

**ENVIRONMENTAL COSTS OF  
RESIDUALS:  
A CHARACTERIZATION OF  
EFFICIENT TAX POLICIES**

Carlos Ocaña Pérez de Tudela  
and  
Joan Pasqual i Rocabert  
92 - 01



WORKING PAPERS

WORKING PAPER 92-01  
January 1992

División de Economía  
Universidad Carlos III de Madrid  
28903 Getafe (Spain)

ENVIRONMENTAL COSTS OF RESIDUALS:  
A CHARACTERIZATION OF EFFICIENT TAX POLICIES

Carlos Ocaña Pérez de Tudela<sup>1</sup>  
and  
Joan Pasqual i Rocabert<sup>2</sup>

ABSTRACT

Durable goods leave residuals after being retired from use. If the environmental costs of the residuals are external to consumers and producers of the good, then overproduction and excess residuals will result. Ad valorem taxes are shown to be ineffective in eliminating this externality. The efficient regulatory policy is shown to be based on a pigouvian tax.

---

**KEYWORDS:** Externalities, Pollution Control, Optimal Taxation, Durable Goods, Residuals.

---

<sup>1</sup> Departamento de Economía. Universidad Carlos III de Madrid.

<sup>2</sup> Departament d'Economia Aplicada. Universitat Autònoma de Barcelona.

## 1.- INTRODUCTION.

Most durable goods leave a residual after being retired from use. Dead batteries and used paper plates, for instance, are residuals. The amount of residuals generated depends on the average life of the good, i.e., on its durability. In turn, the average life of a good is, in many cases, a decision variable controlled to some extent by the producer of the good. Car makers, within some limits, choose the (expected) duration of the cars they make. Likewise, both glass and plastic cups, render similar services while having different durations. Thus, when a firm chooses the average life of its products, it is also indirectly choosing the amount and composition of the residuals that will be generated.

There are two broad categories of costs related to the production and consumption of goods that leave residuals. First, there are production costs. In general, increasing the average duration of a good is costly. Thus, production costs are positively related to the average life of the good<sup>1</sup>.

Second, there are costs originated by the existence of residuals. Residuals have to be recycled, or disposed one way or another. Furthermore, after it has been disposed, a residual may also generate pollution. We refer to the total costs of recycling, disposal and pollution as environmental

---

<sup>1</sup> This is true for the unit cost of production of the good. The cost per unit of service provided, however, may not be related to duration in a monotonous fashion.

costs. Environmental costs are inversely related to the average life of the good. For a given level of total consumption, longer-lived goods will generate less residuals.

While production costs are incurred by the firm, it is less than clear who pays for the environmental costs. Typically, the producer of a durable good assumes no responsibility for the elimination of the residuals of its products. Consumers, in turn, may bear some -typically small- disposal costs. Recycling and pollution costs, however, are not usually paid by the consumers. Throughout this paper it will be a maintained assumption that the costs related to the existence of residuals are not incurred by neither the producers nor the consumers of the good. This generates an externality. Since producers and consumers ignore the costs originated by the residuals, a tendency towards the production of inefficiently short-lived goods can be expected. In other words, there will be too many residuals<sup>2</sup>.

The purpose of this paper is to characterize tax policies that provide an incentive for firms and consumers to produce and buy goods of efficient duration. The framework of analysis is the standard theory of externalities and optimal

---

<sup>2</sup> A similar effect is caused by Planned Obsolescence, Bulow (1986). There is Planned Obsolescence when a firm chooses to produce goods of a duration below the minimum cost duration in order to increase sales. In our analysis, however, strategic behavior is not considered.

pollution control; see Pearce and Turner (1990), Baumol and Oates (1988), Mishan (1982), and Pigou (1920). Our model extends this analysis to a dynamic environment in which firms choose the duration of their products.

Since the average life of a good is not observable *ex ante*, we will only consider tax policies which do not depend on the actual durability of the good taxed. Our main point is simply this: for a tax policy to be effective in reducing residuals it must penalize repeated purchasing of the good. Thus, an efficient tax must be linked to the purchase of the good. The price or the value of the good, which are commonly used as a basis for taxation, are not useful in penalizing repeated purchasing. In summary, a pigouvian tax is also optimal in this dynamic context.

There are two direct consequences of this result. First, an "ad valorem" tax must be relatively ineffective in reducing residuals because it is linked to the total expenditure in the good (but not to the number of units bought). Second, an "specific" tax, one that is charged per unit bought, must be relatively more effective in reducing residuals because it penalizes repeated purchasing of the good.

This line of reasoning is formalized in a simple model of a durable good market; all consumers are assumed to value the good by the services it provides, regardless of its age or

duration. We consider two alternative market structures, monopoly and perfect competition. The same results obtain in both cases.

It is shown that, in the absence of a tax policy, the equilibrium duration of the good is too short and, in consequence, too many residuals are generated. Next, we consider the effect on equilibrium duration of two different tax systems. "Ad valorem" taxes are shown to be neutral in the sense that no incentive is provided to change the equilibrium duration of the goods produced. Thus, an "ad valorem" tax cannot be used as a cure for inefficiently short-lived goods. It is also found that "specific" taxes, paid per unit bought, are potentially non-neutral, and therefore can be used to induce production of longer-lived goods. A characterization of the optimal specific tax is provided: the amount charged per unit of product must be equal to its environmental cost, i.e., a pigouvian tax is optimal.

To summarize, the logic behind these results is simply this. The durability of a good is directly related to the number of times it is bought (and thus replaced). A specific tax exploits this link by penalizing repeated purchases. This creates an incentive to produce and buy longer-lived products. On the other hand, the total expenditure in the good, in equilibrium, does not depend on its durability; since an ad valorem tax is charged on the total expenditure

in the good, it must be neutral with respect to the duration of the good.

The neutrality of the ad valorem tax suggests that it can be used, without distorting the choice of duration, to correct other market imperfections. An example of the optimal regulatory policy for a monopoly will be developed below.

The rest of the paper is organized as follows. First, we provide an example of a two period model in which the external effect of residuals and the basics of the model can be more easily understood. Next, section 3, efficiency in the general model is characterized. Section 4 discusses consumers' behavior; and section 5 gives the characterization of optimal regulatory policies. Finally, section 6 contains some concluding remarks.

## 2.- AN EXAMPLE.

It is convenient to consider first an example with a simplified intertemporal structure. Let's consider a good that can be manufactured to last either 1 or 2 periods. Production costs are  $C(1)$  and  $C(2)$ , respectively. As long as the good remains operative, the services it provides are identical, regardless of age or duration. Thus, there are two alternatives to obtain the services of the good. It is possible to produce the short-lived good for two consecutive periods at a discounted production cost of:

$$TC(1)=C(1)(2+r)/(1+r)$$

where  $r$  is the discount rate. It is also possible to produce the long-lived good at a cost of  $C(2)$ .

The environmental cost of the good, which is independent of its duration, is  $E$ . Thus, the total (social) costs incurred to obtain the services of the good for two periods can be:

$$CC(1)= (C(1)+E)(2+r)/(1+r)$$

or 
$$CC(2)= C(2)+E$$

An efficient duration must minimize the total cost of obtaining the services of the good. Thus, it is efficient to produce the long-lived good if

$$(*) \quad (C(1)+E)/(1+r) \geq C(2)-C(1)$$

i.e., if the total cost of producing one unit of the short-lived good in the second period is larger than the incremental cost of increasing the life of the good from one to two periods.

Let  $P(1)$  and  $P(2)$  be the prices of the short-lived and long-lived good, respectively. The total expenditures required to obtain the services of the good for two periods are:



$$X(1)=P(1)(2+r)/(1+r)$$

and  $X(2)=P(2)$ .

Consumers are indexed by  $m > 0$ , where  $m$  is the reservation value of consumer  $m$ . Thus, consumer  $m$  will demand the short-lived good if  $m \leq X(1) \leq X(2)$ , the long-lived good if  $m \leq X(2) \leq X(1)$ , and will choose not to buy otherwise. We will suppose in this example that consumers' reservation values are distributed uniformly on the interval  $[0, A]$ .

Suppose (\*) does not hold. Then, an efficient allocation requires to produce the good of duration 1; it must be offered at a price equal to its cost, i.e.,  $P(1)=C(1)+E$ , and a total of  $(A-CC(1))$  units must be produced each period. Otherwise, the long-lived good must be produced; its price must satisfy  $P(2)=C(2)+E$ , and a total of  $(A-CC(2))$  units must be produced.

Firms will choose a duration to minimize their total private costs. This choice is independent of the number of firms offering the product. Minimization of production costs will lead to choose the short duration if:

$$(**) \quad C(1)(2+r)/(1+r) \leq C(2)$$

and the long duration otherwise. Thus, if firms ignore environmental costs, inefficiently short-lived goods may be produced. From (\*) and (\*\*) it follows that, if

$0 \leq (1+r)(C(2)-C(1))-C(1) < E$ , it is efficient to manufacture the good to last for two periods but firms will choose a duration of only one period.

Under perfect competition, the price of each duration will be set equal to the private cost of production,  $P(1)=C(1)$  and  $P(2)=C(2)$ . This price is below the true marginal cost. In each case, a total of  $(A-C(1)(2+r)/(1+r))$  and  $(A-C(2))$  units will be produced, respectively. Thus, too many units will be produced.

A monopolist, on the other hand, would choose a price

$$P(1) = [A(1+r)/(2+r) + C(1)]/2$$

if (\*\*) holds, and would set

$$P(2) = (A + C(2))/2$$

otherwise. At these prices, actual production may be above or below the efficient levels, but is always above the equilibrium levels that would obtain should the monopolist take into account the true (marginal) cost.

### 3.- EFFICIENCY.

Consider the market for a durable good  $D$ . Let  $d \in [0, \infty)$  denote the number of years that the good will be in service. While in service, every good generates the same services, regardless of its durability  $d$  or age.

Let  $C(d)$  be the cost of producing one unit of  $D$  that will last  $d$  years.  $C(\cdot)$  is twice continuously differentiable,  $C'(\cdot) > 0$  and  $C''(\cdot) > 0$ . Thus increasing the average duration of the good is costly and the cost increases at an increasing rate. Furthermore, it seems natural to assume that  $C(\infty) = \infty$  and  $C(0) = 0$ .

In order to characterize an efficient duration, we must consider total costs per unit of service provided (instead of the cost per unit of product). We must consider, for instance, the cost per mile of a car, and not the cost of the car itself. Thus, we define the cost of maintaining one unit of duration  $d$  in service forever is:

$$(1) \quad TC(d) = C(d) \sum_{t=1, \dots, \infty} e^{-rdt} = (1 - e^{-rd})^{-1} C(d)$$

where  $r$  is the discount rate.  $TC(d)$  is the discounted production cost of replacing the good every  $d$  years.

Let  $E > 0$  be the environmental cost of one unit of  $D$ , i.e.,  $E$  is the cost of recycling or disposing of one unit of  $D$ .  $E$  is assumed to be independent of the duration  $d$  of the good.

The total environmental cost of maintaining one unit of duration  $d$  in service forever is:

$$(2) \quad TE(d) = E \sum_{t=1, \dots, \infty} e^{-rdt} = (1 - e^{-rd})^{-1} E$$

which is a decreasing function of the duration  $d$ .

Total costs are thus:

$$(3) \quad CC(d) = TC(d) + TE(d)$$

Since, by assumption, the age and duration of the good do not change the quality of the services it provides, an efficient duration  $d^*$  is simply one that minimizes total costs.

$d^*$  can be characterized as follows:

PROPOSITION 1:  $d^*$  is the unique solution of

$$(4) \quad C'(d^*) = re^{-rd^*} CC(d^*).$$

PROOF: Equation (4) is the first order necessary condition obtained from the definition of  $CC(\cdot)$ . The function  $CC(\cdot)$  is not necessarily convex. However, it can be show that it reaches a unique minimum as follows: to characterize  $d^*$ , notice that

$\lim_{d \rightarrow \infty} CC(d) = \lim_{d \rightarrow 0} CC(d) = \infty$ . Hence,  $CC(\cdot)$  must reach a minimum for some  $d^*$  strictly positive and finite. Therefore, a necessary condition for  $d$  to be efficient is that  $CC'(d^*) = 0$ , that is,  $C'(d^*) = re^{-rd^*} CC(d^*)$ . Furthermore, if  $d^*$

solves (1) then  $CC''(d^*) > 0$  so that any solution of (1) is at least a local minimum and there are no local maxima. Continuity of  $CC(\cdot)$  then requires the local minimum to be unique. ////.

Notice that the efficient duration can be defined without regard to demand factors. This is so because, in our model, consumers are concerned with the services provided by the good, not with the good itself. This is a simplifying assumption. There are durable goods, however, which do not conform to the assumption. New cars may be preferred to old cars (even after controlling for the difference in their expected operative lives). This possibility is simply assumed away along this paper.

Let's examine now the choice of duration of a cost-minimizing firm that ignores the social costs of the residuals.

PROPOSITION 2: In the absence of a tax policy inefficiently short-lived goods will be produced.

PROOF: The firm will select a duration  $d'$  to minimize  $TC(d)$ . The first order condition for this problem can be written as:

$$C'(d') = re^{-rd'}(CC(d') - (1 - e^{-rd'})^{-1}E)$$

which implies that  $C'(d') - re^{-rd'}CC(d') < 0$  (i.e.,  $CC(d)$  is decreasing at  $d'$ ). It was show in proposition 1 that  $CC(d)$  is decreasing if, and only if,  $d < d^*$ . Hence  $d' < d^*$  ///.

This result confirms our intuition that, if environmental costs are ignored, then there will be too many residuals. The result holds, with independence of the particular market structure considered, as long as each minimizes its private cost.

#### 4.- CONSUMERS' BEHAVIOR.

Next, we model the demand side of the market. Let  $p(d)$  be the price of a good of duration  $d$ . The total expenditure required to maintain one unit of  $d$  in stock forever is:

$$(5) \quad X(d) = p(d) \sum_{t=1, \dots, \infty} e^{-rdt} = (1 - e^{-rd})^{-1} p(d)$$

Consumers are assumed to be infinitely lived. There is a total of  $N$  consumers and each one demands either 0 or 1 units of the good at every instant  $t$ . We denote consumers by their reservation values  $m$ ; reservation values are distributed on some interval  $[0, M]$ . The cumulative distribution of consumers reservation values is  $F(m)$ . We assume that  $F(\cdot)$  is strictly increasing on the interval  $[0, M]$ . The inverse function of  $F(\cdot)$  is denoted by  $H(\cdot)$ . Hence  $H(F(m)) = m$ .

Consumers will demand the duration  $d$  of smallest required expenditure. Thus, a consumer with a reservation value  $m$  will demand a duration  $d$  such that  $X(d) \leq X(d')$  for every  $d'$ , provided that  $X(d) \leq m$ . If the smallest total expenditure  $X(d)$  is larger than the reservation value  $m$ , then consumer  $m$  will choose not to buy.

Suppose that  $d$  is the duration of smallest required expenditure. Then a total of:

$$N - F((1 - e^{-rd})^{-1}p(d))$$

consumers will demand this duration; and the remaining consumers will choose not to buy. Since the good has to be replaced every  $d$  years, the (average) demand per year is simply:

$$(6) \quad Q(d) = [N - F((1 - e^{-rd})^{-1}p(d))] / d$$

The timing of sales is arbitrary in this model. For simplicity, we have chosen to consider that sales are concentrated at dates  $0, d, 2d, \dots$ . However our results do not depend on the particular demand pattern considered.

## 5.- OPTIMAL TAX POLICIES.

We will be concerned with a particularly simple class of tax policies, which do not depend on the actual duration of the good. This seems to be a realistic requirement for a tax policy since there would be large costs involved in the administration of a duration-dependent tax. Obtaining information on the average life of a product is a costly activity; furthermore, in many instances such information is only available *ex post*.

It will be show that tax policies in the class considered are enough to induce the choice of an efficient duration. Thus, there is no loss of generality in disregarding duration-dependent tax policies.

Specifically, a tax policy is defined as a pair of numbers  $(g,b)$  such that  $g$  is the per unit "ad valorem" tax, and  $b$  is the per unit "specific" tax. The particular case  $g=b=0$  refers to the case of no regulation.

The total private cost of producing one unit of duration  $d$ , net of taxes, is:

$$(7) \quad NC(d) = (1 - e^{-rd})^{-1} (C(d) + b)$$

Notice that the net cost is independent of the *ad valorem* tax. From this observation, and comparing the expressions for  $NC(d)$  and  $CC(d)$ , the following results are obtained:



PROPOSITION 3: i) the value of  $d$  chosen by the firm is unaffected by  $t$ , i.e., an ad valorem tax is neutral with respect to the duration of the good and the amount of residuals produced.

ii) If  $b=E$ , then the firm will choose to produce the optimal duration  $d^*$ .

PROOF: i)  $NC(d)$  does not depend on  $g$ .

ii) Setting  $b=E$ , it follows that  $NC(d)=CC(d)$ . Hence, both functions reach a minimum at  $d^*$ ./../.

A full characterization of the optimal policy requires to specify the concrete structure of the market. We consider in this section the benchmark of a perfectly competitive market. Under perfect competition the equilibrium industry profits, defined by:

$$(8) \quad B(d, P(d), t, b) = (1 - e^{-rd})^{-1} ((1 - g)P(d) - b - C(d)) (N - F(X(d))),$$

$$\text{where } X(d) = (1 - e^{-rd})^{-1} p(d),$$

are equal to zero. Unless the market is closed, so that  $N=F(X(d))$ , the zero profit condition implies that:

$$(9) \quad (1 - g)P(d) = b + C(d)$$

Efficiency in the choice of  $d$  requires to set  $b=E$ . Efficiency in the choice of an industry output level requires

price to be set equal to the (social) marginal cost  $(E+C(d^*))$ . Hence we have the following:

PROPOSITION 4: Under perfect competition the optimal tax policy is  $g=0$  and  $b=E$ .

According to Proposition 4, efficient durations can be obtained by means of a linear tax policy. Our intuition regarding the neutrality (and ineffectiveness) of "ad valorem" taxation is confirmed; a pigouvian tax, charging the average external cost, is all that is needed to restore efficient residual levels.

The neutrality of an *ad valorem* tax suggests that the optimal policy in an imperfectly competitive market will retain, in essence, the simple structure of the policy of Proposition 4. This is illustrated for the case of a monopoly.

The profits of the monopolist are identical to industry profits under perfect competition. It will be assumed that the profit function is concave, so that the behavior of the monopolist can be characterized by means of first order conditions. Optimal policies are then as follows:

PROPOSITION 5: The efficient tax policy for a monopoly is:

$$(10) \quad b=E$$

$$(11) \quad g = [1 - (N - F(Y))^{-1} (1 - e^{-rd^*})^{-1} (E + C(d^*)) f(Y)]^{-1}$$

where  $Y = (1 - e^{-rd^*})^{-1} (E + C(d^*))$

PROOF:  $b=E$  follows from proposition 3. The choice of price must satisfy the necessary condition for a maximum:

$$(1-g)(1-e^{-rd})^{-1}(N-F(X(d))) - (1-e^{-rd})^{-2}[(1-g)P(d) - b - C(d)]f(X(d)) = 0$$

Output levels are efficient if price equals marginal cost, i.e.,  $P(d^*) = E + C(d^*)$ . Substituting this last condition in the first order condition given above, and solving for  $g$ , the result follows immediately.///.

Notice that the optimal value of  $g$ , characterized in the proposition will generally be negative, i.e., an *ad valorem* subsidy may be optimal.

In summary, an *ad valorem* tax affects prices and quantities but is neutral with respect to duration. Thus, a *specific* tax can be used to induce the choice of an efficient duration, and then an *ad valorem* tax or subsidy can be used to affect the price. The analysis of a value added tax would yield similar results.

## 6.- CONCLUDING REMARKS.

To keep the arguments straight, it has been assumed throughout this paper that duration is the only relevant variable in determining environmental costs; likewise, only one instrument of public policy -taxation- has been considered. In this simplified context, the lack of effectivity of most common forms of taxation in controlling residuals pollution can be more easily highlighted.

In practice, the regulatory problem is a bit more complex. The quality of the residuals generated is as important as the quantity. Recycling costs, for instance, are highly sensitive to the composition of the residuals. Likewise, the amount of pollution generated depends on the chemical properties of the residuals. The quality of residuals poses similar problems to those related to the quantity of residuals. Through the choice of a technology, firms indirectly choose the quality of the residuals. Environmental costs are, once again, mostly external to the firm. Hence, a tendency towards an inefficiently low quality may be expected.

Tax policies are relatively ineffective in motivating the adoption of technologies that generate less costly residuals. This is so because criteria of measurement of the quality of the residuals have to be developed for each product and/or product component. Under these circumstances, direct

regulation, in the form of imposed quality standards, may be more effective.

On the other hand, direct regulation does not seem appropriate in motivating extended product durability because the expected life of a product is not *ex ante* observable. Thus, it seems that an efficient policy must adopt a combination of fiscal and direct instruments. Among the first, there are some options not considered in this paper. Of particular interest are discounts and subsidies offered to stimulate the recycling of residuals.

A problem related to the one considered in this paper is the lack of incentives for the firms to invest in research on new -less polluting- technologies. Technological opportunities are taken as given in our model. The environmental benefits of a new technology are external to the users of the technology. Thus, if technologies have to be developed, too little investment in R&D can be expected in the absence of regulation. An optimal tax policy, in this case, must take into account the changing value of the environmental costs.

A final extension of the basic model could consider environmental costs in a general equilibrium framework. The basic insight to be obtained by considering the whole economy is that, as some goods are overproduced, others will be underproduced. The overproduction of some durable goods

implies that less resources will be devoted to the production of non-durable and non-polluting durable. If, for instance, cars production decreases, demand and resources may shift to public transportation.

In consequence, a regulatory policy can be expected to have a primary and a secondary effect. The primary effect is to reduce production in the regulated market. The secondary effect is to increase production in other related markets.

#### REFERENCES

BAUMOL, W. (1988) and W. OATES. "The Theory of Environmental Policy". Cambridge University Press. Cambridge.

BULOW, J. (1986). "An Economic Theory of Planned Obsolescence". Quarterly Journal of Economics, 51: 729-750.

MISHAN, E.J. (1982). "Cost-Benefit Analysis". Allen and Unwin.

PEARCE, D. and K. TURNER (1990). "Economics of Natural Resources and the Environment". Harvester Wheatsheaf. New York.

PIGOU, A. (1920). "The Economics of Welfare". Macmillan. London.