SILESIAN UNIVERSITY OF TECHNOLOGY

FACULTY OF CIVIL ENGINEERING

STUDY OF DIFFERENT STRUCTURES CONSTITUTED BY CERAMICS, STEEL AND CONCRETE

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INDEX

1 - INTRODUCTION

1.1 - AIM AND REASON OF THE WORK

The work deals with “Study of different structures constituted by ceramics, steel and concrete”. Throughout this work we present different types of structures, cantilever beam, beam with two supports, constituted by the materials as stated previously.

The most commonly used construction materials are ceramics, steel and concrete, for this reason, we study these materials in the different structures, in this way we be able to study the differences between them. It also discusses the advantages and disadvantages of each and between them. We will make some conclusions about all the work.

2 - MATERIALS

2.1 - CERAMICS

The ceramics are basically divided into three groups:

• Amorphous ceramics: they are the glasses.
Based on silica, SiO_2 + additives to decrease the melting temperature.

• Traditional ceramics: products made of clay, silica and feldspar.
Clay: $Al_2O_3 \cdot SiO_2 \cdot 6SiO_2$
Silica: SiO_2
Feldspar: $K_2O \cdot Al_2O_3 \cdot 6SiO_2$
o Porous ceramics (bricks, pottery, earthenware)
o Compact ceramic (porcelain, stoneware)
o Refractory ceramic (porcelain for thermal insulation)

• Advanced ceramics:
o Refractory ceramics
o Piezoelectric and ferroelectrics
o Electro-optical
o Abrasive ceramics: nitrides and carbides
o Superconductor ceramics
o Biocompatible ceramics: hidroxyapatite

2.1.1 - Ceramic Bond

Ceramic materials can have different type of structures, crystalline, amorphous or mixed. The properties of ceramic materials mainly depend on the structure having.

It studies the percentage of ionic and covalent character of bond for some ceramic materials for determining the crystalline structure.

For studying this percentage using Pauling’s formula that it can determine the percentage of ionic character of a reaction:

$$Pauling : \% \, Ionic \;\; character = 100 \cdot \left\{ 1 - e^{\frac{-(x_A - x_B)^2}{4}} \right\}$$

For example,

- Material: TiC

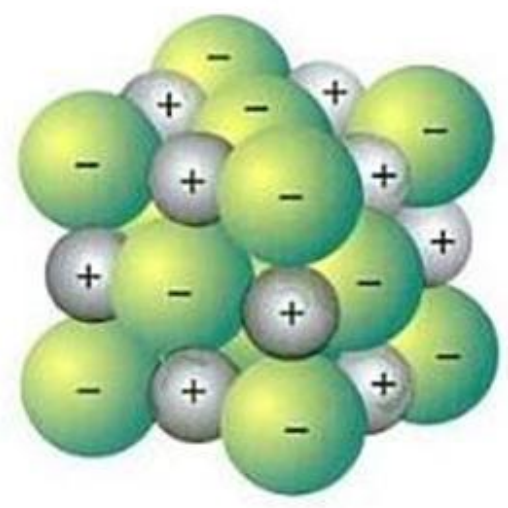
$$\chi_A = 1.3$$
$$\chi_B = 2.5$$
$$\left[\frac{\chi_A - \chi_B}{2} \right]^2 = \left[\frac{1.3 - 2.5}{2} \right]^2 = 0.36$$

$C_C = 0.365$
$C_I = 0.302$

Now, we can show the difference between the types of bond that we can be found, the ionic bonding, the covalent bonding and metallic bonding.

- **Ionic Bond:**

Structure in general and an example of ionic bond



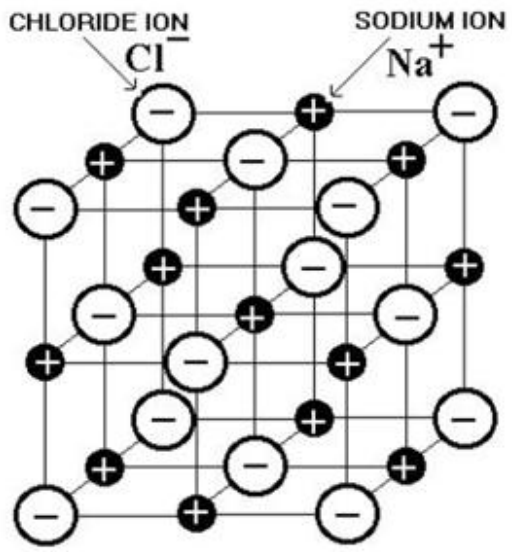


Image 2.1- Example of ionic bond, Cl^-Na^+

- **Covalent Bond:**

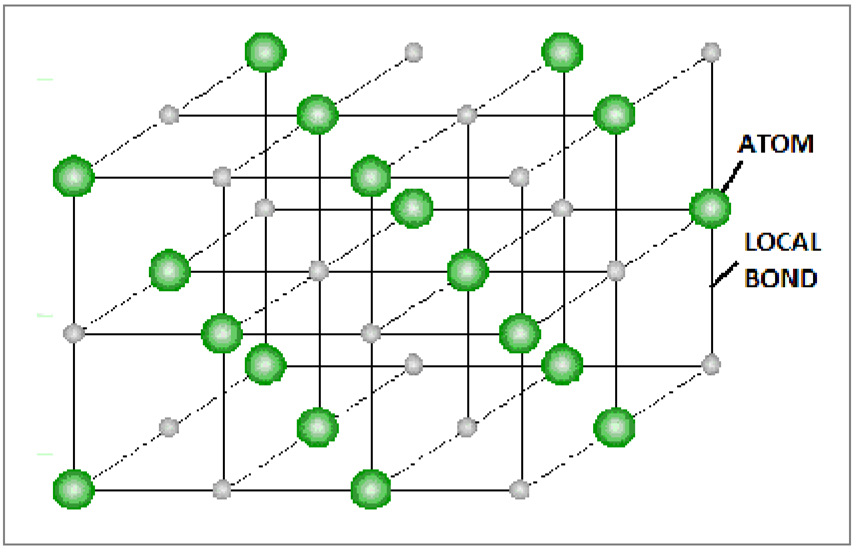


Image 2.2 – Structure in general of Covalent Bond

- **Metallic Bond:**

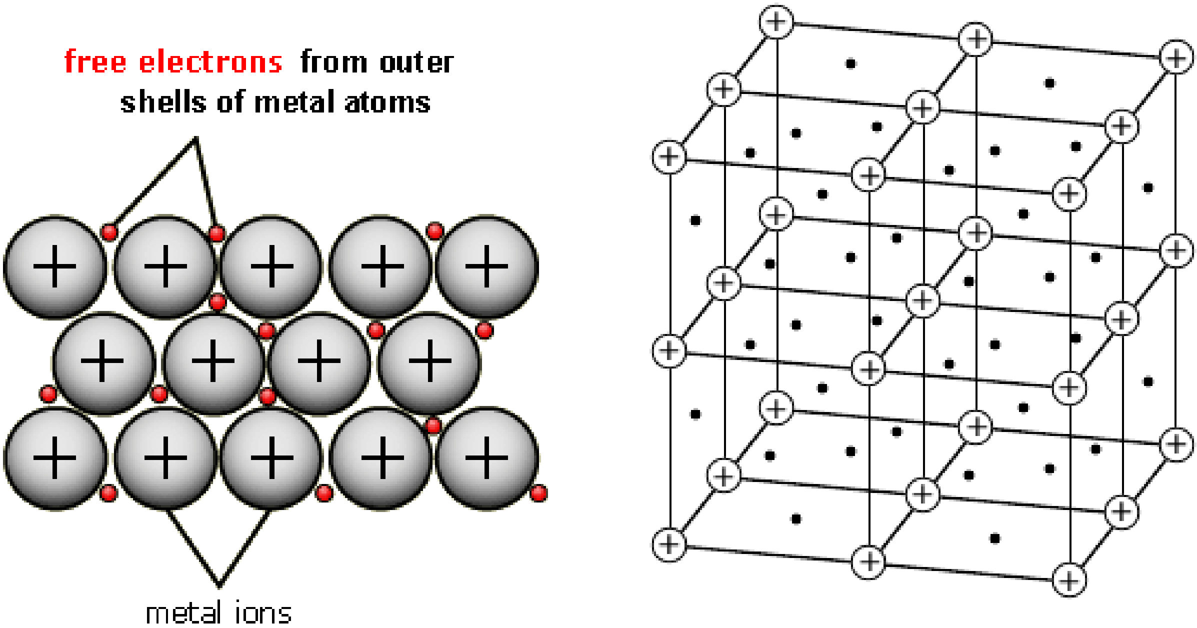


Image 2.3 – Structure in general of Metallic Bond

2.1.2 - Mechanical Properties of Ceramic Materials

- **Young's Modulus**

Young's Modulus, also known as the tensile modulus or elastic modulus, is a measure of the stiffness of an elastic material and is a quantity used to characterize materials. It is defined as the ratio of the uniaxial stress over the uniaxial strain in the range of stress in which Hooke's Law holds. It can be experimentally determined from the slope of a stress-strain-curve created during tensile tests conducted on a sample of the material.

In anisotropic materials, Young's modulus may have different values depending on the direction of the applied force with respect to the material's structure.

Diagram stress – strain. The Young's modulus is represented by the tangent to the curve at every point.

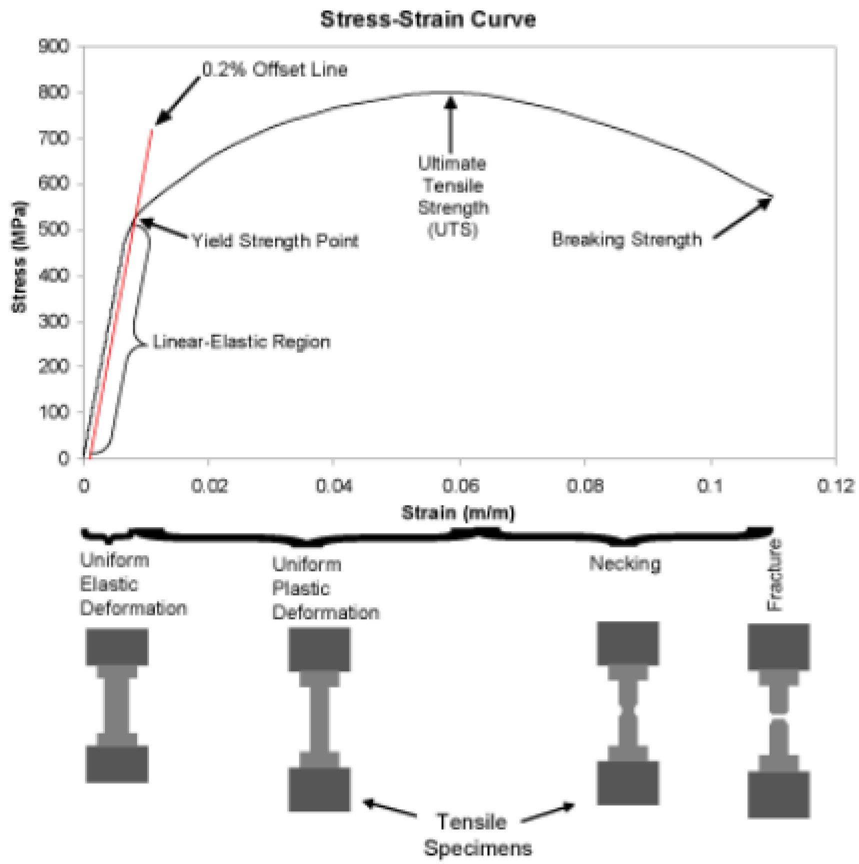


Image 2.4 - Diagram stress – strain

In this diagram stress – strain can observe, according to material, there is a first tranche in which the relationship between stress and deformation is lineal where being fulfilled:

$$\sigma = E \cdot \varepsilon \qquad (2.2)$$

The equation (1.1) known as Hooke's law, where:

E: Young's modulus and is intrinsic property of the material. It is not function of time of load application, and takes very different values depend on material you can use.

ε : The engineering normal strain of a material line element or fiber axially loaded is expressed as the change in length ΔL per unit of the original length L of the line element or fibers. The normal strain is positive if the material fibers are stretched and negative it they are compressed. Thus, we have:

$$\varepsilon = \frac{\Delta L}{L} \qquad (2.3)$$

For the case of ceramic materials, they possess a greater E due to the type of bond ($E_{ceramics} > E_{metallics}$). Ceramics also have a low density, which leads to having high $E_{specific}$. Due to these properties are widely used as fibers in composite materials.

The stress – strain curve can present the following ways:

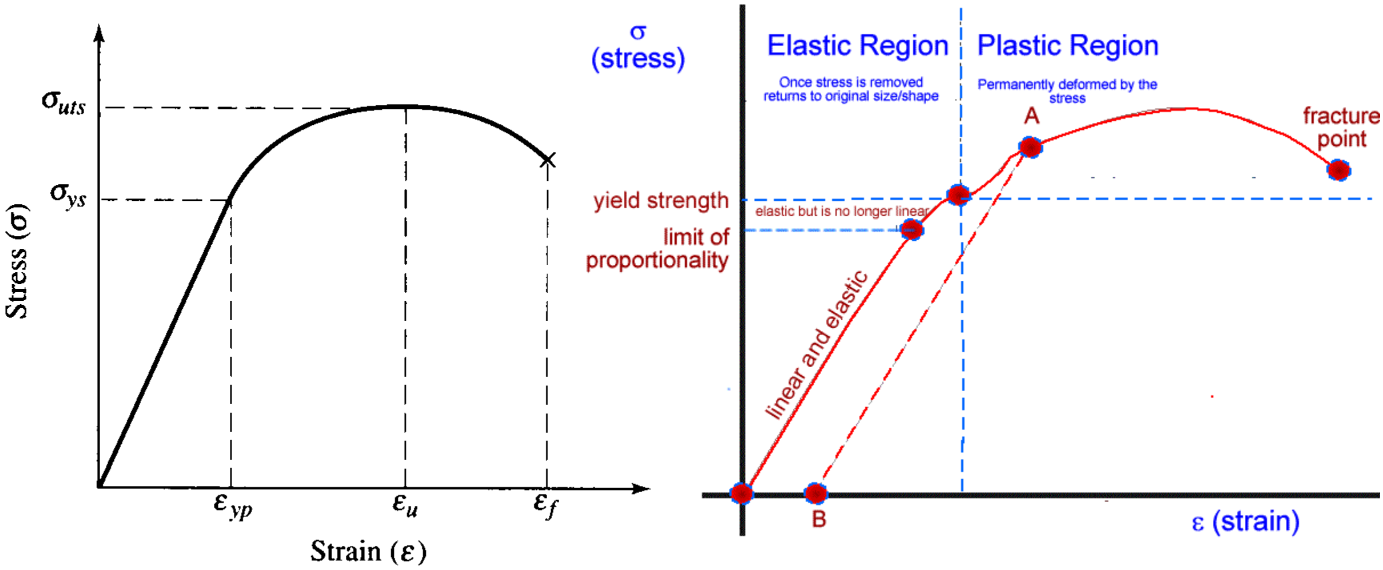


Image 2.5 –The stress – strain curve

Comparing between brittle and ductile materials obtains the following curves:

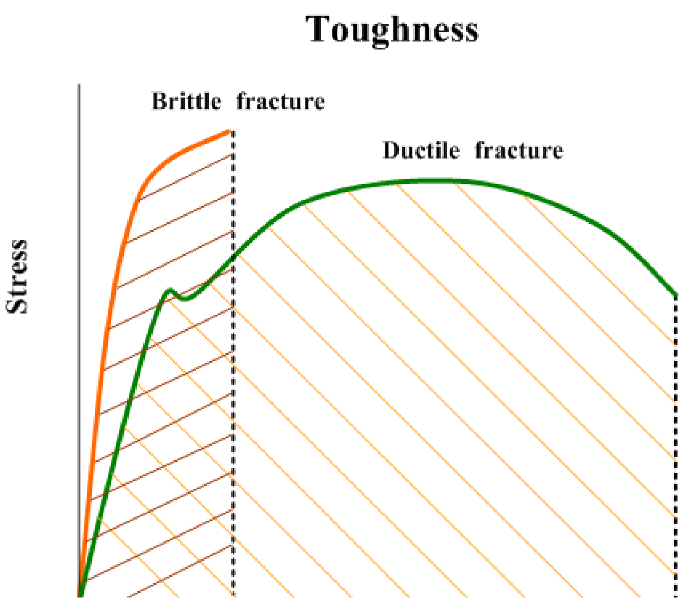


Image 2.6 – Comparing curve between brittle and ductile material

According to strength, ductility and toughness, the curves can be:

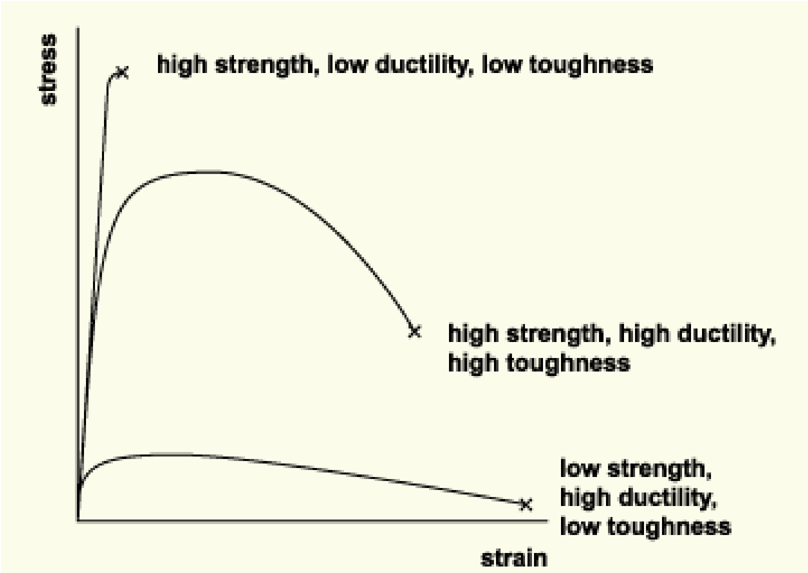


Image 2.7 – Comparing curve between strength, ductility and toughness

- **Hardness**

Hardness is a measure of how resistant solid matter is to various kinds of permanent shape change when a force is applied. Macroscopic hardness is generally characterized by strong intermolecular bonds.

Ceramics are very hard solids, more than metallic materials, this is due to their covalent or ionic bonds are stronger than metal bonds.

For example:

$Hardness_{Covalents}$ (diamond, SiC) > $Hardness_{Ionics}$ (Al_2O_3) > $Hardness_{metallics}$

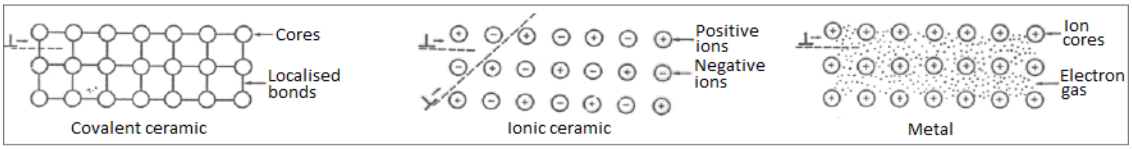


Image 2.8 – Different types of bonds.

The hardness of ceramics materials is a main property of them, as it relates to the ability of the material to withstand penetration of the surface through a combination of brittle fracture and plastic flow.

Often, the hardness is compared with the resistance. This is a habitual concept mistaken with many metallic components and is definitely an incorrect selection criterion with regards to engineering ceramic materials.

Hardness is dependent on ductility, elastic stiffness, plasticity, strain, strength, toughness, viscoelasticity and viscosity.

Hardness measurements in engineering ceramics are generally measured sing a Vickers hardness test. The Vickers test is often easier to use than other hardness tests since the required calculations are independent of the size of the indenter, and the indenter can be used for all materials irrespective of hardness. The basic principle, as with all common measures of hardness, is to observe the questioned material's ability to resist plastic deformation from a standard source. The unit of hardness given by the test is known as the Vickers Pyramid Number (HV) or Diamond Pyramid Hardness (DPH). The hardness number is determined by the load over the surface area of the indentation and not the area normal to the force, and is therefore not a pressure.

Image 2.9 - Vickers hardness tester

Image 2.10 - Rockwell hardness tester

It should be noted that the hardness value quoted for materials is a function of the type of test conducted and the loading conditions employed. Generally, lighter loads typically provide higher hardness values.

In ceramic materials it is necessary analyze other factors to interpreting hardness value. The factors are the amount of porosity in the surface, the grain size of the microstructure and the effects of grain boundary phases.

- **Fracture Toughness**

Fracture Toughness indicates the amount of stress required to propagate a flaw that before existed. It is a very important material property since the occurrence of flaws is not completely avoidable in the processing, fabrication, or service of material or component. Flaws may appear as voids, weld defects, cracks, metallurgical inclusions, design discontinuities or some combination thereof. From the engineering point of view never be able to be totally sure that a material has not any flaw, it is common practice to assume that a flaw of some chosen size will be present in some number of components and use the linear elastic fracture mechanics (LEFM) approach to design critical components. This approach uses the flaw size and features, component geometry, loading conditions and the material property called fracture toughness to evaluate the ability of a component containing a flaw to resist fracture.

Generally use the stress – intensity factor (K) for determining the fracture toughness of most materials. A Roman numeral subscript indicates the mode of fracture; there are three modes of fracture. The most common mode is the **Mode I** that is the condition in which the crack plane is normal to the direction of largest tensile loading, therefore, for remainder of the material we will consider K_I.

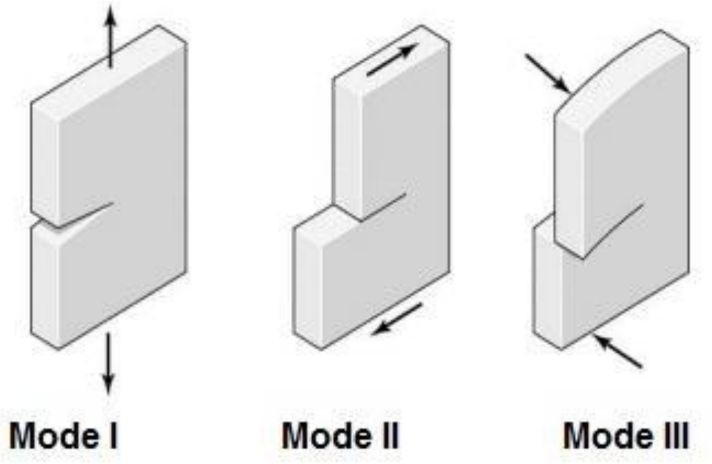


Image 2.11 – Different Modes

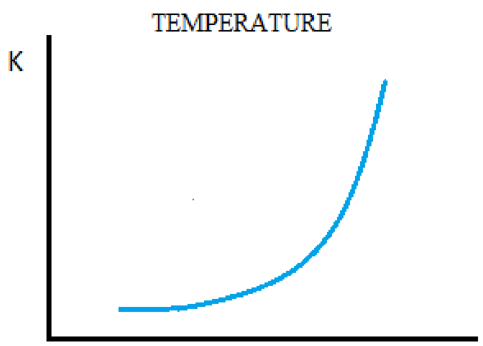
The stress intensity factor may be represented by the following equation:

$$K_I = \beta\sigma\sqrt{\pi \cdot a} \quad (2.4)$$

Where:

K_I: is the fracture toughness
β: is a crack length and component geometry that is different for each specimen and is dimensionless.
σ: is the applied stress.
a: is the crack length.

The stress intensity factor depends on crack size, structural geometry and loading. Also, there influence of different factors as the temperature, the solicitation speed and the thickness.



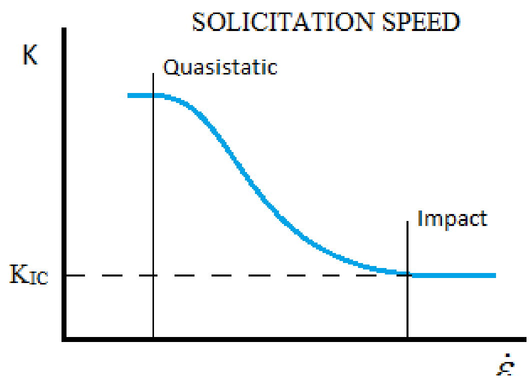


Image 2.12 – Factor Temperature

Image 2.13- Factor Solicitation Speed

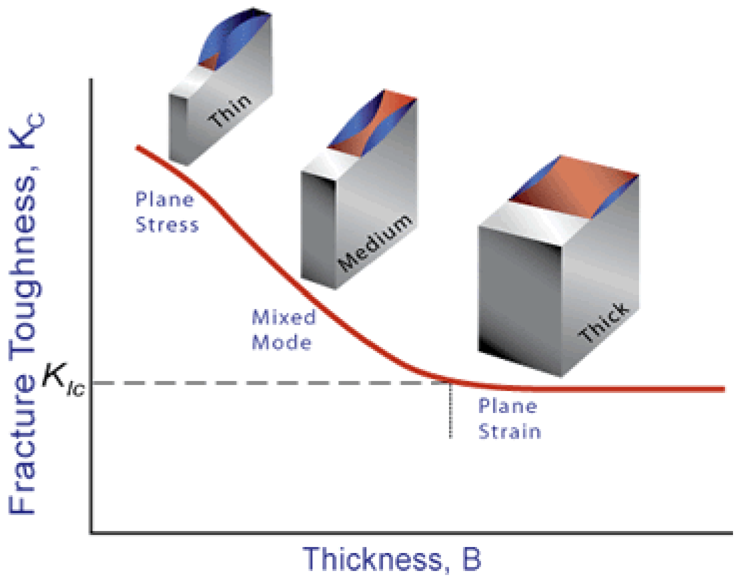


Image 2.14 – Curve obtained between Thicknesses vs. Fracture Toughness

K_{IC} is a true material property which is called the fracture toughness. The stress intensity, K_I, represents the level of "stress" at the tip of the crack and the fracture toughness, K_{IC}, is the highest value of stress intensity that a material under very specific (plane-strain) conditions that a material can withstand without fracture. As the stress intensity factor reaches the K_{IC} value unstable fracture occurs.

The ceramic is characterized by low **K_{IC}**. The fracture occurs due to the high number of defects that act as stress concentrator. In ceramics, the defects are: surface crack, grains large, pores and inclusions. These have low plastic deformation, therefore, the crack propagates and causing brittle fracture and catastrophic.

- **Flexural Strength**

The Flexural Strength, a mechanical parameter for brittle material, is defined as a material's ability to resist deformation under load. The transverse bending test is most common employed, in which a rod specimen having either a circular or rectangular cross-section is bent until fracture using a three point flexural test technique. The flexural strength represents the highest stress experienced within the material at its moment of rupture. It is a measured in terms of stress, here given the symbol **σ**.

Ceramic materials have extremely low ductility; therefore, ceramics do not allow measuring their mechanical properties by conventional tensile test which is widely used for metals. Brittle Materials, including ceramics, are tested by Flexure Test.

There are two standard Flexure Test Methods, 3 point flexure test and 4 point flexure test. The most frequently is 3 point flexure test. In which a specimen with round, rectangular or flat cross-section is placed on two parallel supporting pins. The loading force is applied in the middle by means loading pin.

- **Wear Behavior**

Wear is erosion or sideways displacement of material on a solid surface performed by the action of another surface. The wear is related to interactions between surfaces and, in particular, the removal and deformation of material on a surface as a result of mechanical action of the opposite surface. In the wear, we should think of other aspects of the environmental which cause this effect. These aspects include loads and features such as unidirectional sliding, reciprocating, rolling and impact loads, speed and temperature.

The wear behavior of engineering ceramics is relatively complex and is subject to many variables.

Cracking, plastic deformation, tribochemical interaction, abrasion and surface fatigue have all been identified as wear mechanisms operative in ceramic sliding wear situations.

The individual ceramic microstructures also affect the wear behavior in a fundamental manner.

When one considers the intimate contact of two sliding surfaces where hard particles are either present or formed during sliding, abrasive wear can occur as a consequence of both plastic deformation and fracture mechanisms.

- **Creep**

In materials science, creep is the tendency of a solid material to slowly move or deform permanently under the influence of stresses. It occurs as a result of long term exposure to high levels of stress that are below the yield strength of the material. Creep is more severe in materials that are subjected to heat for long periods, and near melting point. Creep always increases with the temperature.

The rate of this deformation is a function of the material properties, exposure time, temperature and the applied structural load. Depending on the magnitude of the applied stress and its duration, the deformation may become so large that a component can no longer perform its function.

Creep is observed upon tension, compression, torsion and other forms of stress. Under the actual conditions of use of a heat – resistant material, creep occurs under very complex stress conditions. The creep is described by a creep curve, which shows the time dependence of deformation for a constant temperature and a constant applied load or stress.

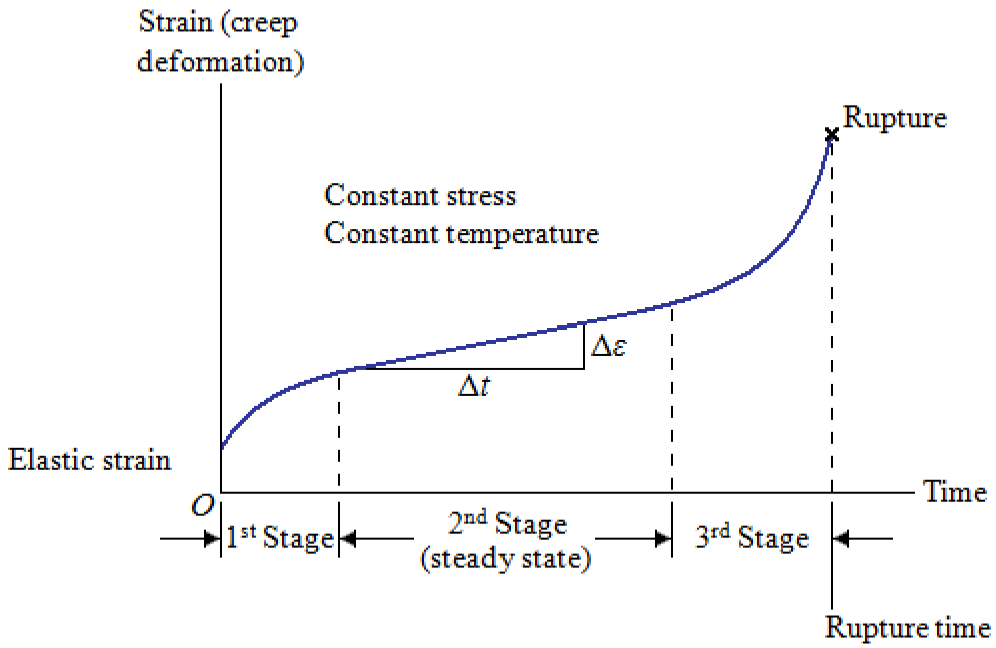


Image 2.15 – Curve of Creep, obtained between Strains vs. Rupture time

The curve is arbitrarily divided into three segments:

- **1st Stage**: The region of high strain rate or Primary Creep
- **2nd Stage**: The region of constant strain rate or Secondary Creep.
- **3rd Stage**: The region of accelerated strain rate.

The attenuation of creep in its first stage is the result of strain hardening, or work hardening. Since the creep is taking place at high temperature, removal of the work hardening, called recovery-creep, is also possible. When the rates of work hardening and recovery-creep become equal, the second stage of creep begins. The transition to the third stage is associated with the accumulation in the material of defects, such as pores and micro fissures, whose formation begins in the first and the second stages.

The Creep is only important in application where high temperature occur. The temperature range in which creep deformations may occur differs in various materials. In general, the effects of creep deformation for ceramics become noticeable at approximately 40-50% of melting point.

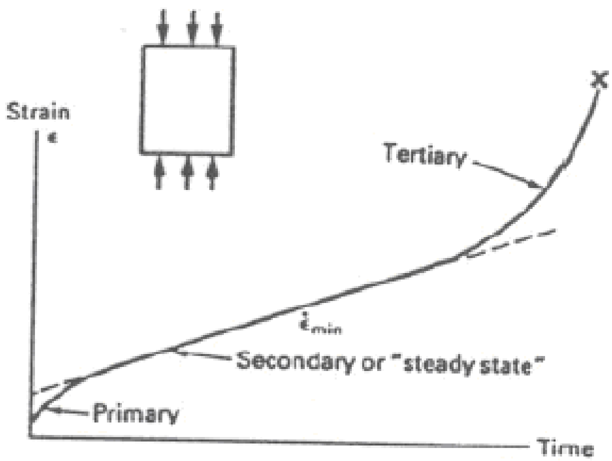


Image 2.16 – A creep curve for a ceramic

- **Thermal Shock**

Thermal shock called the cracking due to rapid temperature change. Ceramics are especially vulnerable to this form of failure, as consequence for their low toughness, low thermal conductivity, and high thermal expansion coefficients. However, they are used in many high temperature applications due to their high melting point.

Thermal Shock Resistance is an ability of material to withstand sharp changes in temperature.

In the case of a ceramic material will be quickly cooled, its surface will reach the temperature of cooling environment and it will tend to contract, thermal contraction. The thermal contraction of skin surface will be impossible as the interior regions of the material are still hot.

This produces to formation of tensile stress in the skin. Such thermal stresses may cause cracks and consequent failure.

The rapid temperature change can produced a crack in the material. If not stopped, the crack will propagate through the material and it will cause the object's structure fails.

Failure due to thermal shock can be prevented by:

- Reducing the thermal gradient seen by the object, by:
  - Changing its temperature more slowly
  - Increasing the material's thermal conductivity
- Reducing the material's coefficient of thermal expansion
- Increasing its strength
- Introducing built in compressive stress
- Decreasing its Young's modulus
- Increasing its toughness, by
  - Crack tip blunting
  - Crack deflection

Thermal shock resistance of material may be calculated in accordance to the equation:

$$R_S = \frac{\lambda \cdot \sigma_F}{\alpha \cdot E} \tag{2.5}$$

Where:

R_S: is the thermal shock resistance
λ: is the thermal conductivity
σ_F: is the flexural strength
α: is the coefficient of thermal expansion
E: is the modulus of elasticity

- **Electrical Properties**

We are talking about electrical properties which are characteristic for ceramic materials:

- Insulating Properties

Ceramics have very low electrical conductivity compared with metals; this is due to his ionic and covalent bonding which does not form free electrons.

- Electrical conductivity

This is the ability of material to conduct electric current.

The electrical conductivities differ by a factor as large as $10^{12}...10^{21}$ between metallic and ceramic materials.

Most of ceramic materials are dielectric (materials, having very low electric conductivity, but supporting electrostatic field).

Electrical conductivity of ceramics varies with the frequency of field applied and also with the temperature. This is due to the fact charge transport mechanisms are frequency dependent.

- Dielectric Strength

This is the ability of a material to prevent electron conductivity at high voltage. It is one of most important dielectric properties

Dielectric Strength is determined as value of electric field strength (units: v/m) at which electron conductivity breakdown occurs.

- Dielectric Constant

The Dielectric Constant – relative (to vacuum) is the ability of material to carry alternating current. It is other important property of dielectric materials. The dielectric constant of vacuum is equal to 1.

Capacitance of a capacitor is directly proportional to the dielectric constant of the dielectric material used in the capacitor.

Dielectric ceramics are used for manufacturing capacitors, insulators and resistors.

- Semi-conducting properties

Ceramics based on ZnO may possess semi-conducting properties when they are appropriately doped.

Normally, the semi-conducting ceramics are prepared by liquid phase sintering with control of grain boundary structure.

Semi-conducting ceramics are used for manufacturing varsities which are resistors with non-linear current-voltage characteristic and they are used for over-voltage protection. The resistors have Positive Temperature Coefficient (PTC).

- Superconducting properties

Generally the ceramics materials have very low electrical conductivity, despite of it, there are some ceramics possessing superconductivity properties (near-to-zero electric resistivity)

Lanthanum (yttrium)-barium-copper oxide ceramic may be superconducting at temperature as high as 138 K. This critical temperature is much higher, than superconductivity critical temperature of other superconductors (up to 30 K).

The critical temperature is also higher than boiling point of liquid Nitrogen (77.4 K), which is very important for practical application of superconducting ceramics, since liquid nitrogen is relatively low cost material.

Such ceramic conductors are called High Temperature Superconductors.

- Piezoelectric properties

The piezoelectric effects are:

- **Generating piezoelectric effect:** Mechanical stress, applied between two surfaces of a solid dielectric part, generates voltage between the surfaces.
- **Motor piezoelectric effect:** Voltage, applied between two surfaces of a solid dielectric part, results in contracting (expanding) of the part.

Some ceramic (lead zirconate titanate, barium titanrate, bismuth titanate, lead magnesium niobate) possess piezoelectric properties.

Piezoelectric ceramics are used for manufacturing various transducers, actuators and sensors like hydrophones, sonar, strain gauges, medical ultrasound equipment.

- Magnetic properties

Magnetic ceramics are prepared by sintering technology from iron oxide and barium/strontium carbonate with small amounts of other metal oxides. The magnetic ceramics are called ferrites.

There are two types of magnetic ceramics (ferrites):

- Isotropic ceramic magnet with equal magnetic properties in all directions;
- Anisotropic ceramic magnets with magnetic properties in the direction of pressing.

Ferrites combine good magnetic properties, high magnetization, with very low electrical conductivity. Low conductivity of ferrites allows reducing energy loss, caused by eddy currents, induced in the material when it works in high frequency magnetic fields.

2.2 - STEEL

Steel is an alloy combined with other elements, the most common of these being carbon. When the steel is combined with carbon, its content is between 0.2% and 2.1% by weight, depending on the grade. Other alloying elements sometimes used are manganese, chromium, vanadium and tungsten.

Carbon and other elements act as a hardening agent, preventing dislocations in the iron atom crystal lattice from sliding past one another. Depending on the amount of alloying elements and the form of their presence in the steel; it is possible to control the qualities such as ductility, hardness and tensile strength of the resulting steel. Steel with increased carbon content can be made harder and stronger than iron, but only steel is also less ductile than iron.

There are alloys with a higher than 2.1% carbon content, these are called cast iron because of their lower melting point and good castability. Steel is different to wrought iron, which can contain a small amount of carbon, but it is included in the form of lag inclusions. There are two distinguishing factors that they are steel's increased rust resistance and better weld ability.

Today, steel is one of the most common materials in the world, with more than 1.3 billion tons produced annually. It is one of main component in buildings, ships, infrastructure, tools, automobiles, appliances, machines and weapons.

2.2.1 - Material Properties of Steel

There are materials that sometimes are added to mixture of iron / carbon to produce steel with specific properties. If we add Nickel and Manganese in the mixture of iron / carbon, the material will have a tensile strength higher and it will appear austenite in the iron / carbon solution that it is more chemically stable. If we add Chromium, the material will have more hardness and its melting temperature will increase. And if we add Vanadium also it will increase hardness while it will reduce the effects of metal fatigue.

To prevent corrosion, it is necessary add to steel at least 11 % chromium, so that a hard oxide forms on the metal surface; this is known as stainless steel.

The density of steel varies based on the alloying constituents; in general, the ranges are between 7.750 and 8.050 kg/m^3 or 7.75 and 8.05 g/cm^3.

Even in the narrow range of concentrations that make up steel, mixtures of carbon and iron can form a number of different structures, with very different properties. Understanding such properties is essential to making quality steel.

There are many types of heat treating processes available to steel. The most common are annealing and quenching and tempering.

- Annealing is the process of heating steel to a sufficiently high temperature to soften it. The temperature required to anneal steel depends on the type of annealing and the constituents of the alloy.

- Quenching and tempering is the process of heating steel, and to continue, quenching it in water or oil. The steel is then tempered, which is just a specialized type of annealing. Finally, the material obtained is more ductile and fracture-resistant metal.

2 .2 .2 - Mechanical Properties of Steel Materials

- **Young's Modulus**

The explication about Young's Modulus was previously presented.

Diagram stress – strain. The Young's modulus is represented by the tangent to the curve at every point. The form stress-strain curve of steel is similar than ceramics.

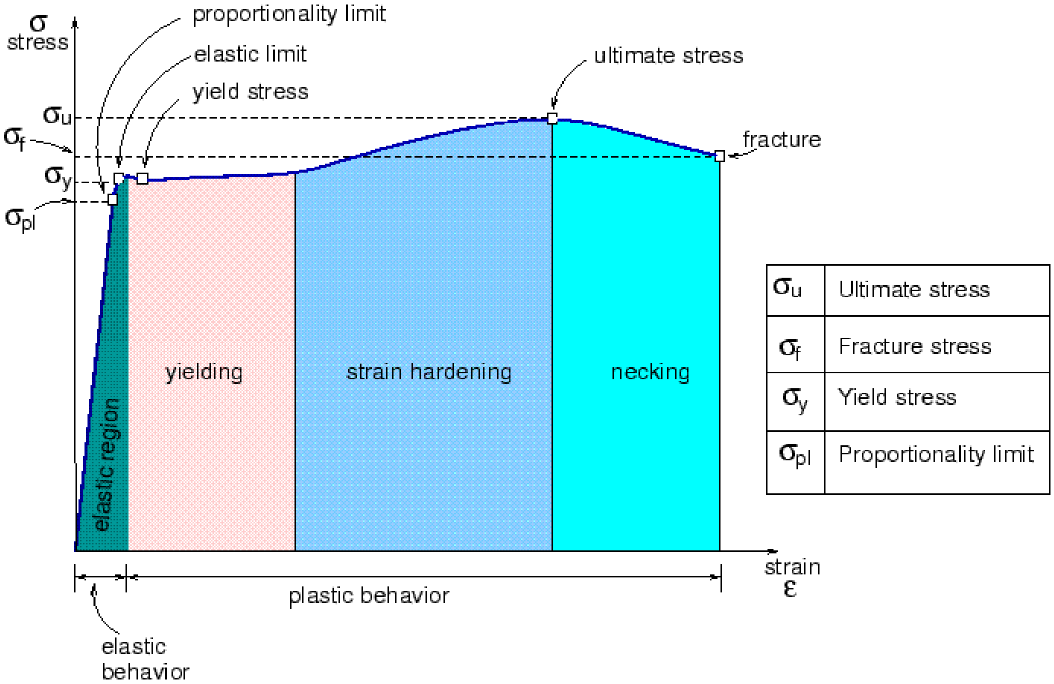


Image 2.17 – Diagram Stress. Stress vs Strain

- **Hardness**

The explication about Hardness was previously presented.

The hardness of the steel varies between the iron and which can be achieved by its alloy or other thermal or chemical procedures among which the best known is perhaps the tempered steel. Applicable to steels with high carbon content, which allows, when superficial, retain a hard core in the piece to avoid brittle fractures. Typical steels with a high degree of surface hardness are used in machining tools, known speed steels containing significant amounts of chromium, tungsten, molybdenum and vanadium.

- **Fracture Toughness**

The explication about Fracture Toughness was previously presented.

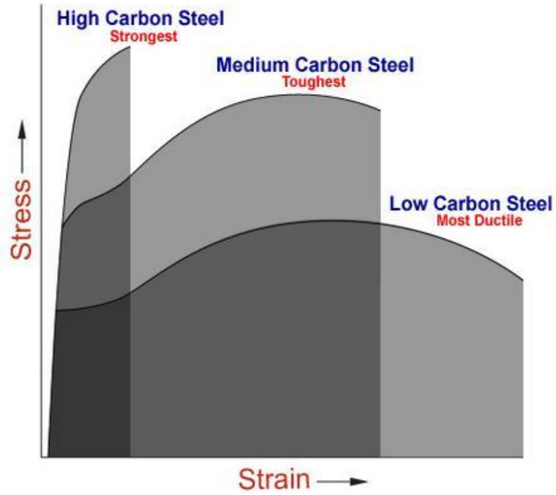


Image 2.18 - Fracture Toughness

There are several variables that have a profound influence on the toughness of a material. These variables are: Strain rate, temperature, n notch effect.

A metal may possess satisfactory toughness under static loads but many fail under dynamic loads or impact. As a rule ductility and, therefore, toughness decrease as the rate of loading increases. Temperature is the second variable to have a major influence on its toughness. As temperature is lowered, the ductility and toughness also decrease. The third variable is termed notch effect, has to do with the distribution of stress. A material might display good toughness when the applied stress is uniaxial; but when a multiaxial stress state is produced due to the presence of a notch, the material might not withstand the simultaneous elastic and plastic deformation in the various directions.

- **Wear Behavior**

For steel, the wear behavior more important is the abrasive wear behavior. This can be considered to be dependent on the deformation behavior of a material which is a strong function of material hardness, ductility and fracture characteristics. Ultimately, these parameters depend on the microstructure of materials. However, due to subsurface deformation during wear, the microstructure in the subsurface regions might be considerably different from that of the bulk. This suggests that wear rate may not follow a similar kind of relation with microstructure unlike other material properties.

Moreover, the configuration and mode of abrasion tests, load, speed, and the nature, shape, size and hardness of the abrasive particles as well as their rake angle and the frequency ofmovement of the specimen on the same abrasive also control the abrasive wear behavior of materials.

- **Creep**

The explication about Creep was previously presented.

In general, in the case of steels of natural hardness, yield strength coincides with the apparent value of the voltage corresponding to the step of provenance. In cases where this step does not appear or appears ill-defined, as usual with cold drawn steels, it is necessary to resort to the conventional value established requirements for higher strength steels to 4200 Kg/cm^2.

For steels of higher resistance, up to 4200 Kg/cm^2, the stress-strain curve can be elasto-plastic or not, depending on the properties of steel and manufacturing processes.

- **Thermal Shock**

The explication about Thermal Shock was previously presented.

Hardened tool steels require not only mechanical properties such as high strength, high ductility, high notch toughness, high wear resistance, etc, but also crack resistance property against thermal shock such as friction and repeated thermal conduction in use, in order to get longer tool lives.

The thermal shock resisting property, friction followed by water cooling to the rubbed surface was employed. In this one-cycle thermal shock test, low carbon steels were smaller in depth of maximum crack than high carbon steels in the same hardness levels, which means that lower carbon steels can provide higher thermal shock resistance on top of better mechanical properties, compared with higher carbon steels.

The thermal shock low carbon steels are more crack resistant than high carbon steels, provided using hardness is the same. On the other hand low carbon steels can have higher hardness than high carbon steel, provided their thermal shock crack resistance is set at the same level.

- **Electrical Properties**

The steel like all metals can conduct electricity. Some types of steel, such as electrical steel, consist of other materials than carbon. For example, 304 stainless steel includes chromium, zinc and carbon. Different metal compositions alter the electrical properties of the steel, making it more suitable for use in electrical work or construction. The three main electrical properties of steel include its electrical conductivity, electrical resistivity and temperature coefficient.

- <u>Electrical Conductivity</u>

Electrical conductivity, measured in Siemens per meter, or S/m, indicates the ability of a material to conduct an electrical charge over a certain distance. Electrical conductivity depends on the amount of free electrons available to transfer a charge. The measurement is the ratio of current density to the strength of an applied electrical field, or how much of the total field actually is flowing through the material rather than through its surrounding medium such as water or air.

- Electrical Resistivity

Resistivity is the opposite of conductivity--the difficulty of a material to carry an electrical current. Materials with high resistivity make excellent electrical insulators because they don't carry a current. Electrical resistivity is measured in ohms/meter, as the reciprocal of conductivity.

- Temperature Coefficient

Because temperature affects electrical conductivity and resistivity, these properties are measured and documented at 20 degrees Celsius. The temperature coefficient determines how much temperature will increase or decrease electrical resistance. According to all About Circuits, steel with 0.5% carbon has a positive temperature coefficient of 3x10^-3. Its resistivity increases with temperature because the molecules of the steel are spread farther apart.

2.3 – CONCRETE

Reinforced concrete is a composite material in which concrete's relatively low tensile strength and ductility are counteracted by the inclusion of reinforcement having higher tensile strength and/or ductility. The reinforcement is usually, though not necessarily, steel reinforcing bars and is usually embedded passively in the concrete before it sets. Reinforcing schemes are generally designed to resist tensile stresses in particular regions of the concrete that might cause unacceptable cracking and/or structural failure. Modern reinforced concrete can contain varied reinforcing materials made of steel, polymers or alternate composite material in conjunction with rebar or not. Reinforced concrete may also be permanently stressed (in compression), so as to improve the behavior of the final structure under working loads.

In addition, the reinforced concrete has been universally accepted because it can be molded essentially into any shape or form, is inherently rigid, and is inherently fire-resistant. With proper protection of the reinforcement, a reinforced concrete structure can be very durable and can have a long life even under harsh climatic or environmental conditions. Reinforced concrete structures have also demonstrated that they can provide a safe haven from the potentially devastating effects of earthquakes, hurricanes, floods, and tornadoes.

For a strong, ductile and durable construction the reinforcement needs to have the following properties at least: high relative strength, high toleration of tensile strain, good bond to the concrete, irrespective of pH, moisture, and similar factors, thermal compatibility, not causing unacceptable stresses in response to changing temperatures.

2.3.1 - Material Properties of Concrete

Concrete is made from small stones and gravel called aggregate, sharp sand, cement and water. The small stone and gravel (aggregate) is the reinforcement and the cement is the matrix that binds it together. Concrete has good ‘strength’ under compression but it is weak in tension. It can be made stronger under tension by adding metal rods, wires, mesh or cables to the composite. The concrete is cast around the rods. This is called reinforced concrete.

The porosity is considered the proportion of voids to the total mass. Influences the strength, density, and permeability of concrete.

The permeability it is the ability of a material to be traversed by liquids or gases. The impermeability of concrete is important for resistance to chemical attack. This depends partly impermeability of excess water in the mixing and subsequent curing of the concrete.

2.3.2 - Mechanical Properties of Concrete Materials

- **Young's Modulus**

As above, the explication about Young's Modulus was previously presented.

Diagram stress – strain. The Young's modulus is represented by the tangent to the curve at every point. The form stress-strain curve of concrete is similar than steel.

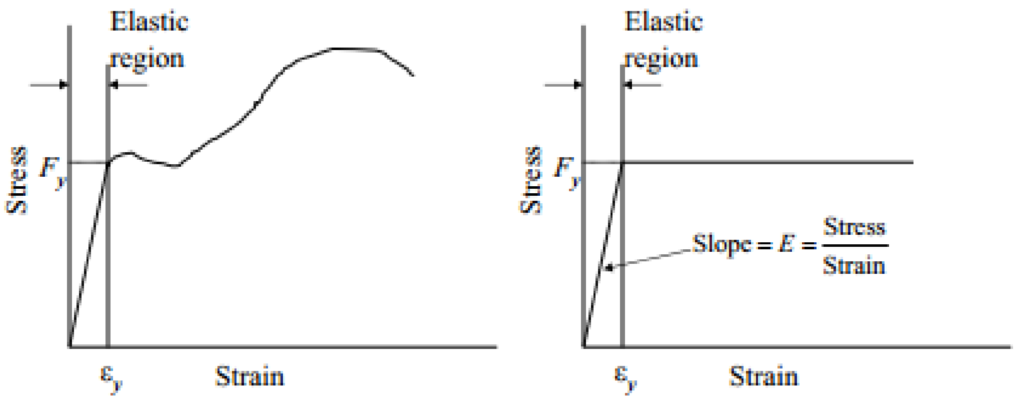


Image 2.19 – The Young's Modulus. (a) Determined by Tensile Test. (b) Idealized

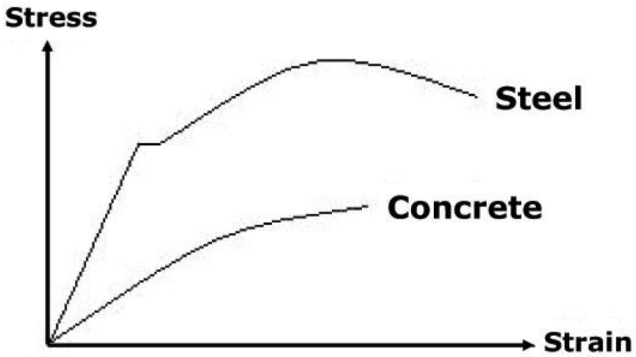


Image 2.20 – Young's Modulus Steel and Concrete

- **Hardness**

The explication about Hardness was previously presented.

The hardness of concrete is related to its resistance. The evolution of concrete strength also depends on the temperature of conservation evolve more rapidly the higher the temperature, then the temperature acts as a catalyst for the hydration reaction cement.

- **Fracture Toughness**

The explication about Fracture Toughness was previously presented.

The concrete has a brittle fracture, which entails that has very little ductility. But the concrete, which is steel-reinforced concrete, has a higher ductility means that is less brittle fracture.

- **Flexural Strength**

The explication about Flexural Strength was previously presented.

There is very little evidence about flexural of structural concrete. Agencies not using flexural strength for field control generally find the use of compressive strength convenient and reliable to judge the quality of the concrete as delivered.

Beam specimens must be properly made in the field. Pavement concretes are stiff (1/2 to 2 ½ inch slump). Consolidate by vibration in accordance with ASTM C 31 and tap side to release bubbles.

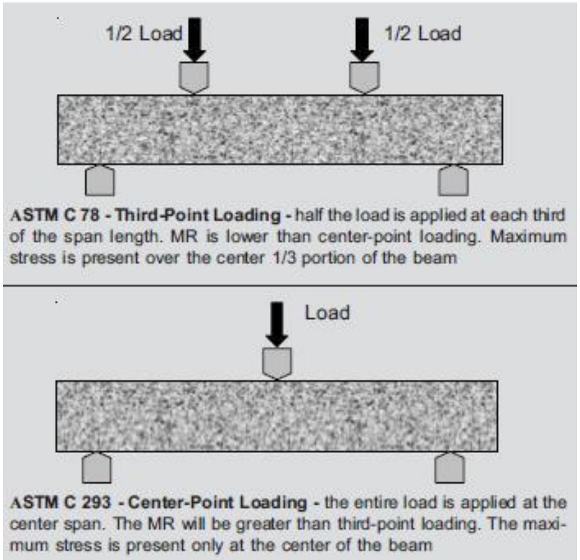


Image 2.21 – Flexural Strength

Specifications about low strengths should take into account the higher variability of flexural strength result.

Another procedure for in-place strength investigation uses compressive strength of cores calibrated by comparison with acceptable placements on either side of the concrete in question.

Flexural tests are extremely sensitive to specimen preparation, handling, and curing procedure. Beam specimens are very heavy, and allowing a beam to dry will yield lower strengths. Beams must be cured in a standard manner, and tested while wet. A short period of drying can produce a sharp drop in flexural strength.

The concrete industry and inspection agencies are much more familiar with traditional cylinder compression tests for control and acceptance of concrete. Flexure can be used for design purposed, but the corresponding compressive strength should be used to order and accept the concrete. Any time trial batches are made, both flexural and compressive tests should be made so that a correlation can be developed for field control.

- **Wear Behavior**

On the surface of reinforced concrete exist, because of its use, a mechanic wear, impact and abrasion and, acting on the concrete lining is a series of harmful agent such as CO_2 – carbonation, chlorides – deicing salts, waters sulfates, freeze/thaw, and other aggressive liquids and gases. This last type of case can lead to oxidation of the rod, initiating a new type of damage with cracks and landslides.

- Abrasive Wear

The abrasion of concrete is defined as the surface wear because of friction or rubbing processes. Particles destroyed by wind may have abrasive effect on concrete surfaces.

Between factors that decrease the resistance of concrete to the abrasive action may be: the exudation of concrete, its compressive strength properties of aggregates finishing procedures, the procedure and the curing time.

- Erosive Wear

The erosion is defined as the damage caused by the abrasive action of moving fluid or solid. The erosion resistance is important in hydraulic structures in which the concrete is subjected to the abrasive action of flowing water which transports solid particles.

The shock action, such sliding or friction can cause wear particles concrete surface.

The amount of erosion depends on the number, speed, size, profile, density and hardness of the moving particles per unit time.

- **Creep**

The explication about Creep was previously presented.

Moderate creep in concrete is sometimes welcomed because it relieves tensile stresses that might otherwise lead to cracking.

Concrete creep is defined as: deformation of structure under sustained load. Basically, long term pressure or stress on concrete can make it change shape. This deformation usually occurs in the direction the force is being applied. Like a concrete column getting more compressed, or a beam bending.

Creep does not necessarily cause concrete to fail or break apart. Creep is factored in when concrete structures are designed.

Factors Affecting Creep: aggregate, mix proportions, age of concrete.

- Influence of Aggregate

Aggregate undergoes very little creep. It is really the paste which is responsible for the creep. However, the aggregate influences the creep of concrete through a restraining effect on the magnitude of creep. The paste which is creeping under load is restrained by aggregate which do not creep. The stronger the aggregate the more is the restraining effect and hence the less is the magnitude of creep. The modulus of elasticity of aggregate is one of the important factors influencing creep.

It can be easily imagined that the higher the modulus of elasticity the less is the creep. Light weight aggregate shows substantially higher creep than normal weight aggregate.

- Influence of Mix Proportions

The amount of paste content and its quality is one of the most important factors influencing creep. A poorer paste structure undergoes higher creep. Therefore, it can be said that creep increases with increase in water/cement ratio. In other words, it can also be said that creep is inversely proportional to the strength of concrete. Broadly speaking, all other factors which are affecting the water/cement ratio are also affecting the creep.

- Influence of Mix Proportions

Age at which a concrete member is loaded will have a predominant effect on the magnitude of creep. This can be easily understood from the fact that the quality of gel improves with time. Such gel creeps less, whereas a young gel under load being not so stronger creeps more. What is said above is not a very accurate statement because of the fact that the moisture content of the concrete being different at different age also influences the magnitude of creep.

Effects of Creep on Concrete

- In concrete beams, creep increases the deflection with time and may be a critical consideration in design.
- In eccentrically loaded columns, creep increases the deflection and can load to buckling.
- In case of statically indeterminate structures and column and beam junctions creep may relieve the stress concentration induced by shrinkage, temperatures changes or movement of support. Creep property of concrete will be useful in all concrete structures to reduce the internal stresses due to non-uniform load or restrained shrinkage.
- In mass concrete structures such as dams, on account of differential temperature conditions at the interior and surface, creep is harmful and by itself may be a cause of cracking in the interior of dams. Therefore, all precautions and steps must be taken to see that increase in temperature does not take place in the interior of mass concrete structure.
- Loss of prestress due to creep of concrete in prestressed concrete structure.

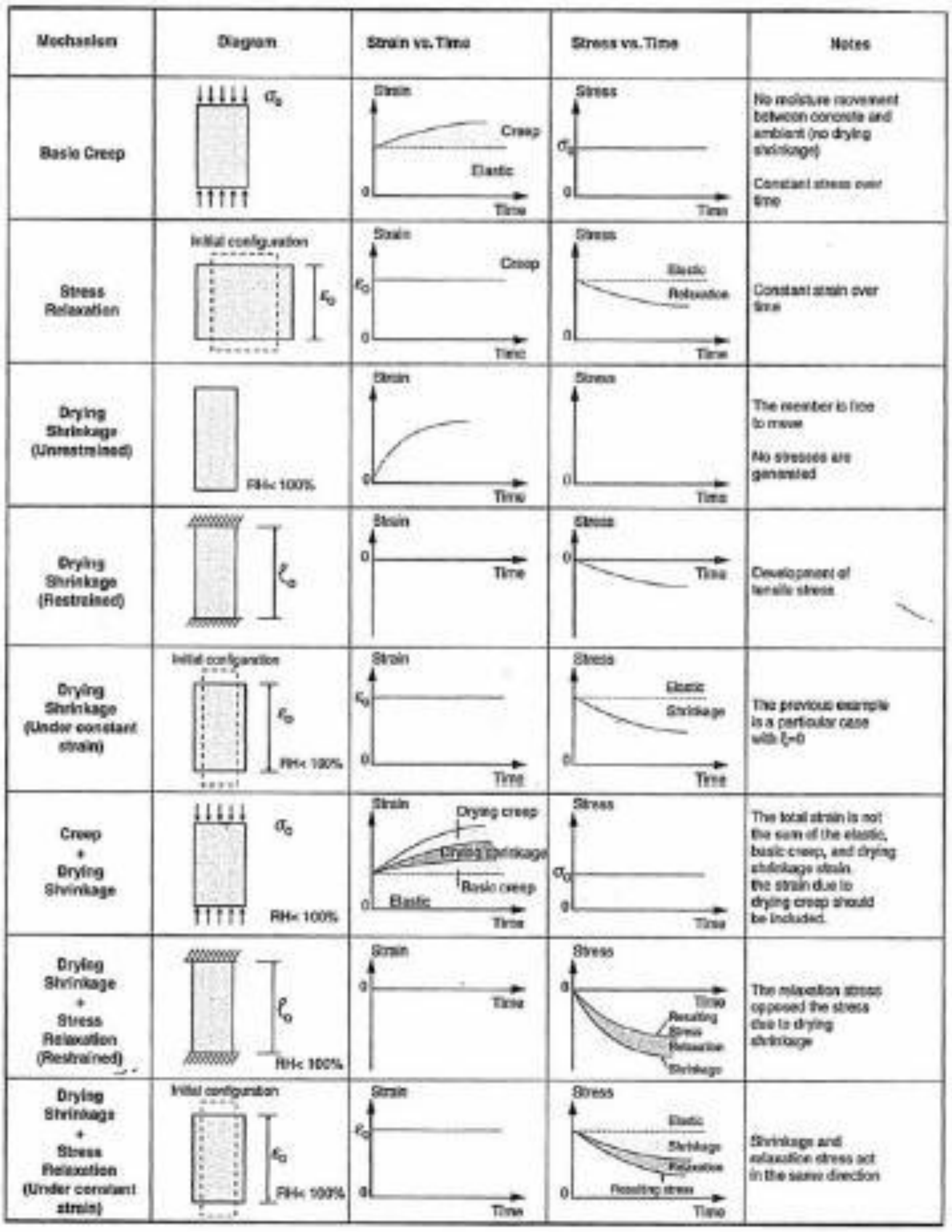


Image 2.22 – Combinations of different load conditions, restrictions and humidity

In contrast to other materials used in civil engineering, such as steel at elevated temperatures or clay, concrete creep is approximately linear if subjected to a stress state which is within the range of operating voltages (approximately of the order 30% of the peak value). Also, creep in concrete is a hereditary phenomenon, with an extended memory (a broader relaxation spectrum), and exhibits the phenomenon of aging (evolution of the mechanical properties and the pore system with age), which is caused, in theory, by the chemical reactions during cement hydration.

- **Thermal Shock**

In general, if concrete has been well prepared for inhibiting explosive spailing, the main damage to concrete caused by fire or elevated temperatures should be loss in mechanical properties. The compressive strength can be broadly maintained within a range of temperature from 20 to 400[degrees]C. Considerable loss in compressive strength occurs between 400 and 600[degrees]C, and most of the original compressive strength before heating may be lost from 600 to 800[degrees]C. Compared with compressive strength, tensile splitting strength suffers a more severe loss under identical temperature, as the latter is more sensitive to thermally induced cracking.

Phenomena that deteriorate the concrete in cold:

- The cycles of freeze – thawing of concrete, or more specifically, the dissolution of the capillary.
- The humidity and temperature changes cause the cracking.
- The use of salt fluxes, which is used to melt the ice formed on the concrete and allow their use, are detrimental to the concrete and its reinforcements.

The concrete always freezes at temperatures below 0ºC.

The melting salts reduce the durability of concrete due to three causes:

- It provides generous amounts of chloride ion to induce reinforcement corrosion.
- The melting ice is an endothermic process. The cooling rate can be up to 14 ° C / min. This causes a thermal shock on the concrete surface that deteriorates.
- The concrete absorbs through its capillary network added to the molten salts accumulate within the concrete after repeated use.

The thermal shock freezes concrete surface and the inner zone, but not the intermediate layers in which the salt concentration in increased.

- **Electrical Properties**

Electrical resistivity is important where concrete will be exposed to corrosive conditions, as corrosion currents will flow more easily in concrete having low resistivity.

Electrical resistivity is an important physical property of cement concrete that affects a variety of applications. Electrical resistivity (or its inverse, conductivity) is important as a measure of the ability of concrete to resist the passage of electrical current.

The electrical resistivity of concrete is an important component of reinforcing steel corrosion cells, as high resistivity of the electrolyte (in this case concrete) will reduce corrosion currents and slow the rate of corrosion. Electrical resistivity is fundamentally related to the permeability of fluids and diffusivity of ions through porous materials such as concrete. Therefore, electrical resistivity can also be used as an indirect measure of the ability of concrete to prevent penetration of chloride salt solutions that may cause corrosion of the reinforcing steel.

Electrical resistivity of concrete is an important consideration with respect to the corrosion of steel in concrete. Corrosion occurs due to the formation of an electrochemical corrosion cell. A corrosion cell must have four elements' in order to function: 1) an anode where the metal is oxidized; 2) a cathode where a reduction process, such as hydrogen evolution or oxygen reduction, occurs; 3) an electrical connection between the anode and cathode; and 4) an ionic conduction path provided by an electrolyte. In the case of metals embedded in concrete the electrolyte for the corrosion cell is the concrete itself. A resistivity of less than 5,000 ohm. cm can support very rapid corrosion of steel 2. If the electrolyte has high resistance to the passage of current, or if the electrolyte is dry and unable to support ionic flow, then corrosion will occur only at a very low rate, if at all.

Characteristics

While the engineering properties and mixing characteristics of conductive concrete and normal concrete are comparable, conductive concrete does have other distinctive characteristics beyond its ability to conduct electricity.

- The conductivity value is stable. The effects of moisture content, hydration time and temperature on conductivity are insignificant.
- It is lightweight: conventionally mixed, conductive concrete has a density of about 70 percent that of normal concrete.
- Conductive concrete is chemically compatible with normal concrete, bonding well with it if used as an overlay.
- Thermal stability is comparable to that of normal concrete.
- The colour of conductive concrete is a darker grey, reflecting its carbon content.

3 - STRUCTURES

3.1 - CANTILEVERBEAM

A cantilever is a beam anchored at only one end. The beam carries the load to the support where it is forced against by moment and shear stress. Cantilever construction allows for overhanging structures without external bracing. Cantilevers can also be constructed with trusses or slabs.

This is in contrast to a simply supported beam such as those found in a post and lintel system. A simply supported beam is supported at both ends with loads applied between the supports.

Cantilevers are widely found in construction, notably in cantilever bridges and balconies. In cantilever bridges the cantilevers are usually built as pairs, with each cantilever used to support one end of a central section. A cantilever in a traditionally timber framed building is called a jetty or fore bay.

Temporary cantilevers are often used in construction. The partially constructed structure creates a cantilever, but the completed structure does not act as a cantilever. This is very helpful when temporary supports, or false work, cannot be used to support the structure while it is being built (e.g., over a busy roadway or river, or in a deep valley). So some truss arch bridges are built from each side as cantilevers until the spans reach each other and are then jacked apart to stress them in compression before final joining. Nearly all cable-stayed bridges are built using cantilevers as this is one of their chief advantages. Many box girder bridges are built segmentally or in short pieces. This type of construction lends itself well to balanced cantilever construction where the bridge is built in both directions from a single support.

These structures are highly based on torque and

rotational equilibrium.

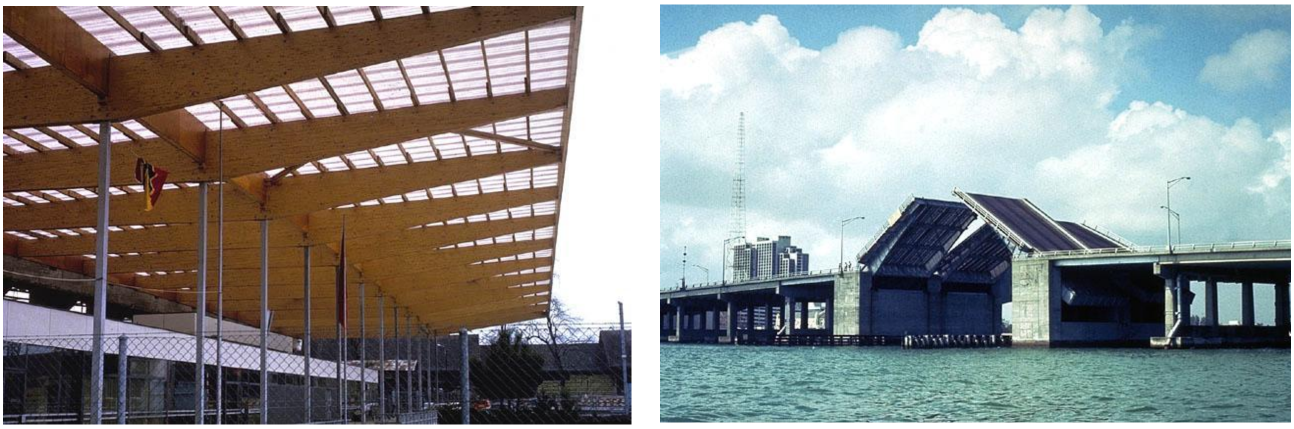


Image 3.1 – Pictures of example

We are going to study the structure of cantilever realized with different materials. The first with a ceramic material, the second with steel and the last with concrete.

We are going to see the behavior with different loads. A point load on the end of cantilever beam, a point load in the middle of cantilever beam and finally a continuous load on the cantilever beam.

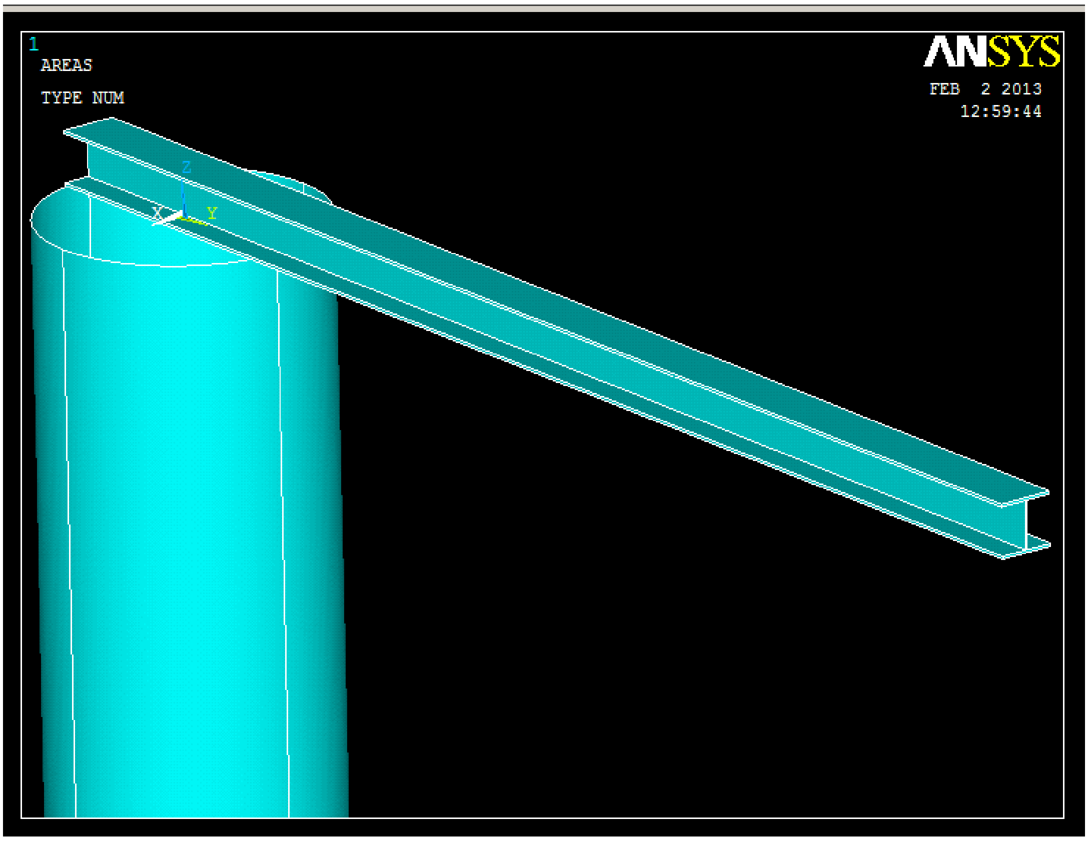


Image 3.2 – The structure of cantilever beam

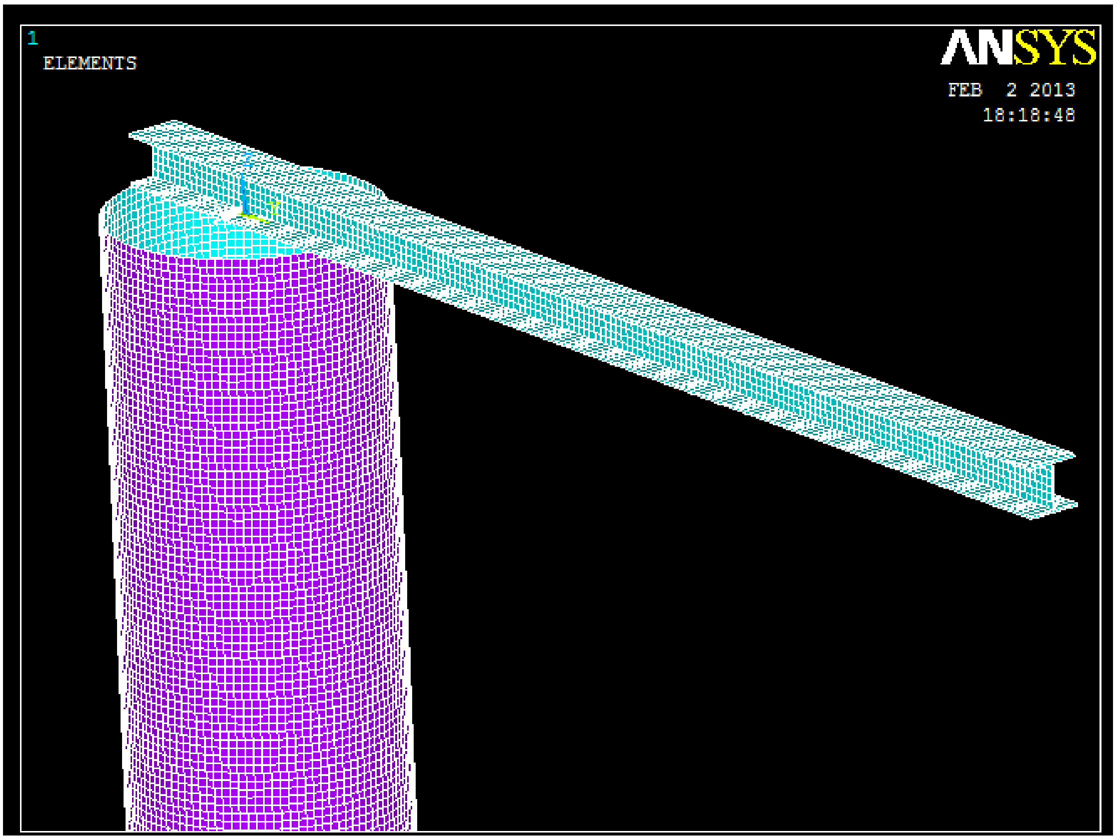
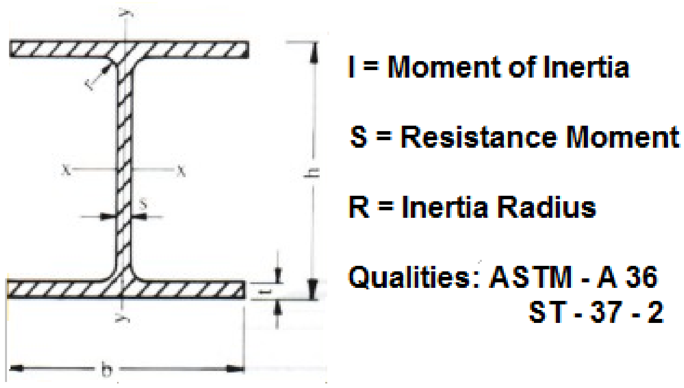


Image 3.3 – Model of structure to can study

3.1.1 - Explanation about beam

The beam which we use has 5 meters as length and it has a profile as:



HEA (I) IPBL	Dimensiones (mm)					Área cm^2	Peso Kg/m	Momento respecto a los ejes					
								EJE-X-X			EJE-Y-Y		
	h	b	s	t	r1			Ix cm^4	Sx cm^3	Rx cm	Iy cm^4	Sy cm^3	Ry cm
100	96	100	5.0	8.0	12	21.2	16.7	349	72.7	4.05	134	26.7	2.51
120	114	120	5.0	8.0	12	25.3	19.9	606	106.0	4.89	231	38.4	3.02
140	133	140	5.5	8.5	12	31.4	27.7	1030	155.0	5.73	389	55.6	3.52
160	152	160	6.0	9.0	15	38.8	30.4	1670	220.0	6.57	615	76.9	3.98
180	171	180	6.0	9.5	15	45.3	35.5	2510	294.0	7.45	924	103.0	4.52
200	190	200	6.5	10.0	18	53.8	42.3	3690	389.0	8.28	1330	133.0	4.98
220	210	220	7.0	11.0	18	64.3	50.5	5410	515.0	9.17	1950	178.0	5.51
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Image 3.4 – Dimensions of our beam

3.1.2 - A load on the end of cantilever beam

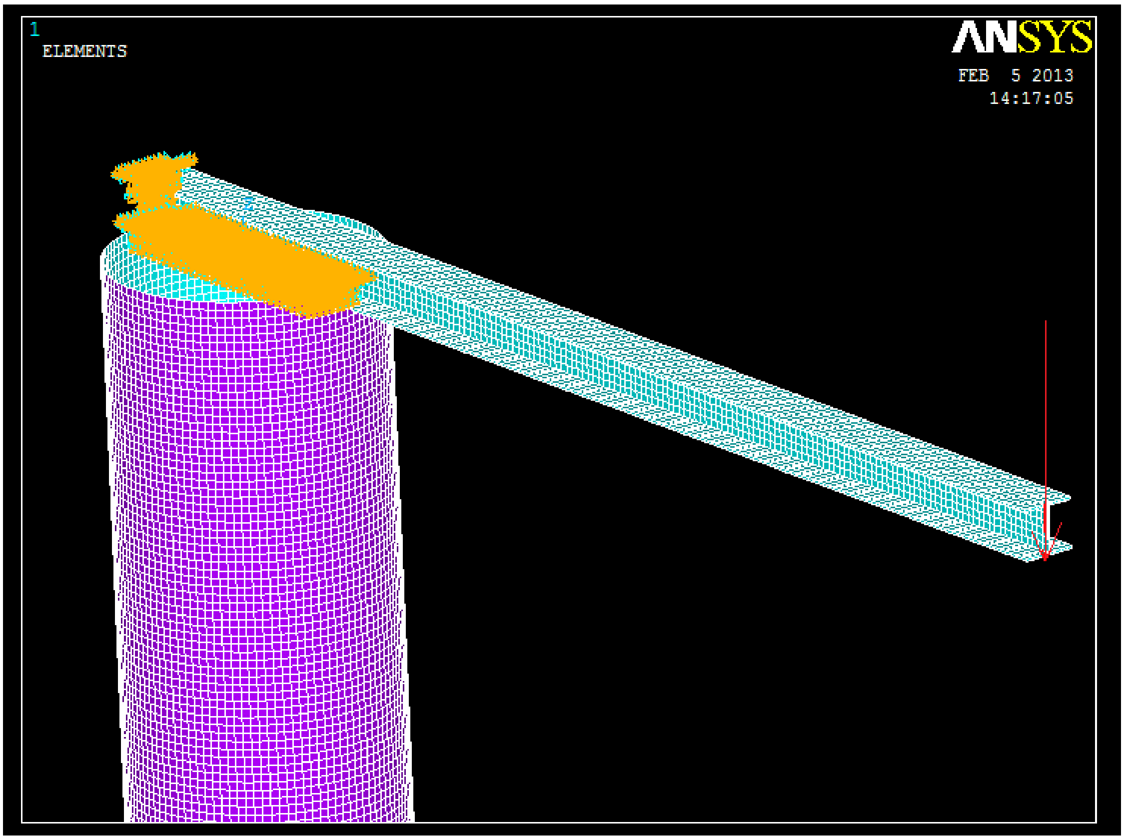


Image 3.5 – Model with a load on the end of cantilever beam

We can calculate the deformation of the beam by three different methods.

- Moment Area Method
- Double Integration Method
- Conjugate Beam Method

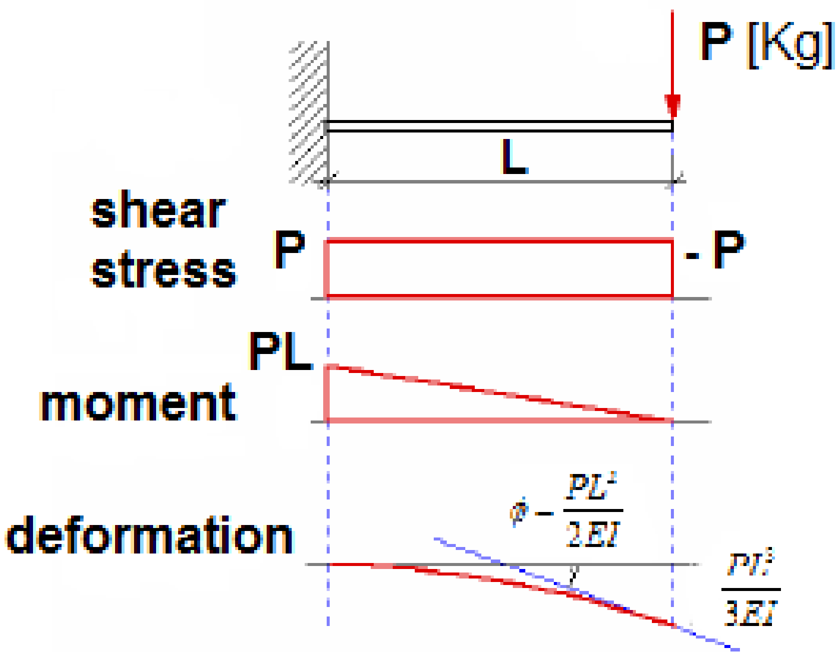
○ <u>Moment Area Method</u>

This method is based on the relationship between the moment and the curvature M and provides practical and efficient means to calculate the slope and curve of the elastic deflection of beams and frames.

This method comprises two theorems:

1 - The change of slope or angle between the tangents at any two points of continuous elastic equals the area of the diagram M/EI between these points.

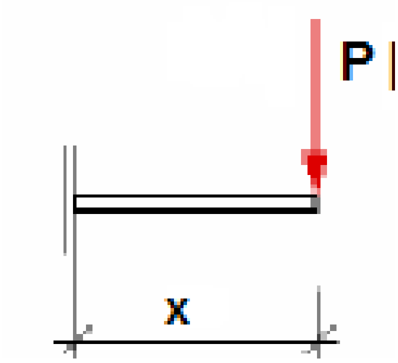
2 – The distance of a point B' of a continuous elastic primitive measured perpendicular to the axis AB at the tangent drawn at another point A' of the curve is equal to B when the area of the diagram M/EI between said points.



We set the external balance

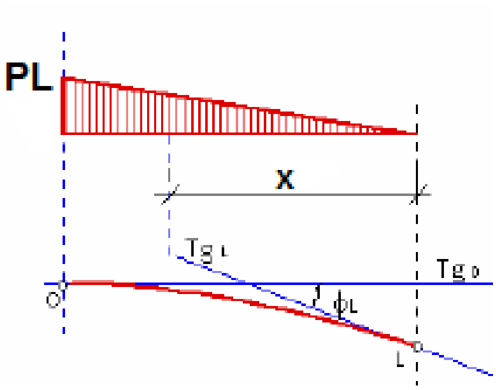
$$Ra = P$$

We determine the general equation of bending moment



M x = - P x

The angle between the tangents drawn to both ends of the beam we get using
Mohr's first theorem.

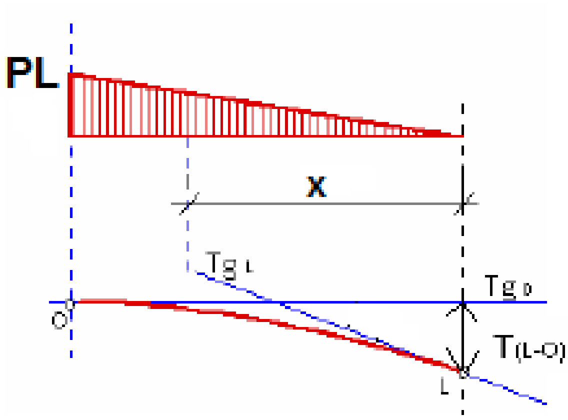


$$\phi_{L0} = -P \cdot L \cdot \frac{L}{2EI} \qquad \phi_{L0} = -\frac{PL^2}{2EI}$$

So:

$$\emptyset_B = \emptyset_{L0} = -\frac{PL^2}{2EI} \tag{3.1}$$

We calculated the shear deviation in O (free end of the beam) with respect to the tangent draw at the other end we can determine the maximum deflection.



$$t_{(L-0)} = -\frac{PL^2}{2EI} \cdot \frac{2L}{3} \qquad t_{(L-0)} = -\frac{PL^3}{3EI}$$

So:

$$Y_{max} = t_{(L-0)} = -\frac{PL^3}{3EI} \quad (3.2)$$

- <u>Double Integration Method</u>

Of the deduction of Mohr's First Theorem expression was obtained:

$$d\emptyset = \frac{1}{EI} \cdot M \cdot dx$$

$$\frac{d\emptyset}{dx} = \frac{M}{EI}$$

The derivative at any point on the curve is equal to the slope of the tangent to the curve at that point.

$$\frac{dy}{dx} = Tg\phi$$

$$Tg\phi \approx \phi$$

$$\phi \approx \frac{dy}{dx}$$

Replacing in the original equation we obtain The Differential Equation of the Elastic Beam.

$$\frac{d}{dx}\frac{dy}{dx} = \frac{M}{EI}$$

$$\frac{d^2y}{dx^2} = \frac{M}{EI} \tag{3.3}$$

Integrating we obtain The General Equation of the Slope

$$\frac{dy}{dx} = \frac{1}{EI}\int M \cdot dx + C_1 \tag{3.4}$$

Integrating again we obtain the general equation of the Arrow

$$y = \frac{1}{EI}\iint M \cdot dx + C_1 + C_2 \tag{3.5}$$

This method allows us to calculate the slope and deflection of the beam at any point. The difficulty lies in clearing the integration constants. This is accomplished by analyzing the support conditions and the deformation of the beam.

With the general equation of bending moment establish the differential equation of the elastic.

$$EI\frac{d^2y}{dx^2} = Px - PL$$

Twice integrating the differential, we obtain:

$$EI\frac{dy}{dx} = \frac{Px^2}{2} - PLx + C_1$$

$$EIy = \frac{Px^3}{6} - \frac{Px^2}{2} + C_1x + C_2$$

According to the deformation of the bean, the slope is zero when X =0.

$$C_1 = 0$$

According to conditions of support, the arrow is zero when X =0.

$$C_2 = 0$$

Then the general equations of angle and arrow are:

- General equation of angle $\quad EI\frac{dy}{dx} = \frac{Px^2}{2} - PLx$

- General equation of the arrow $EIy = \frac{Px^3}{6} - \frac{Px^2}{2}$

The maximum angle is in the right side and it is obtained by replacing X = L in the corresponding equation.

$$\emptyset_B = -\frac{PL^2}{2EI} \qquad (3.6)$$

And the maximum deflection, replacing X=L.

$$Y_{max} = -\frac{PL^3}{3EI} \qquad (3.7)$$

- Conjugate Beam Method

This method is based on the same principles as the area moment method, but differs in its implementation. To generate a new fictitious beam of the same length and with the same conditions of the original beam support, but loaded with the bending moment diagram of the original beam divided by EI. Thus, the angle of the tangent drawn at any point of the elastic beam is given by the actual cutting (Q') of the new beam, and the arrow is determined by calculating the bending moment (M') of the beam fictitious.

As above, setting the following equivalences.

REAL BEAM		FICTITIOUS BEAM	
Moment	M	Load	M / EI
Angle	ϕ	Shear	Q'
Arrow	Y	Moment	M'

We can say that there is an analogy between the relationships of load – shear – moment, and moment – slope – arrow.

With the graph of the bending moment characteristic values and generate

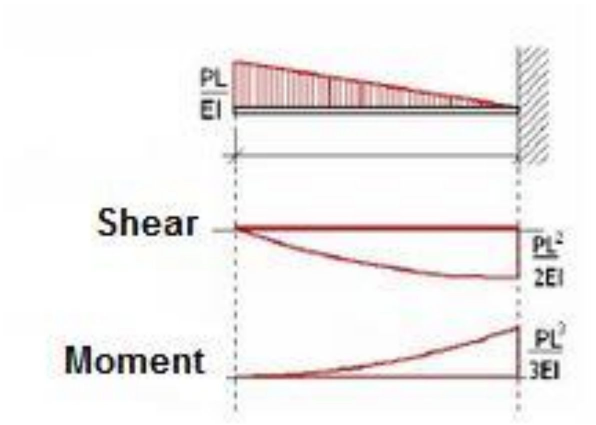
a fictitious beam.

M=P
L

In the fictitious beam will apply like load the bending moment of beam between EI.

The relationship established between the real beam and the fictional beam is that the values of shear and moment in the beam fictional, equals the slope and deflection of the real beam.

But in the particular case of the cantilever beam, the slope and the decline in its support is zero. At this point should not exist R 'or M', so to implement this method it is necessary to reverse the support of the fictitious beam on the other end of the beam, so as to find R 'and M'_{max} in the point corresponding.



$$q' = \frac{M_{max}}{EI} = \frac{PL}{EI}$$

$$\phi_B = R_B = -\frac{PL}{EI}\frac{L}{2}$$

$$\phi_B = -\frac{PL^2}{2EI} \qquad (3.8)$$

$$Y_{max} = M'_{max} = -\frac{PL}{EI}\frac{L}{2}\frac{2L}{3}$$

$$Y_{max} = -\frac{PL^3}{3EI} \qquad (3.9)$$

3.1.2.1 - CERAMIC

We are use as ceramic material, silica which has elastic modulus E= 73GPa and Poisson coefficient ν=0.2.

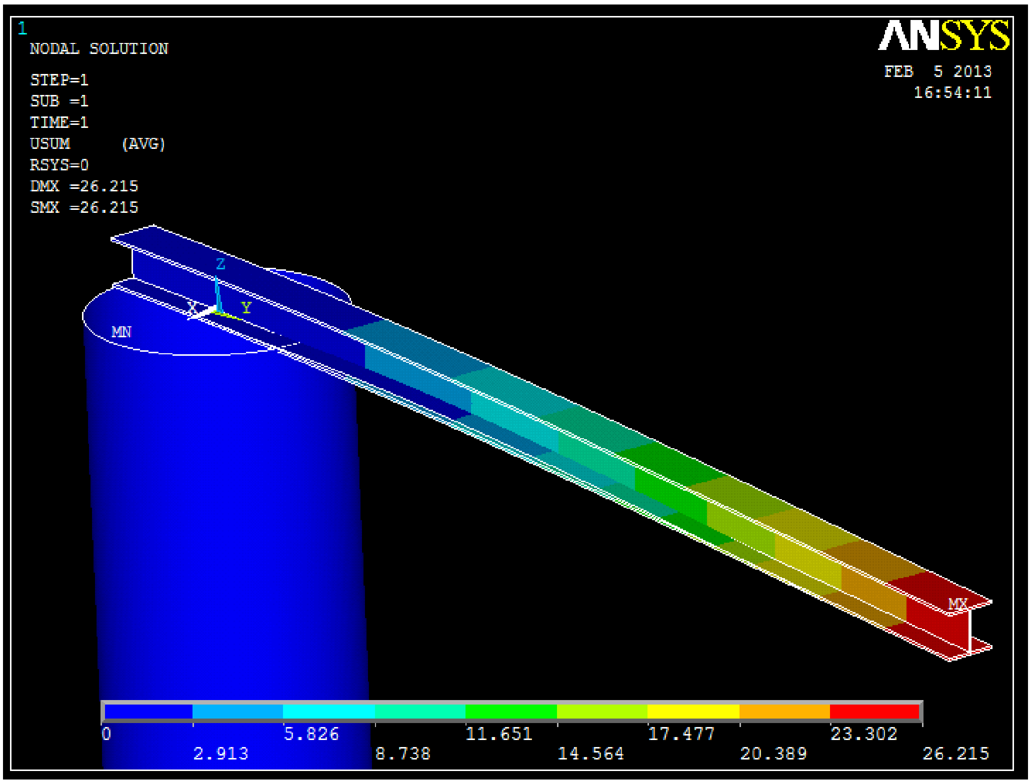


Image 3.6 - Maximum displacement of ceramic beam under boundary conditions indicated

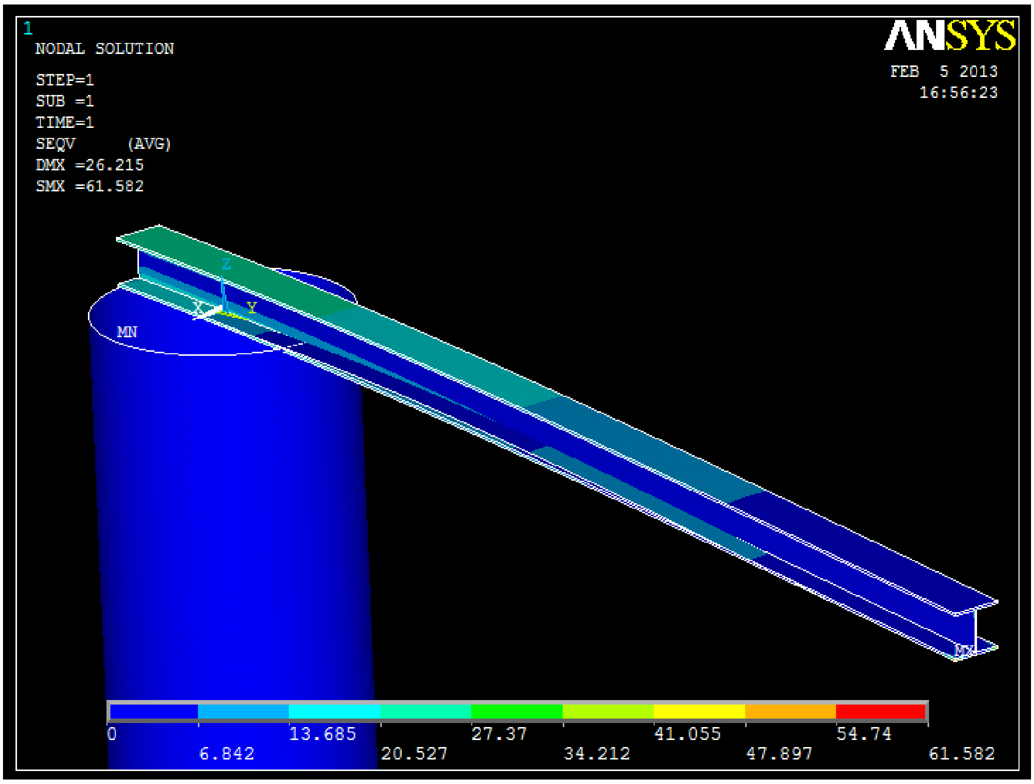


Image 3.7 - Maximum stress of ceramic beam under boundary conditions indicated

3.1.2.2 - STEEL

Now, we are use the steel material which has as elastic modulus E=210GPa and Poisson Coefficient μ =0.3.

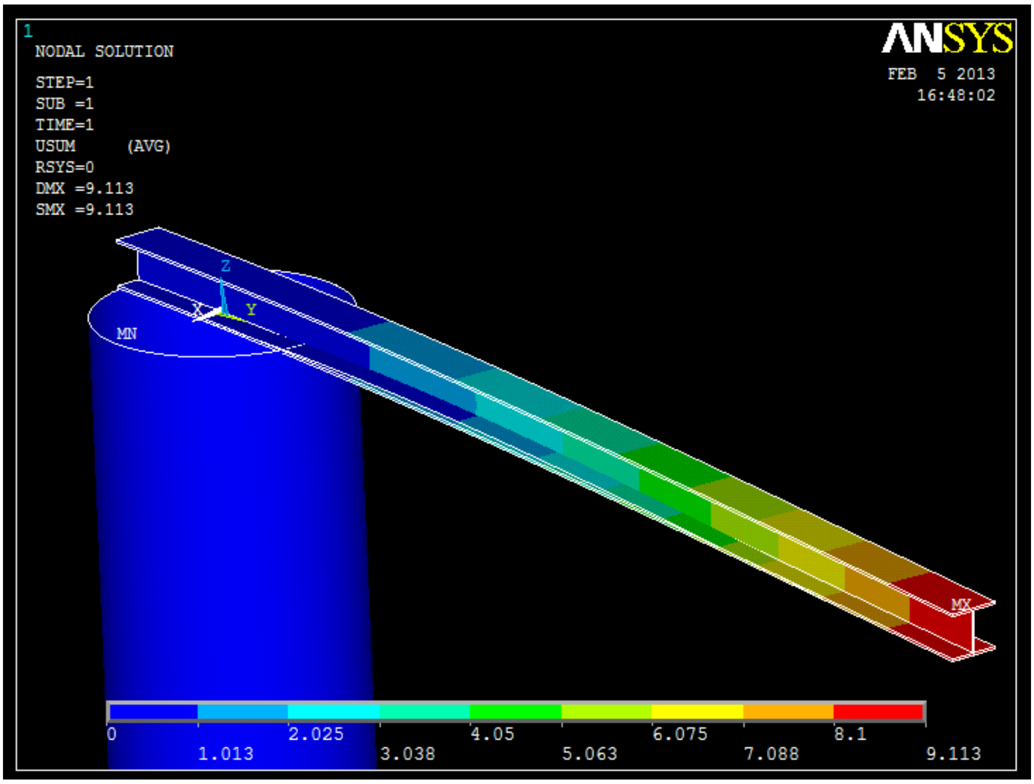


Image 3.8 - Maximum displacement of steel beam under boundary conditions indicated

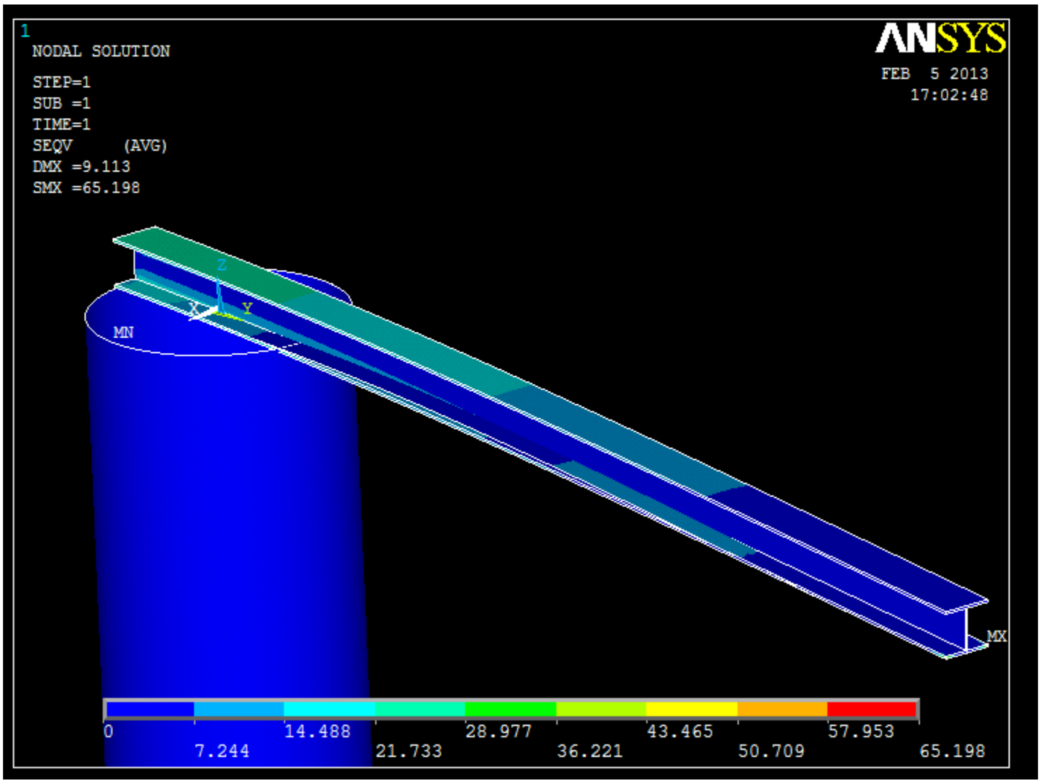


Image 3.9 - Maximum stress of steel beam under boundary conditions indicated

3.1.2.3 - CONCRETE

Now, we are use the concrete material which has as elastic modulus E=27GPa and Poisson Coefficient ν =0.2

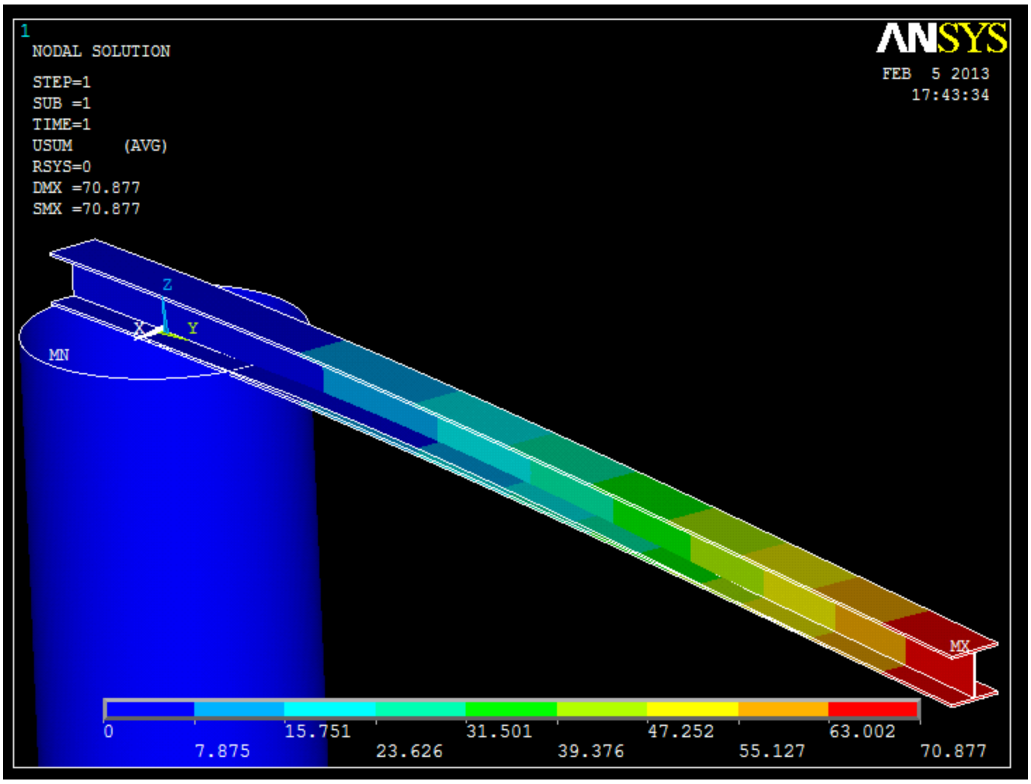


Image 3.10 – Maximum displacement of concrete beam under boundary conditions indicated

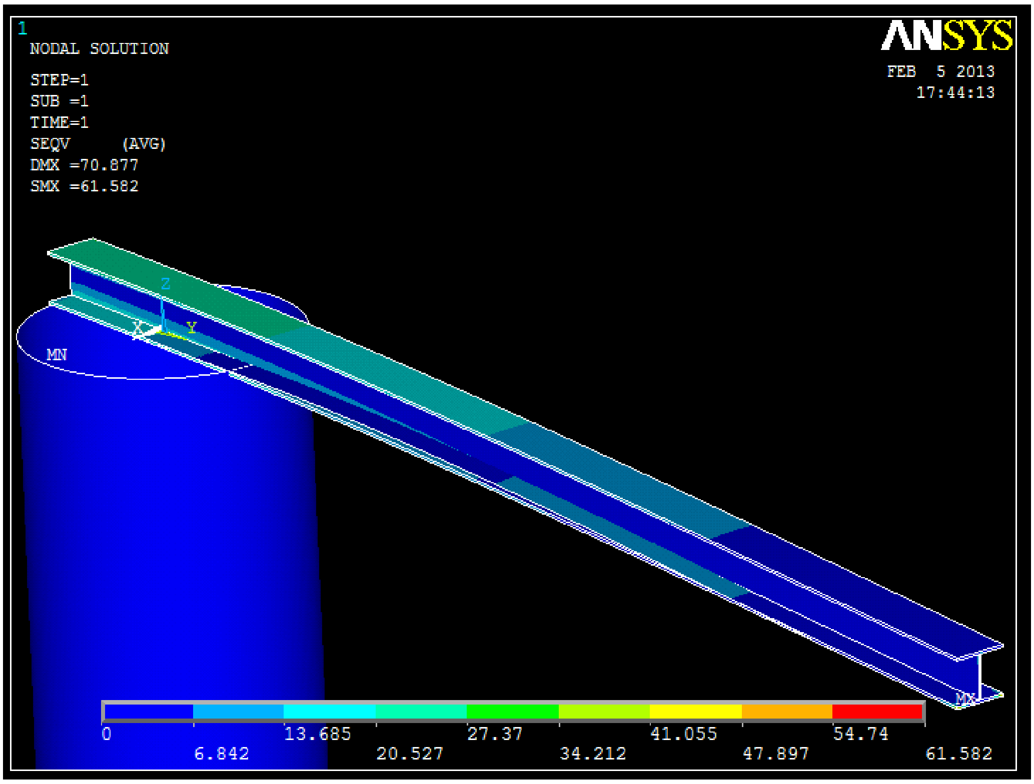


Image 3.11 - Maximum stress of concrete beam under boundary conditions indicated

3.1.3 - A continuous load on the cantilever beam

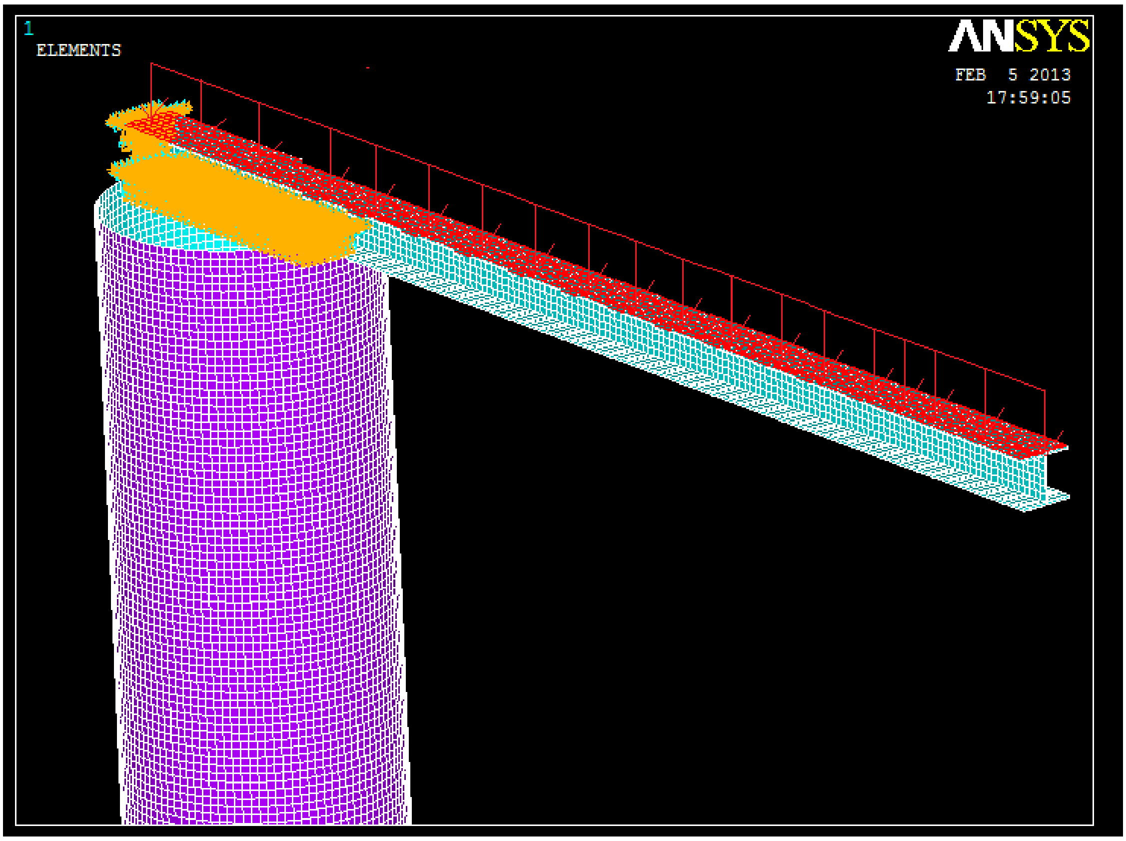
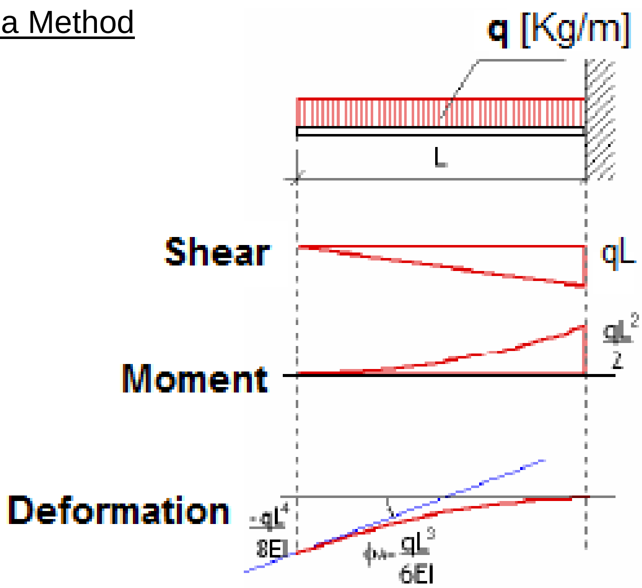


Image 3.12 – Model with a continuous load on the cantilever beam

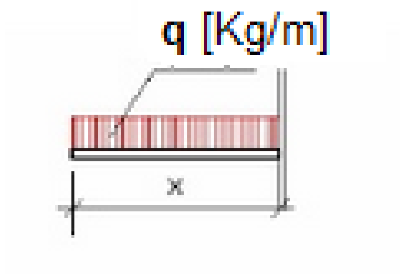
- <u>Moment Area Method</u>



We set the external balance

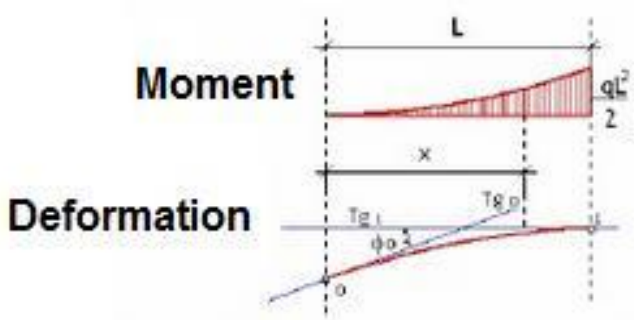
$$Ra = qL$$

We determine the general equation of bending moment



$$Mx = -\frac{qx^2}{2}$$

The angle between the tangents drawn to both ends of the beam we get using Mohr's first theorem.



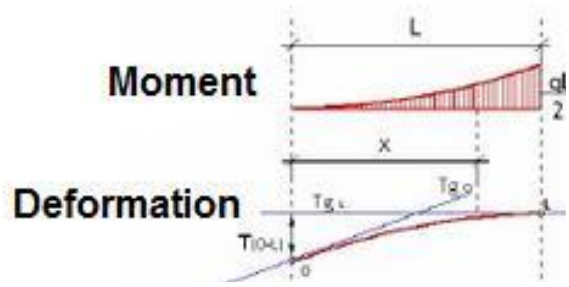
$$\phi_{0L} = -\frac{1}{EI}\int_0^L \frac{qx^2}{2} dx$$

Moment: $\phi_{0L} = \frac{1}{EI} {}_0^L\left[\frac{qx^3}{6}\right]$

Deformation: $\phi_{0L} = -\frac{qL^3}{6EI}$

$$\phi_A = \phi_{0L} = -\frac{PL^3}{6EI} \qquad (3.10)$$

We calculated the shear deviation in O (free end of the beam) with respect to the tangent draw at the other end we can determine the maximum deflection.



$$t_{(0-L)} = -\frac{1}{EI}\int_0^L \frac{qx^2}{2} \cdot x \cdot dx$$

$$t_{(0-L)} = -\frac{1}{EI}\int_0^L \frac{qx^3}{2} dx$$

$$t_{(0-L)} = \frac{1}{EI}\left[\frac{qx^4}{8}\right]_0^L$$

$$Y_{max} = t_{(0-L)} = -\frac{qL^4}{8EI} \qquad (3.11)$$

○ <u>Double Integration Method</u>

With the general equation of bending moment establish the differential equation of the elastic.

$$EI\frac{d^2y}{dx^2} = -\frac{qx^2}{2}$$

Twice integrating the differential, we obtain:

$$EI\frac{dy}{dx} = -\frac{qx^3}{6} + C_1$$

$$EIy = -\frac{qx^4}{24} + C_1x + C_2$$

According to the deformation of the bean, the slope is zero when X =L.

$$C_1 = \frac{qL^3}{6}$$

According to conditions of support, the arrow is zero when X =L.

$$C_2 = \frac{qL^4}{8}$$

Replacing C1 and C2 in the above equations:

- General equation of angle $\quad EI\frac{dy}{dx} = -\frac{qx^3}{6} + \frac{qL^3}{6}$

- General equation of the arrow $$EIy = -\frac{qx^4}{24} + \frac{qL^3}{6}x - \frac{qL^4}{8}$$

The maximum angle is in the right side and it is obtained by replacing X = L in the corresponding equation.

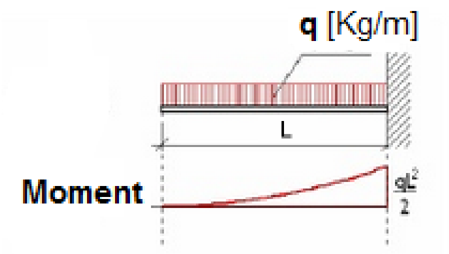
$$\phi_A = \frac{qL^3}{6EI} \quad (3.12)$$

And the maximum deflectino, replacing X=0.

$$Y_{max} = -\frac{qL^4}{8EI} \quad (3.13)$$

- Conjugate Beam Method

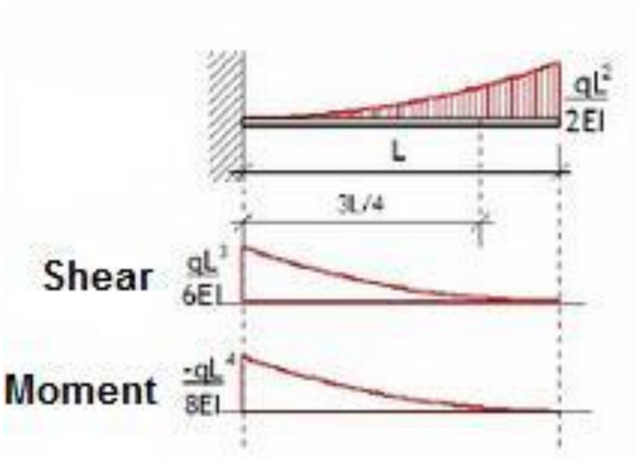
With the graph of the bending moment characteristic values and generate a fictitious beam.



$$Mx = -\frac{qx^2}{2}$$

In the fictitious beam will apply like load the bending moment of beam between EI.

As explained in the previous case, in the case of cantilevered beams, it is necessary invert its support at the other end of the beam for implementing the method.



$$q' = \frac{M_{max}}{EI} = \frac{qL^{\ 2}}{2EI}$$

$$\phi_A = R_{a'} = \frac{qL^2}{2EI}\frac{L}{3}$$

$$\phi_A = \frac{qL^3}{6EI} \qquad (3.14)$$

$$Y_{max} = M'_{max} = \frac{qL^2}{2EI}\frac{L}{3}\frac{3L}{4}$$

$$Y_{max} = \frac{qL^4}{8EI} \qquad (3.15)$$

3.1.3.1 - CERAMIC

We are use as ceramic material, silica which has elastic modulus E= 73GPa and Poisson coefficient ν=0.2.

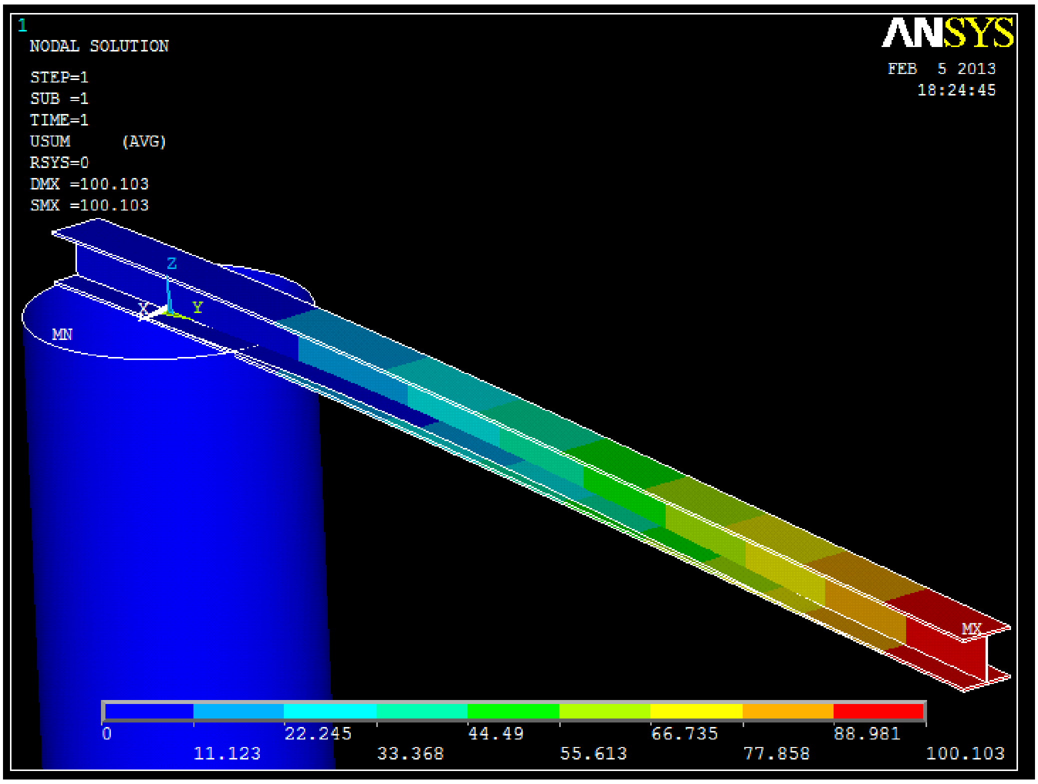


Image 3.13 - Maximum displacement of ceramic beam under boundary conditions indicated

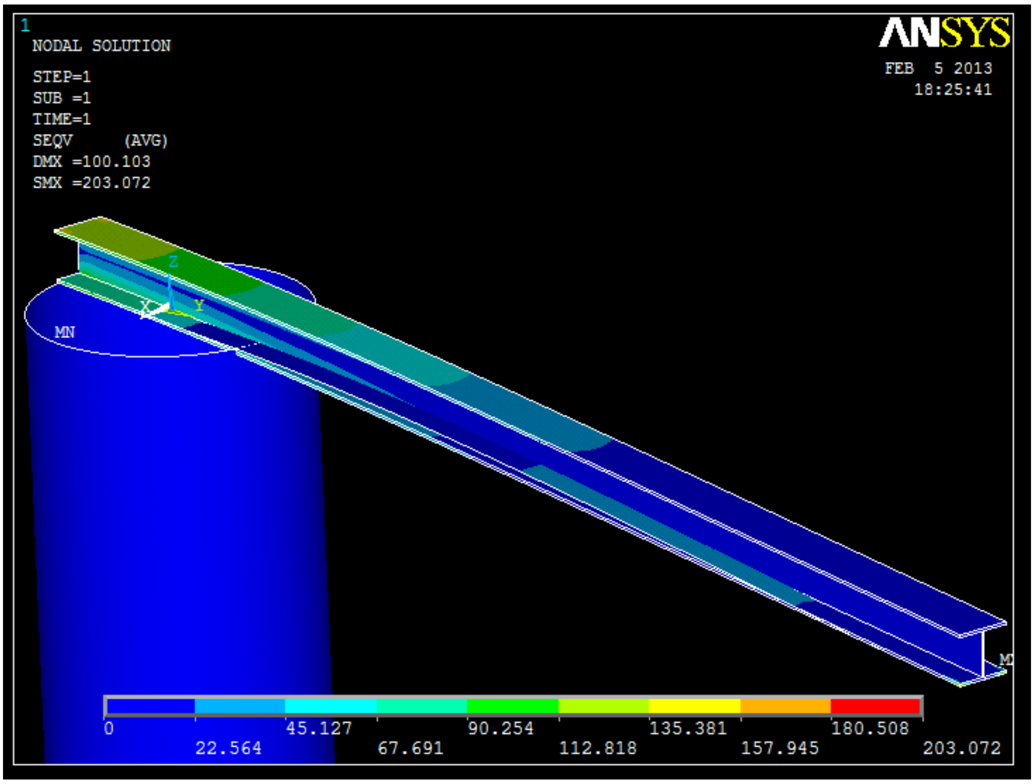


Image 3.14 - Maximum stress of ceramic beam under boundary conditions indicated

3.1.3.2 - STEEL

Now, we are use the steel material which has as elastic modulus E=210GPa and Poisson Coefficient $\upsilon = 0.3$.

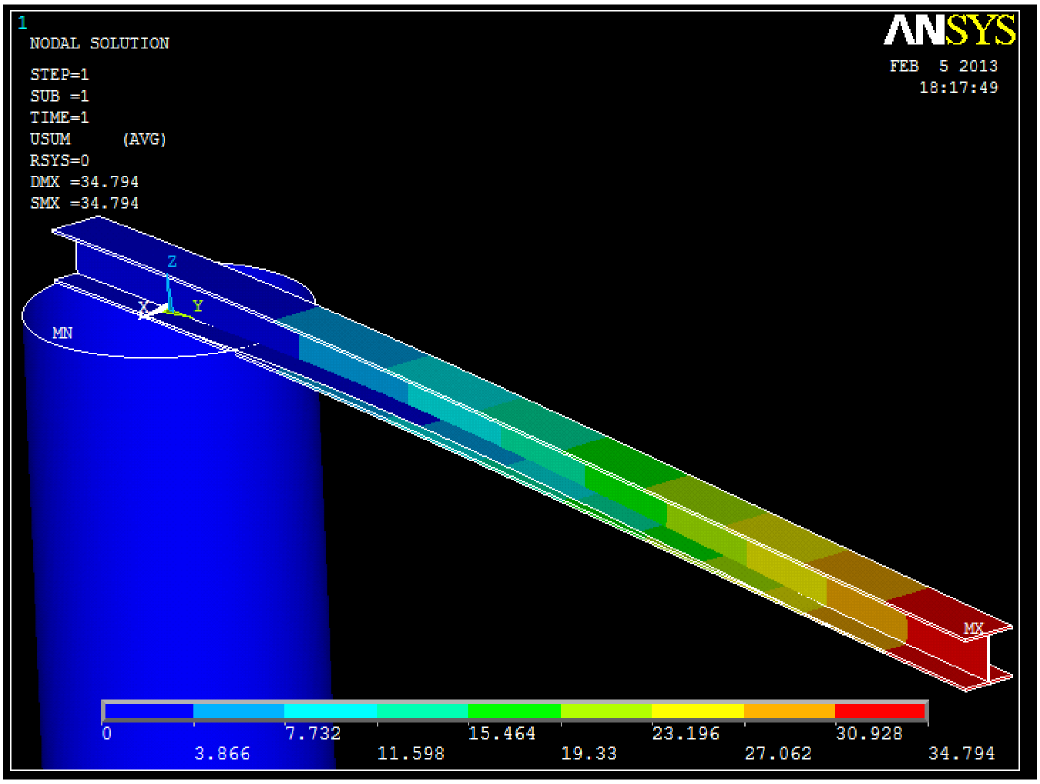


Image 3.15 – Maximum displacement of steel beam under boundary conditions indicated

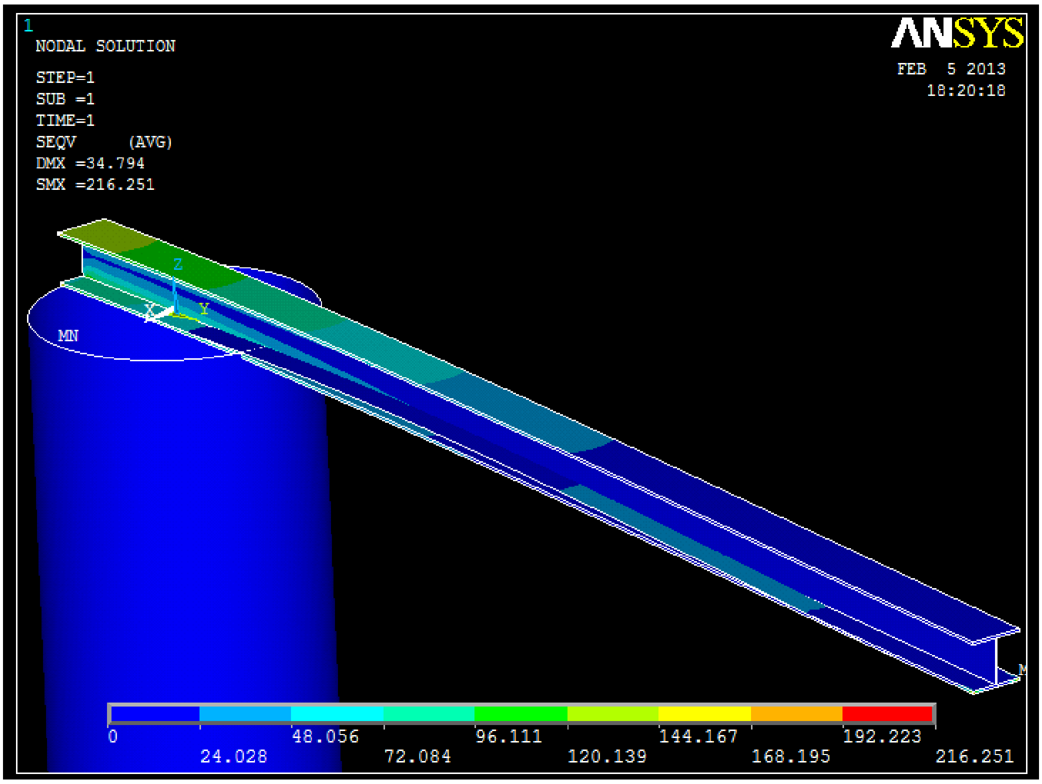


Image 3.16 - Maximum stress of steel beam under boundary conditions indicated

3.1.3.3. - CONCRETE

Now, we are use the concrete material which has as elastic modulus E=27GPa and Poisson Coefficient $\upsilon = 0.2$.

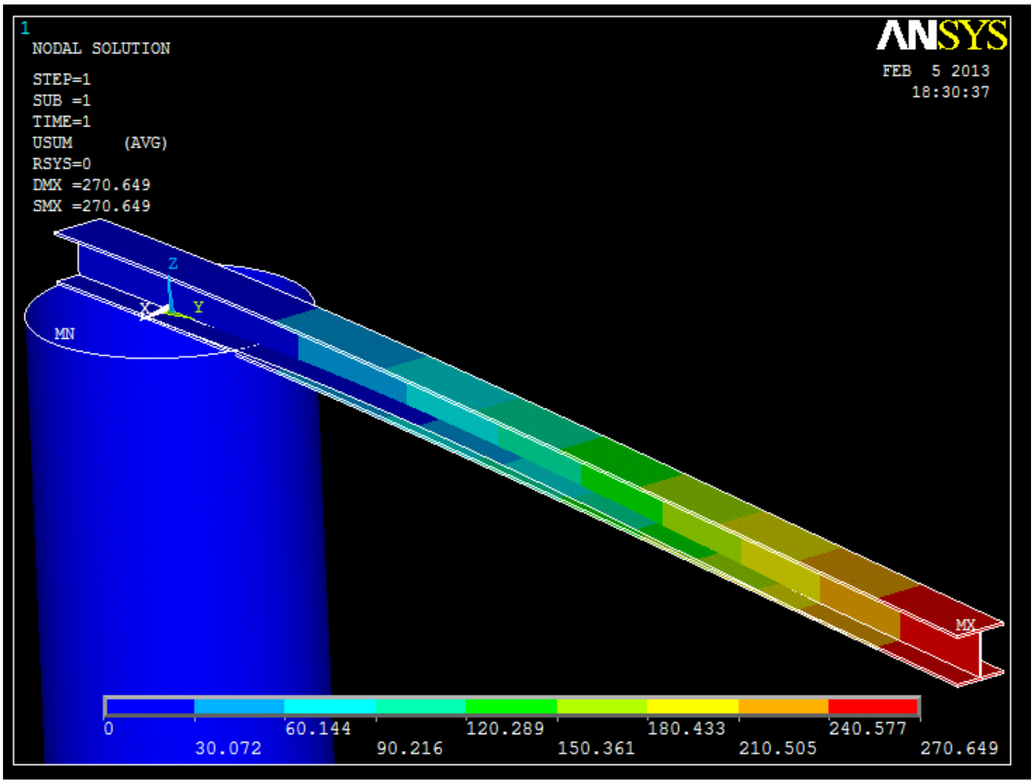


Image 3.17 – Maximum displacement of concrete beam under boundary conditions indicated

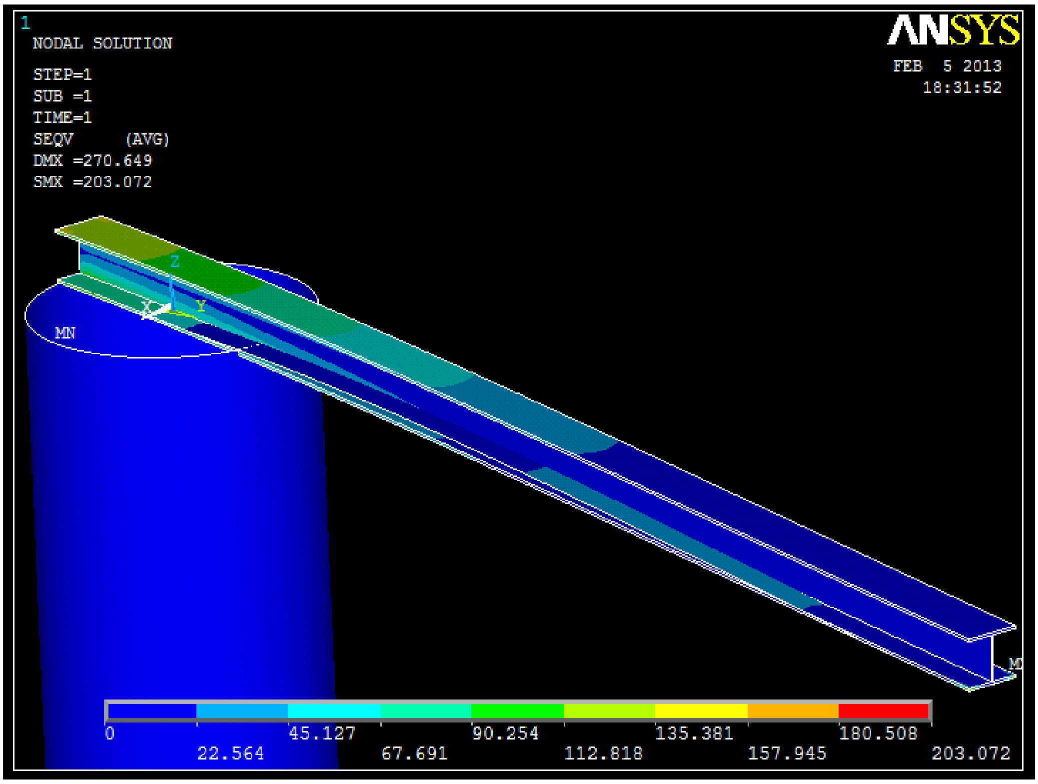


Image 3.18 - Maximum stress of concrete beam under boundary conditions indicated

3.2 – BEAM WITH TWO SUPPORTS

A beam is a structural element that is capable of withstanding load primarily by resisting bending. The bending force induced into the material of the beam as a result of the external loads, own weight, span and external reactions to these loads is called a bending moment.

Beams are traditionally descriptions of building or civil engineering structural elements, but smaller structures such as truck or automobile frames, machine frames, and other mechanical or structural systems contain beam structures that are designed and analyzed in a similar fashion.

Beams generally carry vertical gravitational forces but can also be used to carry horizontal loads (namely, loads due to an earthquake or wind). The loads carried by a beam are transferred to columns, walls, or girders, which then transfer the force to adjacent structural compression members. In light frame construction the joists rest on the beam.

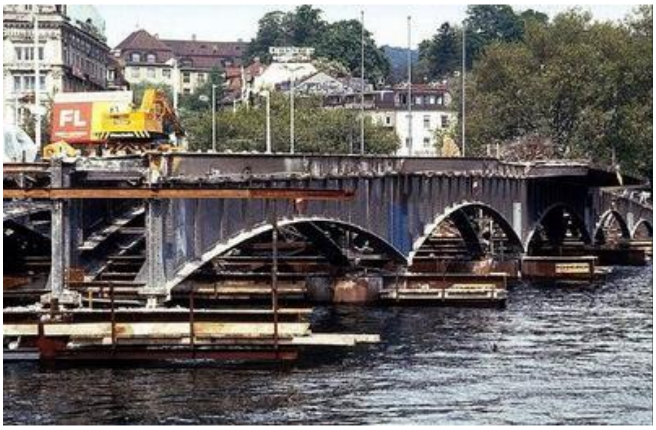


Image 3.19 – Pictures of example

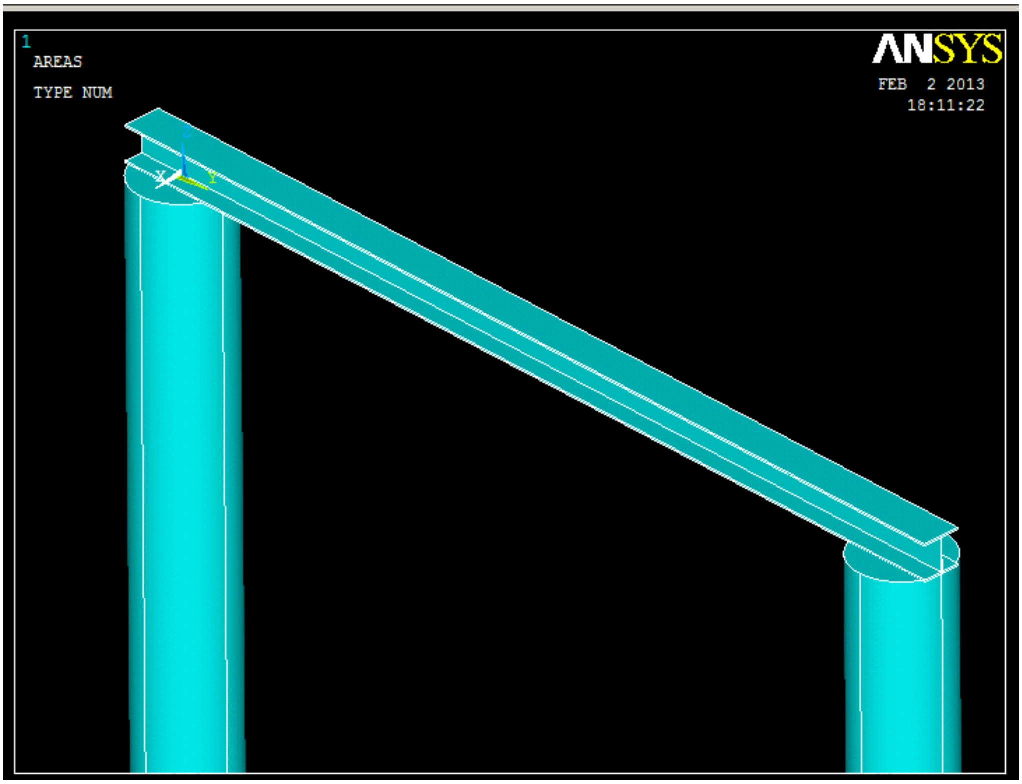


Image 3.20 - The structure of beam with two supports

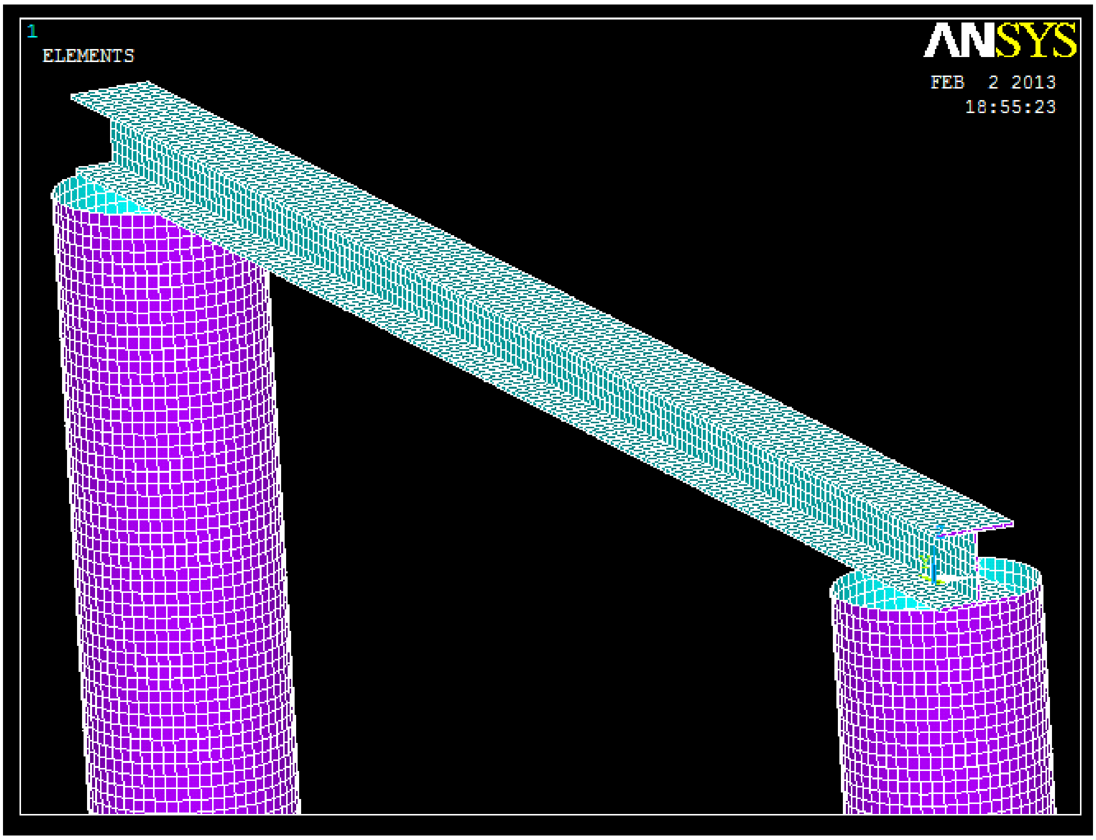


Image 3.21 - Model of structure to can study

3.2.1 - A load in the middle on the cantilever beam

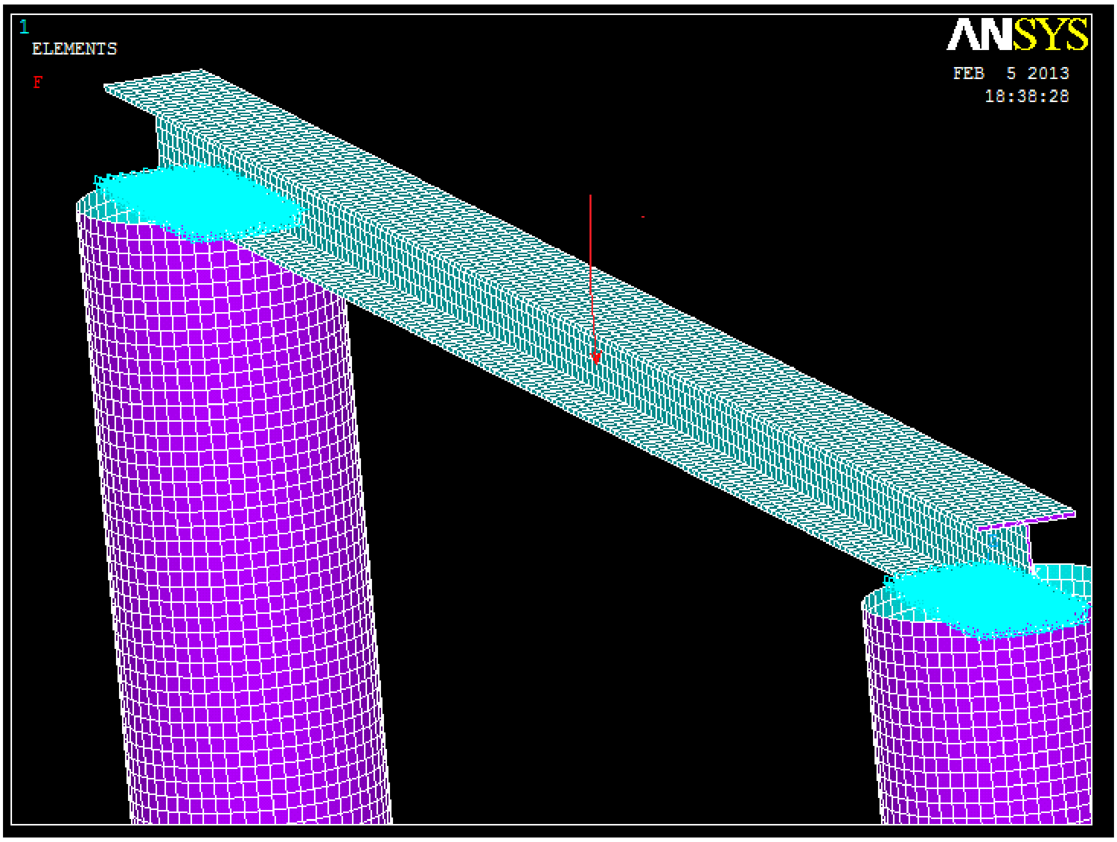
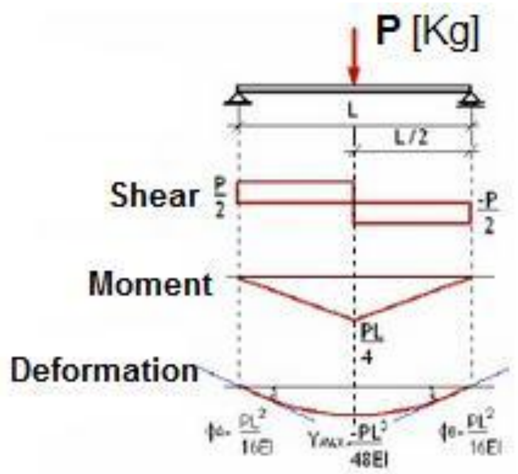
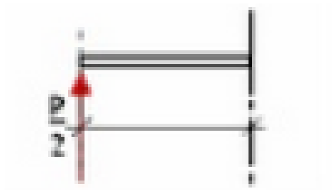


Image 3.22 - Model with a load in the middle of the cantilever beam

o Moment Area Method



We set the external balance



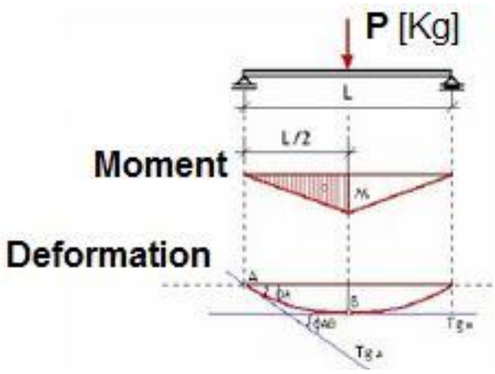
$$R_A = R_B = \frac{P}{2}$$

We determine the general equation of bending moment

$$Mx = \frac{Px}{2}$$

By symmetry of the beam, it follows that the slope plotted at the midpoint of the elastic curve is zero. For the application of Theorems Mohr, we consider a tangent drawn at the left end of the elastic and the tangent drawn at the midpoint thereof.

To determine the values of angle braces calculate the angle between the two tangents.



$$\phi_{AB} = \frac{1}{EI}\int_A^B Mdx$$

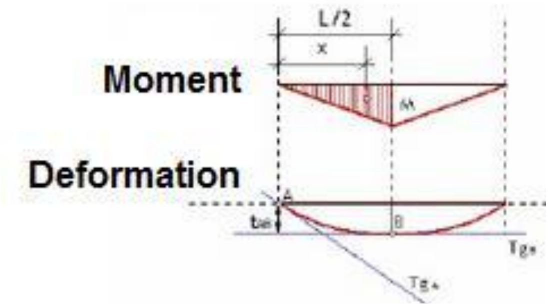
$$\phi_{AB} = \phi_A$$

$$\phi_A = \frac{1}{EI}\int_0^{L/2} \frac{Px}{2} dx$$

$$\phi_A = \frac{1}{EI} \left[\frac{Px^2}{4}\right]_0^{L/2}$$

$$\phi_A = \frac{PL^2}{16EI} \qquad (3.16)$$

And get the maximum deflection calculating the tangential deviation on the left end with respect to the tangent drawn at the midpoint of the elastic curve.



$$Y_{max} = \frac{1}{EI}\int_A^B M \cdot x \cdot dx$$

$$Y_{max} = \frac{1}{EI}\int_0^{L/2} \frac{Px}{2} \cdot x \cdot dx$$

$$Y_{max} = \frac{1}{EI} \left[\frac{Px^3}{6}\right]_0^{L/2}$$

$$Y_{max} = \frac{PL^3}{48EI} \qquad (3.17)$$

- <u>Double Integration Method</u>

As the beam is symmetrical analyze only the first installment. With the general equation of time establish the differential equation of the elastic.

$$EI\frac{d^2y}{dx^2} = \frac{Px}{2}$$

Twice integrating the differential, we obtain:

$$EI\frac{dy}{dx} = \frac{Px^2}{4} + C_1$$

$$EIy = \frac{Px^3}{12} + C_1x + C_2$$

According to the deformation of the bean, the slope of the tangent draw at the center of the beam is zero, namely:

If x = L/2 $\frac{dy}{dx} = 0$

$$0 = \frac{P}{4}\frac{L^2}{4} + C_1$$

$$C_1 = -\frac{PL^2}{16}$$

Then the general equation of angle is:

$$EI\frac{dy}{dx} = \frac{Px^2}{4} - \frac{PL^2}{16}$$

Under the terms of support, the arrow is null in support of life, namely, when X=0.

Therefore C_2= 0

Then the general equation of the arrow is:

$$EIy = \frac{Px^3}{12} - \frac{PL^2}{16}x$$

The maximum arrow substituting in X=L/2

$$\phi_A = \frac{PL^2}{16EI} \quad (3.18)$$

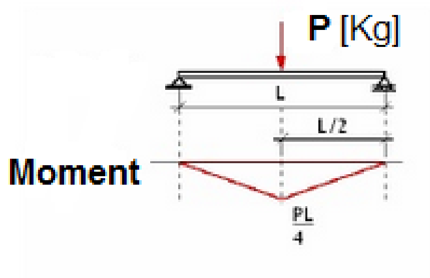
And the maximum deflection, replacing X=L/2

$$Y_{max} = \frac{PL^3}{48EI} \quad (3.19)$$

- <u>Conjugate Beam Method</u>

Real Beam

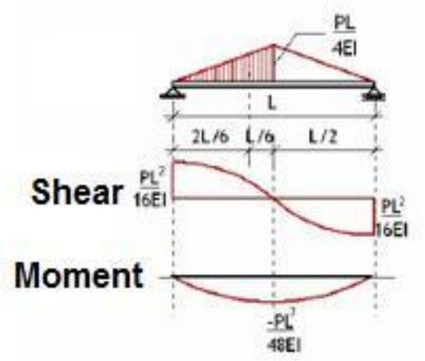
We determine the graph of bending moment and characteristic values.



$$M_{max} = \frac{PL}{4}$$

We generate a fictitious beam and apply such load bending moment given beam divided by EI. And we determine the reactions and the maximum moment, values for the angles in the supports and the maximum decrease of the beam given.

Fictitious Beam



$$M_{max} = q' = \frac{PL}{4EI}$$

$$\phi_A = R_{a'} = \frac{PL}{4EI}\frac{L}{2}\frac{1}{L} = \frac{PL^2}{16EI}$$

$$\phi_A = \frac{PL^2}{16EI} \tag{3.20}$$

This angle value is also valid for the other end, because the beam is symmetrical. And for the same condition, the maximum moment occurs when X = L/ 2.

$$Y_{max} = M_{max} = \frac{PL^2}{16EI}\frac{L}{2} - \frac{PL}{4EI}\frac{1}{2}\frac{L}{2}\frac{1}{3}\frac{L}{2}$$

$$Y_{max} = \frac{PL^3}{48EI} \tag{3.21}$$

3.2.1.1 - CERAMIC

We are use as ceramic material, silica which has elastic modulus E= 73GPa and Poisson coefficient ν=0.2.

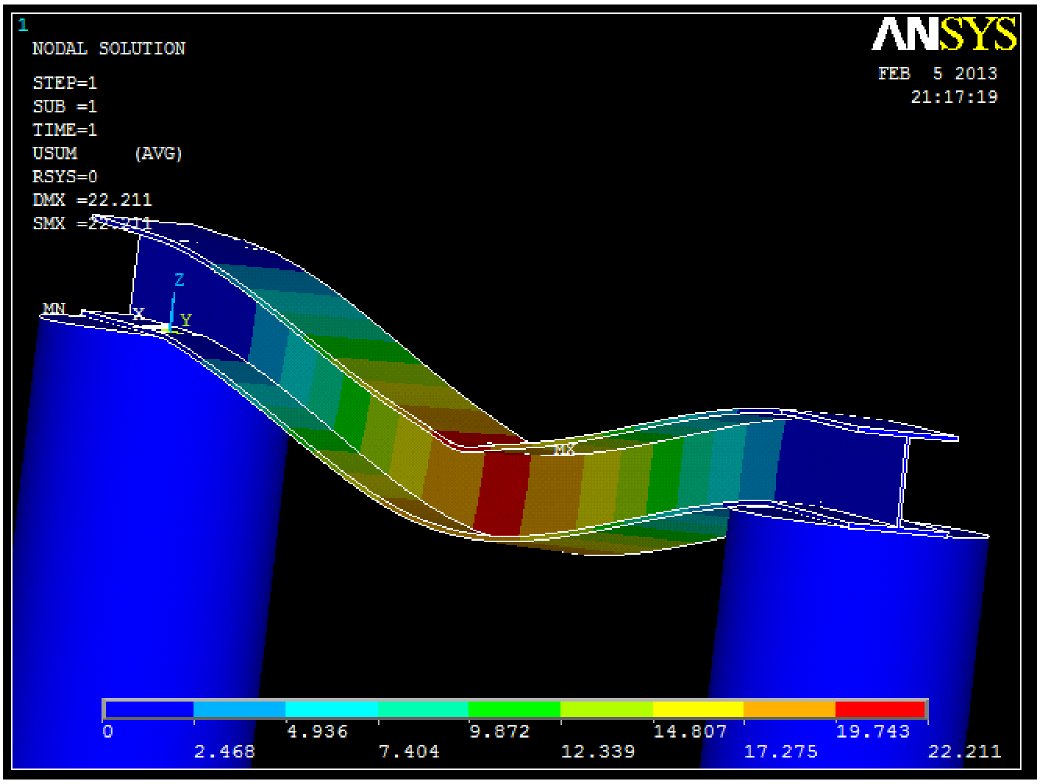


Image 3.23 – Maximum displacement of ceramic beam under boundary conditions indicated

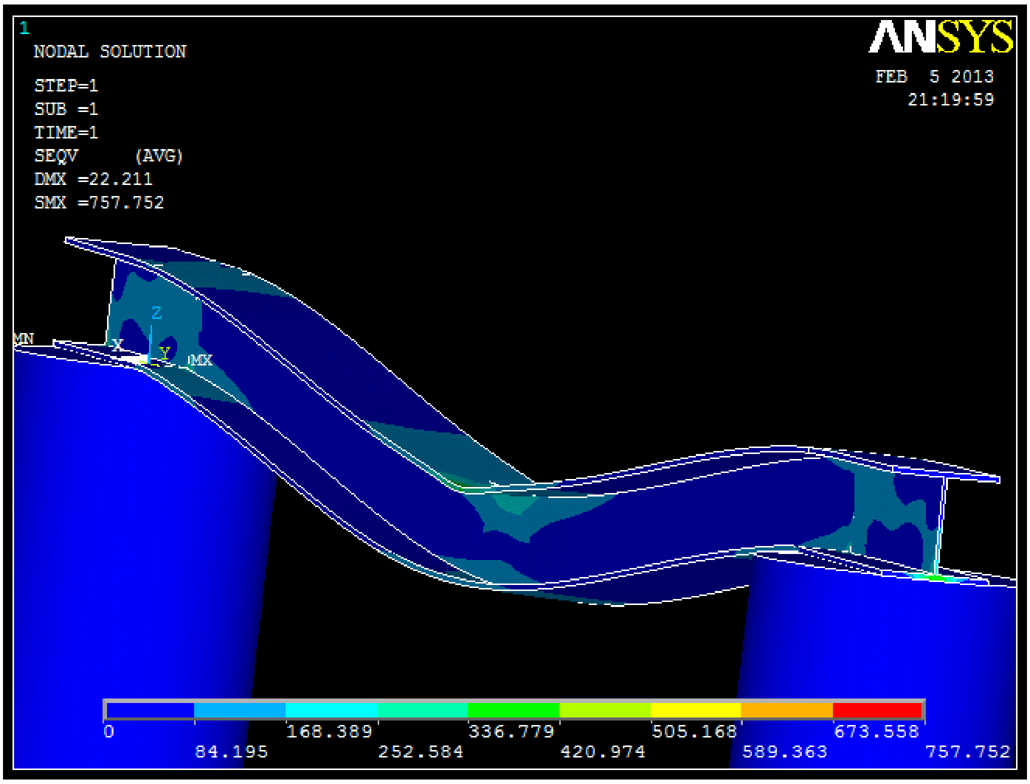


Image 3.24 - Maximum stress of ceramic beam under boundary conditions indicated

3.2.1.2 - STEEL

Now, we are use the steel material which has as elastic modulus E=210GPa and Poisson Coefficient $\upsilon = 0.3$.

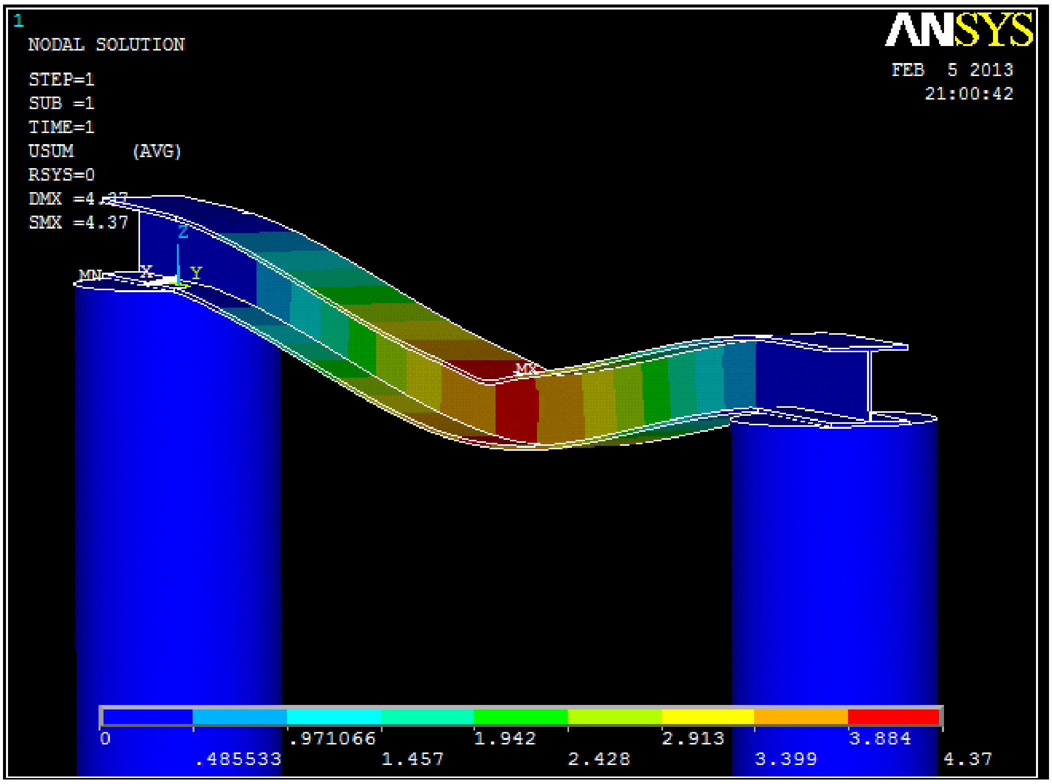


Image 3.25 – Maximum displacement of steel beam under boundary conditions indicated

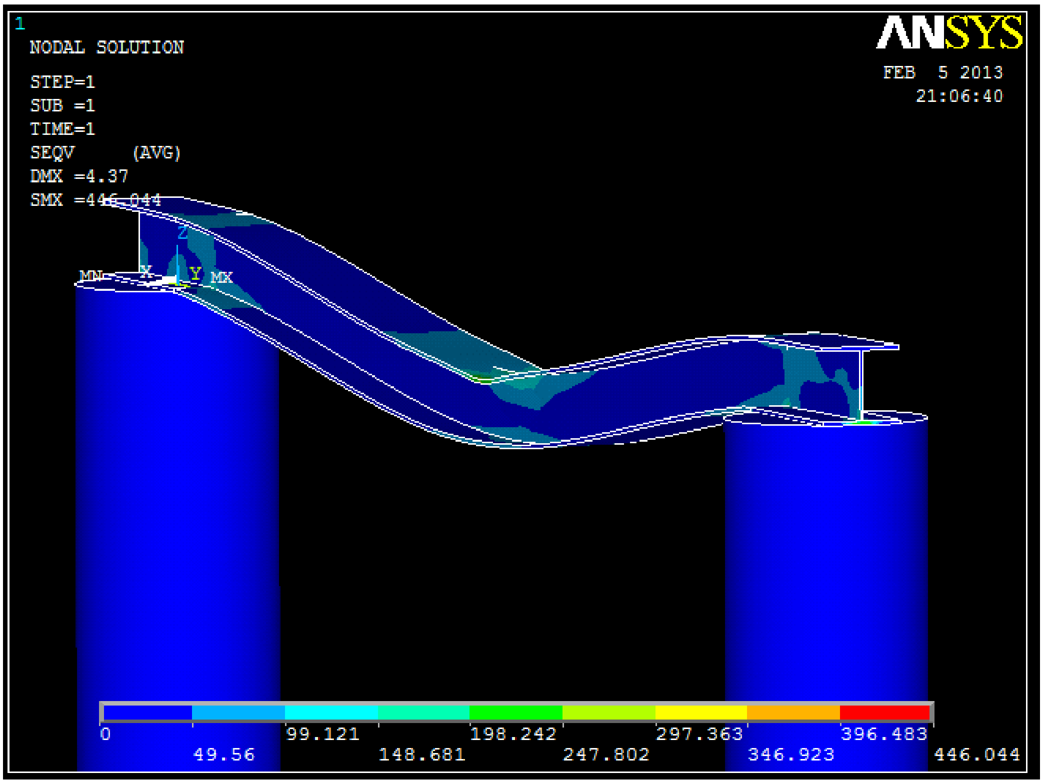


Image 3.26 - Maximum stress of steel beam under boundary conditions indicated

3.2.1.3 - CONCRETE

Now, we are use the concrete material which has as elastic modulus E=27GPa and Poisson Coefficient $\upsilon = 0.2$.

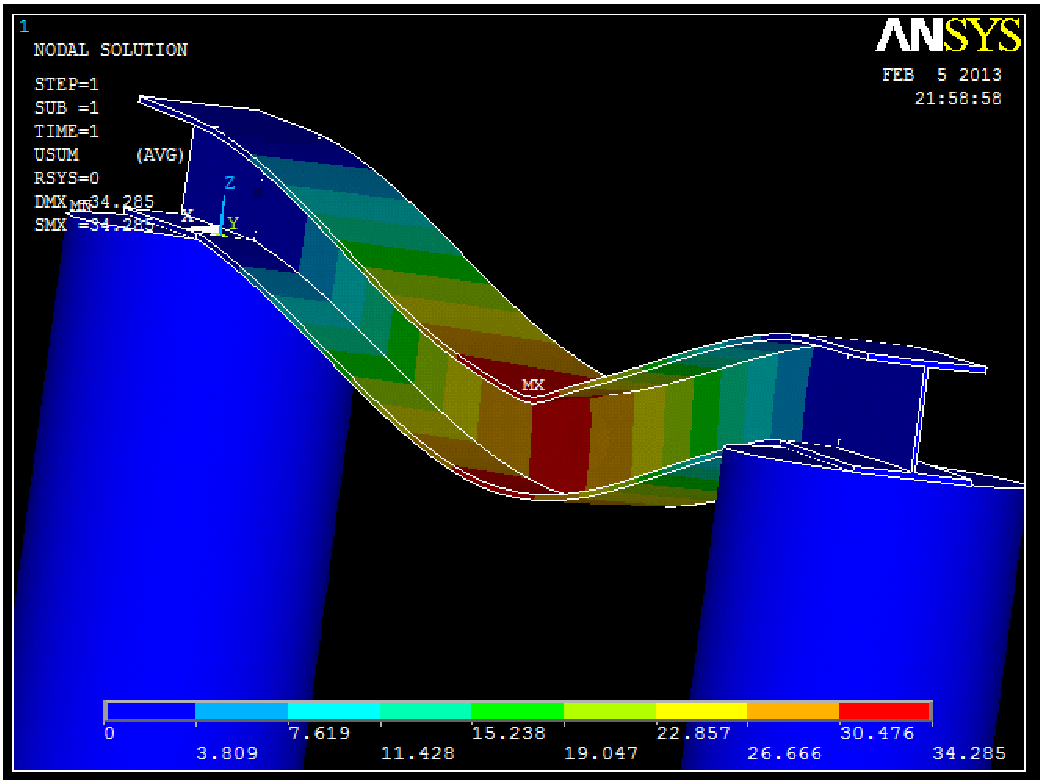


Image 3.27 - Maximum displacement of concrete beam under boundary conditions indicated

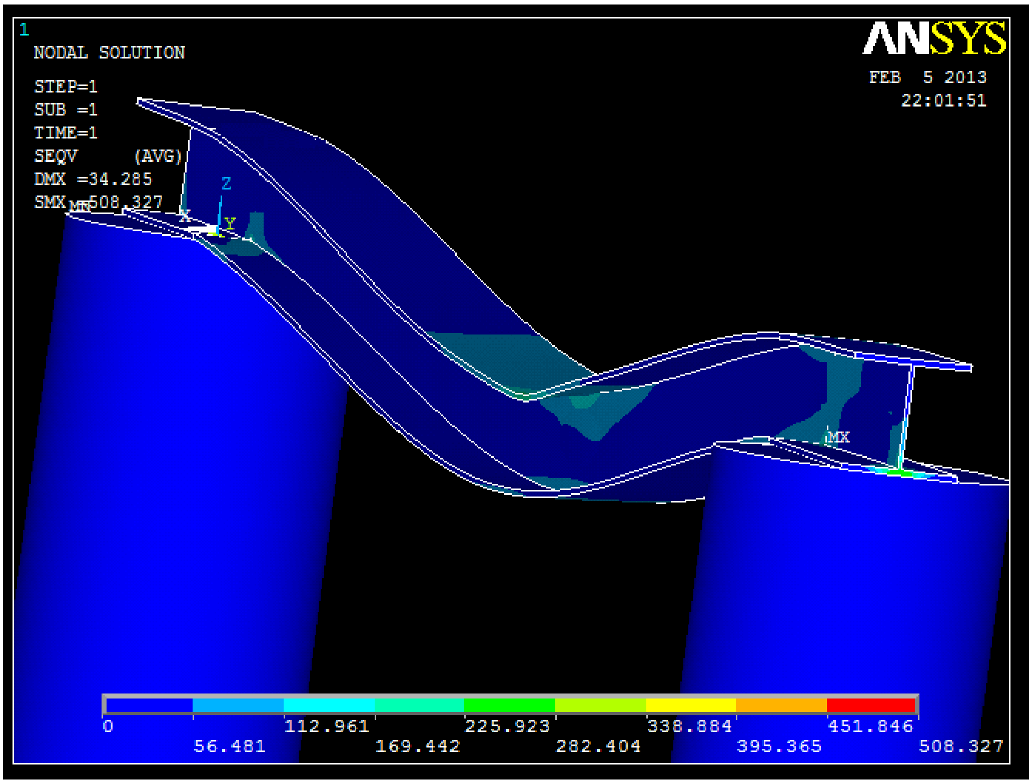


Image 3.28 - Maximum stress of concrete beam under boundary conditions indicated

3.2.2 - A continuous load on the cantilever beam

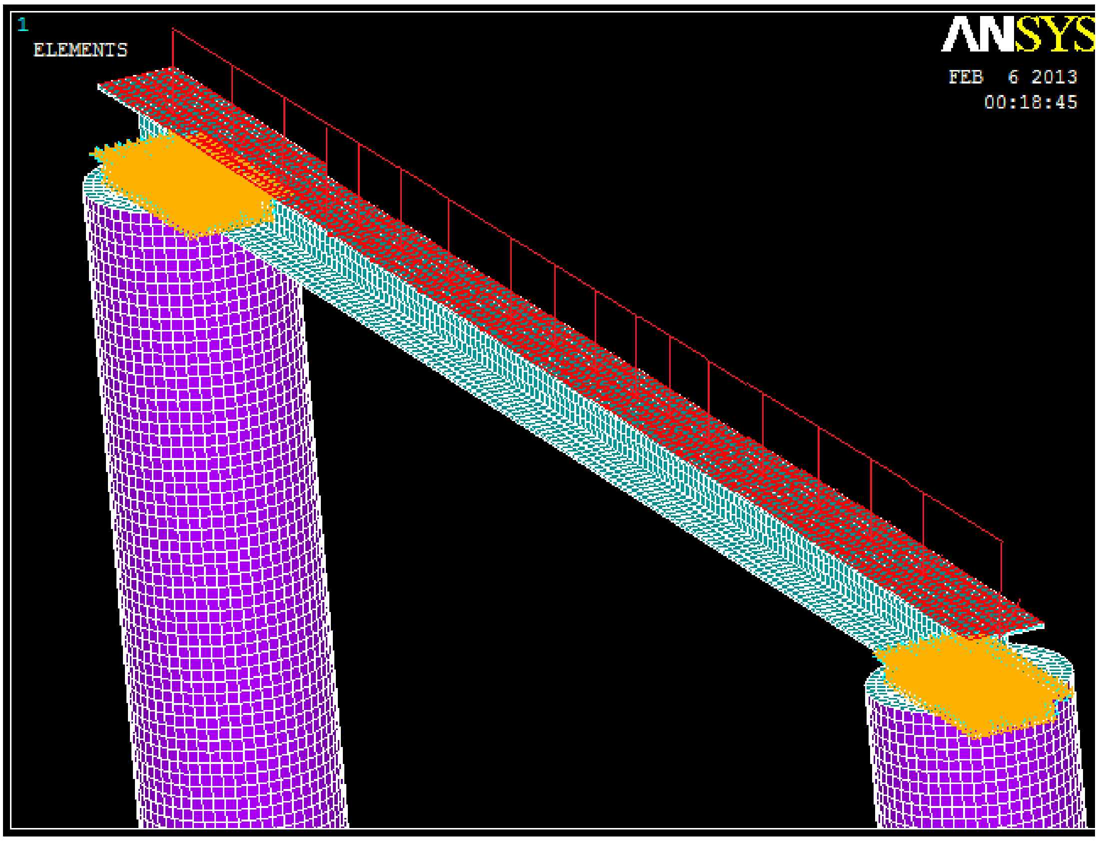
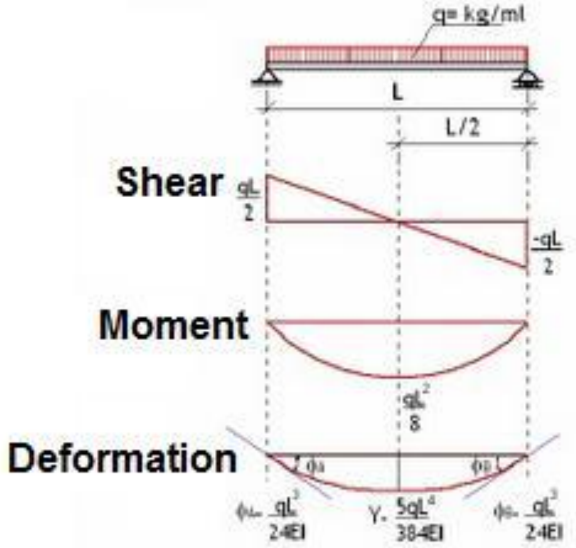
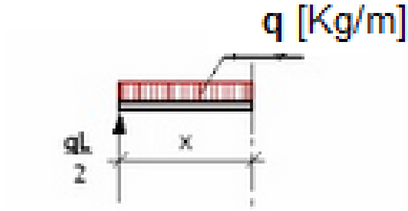


Image 3.29 - Model with a continuous load on the cantilever beam

- Moment Area Method

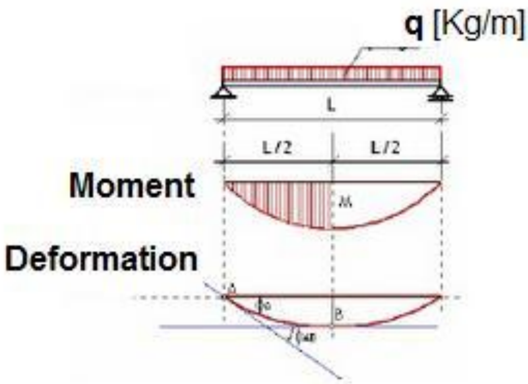


We set the external balance



$$R_A = R_B = \frac{qL}{2}$$

Applying Mohr's first theorem, we can determine the angle to the support by calculating the angle between the tangent drawn at the left end of the elastic and the tangent drawn at the midpoint, which is the tangent slope of zero.



$$\phi_{AB} = \frac{1}{EI}\int_0^{L/2} M dx$$

$$\phi_{AB} = \frac{1}{EI}\int_0^{L/2} \left(\frac{qLx}{2} - \frac{qx^2}{2}\right) dx$$

$$\phi_{AB} = \frac{1}{EI} \Big[\left(\frac{qLx^2}{4} - \frac{qx^3}{6}\right)\Big]_0^{L/2}$$

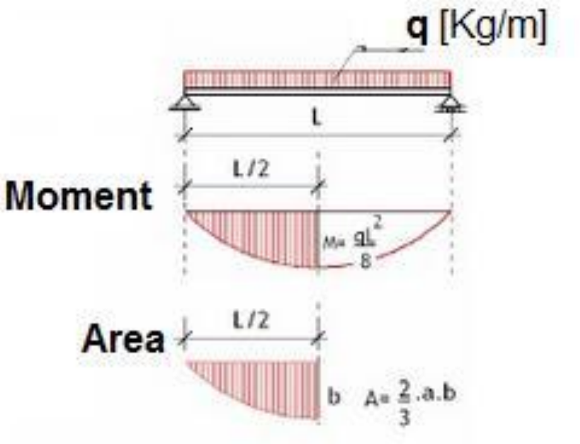
$$\phi_{AB} = \frac{qL^3}{6EI} - \frac{qL^3}{48EI}$$

$$\phi_{AB} = \phi_A$$

Being symmetrical beam follows that this angle value also applies to the right end of it.

Another way is to calculate it through the area

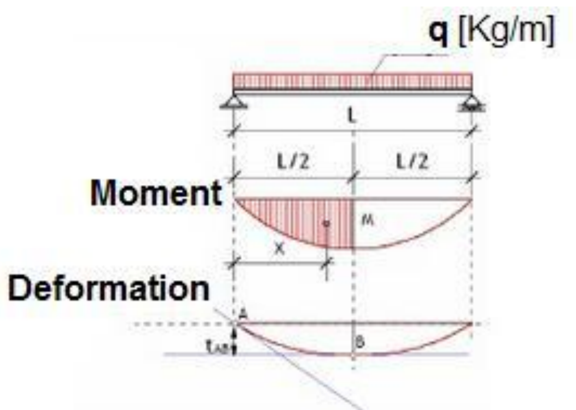
$$\phi_{AB} = \phi_A = \frac{1}{EI} Areabetween A and B$$



$$\phi_A = \frac{1}{EI}\frac{qL^2}{8}\frac{2}{3}\frac{L}{2}$$

$$\phi_A = \frac{qL^3}{24EI} \qquad (3.22)$$

For obtaining the maximum deflection apply the second theorem Mohr. We calculate tangential deviation on the left end of the spring with respect to the tangent drawn at the point of maximum deflection, which in this case corresponds to L / 2.



$$t_{AB} = \frac{1}{EI}\int_A^B M \cdot x \cdot dx$$

$$Y_{max} = \frac{1}{EI}\int_0^{L/2}\left(\frac{qLx}{2} - \frac{qx^2}{2}\right)\cdot x\cdot dx$$

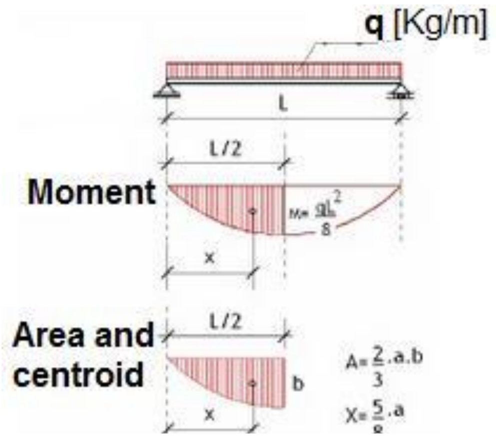
$$Y_{max} = \frac{1}{EI}\int_0^{L/2}\left(\frac{qLx^2}{2} - \frac{qx^3}{2}\right)dx$$

$$Y_{max} = \frac{1}{EI}\Big[\left(\frac{qLx^3}{6} - \frac{qx^4}{8}\right)\Big]_0^{L/2}$$

$$Y_{max} = \frac{qL^4}{48EI} - \frac{qL^4}{128EI}$$

$$Y_{max} = \frac{5qL^4}{384EI} \qquad (3.23)$$

If you know the area and its centroid can perform the calculation as follows:



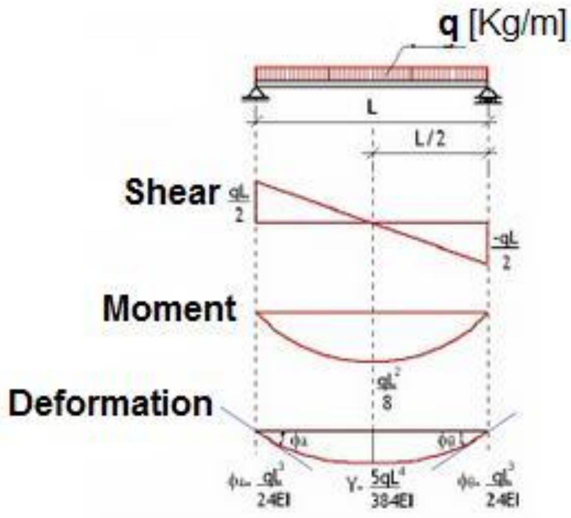
$$t_{AB} = \frac{1}{EI}Area_{AB}\cdot x_A$$

$$t_{AB} = \frac{1}{EI}\frac{qL^2}{8}\frac{2}{3}\frac{L}{2}\frac{5}{8}\frac{L}{2}$$

$$Y_{max} = \frac{5qL^4}{384EI} \qquad (3.24)$$

o Double Integration Method

The differential equation of the elastic beam given by the expression



The moment value of 'x' varies according to the general equation set forth above:

$$Mx = \frac{qLx}{2} - \frac{qx^2}{2}$$

The differential elastic equation for this beam is:

$$EI\frac{d^2y}{dx^2} = \frac{qLx}{2} - \frac{qx^2}{2}$$

Integrating equation we get for every point slope of the elastic

$$EI\frac{dy}{dx} = \int \frac{qLx}{2} - \frac{qx^2}{2}dx$$

$$EI\frac{dy}{dx} = \frac{qLx^2}{4} - \frac{qx^3}{6} + C_1$$

By symmetry is the maximum deflection at the midpoint of the beam, so that the tangent drawn at this point of the elastic slope is zero, namely:

If x=L/2 $\frac{dy}{dx} = 0$

Therefore $0 = \frac{qL}{4}\frac{L^2}{4} - \frac{q}{6}\frac{qL^3}{8} + C_1$

$$C_1 = -\frac{qL^3}{24}$$

Then the general equation of the slope is

$$\phi = \frac{dy}{dx} = \frac{qLx^2}{4EI} - \frac{qx^3}{6EI} - \frac{qL^3}{24EI}$$

The equation of arrow we can obtain integrating the above equation:

$$ELy = \frac{qLx^3}{12} - \frac{qx^4}{24} - \frac{qxL^3}{24} + C_2$$

Under the terms of supporting the arrow is zero when x = 0 or x = L

If x=0 $\qquad C_2 = 0$

If x=L $\qquad 0 = \frac{qLx^3}{12} - \frac{qx^4}{24} - \frac{qxL^3}{24} + C_2$

Therefore $C_2 = 0$

So, the general equation of the arrow is

$$y = \frac{qLx^3}{12} - \frac{qx^4}{24} - \frac{qxL^3}{24} \qquad (3.25)$$

The angles at the supports are obtained by replacing x=0 and x=L in the corresponding equation

$$\phi_A = -\frac{qL^3}{24EI} \qquad (3.26)$$

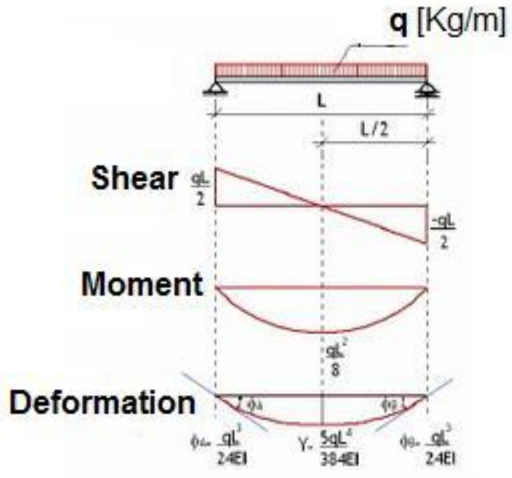
$$\phi_B = \frac{qL^3}{24EI} \qquad (3.27)$$

The maximum arrow replacing in x=L/2

$$Y_{max} = \frac{5qL^4}{384EI} \qquad (3.28)$$

○ <u>Conjugate Beam Method</u>

For the application of this method is necessary to determine the bending moment graph and its characteristic values



$$M_{max} = \frac{qL^2}{8}$$

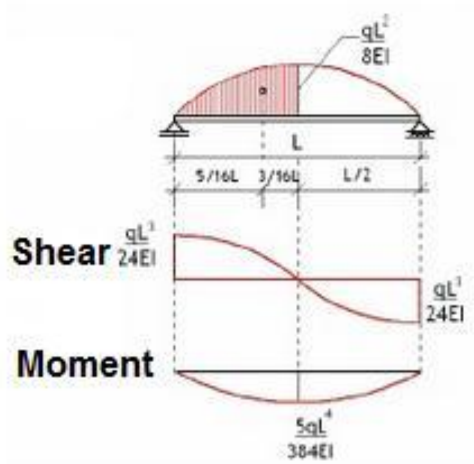
For angle values and generate a beam fictional arrow

Fictitious Beam

Generate a beam and apply as load the bending moment of the beam given divided by EI.

$$q' = \frac{M_{max}}{EI} = \frac{qL^2}{8EI}$$

The shear of the fictitious beam that corresponds to the slope of the tangent drawn to acquire the elastic curve of the real beam, so that the shear graph fictitious beam represents the changes in slope. The angle at the supporting point of the original beam is equal to the fictitious beam reaction.



$$\phi_A = R_{a'} = \frac{qL^2}{8EI}\,\frac{2}{3}\frac{L}{2}$$

$$\phi_A = -\frac{qL^3}{24EI} \qquad (3.29)$$

The bending moment of the fictitious beam corresponds to decline of the real beam when it deformed. In this case, the moment graphic of the fictitious beam deformation values represent the Real beam. As the descent maximum of the beam is at L / 2, we determined the maximum moment of the fictitious beam at that point.

$$Y_{max} = M'_{max} = \frac{qL^3}{24EI}\,\frac{L}{2} - \frac{qL^2}{8EI}\frac{2}{3}\,\frac{L}{2}\frac{3}{8}\frac{L}{2}$$

$$Y_{max} = \frac{5qL^4}{384EI} \qquad (3.30)$$

3.2.2.1 - CERAMIC

We are use as ceramic material, silica which has elastic modulus E= 73GPa and Poisson coefficient ν =0.2.

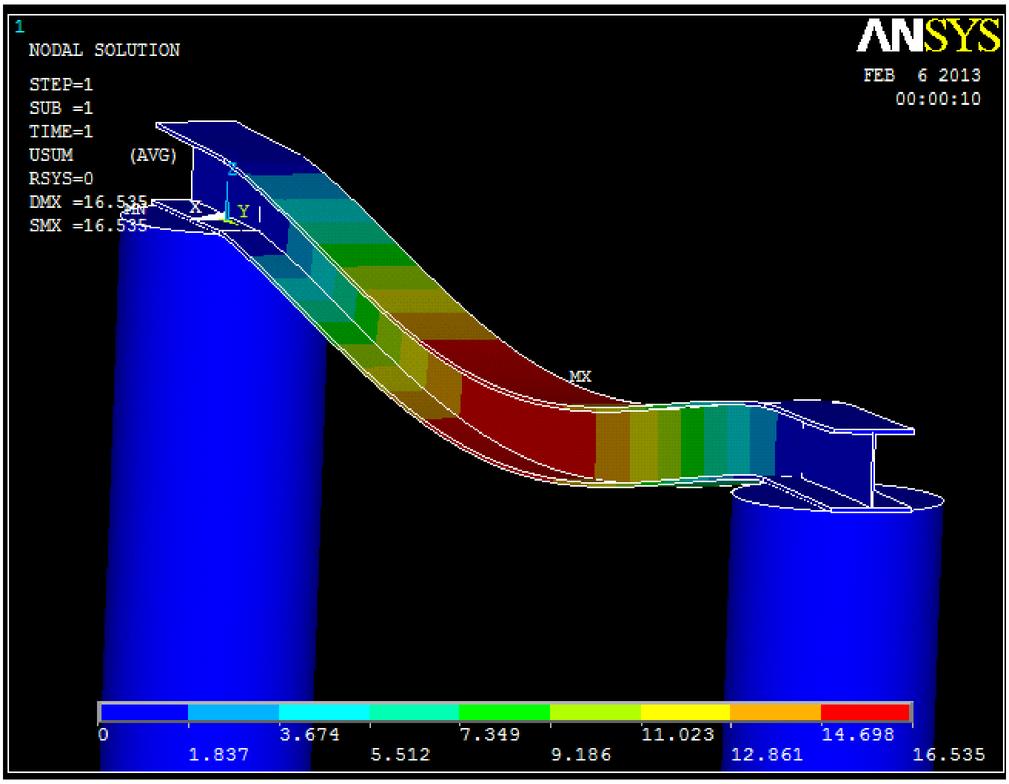


Image 3.30 - Maximum displacement of ceramic beam under boundary conditions indicated

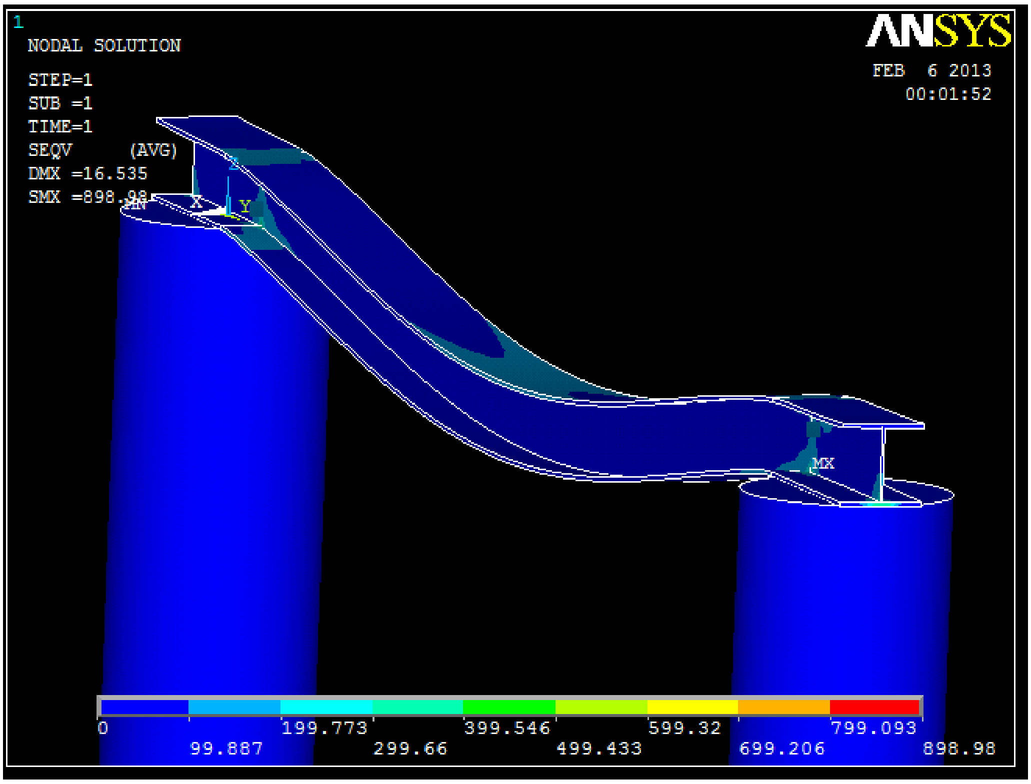


Image 3.31 - Maximum stress of ceramic beam under boundary conditions indicated

3.2.2.2- STEEL

Now, we are use the steel material which has as elastic modulus E=210GPa and Poisson ν Coefficient =0.3.

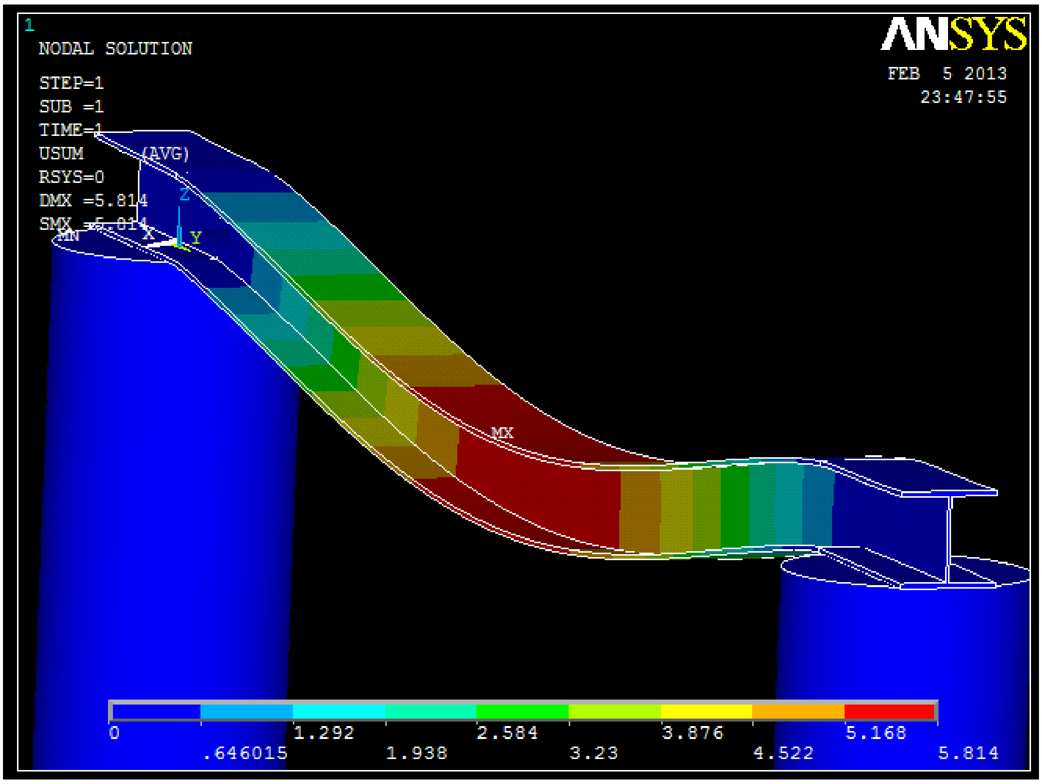


Image 3.32 - Maximum displacement of steel beam under boundary conditions indicated

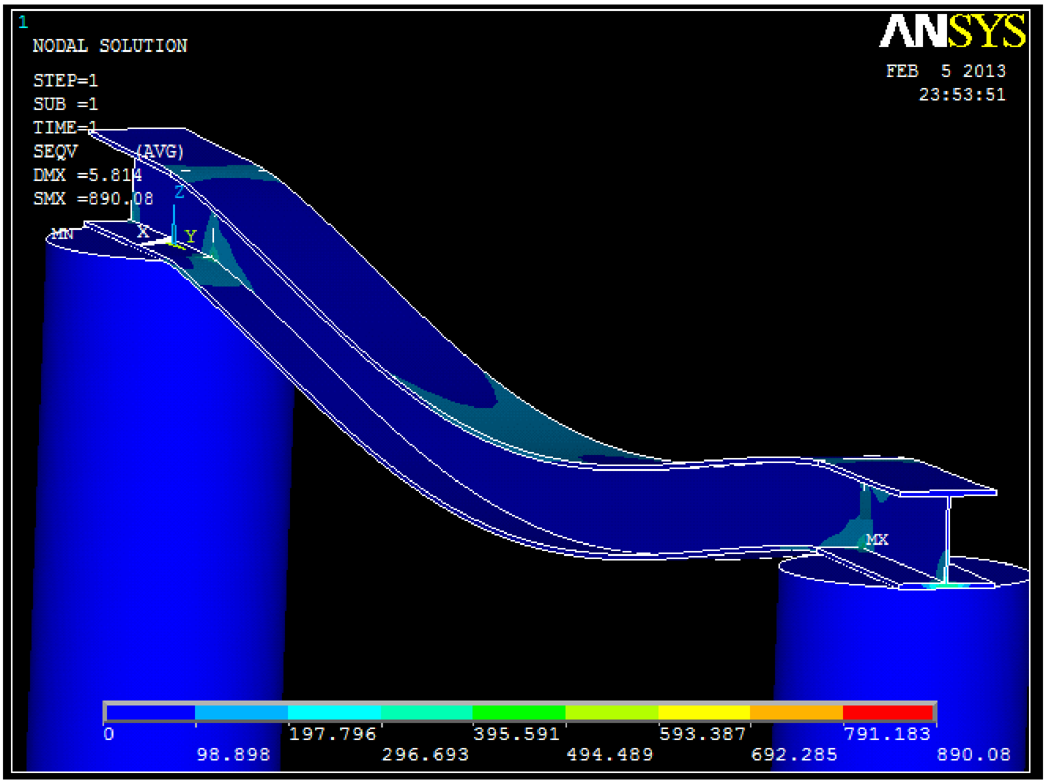


Image 3.33 - Maximum stress of steel beam under boundary conditions indicated

3.2.2.3 – CONCRETE

Now, we are use the concrete material which has as elastic modulus E=27GPa and Poisson Coefficient $\upsilon = 0.2$.

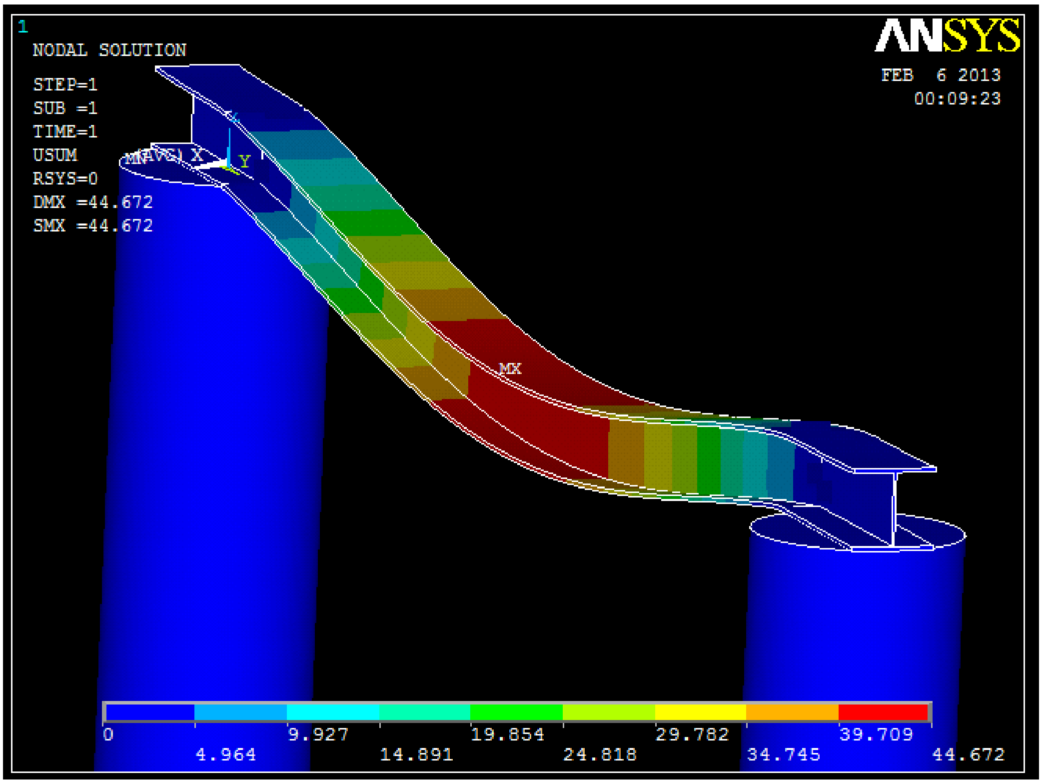


Image 3.34 - Maximum displacement of concrete beam under boundary conditions indicated

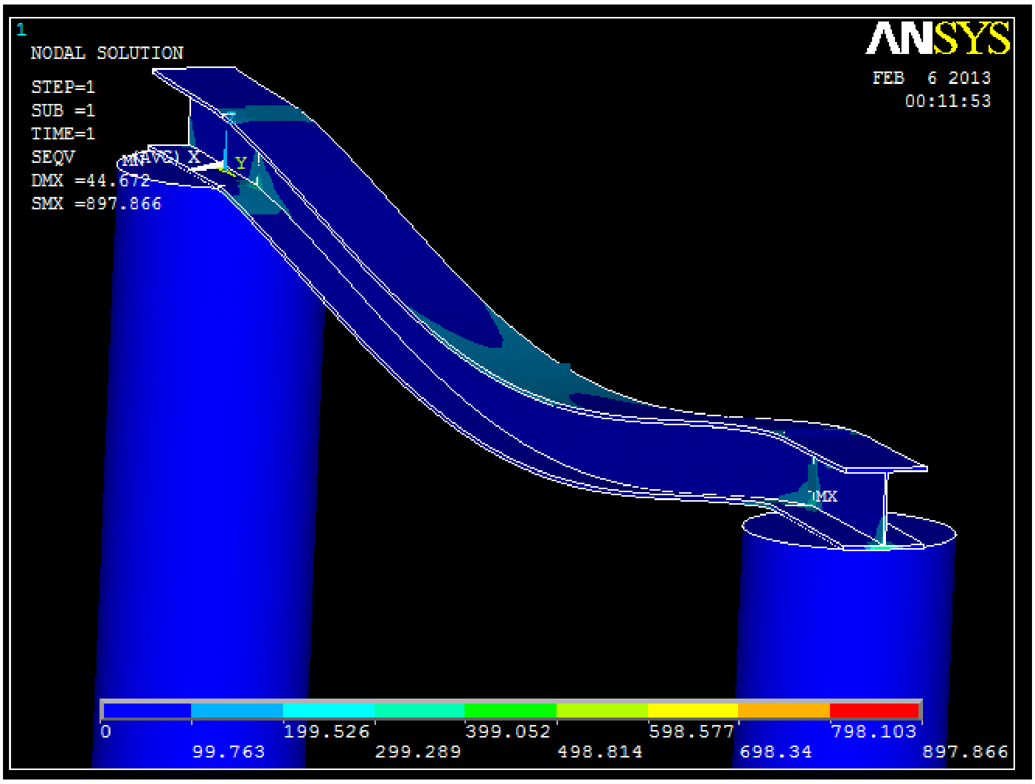


Image 3.35 - Maximum stress of concrete beam under boundary conditions indicated

4 – CONCLUSIONS

We have done the study of different structures constituted by ceramics, steel and concrete. The structures studied have been cantilever beam and beam with two supports for each material. We have calculated the deformation of the beam by the three different methods for each case. The methods are: moment area method, double integration method and conjugate method.

The first step was to design the beam. We looked for the typical beam in construction and we used some standards tables for designing the beam. We choose a double t beam which has 5 meters as length; the other dimensions are explained in the relevant section of the project.

The first structure which we have studied, cantilever beam, we have supposed different loads for each material and we have studied their behavior.

The first load is a load of 1 MPa on the end in the cantilever beam. We have analyzed the behavior with a ceramic material and the result was that it is possible to realize this design because the beam will not break because the result of maximum displacement is less that the maximum displacement possible. We did the same process for steel material and the result was that the beam will resist the load because it will not break for the same reason than the ceramic material. For the concrete material we have done the same and the result was that the beam will break because it is impossible that a beam has a displacement of 70 mm.

But we have to see the value of displacement and the stress to choose the best material in this situation. The material which has the less displacement is the steel material and the material which has the less stress is the ceramic material, but in all conditions we choose the steel material like the best material for this design, because the ceramic material is not very typical for designing the beams and it is more expensive than the steel material which is more typical for designing the beam and is cheaper.

The second load is a continuous load of 1.5MPa on the cantilever beam. We have realized the same process than the other structure. And the result was similar, for the ceramic material and the steel material it is possible this design but for the concrete material it is not possible because the structure will break. So for this structure the best material will be steel again, because it has less maximum displacement and the difference with the maximum stress between ceramic and steel is minimum.

The second structure which we have studied is a beam with two supports and we have supposed different loads for each material and we have studied their behavior.

The first load is a load of 1MPa in the middle on the cantilever beam. The process is the same as above. With this load the result is that it will be possible to realize with ceramic and steel material but with concrete material it will not possible because the structure will break. The best material will be steel material because it is which has the less maximum displacement and the less maximum stress.

The last analysis it is a beam with two supports and a continuous load of 1MPa on the cantilever beam. The result for this structure is the same than the last analyze, but we choose the steel material because with this material the structure has less maximum displacement than the other materials, although with the steel material has more maximum stress than with ceramic material.

For summary of the all analyzes, we can say that the best material for this structures is the steel material which always has the minimum of maximum displacement. In some situation has more maximum stress than ceramic material, but it is the best. The ceramic material is good too but it is more expensive than steel material and it is less usual in this aspect of construction. The worst material like we can see it is concrete material because always has the greatest of maximum displacement and the greatest of maximum stress.

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