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ABSTRACT

Endogenous Capacities and Price Competition: The Role of Demand Uncertainty

This paper analyzes a model of capacity choice followed by price competition under demand uncertainty. Under various assumptions regarding the nature and timing of demand realizations, we obtain general predictions concerning the role of demand uncertainty on equilibrium outcomes. We show that it reduces the multiplicity of equilibria, it may rule out the existence of symmetric equilibria, and it leads to endogenous capacity asymmetries even though firms are ex-ante symmetric. Furthermore, as compared to the certainty equivalent game, demand uncertainty reduces prices and increases consumer surplus, but it also decreases total welfare because of the emergence of idle capacity. By relying on the analysis of firms' reaction functions as well as on the theory of submodular games, we are able to show that a subgame perfect equilibrium always exists and to fully characterize it.

JEL Classification: D43, D80 and L11
Keywords: demand uncertainty, investment, price competition and submodular game

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1 Introduction

In this paper we analyze the effect of introducing demand uncertainty into a model of capacity investments followed by price competition in the product market. In particular, we assume that firms are uncertain about future demand conditions at the investment stage, but are able to observe realized demand prior to competing in prices. We characterize the subgame perfect equilibria of the game under two approaches regarding demand uncertainty, which is modelled as either a discrete or as a continuous random variable. Our analysis focuses on two main issues: "uniqueness versus multiplicity of equilibria" and "existence versus non-existence of symmetric equilibria."

In the absence of demand uncertainty, there exists a continuum of pure-strategy equilibria in all of which total capacity equals total demand. The symmetric candidate equilibrium belongs to the equilibrium set since: (a) reducing capacity would further constrain production without increasing prices and, (b) increasing capacity would not allow the deviant to expand its production as it would just serve residual demand.

The introduction of demand uncertainty has distinct effects depending on the way demand uncertainty is modelled. Under the discrete approach, the pure-strategy symmetric equilibrium survives the introduction of uncertainty as long as none of the demand states is sufficiently likely, and there always exist multiple pure-strategy equilibria that involve asymmetric capacity choices. In contrast, under the continuous approach, equilibrium multiplicity vanishes out and the symmetric equilibrium disappears.

Accordingly, by associating the continuous approach to a more uncertain environment (in the sense that a continuum rather than a finite number of demand states is potentially likely), we find that uncertainty is at the heart of the uniqueness of equilibrium and the non-existence of symmetric equilibria. Intuitively, faced with uncertainty, firms find it more difficult to coordinate on certain equilibria, among which we find the symmetric one.

Irrespective of how demand uncertainty is modelled, a robust conclusion of the analysis is that capacity asymmetries arise endogenously even though firms are ex-ante identical. Such asymmetric market outcomes derive from asymmetries in the returns to investment for large and small firms. In particular, whereas capacity expansions by the large firm only affect its profits when the firm is capacity constrained, capacity expansions by the small firm affect pricing incentives, and thus profits, even when the small firm is selling below capacity. The small firm is discouraged from matching its rival's capacity since that would induce more aggressive pricing from the large firm. Whenever these asymmetries imply that pay-off functions are not differentiable at symmetric capacity pairs, firms' best reply functions do not cross the 45 degree line, and the symmetric equilibrium disappears.

Last, the analysis allows to assess other effects of demand uncertainty, besides reducing multiplicity or ruling out symmetric equilibria. In comparison to the certainty equivalent game, demand uncertainty drives prices down and increases consumer surplus, despite reducing total welfare. The positive effect on prices and consumer surplus is due to the emergence of idle capacity under demand uncertainty, which in turn explains the negative effect on overall welfare.

The analysis of investment choice models followed by product market competition dates back to Kreps and Scheinkman's (1983) seminal paper, which shows that price competition and Cournot outcomes can be reconciled by assuming that firms choose their production capacities prior to engaging in price competition.[1] Although Kreps and Scheinkman (1983) did not analyze the effects of introducing demand uncertainty into the model, they conjectured that *"noise" in the demand function will change this* [equivalence result] *dramatically* (p.337). Since then, several papers have formally analyzed this conjecture.

Similarly to us, Reynolds and Wilson (2000) analyze a game of capacity choice followed by price competition, and model demand as a random variable. Assuming a downward sloping demand, they provide a sufficient and necessary condition for the existence of a symmetric pure-strategy equilibrium in capacity choices, but do not provide a characterization of the equilibria for general demand functions nor demand distributions.[2] Assuming linear demand and a binomial distribution function, they characterize the symmetric (if the extent of demand variation is not too large) and asymmetric pure strategy equilibria of the game. We view our formulation as a complement to Reynolds and Wilson's (2000) in that it improves in terms of generality and tractability at the cost of assuming inelastic demand.[3]

[1] Several papers have assessed the robustness of Kreps and Scheinkman's result to alternative model specifications, such as the rationing rule (Deneckere and Kovenock (1996); Herk (1993)), the pricing rule (Moreno and Úbeda (2006)), the timing of capacity choices (Allen *et al.* (2000); Kovenock and Roy (1998)), the existence of firm cost asymmetries (Deneckere and Kovenock (1996)), or the frequency of firms' interaction in the product market (Benoit and Krishna (1987); Davidson and Deneckere (1986); Staiger and Wolak (1992)), among others.

[2] They note that *a characterization of pure strategy equilibria for general demand functions and demand shock distributions is a challenging problem* (p.131) and conjecture that *there may be restrictions on parameters or functional forms that would allow one to apply the submodular games approach* (p. 132). Price inelasticity, as in our paper, turns out to be one of those.

[3] The assumption of inelastic demand not only makes the analysis tractable. It also has the advantage that the efficient-rationing rule (as in Kreps and Scheinkman (1983)) and the proportional-rationing rule (as in Davidson and Deneckere (1986)) become equivalent. Hence, the lack of robustness of Kreps and Scheinkman's results to the choice of rationing rule does not apply in this context. Several papers that deal

Gabszewicz and Poddar (1997) and Grimm and Zoettl (2006) also consider future demand uncertainty at the investment stage, but assume that firms subsequently compete by choosing quantities rather than prices. The Cournot assumption has important implications on investment incentives and equilibrium outcomes. First, since the (unconstrained) Cournot equilibrium is symmetric for all firms, capacity expansions only affect a firm's profits for those demand realizations at which the firm is selling at capacity. Second, since this implies that the returns to firms' investments are symmetric for large and small firms, the pay-off functions are differentiable at symmetric capacity pairs. Last, with symmetric firms ex-ante and continuous marginal returns to investment, the best reply functions are both symmetric and continuous. Thus, the existence of a symmetric equilibrium in capacity choices is guaranteed.

Our model is also related to Klemperer and Meyer's (1989) seminal paper in which firms facing uncertain demand compete by choosing supply functions.[4] They find that the introduction of demand uncertainty dramatically shrinks the set of Nash equilibria, and conclude that the equilibrium is unique only if the support of the demand distribution function is unbounded. Even though our approach substantially differs from theirs,[5] it is not surprising that our conclusions concerning the effect of demand uncertainty are similar. In both scenarios, demand uncertainty reduces the multiplicity of equilibria since it forces each firm's strategic decision to be optimal against a range of possible demand functions.

From a methodological point of view, our paper is also related to a broad family of analysis which make use of submodularity to prove existence of equilibria. In this respect, ours is similar to Amir and Wooders (2000), who consider a two stage game in which firms first invest in R&D activities that generate spillovers, and then compete in the product market. They show that ex-ante identical firms always engage in different levels of R&D, thus giving rise to asymmetrically sized firms. Applying similar techniques as Amir and Wooders (2000), we show that if the demand distribution function is convex, our game is submodular (i.e. the returns to investment are non-increasing in the rival's capacity choice), allowing us to provide an additional proof of existence of the subgame perfect equilibria. The

with competition under capacity constraints have also adopted the same type of demand we use in this paper (see for instance Compte, Jenny and Rey (2002) and Dechenaux and Kovenock (2003)).

[4]Grant and Quiggin (1996) endogenize capacities within the supply function approach under the assumptions of Cobb-Douglas technology and constant elasticity demand. They focus only on the symmetric equilibrium of the game, whose existence is guaranteed for similar reasons as under the Cournot assumption.

[5]The most important one is probably the fact that, in the supply function approach, firms choose a continuum of price-quantity pairs, whereas in our set-up firms choose a single quantity and a single price, and they do it in different stages.

feature that distinguishes our analysis and theirs is that, whereas they assume submodularity (see assumption A2 in their paper), we are able to pin down the relevant feature of the game (i.e., convexity of the distribution function) which is sufficient to guarantee that the profit function is submodular.

The structure of the paper is as follows. Section 2 describes the model, which we solve backwards in sections 3 (price competition) and 4 (capacity choices). In order to understand the role played by demand uncertainty, section 4 analyzes the game in which demand is known with certainty and then explores two approaches to modelling demand uncertainty: the discrete and the continuous approach. Section 5 explores the effects on equilibrium pricing and investment behavior of changing the timing of demand uncertainty. The last section concludes with a discussion and summary of the main results. The Appendix contains the proofs of the main results of the paper.

2 The Model

We consider a two-stage non-cooperative game between two symmetric firms, $i = 1, 2$. In the first stage of the game, firms simultaneously choose their capacities k_i, $i = 1, 2$, at a constant per-unit cost $c \in (0, 1)$. We let $k^- = \min(k_1, k_2) \leq k^+ = \max(k_1, k_2)$, and refer to the firm with capacity k^- or k^+ as "the small firm" or the "the large firm", respectively. Once investment decisions have been made, information about capacities becomes public knowledge. In the second stage of the game, firms compete in prices to sell an homogenous good, subject to the constraint that each firm's production cannot exceed its capacity limit. We assume that production entails constant marginal costs, and w.l.o.g. normalize them to zero.

There is a mass θ of infinitesimal buyers, each willing to buy one unit as long as the price does not exceed the reservation price, normalized to one. Consumers buy first from the firm with the low price until its capacity has been exhausted. The residual demand faced by the high-priced firm equals total demand minus the capacity of its rival. If firms' prices are equal, consumers split equally between the two firms.[6] Each firm sells its production at its own price.

Demand uncertainty is introduced between the first and second stages of the game. More specifically, firms face uncertain demand at the investment stage, knowing that the number

[6] Since in our setting all consumers have the same value, rationing is not an issue, and the results are independent of the rationing rules used. They are also independent of the tie-breaking rule.

of consumers, θ, will be drawn (before prices are set) from a cumulative distribution function $G(\theta)$. Finally, firms are assumed to be risk neutral and to maximize expected profits.

We proceed by backwards induction in order to find the subgame perfect Nash equilibria of the overall game. Each firm's strategy is a pair specifying its capacity choice and, contingent on realized demand θ, a distribution function over prices given both firms' capacities.

3 Price Competition

In this section, we characterize the equilibrium in the price competition stage. Recall that firms know both the realized value of demand, as well as the capacities chosen in the first stage of the game. The nature of the equilibrium is similar as in the standard capacity-constrained price competition framework (e.g. Osborne and Pitchik (1986)), with the difference being that we assume price-inelastic demand.

Proposition 1 *For given θ and given capacities $k^- \leq k^+$, equilibrium pricing is characterized as follows:*

(i) (Region I) If $\theta \leq k^-$, there exists a unique pure-strategy equilibrium in which both firms set prices equal to (zero) marginal cost and make zero profits.

(ii) (Region II) If $k^- < \theta < k^- + k^+$, a pure strategy equilibrium fails to exist. In the unique mixed strategy equilibrium, the large firm makes expected profits $[\theta - k^-]$, whereas the small firm makes a fraction $\frac{k^-}{\min\{\theta, k^+\}}$ of the large firm's profits.

(iii) (Region III) If $\theta \geq k^- + k^+$, both firms set prices equal to consumers' valuation (which equals 1), and they both sell at capacity.

Proof. See Fabra, von der Fehr and Harbord (2006)'s proof of Proposition 2. ■

Equilibrium pricing behavior depends on the relationship between demand and capacities. For demand realizations in Region I, since both firms have enough capacity to serve total demand, competition drives prices down to marginal cost, and firms make zero profits. For demand realizations in Region III, since there is not enough aggregate capacity to cover demand, the equilibrium price equals consumers' valuation and both firms sell at capacity. For the remaining demand realizations, pure-strategy equilibria fail to exist given that either (i) firms want to price slightly below the rival to sell at capacity at a high price or (ii) want to serve the residual demand at consumers' reservation price. For a given demand realization in Region II, there exists a unique mixed strategy equilibrium such that the two firms mix over a common support, with a lower bound strictly above zero and an upper bound equal to consumers' reservation price. Since the large firm plays a mass point at the upper bound, and since the small firm is pricing below that level with probability one, the large firm's expected profits are the same *as if* it maximized its profits over its residual demand.

To sum up, the first stage capacity choices, together with realized demand, will determine whether competition in the second stage of the game will be *à la* Bertrand (with firms pricing at marginal cost in Region I), *à la* Bertrand-Edgeworth (with firms mixing in Region II) or *à la* Cournot (with firms producing at capacity in Region III).[7]

4 Capacity Choices

In this section we endogenize capacities. As a benchmark, we first consider the case in which demand is known with certainty, and then proceed to introducing demand uncertainty into the model.

4.1 Demand Certainty

We first assume that demand is known at the investment stage. The following Proposition characterizes equilibrium capacity choices.

Proposition 2 *Assume that demand is known to be equal to θ. There is a continuum of pure-strategy subgame perfect equilibria. Specifically, every profile of firms' capacities with $k^+ \in \left(\frac{\theta}{2}, \frac{\theta}{2-c}\right]$ and $k^- = \theta - k^+$ can be sustained by a pure-strategy subgame perfect equilibrium. Hence, there are asymmetric equilibria as well as a symmetric one. Furthermore, in every pure-strategy subgame perfect equilibrium aggregate capacity equals θ.*

Proof. It is a particular case of the proof of Proposition 3 below. ■

Under demand certainty, there exists a continuum of pure-strategy equilibria, in all of which aggregate capacity is equal to total demand, as illustrated by Figure 1. Hence, in any equilibrium capacity is fully utilized and prices are set equal to consumers' reservation value. The only restriction imposed on the set of equilibria is that the large firm need not be too large, since otherwise the small firm would be better off increasing its capacity even at the expense of driving prices below the reservation price. Since the symmetric capacity pair satisfies this condition, it constitutes an equilibrium. Intuitively, the symmetric equilibrium

[7] As shown by Dechenaux and Kovenock (2003), the equilibria in the pricing game induce equivalent outcomes as those that would arise if we allowed firms to costlessly choose the maximum quantity that each is willing to sell at the quoted price, subject to their capacity limits.

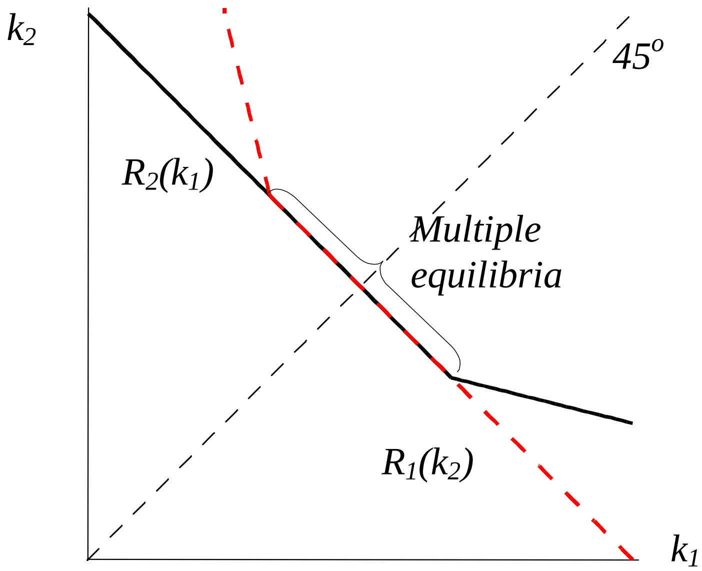

Figure 1: Firms' Reaction Functions at the Investment Stage: certain demand

is sustained by two forces: (a) a firm is discouraged from *reducing* its capacity since this would constrain its production without increasing prices (already set at the reservation level); and (b) a firm is also discouraged from *increasing* its capacity since, while it is costly, it would not lead to an increase in its production (the deviant would become the large firm, so it would be selling the residual demand without exhausting its capacity).

4.2 Discrete Demand Uncertainty

Let us now assume that demand is uncertain at the investment stage. We first model demand uncertainty as deriving from a binomial distribution function.

Proposition 3 *Suppose that demand takes the value $\theta^L > 0$ with probability $\rho \in [0, 1]$ and θ^H with probability $1 - \rho$. Let $\Delta = \frac{\theta^H}{\theta^L} \geq 1$. There exists $\underline{\rho} \in \left(\frac{1-c}{2}, 1 - c\right)$ such that:*

(i) Symmetric equilibria in capacity choices exist if and only if $\rho \in [0, \underline{\rho}) \cup (1 - c, 1]$. For all ρ, there exists a continuum of asymmetric equilibria in capacity choices.

(ii) In any pure-strategy equilibrium, aggregate capacity is θ^L if $\rho \in (1 - c, 1]$, and is θ^H otherwise.

Proof. See the Appendix. ■

Similarly to the certain demand case, the investment game under discrete demand uncertainty generates a continuum of pure-strategy equilibria. These equilibria induce two distinct types of outcomes.

On the one hand, if the probability of the low demand state is sufficiently large (i.e., $\rho \in (1-c, 1])$, firms behave as if demand was known to be low with certainty. In particular, equilibrium aggregate capacity equals low demand, capacity is fully utilized, and prices are equal to consumers' reservation value independently of which demand state is finally realized. Furthermore, for all values of ρ in this range, there exists a symmetric equilibrium in capacity choices. This is so for similar reasons as under demand certainty: (a) capacity reductions would further constrain production without increasing prices, and (b) capacity expansions would not allow the deviant to increase its production in the low demand state (since it would just serve the residual demand), and the marginal gain associated with the increase in production in the high demand state, $1-\rho$, would not cover the extra investment costs, c.

On the other hand, if the probability of the low demand state is sufficiently small (i.e., $\rho \in (0, 1-c])$, firms behave in a similar fashion as when demand is high with certainty. Since aggregate equilibrium capacity is equal to high demand, capacity is fully utilized and prices are at its maximum only when demand turns out to be high. Otherwise, if realized demand is low, there is excess capacity and prices are below their reservation level. This fact implies that a symmetric equilibrium may not exist for some ρ values over this range. In particular, point (a) above may no longer hold: since for low demand firms operate below capacity, a capacity reduction would not constrain the deviant's production but it would lead to higher prices. Clearly, the associated marginal gain has to be balanced against the marginal loss that the deviant suffers from having to reduce its production under high demand, and this ultimately depends on the relative incidence of low and high demand. When low demand is sufficiently probable (i.e. $\rho \in \left[\underline{\rho}, 1-c\right]$), the marginal gain from reducing capacity exceeds its marginal cost, and therefore destroys the candidate symmetric equilibrium. Comparative statics of the asymmetric equilibria show that they approach the symmetric equilibrium as ρ approaches $\underline{\rho}$.

Comparison with the certain demand case allows to derive interesting conclusions regarding the role of demand uncertainty as an equilibrium selection device. If we perturb the certain demand case by introducing demand uncertainty, we obtain distinct conclusions depending on whether we decrease ρ below 1 or raise it above 0. In the first case, demand uncertainty reduces the multiplicity of equilibria, whereas in the second case, it widens it.[8] The intuition runs as follows. When we decrease ρ below 1, we are adding a state with higher

[8]This is true independently of whether we move ρ alone, or on whether we keep expected demand constant by either reducing θ^L as we decrease ρ, or by increasing θ^H as we increase ρ.

demand. Since the small firm would then like to increase her capacity in order to produce more when demand is high, she has to be given a bigger share of aggregate capacity (fixed at θ^L). This shrinks the equilibrium set. In the opposite case, when we increase ρ above 0, we are adding a state with lower demand. Now, since the problem is to discourage the large firm from reducing her capacity, the large firm has to be compensated with a bigger share of aggregate capacity (fixed at θ^H). This expands the equilibrium set.

Last, note that independently of the value of ρ, equilibrium capacity choices are never unique, and that, as long as the change in ρ with respect to the certain demand case is not large enough, the symmetric equilibrium survives the introduction of uncertainty.

However, as we will show in the following section, these two issues - "uniqueness versus multiplicity of equilibria" and "existence versus non-existence of symmetric equilibria" - depend not only on the "amount" of uncertainty (however measured) but also on the way demand uncertainty is modelled. In particular, taking a discrete or a continuous approach to modelling demand uncertainty is not innocuous. In contrast to the discrete approach analyzed so far, the continuous approach shows that the multiplicity of equilibria disappears, and the symmetric equilibrium disappears with it, as we spread probability over a compact set of demand values.

4.3 Continuous Demand Uncertainty

To conclude this section, we assume that demand is distributed according to a continuous distribution function, $G(\theta)$, with full support on $[0,1]$. Based on Proposition 1, we can construct firms' expected profit function at the investment stage as a function of their capacity choices,

$$\pi_i(k_i,k_j) = \begin{cases} \pi^-(k^-,k^+) & \text{if} \quad k_i \leq k_j \\ \pi^+(k^+,k^-) & \text{if} \quad k_i \geq k_j \end{cases} \tag{1}$$

where,[9]

$$\pi^-\left(k^-,k^+\right) = \int_{k^-}^{k^-+k^+} \frac{k^-}{\min\{\theta,k^+\}}\left[\theta-k^-\right]dG(\theta) + \int_{k^-+k^+}^{1} k^- dG(\theta) - ck^- \tag{2}$$

$$\pi^+\left(k^+,k^-\right) = \int_{k^-}^{k^-+k^+} \left[\theta-k^-\right]dG(\theta) + \int_{k^-+k^+}^{1} k^+ dG(\theta) - ck^+ \tag{3}$$

[9]Note that we are implicitly assuming that $k^- + k^+ \leq 1$. It can easily be shown that $k^- + k^+ > 1$ would never constitute a subgame perfect equilibrium. The reason is that the large firm would never sell at capacity, so that it would be better off by reducing its capacity to the point at which aggregate capacity no longer exceeds the maximum demand realization.

Capacity choices affect the value of firms' profits for a given demand realization.[10] For demand realizations in Region I (i.e., below k^-), profits are zero independently of the value of firms' capacities, whereas for demand realizations in Region III (i.e., above $k^- + k^+$), firms' profits are fully determined by their capacity choices. The link between capacity choices and profits becomes more complex for demand realizations in Region II (i.e., in the interval $(k^-, k^- + k^+)$). Over this range, the large firm's profits do not depend on its own capacity, as these are the same as if it served the residual demand with probability one. In contrast, the small firm's profits depend on its own capacity choice, for two reasons: first, it constrains its sales when it prices below the rival; and second, it affects its rival's pricing behavior, ultimately determining its chances of selling at capacity.

The expected profit function $\pi_i(k_i, k_j)$ is everywhere continuous in k_i. Nevertheless, it is not differentiable at symmetric capacity pairs. In particular, along the diagonal, the right-hand derivative is larger than the left-hand derivative. This non-differentiability stems from the asymmetric effects of marginal increases in capacities across firms in Region II: whereas the large firm gains nothing by expanding its capacity, an increase in the small firm's capacity may lead to either a profit gain or a profit loss depending on the strength of the two effects involved: an increase in its capacity allows it to expand its production when it prices below the rival; however, as this also makes the large firm more aggressive,[11] the probability that this occurs is reduced.

The next lemma summarizes some properties of the expected profit function, (1).

Lemma 1 *Suppose that demand is distributed according to a continuous distribution function $G(\theta)$ with full support on $[0,1]$.*

(i) The following are sufficient conditions for the expected profit function to be piece-wise concave: either $G(\theta)$ is convex, or $G(\theta)$ is concave and $G'(\theta)$ is convex.

(ii) The second-order cross derivative of the large firm is negative for all $G(\theta)$.

(iii) The following are sufficient conditions for the second-order cross derivative of the small firm to be negative: either if $G(\theta)$ is convex, or $G(\theta)$ is concave and $k^+ > 2k^-$.

Proof. See the Appendix. ■

[10] Capacity choices also affect the distribution of equilibrium profits. However, the effect of marginal increases in capacity on profits is null since the profit function in continuous in θ.

[11] In more detail, the large firm plays a mass point at the reservation price with probability $1 - \frac{k^-}{\min(\theta, k^+)}$. Hence, the higher k^-, the less likely it is that the large firm prices at the upper bound of the support of firms' mixed strategies.

The first part of the Lemma guarantees that under certain weak conditions on the shape of the demand distribution function,[12] the problem is well-behaved in the sense that both the large and the small firm's expected profit functions are concave. The second part allows to conclude that investments are strategic substitutes for the large firm, whereas the third part identifies the properties of the demand distribution function that make this also true for the small firm. The intuition for these results runs as follows.

An increase in the large firm's capacity allows to expand its production for demand realizations in Region III (i.e. when demand exceeds aggregate capacity). Since the relative incidence of demand realizations in Region III is lower the bigger the small firm, the large firm's marginal returns to investment are decreasing in the small firm's capacity. Hence, conditionally on being the large firm, capacity investments are strategic substitutes irrespectively of how demand is distributed.

Similarly, an increase in the small firm's capacity allows to expand its production in Region III. This effect alone would imply that the small firm's profits exhibit decreasing marginal returns to investment as the large firm's capacity is increased. However, since an increase in the small firm's capacity also affects pricing behavior for demand realizations in Region II (i.e. above the small firm's capacity but below aggregate capacity), it has two additional effects. On the one hand, as the small firm expands its capacity, it increases the probability of being undercut, and the loss in production (i.e., from selling at capacity to serving the residual demand) is greater the bigger the large firm. On the other hand, the probability that the small firm sells at capacity, and therefore benefits from capacity expansions, is increasing in the large firm's capacity, given that the large firm prices less aggressively the bigger its own capacity. Hence, the small firm's marginal returns to investment may increase or decrease depending on the strength of these three effects, an issue which in turn depends on the shape of the demand distribution function as well as on firms' relative sizes. With convex distribution functions, which put more weight on larger demand values, the first two effects dominate, so that capacity investments are strategic substitutes from the small firm's perspective. This is also true with concave distribution functions as long as firms are sufficiently asymmetric. Otherwise, investments may become strategic complements for the small firm.

These properties have implications for the shape of firms' reaction functions. When the demand distribution is convex, both the large and the small firm's best reply functions

[12]These properties are satisfied by a large family of distribution functions. For instance, to name just a few, $G(\theta) = \theta^x$, or $G(\theta) = \frac{1-e^{-x\theta}}{1-e^{-x}}$, independently of the value of x.

are continuous decreasing functions except at one point. Furthermore, around the point of discontinuity, expected profits exhibit non-increasing differences in capacities. Hence, since marginal returns to increasing capacities do not increase with the rival's choice, we can apply the theory of submodular games to the capacity investment game in order to ensure existence of equilibria (Topkins 1979).

Proposition 4 *If demand is distributed according to a convex cdf with full support on* $[0,1]$, *the capacity game is submodular, and hence has a pure strategy Nash equilibrium.*

Proof. See the Appendix. ■

In contrast, if the demand distribution is concave, the large firm's reaction function is negatively sloped, but the slope of the small firm's reaction function may become positively sloped for some capacity values. Since this implies that the small firm's marginal returns to expanding capacity may increase with the rival's choice, we cannot apply submodularity to ensure existence of pure strategy equilibria. Nevertheless, independently of the shape of the demand distribution function, we can guarantee existence of pure-strategy equilibria, as stated in the following proposition.

Proposition 5 *Suppose that demand is distributed according to a cdf,* $G(\theta)$, *with full support on* $[0,1]$. *The following statements hold for the capacity game:*

(i) *Every pure-strategy Nash equilibrium in capacity choices is asymmetric.*[13]

(ii) *If the second order condition (6) is satisfied, then it holds that: a) best reply functions are continuous everywhere except at one point,* $\hat{k} \in (0,1)$, *where* $R_i^+(\hat{k}) > \hat{k} > R_i^-(\hat{k})$, *and b) there exists a unique subgame perfect equilibrium outcome that involves asymmetric capacity choices. Specifically, in equilibrium one firm invests* k^+ *and the other invests* k^-, *with* $k^+ > k^-$, *and* $k^- + k^+ = G^{-1}(1-c) \subset (0,1)$.

Proof. See the Appendix. ■

The proof of Proposition 5 relies on the analysis of firms' best reaction functions. Independently of the shape of the demand distribution function, the best reaction functions,

[13] There also exists a symmetric mixed strategy equilibrium, in which firms randomly choose capacities. The lower bound in the support of firms' strategies is given by the pure-strategy capacity choice of the small firm, whereas the upper bound is strictly below the pure-strategy capacity choice of the large firm. Note that even if the equilibrium is symmetric, firms would still end up asymmetric with positive probability, despite being fully symmetric ex-ante. The symmetric mixed strategy equilibrium is Pareto dominated by any of the two pure-strategy asymmetric equilibria from the point of view of firms' profits.

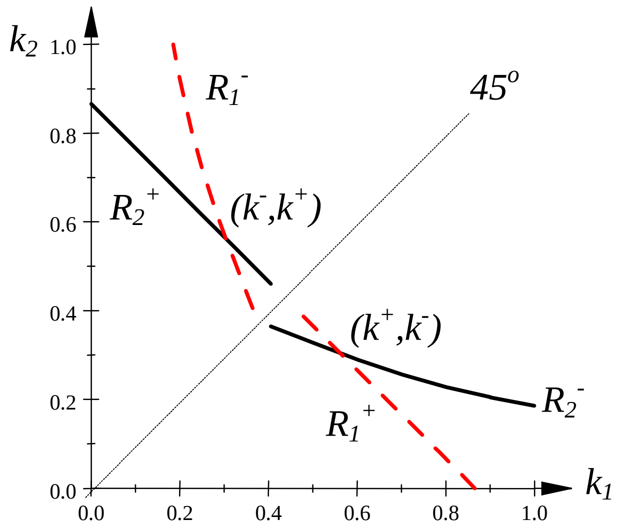

Figure 2: Firms' Reaction Functions at the Investment Stage: continuous uncertainty

which are discontinuous at $\hat{k}$, never cross the 45 degree line given that the marginal returns to investment are asymmetric along the diagonal. This rules out the existence of symmetric equilibria in capacity choices and implies that every pure-strategy equilibrium has to involve asymmetric capacity choices. Indeed, since firms' reaction functions cross twice outside the discontinuity region, $k^- < \hat{k} < k^+$, there exists a unique subgame perfect equilibrium outcome in which the large firm invests k^+ and the small firm invests k^-.[14] Intuitively, the asymmetric endogenous market structure is sustained by the fact that the small firm is discouraged from becoming larger since, if demand falls in Region II, it would make the large firm price more aggressively, thereby reducing its chances of selling at capacity. Figure 2 illustrates firms' reaction functions under continuous demand uncertainty.

Proposition 5 above further shows that, independently of which equilibrium is played, aggregate capacity equals $G^{-1}(1-c)$. Hence, aggregate investment crucially depends on investment costs, as well as on the shape of the distribution of demand uncertainty. In particular, as the degree of convexity of G increases, aggregate equilibrium capacity, $k^- + k^+$, goes up. Firms react by expanding aggregate capacity as the degree of convexity in the distribution function goes up since it implies that larger (smaller) demand realizations become more (less) likely.

We conclude this section by comparing equilibrium outcomes in the game with continuous

[14] Strictly speaking, there exist exactly two asymmetric equilibria, (k^-, k^+) and (k^+, k^-), which only differ in the identity of the large and the small firm. Therefore, these two equilibria are outcome equivalent.

demand uncertainty with those that arise in the certainty equivalent game, i.e. the game in which demand is known to be equal to expected demand in the uncertain game, $E[\theta] = \int_0^1 \theta dG(\theta)$.

Proposition 6 *Comparison of subgame perfect equilibrium outcomes in the game with continuous demand uncertainty versus its certainty equivalent shows that,*

(i) Aggregate capacity is larger under demand uncertainty if and only if $c \in (0, \widehat{c})$, where $\widehat{c}$ is implicitly defined by $G^{-1}(1-\widehat{c}) = E[\theta]$.

(ii) Prices are lower and consumer surplus is higher under demand uncertainty.

(iii) Total welfare is lower under demand uncertainty.

Proof. See the Appendix. ■

Under demand uncertainty, a marginal increase in the large firm's capacity allows it to sell more output at the reservation price whenever demand exceeds aggregate capacity, but it implies an additional investment cost, c. Hence, an increase in c reduces investment, and may ultimately lead to very low investment levels as c approaches consumers' reservation price. In contrast, under demand certainty, firms invest just enough so as to cover demand irrespectively of the unit cost of capacity (as long as it does not exceed consumers' reservation value). Therefore, demand uncertainty generates more investment as compared to the certainty equivalent game if and only if c is sufficiently low with respect to expected demand.

Furthermore, under demand uncertainty, the emergence of idle capacity for some demand realizations drives prices below the reservation price, allowing consumers to retain a positive share of total surplus. However, the emergence of unused capacity also implies that total welfare is reduced since firms could have saved on investment costs ex-post. This contrasts with the certainty equivalent game, which provides more efficient outcomes at the cost of driving consumer surplus to zero.

5 Prices are set before demand uncertainty is resolved

In this section we explore a variation on the main model. In particular, we change the timing of demand uncertainty and assume that demand is realized after (rather than before) prices are set. This formulation has three alternative interpretations which yield mathematical equivalent results. First, demand could simply be uncertain at the pricing stage. Alternatively, demand could be known but it is not possible to change prices as frequently as demand conditions vary (for instance, due to the existence of menu costs, seasonal brochures, etc.). Last, there could exist a fringe of price-taking firms whose supply is stochastic; hence, even if total demand is fixed and known with certainty, the residual demand faced by the strategic players is uncertain at the pricing stage.

Since demand is random at the pricing stage, we need to provide a new characterization of the equilibrium. Consider first the pricing stage. Trivially, if the small firm chooses to be large enough so that its capacity always exceeds the largest possible demand realization, firms would always set prices equal to marginal costs. However, and precisely for this reason, this case would never arise as a subgame perfect equilibrium. If the small firm is always capacity constrained to serve the market alone, the equilibrium differs substantially from the case in which demand is known with certainty before prices are set. In particular, two forces destroy any candidate pure-strategy equilibrium: on the one hand, a higher price translates into higher profits if demand exceeds aggregate capacity; on the other hand, pricing high reduces a firm's expected sales. Hence, one needs to consider equilibria in mixed strategies (see Fabra, von der Fehr and Harbord, 2006).

We find that the large firm's expected profits do not depend on whether demand is realized before or after prices are set. The small firm's profits do however depend on the timing of demand uncertainty, but they still preserve the main feature that accounts for the non-existence of a symmetric equilibrium in capacity choices under continuous demand uncertainty. Namely, around a symmetric capacity pair, the small firm's marginal returns to investment are lower than those of the large firm, since the small firm takes into account that an increase in its capacity would induce a more aggressive pricing behavior by its rival.

Since a full characterization of equilibrium capacity choices when demand is realized after prices are set is out of the scope of the paper, we limit ourselves here to providing the equilibrium characterization under the assumption of uniformly distributed demand.

Lemma 2 *Assume that demand is uniformly distributed on the unit interval and that it is realized after prices are set. Then, there exist a unique subgame perfect equilibrium outcome in which one firm invests k^+ and the other invests k^+, with $k^+ > k^-$ and $k^- + k^+ = 1 - c$. Specifically,*

$$k^+ = \frac{1}{2}\left[\sqrt{3c^2 + 4c + 2} - 3c\right] \text{ and } k^- = [1 - c] - k^+.$$

Proof. See the Appendix. ■

A straightforward comparison of equilibrium outcomes when prices are set either before or after uncertainty is resolved shows that, under the assumption of uniformly distributed demand, aggregate capacity is the same, but prices and profits are higher when firms compete in prices without knowing demand.

6 Conclusions

We have analyzed a game in which firms take investment decisions under demand uncertainty, and then compete in prices subject to capacity constraints. In order to understand the role played by demand uncertainty, we have first characterized equilibrium capacity choices under demand certainty. Demand certainty guarantees the existence of a symmetric equilibrium in capacity choices at which each firm invests just enough so as to serve one half of the market. Nevertheless, there also exist a continuum of asymmetric equilibria in all of which firms' capacities sum up to total market demand.

In contrast, the introduction of demand uncertainty has important implications on equilibrium investment choices, as it may rule out the existence of symmetric equilibria and reduce the multiplicity of equilibria. Intuitively, demand uncertainty strengthens firms' incentives to deviate from a candidate equilibrium, as it gives rise to demand states at which firms' aggregate capacity does not coincide with market demand. If there is excess capacity, firms may have unilateral incentives to cut down on investments, whereas is there is excess demand, firms may have gains from expanding investments. However, while deviating by expanding (contracting) capacity may increase a firm's profit whenever there is excess demand (supply), it also depresses profits in all remaining states. Therefore, for deviations to be profitable, they have to be fined tuned so that the marginal losses do not offset the marginal gains from deviating. Since discreteness in the grid of demand states may stop a large number of deviations from being profitable, the discrete approach to modelling demand uncertainty delivers multiple equilibria. This contrasts with the continuous approach, which predicts a unique equilibrium outcome.

A robust conclusion of the analysis is that the investment incentives induced by price competition give rise to asymmetric marginal returns to investment, which ultimately lead to asymmetric equilibria in capacity choices. Unlike the large firm's capacity, changes in the small firm's capacity have crucial effects on firms' pricing incentives. In particular, a marginal increase in the small firm's capacity would induce more aggressive pricing by the large firm, thereby reducing the small firm's chances of selling at capacity. This result does not depend on how demand uncertainty is modelled and it is also robust to changing the timing of demand uncertainty.

Our formulation contributes to the existing literature on capacity choices and imperfect competition under demand uncertainty in several respects. First, in contrast to the papers that assume Cournot competition, our approach conforms the widely accepted view that firms compete in prices subject to capacity constraints. Second, as compared to the papers that analyze investment decisions followed by Bertrand competition, our approach is appealing in terms of generality and tractability, as it provides a characterization of pure-strategy equilibria for general demand distributions and allows to make use of powerful results within the theory of submodular games. Last, our model is able to generate relevant predictions regarding market structure and the sensitivity of investment decisions and pricing behavior to measurable variables, such as the shape of the demand distribution function, or to market characteristics, such as the timing of demand uncertainty.

Our analysis may shed light on investment incentives and endogenous market structure in a large set of industries characterized by long-lived assets that involve large sunk cost investments, imperfect competition and demand fluctuations. These features are common to most important industries producing commodities, such as steel, chemicals, cement, or electricity, to name just a few. The main implication of this analysis for the empirical work is that the distribution of past or future demand could be used as an additional determinant of the long-run market structure, and thus market power, in this type of industries.

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7 Appendix

Proof of Proposition 3 Let $k_2 = y$ and consider the best reply by firm one to its rival capacity choices. If $y \leq \min(\theta^L, \frac{\theta^H}{2})$ then firm one expected profit is a continuous piecewise function with two regions. In particular,

$$\pi(k,y) = \begin{cases} \rho \min\left\{\frac{k}{y}\left(\theta^L - k\right), k\right\} + ([1-\rho]\, k - ck & 0 \leq k \leq y \\ \rho\left[\theta^L - y\right] + ([1-\rho] \min(\left\{\theta^H - y, k\right\} - ck & y \leq k \leq \theta^H \end{cases}$$

Assume first $\rho \leq \underline{\rho} < [1-c]$ so that the high demand realization is most likely. In region 2 profits are strictly decreasing for any $k > \theta^H - y$, whereas for $k < \theta^H - y$ profits are strictly increasing as $\rho < 1-c$. Thus local maximum is $\theta^H - y$. In region 1, profits are maximized at $k^*(y) = \frac{1}{2}\left[\theta^L + y\frac{1-c-\rho}{\rho}\right] \cdot$ Since $k^*(y) > y$ for any $\rho \leq \frac{1-c}{2}$, the global maximum and hence the best reply is $\theta^H - y$. If $\rho \in \left(\frac{1-c}{2}, 1-c\right)$ there is $\hat{y} \in \left(\frac{\theta^L}{2}, \theta^L\right)$, $\hat{y} = \rho\frac{\theta^L}{3\rho-1+c}$, such that $k^*\left(\hat{y}\right) = \hat{y}$. Thus, for all $y \leq \hat{y}$ the local maxima is y, and the global maximum is $\theta^H - y$. In contrast, for $y \geq \hat{y}$ the local maxima is k^*. To determine the global maxima we need to compare profits at local maxima in (1) and (2). The difference in profits, $\pi(\theta^H - y, y) - \pi(k^*(y), y)$, is a concave function with a maximum below $\frac{\theta^L}{2}$,[15] so that over the relevant range $y \leq \min\left\{\theta^L, \frac{\theta^H}{2}\right\}$ it is strictly decreasing in y. We need to distinguish two cases: 1). $\min\left\{\theta^L, \frac{\theta^H}{2}\right\} = \theta^L$, i.e., $\Delta > 2$. In this case the difference in profits evaluated at $y = \theta^L$ attains a value of zero iff $\rho = \rho'$, so that the global maximum is $\theta^H - y$ for all $\rho \leq \rho'$, where

$$\rho' = \frac{1-c}{2}\left[1 + \sqrt{\frac{\Delta-2}{\Delta-1}}\right].$$

[15] Note that

$$\frac{\partial\left[D\pi = \pi(\theta^H - y, y) - \pi(k^*(y), y)\right]}{\partial y} = \left[\rho\theta^L\right]^2 - y^2\left([1-c+\rho]\right)^2, \text{ with } \frac{\partial^2\left[D\pi\right]}{\partial y \partial \rho} > 0.$$

As firm two increases further her capacity so that $\theta^L \leq y \leq \frac{\theta^H}{2}$, the maximum in region 1 becomes $\frac{1-c}{2\rho}\theta^L$. The global maximum remains $\theta^H - y$ if $\rho \leq \hat{\rho} < \rho'$, where

$$\hat{\rho}(\Delta) = \frac{1-c}{2}\left[1+\sqrt{\frac{\Delta-2}{\Delta}}\right] \in \left(\frac{1-c}{2}, 1-c\right) \text{ with } \hat{\rho}(2) = \frac{1-c}{2}$$

As $\frac{\theta^H}{2}$ belongs to the best reply, a symmetric equilibria exists for any $\rho \leq \hat{\rho}$. Furthermore, as $\theta^H - y$ remains the global maximum for any $y \leq y^* = \theta^H - \frac{\theta^L[1-c]^2}{4\rho[1-\rho-c]}$ with $y^* \geq \frac{\theta^H}{2}$, there is also a continuum of asymmetric equilibria. In particular, if $\rho \leq \frac{1-c}{2}$ equilibria are pairs $(\theta^H - k^+, k^+)$ with $k^+ \in \left[\frac{\theta^H}{2}, \theta^H - y'(\rho)\right]$. If $\rho \in \left(\frac{1-c}{2}, \hat{\rho}\right]$ there are two disjoint sets of equilibria, one made of pairs $(\theta^H - k^+, k^+)$ with $k^+ \in \left[\theta^H - \frac{1-c}{2\rho}\theta^L, y'(\rho)\right]$, and the other one consisting of pairs $(\theta^H - k^+, k^+)$ with $k^+ \in \left[\frac{\theta^H}{2}, \theta^H - \frac{\theta^L(1-c)^2}{4\rho(1-\rho-c)}\right]$. Note that as $\hat{\rho} \to \frac{1-c}{2}$ we have $\lim_{\rho \to \frac{1-c}{2}} \left[\theta^H - \left(\theta^H - \frac{\theta^L(1-c)^2}{4\rho(1-\rho-c)}\right) - \frac{1-c}{2\rho}\theta^L\right] = 0$ so that the two sets of equilibria become connected and hence we get as equilibria exactly those as when $\rho \leq \frac{1-c}{2}$. Consider now case 2). $\min\left\{\theta^L, \frac{\theta^H}{2}\right\} = \frac{\theta^H}{2}$ so that $\Delta \leq 2$. The difference in profits evaluated at $y = \frac{\theta^H}{2}$ is zero iff $\rho = \tilde{\rho}$ where

$$\tilde{\rho}(\Delta) = (1-c)\left(\frac{\Delta}{3\Delta-2}\right) \in \left(\frac{1-c}{2}, 1-c\right) \text{ with } \tilde{\rho}(2) = \frac{1-c}{2}$$

Thus for any $\rho \leq \tilde{\rho}$ the global maximum is $\theta^H - y$, which ensures the existence of a symmetric equilibrium at $\frac{\theta^H}{2}$. As firm two increases its capacity up to θ^L the global maximum remains $\theta^H - y$ for any $y \leq \frac{(1+\rho)\theta^H - \rho\theta^L}{2-c}$, which ensures the existence of a continuum of asymmetric equilibria. Letting $\underline{\rho}$ be equal to $\hat{\rho}$ for $\Delta > 2$ and equal to $\tilde{\rho}$ for $\Delta \leq 2$, we have part i) of proposition follows.

Let now $\rho > 1-c$ so that the low demand realization is most likely. In region 2 profits are strictly decreasing for any k as $\rho > 1-c$. Thus local maximum is y. In region 1, profits are maximized at $k^*(y)$ if it is larger than $\theta^L - y$ and at $\theta^L - y$ otherwise. Since $k^*(y) < \theta^L - y$ for any $y \leq \theta^L \frac{\rho}{1+\rho-c}$, the best reply is $\theta^L - y$ for $0 \leq y \leq \theta^L \frac{\rho}{1+\rho-c}$ and it is $k^*(y)$ for $y \in \left[\theta^L \frac{\rho}{1+\rho-c}, \theta^L\right]$. As $\theta^L \frac{\rho}{1+\rho-c} > \frac{\theta^L}{2}$, a symmetric equilibrium always exist together with a continuum of asymmetric equilibria $(\theta^L - k^+, k^+)$ with $k^+ \in \left(\frac{\theta^L}{2}, \frac{\theta^L \rho}{1+\rho-c}\right]$, which shows ii).

Consider finally $\rho \in \left(\underline{\rho}, 1-c\right)$. Local maximum in region 2 is $\theta^H - y$ as $\rho < 1-c$. In region 1, profits are maximized at $k^*(y)$ which is now larger than $\theta^L - y$ for any $y > \frac{\theta^L}{2}$. Furthermore, there is $\hat{y} \in \left(\frac{\theta^L}{2}, \theta^L\right)$, $\hat{y} = \rho\frac{\theta^L}{3\rho-1+c}$, such that $k^*(\hat{y}) = \hat{y}$. Thus, for all $0 < y \leq \hat{y}$ the local maxima is y, and the global maximum is $\theta^H - y$. To determine the global maxima when $y > \hat{y}$ we must again distinguish two cases.

1). Let $\Delta > 2$. For $\rho \in (\hat{\rho}, \rho')$ the global maximum is $\theta^H - y$ for all $y \in \left[\hat{y}, \theta^L\right]$ as shown above.

As firm two increases further its capacity so that $\theta^L \leq y \leq \frac{\theta^H}{2}$, the maximum in region 1 becomes $\frac{[1-c]}{2\rho}\theta^L$. Comparing profits at the two local maxima, it is straightforward to see that the global maximum is $\theta^H - y$ for any $y \leq \theta^H - \frac{\theta^L(1-c)^2}{4\rho(1-\rho-c)}$ whereas it is $\frac{(1-c)}{2\rho}\theta^L$ otherwise. Since $\theta^H - \frac{\theta^L(1-c)^2}{4\rho(1-\rho-c)} < \frac{\theta^H}{2}$ the best reply does not cross the 45^o line so that a symmetric equilibrium fails to exists.[16] However there is a continuum of asymmetric equilibria. To show this claim we need to characterize the best reply for $y > \frac{\theta^H}{2}$. We first note that the profit function of firm 1 when $\theta \geq y \geq \max\left\{\theta^L, \frac{\theta^H}{2}\right\}$ is a piecewise concave function with three regions given by

$$\pi(k,y) = \begin{cases} \rho\frac{k}{\theta^L}\left(\theta^L - k\right) + (1-\rho)\,k - ck & 0 \leq k < \theta^H - y \\ \rho\min\left\{\frac{k}{\theta^L}\left[\theta^L - k\right], 0\right\} + ((1-\rho)\frac{k}{y}\left(\theta^H - k\right) - ck & \theta^H - y \leq k \leq y \\ (1-\rho)\left(\theta^H - y\right) - ck & y \leq k \leq \theta^L. \end{cases}$$

For capacities in region (1) local maximum is $\frac{1-c}{2\rho}\theta^L$ which equals $\theta^H - y$ at $y = \theta^H - \frac{1-c}{2\rho}\theta^L$. For capacities in (2) the local maximum is $\hat{k}(y) = \frac{\theta^L}{2}\left(\frac{\theta^H(1-\rho)+(\rho-c)y}{\theta^L(1-\rho)+y\rho}\right)$ which is decreasing in y.[17] Since $\hat{k}(y) - \left(\theta^H - y\right)$ is increasing in y, and $\hat{k}(\theta^H - \theta^L) - \theta^L = \frac{\theta^L}{2}\left(\frac{\theta^H(1-c-2\rho) - \theta^L(2-c-3\rho)}{\theta^L(1-2\rho)+\theta^H\rho}\right) < 0$ whereas $\hat{k}(\theta^H) = \frac{\theta^L}{2}\frac{(1-c)\theta^H}{\theta^L(1-\rho)+\theta^H\rho} > 0$, there is $y'(\rho)$ such that the local maximum is $\left[\theta^H - y\right]$ if $y \leq y'$ and it is $\hat{k}(y)$ otherwise. Moreover,

$$y'(\rho) = \frac{2\rho\theta^H - \theta^L(2-\rho-c) + \sqrt{\left((2-c-\rho)\,\theta^L\right)^2 + 4\rho\theta^H\left(\rho\left(\theta^H - \theta^L\right)\right) + c\theta^L}}{4\rho}.$$

Consequently, for any $\rho \in \left(\frac{1-c}{2}, 1\right]$, global maximum is $\frac{1-c}{2\rho}\theta^L$ if $y \leq \theta^H - \frac{1-c}{2\rho}\theta^L$, it is $\left[\theta^H - y\right]$ for $\theta^H - \frac{1-c}{2\rho}\theta^L < y < y'(\rho)$ and it is $\hat{k}(y)$ for $y > y'(\rho)$. Discussion above shows that if firm 1 is the large firm, it is an equilibrium to play $\left(k^+, \theta^H - k^+\right)$ for any $k^+ \in \left[\theta^H - \frac{1-c}{2\rho}\theta^L, y'(\rho)\right]$.

2). Assume now $\Delta < 2$. The best reply is a continuous function everywhere except at one point, $y = \bar{y} \in \left(\frac{\theta^L}{2}, \frac{\theta^H}{2}\right)$. Recall that for $\rho \in [\tilde{\rho}, 1-c)$ there is $\bar{y}(\rho) \in \left(\hat{y}, \frac{\theta^H}{2}\right)$ such that the global maximum is $\theta^H - y$ if $\hat{y} \leq y \leq \bar{y}$ and it is k^* for $\bar{y} \leq y \leq \frac{\theta^H}{2}$, where denoting by r to $1-c-\rho$, we have

$$\bar{y}(\rho) = \rho\frac{r\left(2\theta^H - \theta^L\right) + 2\rho\theta^L + 2\sqrt{r\left(\theta^H - \theta^L\right)\left(\theta^H r - (r-\rho)\,\theta^L\right)}}{(1-c+\rho)^2}$$

Since it never crosses the 45^o line, a symmetric equilibrium fails to exists. Nevertheless there is a continuum of asymmetric equilibria each of them involving total capacity equal to θ^H.

[16]The same is true for $\rho > \rho'$ but now the best reply jumps down at a smaller y. In particular the best reply becomes $\theta^H - y$ for $0 \leq y \leq \overline{y}$ and $k^*(y)$ for $\overline{y} \leq y \leq \theta^L$.

[17]Note that $\hat{k}(y) < \theta^L$ as $\hat{k}(y) < \hat{k}\left(\frac{\theta^H}{2}\right) < \theta^L$ for all $\rho > \frac{\Delta(2-c)-4}{3\Delta-4}$. Since $\hat{\rho} > \frac{\Delta(2-c)-4}{3\Delta-4}$ the statement follows.

The set of asymmetric equilibria depends on the value of Δ. In particular if $1 < \Delta < \frac{5-3c}{3-c}$ then it is an equilibrium to play $\left(k^{+}, \theta^{H}-k^{+}\right)$ for any $k^{+} \in \left[\rho\frac{2\theta^{H}-\theta^{L}}{1-c+\rho}, \frac{\rho\left(\theta^{H}-\theta^{L}\right)+\theta^{H}}{2-c}\right]$; if $\frac{5-3c}{3-c} \leq \Delta \leq 3/2$ it is an equilibrium to play any $k^{+} \in \left[\rho\frac{2\theta^{H}-\theta^{L}}{1-c+\rho}, \max(\theta^{L}, y'(\rho))\right]$, and if $\Delta > 3/2$, the asymmetric equilibria are given by $k^{+} \in \left[\rho\frac{2\theta^{H}-\theta^{L}}{1-c+\rho}, y'(\rho)\right]$ if $\rho \in \left(\tilde{\rho}, \frac{1-c}{2}\frac{1}{\Delta-1}\right)$, and by $k^{+} \in \left[\theta^{H} - \frac{1-c}{2\rho}\theta^{L}, y'(\rho)\right]$ if $\rho \in \left(\frac{1-c}{2}\frac{1}{\Delta-1}, 1-c\right)$. *Q.E.D.*

Proof of Lemma 1

The first order derivatives of (2) and (3) are given by,

$$\frac{\partial \pi^{-}}{\partial k^{-}} = \int_{k^{-}}^{k^{-}+k^{+}} \frac{\theta - 2k^{-}}{\min\{\theta, k^{+}\}} dG(\theta) + 1 - G\left(k^{-}+k^{+}\right) - c \tag{4}$$

$$\frac{\partial \pi^{+}}{\partial k^{+}} = 1 - G\left(k^{-}+k^{+}\right) - c \tag{5}$$

(i) The second order derivatives of (2) and (3) are given by,

$$\frac{\partial^{2} \pi^{-}}{\partial\left[k^{-}\right]^{2}} = -\left[2\int_{k^{-}}^{k^{-}+k^{+}} \frac{dG(\theta)}{\min\{\theta, k^{+}\}} + G'\left(k^{-}+k^{+}\right)\frac{k^{-}}{k^{+}} - G'\left(k^{-}\right)\right] \tag{6}$$

$$\frac{\partial^{2} \pi^{+}}{\partial\left[k^{+}\right]^{2}} = -G'\left(k^{-}+k^{+}\right) < 0 \text{ for all cdf } G.$$

The second-order derivative of the small firm is negative for any convex cdf. Note that if G is convex then $\int_{k^{+}}^{k^{-}+k^{+}} dG(\theta) > G'\left(k^{-}\right)$ which suffices for (6) to be negative. If G is concave but its pdf G' is convex then the result does also hold. To see this note that a sufficient condition for the SOC to be negative is

$$2\int_{k^{-}}^{k^{-}+k^{+}} \frac{dG(\theta)}{k^{+}} + G'\left(k^{-}+k^{+}\right)\frac{k^{-}}{k^{+}} - G'\left(k^{-}\right) > 0.$$

In what follows we show that this inequality holds for a convex pdf. To do so we use an auxiliary result whose statement and proof follows. If G' is convex then

$$2\int_{k^{-}}^{k^{-}+k^{+}} \frac{dG(\theta)}{k^{+}} - G'\left(k^{-}\right) \geq G'\left(k^{-}\right) + \left[k^{+}-k^{-}\right]G''\left(k^{-}\right). \tag{7}$$

Let g be a convex function. Since a convex function is locally Lipschitzian, integration by parts implies

$$\int_{x}^{b}[b-t]\,g'(t)dt - \int_{a}^{x}[t-a]\,g'(t)dt = \int_{a}^{b} g(t)dt - [b-a]\,g(x)$$

Since $g'(t) \geq g'_{+}(x)$ for all $t \in [x,b]$, if we multiply by $[b-t] \geq 0$, $t \in [x,b]$ and we integrate on $[x,b]$ we get,

$$\int_{x}^{b}[b-t]\,g'(t)dt \geq \frac{1}{2}[b-x]^{2} g'_{+}(x). \tag{8}$$

Similarly, since $g'(t) \leq g'_-(x)$ for all $t \in [a,x]$, multiplying both sides by $[t-a] \geq 0$, $t \in [a,x]$ and integrating on $[a,x]$ we get,

$$\int_a^x [t-a]\, g'(t) dt \leq \frac{1}{2}[x-a]^2 g'_-(x). \tag{9}$$

Extracting (9) from (8), we deduce

$$\int_a^b g(t)dt - [b-a]\, g(x) \geq \frac{1}{2}\left[[b-x]^2 g'_+(x) - [x-a]^2 g'_-(x)\right]$$

If x is a point of differentiability for g, then $g'_+(x) = g'_-(x) = g'(x)$ and the inequality above simplifies to

$$\frac{1}{b-a}\int_a^b g(t)dt - g(x) \geq \left[\frac{a+b}{2} - x\right] g'(x)$$

Taking $a = k^-$, $b = k^- + k^+$, $x = k^-$, and $g = G'$, we have

$$\begin{aligned} \int_{k^-}^{k^-+k^+} \frac{dG(\theta)}{k^+} - G'\left(k^-\right) &\geq \left[\frac{k^+ - k^-}{2}\right] G''\left(k^-\right) \text{ and} \\ \int_{k^-}^{k^-+k^+} \frac{dG(\theta)}{k^+} &\geq G'\left(k^-\right) + \left[\frac{k^+ - k^-}{2}\right] G''\left(k^-\right) \end{aligned}$$

Adding up the two inequalities above, the result (7) is derived. Using the derived inequality, the SOC is negative if

$$G'\left(k^-\right) + \left[k^+ - k^-\right] G''\left(k^-\right) + G'\left(k^- + k^+\right)\frac{k^-}{k^+} > 0,$$

which holds trivially as $G'(k^-) + [k^+ - k^-]\, G''(k^-)$ is the linear approximation (the tangent line $y(x) = G'(k^-) + (x - k^-)G''(k^-)$) to G'' at argument k^- passing by $x = k^+$, and it hence satisfies

$$G'\left(k^-\right) + \left[k^+ - k^-\right] G''\left(k^-\right) > G'\left(k^- + k^+\right) > 0$$

as G' is convex.

(ii) The second order cross derivatives of (2) and (3) are given by

$$\begin{aligned} \frac{\partial^2 \pi^-}{\partial k^- \partial k^+} &= -\frac{1}{k^+}\left[\int_{k^+}^{k^-+k^+} \frac{[\theta - 2k^-]}{k^+} dG(\theta) + G'\left(k^- + k^+\right) k^-\right] \\ \frac{\partial^2 \pi^+}{\partial k^+ \partial k^-} &= -G'\left(k^- + k^+\right) < 0 \end{aligned}$$

Independently of the shape of the distribution function, the second-order cross derivative of the small firm is negative for all $k^+ > 2k^-$. If G is convex the result also holds for $k^+ \in [k^-, 2k^-]$. Note that integration by parts allows to rewrite $\frac{\partial^2 \pi^-}{\partial k^- \partial k^+}$ as $-\frac{1}{k^+}H(k^-, k^+)$, where

$$\begin{aligned} H(k^-, k^+) &= \frac{k^+ - k^-}{k^+}\left[G\left(k^- + k^+\right) - G\left(k^+\right)\right] + \frac{k^-}{k^+} G\left(k^+\right) \\ &\quad - \int_{k^+}^{k^-+k^+} \frac{G(\theta)}{k^+} d\theta + G'\left(k^- + k^+\right) k^- \end{aligned}$$

Thus, $H(k^-, k^+) > 0$ if $G'(k^- + k^+) k^- > \int_{k^+}^{k^-+k^+} \frac{G(\theta)}{k^+} d\theta$, which holds for a convex G as

$$\int_{k^+}^{k^-+k^+} \frac{G(\theta)}{k^+} d\theta \leq k^- \frac{G(k^- + k^+)}{k^+} \leq k^- G'\left(k^- + k^+\right),$$

where the first inequality follows from the *Hermite-Hadamard inequality* for convex functions,[18] and the second inequality follows from the properties of convex functions ($G'(y + a)y \geq G(y + a)$). *Q.E.D.*

Proof of Proposition 4

Let $\Delta^+ = \left\{(x, y) \in [0, 1]^2 : x \geq y\right\}$, and $\Delta^- = \left\{(x, y) \in [0, 1]^2 : x \leq y\right\}$. Fix x_1, x_2, y_1, y_2 in $[0, 1]$ with $x_1 > x_2$ and $y_1 > y_2$. If all four points (x_1, y_1), (x_1, y_2), (x_2, y_1) and (x_2, y_2) lie either in Δ^+ or in Δ^- then strict submodularity of the profit function follows from part ii) of Lemma 1. If some of the four points lie in Δ^+ and the rest in Δ^-, then there are three different cases to consider depending on the number of points in each region:

1.- (x_1, y_1), (x_1, y_2), (x_2, y_2) in Δ^+ and (x_2, y_1) in Δ^-, i.e., $x_1 > y_1 > x_2 > y_2$. Decreasing differences requires

$$\pi^+(x_1, y_1) - \pi^+(x_1, y_2) < \pi^-(x_2, y_1) - \pi^+(x_2, y_2),$$

where π^- and π^+ have been defined in (2) and (3).

2.- (x_1, y_1), (x_1, y_2) in Δ^+ and (x_2, y_1), (x_2, y_2) in Δ^-, so that $x_1 > y_1 > y_2 > x_2$. Decreasing differences requires

$$\pi^+(x_1, y_1) - \pi^+(x_1, y_2) < \pi^-(x_2, y_1) - \pi^-(x_2, y_2)$$

3.- (x_1, y_2) in Δ^+ and (x_1, y_2), (x_2, y_1) and (x_2, y_2) in Δ^-, so that $y_1 > x_1 > y_2 > x_2$. Now, we have to show that

$$\pi^-(x_1, y_1) - \pi^+(x_1, y_2) < \pi^-(x_2, y_1) - \pi^-(x_2, y_2)$$

The proofs for the three cases are similar, thus we only provide here the one corresponding to case 3, which is the most elaborated one.[19] Let C stand for the right-hand side of inequality above ($\pi^-(x_2, y_1) - \pi^-(x_2, y_2)$). It is easy to show that C exceeds the following expression,

$$C > -\frac{x_2 [y_1 - x_2]}{y_1} G(y_1) - \frac{x_2}{y_1} \int_0^{x_2} G(\theta + y_1) d\theta + \frac{x_2 [y_2 - x_2]}{y_2} G(y_2) + \frac{x_2}{y_2} \int_0^{x_2} G(\theta + y_2) d\theta.$$

[18] If f is convex then $f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a} \int_a^b f(t)\, dt \leq \frac{f(a)+f(b)}{2}$.

[19] The proofs for the remaining cases are available from the authors upon request.

Let D denote the left-hand side of inequality above which is given by

$$D = -\frac{x_1\left[y_1 - x_1\right]}{y_1}G(y_1) + \int_{x_1}^{y_1} \frac{x_1\left[\theta - x_1\right]}{\theta} dG\left(\theta\right) + \int_{0}^{x_1} \left[G(\theta + y_2) - \frac{x_1}{y_1}G\left(\theta + y_1\right)\right] d\theta$$

For $C - D > 0$, the following is sufficient

$$0 < \frac{1}{y_1}\left[x_1\left[y_1 - x_1\right] - x_2\left[y_1 - x_2\right]\right]G(y_1) + \int_{0}^{x_2} \left[\frac{x_1 - x_2}{y_1}G\left(\theta + y_1\right) - \frac{y_2 - x_2}{y_2}G(\theta + y_2)\right] d\theta$$

$$+ \int_{x_2}^{x_1} x_1\left[\frac{G\left(\theta + y_1\right)}{y_1} - \frac{G(\theta + y_2)}{x_1}\right] d\theta + \frac{x_2\left[y_2 - x_2\right]}{y_2}G\left(y_2\right) - \int_{x_1}^{y_1} \frac{x_1\left[\theta - x_1\right]}{\theta} dG\left(\theta\right)$$

The convexity of the cdf yields $\frac{G(\theta+y_1)}{y_1} \geq \frac{G(\theta+y_2)}{y_2}$, which implies

$$\int_{0}^{x_2} \left[\frac{x_1 - y_2 + y_2 - x_2}{y_1}G\left(\theta + y_1\right) - \frac{y_2 - x_2}{y_2}G(\theta + y_2)\right] d\theta > \int_{0}^{x_2} \frac{x_1 - y_2}{y_1}G\left(\theta + y_1\right) d\theta, \text{ and}$$

$$x_1\left[\frac{G\left(\theta + y_1\right)}{y_1} - \frac{G(\theta + y_2)}{y_2}\right] > x_1\left[\frac{G\left(\theta + y_2\right)}{y_2} - \frac{G(\theta + y_2)}{x_1}\right] > G(\theta + y_2)\left[\frac{x_1 - y_2}{y_1}\right]$$

Note also that integration by parts gives

$$\frac{x_1\left[y_1 - x_1\right]}{y_1}G(y_1) - \int_{x_1}^{y_1} \frac{x_1\left[\theta - x_1\right]}{\theta} dG\left(\theta\right) = \int_{x_1}^{y_1} \frac{x_1^2}{\theta^2}G\left(\theta\right) d\theta > 0$$

Thus, if it holds that

$$\int_{0}^{x_2} \frac{x_1 - y_2}{y_1}G\left(\theta + y_1\right) d\theta > \frac{x_2\left[y_1 - x_2\right]}{y_1}G(y_1),$$

then it will suffice to ensure $C - D > 0$. The convexity of G and the fact that $G(0) = 0$ implies that the *average function* $F(x) \equiv \frac{\int_0^x G(t)dt}{x}$ is *starshaped* ($F(\alpha x) \leq \alpha F(x)$ for $0 \leq \alpha \leq 1$), consequently,

$$\frac{x_2\left[\frac{x_1 - y_2}{y_1}\right] \int_0^{x_2} G\left(\theta + y_1\right) d\theta}{x_2} \geq \frac{x_2 \int_0^{x_2\left[\frac{x_1 - y_2}{y_1}\right]} G\left(\theta + y_1\right) d\theta}{\left[\frac{x_1 - y_2}{y_1}\right] x_2} = \frac{y_1}{x_1 - y_2} \int_0^{x_2\left[\frac{x_1 - y_2}{y_1}\right]} G\left(\theta + y_1\right) d\theta$$

Since $\frac{y_1}{x_1 - y_2} \geq \frac{x_2\left[y_1 - x_2\right]}{y_1}$, and G is increasing, the desired result follows.

Since the game is submodular, existence of a pure strategy equilibrium is guaranteed (see Vives 1990). *Q.E.D.*

Proof of Proposition 5

(i). Along the diagonal, the right-hand derivative is always larger than the left-hand derivative as

$$\frac{\partial^{+}\pi(k,k)}{\partial k^{+}}-\frac{\partial^{-}\pi(k,k)}{\partial k^{-}}=\int_{k}^{2k}\frac{2k-\theta}{k}dG\left(\theta\right)>0 \tag{10}$$

for any G. This implies that k is never a best reply to k and hence no pure strategy equilibrium can be symmetric.

(ii). a). Assuming that the second order derivative (6) is negative, the profit function (1) is piecewise concave and continuous everywhere, in particular at $k^{-}=k^{+}$. Choose an arbitrary but fixed value for k_j. Then, the payoff functions $\pi_i^{-}(\cdot,k_j)$, $\pi_i^{+}(\cdot,k_j)$ are single-peaked on the interval $[0,1]$, with unconstrained maxima at $R_i^{-}\left(k_j\right)$ and $R_i^{+}\left(k_j\right)$,which solve (4) and (5).

Since the marginal revenue function jumps up at symmetric capacity pairs (see equation (10)), to determine the global maxima we must distinguish three regions:

Region A. If $\frac{\partial\pi^{+}(k,k)}{\partial k^{+}}\geq\frac{\partial\pi^{-}(k,k)}{\partial k^{-}}\geq 0$, $R_i^{+}\left(k_j\right)$ is interior and $R_i^{-}\left(k_j\right)$ is constrained. Thus, the global maximum is $R_i^{+}\left(k_j\right)$.

Region B. If $\frac{\partial\pi^{-}(k,k)}{\partial k^{-}}\leq\frac{\partial\pi^{+}(k,k)}{\partial k^{+}}<0$, then $R_i^{-}\left(k_j\right)$ is interior and $R_i^{+}\left(k_j\right)$ is constrained. Thus, the global maximum is $R_i^{-}\left(k_j\right)<k_j$.

Region C. If $\frac{\partial\pi^{+}(k,k)}{\partial k^{+}}\geq 0$ and $\frac{\partial\pi^{-}(k,k)}{\partial k^{-}}\leq 0$, both $R_i^{+}\left(k_j\right)$ and $R_i^{-}\left(k_j\right)$ are interior, we need hence to compare profits at the two candidate. For this purpose, let us first implicitly define k^{*} and k^{**} as,

$$\frac{\partial\pi^{-}(k^{*},k^{*})}{\partial k^{-}}=0\text{ and }\frac{\partial\pi^{+}\left(k^{**},k^{**}\right)}{\partial k^{+}}=0.$$

Given that (10) implies $\frac{\partial\pi^{+}(k^{*},k^{*})}{\partial k^{+}}>0$, it follows from the concavity of π^{+} that $k^{**}>k^{*}$. Using these definitions, in region C the following equations are satisfied,

$$\begin{aligned}\pi_i^{-}\left(R_i^{-}\left(k^{*}\right),k^{*}\right)-\pi_i^{+}\left(R_i^{+}\left(k^{*}\right),k^{*}\right) &< 0\\ \pi_i^{-}\left(R_i^{-}\left(k^{**}\right),k^{**}\right)-\pi_i^{+}\left(R_i^{+}\left(k^{**}\right),k^{**}\right) &> 0.\end{aligned}$$

Furthermore, the difference in profits is a strictly increasing function of $k_j\in\left[k^{*},k^{**}\right]$,

$$\begin{aligned}\frac{d\pi_i^{-}\left(R_i^{-}\left(k_j\right),k_j\right)}{dk_j}-\frac{d\pi_i^{+}\left(R_i^{+}\left(k_j\right),k_j\right)}{dk_j} &= \int_{k_j}^{R_i^{-}+k_j}\frac{1}{k_j^{2}}\left[k_j^{2}+\left[R_i^{-}\right]^{2}-R_i^{-}\theta\right]dG\left(\theta\right)\\ &\quad+\left[G\left(R_i^{+}+k_j\right)-G\left(R_i^{-}+k_j\right)\right]>0\end{aligned}$$

as $R_i^{+}>R_i^{-}$ and $k_j^{2}+\left[R_i^{-}\right]^{2}-R_i^{-}\theta>k_j^{2}+\left[R_i^{-}\right]^{2}-R_i^{-}\left[R_i^{-}+k_j\right]>0$. Therefore, there exists a unique $\hat{k}\in\left(k^{*},k^{**}\right)$ such that

$$\pi_i^{-}\left(R_i^{-}\left(\hat{k}\right),\hat{k}\right)-\pi_i^{+}\left(R_i^{+}\left(\hat{k}\right),\hat{k}\right)=0.$$

At $k_j = \hat{k}$ both R_i^- and R_i^+ are a best reply. For values $k_j < \hat{k}$ the best reply is R_i^+, whereas for $k_j > \hat{k}$ the best reply is R_i^-. In summary, the reaction functions for firm $i = 1, 2$, $i \neq j$, are as follows:

$$R_i(k_j) = \begin{cases} R_i^-(k_j) & \text{if} \quad k_j \geq \hat{k} \\ R_i^+(k_j) & \text{if} \quad k_j \leq \hat{k} \end{cases}$$

Notice that $R_i(k_j)$ is a continuous function everywhere except at one point, $k_j = \hat{k}$.

(ii). b). If G is convex both $R_i^-(k_j)$ and $R_i^+(k_j)$ are decreasing functions. Since $R_i^-(\hat{k}) < R_i^+(\hat{k})$ and $R_i^+(0) = G^{-1}(1-c) < 1$ and $R_i^-(1) > 0$, then they must cross outside the discontinuity region, i.e., $k^- < \hat{k} < k^+$, which guarantees the existence of a Nash equilibrium $((k^-, k^+))$. Finally, since the best replies are identical for both players then $(k_1 = k^+, k_2 = k^-)$ is also an equilibrium as claimed.

If G is concave then $R_i^+(k_j)$ is a decreasing function, and $R_i^-(k_j)$ is also decreasing for all $k_j > 2k_i^-$. We first note that a candidate to equilibrium always exists. Note that substracting equation (5) from (4), a candidate to equilibrium is a pair $\left(k_i^-, k_j^+\right)$ such that

$$\int_{k_i^-}^{k_i^- + k_j^+} \frac{\theta - 2k_i^-}{\min(\theta, k_j^+)} dG(\theta) = 0 \text{ and } k_i^- + k_j^+ = G^{-1}(1-c)$$

Let $G^{-1}(1-c) = A$ and consider the function $H(k_j) : [A/2, A] \to \mathsf{R}$ defined by

$$H(k_j) = \int_{A-k_j}^{A} \frac{\theta - 2(A - k_j)}{\min(\theta, k_j)} G'(\theta)\, d\theta$$

Since $H(A) > 0$ and $H\left(\frac{A}{2}\right) < 0$, by appealing to Bolzano intermediate value theorem, there exists $k_j^* \in \left(\frac{A}{2}, A\right)$ at which $H(k_j^*) = 0$. Taking $k_1^- = A - k_2^*$, then the pair $(k_1^-, k_2^+ = k_2^*)$ is a solution to system made of equations (5) and (4). Now this solution constitutes an equilibrium if it lies outside the discontinuity region in the best reply functions. A sufficient condition for the pair $(k_1^-, k_2^+ = k_2^*)$ to be an equilibrium is that $R_1^-(2k_1^-) = \bar{k}_1 \leq \tilde{k}_1$ where $k_2^+(\tilde{k}_1) = \tilde{k}_2 = 2\tilde{k}_1$. To see this note that $R_1^-(2k_1^-)$ determines the crossing point between the best reply $R_1^-(k_2)$ and the line $k_2 = 2k_1$. For $k_2 > 2k_1^-$ the best reply decreases as $\frac{\partial^2 \pi_1^-}{\partial k_1 \partial k_2}$ becomes negative. Similarly, $R_2^+(\tilde{k}_1)$ determines the crossing between the best reply $R_2^+(k_1)$ and the line $k_2 = 2k_1$. Thus if $\bar{k}_1 \leq \tilde{k}_1$ then $(k_1^-, k_2^+ = k_2^*)$ lies outside the discontinuity region.

Now for $\bar{k}_1 \leq \tilde{k}_1$ it must be the case that $\int_{\bar{k}_1}^{2\bar{k}_1} \left(\frac{2\bar{k}_1 - \theta}{\theta}\right) dG(\theta) + \int_{2\bar{k}_1}^{3\bar{k}_1} \left(\frac{2\bar{k}_1 - \theta}{2\bar{k}_1}\right) dG(\theta) \geq 0$, as, by definition,

$$\begin{aligned}
1-c &= G(3\tilde{\bar{k}}_1) \text{ and} \\
1-c &= G(3\bar{k}_1)+\int\limits_{\bar{k}_1}^{2\bar{k}_1}\frac{2\bar{k}_1-\theta}{\theta}dG\left(\theta\right)+\int\limits_{2\bar{k}_1}^{3\bar{k}_1}\frac{2\bar{k}_1-\theta}{2\bar{k}_1}dG\left(\theta\right),
\end{aligned}$$

and the right hand side of equalities above determines two strictly increasing functions provided that the SOC holds. Thus if

$$\int\limits_{\bar{k}_1}^{2\bar{k}_1}\frac{2\bar{k}_1-\theta}{2\bar{k}_1}dG\left(\theta\right)\geq\int\limits_{2\bar{k}_1}^{3\bar{k}_1}\frac{\theta-2\bar{k}_1}{2\bar{k}_1}dG\left(\theta\right)$$

then the result will follow as

$$\int\limits_{\bar{k}_1}^{2\bar{k}_1}\left(\frac{2\bar{k}_1-\theta}{\theta}\right)G'\left(\theta\right)d\theta\geq\int\limits_{\bar{k}_1}^{2\bar{k}_1}\left(\frac{2\bar{k}_1-\theta}{2\bar{k}_1}\right)G'\left(\theta\right)d\theta.$$

We next show that this is the case by using *Steffensen's Inequality*.[20] Since $G'\left(\theta\right)$ is a non-negative and monotone decreasing function and $0\leq\frac{\theta-2\bar{k}_1}{2\bar{k}_1}\leq 1$ for all $\theta\in\left[2\bar{k}_1,3\bar{k}_1\right]$ then

$$\int_{3\bar{k}_1-d}^{3\bar{k}_1}dG\left(\theta\right)\leq\int\limits_{2\bar{k}_1}^{3\bar{k}_1}\frac{\theta-2\bar{k}_1}{2\bar{k}_1}dG\left(\theta\right)\leq\int\limits_{2\bar{k}_1}^{2\bar{k}_1+d}dG\left(\theta\right),\ \text{for } d=\int\limits_{2\bar{k}_1}^{3\bar{k}_1}\frac{\theta-2\bar{k}_1}{2\bar{k}_1}d\theta=\frac{\bar{k}_1}{4}.$$

Consequently,

$$\begin{aligned}
\int\limits_{2\bar{k}_1}^{3\bar{k}_1}\frac{\theta-2\bar{k}_1}{2\bar{k}_1}G'\left(\theta\right)d\theta &\in \left[G(3\bar{k}_1)-G\left(\frac{11}{4}\bar{k}_1\right),G\left(\frac{11}{4}\bar{k}_1\right)-G(2\bar{k}_1)\right],\ \text{and similarly,} \\
\int\limits_{\bar{k}_1}^{2\bar{k}_1}\frac{2\bar{k}_1-\theta}{2\bar{k}_1}G'\left(\theta\right)d\theta &\in \left[G\left(\bar{k}_1\right)-G\left(\frac{3}{4}\bar{k}_1\right),G(2\bar{k}_1)-G\left(\frac{7}{4}\bar{k}_1\right)\right].
\end{aligned}$$

Since concavity of G implies

$$G(\bar{k}_1)-G\left(\frac{3}{4}\bar{k}_1\right)>G\left(\frac{11}{4}\bar{k}_1\right)-G(2\bar{k}_1)$$

[20] Let $f(x)$ be a nonnegative and monotonic decreasing function in $[a,b]$ and $h(x)$ such that $0\leq h(x)\leq 1$ in $[a,b]$, then

$$\int_{b-d}^{b}f\left(x\right)dx\leq\int\limits_{a}^{b}f(x)h(x)dx\leq\int\limits_{a}^{a+d}f\left(x\right)dx$$

where $d=\int\limits_a^b h(x)dx$. See Gradshteyn and Ryzhik (2000), page 1099.

the result follows. Finally, using the same reasoning as above it is straightforward to see that $k_1^* < \bar{k}_1$, so that the two best replies cross outside the discontinuity region.

Finally, an equilibrium satisfies $1-G\left(k_i^- + k_j^+\right) - c = 0$. Consequently aggregate capacity $k_i^- + k_j^+$ equals $G^{-1}(1-c)$. *Q.E.D.*

Proof of Proposition 6

(i) Subgame perfect equilibrium aggregate capacity is given by $K^u = G^{-1}(1-c)$ in the game with continuous demand uncertainty, and it is given by $K^c = E[\theta]$ in the certainty equivalent game. The difference $K^u - K^c$ is strictly decreasing in c. Furthermore, $\lim_{c\to 0}[K^u - K^c] > 0$ and $\lim_{c\to 1}[K^u - K^c] < 0$. Hence, it follows that there must exist some $\widehat{c}$ such that $K^u > K^c$ if and only if $c \in (0, \widehat{c})$.

(ii) In the certainty equivalent game, subgame perfect equilibrium prices are equal to consumers' reservation price, so that consumer surplus is zero. In the game with continuous demand uncertainty, prices are strictly below the reservation price for $\theta \in (0, G^{-1}(1-c)) \subset [0,1]$, so that consumer surplus is strictly positive. It follows that expected prices must be lower and consumer surplus higher under demand uncertainty.

(iii) Let W^u and W^c denote subgame perfect equilibrium Welfare in the game with demand uncertainty and in the certainty equivalent game, respectively. These can be expressed as,

$$\begin{aligned} W^u &= \int_0^1 \min\left[\theta, G^{-1}(1-c)\right] dG(\theta) - cG^{-1}(1-c), \\ W^c &= [1-c]\int_0^1 \theta dG(\theta). \end{aligned}$$

With the difference being,

$$W^u - W^c = c\left[E[\theta] - G^{-1}(1-c)\right] - \int_{G^{-1}(1-c)}^1 \left[\theta - G^{-1}(1-c)\right] dG(\theta).$$

The above expression is clearly negative if $c \in (0, \widehat{c})$, as the second term is negative and, by point (i) above, the first term is negative as well. Furthermore, since $\frac{\partial[W^u - W^c]}{\partial c} = E[\theta] - G^{-1}(1-c) > 0$ for $c \in (\widehat{c}, 1)$ and $\lim_{c\to 1}[W^u - W^c] = 0$, it follows that $W^u < W^c$ for all c.

Proof of Lemma 2

When demand is uniformly distributed, expected profits become,

$$\begin{aligned} \pi^- &= \frac{k^+}{2}\left[2 - 2k^- - k^+\right]\frac{2-k^-}{2-k^+}\frac{k^-}{k^+} - ck^-, \text{ and} \\ \pi^+ &= \frac{k^+}{2}\left[2 - 2k^- - k^+\right] - ck^+. \end{aligned}$$

The first order derivatives are

$$\begin{aligned}\frac{\partial \pi^-}{\partial k^-} &= \frac{k^- k^+ - k^+ - 6k^- + 3\left[k^-\right]^2 + 2}{2 - k^+} - c,\\ \frac{\partial \pi^+}{\partial k^+} &= \left[1 - k^- - k^+\right] - c.\end{aligned}$$

Along the diagonal,

$$\frac{\partial^+ \pi(k,k)}{\partial k^+} - \frac{\partial^- \pi(k,k)}{\partial k^-} > 0 \tag{11}$$

which rules out the existence of a symmetric equilibrium.

Clearly, the second order conditions are satisfied. Furthermore, the second-order cross derivatives are negative, which guarantees that firms' reaction functions $R_i^-(k_j)$ and $R_i^+(k_j)$ are downward sloping. Furthermore, since $R_i^-(\hat{k}) < R_i^+(\hat{k})$, $R_i^+(0) = 1-c < 1$ and $R_i^-(1) > 0$, then they must cross outside the discontinuity region, i.e., $k^- < \hat{k} < k^+$, which guarantees that a Nash equilibrium exists and that it is the solution to the system of first order conditions above,

$$k^+ = \frac{1}{2}\left[\sqrt{3c^2 + 4c + 2} - 3c\right] \text{ and } k^- = [1 - c] - \frac{1}{2}\left[\sqrt{3c^2 + 4c + 2} - 3c\right]$$

Q.E.D.