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SPATIAL DISTRIBUTION OF PRODUCTION AND INTERNATIONAL TRADE

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Abstract

In this paper we have developed a monopolistic competition model that explains the sizes and locations of cities as a consequence of centrifugal and centripetal forces. Our interest is to present a framework that allows us to study the principal causes that favor agglomeration and those which stop it in current societies, where farmers are not a large proportion of the total population and where international relationships substantially affect the inner structure of a country.

Key Words
Monopolistic Competition; Congestion Costs; Location; Diversity of Goods.

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1. Introduction

In the last few years, economic geography has recovered a relevant role in the world of politics. The continous changes in the frontiers of many countries, or the possibility of economic integration, are facts that have contributed to this general interest. Geography plays an important role in the context of international economics. Clearly, the situation of a country is different if it is adjacent to a world power instead of to a small country. Moreover, it is different to be linked to the core of an international economic area instead of to the periphery. It is important to realize that relationships between countries are affected by spatial dimension. Countries are not dimensionless points, and, in fact, different relations between them imply the emergence of different urban and industrial location patterns. So, for example, Livas Elizondo and Krugman (1992) justify the existence of large cities in developing countries, as the case of Mexico D.C., by the strong backward and forward linkages that emerge from selling fundamentally to the domestic market. They suggest that a more liberal policy would contribute to the existence of more cities, or in other words, cities of a smaller size. At the same time, one can think that a country mainly serving a foreign market will have its industries near borders, in order to spend less on transportation, while if it serves to domestic market, its industries will be located near large markets, which are not necesarily at the borders. In order to analyze these situations it is necessary to previously obtain the principal factors that favor the agglomeration of economic activities.

In the last few years, some articles have tried to explain this fact through formal microecomic models. In fact, Krugman, for example, presents several works in this line of research, where cities emerge from the interactions among individuals. In this essay we present a variation of one of these models (Krugman 1993). In his model, agglomeration emerges from three sources: the existence of economies of scale at a firm level, transportation costs, and the mobility of labor. Increasing returns to scale imply that the production of each good will take place in a single location. On the other hand, the existence of transportation costs means that the best locations for a firm will be those with easy access to markets. These places are those where products are concentrated. But in this model not all factors are mobile, farmers are immobile and they are the centrifugal force that breaks agglomeration.

However, farmers do not seem to be the force that is putting a stop to the growth of cities. Furthermore, if farmers are playing the role of immobile factors such as land it is *ad hoc* to assume that they produce an agricultural good that is mobile, as happens in the cited model. Since the underlying idea is that this mobile agricultural good can make the returns to land in region 1 easily transfereable to region 2, this is not realistic.

We are interested in explaining the formation of cities in the context of an urban society, where peasants are not a strong proportion of the population, and where international trade plays an important role in the configuration pattern of a country.

For these reasons, our model takes off from Krugman's since it maintains the same centripetal forces, but it departs from it regarding centrifugal forces. We introduce two kinds of centrifugal forces, one global and another local. The first one is represented by the existence of foreign countries. The population size of these countries will affect the location and size of cities in our country of interest. If foreign countries are big, our country will mainly serve the exterior market and the cities will appear near borders. On the other hand, if foreign countries are small, our country will mostly sell in the domestic market, and their cities' location will depend on historic events, i.e., they will appear where there are more people, which can depend on historical reasons. The second centrifugal force is due to the existence of congestion costs. Big cities have urban traffic problems, pollution or high housing prices that make small cities more attractive places to live in. All these negative effects of agglomeration are included under the paragraph of congestion costs. When congestion costs are high more cities exist.

The Heckscher-Olin trade model has dominated work in the theory of international trade for years. However, since World War II, the largest and fastest growing component of world trade has been the exchange of manufactures between the industrialized economies, fact that cannot be explained by the H-S model. As a result, a new framework for analyzing trade was needed. Some recent works have given an alternative explanation of international trade. So, Krugman (1979), for example, develops a formal model in which trade is caused by economies of scale instead of differences in factor endowments or technology. His approach differs from that of other formal treatments of trade under increasing returns, which assume that scale economies are external to firms so that markets remain perfectly competitive. In his model the existence of economies of scale implies that both the variety of goods that one country can produce and the scale of its production are constrained by the size of the market. However, if each country trades with other ones, nations will be not constrained by their own market size, because the world market is larger than each individual national market. In this way, each country can specialize in producing a smaller number of products than it would in the absence of trade, but by buying goods it does not make from other countries, each nation can simultaneously increase the variety of goods that its citizens can consume. So, trade offers an opportunity for mutual gain even when countries do not differ in their resources or technology.

Within this same framework, our model is based on a simplest version of Krugman (1980), where gains from trade occur because the world economy produces a greater diversity of goods than would either country alone, offering each individual a wider range of choice.

This paper is organized as follows. In section 2 we introduce the assumptions of the model and the short-run equilibrium. In section 3 we analyze the long-run equilibrium in the case of three cities and in the multiple case. In fact, we discuss the effects that the size of foreign countries, the transportation parameter and congestion costs have over the urban pattern of the country of study. In section 4 we present some welfare implications derived from investments in transportation infrastructure and the existence of trade barriers. Finally, section 5 concludes and suggests some extensions to the model.

2.1. Assumptions of the model

The model consists of a long-narrow economy with three countries, two of them exterior and one interior. In each of the exterior countries there is only one city. The location and size of this city are given, i.e., we assume that they are fixed. We are interested in studying the location and size of cities in the interior country.

One fundamental aspect that distnguishes the relationship between regions from that between countries is the restrictions to the movements of some factors. Usually, factors such as capital or labor have not free mobility between countries. For this reason, in this model we consider that every country has a fixed population, and that it is impossible for a worker to change the country where he is working. But, on the other hand, within our country of interest there are different regions or cities, and the workers in this country can move across them without any restriction. However, these countries are not closed economies, but have trade between them, and we study how this trade affects the location and size of cities in the interior country.

We assume an economy with a large number of potential goods that appear in the utility function in a symmetric way. All consumers have the same CES tastes:

$$U = \left(\sum_i c_i^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

where the elasticity of substitution between any two goods, σ, is greater than 1.

Individuals of the interior country may move across K different cities in their country, but they can not move to foreign cities. In each of the exterior countries there is only one city.

Let λ_j be population in city j at any point in time.

There is only one factor of production: labor. All goods are produced under economies of scale with the same technology

$$L_{ij} = \alpha + \beta x_{ij} \; (\alpha > 0, \beta > 0)$$

where L_{ij} is the number of workers needed to produce x_{ij} units of good i in city j.

We assume full employment in each city at any time, so $\sum_i L_{ij} = \lambda_j$.

In this model we introduce transportation and congestion costs. Following Krugman, they take the iceberg form. This means that a fraction of any good produced by a firm disappears before this good arrives to the consumer. One share melts because of the transportation cost between cities and the other melts because of the negative effects of every city (pollution, housing prices, transportation costs within the city...) that are included under the paragraph of "congestion". So, if a unit is shipped from city j the amount that consumers placed in city k can consume is only $e^{-\tau D_{jk}-\gamma\lambda_k}$, τ being the parameter of transportation costs, γ the parameter of congestion, and D_{jk} the distance between cities j and k. In the particular case where good i is produced in city k any consumer living in the same city can obtain only a proportion $e^{-\gamma\lambda_k}$ of each unit of good i. We can see that the city size affects the loss due to agglomeration. [1]

Finally, we suppose that individuals in the interior country move toward locations with higher real wages, the law of motion being:

$$\frac{d\lambda_k}{dt} = \mu\lambda_k(\omega_k - \bar{\omega}), \ \ \mu > 0,$$

where ω_k is the real wage in city k and $\bar{\omega} = \sum_k \frac{\lambda_k}{\sum_{j\epsilon K}\lambda_j}\omega_k$ is the average real wage in the interior country and $K =$ are the locations in the interior country.

2.2. Short-run equilibrium

Drawing on Starret's spatial impossibility theorem, Fujita indicates that there are only two kinds of models which can explain the endogenous formation of cities: non-price interaction models and non-competitive models. The model discussed here, one of monopolistic competition, is included in the last group.

Economies of scale (due to the existence of fixed costs) in production imply that every

[1] We can treat intra-urban congestion in a more explicit way, such as land consumption and/or traffic congestion in cities. But, such an extension would not significantly change the main conclusions of this paper. Therefore, we take the simplest form of urban congestion.

good is produced in only one location, so that different cities have different goods.

To determine the profit-maximizing behavior of firms it is important to stress the fact that there are two types of demands: the demand of individuals living in the city where the good is produced and the demand of other cities (national or foreign). One must remember that all goods appear in the utility function in a symmetric way, which means that all goods are consumed. The main point is that both demands have the same price elasticity σ. For this reason transportation and congestion costs (which make consumers in different cities pay different prices for the same good) do not alter the strategy of firms. It can be shown that the first order condition implies

$$p_{ij} = w_j \beta \frac{\sigma}{\sigma - 1},$$

where w_j is the wage rate in city j. So, we have that all f.o.b. prices are the same within a city.

Since there is monopolistic competition firms enter the market until profits are zero. All this implies that

$$x_{ij} = \alpha(\frac{\sigma - 1}{\beta}) \text{ for every good } i \text{ and city } j.$$

As every firm produces the same quantity and has the same technology, the number of firms in city j, n_j, will be proportional to its population: $n_j = n\lambda_j$, where n is the number of goods in the whole economy. This value might be obtained by dividing the number of workers in the economy by the number needed in each firm.

In this section we assume that workers cannot move to other locations (neither national nor foreign) and we obtain the wage rate for each city. In order to do this we modify the units of goods such that $p_{ij} = w_j$, which means that β should be equal $\frac{(\sigma-1)}{\sigma}$ in the cost function for each firm.

Suppose that we have a *numeraire* good at $j = 1$. Then, all prices in this location will be 1, and therefore $w_1 = 1$.

Following Krugman we define the true price index at j as

$$T_j = \left[\sum_k \lambda_k (w_k e^{\tau D_{jk} + \gamma \lambda_j})^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{1}$$

We can prove that

$$w_j = \left[\sum_k Y_k (e^{-(\tau D_{jk} + \gamma \lambda_k)} T_k)^{\sigma - 1} \right]^{\frac{1}{\sigma}}, \quad (2),$$

where Y_j is the income of city j and

$$Y_j = \lambda_j w_j. \tag{3}$$

Then, for a given distribution of the population, we can calculate the wage rate for each city. To do so we only need to introduce the parameters of taste for variety (σ) and transportation and congestion parameters (τ and σ, respectively) in the preceeding equations.

3. Long-run equilibrium

We are now interested in knowing what happens in our economy if workers in the interior country can move across its national cities. Actually, we will try to explain how the congestion cost and the sizes of the foreign countries affect the location and size of cities in the country of study. The force that may move workers from one place to another is the real wage, defined as the ratio between the wage rate and price index, namely $\omega_j = w_j T_j^{-1}$. Using the dynamic process described above we know that workers move to cities with real wages above the average real wage, and that they move away from cities with real wages below average.

We can define equilibrium as any distribution of population between the different locations in the interior country such that $\omega_j = \bar{\omega}$ for each j with $\lambda_j \neq 0$ and $\omega_j \leq \bar{\omega}$ otherwise.

We can begin by analytically studying the case of three possible locations in the interior country and later we will discuss the case of more than three cities, in fact, seven locations, in numerical examples.

3.1. The three cities case

Let us suppose three possible locations in our country of interest one in the center of the country and the others near each border. Namely locations 2, 3 and 4.

Let 1 and 5 be the locations of the foreign cities. We assume that the distance between location 1 and 5 is 1, and that the three countries have the same length. Each country is specialized in the production of different goods. But every individual in this economy needs to consume all world goods. For this reason each country imports goods from others, so it can benefit from the total variety of goods and at the same time export goods to other nations. Each location j has a population of λ_j

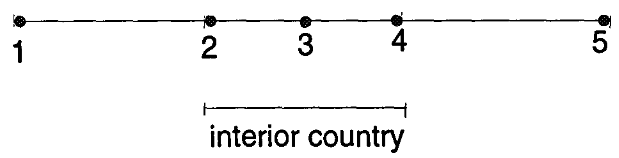

Figure 1. Possible locations: borders and center

We will do two different conjectures related to the location of cities in order to see what are the different forces:

- First, we assume that in our country all workers are concentrated in location 3 ($\lambda_2 = \lambda_4 = 0$) and that $\lambda_1 = \lambda_5$.

As already stated, $w_1 = 1$, and equations (1) to (3) define the wage rates in each location. Using the fact that, by symmetry, $w_5 = 1$ we can write ω_2 and ω_3 as follows (see the appendix C):

$$\omega_2 = \left\{\lambda_1(e^{-\tau D_{12}(\sigma-1)} + e^{-\tau D_{25}(\sigma-1)})(\lambda_1 + \lambda_3 w_3{}^{1-\sigma} e^{\tau D_{13}(1-\sigma)} + \lambda_5 e^{\tau D_{15}(1-\sigma)}\right)^{-1} +$$

$$
\begin{aligned}
& +\lambda_3 w_3 e^{-\tau D_{23}(\sigma-1)}\left(\lambda_1 e^{\tau D_{13}(1-\sigma)}+\lambda_3 w_3{}^{1-\sigma}+\lambda_5 e^{\tau D_{35}(1-\sigma)}\right)^{-1}\Big\}^{\frac{1}{\sigma}}\cdots \\
& \cdots\left\{\lambda_1 e^{\tau D_{12}(1-\sigma)}+\lambda_3 w_3^{1-\sigma} e^{\tau D_{23}(1-\sigma)}+\lambda_5 e^{\tau D_{25}(1-\sigma)}\right\}^{\frac{1}{\sigma-1}} \\
\omega_3 \;=\; & \Big\{\lambda_1\left(e^{-\tau D_{13}(\sigma-1)}+e^{-\tau D_{35}(\sigma-1)}\right)\left(\lambda_1+\lambda_3 w_3{}^{1-\sigma} e^{\tau D_{13}(1-\sigma)}+\lambda_5 e^{\tau D_{15}(1-\sigma)}\right)^{-1}+ \\
& +\lambda_3 w_3\left(\lambda_1 e^{\tau D_{13}(1-\sigma)}+\lambda_3 w_3{}^{1-\sigma}+\lambda_5 e^{\tau D_{35}(1-\sigma)}\right)^{-1}\Big\}^{\frac{1}{\sigma}}\cdots \\
& \cdots\left\{\lambda_1 e^{(\tau D_{13}+\gamma\lambda_3)(1-\sigma)}+\lambda_3 w_3^{1-\sigma} e^{\gamma\lambda_3(1-\sigma)}+\lambda_5 e^{(\tau D_{23}+\gamma\lambda_3)(1-\sigma)}\right\}^{\frac{1}{\sigma-1}}.
\end{aligned}
$$

In general it is not easy to compare these two expressions. There are some terms of the real wage which are bigger in location 2 and other which are bigger in location 3. So, we shall just analyze two symmetrical cases. One is what happens if the population in exterior countries disappears ($\lambda_1 = \lambda_5 \approx 0$) and the other one is what happens if the population in the interior country is very small ($\lambda_1 = \lambda_5 \approx \frac{1}{2}$).

1. When exterior countries are negligible ($\lambda_1 = \lambda_5 \approx 0$) real wages in location 2 and 3 take the form

$$
\begin{aligned}
\omega_2 &= e^{-\tau D_{23}\frac{(2\sigma-1)}{\sigma}} \\
\omega_3 &= e^{-\gamma}.
\end{aligned}
$$

   Using these equations it is easy to prove that

$$
\omega_3 > \omega_2 \text{ if and only if } \frac{\tau(2\sigma-1)}{\sigma} D_{23} > \gamma.
$$

   So, concentration in the center of the country can be an equilibrium if and only if the above condition is verified. In other words, if congestion costs are sufficiently low in relation to transportation costs. We can also see that concentration is more likely when transportation costs (τ) and the elasticity of substitution (σ) are high.

   The explanation that justifies this result is the following: if there is no population in the exterior countries, the only centrifugal force that breaks agglomeration is the congestion cost. Workers in our country can not obtain higher real wages by moving away from concentration because there are no consumers elsewhere who could increase the demand of the good being shipped. So, as τ increases any unilateral deviation pays less, the gains due to this movement decrease and the losses increase. On the other hand, as σ increases the mark-up of any firm is lower, so there is less interest in a deviation from concentration.

In the particular case where $\gamma = 0$ there is always concentration in the central city. This result is equivalent to that obtained by Krugman when peasants, who are the centrifugal force in his model, disappear. And in that case, concentration also increases in τ. However the variety parameter does not affect concentration in his model.

2. We can see now what happens in our three-city economy when population in the interior country disappears ($\lambda_1 = \lambda_5 \approx \frac{1}{2}$). In this case the real wage equations take the form

$$\begin{aligned} \omega_2 &= (e^{-\tau D_{12}(\sigma-1)} + e^{-\tau D_{25}(\sigma-1)})^{\frac{1}{\sigma-1}}(1 + e^{\tau D_{15}(1-\sigma)})^{\frac{-1}{\sigma}}(\frac{1}{2})^{\frac{1}{\sigma-1}} \\ \omega_3 &= (e^{-\tau D_{13}(\sigma-1)} + e^{-\tau D_{35}(\sigma-1)})^{\frac{1}{\sigma-1}}(1 + e^{\tau D_{15}(1-\sigma)})^{\frac{-1}{\sigma}}(\frac{1}{2})^{\frac{1}{\sigma-1}}. \end{aligned}$$

After some algebraic operations it can be shown that

$$\omega_3 > \omega_2 \text{ if and only if } 2e^{-\tau D_{13}(\sigma-1)} > e^{-\tau D_{12}(\sigma-1)}(1 + e^{-\tau D_{12}(\sigma-1)}).$$

Which holds if and only if

$$2 > e^{\frac{\tau}{2}D_{12}(\sigma-1)} + e^{\frac{-\tau}{2}D_{12}(\sigma-1)}.$$

But the last inequality does not hold, since if we define $x = e^{\frac{\tau}{2}D_{12}(\sigma-1)}$ then we can write the above expression as follows $2 > x + \frac{1}{x}$, which is equivalent to $(1-x)^2 < 0$ and we know that $(1-x)^2$ is always greater or equal to zero. So, the real wage in location 3 is always lower than that of location 2 and this means that concentration in the center of the country is not an equilibrium when the exterior countries are very large. There are two possibilities: either there is concentration in one frontier or there is dispersion. In other words, foreign countries are the global centrifugal force in this economy they attract cities of the interior country to the borders. In this case, we obtain that concentration in the center decreases in σ and in τ. The reasons are clear: when transportation costs increase, borders are more attractive, and when goods are better substitutes being as close as possible to big cities (now foreign cities are the biggest) is the only way of increasing the market power.

To better understand the centrifugal and centripetal forces in this model it is interesting to focus not only on the possibility of concentration in the central location but also at the border.

- For this reason, in what follows we will consider that the population in our country is concentrated in location 2 and that we are going to study the relationship between real wages in location 2 and 3.

In this new context we analize the same previous extreme cases.

1. First consider the case in which foreign population is negligible. Concentration in location 2 now implies an asymmetry in the model such that the wage rate in location 5 is not equal to 1, as in the above case. In fact, $w_5 < 1$ because $w_1 = 1$. We know that every firm produces the same quantity and that its f.o.b. prices are equal to the wage rates. Therefore, if f.o.b. prices in location 1 are equal to 1, f.o.b prices in location 5 must be less than 1 if firms wish to sell all the quantity that they produce (because of the higher transportation costs between location 2 and 5).

   If we write the real wage equations under the new conditions we can show (see appendix) that
$$\omega_2 > \omega_3 \text{ if and only if } \gamma < \frac{\tau(2\sigma - 1)}{\sigma} D_{23}$$
   and, for all the values of the parameters, $\omega_3 > \omega_4$. Then, the previous inequality warrants concentration in location 2. This condition is equal to the one we obtained when studying the case of concentration in the center. Moreover, it can be proved that the above condition is true independently of the locations of cities 2 and 3.

   We can therefore conclude that when exterior countries are not important, then concentration in a location takes place when the congestion cost is sufficiently small in relation to transportation cost. In other words, if exterior countries are not important and congestion cost exists, a new city can appear near the biggest one if the distance between them is small.

2. Let us now consider the case $\lambda_1 = \lambda_5 \approx \frac{1}{2}$. In this case, we saw that

$$\omega_3 > \omega_2 (= \omega_4) \text{ iff } \frac{1}{x} + x < 2 \text{ iff } (x-1)^2 < 0.$$

When exterior countries are very big, we can conclude that an even distribution of population between both borders is always an equilibrium [2].

So far we have seen the effects of foreign countries over the location of cities in the country studied in two extreme cases: $\lambda_1 = \lambda_5 \approx \frac{1}{2}$ and $\lambda_1 = \lambda_5 \approx 0$.

Unfortunately, the model is too complicated for analytically solutions in a more general case. For this reason, in what follows we shall use numerical examples. We shall assume that the population in the country studied is 20% of the world population and we will see how the different parameters affect the location and size of cities. First, we consider $\tau = 0.1$ (small transportation cost), $\gamma = 0.1$ (small congestion cost) and $\sigma = 4$ as the status quo. Secondly, we will modify the value of each parameter to see how the configuration of cities changes.

In the status quo we obtain only one stable equilibrium

$$\lambda_2^* = 30.1\% \ \lambda_3^* = 39.8\% \ \lambda_4^* = 30.1\%$$

of the national population.

In order to calculate the equilibria we can fix values for λ_3 from 0 to 1 and study the real wage differential between cities 2 and 4 ($\omega_2 - \omega_4$) against the labor force in city 2 (λ_2). For each value of λ_3 this curve allows us to find possible equilibria between cities 2 and 4 (when $\omega_2 - \omega_4 = 0$ or cases of concentration). For example, in the status quo when $\lambda_3 = 0$ we get the following curve

[2] If we conveniently adjust the values of the parameters, dispersion between the two borders can emerge as a stable equilibrium.

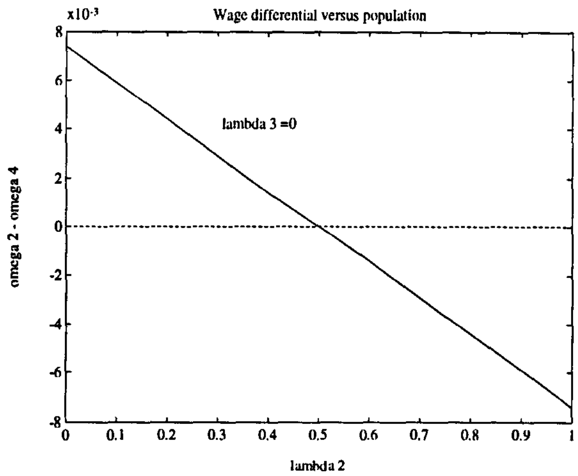

Figure 2. Example of curve $\omega_2 - \omega_4$ versus λ_2.

In this particular case we can see that the only possible equilibria is the distribution of population between the two locations.

In these points (possible equilibria) we must check if ω_3 is equal, greater or lower than ω_2 and ω_4. Let us remember that a distribution of population between cities 2, 3 and 4 is an equilibrium if all cities with a population achieve the same real wage, and if locations without a population (if population ≈ 0) offer a lower real wage than the others.

In the previous example, when we check if an even distribution between locations 2 and 4 is an equilibrium, we obtain that $\omega_3 = 0.9599 > 0.9510 = \omega_2 = \omega_4$. Therefore, even distribution between locations 2 and 4 is not an equilibrium.

With respect to (local) stability three points must be made:

- If we only consider the case of two variables, λ_i and λ_j, any interior equilibrium is stable, with respect to these variables, if the real wage differential, $\omega_i - \omega_j$ against λ_i is downward-sloping at that point. In the previous figure, even distribution is stable.

- If concentration is an equilibrium (and the real wage in that location is strictly higher than in the others) then it is stable.

- If a distribution of population between two locations is a stable equilibrium with respect to movements in these two locations, and its real wages are higher than the real wage in the location where there is no-one then it is stable.

If we only increase transportation parameter, $\tau = 0.5$ we obtain that three (locally) stable equilibria emerge:

$$\lambda_2^* = 100\% \; \lambda_3^* = 0\% \; \lambda_4^* = 0\%$$

$$\lambda_2^* = 0\% \; \lambda_3^* = 100\% \; \lambda_4^* = 0\%$$

$$\lambda_2^* = 0\% \; \lambda_3^* = 0\% \; \lambda_4^* = 100\%.$$

We can see that, when transportation costs are relative high with respect to congestion costs, concentration appears. This result is consistent with that obtained in the limit case where foreign population was close to 0. Transportation cost is therefore a centripetal force, because it favors agglomeration.

If we simply change congestion parameter, $\gamma = 0.5$ we can see that the only stable equilibrium is

$$\lambda_2^* = 32.8\% \; \lambda_3^* = 34.4\% \; \lambda_4^* = 32.8\%.$$

We now observe that the effect of congestion is bigger and this implies more dispersion between the three cities. Congestion cost is another centrifugal force, because it breaks agglomeration.

But each of these two parameters, by itself, does not imply concentration or dispersion. It is the interaction of then both, the relative value of one respect to the other, which implies concentration or dispersion. So, if we simultaneously change $\tau = 0.5$ and $\gamma = 0.5$ the resulting stable equilibrium is

$$\lambda_2^* = 33.2\% \; \lambda_3^* = 33.6\% \; \lambda_4^* = 33.2\%.$$

We have that only when the transportation parameter increases ($\tau = 0.5, \gamma = 0.1$) then concentration exists, but if the congestion parameter also increases ($\tau = 0.5, \gamma = 0.5$) then dispersion exists. We can see that concentration or dispersion emerges as the final equilibrium, as the result of interaction between transportation and congestion costs.

The effect of parameter σ (elasticity of substitution between any two goods) is less clear. On the one hand, it makes the effect of congestion less important. So, for given congestion and transportation parameters, more concentration is obtained as σ increases (as was obtained in the case of no population in foreign countries). On the other hand, when σ increases, the center is less important (as seen in the case of total population in foreign countries, the real wage in the center is progressively lower than the real wage at the borders, as the elasticity of substitution increases). We can observe these ideas in the following examples.

If $\sigma = 1.1$ the unique stable equilibrium is

$$\lambda_2^* = 31.2\% \; \lambda_3^* = 37.6\% \; \lambda_4^* = 31.2\%.$$

If $\sigma = 20$ the unique stable equilibrium is

$$\lambda_2^* = 34\% \; \lambda_3^* = 32\% \; \lambda_4^* = 34\%.$$

And when $\sigma = 40$ then we have two stable equilibria

$$\lambda_2^* = 100\% \; \lambda_3^* = 0\% \; \lambda_4^* = 0\%$$
$$\lambda_2^* = 0\% \; \lambda_3^* = 0\% \; \lambda_4^* = 100\%.$$

We have seen above that if transportation cost is relatively high with respect to congestion cost, $\tau = 0.5, \gamma = 0.1$ and$\sigma = 4$, then concentration emerges in the three possible locations because the effect of congestion is weaker.

If we make $\sigma = 1.1$ we obtain two stable equilibria, namely

$$\lambda_2^* = 60\% \; \lambda_3^* = 40\% \; \lambda_4^* = 0\%$$
$$\lambda_2^* = 0\% \; \lambda_3^* = 40\% \; \lambda_4^* = 60\%.$$

In this latter case we observe that when σ is small (close to 1) the effect of congestion is more important than when $\sigma = 4$, and more dispersion appears.

3.2. The multiple case

In this section we assume that there are seven possible locations in the interior country, locations 2 and 8 being the borders. The cities in the exterior countries are located in 1 and 9 respectively. One of the exterior countries begins in location 1 and finishes in location 2 and the other one begins in location 8 and finishes in location 9. The three countries have the same length and the distance between cities 1 and 9 is 1.

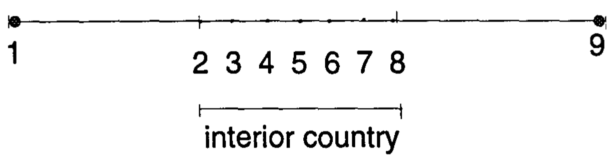

Figure 3. Case of multiple locations

As in the previous section, in our country of interest workers can move across national locations, but they cannot move to foreign cities. Given an initial distribution of the population, we are interested in knowing the long-run equilibrium. To obtain this we must use the law of motion of workers defined above. The law implies that workers move away from cities with real wages -lower the average real wage- and move to cities with real wages higher than average. There are several factors that influence this final equilibrium: transportation parameter τ, congestion cost γ, the taste for variety σ and the national population size. In the following examples we maintain national size at 20% of the world population, as in the above three-cities case, and will explain the different effects of the other parameters. Let us consider an initial distribution arbitrarily chosen

$$\lambda_2 = 8\% \ \lambda_3 = 20\% \ \lambda_4 = 4\% \ \lambda_5 = 20\% \lambda_6 = 8\% \ \lambda_7 = 22\% \ \lambda_8 = 18\%.$$

To easily understand the effects of the different parameters we will keep this initial distribution fixed at the following.

When $\tau = 0.1, \gamma = 0.1, \sigma = 4$ the long-run equilibrium is

$$\lambda_2^* = 3.62\% \ \lambda_3^* = 11.1\% \ \lambda_4 = 17.45\% \ \lambda_5^* = 20.87\% \ \lambda_6^* = 20.54\% \ \lambda_7^* = 16.56\% \ \lambda_8^* = 9.85\%.$$

The initial advantage of the right side of the country implies that the biggest cities appear there in the long equilibrium. Obviously, bigger cities have an initial advantage, which influences the final solution. But there are other factors that influence the result. For example, location 2 has the same initial size as location 6. However, in the long-run, the size of location 6 is about six times that of location 2. This is because of the size of its closest cities.

If we slightly modify $\tau = 0.11$ then the new long-run equilibrium is

$$\lambda_2^* = 0\% \ \lambda_3^* = 5.99\% \ \lambda_4 = 14.76\% \ \lambda_5^* = 21.03\% \ \lambda_6^* = 23.21\% \ \lambda_7^* = 20.74\% \ \lambda_8^* = 14.21\%.$$

Thus, it is clear that the transportation parameter favors concentration, in other words, it is a centripetal force. When $\tau = 0.1$ the biggest city has 20.87% of the national population, while this is about 23.21% when $\tau = 0.11$. And not only does the biggest city increase, but the population is agglomerated in a lower number of cities.

As we could imagine, the congestion parameter is another centrifugal force (in addition to foreign countries). Actually, it is a local centrifugal force, since its effect is to stop the growth of the biggest cities. The congestion parameter does not increase the advantage of any movement to some other market, as the foreign size effect does. We observe that the existence of congestion cost leads to patterns with different sized cities. For example, if $\gamma = 0.11$ the long-run equilibrium is

$$\lambda_2^* = 5.35\% \ \lambda_3^* = 12\% \ \lambda_4^* = 17.13\% \ \lambda_5^* = 19.97\% \ \lambda_6^* = 19.46\% \ \lambda_7^* = 15.93\% \ \lambda_8^* = 10.13\%.$$

In Krugman's model cities were the same size in the long-run equilibrium. So, we can conclude that this model presents more diverse urban configurations as possible equilibria.

With respect to the taste for variety σ, we obtain, in the above case, that when $\sigma = 4.1$ more concentracion appears. So, when $\sigma = 4.1$ the long-run equilibrium is

$$\lambda_2^* = 3.5\% \ \lambda_3^* = 11.02\% \ \lambda_4 = 17.43\% \ \lambda_5^* = 20.89\% \ \lambda_6^* = 20.6\% \ \lambda_7^* = 16.64\% \ \lambda_8^* = 9.92\%.$$

The explanation may be that when variety is less important (σ is higher) moving away from concentration can decrease the market power of the moving firm (as seen in the three cities case). On the other hand, when goods are better substitutes, the bigger the city the fewer goods it imports from others. In other words, the advantage of agglomeration increases.

4. Welfare and policy implications

In the previous section we saw that different transportation parameters involve different spatial configurations. However we have not yet discussed the different welfare levels implied in each parameter value. We assume that our country's government can undertake different policies to improve the utility of its citizens. One of them consists of carrying out investments in infrastructures such as transportation (constructing highways between cities for example) or improving urban structures (offering good public transportation within cities, among others). This kind of study is analytically discussed by Alonso Villar (1994) using a simple model with only one country. In the present paper we merely show some numerical examples of the effect of investments in transportation over national welfare. Furthermore, investments in infrastructures are not the only policy the government can implement. In this section we present some effects on the pattern configuration, and therefore on the social welfare produced by the existence of trade barriers. In fact, we only wish to remark that trade barriers can involve spatial consequences and that consequently this is a factor to take into account when the government implements this kind of policy. We will see that national policies may, undeliberately, encourage development in the biggest cities. We begin by studying the effects of improvements on transportation infrastructure within our country and later we will show what happens if we introduce trade barriers into this model.

In the following examples we can observe how the real income of our country changes with τ. We assume that when one good is delivered from a city in the interior country to a city in the foreign country (or the opposite) two transportation parameters must be taken into account: the parameter of the interior country (τ_1) and that of the exterior country (τ_2). However, we assume that in order to transport one good from a city in one of the exterior countries to the other it is not necessary to cross the interior country. So, in this case the transportation parameter used is that of the exterior countries, which we assume are equal (see the appendix B).

We shall consider the same three-cities economy, where the population of the interior country is 20% or the world population. The rest of the parameters are: $\tau_1 = \tau_2 = 0.5$, $\gamma = 0.1$, $\sigma = 4$. Using these parameter values in the long-run we get three stable equilibria: concentration in each location. The average real wage is about 0.8152 when we consider concentration in location 3 and it is about 0.8213 in the other cases. Real income in the country is the population size multiplied by the average real wage (in an equilibrium all workers obtain the same real wage, so the average real wage coincides with the real wage of every worker). Therefore, in order to compare different real incomes in the same sized country it is enough to compare their average real wages.

If the government of our country invests some resources in transportation in such a way that its new transportation parameter is now $\tau_1 = 0.1$ then, in the long-run, the following stable equilibrium emerge:

$$\lambda_2^* = 28.4\% \; \lambda_3^* = 43.2\% \; \lambda_4^* = 28.4\%.$$

Note that there is more dispersion because transportation decreases. Moreover, investment does not only affect the pattern of configuration, but also welfare. In fact, the average real wage is now 0.8732.

This is only an example of the possibility of improving in the social welfare of a country when the government carries out investments in transportation infrastructure. It would be necessary to introduce a tax into this model in order to properly study the effect of a change in infrastructure, but this is not the principal aim of our paper.

On the other hand, investments in transportation are not the only policy the government can choose. The existence of trade barriers can produce different impacts on the pattern of configuration, which depends, among other things, on the transportation infrastructure.

We assume that this barrier only affects the imports, and not the exports, of the interior country (see the appendix B). When the population in the interior country is 20% and $\tau = 0.1, \gamma = 0.1, \sigma = 4$ we already know that three cities exists in the long-run, the size of both borders being about 30.1% of the national population (central location 39.8%). If the government introduces a trade barrier such that importing goods is now more expensive than before, and if this barrier is sufficiently high, concentration increases[3].

However, if trade barrier is small, dipersion can be obtained (what differs from Livas Elizondo and Krugman (1992)). If imported goods were four times more expensive in the long-run three cities would appear, each border being about 38.7% of the national population (central location 22.6%). And when the trade barrier is higher (imported goods are ten times more expensive) then two cities of identical size emerge at the borders (50%). But, in all cases, the welfare level decreases with the existence of a trade barrier. We are not trying to make a general recommendation. The important point to make that factors such as barriers can distort urban configuration, and this must be a non negligible aspect that a government should take into account when it wants to implement a policy to improve national welfare.

5. Conclusions and extensions

In this paper we have developed a monopolistic competition model that explains the sizes and locations of cities as a consequence of centrifugal and centripetal forces. Our interest was to present a framework which allows us to study the principal causes that favor

[3]This result is consistent with that obtained by Livas Elizondo and Krugman. They justify the existence of large cities in developing countries, as is the case of Mexico D.C., by the strong backward and forward linkages that emerge from import-substituting industrialization policies.

agglomeration and those which hinder it in current societies, where peasants are not a large proportion of the total population and where international relationships substantially affect the inner structure of a country.

The centrifugal forces are due to the existence of foreign countries which our country of interest trades with, and to the effect of congestion costs. On the one hand, foreign nations make concentration in just one city difficult, because each of them constitutes an opposite center of attraction. On the other hand, they attract firms from the interior country to the borders, especially if the interior country is small. This centrifugal force underlines the effects that international trade has over the configuration of cities. Congestion costs are the local centrifugal force that stops the growth of the biggest cities, which are affected by congestion in a higher proportion.

The centripetal forces are due to transportation costs, the existence of economies of scale and the mobility of workers in the national sphere. Transportation costs make concentration easier, because a smaller share is spent in transportation when goods are concentrated in the same city. Increasing returns to scale imply that the production of each good takes place in a single location.

In this context we analyze the effects of different parameters on the urban pattern. We reveal, with some numerical examples, the improvement of society when the government carries out investments in transportation infrastructure, and we also point out the importance of a general analysis of the consequences of international relationships on the national configuration.

This framework allows us to answer some other questions posed as possible extensions. One of them is to obtain more general conclusions on the repercussions of trade barriers on urban configuration and how they depend on the differences in transportation infrastructures of the trading countries. Another unsolved question is what this model says about integration. If we were to have two interior countries instead of one, each of them with different transportation parameters and a different number of workers (and therefore, of firms), it would be interesting to know what the final location of cities would be if there were free mobility of the labor factor between them.

Appendix

A. The wage rate

We begin by solving the following problem:

$$\max \left(\sum_i {c_i^k}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$
$$\text{s.t. } \sum_i p'_{ik} c_i^k = m$$

where, c_i^k is the consumption of good i by an individual of city k, good 1 is numeraire, p'_{ik} is the c.i.f. price paid by this individual for a unit of good i, and $m = w_k$ is this individual's income.

By calculating the first order conditions we obtain that

$$c_i^k = \frac{{p'_{2k}}^{\sigma}}{{p'_{ik}}^{\sigma}} c_2^k.$$

This equation can be rewritten as follows:

$$p'_{ik} c_i^k = \frac{{p'_{2k}}^{\sigma}}{{p'_{ik}}^{\sigma-1}} c_2^k.$$

Using that aggregate consumption $C_i^k = \lambda_k c_i^k$ we can write

$$p'_{ik} C_i^k = \frac{{p'_{2k}}^{\sigma}}{{p'_{ik}}^{\sigma-1}} C_2^k.$$

We define $Y_k = \lambda_k w_k$ as the income of city k. This income is used to pay for goods consumed in this city, i.e. $Y_k = \sum_i p'_{ik} C_i^k$. Combining the above expressions yields

$$Y_k = p'_{2k} C_2^k \left(\sum_i \left(\frac{p'_{2k}}{p'_{ik}} \right)^{\sigma-1} \right).$$

Rearranging, we have

$$p'_{2k} C_2^k = \frac{Y_k {p'_{2k}}^{1-\sigma}}{\sum_j {p'_{jk}}^{1-\sigma} n_j}. \tag{4}$$

Let S_{2k} be the expenditure in city k on goods produced in city 2, namely, $S_{2k} = n_2 p'_{2k} C_2^k$ (we are identifying good 2 with any good produced in city 2).

If we introduce equation (4) into S_{2k}, then we add in k and use that $p'_{jk} = p_{jk}e^{(\tau D_{jk}+\gamma\lambda_k)}$, we have that

$$\sum_k S_{2k} = \lambda_2 \sum_k Y_k (w_2 e^{\tau D_{2k}+\gamma\lambda_k} T_k^{-1})^{1-\sigma}, \tag{5}$$

where

$$\begin{aligned} n_k &= \lambda_k n \text{ for every } k, \\ T_k &= \left[\sum_j \lambda_j (w_j e^{\tau D_{kj}+\gamma\lambda_k})^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \end{aligned}$$

On the other hand, expenditures in each city have to equalize income, which means that

$$\sum_k S_{2k} = w_2 \lambda_2. \tag{6}$$

By equalizing expressions (11)and(12), we have the following equation if and only if $\lambda_2 \neq 0$:

$$w_2 = \left[\sum_k Y_k (e^{-(\tau D_{2k}+\gamma\lambda_k)} T_k)^{\sigma-1} \right]^{\frac{1}{\sigma}}.$$

In the case where $\lambda_2 = 0$ we have to calculate the limit of w_2 when $\lambda_2 \to 0$. The continuity of functions that defines w_2 implies that the above expression is also valid when $\lambda_2 = 0$.

This proof can be repeated for a generic city j.

B. Wage rates in a more general case

Let us consider that the transportation parameter of the interior country (τ_1) differs from that of the exterior countries (τ_2). So, when a good is shipped from one city to another, we must separate the transportation cost into the cost from the initial city to the border, and the cost from the border to the final city. We assume that transportation cost between foreign cities is only affected by τ_2. There are also trade barriers to imports by the interior country that are included by using parameter ρ, as a transportation cost. The above equations obtained in appendix A can be written in the case of 7 possible locations in the country of interest as follows:

$$
\begin{aligned}
T_i &= \{\lambda_1 e^{(\tau_2 D_{i1}+\gamma\lambda_1)(1-\sigma)} + \sum_{j=2}^{8} \lambda_j \big(w_j e^{\tau_2 D_{i2}+\tau_1 D_{2j}+\gamma\lambda_i}\big)^{1-\sigma} + \lambda_9 \big(w_9 e^{\tau_2 D_{i9}+\gamma\lambda_1}\big)^{(1-\sigma)}\}^{\frac{1}{1-\sigma}} \\
T_j &= \Big\{\lambda_1 e^{(\tau_2 \rho D_{12}+\tau_1 \rho D_{2j}+\gamma\lambda_1)(1-\sigma)} + \sum_{j=2}^{8} \lambda_j \big(w_j e^{\tau_2 D_{i2}+\tau_1 D_{2j}+\gamma\lambda_i}\big)^{1-\sigma} \\
&\quad + \lambda_9 \big(w_9 e^{\tau_2 \rho D_{89}+\tau_1 \rho D_{8j}+\gamma\lambda_1}\big)^{(1-\sigma)}\Big\}^{\frac{1}{1-\sigma}} \\
w_j &= \Big\{\lambda_1 \big(e^{-(\tau_2 D_{12}+\tau_1 D_{2j}+\gamma\lambda_1)} T_1\big)^{\sigma-1} + \sum_{j=2}^{8} \lambda_j w_j \big(e^{-(\tau_2 D_{i2}+\tau_1 D_{2j}+\gamma\lambda_i)} T_j\big)^{\sigma-1} \\
&\quad + \lambda_9 w_9 \big(e^{-(\tau_2 \rho D_{89}+\tau_1 \rho D_{8j}+\gamma\lambda_1)} T_9\big)^{(\sigma-1)}\Big\}^{\frac{1}{1-\sigma}} \\
w_9 &= \Big\{\lambda_1 \big(e^{-(\tau_2 D_{12}+\gamma\lambda_1)} T_1\big)^{\sigma-1} + \sum_{j=2}^{8} \lambda_j w_j \big(e^{-(\tau_2 \rho D_{i2}+\tau_1 \rho D_{2j}+\gamma\lambda_i)} T_j\big)^{\sigma-1} \\
&\quad + \lambda_9 w_9 \big(e^{-(\tau_2 D_{89}+\gamma\lambda_1)} T_9\big)^{(\sigma-1)}\Big\}^{\frac{1}{1-\sigma}}
\end{aligned}
$$

where, $i = 1, 9$ and $j = 2, ..., 8$.

C. Calculus of the real wages in the three cities case

Using equations (1) to (3) in the case of three possible locations in interior country 2, 3 and 4 and assuming that in this country people are located in city 3 ($\lambda_2 = \lambda_4 = 0$) and that the population of foreign countries is equal, we have that

$$
\begin{aligned}
T_1 &= \big\{\lambda_1 \big(e^{\gamma\lambda_1}\big)^{1-\sigma} + \lambda_2 \big(w_2 e^{\tau D_{12}+\gamma\lambda_1}\big)^{1-\sigma} + \lambda_3 \big(w_3 e^{\tau D_{13}+\gamma\lambda_1}\big)^{1-\sigma} + \\
&\quad \lambda_4 \big(w_4 e^{\tau D_{14}+\gamma\lambda_1}\big)^{1-\sigma} + \lambda_5 \big(w_5 e^{\tau D_{15}+\gamma\lambda_1}\big)^{1-\sigma}\big\}^{\frac{1}{1-\sigma}} \\
T_2 &= \big\{\lambda_1 \big(e^{\tau D_{12}+\gamma\lambda_2}\big)^{1-\sigma} + \lambda_2 \big(w_2 e^{\gamma\lambda_2}\big)^{1-\sigma} + \lambda_3 \big(w_3 e^{\tau D_{23}\gamma\lambda_2}\big)^{1-\sigma} + \\
&\quad \lambda_4 \big(w_4 e^{\tau D_{24}+\gamma\lambda_2}\big)^{1-\sigma} + \lambda_5 \big(w_5 e^{\tau D_{25}+\gamma\lambda_2}\big)^{1-\sigma}\big\}^{\frac{1}{1-\sigma}} \\
T_3 &= \big\{\lambda_1 \big(e^{\tau D_{13}+\gamma\lambda_3}\big)^{1-\sigma} + \lambda_2 \big(w_2 e^{\tau D_{32}+\gamma\lambda_3}\big)^{1-\sigma} + \lambda_3 \big(w_3 e^{\gamma\lambda_3}\big)^{1-\sigma} +
\end{aligned}
$$

$$
\begin{aligned}
&\lambda_4\left(w_4 e^{\tau D_{34}+\gamma\lambda_3}\right)^{1-\sigma} + \lambda_5\left(w_5 e^{\tau D_{35}+\gamma\lambda_3}\right)^{1-\sigma}\Big\}^{\frac{1}{1-\sigma}} \\
T_4 &= T_2 \\
T_5 &= T_1 \\
w_1 &= 1 \\
w_2 &= \Big\{\lambda_1\left(e^{\tau D_{12}+\gamma\lambda_1}\right)^{1-\sigma} T_1^{\sigma-1} + \lambda_2 w_2 e^{\gamma\lambda_2(1-\sigma)} T_2^{\sigma-1} + \lambda_3 w_3 e^{(\tau D_{23}+\gamma\lambda_3)(1-\sigma)} T_3^{\sigma-1} \\
&\quad + \lambda_4 w_4 e^{(\tau D_{24}+\gamma\lambda_4)(1-\sigma)} T_4^{\sigma-1} + \lambda_5 w_5 e^{(\tau D_{25}+\gamma\lambda_5)(1-\sigma)} T_5^{\sigma-1}\Big\}^{\frac{1}{\sigma}} \\
w_3 &= \Big\{\lambda_1\left(e^{\tau D_{13}+\gamma\lambda_1}\right)^{1-\sigma} T_1^{\sigma-1} + \lambda_2 w_2 e^{(\tau D_{23}+\gamma\lambda_2)(1-\sigma)} T_2^{\sigma-1} + \lambda_3 w_3 e^{(\gamma\lambda_3)(1-\sigma)} T_3^{\sigma-1} + \\
&\quad \lambda_4 w_4 e^{(\tau D_{34}+\gamma\lambda_4)(1-\sigma)} T_4^{\sigma-1} + \lambda_5 w_5 e^{(\tau D_{35}+\gamma\lambda_5)(1-\sigma)} T_5^{\sigma-1}\Big\}^{\frac{1}{\sigma}}.
\end{aligned}
$$

If we introduce expressions T_1, T_2, T_3, T_4, T_5 in w_2, w_3 and take into account that $w_5 = 1$ (by symmetry) and that concentration in location 3 implies that $\lambda_2 = \lambda_4 = 0$, then we can write

$$
\begin{aligned}
w_2 &= \Big\{\lambda_1\left(e^{\tau D_{12}(1-\sigma)} + e^{\tau D_{25}(1-\sigma)}\right)\{\lambda_1 + \lambda_3 w_3^{1-\sigma} e^{\tau D_{13}(1-\sigma)} + \lambda_5 e^{\tau D_{15}(1-\sigma)}\}^{-1} \\
&\quad + \lambda_3 w_3 e^{\tau D_{23}(1-\sigma)}\{\lambda_1 e^{\tau D_{13}(1-\sigma)} + \lambda_3 w_3^{1-\sigma} + \lambda_5 e^{\tau D_{35}(1-\sigma)}\}^{-1}\Big\}^{\frac{1}{\sigma}} \\
w_3 &= \Big\{\lambda_1\left(e^{\tau D_{13}(1-\sigma)} + e^{\tau D_{35}(1-\sigma)}\right)\{\lambda_1 + \lambda_3 w_3^{1-\sigma} e^{\tau D_{13}(1-\sigma)} + \lambda_5 e^{\tau D_{35}(1-\sigma)}\}^{-1} \\
&\quad + \lambda_3 w_3\{\lambda_1 e^{\tau D_{13}(1-\sigma)} + \lambda_3 w_3^{1-\sigma} + \lambda_5 e^{\tau D_{35}(1-\sigma)}\}^{-1}\Big\}^{\frac{1}{\sigma}} \\
T_2 &= \{\lambda_1 e^{\tau D_{12}(1-\sigma)} + \lambda_3 w_3^{1-\sigma} e^{\tau D_{23}(1-\sigma)} + \lambda_5\left(e^{\tau D_{25}}\right)^{1-\sigma}\}^{\frac{1}{1-\sigma}} \\
T_3 &= \{\lambda_1 e^{(\tau D_{12}+\gamma\lambda_3)(1-\sigma)} + \lambda_3 w_3{}^{1-\sigma} e^{\gamma\lambda_3(1-\sigma)} + \lambda_5\left(e^{\tau D_{35}+\gamma\lambda_3}\right)^{1-\sigma}\}^{\frac{1}{1-\sigma}}.
\end{aligned}
$$

To obtain the real wages we must divide the rate wages by the price index. So, we can obtain

$$
\begin{aligned}
\omega_2 &= \Big\{\lambda_1\left(e^{-\tau D_{12}(\sigma-1)} + e^{-\tau D_{25}(\sigma-1)}\right)\left(\lambda_1 + \lambda_3 w_3{}^{1-\sigma} e^{\tau D_{13}(1-\sigma)} + \lambda_5 e^{\tau D_{15}(1-\sigma)}\right)^{-1} + \\
&\quad + \lambda_3 w_3 e^{-\tau D_{23}(\sigma-1)}\left(\lambda_1 e^{\tau D_{13}(1-\sigma)} + \lambda_3 w_3{}^{1-\sigma} + \lambda_5 e^{\tau D_{35}(1-\sigma)}\right)^{-1}\Big\}^{\frac{1}{\sigma}} \dots \\
&\quad \dots \{\lambda_1 e^{\tau D_{12}(1-\sigma)} + \lambda_3 w_3^{1-\sigma} e^{\tau D_{23}(1-\sigma)} + \lambda_5 e^{\tau D_{25}(1-\sigma)}\}^{\frac{1}{\sigma-1}} \\
\omega_3 &= \Big\{\lambda_1\left(e^{-\tau D_{13}(\sigma-1)} + e^{-\tau D_{35}(\sigma-1)}\right)\left(\lambda_1 + \lambda_3 w_3{}^{1-\sigma} e^{\tau D_{13}(1-\sigma)} + \lambda_5 e^{\tau D_{15}(1-\sigma)}\right)^{-1} +
\end{aligned}
$$

$$+\lambda_3 w_3(\lambda_1 e^{\tau D_{13}(1-\sigma)} + \lambda_3 {w_3}^{1-\sigma} + \lambda_5 e^{\tau D_{35}(1-\sigma)})^{-1}\Big\}^{\frac{1}{\sigma}} \ldots$$
$$\cdots \{\lambda_1 e^{(\tau D_{13}+\gamma\lambda_3)(1-\sigma)} + \lambda_3 w_3^{1-\sigma} e^{\gamma\lambda_3(1-\sigma)} + \lambda_5 e^{(\tau D_{23}+\gamma\lambda_3)(1-\sigma)}\}^{\frac{1}{\sigma-1}}.$$

Analogous steps are needed to obtain real wage equations in the case of border concentration.

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