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STRATEGIC PROFIT SHARING BETWEEN FIRMS: A WIN-WIN STRATEGY *

*Roberts Waddle*¹

Abstract

Our companion article developed a clear conceptual framework of profit sharing between two rival firms and studied the positive effects of this strategy on each firm's profit under the assumption that each firm decides unilaterally to give away voluntarily a part of its profit to its rival. This article relaxes partially this assumption by letting only one firm to share its profit whereas the other firm keeps its entire profit.

Contrary to the previous article, we show that no firm wins by adopting such an opportunistic behavior. This suggests that *profit sharing between firms is a win-win (dominant) strategy if both firms are involved and compete in prices.*

Key Words: Profit sharing, Oligopoly, Deviation, Competition.

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Strategic Profit Sharing Between Firms: A Win-Win Strategy¹

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Preliminary- Comments welcome!
(*please, do not circulate*)

¹I'm very grateful to my supervisor José Luis Ferreira for his numerous helpful suggestions. Nevertheless, all remained errors are my own.

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Abstract

Our companion article developed a clear conceptual framework of profit sharing between two rival firms and studied the positive effects of this strategy on each firm's profit under the assumption that *each firm* decides unilaterally to give away voluntarily a part of its profit to its rival. This article relaxes partially this assumption by letting *only one firm* to share its profit whereas the other firm keeps its entire profit.

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JEL Classification: C72, D21, L13.

1 Introduction

In a companion paper (Waddle 2005b), we examined whether and how sharing profits may increase the profit of two firms in a duopoly market.

Our companion paper¹ focused on such a strategy where *both firms* unilaterally decide to give away a fraction of their profits to their rivals. The purpose of this present paper is to relax partially this assumption and to allow *only one firm* to cede a part of its profit whereas the other firm keeps its entire profit and still receives a portion of its rival's profit. In other terms, we had before a two-side profit-sharing while we focus here on just one-side profit-sharing².

Contrary to the previous article, we show that no firm (neither the deviating firm, let alone the loyal firm) wins by adopting such an opportunistic behavior. This suggests that profit sharing between firms is a win-win (dominant) strategy if both firms are involved and compete in prices.

The article proceeds as follows. Section 2 presents the model. Section 3 centers on the second-stage of the game and shows that there exists a unique NE in prices. Section 4 then turns to the first-stage of the game and demonstrates the existence of a unique SPNE. Section 5 concludes with suggestions for future research.

2 The model

We consider here a model similar to the one presented in our companion paper except that we allow *only one firm* to share its profit whereas the other firm keeps its entire profit and still receives a fraction of its rival's profit.

As before, let two firms 1 and 2 in a homogeneous market and suppose that each firm incurs a cost c per unit of production. The market demand function is $q = D(p) = 1 - p$. We assume that firms do not have capacity constraints and always supply the demand they face. Therefore, the profit function of firm i is:

$$\Pi_i = \begin{cases} (p_i - c)q_i & \text{if } p_i < p_j \\ \frac{1}{2}(p_i - c)q_i & \text{if } p_i = p_j \\ 0 & \text{otherwise} \end{cases} \quad i = 1, 2 \ (i \neq j)$$

¹We refer to our companion article (Waddle 2005b) for a discussion of the relation between our work and the literature.

²The terms one-side and two-side profit sharing are inspired by the one-sided, two-sided or multi-sided markets where strategies such as "tying" is often used in at least one platform.

where q_i is the quantity demanded faced by firm i .

Let α_1 denote the part of the profit that firm 1 (the loyal firm) wants to share with firm 2 (the deviating firm). We suppose that $\alpha_1 \in]0, 1[$. Consequently, we can write the *new profit functions* $P_1(p_1(\alpha_1), p_2(\alpha_1))$ and $P_2(p_1(\alpha_1), p_2(\alpha_1))$ (hereafter P_1 and P_2) of each firm as:

$$P_1 = (1 - \alpha_1)\Pi_1(p_1(\alpha_1), p_2(\alpha_1))$$

$$P_i = \Pi_2(p_1(\alpha_1), p_2(\alpha_1)) + \alpha_1\Pi_1(p_1(\alpha_1), p_2(\alpha_1))$$

We consider a two-stage game whose sequences are thus defined. In the first stage of the game, firm 1 chooses α_1 . In the second stage of the game, firms select p_i .

In the *first stage of the game*, for α_1 firms simultaneously solve:

$$\text{Max}_{\alpha_1} \quad P_1 = (1 - \alpha_1)\Pi_1$$

$$\text{Max} \quad P_2 = \Pi_2 + \alpha_1\Pi_1$$

In the *second stage of game*, for p_1 and p_2 firms simultaneously solve:

$$\text{Max}_{p_1} \quad P_1 = (1 - \alpha_1)\Pi_1$$

$$\text{Max}_{p_2} \quad P_2 = \Pi_2 + \alpha_1\Pi_1$$

3 Solving the second-stage of the game

To find the subgame perfect Nash equilibrium (SPNE), we begin by solving subgames in the second-stage. Recall that, in the second stage, firms are looking for prices that maximize their profits.

Proposition 1 *Any prices (p_1, p_2) such that $c < p_1 = p_2 \leq p_m$ cannot be NE in the second stage of the game*

Proof. (p_1, p_2) such that $c < p_1 = p_2 \leq p_m$ are not NE if and only if at least one firm wants to deviate from those prices by fixing a price p'_i above or below. In fact:

$$c \leq p_1 = p_2 = p \leq p_m \Rightarrow \Pi_1 = \Pi_2 > 0$$

$$\Pi_1 = \frac{1}{2} (p_1 - c) (1 - p_1) = \frac{1}{2} (p - c) (1 - p)$$

$$\Pi_2 = \frac{1}{2} (p_2 - c) (1 - p_2) = \frac{1}{2} (p - c) (1 - p)$$

$$P_1 = (1 - \alpha_1) \Pi_1 = (1 - \alpha_1) \frac{1}{2} (p - c) (1 - p)$$

$$P_1 = \frac{1}{2} (1 - \alpha_1) (p - c) (1 - p)$$

$$P_2 = \frac{1}{2} (1 + \alpha_1) (p - c) (1 - p)$$

Since firms' strategies are different, we have to study separately the deviation for both firms. Let us check first for firm 1. Suppose that:

$$i) p_1 = p_2 - \varepsilon \ (\varepsilon > 0) \iff \Pi_1 = (1 - p_1) (p_1 - c) > 0 \text{ and } \Pi_2 = 0$$

$$P'_1 = (1 - \alpha_1) \Pi_1 = (1 - \alpha_1) (1 - p_1) (p_1 - c)$$

If $p_1 \leq p_m$ (monopolistic price), then $p_1 = p - \varepsilon$.

$$\text{For } \varepsilon \text{ very small}^3, P'_1 \simeq (1 - \alpha_1) (1 - p) (p - c) > P_1$$

$P'_1 > P_1 \Rightarrow$ Firm 1 would deviate. In that case, it is useless to check whether or not firm 2 will deviate. In fact, the deviation of one firm is enough to prove the non-equilibrium.

Conclusion: (p_1, p_2) such that $c < p_1 = p_2 \leq p_m$ cannot be NE in the second-stage of the game. ■

Proposition 2 *Any prices (p_i, p_j) such that $c < p_i = p_m < p_j$ cannot be NE in the second stage of the game*

³There is no reason for not to suppose that ε is very small. For instance, firms need to decrease or increase just slightly to get or to lose the entire market.

Proof. (p_1, p_2) s. t. $c < p_2 = p_m < p_1$ ($c < p_1 = p_m < p_2$) constitute a NE if and only if no firm has interest to deviate from those prices by fixing a price p'_i above or below.

$$A.- c < p_2 = p_m < p_1 \Rightarrow \Pi_1 = 0 \text{ and } \Pi_2 = (p_2 - c)(1 - p_2) > 0$$

$$P_1 = 0$$

$$P_2 = \Pi_2 = (p_2 - c)(1 - p_2)$$

Since prices p_1 and p_2 are different, we have to study separately the deviation for both firms. Let us check first for firm 1. Suppose that:

$$i) p_1 = p_2 - \varepsilon \ (\varepsilon > 0) \iff \Pi_1 = (1 - p_1)(p_1 - c) \text{ and } \Pi_2 = 0$$

$$P'_1 = (1 - \alpha_1)\Pi_1 = (1 - \alpha_1)(1 - p_1)(p_1 - c) > P_1 = 0$$

$P'_1 > P_1 \Rightarrow$ Firm 1 would deviate and therefore $c < p_2 = p_m < p_1$ cannot be a NE.

$$B.- c < p_1 = p_m < p_2 \Rightarrow \Pi_1 = (p_1 - c)(1 - p_1) \text{ and } \Pi_2 = 0$$

$$P_1 = (1 - \alpha_1)(p_1 - c)(1 - p_1)$$

$$P_2 = \alpha_1\Pi_1 = \alpha_1(p_1 - c)(1 - p_1)$$

Since prices p_1 and p_2 are different, we have to study separately the deviation for both firms. Let us check first for firm 2. Suppose that:

$$i) p_2 = p_1 - \varepsilon \ (\varepsilon > 0) \iff \Pi_2 = (1 - p_2)(p_2 - c) \text{ and } \Pi_1 = 0$$

$$P'_2 = \Pi_2 = (1 - p_2)(p_2 - c) = (1 - p_1 + \varepsilon)(p_1 - c - \varepsilon)$$

$$\text{For } \varepsilon \text{ very small, } P'_2 \simeq (1 - p_1)(p_1 - c) > P_2$$

$P'_2 > P_2 \Rightarrow$ Firm 2 would deviate and therefore $c < p_1 = p_m < p_2$ cannot be a NE.

Conclusion: Any prices (p_i, p_j) such that $c \leq p_i = p_m < p_j$ cannot be a NE in the second-stage of the game. ■

Proposition 3 *Any prices (p_1, p_2) such that $p_1 = p_2 = c$ is NE in the second stage of the game*

Proof. (p_1, p_2) s.t. $p_1 = p_2 = c$ is NE if and only if no firm has interest to deviate from those prices to fix a price p'_i above or below.

$$p_1 = p_2 = c \Rightarrow \Pi_1 = 0 \text{ and } \Pi_2 = 0$$

$$P_1 = (1 - \alpha_1) \Pi_1 = 0$$

$$P_2 = \Pi_2 + \alpha_1 \Pi_1 = 0$$

Since firms' strategies are different, we have to study separately the deviation for both firms. Let us check first for firm 1. Suppose that:

$$i) p_1 = p_2 - \varepsilon \text{ (} p_1 < p_2 \text{ and } p_1 < c) \Rightarrow \Pi_1 < 0 \text{ and } \Pi_2 = 0$$

$$P'_1 = (1 - \alpha_1) \Pi_1 < 0$$

$$\Rightarrow P'_1 < P_1 = 0 \Rightarrow \text{Firm 1 has no interest by fixing a price below } p_2$$

$$ii) p_1 = p_2 + \varepsilon \text{ (} p_1 > p_2 = c) \iff \Pi_2 = (1 - p_2) (p_2 - c) = 0 \text{ and } \Pi_1 = 0 \\ \text{(firm 1 does not produce)}$$

$$P''_1 = 0 = P_1 \Rightarrow \text{Firm 1 has no interest by fixing a price above } p_2$$

Let us check now for firm 2. Suppose that:

$$i) p_2 = p_1 - \varepsilon \text{ (} p_1 < p_2 \text{ and } p_1 < c) \Rightarrow \Pi_2 < 0 \text{ and } \Pi_1 = 0$$

$$P'_2 = \alpha_1 \Pi_1 = 0$$

$$\Rightarrow P'_2 = P_2 = 0 \Rightarrow \text{Firm 2 has no interest by fixing a price below } p_1$$

$$ii) p_2 = p_1 + \varepsilon \text{ (} p_1 > p_2 = c) \iff \Pi_1 = (1 - p_1) (p_1 - c) = 0 \text{ and } \Pi_2 = 0 \\ \text{(firm 2 does not produce)}$$

$$P''_2 = 0 = P_2 \Rightarrow \text{Firm 2 has no interest by fixing a price above } p_1$$

Conclusion: $\forall \alpha_1 \in]0, 1[$, any prices (p_1, p_2) s.t. $p_1 = p_2 = c$ constitute a NE in the second-stage of the game. ■

The second-stage being entirely solved and NE being found, we can thus move to the first-stage of the game in order to find $SPNE_a$

4 Solving the first-stage of the game

In the first-stage of the game, firms choose the α_i optimal maximizing their profit to share with their rival.

Solving backwards, we have solved the second-stage of the game in the previous section and have found the NE_a in prices summarized below:

i) $(p_1, p_2) : p_1 = p_2 = c$ if $0 < \alpha_1 < 1$ with:

$$\begin{cases} P_1 = 0 \\ P_2 = 0 \end{cases}$$

Now, in the current section, we draw our attention to the first-stage of the game searching for $SPNE_a$ in α_1 .

Proposition 4 *The strategies $(\alpha_1, p_1(\alpha_1)), p_2(\alpha_1)$ s.t.:*

- i) $\alpha_1 \in]0, 1[$
 - ii) $p_1^* = p_2^* = c$ if $0 < \alpha_1 < 1$
- are $SPNE_a$ of the game*

Proof. The strategies $(\alpha_1, p_1(\alpha_1)), p_2(\alpha_1)$ s.t. i) and ii) are satisfied, are $SPNE_a$ if and only if no firm has interest to deviate from those prices by choosing a α'_i above or below.

Let us check for firm 1. For instance, suppose that:

$$i) \alpha'_1 < \alpha_1 \Rightarrow 0 < \alpha'_1 < 1 \Rightarrow$$

$$P'_1 = 0 = P_1 \Rightarrow \text{Firm 1 does not deviate.}$$

$$ii) \alpha'_1 > \alpha_1 \Rightarrow 0 < \alpha'_1 < 1 \Rightarrow$$

$$P''_1 = 0 = P_1 \Rightarrow \text{Firm 1 does not deviate.}$$

Conclusion: The strategies $(\alpha_1, p_1(\alpha_1)), p_2(\alpha_1)$ s.t. i) and ii) are satisfied, are $SPNE_a$. ■

5 Conclusion

This paper has shown how (one-side) profit sharing between two firms in a homogeneous market may be deceitful. After all, it shed light on that such an opportunistic behavior is not at all profitable neither to the deviating firm, nor to the loyal one. It has thus suggested that our theory of (two-side) profit sharing between firms is a win-win (dominant) strategy if firms compete in prices.

There are many dimensions along which this simple model can be enriched. For instance, a natural one is the extension of our model to the Cournot, Stackelberg models and the like. *Profit Sharing Between Firms: A Lose-Lose Strategy* (Waddle 2005e) focuses on this issue.

6 References

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