming more precise. Under decentralization both decisions are less biased and communication is more precise. The externality $\phi_{12}(s)$ is almost always greater under decentralization.[31] This externality reflects the high responsiveness to local conditions (to increase revenues) which also reduces the coordination what is translated into higher production costs.
The expected profits for a particular product in division 1 are,

$$E[\Pi_1] = K(\beta) - \left[1 - e_1 \underbrace{[1-\Lambda_1-\Gamma_{11}(1-V_1)]}_{\text{due to 1 information}} + e_2 \underbrace{[\Lambda_1+\Gamma_{21}(1-V_2)]}_{\text{due to 2 information}} + \mu C(e_1)\right]\sigma_\theta^2. \quad (2.33)$$

Expected value generated by division 1:

$$\begin{aligned}E[\pi_{1A}]+E[\pi_{1B}] &= K_A-\sigma_A^2+e_{1A}\psi_{1A}\sigma_A^2-\mu_A(t_{1A})C(e_{1A})\sigma_A^2,\\ &+ K_B-\sigma_B^2+e_{1B}\psi_{1B}\sigma_B^2-\mu_B(1-t_{1A})C(e_{1B})\sigma_B^2.\end{aligned}$$

Expected value appropriated by local manager in division 1:

$$\begin{aligned}K-\sigma_A^2+e_{1A}\tilde{\psi}_{11A}\sigma_A^2+e_{2A}\tilde{\psi}_{21A}\sigma_A^2-\mu_A(t_{1A})C(e_{1A})\sigma_A^2,\\ K-\sigma_B^2+e_{1B}\tilde{\psi}_{11B}\sigma_B^2+e_{2B}\tilde{\psi}_{21B}\sigma_B^2-\mu_B(1-t_{1A})C(e_{1B})\sigma_B^2.\end{aligned}$$

2.7.6 Headquarters Allocates Resources

2.7.6.1 Assumption

Sufficient Assumption for $\Pi_{t_At_A} \leq 0$ (and when managers allocate t) is:

A1- $\mu(t)\mu''(t)C(e)C''(e) > C'(e)^2\mu'(t)^2$ for all t and e.

This assumption holds for functions $\mu(t) = \overline{\mu}\left(\frac{0.5}{t}\right)^b$ with $b \in [0,1]$ and $\overline{\mu} \in \mathbb{R}_+$, and $C(e) = -(e+\log(1-e))$.

[31] Only if $s \to 0$ and $\beta \to +\infty$ the externality is a little greater under centralization.

2.7.6.2 Managers' Choices

Each manager solves:

$$\max_{e_{1A},e_{1B}} 2K-\sigma_A^2-\sigma_B^2+(e_{1A}\tilde{\psi}_{11A}+e_{2A}\tilde{\psi}_{21A}-\mu_A(t_{1A})C(e_{1A}))\,\sigma_A^2,$$
$$+(e_{1B}\tilde{\psi}_{11B}+e_{2B}\tilde{\psi}_{21B}-\mu_B(1-t_{1A})C(e_{1B}))\,\sigma_B^2.$$

The values of e_A and e_B are determined by $\tilde{\psi}\sigma^2-\mu(t)C'(e)\sigma^2=0$. The comparative static are: i- $\frac{\partial e}{\partial s}=\frac{\partial\tilde{\psi}}{\partial s}\frac{1}{\mu C''(e)}<0$; ii- $\frac{\partial e}{\partial t}=-\frac{\mu'(t)C'(e)}{\mu(t)C''(e)}>0$; $\frac{\partial e}{\partial\sigma^2}=0$; and $\frac{\partial e_A}{\partial s_B}=0$. Also

$$\frac{\partial^2 e}{\partial s\partial t}=-\frac{\partial\tilde{\psi}}{\partial s}\frac{1}{[\mu C''(e)]^2}\frac{\mu'(t)}{C''(e)}[C''(e)^2-C'(e)C'''(e)]>0 \qquad \text{if } e>0.5,$$
$$\frac{\partial^2 e}{\partial t^2}=-\frac{C'(e)C''(e)[\mu(t)\mu''(t)-\mu'(t)^2]-\mu'(t)^2\frac{C'(e)}{C''(e)}[C''(e)^2-C'''(e)C'(e)]}{[\mu C''(e)]^2}<0,$$
$$\frac{\partial^2 e}{\partial s^2}=\left[\frac{\partial^2\tilde{\psi}}{\partial s^2}-\left(\frac{\partial\tilde{\psi}}{\partial s}\right)^2\frac{C'''(e)}{\mu C''(e)^2}\right]\frac{1}{\mu C''(e)}\gtrless 0.$$

Where $\frac{\partial^2 e}{\partial s^2}\le 0$ for μ sufficiently small. We can define $t_B=1-t_A$ and we have $\frac{\partial e_B}{\partial t_A}=\frac{\mu'(1-t_A)C'(e_B)}{\mu(1-t_A)C''(e_B)}<0$.

2.7.6.3 Headquarters' Design

The HQ solves:

$$\max_{s_A,s_B,t_A} E[\pi_{1A}]+E[\pi_{1B}]=K-\sigma_A^2+(e_{1A}\psi_{1A}-\mu_A(t_{1A})C(e_{1A}))\,\sigma_A^2,$$
$$+K-\sigma_B^2+(e_{1B}\psi_{1B}-\mu_B(1-t_{1A})C(e_{1B}))\,\sigma_B^2. \tag{2.34}$$

The first order conditions are,

$$\sigma_A^2\left[\frac{\partial\psi_A}{\partial s_A}e_A+\frac{\partial e_A}{\partial s_A}(\psi_A-\tilde{\psi}_A)\right]=0,$$
$$\sigma_B^2\left[\frac{\partial\psi_B}{\partial s_B}e_B+\frac{\partial e_B}{\partial s_B}(\psi_B-\tilde{\psi}_B)\right]=0,$$
$$-\mu'(t_A)C(e_A)\sigma_A^2+\frac{\partial e_A}{\partial t_A}(\psi_A-\tilde{\psi}_A)\sigma_A^2+\mu'(1-t_A)C(e_B)\sigma_B^2+\frac{\partial e_B}{\partial t_A}(\psi_B-\tilde{\psi}_B)\sigma_B^2=0.$$

The second order conditions are

$$\begin{pmatrix} \Pi_{s_As_A} & 0 & \frac{\partial\psi_A}{\partial s_A}\frac{\partial e_A}{\partial t_A}+\frac{\partial^2 e_A}{\partial s_A\partial t_A}(\psi_A-\tilde{\psi}_A) \\ 0 & \Pi_{s_Bs_B} & \frac{\partial\psi_B}{\partial s_B}\frac{\partial e_B}{\partial t_A}+\frac{\partial^2 e_B}{\partial s_B\partial t_A}(\psi_B-\tilde{\psi}_B) \\ \sigma_A^2[\frac{\partial\psi_A}{\partial s_A}\frac{\partial e_A}{\partial t_A}+\frac{\partial^2 e_A}{\partial s_A\partial t_A}(\psi_A-\tilde{\psi}_A)] & \sigma_B^2[\frac{\partial\psi_B}{\partial s_B}\frac{\partial e_B}{\partial t_A}+\frac{\partial^2 e_B}{\partial s_B\partial t_A}(\psi_B-\tilde{\psi}_B)] & \Pi_{t_At_A} \end{pmatrix}\begin{pmatrix} ds_A \\ ds_B \\ dt_A \end{pmatrix}.$$

Where $\Pi_{t_At_A} \equiv \sigma_A^2\Big[-\mu_A''C_A - \mu_A'C_{1A}\frac{\partial e_A}{\partial t_A} + \frac{\partial^2 e_A}{\partial t_A^2}(\psi_A - \tilde{\psi}_A)\Big] + \sigma_B^2\Big[-\mu_B''C_B + \mu_B'C_{1B}\frac{\partial e_B}{\partial t_A} + \frac{\partial^2 e_B}{\partial t_A^2}(\psi_B - \tilde{\psi}_B)\Big]$.[32] We can split $\Pi_{t_At_A}$ into two parts $\Pi_{t_At_A} \equiv \Pi^A_{t_At_A} + \Pi^B_{t_At_A}$. Also $\Pi_{s_As_A} \equiv \frac{\partial^2\psi_A}{\partial s_A^2}e_A + \frac{\partial\psi_A}{\partial s_A}\frac{\partial e_A}{\partial s_A} + \frac{\partial^2 e_A}{\partial s_A^2}(\psi_A - \tilde{\psi}_A) + \frac{\partial e_A}{\partial s_A}\frac{\partial(\psi_A - \tilde{\psi}_A)}{\partial s_A}$ (≤ 0 in our relevant domain). The determinant is defined by $|J| \equiv \Pi_{s_As_A}[\Pi_{s_Bs_B}\Pi_{t_At_A} - \Pi^2_{s_Bt_B}] - \Pi_{s_Bs_B}\Pi^2_{s_At_A}$ or
$|J| \equiv \Pi_{s_As_A}[\Pi_{s_Bs_B}\Pi^B_{t_At_A} - \Pi^2_{s_Bt_B}] + \Pi_{s_Bs_B}[\Pi_{s_As_A}\Pi^A_{t_At_A} - \Pi^2_{s_At_A}] < 0$

$$\begin{pmatrix} 0 \\ 0 \\ \mu_A'(t_A)C(e_A) - \frac{\partial e_A}{\partial t_A}(\psi_A - \tilde{\psi}_A) \end{pmatrix}(d\sigma_A^2) = \begin{pmatrix} 0 \\ 0 \\ -\Pi_{t_A\sigma_A^2} \end{pmatrix}(d\sigma_A^2).$$

Then, we have $\Pi_{s_As_A} \leq 0$, $\Pi_{s_Bs_B} \leq 0$, $\Pi_{t_At_A} \leq 0$ but $\Pi_{s_At_A} \geq 0$, $-\Pi_{t_A\sigma_A^2} \leq 0$ and $\Pi_{s_Bt_A} \leq 0$. I took common factor $\sigma_A^2\sigma_B^2$, then:

$$\frac{\partial s_A}{\partial \sigma_A^2} = \frac{-\Pi_{t_A\sigma_A^2}[-\Pi_{s_At_A}\Pi_{s_Bs_B}]}{|J|} > 0, \tag{2.35}$$

$$\frac{\partial t_A}{\partial \sigma_A^2} = \frac{-\Pi_{t_A\sigma_A^2}[\Pi_{s_As_A}\Pi_{s_Bs_B}]}{|J|} > 0, \tag{2.36}$$

$$\frac{\partial s_B}{\partial \sigma_A^2} = -\frac{-\Pi_{t_A\sigma_A^2}[\Pi_{s_Bt_A}\Pi_{s_As_A}]}{|J|} < 0. \tag{2.37}$$

Finally, the headquarters chooses to centralize or decentralize decision rights in each market considering the one that generates higher value, i.e., higher $[\psi_Ae_A - \mu_AC(e_A)]\sigma_A^2 + [\psi_Be_B - \mu_BC(e_B)]\sigma_B^2$. If $\overline{\mu}$ is sufficiently low and $\sigma_A^2 \sim \sigma_B^2$, the headquarters decentralizes decision rights about both products. If σ_A^2 increases, s_A and t_A increase and s_B decreases, making more likely that the firm prefers to centralize decision rights of product B (see equations (2.35), (2.36), and (2.37)). There is a cutoff $\tilde{\sigma}_A^2$ above which the headquarters decentralizes decision making about product A and centralizes decision making about product B.

2.7.7 Managers Allocate Resources

2.7.7.1 Managers' Choices

Each manager solves:

$$\max_{e_{1A},e_{1B},t_{1A}} 2K - \sigma_A^2 - \sigma_B^2 + (e_{1A}\tilde{\psi}_{11A} + e_{2A}\tilde{\psi}_{21A} - \mu_A(t_{1A})C(e_{1A}))\,\sigma_A^2,$$
$$+ (e_{1B}\tilde{\psi}_{11B} + e_{2B}\tilde{\psi}_{21B} - \mu_B(1 - t_{1A})C(e_{iB}))\,\sigma_B^2.$$

[32]Note that, replacing the expression of $\frac{\partial e_A}{\partial t_A}$, each term in brackets can be re-expressed as $\Big[-\frac{1}{\mu_AC_{A11}}(\mu_A\mu_A''C_AC_{A11} - \mu_A'^2C_{1A}^2) + \frac{\partial^2 e_A}{\partial t_A^2}(\psi_A - \tilde{\psi}_A)\Big] < 0$ guaranteeing that $\Pi_{t_At_A} \leq 0$.

The first order conditions are

$$\begin{aligned}
\tilde{\psi}_A\sigma_A^2 - \mu_A(t_A)C_A'\sigma_A^2 &= 0,\\
\tilde{\psi}_B\sigma_B^2 - \mu_B(1-t_A)C_B'\sigma_B^2 &= 0,\\
-\mu_A'(t_A)C(e_A)\sigma_A^2 + \mu_B'(1-t_A)C(e_B)\sigma_B^2 &= 0.
\end{aligned}$$

Differentiating the first order conditions we have,

$$\begin{pmatrix} -\mu_A(t_A)C_A''\sigma_A^2 & 0 & -\mu_A'(t_A)C_A'\sigma_A^2 \\ 0 & -\mu_B(1-t_A)C_B''\sigma_B^2 & \mu_B'(1-t_A)C_B'\sigma_B^2 \\ -\mu_A'(t_A)C_A'\sigma_A^2 & \mu_B'(1-t_A)C_1(e_B)\sigma_B^2 & -(\mu_A''(t_A)C(e_A)\sigma_A^2 + \mu_B''(1-t_A)C(e_B)\sigma_B^2) \end{pmatrix} \begin{pmatrix} de_A \\ de_B \\ dt_A \end{pmatrix}.$$

$$\begin{pmatrix} 0 & -\frac{\partial\tilde{\psi}_A}{\partial s_A}\sigma_A^2 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\partial\tilde{\psi}_B}{\partial s_B}\sigma_B^2 \\ \mu_A'(t_A)C(e_A) & 0 & \mu_B'(t_B)C(e_B) & 0 \end{pmatrix} \begin{pmatrix} d\sigma_A^2 \\ ds_A \\ d\sigma_B^2 \\ ds_B \end{pmatrix}.$$

where

$$\begin{aligned}|J| = -\mu_A(t_A)C_A''\sigma_A^2\Bigg[\mu_B(1-t_A)C_B''\sigma_B^2\bigg(\mu_A''(t_A)C(e_A)\sigma_A^2 + \mu_B''(1-t_A)C(e_B)\sigma_B^2\bigg) - [\mu_B'(1-t_A)C_B'\sigma_B^2]^2\Bigg] \\ +\mu_A'(t_A)C_A'\sigma_A^2\Big[\mu_A'(t_A)C_A'\sigma_A^2\mu_B(1-t_A)C_B''\sigma_B^2\Big]\end{aligned}$$

Assumption $A1$, i.e., $\mu(t)\mu''(t)C(e)C''(e) > C'(e)^2\mu'(t)^2$ for all t and e, is a sufficient condition for $|J| < 0$. The comparatives static respect to s_A are

$$\frac{\partial e_A}{\partial s_A} = \frac{\partial\tilde{\psi}_A}{\partial s_A}\frac{\sigma_B^2\Big[\mu_B C_B''\mu_B''C(e_B) - [\mu_B'C_B']^2\Big] + \mu_B C_B''\mu_A''C(e_A)\sigma_A^2}{\sigma_B^2\mu_A C_A''\Big[\mu_B C_B''\mu_B''C(e_B) - [\mu_B'C_B']^2\Big] + \mu_B C_B''\Big[\mu_A''C(e_A)\mu_A C_A'' - (\mu_A'C_A')^2\Big]\sigma_A^2} < 0.$$

$$\frac{\partial e_B}{\partial s_A} = -\frac{\partial\tilde{\psi}_A}{\partial s_A}\frac{\mu_A'(t_A)C_A'\mu_B'(1-t_A)C_B'\sigma_A^2}{\sigma_B^2\mu_A C_A''\Big[\mu_B C_B''\mu_B''C(e_B) - [\mu_B'C_B']^2\Big] + \mu_B C_B''\Big[\mu_A''C(e_A)\mu_A C_A'' - (\mu_A'C_A')^2\Big]\sigma_A^2} > 0.$$

$$\frac{\partial t_A}{\partial s_A} = -\frac{\partial\tilde{\psi}_A}{\partial s_A}\frac{\mu_A'(t_A)C_A'\mu_B(1-t_A)C_B''\sigma_A^2}{\sigma_B^2\mu_A C_A''\Big[\mu_B C_B''\mu_B''C(e_B) - [\mu_B'C_B']^2\Big] + \mu_B C_B''\Big[\mu_A''C(e_A)\mu_A C_A'' - (\mu_A'C_A')^2\Big]\sigma_A^2} < 0.$$

The comparatives static respect to σ_A^2 are

$$\frac{\partial e_A}{\partial \sigma_A^2} = \frac{\mu_A'(t_A)^2 C(e_A) C_A' \mu_B (1-t_A) C_B''}{\sigma_B^2 \mu_A C_A'' \Big[\mu_B C_B'' \mu_B'' C(e_B) - [\mu_B' C_B']^2\Big] + \mu_B C_B'' \Big[\mu_A'' C(e_A) \mu_A C_A'' - (\mu_A' C_A')^2\Big] \sigma_A^2} > 0.$$

$$\frac{\partial e_B}{\partial \sigma_A^2} = -\frac{\mu_A'(t_A) C(e_A) \mu_A(t_A) C_A'' \mu_B' (1-t_A) C_1(e_B)}{\sigma_B^2 \mu_A C_A'' \Big[\mu_B C_B'' \mu_B'' C(e_B) - [\mu_B' C_B']^2\Big] + \mu_B C_B'' \Big[\mu_A'' C(e_A) \mu_A C_A'' - (\mu_A' C_A')^2\Big] \sigma_A^2} < 0.$$

$$\frac{\partial t_A}{\partial \sigma_A^2} = -\frac{\mu_A'(t_A) C(e_A) \mu_A(t_A) C_A'' \mu_B (1-t_A) C_B''}{\sigma_B^2 \mu_A C_A'' \Big[\mu_B C_B'' \mu_B'' C(e_B) - [\mu_B' C_B']^2\Big] + \mu_B C_B'' \Big[\mu_A'' C(e_A) \mu_A C_A'' - (\mu_A' C_A')^2\Big] \sigma_A^2} > 0.$$

2.7.7.2 Headquarters' Design

the HQ's problem is $\max_{s_A,s_B} E[\pi_{1A}] + E[\pi_{1B}]$

$$\max_{s_A,s_B} K - \sigma_A^2 + (e_{1A}\psi_{1A} - \mu_A(t_{1A})C(e_{1A}))\,\sigma_A^2 + K - \sigma_B^2 + (e_{1B}\psi_B - \mu_B(1-t_{1A})C(e_{1B}))\,\sigma_B^2. \quad (2.38)$$

the first order conditions are

$$\begin{aligned}
\frac{\partial \psi_A}{\partial s_A} e_A + \frac{\partial e_A}{\partial s_A}(\psi_A - \tilde{\psi}_A) + \frac{\sigma_B^2}{\sigma_A^2}\frac{\partial e_B}{\partial s_A}(\psi_B - \tilde{\psi}_B) = 0,\\
\frac{\partial \psi_B}{\partial s_B} e_B + \frac{\partial e_B}{\partial s_B}(\psi_B - \tilde{\psi}_B) + \frac{\sigma_A^2}{\sigma_B^2}\frac{\partial e_A}{\partial s_B}(\psi_A - \tilde{\psi}_A) = 0.
\end{aligned}$$

Notice that these first order conditions capture the effects of s_A on ψ_A and on e_A like when the headquarters allocate resources. However, it also captures the indirect effects of s_A on e_A and e_B through a change in t_A. This indirect effects are of second order magnitude and do not modify the main trade-off when choosing the organizational design.

The headquarters chooses to centralize or decentralize decision rights considering the sum of $[\psi_A e_A - \mu_A C(e_A)]\sigma_A^2 + [\psi_B e_B - \mu_B C(e_B)]\sigma_B^2$. If $\overline{\mu}$ is sufficiently low and $\sigma_A^2 \sim \sigma_B^2$, the headquarters decentralizes decision rights about both products. Since both products are similar in terms of returns to differentiation the headquarters follows a strategy of product differentiation in both products. If σ_A^2 increases, s_A and t_A increase and s_B decreases, making more likely that the firm prefers to centralize decision rights of product B. There is a cutoff $\hat{\sigma}_A^2$ above which the headquarters decentralizes decision making about product A and centralizes decision making about product B.

Note that the change in the organizational design that yields decentralization in product A and centralization in product B has an discrete jump in the optimal shares. The jump in s arises because the headquarters centralizes product B which non-locally reduces s_B; this also modifies $\psi_B - \tilde{\psi}_B$ and, consequently, provides incentives to reduce s_A and to increase s_B.

2.7.7.3 Redefining Effects

Given an efficient allocation of resources, t_A^*, the optimal s is defined by $\sigma_A^2[\frac{\partial \psi_A}{\partial s_A}e_A + \frac{\partial e_A}{\partial s_A}(\psi_A - \tilde{\psi}_A)] = 0$. However, if local managers allocate resources the first order condition becomes:

$$\sigma_A^2[\frac{\partial \psi_A}{\partial s_A}e_A + \frac{\partial e_A}{\partial s_A}(\psi_A - \tilde{\psi}_A)] + \frac{\partial t_A}{\partial s_A}\frac{\partial e_A}{\partial t_A}[\psi_A\sigma_A^2 - \tilde{\psi}_A\sigma_A^2] + \frac{\partial t_A}{\partial s_A}\frac{\partial e_B}{\partial t_A}[\psi_B\sigma_B^2 - \tilde{\psi}_B\sigma_B^2],$$
$$+\frac{\partial t_A}{\partial s_A}[-\mu_A'(t_A)C_A(e_A)\sigma_A^2 + \mu_B'(1-t_A)C_B(e_B)\sigma_B^2] = 0.$$

with $\frac{\partial t_A}{\partial s_A} < 0$ and $\frac{\partial t_A}{\partial s_B} > 0$ leading to

$$\begin{aligned}
\frac{\partial \psi_A}{\partial s_A}e_A + \frac{\partial e_A}{\partial s_A}(\psi_A - \tilde{\psi}_A) &= -\frac{\partial t_A}{\partial s_A}\frac{\sigma_B^2}{\sigma_A^2}\Big[\frac{\sigma_A^2}{\sigma_B^2}\frac{\partial e_A}{\partial t_A}[\psi_A - \tilde{\psi}_A] + \frac{\partial e_B}{\partial t_A}[\psi_B - \tilde{\psi}_B]\Big],\\
\frac{\partial \psi_B}{\partial s_B}e_B + \frac{\partial e_B}{\partial s_B}(\psi_B - \tilde{\psi}_B) &= -\frac{\partial t_A}{\partial s_B}\Big[\frac{\sigma_A^2}{\sigma_B^2}\frac{\partial e_A}{\partial t_A}[\psi_A - \tilde{\psi}_A] + \frac{\partial e_B}{\partial t_A}[\psi_B - \tilde{\psi}_B]\Big].
\end{aligned}$$

calling $\varphi \equiv \frac{\sigma_A^2}{\sigma_B^2}\frac{\partial e_A}{\partial t_A}[\psi_A - \tilde{\psi}_A] + \frac{\partial e_B}{\partial t_A}[\psi_B - \tilde{\psi}_B]$. It is easier to separate the indirect effect of s_A and s_B on t_A from the direct effect of s_A on e_A and s_B on e_B. This alternative expression is used in the following section.

2.8 Appendix C: Extension

2.8.1 Extension in Section 2.4.1: Coordination

The first order conditions of the Headquarters problem are

$$-\frac{\partial \mu_A}{\partial t_A}\sigma_A^2\left[C(e_A) - \frac{\partial e_A}{\partial \mu_A}[\psi_A - \mu_A C'(e_A)]\right] = -\frac{\partial \mu_B}{\partial t_B}\sigma_B^2\left[C(e_B) - \frac{\partial e_B}{\partial \mu_B}[\psi_B - \mu_B C'(e_B)]\right], \tag{2.39}$$

$$\sigma_A^2\left[e_A\frac{\partial \psi_A}{\partial s_A} + \frac{\partial e_A}{\partial s_A}[\psi_A - \mu_A C'(e_A)]\right] = 0, \tag{2.40}$$
$$\sigma_B^2\left[e_B\frac{\partial \psi_B}{\partial s_B} + \frac{\partial e_B}{\partial s_B}[\psi_B - \mu_B C'(e_B)]\right] = 0. \tag{2.41}$$

$$K'(\beta_A) - \sigma_A^2\left[e_A\frac{\partial \psi_A}{\partial \beta_A} + \frac{\partial e_A}{\partial \beta_A}[\psi_A - \mu_A C'(e_A)\right] = 0, \tag{2.42}$$
$$K'(\beta_B) - \sigma_B^2\left[e_B\frac{\partial \psi_B}{\partial \beta_B} + \frac{\partial e_B}{\partial \beta_B}[\psi_B - \mu_B C'(e_B)\right] = 0. \tag{2.43}$$

First, we could replace the optimal effort choice $\mu_A C'(e_A) = \tilde{\psi}$ into all first order conditions. Once t_A is chosen, the decisions of (s,β,g) are described by Rantakari (2010). Given the convexity of $\mu(t)$, the decision of t_A depends directly on who makes effort choice. In either case there exists increasing relations $\tilde{\sigma}_B^2(\sigma_A^2)$ and $\tilde{\sigma}_A^2(\sigma_B^2)$ such that product A is centralized if $\sigma_A^2 < \tilde{\sigma}_A^2(\sigma_B^2)$ and decentralized if $\sigma_A^2 \geq \tilde{\sigma}_A^2(\sigma_B^2)$, and product B is centralized if $\sigma_B^2 < \tilde{\sigma}_B^2(\sigma_A^2)$ and decentralized if $\sigma_B^2 \geq \tilde{\sigma}_B^2(\sigma_A^2)$.

If local division managers control resources, the first order conditions are

$$\sigma_A^2\left[e_A\frac{\partial\psi_A}{\partial s_A}+\frac{\partial e_A}{\partial s_A}[\psi_A-\mu_A C'(e_A)]\right] = -\frac{\partial t_A}{\partial s_A}\varphi, \tag{2.44}$$

$$\sigma_B^2\left[e_B\frac{\partial\psi_B}{\partial s_B}+\frac{\partial e_B}{\partial s_B}[\psi_B-\mu_B C'(e_B)]\right] = -\frac{\partial t_A}{\partial s_B}\varphi. \tag{2.45}$$

$$K'(\beta_A)-\sigma_A^2\left[e_A\frac{\partial\psi_A}{\partial\beta_A}+\frac{\partial e_A}{\partial\beta_A}[\psi_A-\mu_A C'(e_A)\right] = -\frac{\partial t_A}{\partial\beta_A}\varphi, \tag{2.46}$$

$$K'(\beta_B)-\sigma_B^2\left[e_B\frac{\partial\psi_B}{\partial\beta_B}+\frac{\partial e_B}{\partial\beta_B}[\psi_B-\mu_B C'(e_B)\right] = -\frac{\partial t_A}{\partial\beta_B}\varphi, \tag{2.47}$$

with

$$\varphi \equiv \sigma_A^2\frac{\partial\mu_A}{\partial t_A}\frac{\partial e_A}{\partial\mu_A}\left[\psi_A-\mu_A C'(e_A)\right]-\sigma_B^2\frac{\partial\mu_B}{\partial t_B}\frac{\partial e_B}{\partial\mu_B}\left[\psi_B-\mu_B C'(e_B)\right]. \tag{2.48}$$

We can replace the optimal allocation of time chosen by managers into φ. There exists increasing relations $\tilde{\sigma}_{BM}^2(\sigma_A^2)$ and $\tilde{\sigma}_{AM}^2(\sigma_B^2)$ such that decision making about product A is centralized if $\sigma_A^2 < \tilde{\sigma}_{AM}^2(\sigma_B^2)$ and decentralized if $\sigma_A^2 \geq \tilde{\sigma}_{AM}^2(\sigma_B^2)$, and decision making about product B is centralized if $\sigma_B^2 < \tilde{\sigma}_{BM}^2(\sigma_A^2)$ and decentralized if $\sigma_B^2 \geq \tilde{\sigma}_{BM}^2(\sigma_A^2)$.

When $\sigma_A^2 > \sigma_B^2$ then $\varphi > 0$, and thus the firm prefers to allocate more resources in product A than implemented by local managers, i.e., $t_A < t_A^*$. The right hand side of equations (2.44) and (2.46) are positive and the right hand side of equations (2.45) and (2.47) are negative. The headquarters aligns incentive, s, and integrate divisions, β, to affect the resource allocation choice of local managers. When $\sigma_A^2 > \sigma_B^2$, the headquarters reduces incentive alignment of product A, i.e., ∇s_A, increases incentive alignment of product B, i.e., Δs_B, reduces integration of product A, i.e $\nabla\beta_A$, and increases integration of product B, i.e., $\Delta\beta_B$. In other words, local managers put too much resources in product B, because they underestimate the opportunity cost of resources. The headquarters finds less important to provide incentives for information acquisition in product B and then decentralization appears more profitable.

2.8.2 Extension in Section 2.4.2: Delegation

The headquarters objective function is:

$$E[\pi_{iA}]+E[\pi_{iB}]=\left\{K_A-[1-(e_A+\alpha e_B)\psi_A+\mu_A C(e_{iA})]\,\sigma_A^2\right\}+\left\{K_B-[1-(e_B+\alpha e_A)\psi_B+\mu_B C(e_{iB})]\,\sigma_B^2\right\}.$$

When the headquarters control resources, the first order conditions are

$$-\frac{\partial\mu_A}{\partial t_A}\sigma_A^2\left[C(e_A)-\frac{\partial e_A}{\partial\mu_A}[\psi_A+\frac{\sigma_B^2}{\sigma_A^2}\alpha\psi_B-\mu_A C'(e_A)]\right]=-\frac{\partial\mu_B}{\partial t_B}\sigma_B^2\left[C(e_B)-\frac{\partial e_B}{\partial\mu_B}[\psi_B+\frac{\sigma_A^2}{\sigma_B^2}\alpha\psi_A-\mu_B C'(e_B)]\right]. \tag{2.49}$$

$$\sigma_A^2\left[(e_A+\alpha e_B)\frac{\partial\psi_A}{\partial s_A}+\frac{\partial e_A}{\partial s_A}[\psi_A+\frac{\sigma_B^2}{\sigma_A^2}\alpha\psi_B-\mu_A C'(e_A)]\right] = 0, \tag{2.50}$$
$$\sigma_B^2\left[(e_B+\alpha e_A)\frac{\partial\psi_B}{\partial s_B}+\frac{\partial e_B}{\partial s_B}[\psi_B+\frac{\sigma_A^2}{\sigma_B^2}\alpha\psi_A-\mu_B C'(e_B)]\right] = 0. \tag{2.51}$$

Once t_A is chosen, the effort exerted by local managers is given by $\tilde{\psi}_A=\mu_A C'(\hat{e}_A)$. Note that the inefficiency in effort choice is given by $\psi_A-\tilde{\psi}_A+\frac{\sigma_B^2}{\sigma_A^2}\alpha\psi_B$.
If local division managers control resources, the first order conditions are

$$\sigma_A^2\left[(e_A+\alpha e_B)\frac{\partial\psi_A}{\partial s_A}+\frac{\partial e_A}{\partial s_A}[\psi_A+\frac{\sigma_B^2}{\sigma_A^2}\alpha\psi_B-\mu_A C'(e_A)]\right] = -\frac{\partial t_A}{\partial s_A}\varphi, \tag{2.52}$$
$$\sigma_B^2\left[(e_B+\alpha e_A)\frac{\partial\psi_B}{\partial s_B}+\frac{\partial e_B}{\partial s_B}[\psi_B+\frac{\sigma_A^2}{\sigma_B^2}\alpha\psi_A-\mu_B C'(e_B)]\right] = -\frac{\partial t_A}{\partial s_B}\varphi. \tag{2.53}$$

with

$$\varphi \equiv \sigma_A^2\frac{\partial\mu_A}{\partial t_A}\frac{\partial e_A}{\partial\mu_A}\left[\psi_A+\frac{\sigma_B^2}{\sigma_A^2}\alpha\psi_B-\mu_A C'(e_A)\right]-\sigma_B^2\frac{\partial\mu_B}{\partial t_B}\frac{\partial e_B}{\partial\mu_B}\left[\psi_B+\frac{\sigma_A^2}{\sigma_B^2}\alpha\psi_A-\mu_B C'(e_B)\right] \tag{2.54}$$

However, the effort choice is now given by $\tilde{\psi}_A+\frac{\sigma_B^2}{\sigma_A^2}\alpha\tilde{\psi}_B=\mu_A C'(\hat{e}_A)$, and the inefficiency in effort choice is given by $\psi_A-\tilde{\psi}_A+\frac{\sigma_B^2}{\sigma_A^2}\alpha(\psi_B-\tilde{\psi}_B)$. The headquarters faces a trade-off between allocating resources efficiently and internalizing the learning externality. If the firm is organized by regional divisions and resource allocation is delegated to local managers, there is an inefficiency in resource allocation but local managers internalize that learning about one product has a positive externality in learning about the other product.

Chapter 3

Product Reliability, Consumers' Complaints and Market Performance: The case of Consumers' Associations.

3.1 Introduction

In their dealings with retailers and suppliers, regulations and warranties ensure that consumers can seek a repair, a replacement or a refund if the good they have purchased is faulty. The evidence, however, indicates that few consumers pursue any form of compensation (Best and Andreasen, 1977; Huppertz, 2007), suggesting that, for most consumers, claiming costs are high (see Huppertz (2007), which is based on Hirschmann (1970)) and providing a rationale for the role that consumers' associations play in helping consumers channel their complaints. In this paper, we analyze the monopolist's pricing and product reliability problem when consumers are entitled to product replacement and we assess the implications of a decrease in consumers' claiming costs due to, for instance, the appearance of consumers' associations. Our results suggest that consumers' associations could, instead, lower product reliability.

Our results hinge on how the firm deals with the replacement decision of consumers. In its choice of failure rate, the firm internalizes three effects. On the one hand, the failure rate affects directly the cost of production, through the manufacturing cost, and indirectly, through the number of units it must produce to replace the claimed faulty ones (including the associated indirect cost per replacement). On the other hand, the failure rate affects the expected utility of consumers because it modifies the probability of consuming a non-faulty good either when buying or when receiving a replacement of a faulty unit. When choosing the failure rate, the firm's trade-off among these three effects depends on the consumers' claiming cost. A decrease in the claiming cost implies more units being replaced and thus a higher cost of production, and a higher expected utility both from complaint and consumption, which implies a higher demand. Alone, the manufacturing cost of the replacement units would make the firm increase the level of its product reliability, but the presence of the expected utility effect may yield opposite results.

To study the effect of a reduction in the consumers' claiming cost on the firm's decisions, we construct a model with a monopolist choosing the price and reliability of the product it manufactures. The product's reliability is defined by the probability that the product is defective. Replacing a faulty product, and providing a more reliable one are costly actions for the firm. Consumers derive a high utility from consumption when the good does not break down, and a low utility otherwise. If the good is faulty, consumers choose whether to seek a replacement, incurring a claiming cost. Consumers differ in their cost to file a complaint. Within this set-up, we propose a comparative static analysis to understand the impact of the consumers' associations (as a way to reduce the claiming costs) on market performance. We show that the monopolist can optimally respond to a decrease in transaction costs by decreasing the product's reliability.

In our environment, there is sorting of consumers: those with low claiming costs are the ones with high willingness to pay and the ones who are prompt to complain if the product fails. Those with high claiming costs are the ones with low willingness to pay and the ones who do not complain when receiving a defected unit. The firm will follow either a high-pricing strategy, allowing only consumers with high willingness to pay to purchase the product, or a low-pricing strategy, allowing all consumers to buy the product. The dispersion of the consumers' willingness to pay is negatively related with the consumers' claiming cost, and, in line with Johnson and Myatt (2006), in this setting "*firm's profits are a U-shaped function of dispersion [of consumers' willingness to pay]. High dispersion is complemented by niche production [and high price] and low dispersion is complemented by mass-market supply [and low price].*" (brackets added)

On the one hand, low consumers' claiming cost is associated with high dispersion of consumers' willingness to pay, high price, and low demand. Firm's profit and product's failure rate are decreasing functions of consumers' claiming cost. The claiming cost is an unavoidable transaction cost affecting all consumers. When it is low, a marginal increment in transaction cost generates a reduction in consumers' willingness to pay which is compensated by an increase in product's reliability. Both effects reduce firm's profit. On the other hand, high consumers' claiming cost is associated with low dispersion of consumers' willingness to pay, low price, and high demand. Firm's profit and product's failure rate are increasing functions of consumers' claiming cost. When it is high, a marginal increment in claiming cost generates a reduction in consumers' willingness to complain, reducing the firm's liability cost and thus its unit cost. That is, a higher consumers' claiming cost reduces the number of complaints and the unit cost, making the failure rate to go up.

The result that product's reliability may decrease when consumers' claiming cost decreases arises when the firm changes its strategy as an optimal response for a discrete reduction in consumers' claiming cost. This result is more likely when the firm's cost of receiving complaints is sufficiently low: the lower the firm's cost of receiving complaints, the lower the liability cost associated to replacements and the higher the weight the firm puts on consumer's willingness to pay.

One way to reduce consumers' claiming cost is with the entry of consumer protection agencies, e.g., consumers' associations. In the 1960s, consumers' associations (hereafter, CAs) became more common.[1] Since then, CAs have been bridging between consumers, firms, and law makers: they have demanded to pass laws that add rights to consumers, they have provided more reliable information to consumers, and they have represented the consumers' interests by direct action. With the appearance of internet, some independent websites started to provide devices to help consumers to voice their complaints and to help to mediate between consumers and firms.[2] Consumers participate in the CAs as a substitute or a complement to costly direct legal actions. Some governmental agencies, like the National Highway Traffic Safety Administration (NHTSA) in the US car market, also play the role of protecting consumers' rights: they collect information about automobiles (cars, trucks, motorbikes, etc) and force companies to issue recalls to repair, replace, or repurchase the defective automobiles.

In the US car market, the NHTSA is the governmental agency that rules and controls safety standards. In 1995, the NHTSA introduced on-line complaints, lowering significantly consumers' claiming costs. Data from the NHTSA shows that the number of car complaints and the number of recalls issued have substantially increased since 1995. Firms could have adjusted product reliability after 1995 as a response to this change. However, internet filings were permitted for cars manufactured both before and after 1995. This allows us to interpret the difference in the number of recalls for cars manufactured before and after 1995 as a change in product's reliability. The evidence appears consistent with a reduction in the firm's choice of reliability.

Although the CAs have become increasingly common, the theoretical and empirical literatures have not analyzed their effects on market performance. Few exceptions are Inderst and Ottaviani (2009), showing that the imposition of minimum post-sale return policies on sellers improves welfare and consumers' surplus when the proportion of credulous buyers is high, and Xinyu (Forthcoming JLEO), showing that when firm's product liability is duty to recall there is a higher rate of consumers' response to recall but the firm has fewer incentives to make these recalls voluntarily. Simon (1981) studies the impact of costly litigation and imperfect information about product quality and the outcome of a lawsuit on the existence of negligent firms. In her environment, a reduction in consumers' litigation cost fosters firms to increase product reliability. She models imperfect information about product quality in consumer side, while we introduce asymmetric information about consumer's valuation or consumer's claiming cost in the firm side. She also ignores the possibility of not buying, banning the demand effect.

Our paper also relates to other strands of literature. First, to the vast literature on product liability. Murthy and Djamaludin (2002) and Huang, Liu, and Murthy (2007) analyze how warranties may

[1] The first two organizations to focus on consumers' rights were the National Consumer League founded in 1899 and the Consumer Union founded in 1936. It was not until the 60s, however, that consumers' associations became popular and common in a number of countries.

[2] For instance, the website getsatisfaction.com is used by more than 40.000 firms to handle complaints and mediate between consumers and firm. Other websites are fearlessrevolution.com in US, miqueja.es, quejasonline.com and reclamacionesconsumidor.com in Spain.

affect the firm's choice of product reliability and demand.[3] Oi (1973) analyzes how a change in product's liability from consumer to producer can affect quality negatively. Daughety and Reinganum (2008) work on the relation between signaling price and information disclosure when product's safety is private information. Daughety and Reinganum (1995) study the relation between R&D phase and product's safety when price may work as a signal of product's safety. Daughety and Reinganum (2005) analyze the relation between safety, R&D and confidential settlements. My contribution is to endogenize the number of claimants among buyers as a function of price. The relationship between price and claimants is translated to the firm's expected marginal cost and to the optimal choice of product's reliability.

Second, our paper also relates to the literature on consumers' complaint behavior (CCB), which focuses on the consumers' reaction to dissatisfaction (Chebat, Davidow, and Codjovi, 2005; Owens and Hausknecht, 1999). In our paper, we focus on complaints filed against the firm or third parties (CAs). We do assume that the firm undergoes an additional transaction cost when the consumer files a complaint and the information becomes public. We contribute to this literature showing that the number of complaints is endogenously determined and that the product's reliability can either increase or decrease when reducing the consumers' claiming cost.[4]

The paper is structured as follows. Section 3.2 introduces the model and explains the firm's strategies. Section 4.3 states the equilibrium and the main proposition. Section 3.4 illustrates the main results with a numerical example. Section 3.5 provides evidence about the effect of reducing claiming cost in the US car market. Finally, Section 3.6 concludes.

3.2 Model

We consider the problem of a monopolist choosing its product's price p and failure rate $x \in [0,1]$ to maximize its profit, while anticipating that consumers may request a replacement if the product they have purchased is faulty. The firm grants all replacement requests (there is binding legislation) by exchanging faulty products for new ones. Consumers are heterogeneous in their cost of requesting a replacement, which may lead to some consumers scrapping faulty products. Obviously, a replacement itself may fail as well and the consumer may again request a replacement, which will, again, be granted.[5]

[3] Spence (1977) analyzes the effect of consumers' misperceptions on product quality and characterizes the conditions under which a warranty can work as a signal of product quality. He suggests different types of interventions to solve the subprovision of quality. However, he ignores any type of consumers' claiming cost.

[4] There are other ways to affect the consumers' claiming cost, such as Class Action Lawsuits (Hensler, 2001; Klement and Neeman, 2004) and Small Courts (Best and Andreasen, 1977). Also, there are other mechanisms to motivate a provision of reliability. Klein and Leffler (1981) study the choice of product reliability by the firm in a dynamic framework. Repeated purchases, reputation, and brand name may ensure the provision of high quality goods by the firm. Finally, Greif, Milgrom, and Weingast (1994) describe how merchants create guild to improve the terms of trade, such as the level of product quality/reliability. We work on a static model with perfect information where reputation does not guarantee the provision of product reliability.

[5] Our model resemblesBar-Isaac, Caruana, and Cuñat (2010). In their paper, a marketing strategy transmits, prior to purchasing, information to the consumer about how suitable the product is, informing him whether the match is good or

3.2.1 Consumers and Demand

Consumers are indexed by i and are heterogeneous in their cost of requesting a replacement, $k_i \in \mathbb{R}_+$. We assume k is exponentially distributed, with cumulative distribution $G(k,\lambda)$.[6] Consumers derive utility $v \geq 0$ from consuming one unit of a non-faulty good. If the good is faulty or the consumer chooses not to purchase it, the consumer derives a utility of 0.[7] Each consumer i makes two sequential decisions, whether to purchase one unit of the product and whether to pursue a replacement if the product breaks down. The consumer pursues a replacement if $v(1-x)-k_i \geq 0$. Notice that the consumer's replacement decision is the same independent of whether the product being replaced is the original product or a replacement. This implies that the expected utility consumer i derives from replacing faulty products is given by

$$x\frac{\max\{v(1-x)-k_i,0\}}{1-x}. \tag{3.1}$$

Then, given (3.1), the consumer purchases one unit of the product if

$$EU_i(p,x) = v(1-x) - p + x\frac{\max\{v(1-x)-k_i,0\}}{1-x} \geq 0. \tag{3.2}$$

Figure 3.1.a depicts the maximum willingness to pay of each consumer type k_i given a failure rate x. It shows that those consumers who purchase the good and then request replacements are those with lowest k, while those with highest k prefer to purchase the good and will not replace it if it is faulty. The characterization of the actual consumption choice depends on p. If p is high (e.g., equal to p_H in the figure), all consumers who purchase replace the product if it is faulty, with their willingness to pay only depending on x by the expected replacement cost. Instead, if p is low (e.g., p_L), some consumers purchase and replace while others purchase and scrap.

We next obtain the demand function. We aggregate the consumers' choices for all k, but separate them into two groups, depending on whether they will request a replacement if the good results to be faulty. Those consumers with $k_i \leq v(1-x)$ anticipate replacing a product if it is faulty. We label them *claimants*. Among these consumers, those with $k_i \leq (v-p)(1-x)/x$ purchase the product. Instead, the consumers with $k_i \geq v(1-x)$ anticipate not replacing a faulty product. We label these *non-claimants*. All consumers who are non-claimants make the same purchasing decision. If $v(1-x) \geq p$, all non-claimants purchase the product, while otherwise, all non-claimants do not. (Notice that the difference in thresholds $v(1-x)$ and $(v-p)(1-x)/x$ can be rewritten as $(1-x)(p-v(1-x))/x$, which implies that the demand cut-offs are solely determined by the comparison of p and $v(1-x)$.) That is, the demand function of claimants, Q^C, and non-claimants,

bad. The consumer learns the result of the match after purchasing. Here the consumer anticipates that the product can work (good) or fail (bad) and the buyer chooses to request a replacement or not after buying.

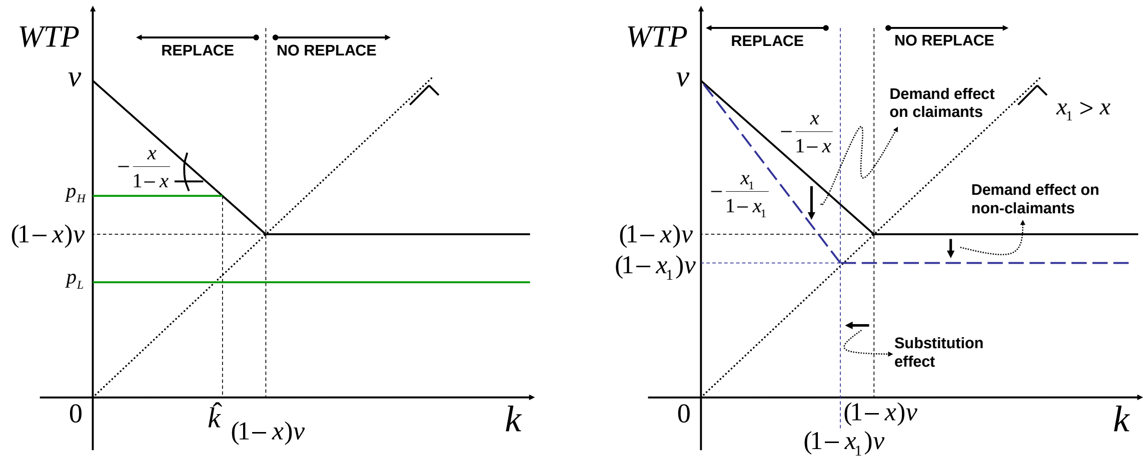
[6] We can generalize our results for any family of continuous and differential distributions with increasing reverse hazard rate that can be ranked according to the reverse hazard rate criteria.

[7] Our results also hold if, instead, $v \sim U[0,1]$ and $k \in \mathbb{R}_+$. The proofs are available upon request.

Q^{NC}, is given, respectively, by

$$Q^C = \begin{cases} G(\frac{(v-p)(1-x)}{x}, \lambda) & \text{if } p > v(1-x), \text{ and} \\ G(v(1-x), \lambda) & \text{if } p \leq v(1-x). \end{cases} \qquad Q^{NC} = \begin{cases} 0 & \text{if } p > v(1-x), \text{ and} \\ 1 - G(v(1-x), \lambda) & \text{if } p \leq v(1-x). \end{cases}$$

Exponential distributions are ordered by λ. A distribution with a higher λ is first order stochastically dominated by another distribution with a lower λ. If λ is higher, the exponential distribution moves to the left and consumers have lower claiming costs. Also, an exponential distribution with a higher λ is dominated in terms of reverse hazard rate by another exponential distribution with lower λ. This property guarantees that the firm charges a higher price to a population of consumers with lower claiming costs. We next turn to the problem of the firm.



(a) Maximum willingness to pay.

(b) Demand and substitution effects.

Figure 3.1: Willingness to pay (a) and demand and substitution effects (b).

3.2.2 Monopolist's Problem

We let $c(x)$ denote the marginal cost of production, which is constant for a given x. We assume $c(x) > 0$, $c'(x) < 0$ and $c''(x) > 0$. We also assume that $xc(x)$ is weakly increasing in x. This assumption implies that $xc(x)/(1-x)$, i.e., the expected cost of manufacturing replacement units is increasing in the failure rate.[8] When serving replacement requests, the firm also incurs a per-unit replacement cost $c_r \geq 0$.[9] This implies that the cost of selling one unit of the good to a claimant consumer is $c(x) + x(c(x) + c_r)/(1-x)$, while selling the same unit to a non-claimant is only $c(x)$. To ensure that a solution to the firm's maximization problem exists, we assume there exists a $c_r > 0$ such that, for some $x \in [0,1]$, $v(1-x) - c(x) - x\frac{c(x)+c_r}{1-x} > 0$. This assumption says that the firm can make positive profits if it sells to all consumers, including those with lowest willingness-to-pay

[8]The family of functions $c(x) = \frac{\beta}{x^{\alpha}}$, with $\alpha \in [0,1]$ and $\beta \in \mathbb{R}_+$, satisfies this assumption.

[9]The parameter c_r may account for a variety of costs, from administrative/shipping and handling costs to reputational losses.

$v(1-x)$.

Then, given the demand function in (3.3), the monopolist's maximization problem is given by

$$\Pi(p,x)=\begin{cases}\left[p-c(x)-x\frac{c(x)+c_r}{1-x}\right]\left[1-e^{-\lambda\frac{(v-p)(1-x)}{x}}\right] & \text{if } p>v(1-x), \text{ and}\\ \\ p-c(x)-x\frac{c(x)+c_r}{1-x}\left[1-e^{-\lambda v(1-x)}\right] & \text{if } p\leq v(1-x).\end{cases}$$

We say that the firm follows a high-pricing strategy if p and x are such that $p>v(1-x)$; otherwise, we say that the firm follows a low-pricing strategy. Notice that $p=v(1-x)$ dominates all other p and x that constitute a low-pricing strategy as the objective function depends positively on p.

The firm faces a trade-off in its choice of price and failure rate. When following a low-pricing strategy the marginal consumer is a non-claimant whose willingness to pay is low and depends only on the probability of buying a non-defected unit; reducing the failure rate increases considerably his willingness to pay. This effect defines the demand effect on non-claimants in Figure 3.1.b. The marginal cost of selling this unit, however, is only its production cost, which increases as failure rate decreases. In contrast, when following a high-pricing strategy the marginal consumer is a claimant whose willingness to pay is high and depends both on the failure rate and on his own claiming cost. Reducing the failure rate increases his willingness to pay but this increment decreases as his claiming cost decreases. This effect defines the demand effect on claimants in Figure 3.1.b. The marginal cost of selling this unit incorporates a production cost and a replacement cost and it increases as the failure rate decreases.[10]

3.3 Equilibrium

We next find the monopolist's optimal strategy for a given distribution of consumers' claiming cost and then analyze the effect of a reduction in the consumers' claiming cost, i.e., the effect of an increase in λ.

The firm's optimal choice of p and x depends crucially on the parameter λ. The intuition is as follows. If λ is such that consumers have low claiming costs, they are more likely to complain when receiving a defected unit and there is high dispersion in the consumers' willingness pay. If this is the case, the firm does not find rewardable to reduce the price to $v(1-x)$ so as to increase demand and it follows a high-pricing strategy. If, instead, consumers have high claiming costs, there is low dispersion in consumers' willingness pay, and the firm finds rewardable to reduce the price to $v(1-x)$ to increase demand and thus follows a low-pricing strategy. The next proposition characterizes the cut-off in the expected consumers' claiming cost above which the firm chooses to follow a low-pricing strategy.

[10]The unit cost that incorporates the production and replacement costs has a U-shaped function of product's failure rate, i.e., $c(x)+x(c(x)+c_r)/(1-x)$ has a U-shaped function of x. When following a high-pricing strategy the firm always chooses a failure rate in the decreasing part.

Proposition 6. *There exists a cut-off in the parameter values characterizing the consumers' claiming cost distribution, $\hat{\lambda}$, such that the firm follows a low-pricing strategy if the consumers' claiming costs are sufficiently high ($\lambda < \hat{\lambda}$) and follows a high-pricing strategy otherwise. The firm's profit function has a U-shaped relation with λ, with a minimum value at the cutoff $\hat{\lambda}$.*

The demands of claimants and non-claimants are the only terms in the firm's profit function that depend on the distribution of the consumers' claiming cost. If the firm follows a high-pricing strategy, only claimants purchase the product and the demand increases when the distribution of consumers' claiming cost is moved to the left, i.e., when λ increases. For this reason, when following a high-pricing strategy, firm's profits are increasing in λ. If, instead, the firm follows a low-pricing strategy, both claimants and non-claimants purchase the product and increments in the consumers' claiming cost do not affect the number of units sold. At most, a claimant may become non-claimant (only the substitution effect in Figure 3.1.b). In this case, when following a low-pricing strategy, the firm saves part of the liability cost and the firm's profit decreases in λ. Under certain conditions, there is a cut-off where the firm's profits coincide with both strategies.
Proposition 6 provides a rationale for the firm's optimal strategy when subject to product replacement. If the firm follows a low-pricing strategy, its profit increases when consumers have higher claiming costs. If it were possible for the firm to affect consumers' claiming costs, the firm would have clear incentives to provide a tiresome procedure for requiring its customer service to discourage consumers from complaining. If, instead, the firm follows a high-pricing strategy, we observe the opposite result. The firm's profit decreases with the consumers' claiming cost and the firm would provide devices to aid consumers complain.
We argue in the Introduction that the empirical evidence shows that many buyers do not request a replacement despite having received a defective product. In our environment, this evidence is consistent with the firm following a low-pricing strategy so that non-claimants also purchase the product. In Proposition 7, we show that, if the firm's cost of receiving complaints is not too high, the firm may reduce its product's reliability when the consumers' claiming cost is reduced.

Proposition 7. *If the firm's liability cost of receiving complaints is not too high, there exists a reduction in the consumers' claiming cost to which the firm responds by reducing the level of its product's reliability.*

The intuition behind the formal proof is as follows. If consumers' claiming costs are infinitely high (λ is low), there are only non-claimants. If this is the case, the firm follows a low-pricing strategy and the product's reliability is determined by $-c'(x) = v$. The failure rate monotonically decreases with consumers' claiming cost. On the other hand, if the consumers' claiming cost is zero ($\lambda \to +\infty$), there are only claimants. If this is case, the firm follows a high-pricing strategy and the product's reliability is determined by $-c'(x) = (c(x_a) + c_r)/(1 - x_a)$. The failure rate monotonically increases with consumers' claiming cost. The change in the strategy from low-pricing to high-pricing at the cut-off $\hat{\lambda}$ implies that the failure rate has a U-shaped function of consumers' claiming cost. If

$\frac{c(x_a)+c_r}{1-x_a} < v$, for any λ, there exists a $\lambda' < \lambda$ such that $x(\lambda') > x(\lambda)$, which means that there exists a reduction in consumers' claiming costs, represented by moving the distribution from λ to λ', for which the firm reduces product's reliability.

To understand this result, we consider first a marginal reduction in the consumers' claiming cost. Under a high-pricing strategy, the consumers claiming cost represents a transaction cost. A marginal reduction in consumers' claiming cost generates a reduction in transaction cost. The firm increases the price and reduces the product's reliability, making both demand and profits increase. Under a low-pricing strategy, a marginal reduction in the consumers' claiming cost generates an increment in complaints and in the firm's costs. The firm increases product reliability and price and the mark-up and profit decrease.

However, for a discrete reduction in consumers' claiming cost, we have a new insight. For a high pricing strategy, there is nothing new. For a low-pricing strategy, a discrete reduction in consumers' claiming cost leads to a change in the firm's strategy, from low to high-pricing. This non-local increment in price generates a reduction in demand, a change in product reliability, and a change in profits. For a large enough discrete reduction in consumers' claiming costs, the firm's profit increase and the product reliability decreases. The reason is that once consumers have an inexpensive way of guaranteeing a well-functioning product, either because the good does not fail or because it is replaced at a low cost, reliability loses its value for consumers.

Proposition 7 states a clear result: a discrete reduction in consumers' claiming cost might generate a reduction in product's reliability and an increase in firm's profit. The policy implication of Proposition 7 is that any policy that helps consumers to file complaints pursues to increase product reliability but it could get exactly the opposite effect.

This result, that product reliability can decrease when there is a reduction in consumers' claiming cost, is more likely when the firm has a low cost of receiving complaints. Under a high-pricing strategy, the replacement effect is what motivates the firm to increase product reliability. While the demand effect is only determined by consumers' willingness to pay, the replacement effect is determined by the firm's cost of facing complaints. When the firm has a low liability cost, it has more incentives to reduce reliability if consumer' claiming cost decreases.

3.4 Example

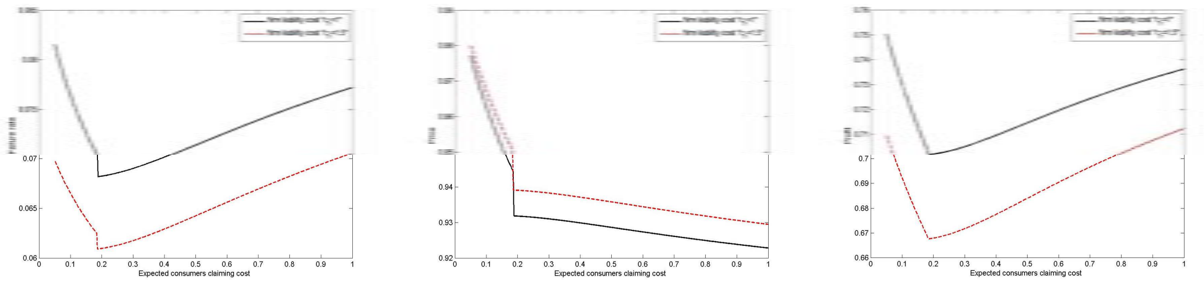
In this section we run a numerical example assuming that $v = 1$ and that the unit production cost function is $c(x) = \frac{\beta}{x}$, with $\beta = 0.01$.[11] In Figures 3.2.a, 3.2.b and 3.2.c we plot product's failure rate, price, and firm's profit, as a function of consumers' claiming cost for different values of c_r, i.e., the black line for $c_r = 1$ and the red dashed-line for $c_r = 1.5$. First, note that the optimal price decreases when the expected consumers' claiming cost increases (Figure 3.2.b). Moreover, both firm's profit and product's failure rate are U-shaped functions of expected consumers' claiming cost

[11] The cost function $c(x) = \frac{\beta}{x}$ satisfies technical specifications according to Huang, Liu, and Murthy (2007).

(Figures 3.2.a and 3.2.c). The lowest level of both functions coincides with the threshold above which the firm follows a low-pricing strategy.

From this example we can quantify our effect. Assume that firm's liability cost is 1, i.e., $c_r = 1$, and that on average the consumers' claiming cost is 60% of their valuation of a non-defected product. If this is the case, any reduction of more than 75% on consumers' claiming cost will generate an increment in product's failure rate. For instance, a reduction of 80% of consumers claiming cost generates an increment in product's failure rate of around 3%. This increment in failure rate jumps to 5.2% for a reduction of 83% of consumers' claiming cost.

This example highlights that the introduction of consumer associations, or any device that helps consumers voice their complaints, leads to either a reduction or an increase in the failure rate.[12] This change in failure rate depends on the magnitude of consumers' claiming cost reduction and on the level of firm's liability cost.



(a) Product failure rate (b) Price (c) Profit

Figure 3.2: Failure rate, price and profit as a function of consumers' claiming cost.

3.5 The Effect of Claiming Cost Reductions in the US Car Market

The National Highway Traffic Safety Administration (NHTSA) is a US governmental agency that has the authority to issue vehicle safety standards and to require manufacturers to recall vehicles that have safety-related defects or do not meet Federal safety standards.[13] Consumers use the NHTSA service as an additional device to voice their complaints. When a car company does not attend their complaints, consumers can either privately sue the firm, file a complaint with a consumers' association, file a complaint with the NHTSA, or do nothing.

In 1995, the NHTSA introduced two important modifications to its filing process, which significantly decreased the consumers' claiming costs: it released an electronic mailing address to which consumers could mail complaints and it launched a website application that allowed consumers to

[12] As pointed out in the introduction, consumers' associations have been working for consumers' sake in different dimensions. One of them is to decrease consumers' claiming cost helping them to exercise their rights. In our setup a CA affects consumers in two different ways: i) by moving consumers' claiming cost distribution downward, and ii) by providing a "legal" help when a problem arises (which is equivalent to provide a lower claiming cost). In this paper, these two effects are not distinguished.

[13] See Rupp and Taylor (2002) for a detailed explanation of a recall process.

file their complaints on-line.[14] Since then, the number of complaints and the number of recalls have increased, and the correlation between them is positive.

Figure 3.3 plots the number of recalls and complaints per car model-year accumulated since the year the car was manufactured until 2008.[15] For instance, cars produced in 1994 suffered 50 recalls and cars produced in 1995 suffered 78 recalls until 2008. Notice that the on-line devices had been available since 1995 for those consumers who had bought a car before and for those who bought it after 1995. The figure shows that the accumulated number of recalls for cars produced after 1995 is substantially higher than for those cars produced before 1995. The increment in recalls for cars produced after 1995 may be consistent with the following effects: car companies have decreased cars' reliability, consumers' complaints help disclose more information, car companies have introduced new car models (i.e., new car segments like Minivan), the NHTSA has tightened its controls and/or standards, or, more likely, a combination of all of the above. The informational and reliability change effects are more likely to dominate because we have not found evidence of a change in the NHTSA's regulations and, although we do not have data on the number of models in the market, it is quite unlikely that it changes discontinuously in a particular year.

We provide evidence consistent with the existence of informational and reliability change effects. A clear way to see if there is a reliability effect is to compare the number of recalls for cars sold immediately before and after 1995. Notice that the reduction of consumers' claiming cost is available for consumers who bought a car either in 1994 or 1995. The data, however, show that the number of recalls were much more higher in 1995 with respect to 1994. If cars' reliability is the same along the whole period, we must observe, ceteris paribus, the same number of recalls for cars produced in 1994 and 1995. This fact is consistent with our predictions that a reduction in consumers' claiming cost can motivate the firm to decrease the reliability of the product manufactured.

To reinforce our analysis, in Table 3.1 we report the number of recalls for car models-year aggregated in periods of 6 years and its distribution over time. Cars produced between 1995 and 2000 suffered 429 recalls; half of them took place in the same year that these new cars were launched.[16] Given that the car's life expectancy is around 10 years, we expect that cars produced before 1982 received no effect of the introduction of on-line devices, while cars produced between 1983 and 1994 received only an informational effect. For this reason, cars produced between 1989-1994 are a control group that captures only the informational effect. Cars produced in 1995 or after have received an informational and reliability change effect.

[14] These on-line devices were available since the end of 1994.

[15] 2008 is the last year of data available. The data correspond to the six largest US automobile manufacturers: Ford, General Motors, Chrysler, Toyota, Nissan and Honda. Since 1970, these companies have accounted for around 90% of US car market. However, this aggregated share has been slightly reduced since mid 2000. The NHTSA provides data of complaints as from 1984 but it was not until 1995 that consumers' complaints became sizable. The evolution of complaints and complaints/sales have the same pattern. For robustness, we should control recalls by the number of car models. However, we do not have this data yet.

[16] Since the average car life expectancy is 10 years, the serie is truncated for car models in the subperiod 1995-2000. The number of recalls is higher in spite of this truncation. However, this does not constitute a problem because the introduction of internet devices has helped disclose information earlier.

Table 3.1: Number of recalls per model and its distribution over time.

		time elapsed since model launched			
cars produced in	recalls	same year	1-5 years	6-10 years	rest
1977-1982	**248**	73.39%	22.58%	**3.63**%	0.40 %
1983-1988	**234**	52.14%	39.32%	**8.12**%	0.43 %
1989-1994	**252**	44.05%	40.87%	**12.70**%	2.38 %
1995-2000	**429**	50.12%	44.76%	**5.13**%	-

We first provide evidence of our claim that consumers' complaints help to disclose more information. Consistent with our statements, for cars produced in the period 1977-1982, around 4% of recalls took place six years after new cars were launched or later (See column 5 and 6 in Table 3.1). This proportion raises to 8.5% and 15% for cars produced in 1983-1988 and 1989-1994, respectively. Note that the accumulated number of recalls of cars produced after 1983 may have captured an informational effect when the NHTSA introduced the on-line devices to complain. The difference in the proportion of recalls that took place six years after manufacture or later for subperiods 1977-1982 and 1989-1994 is statistically significant at standard levels, showing that the on-line devices helped to disclose some additional information.[17] This new information allowed many recalls that would have not been done otherwise.

We now provide evidence of the reliability change effect. Aggregating in subperiods and comparing the trend in recalls for cars produced before (between 1989-1994) and after (1995-2000), the increment in the number of recalls after the NHTSA introduced on-line applications to file consumers complaints is statistically significant.[18] This evidence is consistent with our theoretical results.

Concluding, we observe that there is clear evidence of the effect of the NHTSA's website application and e-mail on complaints and recalls. These inexpensive devices to voice consumers' complaints generate an increment in recalls. We claim that these increments are due to more information disclosure and to a reduction in cars' reliability. The increment in the number of recalls per year-model after 1995 conciliates our theoretical results.[19]

[17] The statistic comparing the proportion in the period 1983-1988 (which captures only partially the information effect) with 1977-1982 is $|t| = 2.05$ and comparing the period 1989-1994 with 19877-1982 is $|t| = 5.11$. If we consider only recalls initiated in the first 5 years since launched, the increment in recalls from 1989-1994 to 1995-2000 is from 214 to 409 recalls, almost 100%.

[18] The statistic of testing that the means of recalls are the same for subperiods 1989-1994 and 1995-2001 is $|t| = 4.7$, rejecting the hypothesis that they are equal (a one tail test is also rejected). Assuming that there is only an informational effect for cars produced in the subperiod 1989-1994, and a combination of informational and reliability change effects in the subperiod 1995-2001, the test is consistent with the existence of a reliability change effect.

[19] We are aware of the limitations of the data. We have made some controls by unit sold and the results remain the same. However, we could not get any control by number of models sold by each company. As mentioned above, even when the new models may account for part of the change in complaints and recalls, it is unlikely that there is a discontinuous change in the number of models. Finally, we must be cautious when interpreting the data with our theoretical findings since our model lacks the strategic interaction among firms that is present in the U.S. car market.

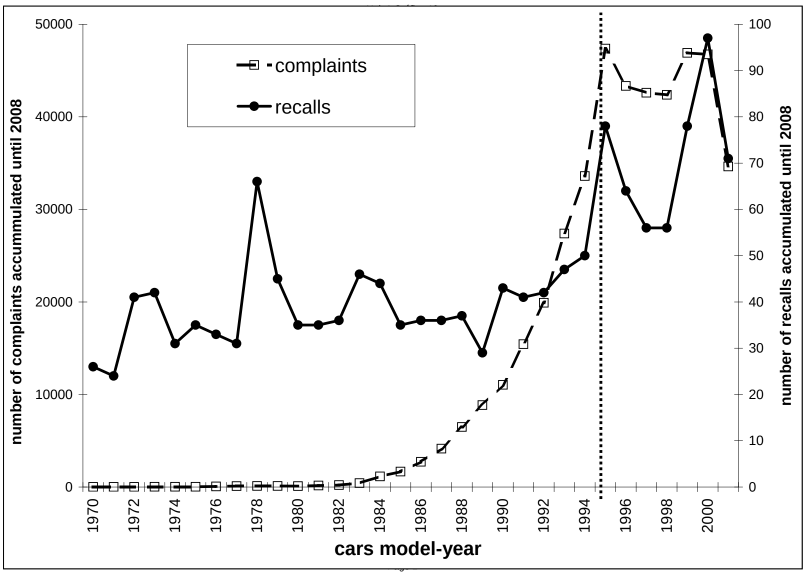


Figure 3.3: Number of recalls and complaints accumulated until 2008 in the U.S. car Market per model-year.

3.6 Conclusion

Consumers' associations goals are, among others, to help consumers voice their complaints and enforce the provision of reliable products. In this paper, we show that these goals may not be aligned: the firm may decide to produce a less reliable product if more consumers request a replacement of a defected product.

The evidence from U.S. car market shows that after a reduction in claiming costs in 1995, the number of complaints and recalls has clearly increased. Complaints were permitted for the cars manufactured before that year, but the number of recalls increased much more for cars produced after 1995. The evidence appears inconsistent with the firms' choice of reliability increasing or being constant, even when controlling for the effect of more information disclosure. All this evidence goes in the direction of our results.

The main contribution in terms of policy implication of this paper is to warn consumers' associations that reliability is an endogenous decision. Product reliability depends on both consumers' claiming costs and firm's replacement costs. An increase in consumers' complaints may be followed by a reduction in product's reliability, which in turn generates additional complaints. Then, observing a sharp increment in complaints is not necessarily a good market signal. This paper also points out that the expected number of complaints is endogenously determined by the price and product reliability.

3.7 Appendix

3.7.1 Change of Variable for High-pricing Strategy

To find the solution when $p > v(1-x)$ we make a simple transformation: define $k = \frac{(v-p)(1-x)}{x}$ implying that $p = v - \frac{xk}{1-x}$. Since $p \in [0,v] \subset \mathbb{R}_+$ and $x \in (0,1]$, then $k \in \mathbb{R}_+$. The problem is re-expressed as

$$\max_{k,x} \Pi(k,x) = \left[v - \frac{xk}{1-x} - c(x) - x\frac{c(x)+c_r}{1-x}\right]\left[1-e^{-\lambda k}\right]. \tag{3.3}$$

This expression allows us to find, first, the optimal choice of x as a function of k, and second, the value of k that maximizes firm's profit. For any k there exists $x_k(k) = \arg\min\left[\frac{c(x)+x(c_r+k)}{1-x}\right]$ with $x_k'(k) < 0$, i.e., if k increases x_k decreases. The minimum of $\frac{c(x)+x(c_r+k)}{1-x}$ is increasing in k. The value of k is defined by the first order condition of equation (3.3),

$$\frac{1-x}{x}\left[v - c(x) - x\frac{c(x)+c_r+k}{1-x}\right] = \frac{\left[1-e^{-\lambda k}\right]}{\lambda e^{-\lambda k}}. \tag{3.4}$$

The left hand side is decreasing in k if $c(x)$ is sufficiently convex.[20] The right hand side is increasing in k. The value of k is uniquely defined. Finally, the second order condition respect to k is

$$\Pi_{kk} = -\frac{2x}{1-x}\lambda e^{-\lambda k} - \lambda^2 e^{-\lambda k}\left[v - c(x) - x\frac{c(x)+c_r+k}{1-x}\right] < 0. \tag{3.5}$$

3.7.2 Proof of Proposition 6

Proof. The exponential distribution has the following properties: if $\lambda \to +\infty$, $G(k,\lambda)$ is degenerated in $k=0$, and, if $\lambda \to 0$, the cumulative is $G(k,\lambda) \sim 0$ for any $k < +\infty$ (consumers have a k too high and never complain).

For any $0 < \lambda < +\infty$, the profit function is discontinuous at $p = v(1-x)$. However, we can consider the expressions under a low-pricing strategy and under a high-pricing strategy separately. By the envelope theorem $\Pi(p = v(1-x), \lambda)$ is decreasing in λ and $\Pi(p > v(1-x), \lambda)$ is increasing in λ. For details of the high-pricing strategy see Appendix 3.7.1.

Note that if $\lambda \to 0$ there is only non-claimants and $\Pi(p = v(1-x)) > 0 = \Pi(p > v(1-x))$. If $\lambda \to +\infty$ there is only claimants and $0 < \Pi(p = v(1-x)) < \Pi(p = v - \varepsilon)$, for some $\varepsilon \geq 0$. Assumption A1 guarantee an equilibrium with positive profits that is characterized by at least one these two strategies for any λ. The existence of the cutoff $\hat{\lambda}$ such that $\Pi(p = v(1-x), \hat{\lambda}) = \Pi(p > v(1-x), \hat{\lambda})$

[20] Recall that $x'(k) < 0$, which depends on $c''(x)$. The condition for the left hand side to be decreasing in k is that $\frac{v-c(x)-x\frac{c(x)+c_r+k}{1-x}}{x^2(1-x)c''(x)} - 1 < 0$. It is sufficient that the slope $x'(k) = -\frac{1}{(1-x)c''(x)}$ not to be so steep, i.e., $c''(x)$ sufficiently high. Note that the mark-up can not be too high in this equilibrium, given that if v is high, the firm would prefer to expand demand choosing a price $p = v(1-x)$.

is guaranteed. □

3.7.3 Proof of Proposition 7

Recall that higher λ implies lower consumers claiming cost.

Proof. We show that x is a decreasing function of λ when the firm follows a low-pricing strategy, i.e., $p = v(1-x)$, and x is an increasing function of λ when the firm follows a high-pricing strategy, i.e., $p > v(1-x)$. Finally, we guarantee conditions such that $x(\lambda \to 0)$ under a low-pricing strategy is lower than $x(\lambda \to +\infty)$ under a high-pricing strategy, such that there exists an increment in λ (i.e., a reduction in consumers claiming cost) that generates an increase in the failure rate.

Since the objective function is submodular in x and λ when the firm follows a low-pricing strategy, the failure rate is decreasing in λ.[21] When the firm follows a high-pricing strategy, we state the monotonicity x respect to λ from the first order condition respect to k in equation (3.4) and from the monotonicity of $x'(k) < 0$. The right hand side in equation (3.4) increases in λ since the derivative of the reverse hazard rate respect to λ is positive, i.e., $-\frac{e^{k\lambda}}{\lambda^2}\left(1-e^{-k\lambda}-k\lambda\right) > 0$, since $1-e^{-k\lambda}-k\lambda < 0$. Any increment in λ generates a reduction in k and thus an increase in x. There is a monotonic relationship between λ and x, i.e., the higher the λ the higher the x.

In the limit, if $\lambda \to +\infty$, consumers have zero claiming cost: the firm chooses $k \to 0$ and $p = v - \varepsilon$ for some $\varepsilon \geq 0$; and the failure rate is determined by $-c'(x_a) = \frac{c(x_a)+c_r}{1-x_a}$. On the other hand, if $\lambda \to 0$, most consumers have extremely high claiming cost: the firm chooses $p = (1-x)v$ and the failure rate is determined by $-c'(x_b) = v$. $x_a \leq x_b$ if and only if $\frac{c(x_a)+c_r}{1-x_a} \geq v$. Note that, since $c'(x) < 0$ and $c''(x) > 0$, the expression $\frac{c(x)+c_r}{1-x}$ has a global minimum at x_a for a given c_r. This minimum $\frac{c(x_a(c_r))+c_r}{1-x_a(c_r)}$ is increasing in c_r and x_a is decreasing in c_r. Since v is constant and $\frac{c(x_a(c_r))+c_r}{1-x_a(c_r)}$ is increasing in c_r, there exists $\hat{c}_r(v) \in \mathbb{R}_+$ such that $\frac{c(x_a)+\hat{c}_r(v)}{1-x_a} = v$ for which $x_a = x_b$. Then, $x_a < x_b$ if and only if $c_r > \hat{c}_r$. The threshold $\hat{c}_r(v)$ is increasing in v.

Concluding, the product's failure rate has a U-shaped relation with λ, with a jump in the minimum x at $\hat{\lambda}$. There exists a cutoff $\hat{\lambda}$ such that the firm follows a low-pricing strategy if $\lambda < \hat{\lambda}$. Finally, it is guaranteed for $c_r < \hat{c}_r(v)$ that there exists a reduction in consumers claiming cost, i.e., $\Delta\lambda > 0$, such that product reliability decreases, i.e., $\frac{\Delta x}{\Delta\lambda} > 0$. The proof is complete. □

[21] Assumption that $xc(x)$ is non-decreasing in x is sufficient condition. The cross derivative is $\frac{\partial^2 \Pi}{\partial x \partial \lambda} = -ve^{\lambda v(1-x)}[\lambda vx(c(x)+c_r)+(c(x)+c_r)+c'(x)x] \leq 0$, which is guaranteed if $(c(x)+c_r)+c'(x)x > 0$.

Chapter 4

Hotelling Competition for a Consumer with Unknown Taste

(joint work with Daniel García-González)

4.1 Introduction

Preferences for most products are in constant change, so that they are difficult to predict for firms. Moreover, consumers may also be uncertain over which products they prefer from an ex-ante point of view, i.e., before observing or tasting the products. In spite of having vertical advantages, like in brands positioning, product quality and/or in cost of production, firms might have little market success when having a bad foresight of consumer's taste. For instance, Honda's market losses in late 80s in Japan are attributed to a bad response to a shift in demand's preferences. A dynamic and changing clothing markets is another good example of these situations.[1] In this paper, we study how the presence of ex-ante firms' heterogeneity and the acquisition of informative signals affect the degree of horizontal differentiation, profits and consumer surplus when firms are uncertain about consumer's taste.

We analyze a Hotelling's duopoly game of location-then-price-competition choice with quadratic transportation cost and an ex-ante advantaged firm, under the assumption that firms are uncertain about consumer's taste.[2] We show that both factors –firm advantage and informative signals- can foster competition increasing consumer surplus in spite of decreasing profits and welfare.

In the benchmark model, there is one consumer with unitary demand and imperfect information about her most preferred variety. Her willingness to pay for a product decreases with the distance between product's variety and her most preferred one, with quadratic transportation costs. In the supply side, two firms simultaneously and costlessly choose a product location/variety. Af-

[1] For the evidence of Honda see Brickley, J., Smith, C., Zimmerman, J.(2001) and for a description of the apparel market see Uzzi (1997).

[2] The firm advantage is common knowledge and may stem from costs reduction or better quality.

ter consumer's preference and products' varieties are disclosed, Bertrand competition with perfect information defines the equilibrium price and the allocation.

As in the traditional Hotelling model, anticipating a fierce Bertrand competition in prices, firms choose to differentiate their products ex-ante. Given the timing of the model, strategic interaction moves upstream to the location stage. We analyze whether firms can coordinate their strategies in the location stage in order to soften competition and to make higher expected profits. Uncertainty about consumer's taste fosters differentiation. We find two types of equilibria. Some equilibria are characterized by a split of the market, so that each firm sells customers located in a given subset with probability one, and does not try to sell to customers located outside. The remaining equilibria exhibit more competition, since at least one firm will try to sell to any particular customer.

An equilibrium with a market partition has higher expected price, profits, welfare and lower expected consumer surplus when compared with an equilibrium without market partition. We call the former an equilibrium with low competition and the latter an equilibrium high competition. In each equilibrium (of any type) one of the firms is leading the market if it sells with higher probability and makes higher expected profit than the other firm. We analyze how equilibria are affected when we add to the benchmark model two factors: the presence of firms' heterogeneity, interpreted as different unit cost (low or high cost) or brand positioning, and the provision of informative signals. We additionally distinguish between private and public signals.

In the expanded model, when its cost advantage is sufficiently high, a firm is leading the market in any of the equilibria described above. Having a high advantage in cost makes the firm more aggressive and it cannot commit to accommodate. Additionally, if a firm can acquire a costly private signal about consumer's taste, it uses the information choosing a location that is likely to match consumer's preference. If information is sufficiently cheap and a firm has a cost advantage, there only exists equilibria with high competition. An equilibrium with low competition cannot be sustained because a firm with cost advantage cannot commit to restrict his product variety to a subset of preferences. Its informative signal and its cost advantage increase the opportunity cost of coordination; and the likelihood that both firms locate close to each other (thereby competing more fiercely) increases.

If a signal is public, that is, if the realization of a signal is common knowledge, equilibria with low competition can be sustained for any range of parameters. While the firm with cost advantage has incentive to locate its product following the realization of the public signal, the firm with cost disadvantage has incentive to locate its product with the opposite variety. Both firms get higher expected profits coordinating.

With this framework we seek to understand the functioning of markets where uncertainty may be important, like the high competitive environment of New York apparel industry described by Uzzi (1997) where heterogeneous firms acquire private information about uncertain preferences. Finally, our model can be mapped into a procurement second-price auction in which the consumer values pairs of price and positioning and chooses the one that yields higher ex-post surplus. In procurements, the consumer, e.g., the state, may have to decide whether to allow for private market inquiries, or whether to make private or public announcements about her preference. Against common wisdom, increasing transparency through public announcements may reduce its consumer surplus.

The implications of our model for procurements and market design are the following. First, making public announcements can foster firms' coordination which softens competition.[3] Second, allowing for private inquiries about market conditions can foster competition even when it can favor a particular supplier that already has a cost advantage. Finally, the existence of ex-ante asymmetries among firms reduces the set of parameters for which low competition equilibria may be sustained, and thus, may increase consumer's surplus.

4.1.1 Literature

This paper relates to the literature on product differentiation with asymmetric information. Several papers study the relation of product differentiation and asymmetric information on the demand side. Shaked and Sutton (1982) analyze entry and vertical differentiation, Bester (1998) investigates how product differentiation (both vertical and horizontal) of experienced goods is used to attract uninformed consumers, and Schultz (2004) analyzes the impact of transparency on competition.

However, our paper relates closely to the literature that focus on uncertainty on the supply side. This approach is analyzed in Moscarini and Ottaviani (2001) assuming fixed firms' locations and provision of informative signals to the uninformed buyer about her own preference. We study the endogenous location decision of firms that can obtained additional information about consumer's preference. Casado-Izaga (2000), Meagher and Zauner (2004) and Meagher and Zauner (2005) study location-then-price competition in a symmetric firms' environment with heterogeneous consumers. Out paper differs from theirs in that we study heterogeneous firms and we allow these firms to acquire informative signals.

Our approach allows us also to make predictions for procurement auctions. In this literature, Ganuza (2004) analyzes the provision of public signals by the seller in a single unit private value auction. However, he characterizes the setup with fix locations where the seller chooses the amount of public information. Our approach focuses on buyer's side, i.e., procurement or reverse auctions, and the buyer can provide either private or public signals to the sellers whose locations are endogenously determined.

4.2 Benchmark Model

On the demand side there is one consumer that maximizes her utility by buying one unit of an indivisible product. A product is defined by a unit price p_i ($\in \mathbb{R}_+$) and a variety θ_i ($\in [0,1]$). The

[3]General guidelines would direct suppliers in the right direction, discharging all characteristics that are, for sure, not desirable. However, specific guidelines listing which specifications are desirable can soften competition.

consumer has imperfect information about her most preferred variety θ which takes a value from $\{0, \frac{1}{2}, 1\}$. A priori, each value realizes with the same probability. The consumer derives utility v of buying one unit of her most preferred variety θ, and her willingness to pay for a product i decreases as its variety θ_i differs from her bliss point θ. We assume quadratic transportation costs. The net utility of buying a product with variety θ_i at price p_i is $U(\theta_i, p_i) = v - (\theta - \theta_i)^2 - p_i$. The reservation utility is zero.

On the supply side there are two firms with constant unit cost. Each firm is risk neutral and maximizes expected profits choosing a variety θ_i and a price p_i for his product. Defining product variety is costless and allows for product differentiation. For some exogenous reason (technological, legal or market advantage) firm 1 has an advantage. This advantage is common knowledge, and we model it through the assumption that firm 1 has zero unit cost and firm 2 has a constant unit cost c in $[0, \bar{c}]$.[4] Assumption 1 guarantees that the consumer always buys one unit of a product.

Assumption 1: $v > 1 + \bar{c}$.

After firms simultaneously choose their products' varieties, all information about products' varieties (θ_1, θ_2) and consumer's tastes θ is disclosed and firms simultaneously choose their prices à la Bertrand.[5] The consumer buys one unit of a product to the firm from which she derives higher surplus. In case that she derives the same surplus in both products, the consumer buys from the firm that yields positive profits. In case that both firms get zero equilibrium profits she picks one randomly.

The timing of the model is as follows: first, the consumer's taste θ realizes but it is not observed. Second, each firm chooses the characteristics of his product θ_i ($\in [0, 1]$) to offer in the market. Third, after observing each other firm's product (θ_1, θ_2) and the consumer's preferences θ, firms simultaneously choose their prices.[6] Finally, the consumer buys one unit of the product from the firm she derives higher surplus and payoffs are delivered.[7]

[4]Note that this specification of firms' differences can also be interpreted as vertical differentiation through consumer's idiosyncratic preferences, that is the consumer valuation v^i differs for different brand i. Actually, for any combination of (v^i, c^i) there is an equivalent vertical differentiation either on preferences v^i or on unit costs c^i. For the case where $c^1 = c^2$ and $v^1 > v^2$, Assumption 1 becomes $v^2 > 1$.

[5]In the Hotelling model firms simultaneously choose a location and, observing locations, firms simultaneously choose their prices, and finally the consumer's preference realizes (or there are heterogeneous consumers). In this paper, like in Meagher and Zauner (2004), firms simultaneously choose locations, and, observing consumer's preference and products' locations, firms simultaneously choose their prices. This assumption makes our paper comparable with studies of procurement auctions in markets with uncertain consumer's preferences. We assume that a firm can rapidly adjust his price up or down, and we rely on Bertrand competition as a shortcut for quick market learning about consumer's preference and other products' locations. Finally, notice that Assumption 1 with Bertrand competition guarantee that the market is fully covered.

[6]Alternatively, we can model this step with a reverse auction or procurement.

[7]Other papers in the literature model allow firms to choose their prices without observing consumer's preference. We have developed in Appendix 4.8 an alternative model where firms simultaneously choose location and price, and we characterize the unique equilibrium of the game. We compare our results with this benchmark.

4.3 Equilibrium

We look for a Perfect Bayesian Nash Equilibrium. A strategy for a firm is defined by a variety for his product, θ_i, and a price, $p_i(\theta_1, \theta_2, \theta)$ for each realization of products varieties and consumer taste.

In the last step, firms face price competition à la Bertrand with perfect information. Let's define the difference in firms' production cost $\Delta c \equiv c_2 - c_1 = c$ and the difference in consumer's willingness to pay between products $\Delta v \equiv [v - (\theta - \theta_2)^2] - [v - (\theta - \theta_1)^2]$. The consumer buys one unit of product i if $i = \arg\max_j\{U_j\}$.[8] The following Lemma says that at most one firm makes positive profits ex-post; namely the firm with better market advantage, weighting cost and willingness to pay.

Lemma 5. *Given c, θ_1, θ_2 and θ, price competition à la Bertrand leads, in equilibrium, to the following results: If $c < \Delta v$, prices are $p_1 = 0$ and $p_2 = \Delta v$, and profits are $\Pi_1 = 0$ and $\Pi_2 = \Delta v - c > 0$. If $c > \Delta v$, prices are $p_2 = c$ and $p_1 = c - \Delta v$, and profits are $\Pi_2 = 0$ and $\Pi_1 = c - \Delta v > 0$. If $c = \Delta v$, prices are $p_2 = c$ and $p_1 = 0$ and profits are $\Pi_2 = 0$ and $\Pi_1 = 0$.*

The proof of Lemma 5 follows the Bertrand's argument of undercutting prices with differentiated products until one firm has no incentives to reduce his own price because in doing so the firm will get losses. Lemma 5 says that if $c \leq \Delta v$, the price is restricted for product 1 which equals its marginal costs, i.e., $p_1 = 0$. Firm 2 sells to a positive margin with price $p_2 = \Delta v$ $(\geq c)$. However, if $c \geq \Delta v$, the price is restricted for product 2 which equals its marginal costs, i.e., $p_2 = c$. Firm 1 sells and has positive profits with price $p_1 = c - \Delta v$ (≥ 0).

In the previous stage, anticipating the price competition each firm forms beliefs about the strategy of the other firm, and chooses his variety θ_i maximizing expected profits. For instance, firm 1 solves

$$\max_{\theta_1} E[\Pi_1] \quad = \quad \max_{\theta_1} E[\Pi_1 | c \geq \Delta v] Pr(c \geq \Delta v). \tag{4.1}$$

Conditional on selling, each firm's profit increases in the distance between products' varieties. Firm 1 has a cost advantage that provides some positive profits in case that both firms choose a similar variety. Anticipating the Bertrand price competition, firms have incentives to soften ex-post competition through product differentiation. This effect represents the strategic effect in Hotelling's model.

It is worth noting that, in equilibrium, each firm i chooses the expected consumer's best variety conditional on selling when those varieties realize, i.e., $E_i[\theta|\text{ i } sells]$. $E_i[\theta|\text{ i } sells]$ is a fix point in the following sense: choosing $\theta_1 = E_1[\theta|\text{ i } sells]$ firm 1 sells the product in some states of the world [of consumer's tastes and the other firm's strategy], and the expected consumer's ideal variety in

[8] It is worth noting that firms differences in cost $c_1 < c_2$ are equivalent to vertical differences in consumer's preferences, i.e., $v_1 > v_2$.

those states where firm 1 sells is equal to θ_1. A firm may use a mixed strategy randomizing among different varieties, with each variety satisfying $\theta_i = E_i[\theta|\text{ i } sells]$. For details see Appendix 4.7.1.
There are multiple equilibria in this model that we classify in two types, according to the degree of competition: *low* vs *high* competition. If consumer's taste space can be divided in two disjoint subsets such that one firm sells with probability one in all values of one subset and does not sell otherwise and viceversa, we say that there is an equilibrium with low competition. The remaining equilibria exhibit more competition, since at least one firm will try to sell to any particular customer, and we say that these equilibria are characterized by high competition. For example, if firm 1 chooses a variety $\theta_1 = \frac{1}{4}$ and firm 2 chooses a variety $\theta_2 = 1$, firm 1 sells and makes positive profits with probability one if consumer's taste is $\theta = 0$ or $\theta = \frac{1}{2}$; firm 2 sells and gets positive profits with probability one if consumer's taste is $\theta = 1$. Notice that the expected consumer's taste when firm 1 sells is $E_1[\theta|\text{ 1 } sells] = \frac{(\frac{1}{3}*0+\frac{1}{3}*\frac{1}{2})}{\frac{1}{3}+\frac{1}{3}} = \frac{1}{4}$ which is firm 1's variety. This is true also for firm 2, i.e., $E_2[\theta|\text{ 2 } sells] = 1$. The strategic effect of separating products to increase prices is maximal and the demand effect of competing for consumers' types is represented by the dispute for selling to the consumer when $\theta = \frac{1}{2}$. In our example, this dispute is resolved in favor of firm 1 with $\theta_1 = \frac{1}{4}$.
In an equilibrium with high competition, one firm locates at $\frac{1}{2}$, e.g., $\theta_1 = \frac{1}{2}$ and the other firm randomizes in the corners, e.g., $\sigma_2 = (\sigma_2(0), \sigma_2(1)) = (0.5, 0.5))$. The demand effect is stronger for both firms and the strategic effect generates a randomization between equilibria.
Let's define σ_i as a mixed strategy of firm i and $\sigma_i(x)$ the probability that firm i chooses a variety equals to x. In the following proposition we describe the equilibria if $c = 0$, although the result extends for an open interval of costs. (Proof in Appendix 4.7.2)

Proposition 8. *If $c = 0$, the duopoly location-then-price game under consumer's taste uncertainty has the following equilibria in products varieties,*

8.a Equilibria with low competition are characterized by the following product varieties: $\{(\theta_1 = \frac{1}{4}, \theta_2 = 1), (\theta_1 = \frac{3}{4}, \theta_2 = 0), (\theta_1 = 0, \theta_2 = \frac{3}{4}), (\theta_1 = 1, \theta_2 = \frac{1}{4})\}$.

8.b Equilibria with high competition are characterized by the following product varieties: $\{(\theta_1 = \frac{1}{2}, \sigma_2 = (\sigma_2(0), \sigma_2(1)) = (0.5, 0.5)); (\sigma_1 = (\sigma_1(0), \sigma_1(1)) = (0.5, 0.5), \theta_2 = \frac{1}{2})\}$.

Equilibria with low competition share a pattern. In all of them, one firm is in one corner, e.g., $\theta_2 = 1$, selling only when consumer's taste falls on that corner, and the other firm is in the middle of the other corner and $\frac{1}{2}$, e.g., $\theta_1 = \frac{1}{4}$, selling when consumer's taste falls in that corner or in $\frac{1}{2}$.
Equilibria with high competition offer a different pattern. One firm is in the middle $\frac{1}{2}$ and the other firm is randomizing with probability $\frac{1}{2}$ between the extremes. The firm that randomizes only sells if his choice coincides with consumer's taste realization, otherwise the other firm sells the product.
Equilibria with low competition show the maximum strategic effect of Hotelling's model, while the equilibria with high competition shows a mix of demand effect and strategic effect. Consequently, it is not surprising that the expected price is higher, firms' expected profits are higher and expected

consumer's surplus is lower in equilibria with low competition than in equilibria with high competition. Since firms divide the market ex-ante, the price competition is not tough enough to reduce prices.

In all equilibria, however, there is one firm that is better positioned. This positioning is related to which firm sells more often and makes higher expected profits. We say that a firm is *leading* the market if it is following the strategy that allow him to sell with higher probability. Otherwise we say that a firm is *accommodating* in the market. A firm makes higher expected profits when leading a market than when accommodating, even when comparing across equilibria of different types. That is, a firm gets higher expected profits when leading the market in an equilibrium with high competition than when accommodating the market in an equilibrium with low competition.

If firm 1's cost advantage c is low, any equilibrium can be sustained. However, if c is higher, the equilibrium set shrinks in favor of firm 1, since now firm 1 is leader in all equilibria. This is summarized in the following proposition.

Proposition 9. *If $c > \frac{1}{16}$, there is no equilibrium with low competition where firm 1 is accommodating, i.e., neither $\theta_1 = 0$ nor $\theta_1 = 1$ pure strategies can be hold as an equilibrium. If $c > \frac{1}{8}$ there is no equilibrium with high competition where firm 1 is accommodating, i.e., there is no equilibrium where firm 1 plays a mixed strategy randomizing in $\{0,1\}$ and firm 2 chooses a variety $\theta_2 = \frac{1}{2}$.*

The proof is quite simple and derives from the following reasoning: If $c > \frac{1}{16}$ the equilibrium $\theta_1 = 0$ and $\theta_2 = \frac{3}{4}$ can not be sustained, since firm 1 has incentives to deviate to $\theta_1 = \frac{1}{4}$. Similarly, if $c > \frac{1}{8}$ the equilibrium where firm 2 variety is $\theta_2 = \frac{1}{2}$ and firm 1 randomizes in $\{0,1\}$ can not be hold, since firm 1 has incentives to move to $\theta_1 = \frac{1}{2}$.[9]

It is not surprising that with high cost advantage, firm 1 guarantees higher expected profits. The firm becomes more aggressive and cannot commit to accommodate in an equilibrium since its expected profits from deviating are high. In all equilibria firm 1 is leading the market, and firm 2 follows an accommodating strategy having positive profits only when matches consumer taste in one corner. However, there are still equilibria with low and high competition. In the following Section we allow firms to acquire a costly and private signal about consumer's taste before choosing their varieties, and we see how the access to private information changes the equilibrium set.

4.4 Information Acquisition

Suppose now that, before choosing its own variety, each firm simultaneously acquires a costly private signal s about the consumer's optimal variety θ. A signal of precision $q \in [\frac{1}{3}, 1]$ costs $k(q)$, with $k'(q) > 0$, $k''(q) > 0$, $\lim_{q \to \frac{1}{3}} k'(q) = 0$ and $\lim_{q \to 1} k'(q) = +\infty$. The precision defines the probability with which the signal reveals the state of the world θ and with probability $1-q$ the

[9] We restrict to the set $\bar{c} < \frac{1}{4}$. If $\bar{c} \geq \frac{1}{4}$ firm 1 sells with probability one if $\theta_1 = \frac{1}{2}$ yielding only trivial equilibria.

signal is a random draw from the complement set of the same distribution of θ.[10] Firms' signals are independently distributed.

In this case, the timing of the model is as follows: first, the consumer's taste θ realizes but it is not observed. Second, each firm independently and simultaneously decides the precision (quality) q_i of his private signal s_i at cost $k(q_i)$ to be acquired. Third, without knowing the precision of the signal of the other firm, each firm chooses the characteristics of the product to offer in the market θ_i ($\in [0,1]$). Forth, observing all products varieties (θ_1, θ_2) and the consumer's preferences θ, firms simultaneously choose their prices. Finally, the consumer buys one unit of the product from the firm she derives higher utility and payoffs are delivered.

A strategy for a firm is defined by information acquisition, q_i, and a product variety for each realization of his signal, i.e., $\theta_i(s_i)$, and a price for each combination on consumer's taste and products' varieties, $p_i(\theta_1, \theta_2, \theta)$. Each firm forms beliefs about the quality of the signal and the product's variety (conditional on signal's realization) of the other firm $(\hat{q}_j, \theta_j(s_j, \hat{q}_j))$. Conditioning on those beliefs, a firm maximizes expected profits by choosing the precision of its own signal q_i, and then, its own product's variety $\theta_i(s_i, q_i, \hat{q}_j, \theta_j(s_j, \hat{q}_j))$ for each realization of the signal.

Acquiring private information does not modify the condition derived above regarding the location of a firm given its expected demand. Indeed, the firm will choose the product variety that equals the expected consumer's taste conditional on the signal and the fact that the firm is actually selling, i.e., $\theta_i = E_i[\theta |\text{ i } sells]$. Despite the introduction of informative signals, there are equilibria with low and high competition. However, each firm uses now the information available to improve expected product's fit with consumer taste and thus his expected profits. Strategies vary only slightly when receiving a signal s_i with precision q_i. For instance, one equilibrium with low competition led by firm 1 has the following strategies on varieties:

$$\theta_2 = 1, \text{ and } \theta_1(s_1) = \begin{cases} \frac{1-q_1}{2(2q_1+1-q_1)} & \text{if } s_1 = 0, \\ \frac{q_1}{2q_1+1-q_1} & \text{if } s_1 = \frac{1}{2}, \\ \frac{1}{4} & \text{if } s_1 = 1. \end{cases} \tag{4.2}$$

Since firm 2 only sells when $\theta = 1$, there is no benefit of acquiring information, and $q_2^* = \frac{1}{3}$ (uninformative signal) and $\theta_2 = E_2[\theta |\text{ 2 } sells] = 1$. Firm 1, however, has an incentive to acquire information to match consumer taste either when $\theta = 0$ or when $\theta = \frac{1}{2}$. With this information, firm 1 modifies $\theta_1(s_1 = 0)$ and $\theta_1(s_1 = \frac{1}{2})$ as described in Equation (4.2). On the other hand, one equilibrium with

[10] That is, if $\theta = 0$, the signal is 0 with probability q and, is 0.5 or 1 with probability $\frac{1-q}{2}$. We use a simplified version of a technology where the signal is the true value with probability $\hat{q}$ and a random draw from the original distribution with probability $1-\hat{q}$. This simplification implies that we have $q \geq \frac{1}{3}$, where a signal with $q = \frac{1}{3}$ is uninformative.

high competition led by firm 1 has the following strategies on varieties:

$$\sigma_2(s_2) = \begin{cases} \sigma_2(0) = 1 & \text{if } s_2 = 0, \\ (\sigma_2(0), \sigma_2(1)) = (0.5, 0.5) & \text{if } s_2 = \frac{1}{2}, \\ \sigma_2(1) = 1 & \text{if } s_2 = 1. \end{cases} \quad \text{and}$$

$$\theta_1(s_1) = \begin{cases} \frac{(1-q_1)(1+\frac{3}{2}(1-q_2))}{4q_1+(1-q_1)(2+\frac{3}{2}(1-q_2))} & \text{if } s_1 = 0, \\ \frac{1}{2} & \text{if } s_1 = \frac{1}{2}, \\ \frac{4q_1+(1-q_1)}{4q_1+(1-q_1)(2+\frac{3}{2}(1-q_2))} & \text{if } s_1 = 1. \end{cases}$$

In this equilibrium both firms have incentives to acquire information. When the cost advantage is high, firm 1 sells if consumer's taste is $\theta = \frac{1}{2}$ or if the signal realization is correct ($s_1 = \theta$), or both. On the contrary, firm 2 sells only if consumer taste is in one extreme, firm 2 variety is in that extreme, and the signal of firm 1 is incorrect.[11] Notice that firm 2 chooses a variety, say $\theta_2 = 0$, when receiving a correct signal or when firm 2 randomly peaks that variety if the signal delivers $s_2 = \frac{1}{2}$. In order to sell, firm 2 must choose a variety of the product as far as possible of the variety of the product of firm 1, and this is more likely to happen when firm 1 receives a wrong signal and firm 2 receives the right signal about consumer's taste.

The use of information differs in equilibria with low and high competition. In the former, only firm 1 acquires information to improve the match of his product with consumer's taste in all states that firm 1 sells. However, with high competition, firms acquire information to steal demand from the other firm. They want to match consumer taste in the corners (i.e., $\{\theta = 0, \theta = 1\}$). Firm 2 sells only if firm 1 receives a wrong signal.

When information is cheap, competition becomes fierce since equilibria with low competition can not be sustained. Assume that firm 2 locates its product in one corner, say $\theta_2 = 1$, and firm 1 receives a realization of the signal indicating that the consumer taste is in the corner where firm 2 is located, i.e., $s_1 = 1$. Firm 1 has two options: to choose a product far from the corner, and sell if the signal is incorrect, or to choose a product where firm 2 is located making some positive profits with probability one (is the signal is correct or not). If information is cheap and the signal indicates that it is quite likely that the consumer's taste is located where firm 2 is, the latter option is more profitable. Firm 1 can not commit not to compete.

Defining q_1^{LC} the optimal amount of information that firm 1 would acquire in an equilibrium with low competition. The following Proposition summarizes our result (proof in Appendix 4.7.3).

Proposition 10. *If $q_1^{LC} > 1 - \frac{16}{9}c$ and c is sufficiently high, there is no equilibrium with low competition.*

Thus, only high competitive equilibria survive. In these equilibria the expected price paid by the

[11] If acquiring an informative signal is too expensive, the equilibrium is similar to the one described in Section 4.3: firm 2 sells if the signal in the corner is correct with probability one. If this is the case, strategy of firm 1 varies a little.

consumer is lower, in spite of the possibility of having a bad match between products' varieties and consumer's taste.

This result provides a rationale for the buyer to provide cheap private signals to the supply side. This result applies to procurements where the states builds some public goods (like a bridge, road, highway, dam, etc.). The recommendation is the following: allow suppliers to make their own private inquiries to capture as much information as possible. In this way, an equilibrium with high competition is guaranteed.

4.5 Public Information

We have shown that providing cheap private information when one firm has a cost advantage may generate that only equilibria with high competition can be sustained. Since the firm with cost advantage (firm 1) cannot commit to keep in their own region the equilibrium with low competition can not survive.

There is however some important insight. Suppose that firm 1 acquires information and designs his product using the information like in the equilibrium with high competition. If the signal realization is either $s_1 = 0$ or $s_1 = 1$ and it is publicly observed, firm 2 will anticipate firm 1 behavior and will design a product with opposite characteristic of signal realization. Actually, firm 2 has less incentives to acquire information, since, in this case, firm 2 maximizes both the probability to sell and the expected profits conditioning on selling. Disclosing information allows firms to coordinate and reduce competition.

Without commitment, disclosing truthfully the realization of the signal can be difficult. Nevertheless, a public signal would perfectly coordinate firms' strategies, reducing competition when an extreme taste is delivered. This coordination partially reduces market competition and increases expected prices. When the signal delivers either $s = 0$ or $s = 1$, there exists two disjoint subsets such that one firm sells with probability one for all values of one subset and with probability zero in the other subset. When the signal delivers $\theta = \frac{1}{2}$ there is high competition.

If public information is sufficiently good, i.e., if q_p is high, equilibria with low competition can not survive. If public information is good, one equilibrium with coordination has the following firms' strategies in choosing varieties:

$$\sigma_2(s_p) = \begin{cases} \sigma_2(1) = 1 & \text{if } s_p = 0, \\ (\sigma_2(0), \sigma_s(1)) = (0.5, 0.5) & \text{if } s_p = \frac{1}{2}, \\ \sigma_2(0) = 1 & \text{if } s_p = 1. \end{cases} \quad \text{and } \theta_1(s_1) = \begin{cases} \frac{(1-q_p)}{2(1+q_p)} & \text{if } s_p = 0, \\ \frac{1}{2} & \text{if } s_p = \frac{1}{2}, \\ \frac{(1+3q_p)}{2(1+q_p)} & \text{if } s_p = 1. \end{cases}$$

Firms coordinate their strategies in the extremes when the signal indicates that and, in turn, competition is relaxed. Firms get higher expected profits with public signals than with private signals. The consumer pays a higher expected price with public signals than with public signals. This result is summarized in the following proposition.[12]

Proposition 11. *If information about consumer's taste is sufficiently good, expected profits and expected price are bigger with public information but the expected price is also bigger, and thus expected consumer surplus is bigger with private information.*

Proposition 11 says that when information is sufficiently good, firms appropriate more of the surplus generated with a public signal. Coordinating their actions with a public signal, firms relax competition that allow them to get a higher price when selling. Concluding, if the consumer has some information about her own preferences it is better to allow private inquiries instead of revealing the information directly to the market in a way to avoid coordination.
If the State needs to buy a product, Proposition 11 recommends first to hide any private information and, second, to foster private inquiries. If possible, in a procurement the State must transmit some information to each bidder privately. However, it must avoid public announcements that allow for coordination. With private information, not only the state increases her expected surplus but also pays a lower expected price.

4.6 Conclusion

In this paper we study the role of firms heterogeneity and information in competitive environments whenever there is uncertainty about consumer's taste. We have shown that the degree of competition depends positively on firms heterogeneity and the precision of private signals. We have also shown that public information may foster firms coordination which in turn relaxes market competition.
Our environment can be used for analyzing dynamic and changing markets, like the clothing market, as wells as second-price auctions in procurement of goods that may be difficult to define ex-ante. Allowing for firms heterogeneity and private inquiries can foster competition which reduces the expected price and increases the expected consumer surplus.

[12]Two independent signal reveals more information than one public signal, which may yield that welfare is higher with private signals. If there is only one signal, a public signal generates greater welfare but still lower consumer surplus than a one private signal (for instance, to the more efficient firm). This private signal ensure an equilibrium with high competition. A public signal foster coordination, which is good to guarantee a good match between consumer's taste and, at least, one product in the market. This improvement can be made at a cost of reducing competition. The consumer pays a higher expected price with public signals that might outweigh the increment in expected utility due to a good fit between product variety and consumer taste.

4.7 Appendix

4.7.1 Fixed Point Strategy

Suppose that in equilibrium a firm, e.g., firm 1, sells in some (finite) events x with probability $f(x)$. In those events, the profit of firm 1 is $c+(\theta-\theta_2)^2-(\theta-\theta_1)^2$. Firm 1's expected profit is

$$\sum_x [c+(\theta-\theta_2)^2-(\theta-\theta_1)^2]f(x). \tag{4.3}$$

In equilibrium a change θ_1 have two effects: it modifies the conditional expected profit $c+(\theta-\theta_2)^2-(\theta-\theta_1)^2$, and affects the number of events where the firm sells a product. However, if a marginal increment in θ_1 increases the number of events where the firm sells, the firm makes zero expected profits in those events. Notice that the firm sells in an event if $c+(\theta-\theta_2)^2-(\theta-\theta_1)^2 \geq 0$ on that event, which is exactly the expected profits when selling. For this reason, in equilibrium we expect that the first order condition characterize the optimal θ_1.[13]

$$2\sum_x [-\theta+\theta_1]f(x)=0, \quad \longrightarrow \quad \theta_1 = E_1[\theta|\ 1\ sells] = \frac{\sum_x \theta f(x)}{\sum_x f(x)}. \tag{4.4}$$

4.7.2 Proof of Proposition 8

Let's start with equilibria in pure strategies. Given the rule of $E_i[\theta|\ \text{i}\ sells]$ a firm sells in one, two or three states. If one firm sells in three state it requires that both firms are located at $\frac{1}{2}$, but the other has incentives to move to one corner. If one firm sells in two states his choice must be $\frac{1}{4}$ (or $\frac{3}{4}$), and the best response of the other firm is 1 (or 0). Finally, if both sells when preferences are at $\frac{1}{2}$, their $E_i[\theta|\ \text{i}\ sells]$ is $\frac{1}{6}$ or $\frac{5}{6}$ but cannot be an equilibrium because only firm 1 sells with these strategies. (given his cost advantage)

Let's analyze those equilibria where one firm plays a pure strategy, say firm 1, and the other firm may randomize between products varieties, say firm 2. However, each variety that is played with positive probability must satisfy $E_i[\theta|\ \text{i}\ sells]$ to be part of a maximum and there should not be another point where the firm gets higher expected profit.

Assume that firm 1 plays a pure strategy θ_1 in $[0, 0.5)$. Firm 2 can sell with positive probability to the left or to the right of θ_1. Lets analyze first the interval in the right and focus on $[0.5, 1]$, since the rest $(\theta_1, 0.5)$ is dominated. Firm 2 will sell only if $\theta = 0.5$ or $\theta = 1$. For some strategies, firm 2 only sells if $\theta = 1$, in which case firm's 2 best response is $\theta_2 = 1$. For some other strategies, firm 2 sells a unit of the product if the state is $\theta = 0.5$ or $\theta = 1$, and the highest expected profit is achieved at $\theta_2 = \frac{3}{4}$. Second, In the interval in the left, firm 2 sells only when $\theta = 0$ and $\theta_2 = 0$. So any randomization must be done in $\{0, \frac{3}{4}\}$ or $\{0, 1\}$. The best response of firm 1 to these strategies

[13]The second effect enter into the first order condition as $\sum_y [c+(\theta-\theta_2)^2-(\theta-\theta_1)^2]\frac{\partial f(y)}{\partial \theta_1}$, but either $[c+(\theta-\theta_2)^2-(\theta-\theta_1)^2]$ or $\frac{\partial f(y)}{\partial \theta_1}$ is zero.

is $\theta_1 = 0.5$. And finally, the best response of firm 2 to firm 1 strategy $\theta_1 = 0.5$ is $\theta_2 = 0$, $\theta_2 = 1$ or any randomization to between them (since firm 2 yields the same expected profit in both $\theta_2 = 0$ and $\theta_2 = 1$).

4.7.3 Proof Proposition 10

No equilibrium with low competition exists if $q_1 > 1 - \frac{16}{9}c$. We prove Proposition 10 showing that, if $q_1 > 1 - \frac{16}{9}c$, no equilibrium with low competition can exists since firm 1 has incentives to deviate. In the equilibrium with low competition firm 2 plays $\theta_2 = 1$ and firm 1 uses the signal, playing $\theta_1(s_1 = 1) = \frac{1}{4}$ since this is the expected value of θ conditioning on selling.
However, suppose that firm 1 deviates and sell for all values of θ given the realization of the signal of precision q_1 is $s_1 = 1$. If this is the case, the variety of firm 1 must be $E_1[\theta|\ 1\ sells] = \frac{1+3q_1}{4}$ since firm 1 always sells. So we need to check that consumer will choose to buy product 1 in all states and that firm 1 gets higher profits deviating to this state.
It is straightforward to see that a consumer with $\theta = 0$ or $\theta = \frac{1}{2}$ buys product 1. However, a consumer with $\theta = 1$ buys product 1 only if $c > \Delta v$ that we can express as

$$c > \left[\left(1 - \frac{1+3q_1}{4}\right)^2\right]. \tag{4.5}$$

This condition requires that $q_1 > 1 - \frac{4}{3}\sqrt{c}$ for the consumer to prefer product 1 instead of product 2. Firm 1, instead, prefers to play $\theta_1(s_1 = 1) = \frac{1+3q_1}{4}$ instead of $\theta_1(s_1 = 1) = \frac{1}{4}$ only if his expected profits are greater in the former than in the latter that we can express as

$$\begin{aligned} q_1\left[c - \left(1 - \frac{1+3q_1}{4}\right)^2\right] + \frac{1-q_1}{2}\left[c + 1 - \left(\frac{1+3q_1}{4}\right)^2\right] + \frac{1-q_1}{2}\left[c + \frac{1}{4} - \left(\frac{1}{2} - \frac{1+3q_1}{4}\right)^2\right], \\ > \frac{1-q_1}{2}\left[c + 1 - \left(\frac{1}{4}\right)^2\right] + \frac{1-q_1}{2}\left[c + \frac{1}{4} - \left(\frac{1}{4}\right)^2\right]. \end{aligned}$$

This condition is satisfied if $q_1 > 1 - \frac{16}{9}c$. Given that $c < \frac{9}{16}$, any q_1 that satisfies $q_1 > 1 - \frac{16}{9}c$, also satisfies $q_1 > 1 - \frac{4}{3}\sqrt{c}$, and we claim that if $q_1 > 1 - \frac{16}{9}c$, firm 1 has incentives to deviate from the equilibrium with low competition. This reveals that firm 1 would be better if he can tie his hands when acquiring information. Given that information is not verifiable, we have that only an equilibrium with high competition survives.

4.7.4 Proof Proposition 11

4.7.4.1 Consumer surplus.

We show that $CS(public) - CS(private) < 0$.
$CS(public) = v - \frac{5}{3}\frac{1-q_p}{8} - \frac{2}{3}q_p(c+1) - \frac{1}{3}(c+\frac{1}{4})$, and $CS(private) = v - \frac{2}{3}cq - q\frac{1}{2}(1-q) - \frac{c}{3} - \frac{1}{12} - \frac{(1-q)^2}{2}(c+1) - \frac{(1-q)(1+3q)}{12}\frac{5}{4}$. And the difference is negative for $q > 0.5$ (if the signal is relatively

good), since

$$CS(public) - CS(private) = -c(q - \frac{1}{2}) + \frac{19}{48} - \frac{3}{4}q - \frac{39}{48}q^2 < 0.$$

4.7.4.2 Expected price.

We show that $EP(public) - EP(private) > 0$.
$EP(public) = \frac{1}{2} + qc + \frac{q}{4}$, and $EP(private) = -\frac{qc}{3} + \frac{5c}{6} + \frac{7}{24} + \frac{5q}{12} - q^2(\frac{15}{24} - \frac{c}{2})$. And the difference is positive for $q > 0.5$ (if the signal is relatively good), since

$$EP(public) - EP(private) = \frac{5-4q}{24} + q^2(\frac{15}{24} - \frac{c}{2}) + \frac{c(8q-5)}{6} > 0.$$

4.7.4.3 Welfare: two private signals versus one public signal.

Comparing the case of two private **Welfare** with public signal is:

$$W_{public} = v - \frac{1}{3}\left\{\frac{1-q}{8} + (1-q)(\frac{1}{2})^2\right\} - \frac{1}{3}(1-q)c.$$

Welfare with private signal and high competition is:

$$W_{private} = v - \frac{2}{3}\left[\frac{3(1-q)^2}{8}[\frac{1}{4}+1\right] - \frac{1-q}{12} - \frac{(1-q)\text{Ç}(1+3q)}{6}c.$$

Calling $q_1 = q + x$, where x stands for the difference in signals' precision, and ignoring the first order effect we get:

$$W_{public} - W_{private} = \frac{15q^2 - 20q + 5}{48} - \frac{c}{2}(1-q) + \frac{5+8c}{16}x^2 + \frac{5q-13}{24} - \frac{2}{3}cx.$$

4.7.4.4 One public signal versus one private signal

There is a threshold in q above which the consumer surplus is higher and welfare is lower with private signal than with public signal. We omit the second order effects since they are very small.
Welfare with public signal is:

$$W_{public} = v - \frac{1}{3}\left\{\frac{1-q}{8} + (1-q)(\frac{1}{2})^2\right\} - \frac{1}{3}(1-q)c.$$

Welfare with private signal and high competition is:

$$W_{private} = v - \frac{2}{3}\left[\frac{1-q_1}{4}[\frac{1}{4}+1\right] - \frac{1-q_1}{3}(\frac{1}{2})^2 - \frac{(1-q)}{3}c.$$

The **difference in welfare** is $W_{public} - W_{private} = \frac{1-q}{12} > 0$.

The **consumer surplus** with public information is

$$CS_{public} = v - \frac{7}{24} - \frac{11}{24}q - \frac{2}{3}qc - \frac{c}{3}.$$

Consumer surplus with private signal and high competition is:

$$CS_{private} = v - \frac{11}{24} + \frac{1}{24}q - \frac{1}{3}qc - \frac{2c}{3}.$$

The **difference in consumer surplus** is $CS_{public} - CS_{private} = \frac{1}{3}[(c+\frac{1}{2})(1-q)-q] < 0$, for $q > 0.5$ (since $c < 0.25$).

The **expected price** with public information is

$$EP_{public} = \frac{1}{2} + qc + \frac{q}{4}.$$

The expected price with private signal and high competition is:

$$EP_{private} = \frac{1}{3} + \frac{1}{3}q + \frac{1}{3}qc + \frac{2c}{3}.$$

The **difference in consumer surplus** is $EP_{public} - EP_{private} = \frac{1-q}{12}[1-8c(1-q)] > 0$, for $q > 0.5$ (since $c < 0.25$).

4.7.5 Expected Profits, Utility and Prices with No Information Acquisition

In each type of equilibrium the expected profits are:

1- Low competition led by firm 1 ($\theta_1 = \frac{1}{4}, \theta_2 = 1$): $\Pi_1 = \frac{1}{3}\left[1+\frac{1}{8}+2c\right]$ and $\Pi_2 = \frac{1}{3}\left[\frac{9}{16}-c\right]$. Expected Utility (before price) is $EU = v - \frac{2}{3}\frac{1}{16}$, expected price is $Ep = \frac{2}{3}c + \frac{1}{3}(1+\frac{11}{16})$, and expected consumer surplus is $CS = v - \frac{2}{3}c - \frac{1}{3}\frac{29}{16}$.

2- Low competition led by firm 2 ($\theta_1 = 0, \theta_2 = \frac{3}{4}$): $\Pi_1 = \frac{1}{3}\left[\frac{9}{16}+c\right]$ and $\Pi_2 = \frac{1}{3}\left[1+\frac{1}{8}-c\right]$. Expected Utility (before price) is $EU = v - \frac{1}{3}\frac{1}{16}$, expected price is $Ep = \frac{1}{3}c + \frac{1}{3}(1+\frac{11}{16})$ and expected consumer surplus is $CS = v - \frac{1}{3}c - \frac{1}{3}\frac{29}{16}$.

3- High competition led by firm 1 ($\theta_1 = \frac{1}{2}, (\sigma_2(0), \sigma_2(1)) = (0.5, 0.5)$): $\Pi_1 = \frac{1}{3}[1+2c]$ and $\Pi_2 = \frac{1}{3}\left[\frac{1}{4}-c\right]$. Expected Utility (before price) is $EU = v - \frac{1}{3}\frac{1}{4}$, expected price is $Ep = \frac{2}{3}c + \frac{1}{3}\frac{5}{4}$ and expected consumer surplus is $CS = v - \frac{2}{3}c - \frac{1}{3}\frac{3}{2}$.

4- High competition led by firm 2 ($(\sigma_1(0), \sigma_1(1)) = (0.5, 0.5), \theta_2 = \frac{1}{2}$): $\Pi_1 = \frac{1}{3}\left[\frac{1}{4}+c\right]$ and $\Pi_2 = \frac{1}{3}[1-2c]$. Expected Utility (before price) is $EU = v - \frac{1}{3}\frac{1}{4}$, expected price is $Ep = \frac{1}{3}c + \frac{1}{3}\frac{5}{4}$ and expected consumer surplus is $CS = v - \frac{1}{3}c - \frac{1}{3}\frac{3}{2}$.

The difference in expected price from low competitive equilibrium led by firm 2 and high competitive equilibrium led by firm 1 is: $\Delta Ep = \frac{1}{3}\left(\frac{7}{16}-c\right)$.

UPPER BOUND FOR c: If firm 1 is leading an equilibrium with high competition, it may have incentives to deviate to $\theta_1 = \frac{3}{5}$ if $c > \frac{4}{25}$. Firm 2 is randomizing in $\{0,1\}$, then with $\theta_1 = \frac{3}{5}$ firm 1 sells if $\theta = 1$ with probability one and if $\theta = 0$ with probability $\frac{1}{2}$. However, if $c \leq \frac{1}{5}$ the expected profit firm 1 gets when $\theta_1 = \frac{1}{2}$ is higher than when $\theta_1 = \frac{3}{5}$. So $c < 0.2$.

4.7.6 Expected Profits and Prices with Information Acquisition

4.7.6.1 Equilibrium with low competition

If firm 1 does acquire a signal with q_1 not so high, there is one equilibrium where: i- firm 2 chooses one corner, for instance $\theta_2 = 1$, acquiring no information; and ii) firm 1 chooses to offer a product in $[0, 0.5]$ depending on the realization of the signal, i.e $\theta_1(s_1)$. For the case $\theta_2 = 1$, firm's 1 strategy is defined by:

$$\theta_2 = 1, \text{ and } \theta_1(s_1) = \begin{cases} \frac{1-q_1}{2(2q_1+1-q_1)} & \text{if } s_1 = 0, \\ \frac{q_1}{2q_1+1-q_1} & \text{if } s_1 = \frac{1}{2}, \\ \frac{1}{4} & \text{if } s_1 = 1. \end{cases}$$

Let's calculate the expected profits of firm 2 recalling that $\theta_1(0) = \frac{1}{2} - \theta_1(\frac{1}{2})$.[14]

$$\Pi_2 = \frac{1}{3}\left[-c + q_1(\frac{3}{4})^2 + \frac{1-q_1}{2}(1-\theta_1(0))^2 + \frac{1-q_1}{2}(\frac{1}{2}+\theta_1(0))^2\right]. \tag{4.6}$$

Profits for firm 1 is[15]

$$\begin{aligned} \Pi_1 &= \frac{1}{3}(c+1) - \frac{1}{3}\left\{q_1\theta_1(0)^2 + \frac{1-q_1}{2}\theta_1(\frac{1}{2})^2 + \frac{1-q_1}{2}(\frac{1}{2})^2\right\}, \\ &+ \frac{1}{3}(c+\frac{1}{4}) - \frac{1}{3}\left\{q_1(\frac{1}{2}-\theta_1(\frac{1}{2}))^2 + \frac{1-q_1}{2}(\frac{1}{2}-\theta_1(0))^2 + \frac{1-q_1}{2}(\frac{1}{2})^2\right\}. \end{aligned}$$

knowing that $\theta_1(0) = \frac{1}{2} - \theta_1(\frac{1}{2})$,

$$\frac{\partial \Pi_1}{\partial q_1} = -\frac{2}{3}\left[\theta_1(0)^2\right] + \frac{1}{3}\left[(\frac{1}{2}-\theta_1(0))^2 + (\frac{1}{2})^2\right].$$

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$$\Pi_2 = \frac{1}{3}0 + \frac{1}{3}0 + \frac{1}{3}\left\{q_1\left[(\frac{3}{4})^2 - c\right] + \frac{1-q_1}{2}\left[(1-\theta_1(0))^2 - c\right] + \frac{1-q_1}{2}\left[(1-\theta_1(\frac{1}{2}))^2 - c\right]\right\}.$$

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$$\begin{aligned} \Pi_1 &= \frac{1}{3}\left\{q_1\left[c+1-\theta_1(0)^2\right] + \frac{1-q_1}{2}\left[c+1-\theta_1(\frac{1}{2})^2\right] + \frac{1-q_1}{2}\left[c+1-(\frac{1}{2})^2\right]\right\}, \\ &+ \frac{1}{3}\left\{q_1\left[c+\frac{1}{4}-(\frac{1}{2}-\theta_1(\frac{1}{2}))^2\right] + \frac{1-q_1}{2}\left[c+\frac{1}{4}-(\frac{1}{2}-\theta_1(0))^2\right] + \frac{1-q_1}{2}\left[c+\frac{1}{4}-(\frac{1}{2})^2\right]\right\} + \frac{1}{3}0. \end{aligned}$$

For q_1 not so high, it is true that $\frac{2}{3}\theta_1(0)^2 < \frac{1}{3}(\frac{1}{2}-\theta_1(0))^2$, implying that $\frac{\partial\Pi_1}{\partial q_1} \geq \frac{1}{12}$. Since $\frac{\partial\theta_1(0)}{\partial q_1} < 0$ we can show that $\frac{\partial^2\Pi_1}{\partial q_1^2} > 0$. We require the cost function to be sufficiently convex such that we find a maximum with the first order condition.

In an equilibrium with low competition led by firm 1 **the expected price** is:[16]

$$EP_{LC} = \frac{1}{3}\frac{5}{4} + \frac{2}{3}\left\{c - q_1\theta_1(0)^2 - \frac{1-q_1}{2}[\frac{1}{16} + (\frac{1}{2} - \theta_1(0))^2]\right\} + \frac{1}{3}\left\{q_1\frac{9}{16} + \frac{1-q_1}{2}[(1-\theta_1(0))^2 + (1-\theta_1(\frac{1}{2}))^2]\right\}.$$

The consumer gets the following expected gross utility (before price):[17]

$$EU = v - \frac{2}{3}\left[q_1\theta_1(0)^2 + \frac{1-q_1}{2}[\theta_1(\frac{1}{2})^2 + \frac{1}{16}]\right]$$

Expected Consumer Surplus under **low competition** is:

$$v - \frac{1}{3}\frac{5}{4} - \frac{2}{3}c - \frac{1}{3}\frac{9}{16}q_1 - \frac{2}{3}\frac{1-q_1}{2}[(1-\theta_1(0))^2 + (\frac{1}{2} - \theta_1(\frac{1}{2}))^2]$$

4.7.6.2 Equilibrium with high competition

In this case firm 1 acquires a signal with q_1 and firm 2 acquires a signal with q_2. The equilibrium is characterized by: i- firm 2 chooses one corner, following the signal realization when possible; and ii) firm 1 chooses to offer a product in $[0,1]$ depending on the realization of the signal, i.e $\theta_1(s_1)$. Firms' strategies are defined by:

$$\sigma_2(s_2) = \begin{cases} 0 & \text{if } s_2 = 0, \\ (\sigma_2(0), \sigma_2(1)) = (0.5, 0.5) & \text{if } s_2 = \frac{1}{2}, \\ 1 & \text{if } s_2 = 1. \end{cases}, \text{ and } \theta_1(s_1) = \begin{cases} \frac{(1-q_1)(1+\frac{3}{2}(1-q_2))}{4q_1+(1-q_1)(2+\frac{3}{2}(1-q_2))} & \text{if } s_1 = 0, \\ \frac{1}{2} & \text{if } s_1 = \frac{1}{2}, \\ \frac{4q_1+(1-q_1)}{4q_1+(1-q_1)(2+\frac{3}{2}(1-q_2))} & \text{if } s_1 = 1. \end{cases}$$

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$$EP_{LC} = +\frac{1}{3}\left\{q_1[c + \frac{1}{4} - (\frac{1}{2} - \theta_1(\frac{1}{2}))^2] + \frac{1-q_1}{2}[c + \frac{1}{4} - \frac{1}{16}] + \frac{1-q_1}{2}[c + \frac{1}{4} - (\frac{1}{2} - \theta_1(0))^2]\right\},$$
$$+\frac{1}{3}\left\{q_1[c + 1 - \theta_1(0)^2] + \frac{1-q_1}{2}[c + 1 - \frac{1}{16}] + \frac{1-q_1}{2}[c + 1 - \theta_1(\frac{1}{2})^2]\right\}, +\frac{1}{3}\left\{q_1[(\frac{3}{4})^2] + \frac{1-q_1}{2}[(1-\theta_1(0))^2] + \frac{1-q_1}{2}[(1-\theta_1(\frac{1}{2}))^2]\right\}.$$

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$$EU = v - \frac{1}{3}\left[q_1\theta_1(0)^2 + \frac{1-q_1}{2}\theta_1(\frac{1}{2})^2 + \frac{1-q_1}{2}(\frac{1}{4})^2\right] - \frac{1}{3}\left[q_1(\frac{1}{2} - \theta_1(\frac{1}{2}))^2 + \frac{1-q_1}{2}(\frac{1}{2} - \theta_1(0))^2 + \frac{1-q_1}{2}(\frac{1}{4})^2\right] - \frac{1}{3}0.$$

Recalling that $\theta_1(1) = 1-\theta_1(0)$, the expected profits of firm 2 are,[18]

$$\Pi_2 = \frac{1-q_1}{3}\frac{1+3q_2}{4}[\theta_1(1)^2+\frac{1}{4}-2c].$$

Firm's 1 profit function is[19]

$$\frac{\partial \Pi_1}{\partial q_1} = \frac{2}{3}\left[\frac{c}{2}\frac{1}{8}-\theta_1(0)^2-\frac{3}{8}(1-q_2)[c-(1-\theta_1(0))^2]\right]+\frac{1}{3}\left(\frac{1}{2}-\theta_1(0)\right)^2.$$

For calculating $\frac{\partial^2\Pi_1}{\partial q_1 \partial q_2}$ notice that $\theta_1(0)$ also depends on q_2

In an equilibrium with high competition led by firm 1 the **expected price** is:[20]

$$EP_{HC} = \frac{2}{3}\Big\{q_1[c-\theta_1(0)^2]+q_1\frac{3(1-q_2)}{4}+\frac{1-q_1}{2}\frac{3q_2-1}{2}][\theta_1(\frac{1}{2})^2+\theta_1(1)^2]+2\frac{1-q_1}{2}\frac{3(1-q_2)}{4}[c+1]\Big\},$$
$$+\frac{1}{3}\Big\{[c+\frac{1}{4}]-2\frac{1-q_1}{2}(\frac{1}{2}-\theta_1(0))^2\Big\}.$$

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$$\Pi_2 = \frac{1}{3}\left\{q_2\frac{1-q_1}{2}[\theta_1(1)^2-c]+\frac{1}{2}\frac{1-q_2}{2}\frac{1-q_1}{2}[\theta_1(1)^2-c]+\frac{1-q_1}{2}\frac{1+3q_2}{4}[\frac{1}{4}-c]\right\}+\frac{1}{3}0,$$
$$+\frac{1}{3}\left\{q_2\frac{1-q_1}{2}[(1-\theta_1(0))^2-c]+\frac{1}{2}\frac{1-q_2}{2}\frac{1-q_1}{2}[(1-\theta_1(0))^2-c]+\frac{1-q_1}{2}\frac{1+3q_2}{4}[\frac{1}{4}-c]\right\}.$$

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$$\Pi_1 = \frac{1}{3}\left\{q_1\left[c-\theta_1(0)^2+\frac{3}{4}(1-q_2)\right]+\frac{1-q_1}{2}\left[c-\frac{1}{4}+\frac{3}{4}(1-q_2)\right]+\frac{1-q_1}{2}\frac{3}{4}(1-q_2)\left[c+1-\theta_1(1)^2\right]\right\},$$
$$\frac{1}{3}\left\{q_1\left[c-(1-\theta_1(1))^2+\frac{3}{4}(1-q_2)\right]+\frac{1-q_1}{2}\left[c-\frac{1}{4}+\frac{3}{4}(1-q_2)\right]+\frac{1-q_1}{2}\frac{3}{4}(1-q_2)\left[c+1-(1-\theta_1(0))^2\right]\right\},$$
$$+\frac{1}{3}\left\{c+\frac{1}{4}-\frac{(1-q_1)}{2}(\frac{1}{2}-\theta_1(0))^2-\frac{(1-q_1)}{2}(\theta_1(1)-\frac{1}{2})^2\right\}.$$

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$$EP_{HC} = \frac{1}{3}\Big\{q_1[q_2+\frac{1-q_2}{4}][c-\theta_1(0)^2]+q_1\frac{3(1-q_2)}{4}[c+1-\theta_1(0)^2]+\frac{1-q_1}{2}[q_2+\frac{1-q_2}{4}][\theta_1(\frac{1}{2})^2],$$
$$+\frac{1-q_1}{2}\frac{3(1-q_2)}{4}[c+1-\theta_1(\frac{1}{2})^2]+\frac{1-q_1}{2}[q_2+\frac{1-q_2}{4}][\theta_1(1)^2]+\frac{1-q_1}{2}\frac{3(1-q_2)}{4}[c+1-\theta_1(1)^2]\Big\},$$
$$+\frac{1}{3}\Big\{q_1[c+\frac{1}{4}]+\frac{1-q_1}{2}[c+\frac{1}{4}-(\frac{1}{2}-\theta_1(0))^2]+\frac{1-q_1}{2}[c+\frac{1}{4}-(\theta_1(1)-\frac{1}{2})^2]\Big\},$$
$$+\frac{1}{3}\Big\{q_1[q_2+\frac{1-q_2}{4}][c-(1-\theta_1(1))^2]+q_1\frac{3(1-q_2)}{4}[c+1-(1-\theta_1(1))^2]+\frac{1-q_1}{2}[q_2+\frac{1-q_2}{4}][(1-\theta_1(\frac{1}{2}))^2],$$
$$+\frac{1-q_1}{2}\frac{3(1-q_2)}{4}[c+1-(1-\theta_1(\frac{1}{2}))^2]+\frac{1-q_1}{2}[q_2+\frac{1-q_2}{4}][(1-\theta_1(0))^2]+\frac{1-q_1}{2}\frac{3(1-q_2)}{4}[c+1-(1-\theta_1(0))^2]\Big\}.$$

The consumer gets the following expected gross utility (before price)[21]

$$EU = v - \frac{2}{3}\left[q_1\theta_1(0)^2 + \frac{1-q_1}{2}\frac{3(1-q_2)}{4}[\frac{1}{4} + (1-\theta_1(0))^2\right] - \frac{2}{3}\frac{1-q_1}{2}(\frac{1}{2} - \theta_1(0))^2.$$

Expected Consumer Surplus under under **high competition** is:

$$v - \frac{2}{3}cq_1 - \frac{2}{3}q_1\frac{3}{4}(1-q_2) - \frac{1}{3}(c+\frac{1}{4}) - \frac{2}{3}3(1-q_1)\frac{1-q_2}{4}(c+1) - \frac{2}{3}\frac{1-q_1}{2}\frac{1+3q_2}{4}[\frac{1}{4} + (1-\theta_1(0))^2].$$

for the extreme case that there is perfect information about the taste, the expected net utility under low competition is $v - \frac{1}{3}(1+\frac{1}{4}) - \frac{2}{3}c - \frac{1}{3}\frac{9}{16}$ while for high competition is $v - \frac{1}{3}(c+\frac{1}{4}) - \frac{2}{3}c$ and the difference (low- high) is $-\frac{1}{3}(1-c) - \frac{1}{3}\frac{9}{16}$ which accounts only for the absence of competition, since perfect information generates a perfect match between at least one product and consumer's taste. Notice that the difference is lower when the cost advantage is higher. However, this cost advantage could be necessary for having an equilibrium with high competition.

4.7.7 Firm's incentives to Play Pure Strategies

Notice these strategies.

$$\sigma_2(s_2) = \begin{cases} 0 & \text{if } s_2 = 0, \\ (\sigma_2(0), \sigma_2(1)) = (0.5, 0.5) & \text{if } s_2 = \frac{1}{2}, \\ 1 & \text{if } s_2 = 1. \end{cases}, \text{ and } \theta_1(s_1) = \begin{cases} \frac{(1-q_1)(1+\frac{3}{2}(1-q_2))}{4q_1+(1-q_1)(2+\frac{3}{2}(1-q_2))} & \text{if } s_1 = 0, \\ \frac{1}{2} & \text{if } s_1 = \frac{1}{2}, \\ \frac{4q_1+(1-q_1)}{4q_1+(1-q_1)(2+\frac{3}{2}(1-q_2))} & \text{if } s_1 = 1. \end{cases}$$

Could firm 1 do better when the signal is $s_1 = \frac{1}{2}$ by randomizing between the left and the right of $\frac{1}{2}$. If this is case, by the rule of $E_1[\theta|sell]$, firm 1 randomizes between:

$$\theta_1^a(s_1 = \frac{1}{2}) = \frac{q_1 + \frac{3}{4}(1-q_1)(1-q_2)}{1 + q_1 + \frac{3}{4}(1-q_1)(1-q_2)}, \quad \text{and} \quad \theta_1^b(s_1 = \frac{1}{2}) = \frac{1}{1 + q_1 + \frac{3}{4}(1-q_1)(1-q_2)}. \tag{4.7}$$

With $\theta_1^a < 0.5 < \theta_1^b$. However, if $\theta = 0$ the consumer prefers θ_1^a to $\theta_2 = 0$ only if the value of q_1 satisfies $q_1 < \frac{1}{1-\frac{3}{4}(1-q_2)}[\frac{\sqrt{c}}{1-\sqrt{c}} - \frac{3}{4}(1-q_2)]$.

4.7.8 Expected Prices with Public Signals

Let's assume that there is a public signal s that is the true value θ with probability q and a random draw from the complementary set. If the signal precision q is not so high, there are equilibria with

[21]

$$EU = v - \frac{1}{3}\left[q_1\theta_1(0)^2 + \frac{1-q_1}{2}\frac{3(1-q_2)}{4}(\frac{1}{2})^2 + \frac{1-q_1}{2}\frac{3(1-q_2)}{4}\theta_1(1)^2\right] - \frac{1}{3}\left[q_1 0 + 2\frac{1-q_1}{2}(\frac{1}{2} - \theta_1(0))^2\right],$$
$$-\frac{1}{3}\left[q_1(1-\theta_1(1))^2 + \frac{1-q_1}{2}\frac{3(1-q_2)}{4}(\frac{1}{2})^2 + \frac{1-q_1}{2}\frac{3(1-q_2)}{4}(1-\theta_1(0))^2\right].$$

low competition. However, if the q is high enough, the only equilibria that survives is the one where firms choose the following varieties.

$$\sigma_2(s_p) = \begin{cases} 1 & \text{if } s_p = 0, \\ (\sigma_2(0), \sigma_s(1)) = (0.5, 0.5) & \text{if } s_p = \frac{1}{2}, \\ 0 & \text{if } s_p = 1. \end{cases} \quad \text{and } \theta_1(s_1) = \begin{cases} \frac{(1-q_p)}{2(1+q_p)} & \text{if } s_p = 0, \\ \frac{1}{2} & \text{if } s_p = \frac{1}{2}, \\ \frac{(1+3q_p)}{2(1+q_p)} & \text{if } s_p = 1. \end{cases}$$

the **expected price** paid by the consumer is:[22]

$$EP_{public} = \frac{2}{3}\Big\{q_p[c+1] - q_p[\theta_1(0)^2] + \frac{1-q_p}{2}\frac{1}{2} + \frac{1-q_p}{2}[\theta_1(1)^2 - c]\Big\},$$
$$+\frac{1}{3}\Big\{q_p(c+\frac{1}{4}) + \frac{1-q_p}{2}[c+\frac{1}{4} - (\frac{1}{2} - \theta_1(0))^2] + \frac{1-q_p}{2}[c+\frac{1}{4} - (\theta_1(1) - \frac{1}{2})^2]\Big\}.$$

Expected utility before price is:

$$EU = v - \frac{1}{3}\Big\{2q_p\theta_1(0)^2 + \frac{1-q_p}{2}\frac{1}{4} + (1-q_p)(\frac{1}{2} - \theta_1(0))^2\Big\}$$

the difference between utility and price are:

$$v - \frac{5}{3}\frac{1-q_p}{8} - \frac{2}{3}q_p(c+1) - \frac{1}{3}(c+\frac{1}{4})$$

Notice that if q is high, the difference in expected utility is higher under any type of competition with private information than with public information.

4.8 Appendix B: Bayesian Pricing Game

Setup: One consumer with taste θ ($\in \{0; 0.5; 1\}$) must buy one unit of a product. The prior is that each value of $\theta \in \{0; 0.5; 1\}$ realizes with the same probability. The utility that the consumer with taste θ derives of buying a product with location θ_i ($\in [0,1]$) at price p_i ($\in \mathbb{R}_+$) is $U(p_i, \theta_i) = v - (\theta - \theta_i)^2 - p_i$. We assume $v \sim +\infty$, i.e., the buyer always buys a unit of the product. The prior is that $\theta \in \{0; 0.5; 1\}$ with the same probability. Two homogeneous firms with zero unit cost simultaneously choose a location and a price. After observing both products locations and prices the consumer chooses from which firm she buys one unit of a product. The timing is that, first, firms choose (θ_i, p_i) simultaneously without knowing the consumer's location θ, and second,

[22]

$$EP_{public} = \frac{1}{3}\Big\{q_p[c+1-\theta_1(0)^2] + \frac{1-q_p}{2}[\frac{1}{2}(\frac{1}{4} - c) + \frac{1}{2}(c+1-\frac{1}{4})] + \frac{1-q_p}{2}[\theta_1(1)^2 - c]\Big\},$$
$$+\frac{1}{3}\Big\{q_p(c+\frac{1}{4}) + \frac{1-q_p}{2}[c+\frac{1}{4} - (\frac{1}{2} - \theta_1(0))^2] + \frac{1-q_p}{2}[c+\frac{1}{4} - (\theta_1(1) - \frac{1}{2})^2]\Big\},$$
$$+\frac{1}{3}\Big\{q_p[c+1-(1-\theta_1(1))^2] + \frac{1-q_p}{2}[\frac{1}{2}(\frac{1}{4} - c) + \frac{1}{2}(1+c-\frac{1}{4})] + \frac{1-q_p}{2}[(1-\theta_1(0))^2 - c]\Big\}.$$

the consumer chooses from which firm to buy one unit of a product maximizing her utility. In case of being indifferent between both products, the consumer randomly peaks one firm.

Theorem 1. *There exists a unique equilibrium $\theta_i = \frac{1}{2}$ $\forall i$, $p_i \sim F(p)$, $F(p) = 1 - \frac{1}{p}$ for $p \in [1, +\infty)$.*

First we prove the following lemma

Lemma 6. *In equilibrium, the density function of the mixed strategy $f(p)$ is decreasing function of p.*

Proof. Recall that $\max_p \Pi = \max_p \frac{1}{3} p \sum_k [1 - F_k(p)]$. Suppose that $f(p)$ is increasing. The first order condition is

$$\sum_k [1 - F_k(p)] - p \sum_k f_k(p) = 0.$$

Since the function $f(p)$ represent a mixed strategy, it must satisfy the first order condition for all p in the domain. In particular for p and $p+\varepsilon$: $\sum_k [1 - F_k(p+\varepsilon)] = (p+\varepsilon) \sum_k f_k(p+\varepsilon)$. Subtracting the first order conditions for p and $p+\varepsilon$ we have

$$\sum_k \underbrace{[F_k(p) - F_k(p+\varepsilon)]}_{<0 \text{ since } F'(p)>0} = \underbrace{p[\sum_k f_k(p+\varepsilon) - f_k(p)]}_{>0 \text{ since by assumption } f'(p)>0} + \underbrace{\varepsilon \sum_k f_k(p+\varepsilon)}_{>0}.$$

but this is a contradiction. Then $f(p)$ is decreasing. □

Proof. $\theta_i = \frac{1}{2}$ $\forall i$ is an equilibrium. Let $D(p|\frac{1}{2})$ be the expected demand given the other firm is located at $\frac{1}{2}$. Given this, $F(p)$ is uniquely identified at $F(p) = 1 - \frac{1}{p}$. Note that: $\Pi = p[1 - F(p)] = p[1 - (1 - \frac{1}{p})] = 1$, and that $E[p_1 | p_1 \leq p_2] = 2$.

Lemma 7. *There exists no other equilibrium.*

We prove this by showing that if $\theta_i < \frac{1}{2}$ (W.L.O.G.), the best response of the other player is to choose $\hat{\theta} = BR_j(\theta_i) \in (\theta_i, \frac{1}{2})$. The Best Response mapping has a unique $F(p)$ at $\frac{1}{2}$. The firm chooses $\hat{\theta}$. The demand

$$D(p|\hat{\theta}) = \frac{1}{3}\left[F(p + \theta_1^2 - \hat{\theta}^2) + F(p + (\theta_1 - \frac{1}{2})^2 - (\hat{\theta} - \frac{1}{2})^2) + F(p + (\theta_1 - 1)^2 - (\hat{\theta} - 1)^2)\right].$$

We first prove that (2.a) $\theta_i \leq \hat{\theta}$: by contradiction. Suppose $\theta_i > \hat{\theta}$ and choosing the value of $\hat{\theta}$ to maximize the expected demand:

$$-2\hat{\theta} f(p + \theta_1^2 - \hat{\theta}^2) - 2(\hat{\theta} - \frac{1}{2}) f(p + (\theta_1 - \frac{1}{2})^2 - (\hat{\theta} - \frac{1}{2})^2) - 2(\hat{\theta} - 1) f(p + (\theta_1 - 1)^2 - (\hat{\theta} - 1)^2) = 0.$$

And we obtain a value of $\hat{\theta}$ equal to,

$$\frac{1}{2} < \hat{\theta} = \frac{\frac{1}{2}f(p+(\theta_1-\frac{1}{2})^2-(\hat{\theta}-\frac{1}{2})^2)+f(p+(\theta_1-1)^2-(\hat{\theta}-1)^2)}{f(p+\theta_1^2-\hat{\theta}^2)+f(p+(\theta_1-\frac{1}{2})^2-(\hat{\theta}-\frac{1}{2})^2)+f(p+(\theta_1-1)^2-(\hat{\theta}-1)^2)}.$$

Since $f(p)$ is decreasing, implies that $f(p+\theta_1^2-\hat{\theta}^2) < f(p+(\theta_1-1)^2-(\hat{\theta}-1)^2)$ (recall that that $\hat{\theta} < \theta_1$), but $\frac{1}{2} < \hat{\theta}$ generates a contradiction, and $\theta_1 < \hat{\theta}$.
Second we prove that (2.b) $\theta_i < \hat{\theta}$. Suppose that $\theta_i = \hat{\theta}$

$$-2\hat{\theta}f(p)-2(\hat{\theta}-\frac{1}{2})f(p)-2(\hat{\theta}-1)f(p)=0.$$

which implies that $\hat{\theta}=\frac{1}{2}$, but we have assumed that $\theta_i < \frac{1}{2}$, then $\hat{\theta}=\theta_i$ yields a contradiction.
Third we prove that (2.c) $\hat{\theta} \leq \frac{1}{2}$. For any $\theta_i < 0.5$ the first order condition respect to $\hat{\theta}$ is also lower than $\frac{1}{2}$. Recall that we have proved that $\hat{\theta} > \theta_i$. In the first order condition respect to $\hat{\theta}$ note that $f(p+\theta_1^2-\hat{\theta}^2) > f(p+(\theta_1-1)^2-(\hat{\theta}-1)^2)$

$$-2\hat{\theta}f(p+\theta_1^2-\hat{\theta}^2)-2(\hat{\theta}-\frac{1}{2})f(p+(\theta_1-\frac{1}{2})^2-(\hat{\theta}-\frac{1}{2})^2)-2(\hat{\theta}-1)f(p+(\theta_1-1)^2-(\hat{\theta}-1)^2)=0.$$

now

$$\frac{1}{2} > \hat{\theta} = \frac{\frac{1}{2}f(p+(\theta_1-\frac{1}{2})^2-(\hat{\theta}-\frac{1}{2})^2)+f(p+(\theta_1-1)^2-(\hat{\theta}-1)^2)}{f(p+\theta_1^2-\hat{\theta}^2)+f(p+(\theta_1-\frac{1}{2})^2-(\hat{\theta}-\frac{1}{2})^2)+f(p+(\theta_1-1)^2-(\hat{\theta}-1)^2)}.$$

Notice that we have proved that $\hat{\theta} \in (\theta_i, \frac{1}{2})$. The proof is complete.
Finally, note that when both firms locates in the same point, e.g., $\theta_i = \frac{1}{2}$, given the mixed strategy $F(p)$ of the other firm, the first order condition respect to p is

$$[1-F(p)]-pf(p)=0, \quad \rightarrow \quad -\frac{1}{p}=\frac{\partial \log[1-F(p)]}{\partial p}.$$

And we conclude that $F(p)=1-\frac{1}{p}$. □

4.8.1 Comparison

In this setup firms locate at 0.5 and charge high expected prices. A simple comparison with our benchmark model yields an straithforward conclusion. Our setup drives lower expected prices and lower losses due to location. In our setup the expected price paid by the buyer is $\frac{9}{16}$ and the welfare loss due to location is $\frac{1}{16}\frac{2}{3}$. In the alternative model presented in this appendix, the expected price is 2 and the loss due to location is $\frac{1}{4}\frac{2}{3}$.

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