UNIVERSIDAD CARLOS III DE MADRID

Tesis doctorales

DEPARTAMENTO DE ECONOMÍA

ESSAYS ON AUCTION THEORY

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# Contents

**Acknowledgements**

I am deeply grateful to M. Angeles de Frutos for her guidance, encouragement and generous help. This thesis has significantly benefited from her insightful comments and discussions.

This work has also received comments and suggestions by Rafael Crespí, Luciano de Castro, Juan José Dolado, Juan-José Ganuza, Ángel Hernando, Diego Moreno, Pau Olivella, Sander Onderstal, and Georges Siotis.

I am especially indebted to Juan-José Ganuza, whose comments on a preliminary version of the first two chapters helped me to improve substantially the interpretation of its main results.

Earlier versions of the first three chapters were presented in the XXII Jornadas de Economía Industrial (Barcelona, 2006), 2007 ENTER Jamboree (Mannheim, 2007), 22th EEA Meeting (Budapest, 2007), XV Foro de Finanzas (Palma de Mallorca, 2007), XXXIII Simposio de Análisis Económico (Granada, 2007). I would also like to thank the comments from the respective audiences.

The usual disclaimer applies.

*Küyen, Antü*
*A mi luna y mi sol, Yolanda y Gabriel.*

# Chapter 1

# How to Sell to Buyers with Crossholdings

**Abstract.** This paper characterizes the optimal selling mechanism in the presence of horizontal crossholdings. We find that the optimal mechanism imposes a discrimination policy against the stronger bidders so that the seller's expected revenue is increasing in both the common crossholding and the degree of asymmetry in crossholdings. Furthermore, it can be implemented by a sequential procedure that includes a price-preferences scheme and the possibility of an exclusive deal with the weakest bidder. We also show that a simple sequential negotiation mechanism, although suboptimal, yields a larger seller's expected revenue than both the first-price and the second-price auctions.

*Keywords*: optimal auctions, crossholdings, asymmetric auctions, private values
*JEL Classification*: C72, D44, D82, G32, G34

## 1.1 Introduction

Auctions in which bidders have crossholdings in other bidders' surplus are very frequent in practice, as there are many cases that resemble a contest with *horizontal* crossholdings. For instance, it is usual in some markets for competing firms to hold shares in one other, or for an important proportion of a company's ownership to belong to non-controller block shareholders, which in turn also hold a controlling stake in a rival company.[1]

Unlike the standard auctions, the presence of horizontal crossholdings introduces counter-value incentives on bidders because they get a payoff not only when they win, but also when they lose the auction. Since the loser bidder appropriates a proportion of the winning surplus, he cares about the valuation and the price paid by the winner bidder. Thus, losing transforms the bidder into a *minority buyer*, which induces a *less* aggressive bidding behavior from him. That is, the incentive to lose counteracts the natural bidder's incentives to raise his bid in order to obtain the object.

The previous literature has studied this kind of auction in a framework where signals are independently distributed and values may be interdependent. This literature has shown that the less aggressive bidding behavior induced by horizontal toeholds produces the classical result of revenue equivalence between standard auctions (Myerson (1981), Riley and Samuelson (1981)) no longer holding, even when bidders have symmetric crossholdings. A seller interested in maximizing her expected revenue should therefore not be indifferent with respect to the mechanism used to assign the object. Consequently, design of an optimal selling mechanism should be a very relevant question for her.

In this paper, we address this question and characterize the optimal selling mechanism in the presence of horizontal crossholdings. To this end, we follow the mechanism design methodology introduced by Myerson (1981) in a setup with independent private values and independently distributed signals. In addition, our modelling strategy allows us to study issues which have not been considered so far.

Our approach is a *normative* one, instead of a *positive* one, which has been the focus of most of the previous literature. In general, this literature compares some standard auctions in terms of the expected revenue that they yield. As mentioned before, the main conclusion is that the revenue equivalence breaks down in the presence of

[1] For the case of direct cross-ownership, Claessens et al. (1998) document the fact that other companies (non-affiliated) constitute one of the most important blockholders in the corporate ownership in various Asian countries. For the case of indirect cross-ownership, Hansen and Lott (1996) report that the portfolios held by institutional investors in the U.S. include shares in competing firms in some markets like the computer industry and the automobile industry. Similarly, Brunello et al. (2001) and Becht and Roell (1999) describe how the pyramidal groups are a very frequent structure for corporate ownership in Italy, France and Belgium.

horizontal crossholdings.[2] In contrast, we do not assume the existence of a particular auction format for exogenous reasons, but characterize how the maximizing expected revenue mechanism should be and how this mechanism could be implemented.

One of the few papers that is normative as ours is that of Chillemi (2005). He characterizes the optimal selling mechanism in the presence of horizontal crossholdings, when bidders have positive and symmetric toeholds. His results show that the optimal mechanism is such that the seller's expected revenue is increasing in the common degree of crossholdings since she can extract a higher surplus from the loser bidder. Our work generalizes these results, as we allow for two types of agents: bidders with asymmetric toeholds and bidders without toeholds. The presence of these bidders results in an optimal allocation rule with a double bias. Firstly, among bidders with positive crossholdings, the optimal mechanism discriminates against the bidder with the highest crossholding; and secondly, this mechanism discriminates in a larger degree against the bidder without crossholding. We conclude that this procedure is such that the seller's expected revenue is increasing not only in the size of a common crossholding, but also in the degree of asymmetry of these toeholds.

Consequently, when we make endogenous the bidders' decision about buying/selling crossholdings, we find that their best decision is to transfer no ownership at all. That is, ex ante identical bidders will prefer to keep this symmetry in order to avoid the discrimination policy imposed by the optimal allocation rule. A similar conclusion emerges when we compare this optimal non-transference of crossholdings with two joint bidding strategies: an illegal bid rigging and a legal consortium. In that case, we show that when the seller can design an optimal mechanism as a reaction to these agreements between bidders, the latter will also prefer to remain as symmetric players whenever the informational advantage of collusion generated by its opacity disappears.

Our results concerning the bias against the stronger bidders are analogous to those of the literature about optimal auctions with bidders asymmetrically informed (Povel and Singh (2004) and Povel and Singh (2005)). For instance, Povel and Singh (2005) analyze the case of takeover contests with a general value model that allows a private and a common value environment. They characterize the optimal selling procedure that a target company should design when it faces outside bidders (without vertical toeholds) who are asymmetrically informed, and also conclude as to the optimality of discriminating against the strongest bidder. Similarly, in this paper we find that in the presence of horizontal crossholdings, the optimal mechanism also imposes a heavier discrimination policy on the stronger players of the game. In our model, the strength of each bidder is given by a stochastic comparative advantage resultant from the degree

[2] See Chillemi (2005) and Ettinger (2002) for private values; and Dasgupta and Tsui (2004) for private and interdependent values.

in which each bidder appropriates of his own surplus. The asymmetric cross-ownership structure here assumed is therefore, a central element in explaining the properties of the optimal discrimination allocation rule, and in particular, the monotonicity of its biases with respect to the ranking of advantaged bidders. As did Povel and Singh (2005), we also prove that the optimal mechanism may also be implemented by a two-stage procedure. In the first stage, the seller invites the stronger bidders to participate in a second stage, in a modified first price auction with personalized reserve prices. If both of them reject participation, the object is awarded to the weakest bidder via an exclusive deal for a price which he will always accept. Otherwise, a modified first price auction takes place with the accepting bidders (which will always include the weakest bidder), where the discrimination policy is implemented through a price-preferences scheme.

A central property of the optimal mechanism is that it has to be able to balance out two opposite effects on seller's revenues properly. Since the discrimination policy induces the stronger bidders with high signals to reveal the truth, this enables the seller to extract more value from these bidders and thus, increase her expected revenue. However, this incentive devise is based on a threat with potential costs in terms of efficiency (and thus in terms of creation of value) if it had to be materialized. If the signals of the stronger bidder(s) are not sufficiently high so as to meet the more demanding requirements of the discrimination policy, the seller will have to carry out this threat and assign the object to a weaker bidder, with the risk that his value be smaller than those of the excluded bidder(s). In consequence, the seller's revenue may decrease due to a less ex post creation of value. Notice that it is analogous to the reserve price practice, although here the negative effect on decreasing the creation of value is less severe. This is because the eventual cost of the threat is only to sell the object to a bidder with a smaller value than the excluded bidder, but with a value larger than the seller's one. In contrast, with a reserve price, the object is withdrawn from the auction and is kept in the seller's hands, which in our model always will be worse in terms of created value.

Finally, it is shown that a more simple sequential negotiation mechanism, although suboptimal, yields a larger seller's expected revenue than both the first-price and the second-price auctions. This finding is explained by the fact that this procedure considers exclusive deals with a timing that gives priority to the stronger bidders, as an attempt to extract surplus selectively, and thus, to replicate the main property of the optimal mechanism.

The remainder of this paper proceeds as follows. Section 1.2 constructs a model of auctions with horizontal crossholdings. Section 1.3 characterizes and discusses the properties of the optimal selling mechanism from the seller's viewpoint. The effects of this procedure on the bidders' participation strategies are analyzed in the next section. The implementation of the optimal mechanism via auctions and negotiations is examined in Section 1.5. Conclusions and extensions are discussed in Section 1.6. All the proof are gathered in the Appendix.

## 1.2 The Model

We have a seller who wants to sell a single object to one of three risk-neutral bidders. The value of the object to bidder $i$ is $t_i$, which is private information, but the seller and the other bidders know that it is independently and identically distributed according to the c.d.f. $F$ with support $[\underline{t}, \overline{t}]$, density $f$ and hazard rate $H(t_i) = f(t_i)/(1 - F(t_i))$.[3] Denote by $t_0$ the seller's value, which is assumed common knowledge and normalized to zero.

A horizontal crossholding of bidder $i$ is defined as a partial participation of this bidder in another bidder's surplus, and we suppose the following ownership link structure. Bidders 1 and 2 have crossholdings in each other, and bidder 3 has no crossholdings in the other bidders' surplus. The parameter $\theta_i$ represents the share of bidder $i$ in bidder $j$'s surplus, for all $i, j = 1, 2$ and $i \neq j$. Thus, $(1 - \theta_j)$ represents the participation of bidder $i$ in his own surplus. Crossholdings are assumed common knowledge, with $1/2 > \theta_1 \geq \theta_2 \geq 0$. Finally, no ownership links between bidders and the seller are considered.

It is worthy to make some remarks about the main assumptions of the model. First, the adoption of the simplest valuation and information environment, i.e. the independent private value framework, has the following justification. Since we want to focus on the effects generated by the asymmetry stemming *only* from the different initial stakes held by each bidder, we abstract away from any other sources of asymmetry such as those caused by the valuation and information environment. Consequently, we assume identically distributed signals. For a similar reason, we also work with private valuations instead of interdependent ones. Since the presence of common values introduces an extra source of less aggressive bidding behavior -a different one from that induced by crossholdings-, we prefer to examine a simpler valuation setting in order to establish more clearly the effects of crossholdings on the optimal mechanism.[4]

Second, although at first glance, our modelling strategy regarding the number of bidders and the ownership structure seems to be very *ad hoc*, it indeed allows us to analyze, in a very simple way, matters which have not been considered so far by

[3] We focus on the regular case, i.e., increasing hazard rates, as it is standard in auction theory.

[4] Although we recognize, of course, the importance of characterizing this mechanism under a richer environment, this constitutes an extension of our basic model that should be the aim of future works.

the received literature. In fact, the scarce literature with a normative approach as our work (e.g. Chillemi (2005)) characterizes the optimal selling procedure when bidders possess *positive* and *symmetric* stakes in their rivals. In contrast, our model generalizes this analysis, as it considers two types of agents: bidders with *asymmetric* crossholdings and bidders *without* crossholdings. We shall see that these novelties concerning the ownership structure are crucial to attaining two remarkable results: to obtain an optimal discriminatory allocation rule and to identify properly the source and nature of the biases imposed by such a policy.

## 1.3 The Optimal Selling Mechanism

We restrict our attention to a special class of mechanisms: the *direct revelation mechanisms*. Denote by $t$ the vector of signals realizations, i.e., $t = (t_1, t_2, t_3)$, with support $T$. Similarly, denote by $t_{-i}$ the vector of signal realizations of all bidders except bidder $i$ and $T_{-i}$ its corresponding support. Let $p_i(t)$ be the probability with which the optimal mechanism allocates the object to bidder $i$, given the vector of reported signal realizations $t$, and let $x_i(t)$ be the payment from bidder $i$ to the seller. Let $Q_i(t_i)$ be bidder $i$'s conditional probability of winning given that he observes $t_i$, i.e., $Q_i(t_i) \equiv \int_{T_{-i}} p_i(t_i, t_{-i}) f(t_{-i}) dt_{-i}$. Bidder $i$'s expected payoff, conditional on signal $t_i$ and announcement $\widehat{t_i}$, is then given by[5]

$$U_i(\widehat{t_i}/t_i) \equiv \int_{T_{-i}} \left[(1-\theta_j)(t_i p_i - x_i) + \theta_i(t_j p_j - x_j)\right] f(t_{-i}) dt_{-i}$$

for $i, j = 1, 2$, $i \neq j$, and

$$U_3(\widehat{t_3}/t_3) \equiv \int_{T_{-3}} \left[t_3 p_3 - x_3\right] f(t_{-3}) dt_{-3}$$

for all $t_i, \widehat{t_i} \in \left[\underline{t}, \overline{t}\right]$, $i = 1, 2, 3$. We define the truthtelling payoff as $V_i(t_i) \equiv U_i(t_i/t_i)$ and the seller's expected revenue when all bidders tell the truth as

$$U_0 \equiv \sum_{i=1}^{3} \int_T x_i(t) f(t) dt$$

Following Myerson (1981) (see details in Appendix A), we can rewrite the seller's expected payoff as

$$U_0 = \sum_{i=1}^{3} \left[-V_i(\underline{t}) + \int_T c_i(t_i) p_i(t) f(t) dt\right] \tag{1.1}$$

[5] For simplicity, we have omitted the arguments of $p_i$ and $x_i$, such that $p_i = p_i(\widehat{t_i}, t_{-i})$ and $x_i = x_i(\widehat{t_i}, t_{-i})$, for all $i$.

where $c_i(t_i)$, bidder $i$'s marginal revenue, is defined as[6]

$$c_i(t_i) \equiv \begin{cases} t_i - (1-\theta_j)\frac{1}{H(t_i)} & \text{for } i,j = 1,2,\ i \neq j \\ t_3 - \frac{1}{H(t_3)} & \text{otherwise} \end{cases}$$

Hence, the optimal mechanism solves the following problem:

$$\max_{p_i, V_i(\underline{t})} \quad U_0$$

$$s.t.$$

$$V_i(\underline{t}) \geq 0, \text{ for all } i \tag{1.2}$$

$$Q_i'(t_i) \geq 0 \text{ for all } t_i \in \left[\underline{t}, \overline{t}\right] \text{ and for all } i. \tag{1.3}$$

$$\sum_{i=1}^{3} p_i(t) \leq 1 \text{ and } p_i(t) \geq 0, \text{ for all } i \text{ and for all } t \in T \tag{1.4}$$

where (1.2) is a sufficient condition for bidder $i$'s participation constraint, (1.3) is one of the two sufficient conditions for the incentive compatibility constraints of the bidders and (1.4) corresponds to the feasibility constraints. Notice that when there exist crossholdings, the bidders' reservation utilities are no longer exogenous. The reason for this is the fact that now what a bidder with positive crossholdings can get when refusing to participate in the auction depends on the rule used to assign the object among the active bidders. The seller will then take advantage of this phenomenon by designing an alternative mechanism that induces the participation constraint that maximizes her expected revenues. This can be attained by means of an *optimal threat* that allows us to find the minimum reservation utility of a bidder with crossholdings such that he prefers to participate in the auction. Given our ownership structure, this optimal threat consists of selling for sure the object to the bidder without crossholdings (bidder 3) whenever a bidder with crossholdings (either bidders 1 or 2) decides not to participate in the auction.[7] Notice that such a threat constitutes the maximum punishment against the nonparticipating bidder. In fact, the execution of the threat implies that the seller fully appropriates the nonparticipating surplus stemming from the crossholdings and thus, all bidders exhibit the same *zero* reservation utility. Notice finally that the commitment capacity of the seller is critical to the successful of the procedure, especially because of the materialization of the threat may not be ex post optimal.[8]

[6] See Bulow and Roberts (1989) for an interpretation of the bidder $i$'s marginal revenue concept.

[7] This result is formally derived in the Appendix A.

[8] The endogenous nature of the reservation utilities and its consequences for the participation constraints can also have other sources. For instance, Jehiel, Moldavanu and Stacchetti (1996, 1999) indentify a similar phenomenon when there are auctions with *externalities* between bidders. They

### 1.3.1 Optimal allocation rule

**Lemma 1** *The optimal mechanism sets $V_i(\underline{t}) = 0$ and*

$$p_i(t) = \begin{cases} 1 & \textit{if } c_i(t_i) > \max\{0, \max_{j \neq i} c_j(t_j)\} \\ 0 & \textit{otherwise} \end{cases}$$

*for all $i$, and for all $t \in T$.*

Notice that bidder $i$'s marginal revenue is higher than bidder $j$'s if and only if $t_i > z_{ij}(t_j) \equiv c_i^{-1}(c_j(t_j))$ for all $i \neq j$. Likewise, we define $t_i^* \equiv c_i^{-1}(0)$ as the threshold signal for which bidder $i$'s marginal revenue is higher than the seller's. Then, since $c_i$ and its inverse function are well-behaved, it is equivalent to say that the optimal mechanism sets $V_i(\underline{t}) = 0$ and

$$p_i(t) = \begin{cases} 1 & \text{if } t_i > \max\{t_i^*, \max_{j \neq i} z_{ij}(t_j)\} \\ 0 & \text{otherwise} \end{cases} \tag{1.5}$$

for all $i$, and for all $t \in T$.

### 1.3.2 Properties of the optimal mechanism

With horizontal crossholdings, the optimal rule implies a *discriminatory policy* as $z_{ij}(t_j) \neq t_j$.[9] By analyzing the properties of the functions $z_{ij}$, one can characterize the nature of the biases involved in the optimal mechanism and find out under which circumstances it is revenue maximizing to sell the object to each bidder. This is the content of the next lemma.

**Lemma 2** *The discriminatory policy functions $z_{ij}$ have the following properties:*
*(i) The functions $z_{3j}(t_j)$ and $z_{12}(t_2)$ are strictly increasing in $t_j$ and $t_2$, respectively.*
*(ii) The functions $z_{13}(t_3)$, $z_{23}(t_3)$ and $z_{21}(t_1)$ are non-decreasing in $t_3$ and $t_1$, respectively.*
*(iii) At $t_1 = t_2 = t_3 = \underline{t}$, $z_{32}(\underline{t}) > z_{31}(\underline{t}) > \underline{t}$, $z_{12}(\underline{t}) > z_{13}(\underline{t}) = \underline{t}$, and $z_{21}(\underline{t}) = z_{23}(\underline{t}) = \underline{t}$.*
*(iv) For all $z_{ij}(t_j)$, there exists a unique signal $t_j = \sigma > \underline{t}$ such that $z_{ij}(\sigma) = \sigma$, which*

show that a revenue maximazing procedure in this context has to include an optimal threat that induces bidders to participate in the auction by guaranting to the *critical type* (the lowest type in our case) the lowest possible reservation utility. Consequently, if the externalities are *negative*, the seller will threat with selling for sure to the bidder who imposes the worst damage to the nonparticipating bidder. In contrast, if the externalities are *positive*, the optimal threat implies that the seller keeps the object.

[9] The analysis here is analogous to McAfee and McMillan (1989). They introduce the concept of an optimal discriminatory function in the context of an asymmetric procurement when a government faces domestic and foreign firms with different comparative advantages in costs.

*is* $\sigma = \bar{t}$.

*(v)* $z_{32}(t) > z_{31}(t) > t$, $z_{12}(t) > t \geq z_{13}(t)$, *and* $z_{23}(t) \leq z_{21}(t) \leq t$, *for all* $t < \bar{t}$.

Lemma 2 describes two properties of the optimal mechanism. First, it points out that at the optimal mechanism all bidders must experience some degree of discrimination when facing another rival, either a positive or a negative one. Second, for bidders 3 and 1 there exist a non-zero probability interval of signals with which these bidders lose no matter the signal of their opponents. We discuss now the intuition and implications of these properties.

*Bias against the bidder with the highest crossholding.* Among the bidders with ownership links, the optimal mechanism is biased against the bidder with the *highest* crossholding, because he wins the object only if his signal is sufficiently higher than the signal of the other bidder with crossholding. For instance, if $t_1$ and $t_2$ are uniformly distributed in the interval $\left[\underline{t}, \bar{t}\right]$, for bidder 1 to win it is needed that $t_1 > z_{12}(t_2) = (1-\alpha_2)t_2+\alpha_2\bar{t} > t_2$, where $\alpha_2 \equiv (\theta_1-\theta_2)/(2-\theta_2)$, $0 < \alpha_2 < 1/2$. The intuition of this bias is that the bidder with the higher crossholding exhibits a larger appropriability of his own surplus, which gives him an informational advantage over his rival with the smaller crossholding.[10] Thus, bidder 1 is the strong player that has more incentives to under-report signals. The optimal mechanism then encourages this bidder to reveal high signals by imposing a discriminatory policy against him.[11]

*Bias against the bidder without crossholding.* In order to compare the treatment given by the optimal selling mechanism to bidders with and without ownership links, assume that the three bidders receive the same signal, i.e., $t_1 = t_2 = t_3 = t$. It is easy to check that $z_{32}(t) > z_{31}(t) > t$ for all $t$,[12] which implies that the optimal mechanism imposes a bias against the bidder *without* crossholding, because his probability of winning against the bidders with crossholdings is zero when all of them receive the same signal. The intuition behind this bias is that this bidder exhibits a complete appropriability of his own surplus, and thus, he is the most advantaged player of this game in terms of some informational measure. Since bidder 3 has the largest incentives to under-report high signals, the seller has to force him to tell the truth reducing his winning probability to the largest extent if he reports low signals. Hence, the seller will be more demanding with bidder 3 than any of his rivals when awarding the object.[13]

[10] Formalization of this intuition in terms of stochastic dominance is provided later on.

[11] An alternative explanation is that the bidder with the higher crossholding enjoys the higher losing surplus, and thus, has more incentives to under-report signals. In line with this interpretation, the seller could extract more losing bidder's surplus from him. Nevertheless, this interpretation no longer holds when we consider the bias imposed against the bidder without crossholding.

[12] Or equivalently, $c_2(t) > c_1(t) > c_3(t)$.

[13] An alternative interpretation is that bidder 3 does not face counter-value incentives because he does not have crossholdings at all. In consequence, he can adopt a less aggressive bidding behavior and still defeat his rivals.

*Exclusion of sufficiently low signals.* Notice that for sufficiently low signal reports, the probability of winning for bidder 1 and bidder 3 is null: since $z_{12}$ and $z_{3j}$ are strictly increasing functions, if $t_1 < z_{12}(\underline{t})$ then $p_1(t) = 0$ and if $t_3 < z_{3j}(\underline{t})$ then $p_3(t) = 0$. For instance, when the signals are uniformly distributed in the interval $\left[\underline{t}, \bar{t}\right]$, we have that $z_{12}(\underline{t}) = (1 - \alpha_2)\underline{t} + \alpha_2\bar{t}$ and $z_{3j}(\underline{t}) = (1 - \beta_j)\underline{t} + \beta_j\bar{t}$, where $\beta_j \equiv \theta_i/2$, $0 < \beta_j < 1/4$, and $i, j = 1, 2$, $i \neq j$. It is clear that these upper bounds are higher than $\underline{t}$, which means that the probability that some types of bidders 1 and 3 lose for sure is positive. However, notice that these upper bounds are *not* larger than $\bar{t}$, which implies that the optimal mechanism does not exclude completely any of these two bidders; it only ignores reported signals that are sufficiently low, and encourages them to reveal high ones (and thus to pay high transfers).

*Monotonicity of the bias.* The optimal rule sets the following ranking of *favored* bidders (in descending order): (1) the bidder with the smallest (positive) crossholding, i.e., bidder 2, (2) the bidder with the highest crossholding, i.e., bidder 1, and (3) the bidder without crossholdings, i.e., bidder 3. Notice that there is an apparent *non-monotonicity* in the discrimination introduced by the optimal rule, as this ranking is not monotonic with the ranking of bidders' crossholdings. The next proposition shows however that indeed one can identify a monotonic relationship between the degree of bias against each bidder and their level of some stochastic advantage in the game.

**Proposition 3** *At the optimal mechanism, it is verified that:*
*(i) The larger the proportion of own surplus appropriated by bidder i, the higher the stochastic advantage of this bidder in terms of hazard rate dominance.*
*(ii) As a consequence of (i), the larger the proportion of own surplus appropriated by bidder i, the heavier the discriminatory policy imposed against him.*

The proof of Proposition 1 points out that bidder $i$ will be favored against bidder $j$ if and only if the modified distribution function of his valuations is *hazard rate dominated* by the modified distribution function of his rival's valuations. This means that the higher the stochastic advantage of a bidder, the higher the degree of negative discrimination that the optimal mechanism imposes on this bidder. This interpretation of the problem allow us to restate the standard result that in an asymmetric auction the optimal rule is such that the stronger bidders are more discriminated against. Since in our model, the source of this stochastic asymmetry between bidders is the proportion of their own surplus that they retain, this implies that the optimal mechanism establishes a scheme of biases that is indeed increasing with that proportion.

*Extraction versus creation of value.* A central property of the optimal mechanism is that it induces a trade-off for the seller between extraction and creation of value. On the one hand, the discrimination policy encourages the stronger bidders with high signals to reveal the truth. This enables the seller to extract more value from these bidders and so, increase her expected revenue. On the other hand, this incentive devise is based on a threat with potential costs in terms of efficiency (and thus in terms of creation of value) whenever it has to be materialized. If the signals of the less favored bidder(s) are not sufficiently high to meet the more demanding requirements of the discrimination policy, the seller will have to execute this threat and to award the object to another bidder, with the risk that his value be smaller than that of the excluded bidder(s). In consequence, the seller revenues may decrease due to a less ex post creation of value. The optimal mechanism must therefore balance out these two opposite effects properly in order to maximize the seller's revenues. Notice that it has a similar effect to the reserve price practice, although here the negative effect on decreasing the creation of value is less severe. This is because the cost of the threat is only to sell the object to a bidder with a smaller value than the excluded bidder, but with a larger value than the seller's one. In contrast, with a reserve price, the object is withdrawn from the auction and kept in the seller's hands, which in our model is always a loss.

*Effects on the seller's expected revenue.* The optimal mechanism internalizes the fact that bidders with crossholdings want the object to be sold as they also get a share of the winning surplus whenever they lose the auction and the winner is different from the bidder without crossholdings, i.e., bidder 3. This allows the seller to extract some of the surplus from losing bidders. Furthermore, this mechanism is also sensitive to opportunities for strengthening the optimal discrimination policy given by changes in the ownership structure. Two results follow from these two phenomena. First, the seller increases her expected revenue when the intensity of a common crossholding increases because both the losing bidder's surplus is higher and a more severe bias can be imposed against bidder 3 as the comparative stochastic advantage of this bidder increases. The next proposition formalizes this result.

**Proposition 4** *If the degree of crossholding is symmetric ($\theta_1 = \theta_2 = \theta > 0$), then the seller's expected revenue is increasing in the common degree of ownership links.*

Moreover, the seller also increases her expected revenue when the degree of asymmetry in the crossholdings is higher because she can strengthen the discrimination policy against the bidder who appropriates his own surplus more, improving her ability to extract surplus *selectively* from each bidder. This is the content of the following proposition.

**Proposition 5** *Suppose that $(\theta_1 + \theta_2)$ is constant. Let us define the degree of asymmetry in crossholdings as $\Delta \equiv (\theta_1 - \theta_2)$. Then the seller's expected revenue is strictly increasing in this degree of asymmetry.*

## 1.4 Bidders' participation strategies

Whereas the last two propositions stress the positive effect that crossholdings have on the seller's expected revenues, they imply however opposite consequences from the bidders' point of view. In fact, as the game played by the seller and the bidders constitutes a zero-sum game in expected terms, these two properties induce indeed an extreme aversion toward crossholdings in bidders. Thus, if bidders without crossholdings had the alternative to transfer minority stakes of ownership between them in a previous stage to that in which the optimal procedure is implemented, they would prefer to remain with the original ownership structure. This corner solution for the case in which values are uniformly distributed in the unitary interval is established in the next proposition.

**Proposition 6** *Suppose that $t_i$ is uniformly distributed in the interval $[0, 1]$ for all $i$ and consider a game with the following timing:*
*Stage 1. Two of the bidders (say bidders 1 and 2) can unilaterally choose a couple $(\theta_i, \theta_j)$ for $i, j = 1, 2$, $i \neq j$ with $\theta_i, \theta_j \in [0, 1/2)$.*
*Stage 2. Each bidder observes a realization of his value $t_i$ and participates in an auction which corresponds to the optimal mechanism described by Lemma 1.*
*Then the Subgame Perfect Nash equilibrium of this game is such that it is optimal for these two bidders to choose $(\theta_i^*, \theta_j^*) = (0, 0)$.*

The result of Proposition 4 means that when bidders know in advance that the seller will design an optimal mechanism as a response to their ownership structure, they will anticipate this behavior and will prefer to face a mechanism that provides them with a symmetric treatment. That is, in order to avoid the biases imposed by the optimal mechanism when two of the three bidders have crossholdings, they will prefer to continue being symmetric players and thus, it will be optimal to transfer no minority ownership between them.

The next proposition compares this optimal non-transference of crossholdings strategy with two joint bidding strategies: an illegal bidding ring and a legal bidding consortium.

**Proposition 7** *Suppose that $t_i$ is uniformly distributed in the interval $[0, 1]$ for all $i = 1, 2, 3$ and consider a game with the following timing:*
*Stage 1. The seller calls for bidders to participate in an auction mechanism whose rules will be optimally designed in Stage 3.*
*Stage 2. Two of the bidders (say bidders 1 and 2) decide about three possible participation strategies: (i) Forming an illegal, efficient and equal profit-sharing bidding ring*

*($S_1^j$), (ii) Forming a legal, efficient and equal profit-sharing bidding consortium ($S_2^j$), or (iii) Unilaterally choosing a couple $(\theta_j, \theta_k)$ with $\theta_j, \theta_k \in [0, 1/2)$ ($S_3^j$); for $j, k = 1, 2$, $j \neq k$.*
*Stage 3. The seller designs and implements the optimal selling mechanism according to the observed bidders' participation strategies.*
*Stage 4. Each bidder observes a realization of his value $t_i$ and participates in the auction designed in the previous stage.*
*Then for bidder $j = 1, 2$, it is verified that:*
*1. The bidder $j$'s ex-ante truthtelling payoffs yield from each participation strategy are ranked as follows: $V_j(t_j, S_1^j) > V_j(t_j, S_3^{j*}) > V_j(t_j, S_2^j)$ with $S_3^{j*} = (\theta_j^*, \theta_k^*) = (0, 0)$.*
*2. The Subgame Perfect Nash equilibrium of this game is such that it is optimal for these two bidders to choose $S_1^j$ if the illegal collusion can not be detected. Otherwise, the optimal decision is $S_3^{j*}$.*

The illegal collusive practice dominates therefore the other strategies so long as the ring is not discovered by the seller, as it allows its members to benefit from an informational advantage.[14] Nevertheless, if this practice can be detected for sure by the seller, she will internalize this asymmetry in the optimal mechanism design stage. Notice that in that case the bidding ring becomes strategically equivalent to the consortium as both of them generate the same informational asymmetry, but the extra advantage given by the opacity of the first collusive arrangement vanishes. In consequence, bidders prefer to remain being symmetric players in order to avoid a discriminatory policy against the stronger one (either the ring or the consortium). As shown in Proposition 4, the optimal strategy for bidders in that case is the absolute non-transference of crossholdings.[15]

## 1.5 How to sell? Auctions vs. Negotiations

In this section we state two results regarding the implementation of the optimal selling mechanism. First, we show that the optimal allocation rule can be implemented using a sequential procedure based mainly on non-standard *auctions*.[16] Second, as to put this auction-based mechanism into practice may be too much complicated, we propose a simpler procedure based on sequential *negotiations*, which, although suboptimal, replicates the main properties of the optimal one.

[14] Specifically, since the relevant valuation for the efficient ring is the maximum between $t_1$ and $t_2$, the ring's valuation distribution function hazard-rate dominates the one of bidder 3.

[15] This can also happen if the illegal nature of the ring deters the bidders' participation.

[16] For the sake of simplicity, to find such an optimal mechanism we assume that $\underline{t}$ is sufficiently high such that $\underline{t}H(\underline{t}) \geq 1$. This implies that it will never be revenue maximinzing for the seller to set a reserve price, and therefore, she always will assign the object to some bidder.

### 1.5.1 The auction-based selling procedure

Our claim is that the properties of the optimal mechanism can be replicated by the following sequential procedure:

*Stage I.* Call for strong bidders and a (possible) exclusive deal.

Seller invites the strong bidders (3 and 1) to participate in a first-price auction (FPA) with *personalized* reserve prices $\underline{b_3}$ and $\underline{b_1}$, respectively. If both bidders reject participating, the object is offered exclusively to bidder 2 at a price $\underline{b_2}$ such that he will never reject the deal.

*Stage II.* Competitive bidding process with the accepting bidders.

In this stage, we may have three cases:

*II.1.* If in Stage I both bidder 1 and bidder 3 are willing to participate, there is a modified FPA between all bidders such that bidder $i$ wins if and only if $b_i > \max_{j \neq i} \widetilde{z_{ij}}(b_j)$ and loses otherwise. The functions $\widetilde{z_{ij}}$ correspond to *price-preferences* that this modified auction introduces in order to replicate the optimal discrimination policy represented by the functions $z_{ij}$ described in the previous section. Notice that thanks to the revelation principle, the optimal allocation rule is expressed in terms of signals which in practice are not observed by the seller. Thus, the price-preferences play the role of translating the optimal discrimination policy to a procedure based on bidders' information actually observed by the seller, which are the bids.[17]

*II.2.* If in Stage I only bidder 3 accepts participation, there is a modified FPA between bidder 3 and bidder 2 such that bidder 3 wins if and only if $b_3 > \widetilde{z_{32}}(b_2)$ and bidder 2 wins otherwise.

*II.3.* If in Stage I only bidder 1 accepts participation, there is a modified FPA between bidder 1 and bidder 2 such that bidder 1 wins if and only if $b_1 > \widetilde{z_{12}}(b_2)$ and bidder 2 wins otherwise.

We call this process a modified FPA not only because of the presence of personalized reserve prices, but also because the price-preferences $\widetilde{z_{ij}}$ imply that finally the winner may not be the bidder who submits the highest bid.

The optimal participation and bidding strategies of each bidder and for each stage are stated in the Appendix (see Lemma 3), where a Bayesian Nash equilibrium of this sequential mechanism is fully characterized. The following proposition shows that the mechanism proposed in fact implements the optimal one as it satisfies two conditions: (i) the lowest type bidder gets his reservation payoff, and (ii) the implicit allocation rule coincides with the optimal one.

**Proposition 8** *The sequential selling procedure is optimal.*

[17] McAfee and McMillan (1989) also analyze the implementation of the optimal discrimination policy through price-preferences in a model of asymmetric government procurements.

### 1.5.2 The negotiation-based selling procedure

Given the potential complications of putting the auction mechanism suggested into practice, it would be interesting to analyze whether another more simple procedure, although suboptimal, may replicate some of the properties of the optimal one. Furthermore, it would be useful to compare this alternative mechanism with some of the auction formats most used in the real world. In line with that analysis, we show that indeed a more simple sequential *negotiation* procedure generates higher expected revenue for the seller than both the FPA and the SPA. The following proposition illustrates this result with two bidders and uniformly distributed valuations.

**Proposition 9** *Suppose that $t_i$ is uniformly distributed in the interval $[0,1]$ for all $i = 1, 2$, and $\theta_1 > \theta_2 > 0$. Then, consider the following sequential procedure:*
*Stage I. Negotiation with bidder 1.*
*I.1. The seller makes a take it-or-leave it offer $\rho_1$ to bidder 1.*
*I.2. Bidder 1 observes a realization of his signal $t_1$ and accepts or rejects this offer. If he accepts, the object is sold to him and the game ends.*
*Stage II. Negotiation with bidder 2.*
*II.1. If bidder 1 rejects the deal, the seller makes a new take it-or-leave it offer $\rho_2$ to bidder 2.*
*II.2. Bidder 2 observes a realization of his signal $t_2$ and accepts or rejects this offer. If he accepts, the object is sold to him. Otherwise, the object is kept by the seller.*
*Then,*
*1. The Subgame Perfect Nash equilibrium of this game is such that it is optimal for the seller to set $\rho_1^* > \rho_2^*$.*
*2. At the equilibrium, this mechanism yields a larger seller's expected revenue than both the FPA and the SPA.*

The intuition behind this last finding is straightforward. Since the procedure proposed has a negotiation timing that gives priority to bidders according to their own surplus appropriated, it replicates the main property of the optimal mechanism: to impose a discriminatory policy against the stronger bidders.

From the practical point of view, the sequential procedure exhibits realistic properties, as it is frequent the use of rounds of exclusive and preferential negotiations to sell some items. This situation is especially present in the takeover contests, in which the target firm (the board of directors or a special committee) negotiates sequentially and exclusively with the possible raiders. In general, the timetable of these negotiations favors the buyer who is considered the strongest one because of some advantage like a better knowledge of the firm (for instance, a management buy-out), a participation in the target's ownership (a toehold), or something else. This implies that in the real world the seller is indeed able to commit to the rules of the mechanism, even though this may be inefficient ex post. As Povel and Singh (2006) document for the takeover battles, there exists plenty of protection devices aimed to mitigate the opportunistic behavior from the seller and thus, to sustain the deal that had been done previously.[18]

## 1.6 Concluding Remarks

We characterize the optimal selling mechanism in the presence of horizontal crossholdings, in a setting with independent private values and independently distributed signals. In this environment, the strength of each bidder is given by a stochastic comparative advantage resultant from the degree in which each bidder appropriates from his own surplus. The asymmetric cross-ownership structure here assumed is therefore, a central element to explain the properties of the optimal allocation rule. In particular, this asymmetry is crucial to the fact that this procedure discriminates against both the bidder with the highest crossholding and the bidder without crossholding, with the last bias being the most severe.

Furthermore, at the optimal mechanism the seller's expected revenue is increasing not only in the size of a common crossholding, but also in the degree of asymmetry of these crossholdings. These results have two different consequences for the participants in the auction. For the seller, this implies that she will benefit from larger cross-ownership links as it is possible to extract more surplus from the losing bidders whenever he is a bidder with crossholdings, and improve the selectivity of the discriminatory policy as crossholdings become more asymmetric. From the bidders' point of view, the main implication is that when we make endogenous their decision about buying/selling crossholdings, we find that their best decision is to transfer no ownership between them. One of the possible interpretations of this result is that the crossholdings observed in practical auctions are consequence of the fact that the mechanism used by the seller is different from the optimal one. It is likely that for simplicity, regulation issues or from ignorance, the seller decides to apply a standard auction, which in contrast to the optimal mechanism can benefit (hurt) the bidders (seller) as the cross-ownership links are higher.

[18] Some of these deal protection devices are termination fees, lock-up clauses and poison pills. Recent cases include the sale of the Norwegian Tandberg Television, and the takeover battle for the Spanish tollway operator Europistas. In both cases, the target paid a compensation for revoking a previous exclusive deal in favor of a subsequent buyer. The termination fees were USD 18 million and € 131 million, respectively (see El País, Negocios, November 19, 2006, p. 3; El Economista, August 9, 2006; Tanderberg Television Recommends Ericsson's Offer, http://www.tanderbergtv.com/newsview.ink?newsid=398; Atlanta Business Chronicle, February 26, 2007, http://atlanta.bizjournals.com/atlanta/stories/2007/daily1.html).

We show that the optimal allocation rule may be implemented by a sequential procedure that includes a price-preferences scheme and the possibility of an exclusive deal with the weakest bidder. This selling procedure counterbalances properly two opposite effects on the seller's revenues, which arise from the trade-off between extraction and creation of value induced by the optimal mechanism. Interestingly, it is also found that another more simple sequential negotiation procedure, although suboptimal, replicates the main property of the optimal mechanism and dominates the first-price and the second-price auctions in terms of seller's revenues.

The analysis performed in this paper can be extended, at least, in two directions. First, a natural issue is how the properties of the optimal mechanism could change when more complex valuation and information environments are considered, especially due to the extra source of less aggressive bidding behavior introduced by the winner's curse phenomenon. Finally, since the effects induced by vertical crossholdings on the aggressiveness of bidders are opposite those provoked by horizontal crossholdings, finding out what is the optimal selling mechanism in that case also seems to be a relevant extension.

## 1.7 Appendix

**Appendix A: The optimal mechanism problem.**

The optimal mechanism solves the following problem:

$$\max_{x_i \in \mathbb{R},\ p_i \in [0,1],\ \varphi^i \in [0,1]^2} \quad U_0 \tag{1.6}$$

$$s.t.$$

$$V_i(t_i) \ \geq \varphi^i \overline{u}^i \quad \forall t_i \in \left[\underline{t}, \overline{t}\right],\ i = 1,2,3 \tag{1.7}$$

$$V_i(t_i) \ \geq U_i(\widehat{t_i}/t_i) \quad \forall t_i, \widehat{t_i} \in \left[\underline{t}, \overline{t}\right],\ i = 1,2,3 \tag{1.8}$$

$$\sum_{i=1}^{3} p_i(t) \leq 1 \text{ and } p_i(t) \geq 0,\ i = 1,2,3, \forall t \in T \tag{1.9}$$

where (1.6) is the seller's expected revenue, (1.7) and (1.8) represent bidders' participation constraints and incentive compatibility constraints, respectively, and (1.9) corresponds to the feasibility constraints. First, notice that since there exist crossholdings, the original participation constraints consider endogenous reservation utilities that depends on the allocation rule adopted by the seller in case of non-participation of one bidder. This rule is represented by $\varphi^i = (\varphi^i_j, \varphi^i_k)$, the vector of probabilities with which the seller assigns the object to bidder $j$ or bidder $k$ if bidder $i$ does not participate in the auction. Similarly, $\overline{u}^i = (\overline{u_j}^i, \overline{u_k}^i)$ represents the vector of outside opportunity utilities of bidder $i$ when bidder $j$ or bidder $k$ gets the object. Given the cross-ownership structure assumed, it is clear that $\overline{u_2}^1 > \overline{u_3}^1$, $\overline{u_1}^2 > \overline{u_3}^2$ and $\overline{u_1}^3 = \overline{u_2}^3$. Hence, it must be optimal that $\varphi^1 = (\varphi_2^1, \varphi_3^1) = (0,1)$ and $\varphi^2 = (\varphi_1^2, \varphi_3^2) = (0,1)$. As we normalize $\overline{u_3}^1 = \overline{u_3}^2 = \overline{u_1}^3 = \overline{u_2}^3 = 0$, all of this implies that the *zero* reservation utility for all bidders is optimal as well.

Second, following Myerson (1981), it is possible to show that the incentive compatibility constraints are satisfied if and only if

(i) $\frac{\partial V_i(t_i)}{\partial t_i} = \begin{cases} (1-\theta_j)Q_i(t_i) & \text{if } \theta_j,\theta_i > 0 \\ Q_i(t_i) & \text{if } \theta_j,\theta_i = 0 \end{cases}$

for $i, j = 1, 2$, $i \neq j$

(ii) $\frac{\partial V_3(t_3)}{\partial t_3} = Q_3(t_3)$

(iii) $\frac{\partial Q_i(t_i)}{\partial t_i} \geq 0$ for all $i$.

Using these conditions, straightforward computations allow us to rewrite the seller's expected payoff and to simplify the maximization problem as presented in Section 1.3.

**Appendix B: Proofs.**

*Proof of Lemma 1* Clearly from (1.1), it is in the seller's interest to make $V_i(\underline{t}) = 0$ for all $i$ because $V_i(\underline{t}) > 0$ is suboptimal and setting $V_i(\underline{t}) < 0$ violates the Participation Constraint. On the other hand, $H'(t_i) > 0$ implies $c_i'(t_i) > 0$ and thus $\frac{\partial p_i(t)}{\partial t_i} \geq 0$ , so that $Q_i'(t_i) \geq 0$ is satisfied for all $i$. Finally, since $t_0 = 0$, the optimal allocation rule is found by comparing for a given $t = (t_1, t_2, t_3)$ the terms $c_1(t_1)$, $c_2(t_2)$ and $c_3(t_3)$ whenever they are positive. The solution sets $p_i(t) = 1$ iff $c_i(t_i) > \max\{0, \max_{j \neq i} c_j(t_j)\}$ .□

*Proof of Lemma 2* (i) We only show the claim for $z_{31}$; the remaining cases are similar and hence omitted. Notice that by definition, $z_{31}(t_1) \equiv c_3^{-1}(c_1(t_1))$. Then, $z_{31}'(t_1) = c_3^{-1\prime}(c_1(t_1))c_1'(t_1) > 0$ follows from the fact that both $c_i$ and its inverse are strictly increasing functions for all $i$.

(iii) By definition, $z_{12}(\underline{t}) \equiv c_1^{-1}(c_2(\underline{t})) > c_1^{-1}(c_3(\underline{t})) \equiv z_{13}(\underline{t})$, where the inequality follows from the fact that $c_2(\underline{t}) > c_3(\underline{t})$ and the inverse of $c_1$ is a strictly increasing function. Notice, however, that $z_{13}(\underline{t}) \equiv c_1^{-1}(c_3(\underline{t})) < c_1^{-1}(c_1(\underline{t})) = \underline{t}$, which is not possible and so, we must impose a truncation such that we define

$$z_{13}(t_3) = \begin{cases} \underline{t} & \text{if } \underline{t} \leq t_3 < z_{31}(\underline{t}) \\ c_1^{-1}(c_3(t_3)) & \text{otherwise} \end{cases}$$

Using the same arguments, we can verify that the other cases also hold, which includes the following definition for bidder 2

$$z_{2j}(t_j) = \begin{cases} \underline{t} & \text{if } \underline{t} \leq t_j < z_{j2}(\underline{t}) \\ c_2^{-1}(c_j(t_j)) & \text{otherwise} \end{cases}, \text{ for all } j \neq 2$$

(ii) According to the definitions of the discrimination policy functions provided in the proof of (iii), and using the same arguments applied in (i), the desired result follows directly.

(iv) The claim is only proved for $z_{31}$. First, from (iii) we know that $z_{31}(\underline{t}) > \underline{t}$. Second, notice that $z_{31}(\overline{t}) \equiv c_3^{-1}(c_1(\overline{t})) = \overline{t}$, where the last equality follows from the fact that $c_1(\overline{t}) = c_3(\overline{t})$. Since $z_{31}$ is a strictly increasing function, all of that implies that $z_{31}$ has a unique fixed point $\sigma = \overline{t}$.

(v) This is a direct consequence of results (i)-(iv).□

*Proof of Proposition 1* Let us define $J_i(t_i) \equiv \frac{H(t_i)}{s_i}$, the *modified* hazard rate of bidder $i$'s value distribution function, where $s_i$ is the proportion in which bidder $i$ appropriates his own surplus, i.e., $s_1 = 1 - \theta_2$, $s_2 = 1 - \theta_1$ and $s_3 = 1$. Denote by $G_i$ its corresponding c.d.f.. Since $c_i(t)$ is increasing with $t$ then $z_{ij}(t) \gtreqless t$ iff $c_i(t) \lesseqgtr c_i(z_{ij}(t)) = c_j(t)$, where the last equality follows from the implicit definition of $z_{ij}$. It is easy to check that this inequality is equivalent to $J_j(t) \gtreqless J_i(t)$ for all $t$, and for all $i \neq j$, which means that $z_{ij}(t) < t$ iff $G_j \succ_{HRD} G_i$ (i.e., $G_j$ hazard rate dominates $G_i$). Since $s_3 > s_1 > s_2$ implies that $G_3 \succ_{HRD} G_1 \succ_{HRD} G_2$, the desired result follows.□

*Proof of Proposition 2* From (1.1), when $\theta_1 = \theta_2 = \theta > 0$, we obtain that $\frac{\partial U_0}{\partial \theta} = \sum_{i=1}^{2} \int_T \left[\frac{1}{H(t_i)}\right] p_i(t) f(t) dt \geq 0$ because $H(t_i) > 0$ and $p_i(t) \geq 0$ for all $t$ and $i$.□

*Proof of Proposition 3* Given some $\theta_1$ and $\theta_2$, from conditions (i)-(iii) of Appendix A and Lemma 1, the seller's expected revenue evaluated according to the optimal mechanism is given by

$$\begin{aligned} V_0^\theta \; = & \; \sum_{\substack{i=1 \\ i\neq j}}^{2} \int_T \left[(1-\theta_j) t_i p_i(t) + \theta_i t_j p_j(t)\right] f(t) dt + \int_T t_3 p_3(t) f(t) dt \\ & - \sum_{\substack{i=1 \\ i\neq j}}^{2} (1-\theta_j) \int_{\underline{t}}^{\overline{t}} \int_{\underline{t}}^{t_i} Q_i(s_i) ds_i f(t_i) dt_i - \int_{\underline{t}}^{\overline{t}} \int_{\underline{t}}^{t_3} Q_3(s_3) ds_3 f(t_3) dt_3 \end{aligned}$$

Consider an increase and a decrease of $\varepsilon$ in $\theta_1$ and $\theta_2$ respectively, with $0 < \varepsilon < \theta_2$. Then, the seller's expected revenue if the ownership link parameters are $\widetilde{\theta}_1 = \theta_1 + \varepsilon$ and $\widetilde{\theta}_2 = \theta_2 - \varepsilon$, but she follows the optimal allocation rule for $\theta_1$ and $\theta_2$, can be reduced to

$$V_0^{\widetilde{\theta}} = V_0^\theta + \varepsilon \int_{\underline{t}}^{\overline{t}} \int_{\underline{t}}^{u} \left[Q_2(s) - Q_1(s)\right] ds f(u) du \tag{1.10}$$

where $Q_i(s) = \Pr(t_j < z_{ji}(s)) \Pr(t_i^* < s)$ for $i, j = 1, 2$ $i \neq j$. Note that $t_1^* > t_2^*$ implies that $\Pr(t_2^* < s) \geq \Pr(t_1^* < s)$, and from Lemma 2 it follows that $F(z_{21}(s)) < F(z_{12}(s)))$

for all $s \in [\underline{t}, \bar{t})$. All of that implies that $Q_1(s) \leq Q_2(s)$ for a given signal $s$. As long as the exclusion of both bidders is not possible for all $s$, the last result implies from (1.10) that $V_0^{\widetilde{\theta}} > V_0^{\theta}$. That is, the expected revenue can become larger as asymmetry increases, without changing the allocation rule. Therefore, the seller may additionally increase her expected revenue by switching to the optimal allocation rule.□

*Proof of Proposition 4* Applying backward induction, firstly we have to find the Nash equilibrium of Stage 2. Since we know this equilibrium from Lemma 1 and the fact that the optimal mechanism induces a truthtelling bidders' strategy via the incentive compatibility constraint, we only concentrate on the Nash equilibrium of the complete game. To this end, we previously need to characterize the objective function for bidder $i$ = 1,2 in Stage 1, using the equilibrium of Stage 2. From the definition of $Q_i(s_i)$, conditions (i)-(iii) of Appendix A and Lemma 1, we obtain that when $t_i$ is uniformly distributed in the unitary interval, the truthtelling payoff is given by $V_i(t_i) = (1-\theta_j)\int_{T_{-i}}[t_i - z_i(t_{-i})]\, 1_{\{t_i \geq z_i(t_{-i})\}} dt_{-i}$ where

$$\begin{aligned} z_i(t_{-i}) &= \inf\{s_i : c_i(s_i) \geq 0 \text{ and } c_i(s_i) \geq c_j(t_j) \text{ for all } j \neq i\} \\ &= \max\left\{t_i^*, (1-\alpha_j)t_j + \alpha_j, \frac{t_3}{1-\beta_i} - \frac{\beta_i}{1-\beta_i}\right\} \text{ for } i \neq j \end{aligned}$$

is the infimum of all winning values for $i$ against $t_{-i}$ and $t_i^* = (1-\theta_j)/(2-\theta_j)$, with $\alpha_j$ and $\beta_i$ defined as in Section 1.3. After integrating, we obtain the truthtelling payoff of bidder $i$ at the interim state. For the sake of presentation, we omit this expression here, but we represent it using the generic function $v_i(t_i, \theta_i, \theta_j)$ for the term $\int_{T_{-i}}[t_i - z_i(t_{-i})]\, dt_{-i}$ as follows:

$$\begin{aligned} V_i(t_i, \theta_i, \theta_j) &= (1-\theta_j)[v_i(t_i, \theta_i, \theta_j)1_{\{t_i > z_i(t_{-i}) = t_i^*\}} \\ &\quad + v_i(t_i, \theta_i, \theta_j)1_{\{t_i > z_i(t_{-i}) = (1-\alpha_j)t_j + \alpha_j\}} \\ &\quad + v_i(t_i, \theta_i, \theta_j)1_{\{t_i > z_i(t_{-i}) = t_3/(1-\beta_i) - \beta_i/(1-\beta_i)\}}] \end{aligned}$$

Taking expectation with respect to $t_i$, we get the ex-ante truthtelling payoff for bidder $i$, which we summarize as:[19]

$$V_i(\theta_i, \theta_j) \equiv E_{t_i} V_i(t_i, \theta_i, \theta_j) = \int_{t_i = t_i^*}^{1} V_i(t_i, \theta_i, \theta_j)$$

[19] The explicit expressions of $V_i(t_i, \theta_i, \theta_j)$ and $V_i(\theta_i, \theta_j)$ are available on request.

Hence, at Stage 1 bidder $i$ has to solve the following program:

$$\max_{(\theta_i,\theta_j)} V_i(\theta_i,\theta_j)$$
$$s.t.$$
$$0 \leq \theta_i < 1/2$$
$$0 \leq \theta_j < 1/2$$

for $i,j = 1,2, i \neq j$. Finally, we can check that $\frac{\partial V_i(\theta_i,\theta_j)}{\partial \theta_i} < 0$ and $\frac{\partial V_i(\theta_i,\theta_j)}{\partial \theta_j} < 0$ for all $\theta_i, \theta_j \in [0, 1/2)$. This implies that this program has only a corner solution such that $\theta_i^* = \theta_j^* = 0$, which completes the proof.$\square$

*Proof of Proposition 5* Applying backward induction, firstly we need to characterize the BNE resulting from Stage 4 for the two possible optimal selling mechanisms that can be implemented in Stage 3. Since these mechanisms satisfy the incentive compatibility constraint it will be in the best interest of the bidders to follow a truthtelling strategy. Then, we characterize the possible optimal mechanisms in Stage 3.

First, if bidders 1 and 2 decide to form an efficient consortium, the seller will design a mechanism taking into account that the relevant valuation for the coalition is $t_C = \max\{t_1, t_2\}$. Since the consortium's value distribution hazard-rate dominates the bidder 3's, the seller will design an optimal auction with asymmetric bidders so that it will impose a bias against the strongest player of the game, i.e., the consortium. Following the same methodology in Proof of Proposition 4, we obtain that the consortium's truthtelling payoff is given by $V_c(t_c) = \int_{t_3} [t_c - z_c(t_3)]\, 1_{\{t_c \geq z_c(t_3)\}} dt_3$ where[20]

$$z_c(t_3) = \max\left\{\sqrt{\frac{1}{3}}, \frac{2t_3 - 1}{2} + \frac{2\sqrt{t_3(t_3-1)+1}}{3}\right\}$$

After integrating, the consortium's truthtelling payoff at the interim state is given by

$$V_c(t_c) = \begin{cases} t_c - 0.67601 & \text{if } t_c \geq z_c(t_3) \\ 0 & \text{otherwise} \end{cases}$$

and the ex-ante consortium's truthtelling payoff is $E_{t_c} V_c(t_c) = 0.08769$. Under the equal profit-sharing rule, each partner of the consortium gets in expected terms $V_j(t_j, S_j^2) \equiv \frac{1}{2} E_{t_c} V_c(t_c) = 0.043845$.

Second, if bidders 1 and 2 decide to form an efficient (but illegal) bidding ring, the seller (and also bidder 3) is not aware of the existence of this coalition when designing the optimal mechanism. In particular, whereas the seller believes that she

[20] Notice that $c_c^{-1}(c_3(t_3)) = \frac{2t_3-1}{2} \pm \frac{2\sqrt{t_3(t_3-1)+1}}{3}$. For the computations, we only consider the positive root.

faces three symmetric bidders, the ring has an informational advantage similar to the consortium case because its relevant valuation is $t_R = \max\{t_1, t_2\}$. Thus, the seller incorrectly designs a standard optimal mechanism with symmetric bidders (Myerson (1981)). Assuming that this procedure is implemented by a second-price auction with a reserve price, the ring's truthtelling payoff is given by

$$V_R(t_R) = \int_{t_3} [t_R p_R(t_R, t_3) - x_R(t_R, t_3)] \, 1_{\{t_R \geq z_R(t_3)\}} dt_3$$

where $z_R(t_3) = \max\{\frac{1}{2}, t_3\}$,

$$p_R(t_R, t_3) = \begin{cases} 1 & \text{if } t_R \geq z_R(t_3) \\ 0 & \text{otherwise} \end{cases}$$

and

$$x_R(t_R, t_3) = \begin{cases} z_R(t_3) & \text{if } p_R(t_R, t_3) = 1 \\ 0 & \text{otherwise} \end{cases}$$

The ring's truthtelling payoff at the interim state is then given by

$$V_R(t_R) = \begin{cases} \frac{t_R^2}{2} - \frac{1}{8} & \text{if } t_R \geq z_R(t_3) \\ 0 & \text{otherwise} \end{cases}$$

and its corresponding ex-ante truthtelling payoff is $E_{t_R} V_R(t_R) = 0.140625$. Each member of the ring obtains then, in expected terms, $V_j(t_j, S_j^1) \equiv \frac{1}{2} E_{t_R} V_R(t_R) = 0.0703125$.

Third, from Proposition 4 we know that both bidders optimally choose zero crossholdings when deciding about the transfer of crossholdings between them, such that $S_3^{j*} = (\theta_j^*, \theta_k^*) = (0, 0)$ for $j, k = 1, 2,\ j \neq k$. Thus, the seller now correctly designs a standard optimal mechanism with symmetric bidders. In that case, each bidder gets $V_i(t_i) = \int_{T_{-i}} [t_i - z_i(t_{-i})] \, 1_{\{t_i \geq z_i(t_{-i})\}} dt_{-i}$ where $z_i(t_{-i}) = \max\{1/2, \max_{j \neq i} t_j\}$. The bidder $i$'s truthtelling payoff at the interim state becomes

$$V_i(t_i) = \begin{cases} \frac{t_i^3}{3} - \frac{1}{24} & \text{if } t_i \geq z_i(t_{-i}) \\ 0 & \text{otherwise} \end{cases}$$

and its corresponding ex-ante truthtelling payoff is given by $V_i(t_i, S_3^{i*}) \equiv E_{t_i} V_i(t_i) = 0.05729166$. We can therefore establish the following ranking for bidders 1 and 2: $V_j(t_j, S_1^j) > V_j(t_j, S_3^{j*}) > V_j(t_j, S_2^j)$.

Hence, it follows directly that at the participation decision stage (Stage 2) these two bidders prefer the strategy $S_1^j$ whether the existence of the bidding ring is unknown by the seller, as the coalition can take advantage of the informational asymmetry. However, if the ring can be discovered with certainty, the bidding ring's strategy becomes, from a bidder's viewpoint, similar to the consortium strategy. In that case, it is clear from the ex-ante truthelling payoffs that the zero crossholding strategy dominates both joint bidding practices.□

In order to demonstrate Proposition 6, we need previously to state the next auxiliary result.

**Lemma 10** *A Bayesian Nash Equilibrium of the sequential procedure is the following one:*
*Bidder 3' strategy. Accept participation in Stage II if and only if* $t_3 \geq z_{32}(\underline{t})$*; and in Stage II bid:*

$$b_3^{II.1}(t_3) = E\left[\max\left\{z_{32}(t_2), z_{31}(t_1)\right\} | \max\left\{z_{12}(t_2), t_1\right\} < z_{13}(t_3)\right]$$

*and*

$$b_3^{II.2}(t_3) = E\left[z_{32}(t_2)|t_2 < z_{23}(t_3)\right]$$

*Bidder 1' strategy. Accept participation in Stage II if and only if* $t_1 \geq z_{12}(\underline{t})$*; and in Stage II bid:*

$$b_1^{II.1}(t_1) = E\left[\max\left\{\frac{z_{12}(t_2) - \Theta_1 t_2}{1-\Theta_1}, z_{13}(t_3)\right\} | \max\left\{t_2, z_{23}(t_3)\right\} < z_{21}(t_1)\right]$$

*and*

$$b_1^{II.3}(t_1) = E\left[\frac{z_{12}(t_2) - \Theta_1 t_2}{1-\Theta_1}|t_2 < z_{21}(t_1)\right]$$

*Bidder 2' strategy. Accept the offer to pay* $\underline{b_2}$ *in Stage I; and in Stage II bid:*[21]

$$b_2^{II.1}(t_2) = \frac{\underline{t}L(\underline{t})}{L(t_2)} + E\left[\max\left\{\frac{z_{21}(t_1) - \Theta_2 t_1}{1-\Theta_2}, z_{23}(t_3)\right\} | \max\left\{t_1, z_{13}(t_3)\right\} < z_{12}(t_2)\right]$$

$$b_2^{II.2}(t_2) = \frac{\underline{t}F(z_{32}(\underline{t}))}{F(z_{32}(t_2))} + E\left[z_{23}(t_3)|t_3 < z_{32}(t_2)\right]$$

$$b_2^{II.3}(t_2) = \frac{\underline{t}L_1(\underline{t})}{L_1(t_2)} + E\left[\frac{z_{21}(t_1) - \Theta_2 t_1}{1-\Theta_2}|t_1 \in [z_{12}(\underline{t}), z_{12}(t_2))\right]$$

*where* $\Theta_i \equiv \frac{\theta_i}{1-\theta_j}$ *for* $i, j = 1, 2,\ i \neq j$*. These are equilibrium strategies if the seller designs a modified FPA with the following characteristics:*
$\underline{b_3} = z_{32}(\underline{t})\underline{b_1} = z_{12}(\underline{t})$
$\underline{b_2}$ *such that* $\Gamma_2(\underline{t}) = 0$
$\widetilde{z_{ij}}(b) = b_i(z_{ij}(b_j^{-1}(b)))$
*where* $\Gamma_i(.)$ *represents the bidder i's expected truthtelling payoff (i.e. the average across all stages) in this sequential mechanism.*

[21] Notice that $L(t) = F^{1-\Theta_2}(z_{12}(t))F(z_{32}(t))$ and $L_1(t) = F^{1-\Theta_2}(z_{12}(t))$.

*Proof of Lemma 10* We only demonstrate that these candidate bidding functions constitute an equilibrium for the most general case: bidder 1. Define:
$b_1^k(t_1)$, the candidate bidding function for bidder 1 in Stage $k$, as follows

$$b_1^k(t_1) = \begin{cases} b_1^{II.1}(t_1) & \text{if } t_1 > z_{12}(\underline{t}) \text{ and } t_3 > z_{32}(\underline{t}) \text{ (Stage II.1)} \\ b_1^{II.3}(t_1) & \text{if } t_1 > z_{12}(\underline{t}) \text{ and } t_3 < z_{32}(\underline{t}) \text{ (Stage II.3)} \\ 0 & \text{otherwise (Stage II.2 or Stage I)} \end{cases}$$

$q_i^k(t_i, t_{-i})$ as the probability that bidder $i$ gets the object in Stage $k$,
$\Lambda_1^k(\widehat{t_1}/t_1)$, bidder 1's expected payoff in Stage $k$ when he observes $t_1$, but plays the strategy as if his signal were $\widehat{t_1}$, as follows

$$\int_{T_{-1}} \left\{(1-\theta_2)\left[t_1 - b_1^k(\widehat{t_1})\right] q_1^k(\widehat{t_1}, t_{-1}) + \theta_1 \left[t_2 - b_2^k(t_2)\right] q_2^k(\widehat{t_1}, t_{-1})\right\} f(t_{-1})dt_{-1}$$

$\Gamma_1^k(t_1) \equiv \Lambda_1^k(t_1/t_1)$, bidder 1's truthtelling payoff in Stage $k$, and
$P_1^k(t_1) \equiv \int_{T_{-1}} q_1^k(t_1, t_{-1}) f(t_{-1})dt_{-1}$, bidder 1's probability of winning in Stage $k$, conditional on the realization $t_1$.

Let us organize this proof in two steps.

*Step 1.* Notice that the payoff function $\Gamma_1^k$ corresponds to the particular case of the truthtelling payoff function $V_1$ defined in Section 1.3 when the optimal payment is $x_1(t_1, t_{-1}) = b_1^k(t_1)$ and the optimal allocation rule is $p_1(t_1, t_{-1}) = q_1^k(t_1, t_{-1})$.[22] It follows then directly from conditions (i) and (iii) of Appendix A that the incentive compatibility constraint $\Gamma_1^k(t_1) \geq \Lambda_1^k(\widehat{t_1}/t_1)$ for all $t_1, \widehat{t_1} \in \left[\underline{t}, \overline{t}\right]$ and $k$, is satisfied if $\frac{\partial \Gamma_1^k(t_1)}{\partial t_1} = (1-\theta_2)P_1^k(t_1)$ and $\frac{\partial P_1^k(t_1)}{\partial t_1} \geq 0$ for all $k$.

*Step 2.* We show now that the strategy $b_1^k(t_1)$ satisfies these two sufficient conditions and therefore is an equilibrium bidding strategy of this game. First, notice that $b_1^k(t_1)$ is increasing for stages II.1 and II.3 and, since by construction $\widetilde{z_{ji}}$ implements the optimal allocation rule, we have that

$$P_1^k(t_1) = \begin{cases} F(z_{21}(t_1))F(z_{31}(t_1)) & \text{if } t_1 > z_{12}(\underline{t}) \text{ and } t_3 > z_{32}(\underline{t}) \text{ (Stage II.1)} \\ F(z_{21}(t_1)) & \text{if } t_1 > z_{12}(\underline{t}) \text{ and } t_3 < z_{32}(\underline{t}) \text{ (Stage II.3)} \\ 0 & \text{otherwise (Stage II.2 or Stage I)} \end{cases} \tag{1.11}$$

Notice that $\frac{\partial P_1^k(t_1)}{\partial t_1} \geq 0$ is satisfied both for each stage and across stages, as by assumption $f(z_{i1}(t_1)) > 0$, $F(z_{i1}(t_1)) > 0$ and by Lemma 2 $z_{i1}'(t_1) > 0$, for all $t_1 > z_{12}(\underline{t})$. We prove now that the second sufficient condition also holds. If $t_1 > z_{12}(\underline{t})$ and $t_3 > z_{32}(\underline{t})$ (Stage II.1), it can be checked after some computations that

[22] In particular, notice that since $b_i^k(t_1) = 0$ when $q_i^k(t_i, t_{-i}) = 0$, we can factorize the surplus of bidder 1 and 2 in terms of $q_i^k(t_i, t_{-i})$.

$\frac{\partial\Gamma_1^{II.1}(t_1)}{\partial t_1} = (1-\theta_2)F(z_{21}(t_1))F(z_{31}(t_1)) = (1-\theta_2)P_1^k(t_1)$, where the second equality follows from (1.11). Using the same argument, a similar result holds for Stage II.3. Finally, when $t_1 < z_{12}(\underline{t})$, bidder 1 does not participate in the auction. Noting that $z_{21}(t_1) = \underline{t}$ for all $t_1 < z_{12}(\underline{t})$, we can verify that $\frac{\partial\Gamma_1^k(t_1)}{\partial t_1} = 0 = (1-\theta_2)P_1^k(t_1)$, where the second equality follows from (1.11), which completes the proof.□

We are now prepared to demonstrate Proposition 6.

*Proof of Proposition 6* From Lemma 1, we know that a selling procedure is optimal if it satisfies two conditions: (1) the bidder with the lowest possible signal realization gets his reservation payoff (which in Appendix A we have showed to be optimally the same for all bidders and normalized to zero), and (2) it uses the optimal allocation rule. Notice that by construction, the sequential selling procedure satisfies both conditions. First, the payoff for either bidder with signal $\underline{t}$ is zero: (i) bidder 3 does not participate in Stage II (because $z_{32}(\underline{t}) > \underline{t}$) and thus, he gets $\Gamma_3^k(\underline{t}) = 0$ for all stage $k$; (ii) bidder 2 loses the auction for sure if some other bidder agrees to participate in Stage II and thus, he has a positive expected payoff. Otherwise, he pays $\underline{b_2}$ in the exclusive deal, which by construction, guarantees that the average payoff across all stages in the sequential mechanism for the lowest-type is $\Gamma_2(\underline{t}) = 0$; and (iii) bidder 1 neither participates in Stage II (because $z_{12}(\underline{t}) > \underline{t}$), and result (ii) also ensures that he gets in expected terms (as average across all stages) $\Gamma_1(\underline{t}) = 0$. Second, the allocation rule is the optimal one as we can check that $b_i > \widetilde{z_{ij}}(b_j)$ iff $t_i > z_{ij}(t_j)$ using the definition of $\widetilde{z_{ij}}(.)$.□

*Proof of Proposition 7* Applying backward induction, firstly we need to characterize the NE resulting from Stage II. In this stage, bidder 2 accepts the offer if $(1-\theta_1)(t_2-\rho_2) > 0$, i.e., if $t_2 > \rho_2$, and rejects otherwise. The seller therefore has to optimally choose the offer given by $\rho_2^* = \arg\max_{\rho_2}(1-\rho_2)\rho_2$. After solving, we get that $\rho_2^* = 1/2$ and the optimal seller's expected revenue from this stage is equal to 1/4.

In Stage I, bidder 1 accepts any seller's offer if his payoff is larger than the expected payoff at the equilibrium of stage II. That is, if $(1-\theta_2)(t_1-\rho_1) > E_{t_2}[\theta_1(t_2-\rho_2^*)] = \theta_1/8$, which is equivalent to the condition $t_1 > \rho_1 + \theta_1/8(1-\theta_2)$. Thus, the seller's optimal offer is characterized by

$$\rho_1^* = \arg\max_{\rho_1}\left[(1-(\rho_1+\frac{\theta_1}{8(1-\theta_2)}))\rho_1 + (\rho_1+\frac{\theta_1}{8(1-\theta_2)})\frac{1}{4}\right]$$

The solution is given by $\rho_1^* = 5/8 - \theta_1/16(1-\theta_2)$, which yields an optimal seller's expected revenue equal to $(100-\lambda(12-\lambda))/256$, where $\lambda \equiv \theta_1/(1-\theta_2)$. Hence, and since $\lambda < 1$, it is easy to verify that $\rho_1^* > \rho_2^*$, which proves the first statement of the proposition.

In order to show the second result, notice that in the presence of asymmetric crossholdings it is not possible to find an analytical expression for the equilibrium bidding strategy in both FPA and SPA, and thereby, it is neither possible to obtain a closed expression for the seller's revenue (see Section 5, Dasgupta and Tsui (2004)). Notice however that we can perform a comparison with both FPA and SPA without crossholdings, which yield a larger expected revenue than their versions with crossholdings due to the fact that these ownership links hurt the seller (see Proposition 1 and Section 4, Chillemi (2005)). So, it is enough to show that the expected revenue of the sequential mechanism proposed exceeds the expected revenue for both FPA and SPA without crossholdings, which thanks to the Revenue Equivalence Theorem is the same for both auction formats and equal to 1/3. Since $\lambda < 1$, the worst case for our sequential negotiation mechanism is when $\lambda \to 1$, in which case the expected revenue for the seller converges to $89/256 > 1/3$, implying that the second part of the proposition holds.□

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# Chapter 2

# Optimal Takeover Contests with Toeholds

**Abstract.** This paper characterizes how a target firm should be sold when the possible buyers (bidders) have prior stakes in its ownership (toeholds). We find that the optimal mechanism needs to be implemented by a non-standard auction which imposes a bias against bidders with high toeholds. This discriminatory procedure is so that the target's average sale price is increasing in both the size of the common toehold and the degree of asymmetry in these stakes. It is also shown that a simple mechanism of sequential negotiation replicates the main properties of the optimal procedure and yields a higher average selling price than the standard auctions commonly used in takeover battles.

*Keywords*: optimal auctions, takeovers, toeholds, asymmetric auctions
*JEL Classification*: C72, D44, D82, G32, G34

## 2.1 Introduction

Auctions in which bidders have stakes in the seller's surplus are not rare in the real world, as there are many examples that resemble a bidding competition with vertical toeholds. Takeover contests provide one of the clearest illustrations, since block shareholders compete among themselves or with an outside investor to gain the control of a company, while the minority shareholders play the role of *pure* sellers.[1]

In order to illustrate some of the features of takeover battles, consider the next real life example. In 2006, the Spanish tollway operator Europistas was the target of a takeover battle between two bidders. Firstly, the group Isolux Corsán submitted an offer for 100% of the ownership, consisting of 4.8 euros per share. At this stage, Cintra, one of the principal block shareholders of the target firm, attained an agreement with Isolux. According to the deal, Cintra committed itself to participate in this tender offer and sell irrevocably its 27.1 per cent stake for a price of 5.13 euros per share. In less than 24 hours, a second buyer emerged: a bidding consortium formed by the constructor conglomerate Sacyr Vallehermoso and three Basque saving banks grouped in the society Telekutxa. While Isolux Corsán was an outside bidder, Telekutxa held a 32.4 per cent stake in the capital of Europistas. The final tender offer of this consortium rose to 9.15 euros per share, that is, an improvement of 78.36% with respect to the first offer. This implied that Cintra was trapped in the pre-sale agreement reached with Isolux, which impeded it from taking advantage of the substantially better tender offer made by the consortium led by Sacyr. Finally, Cintra paid 131 million euros to Isolux as a compensation to recover its freedom to sell its stake to the bidding consortium, which was the winner of the contest and thus, took over Europistas.

This case highlights some interesting issues. First, unlike standard auctions, the presence of vertical toeholds introduces countervailing incentives on bidders because they can get a payoff not only when they win, but also when they lose the auction. In fact, since the losing bidder owns a proportion of the seller's surplus, he cares about the payment *received* by the seller. In the context of a takeover battle, as the winner bidder must buy all the shares, losing transforms a bidder with a toehold into a *minority seller*. This implies that, conditional on losing, a toehold induces a *more* aggressive bidding behavior. In addition, holding stakes in the target firm also means, by comparison with the outside bidders, lower costs of overbidding when winning, as the amount of shares to be bought is smaller. Consequently, toeholds strengthen the standard incentive to increase bids present in any auction, but now with the intention

[1] Other examples are creditors' bidding in bankruptcy auctions, or the negotiation of a partnership's dissolution. Also, a situation in which firms are related vertically, e.g. if a buyer firm hold shares in a supplier firm.

of selling at a higher price.[2] Second, the aforementioned Europistas case illustrates the large costs that an incorrect choice of selling procedure may impose on the nonbidding shareholders' wealth.[3] Nonbidding shareholders of a target company - by means of the board of directors or a special committee - should therefore pay attention to the selling mechanism to be used.

The auction literature has studied takeovers using different valuation environments, but assuming always that signals are independently distributed. The main conclusion is that the more aggressive bidding behavior induced by toeholds leads to the break-down of the Revenue Equivalence Theorem (Myerson 1981, Riley and Samuelson 1981) even when bidders possess symmetric stakes. As a result, the equivalence between standard auctions no longer holds, as several papers have shown. In particular, Singh (1998), when analyzing a game in which a toehold bidder and an outside bidder compete to gain control of a company in a private values framework, has shown the superiority of the second-price auction over the first-price auction. The major insight stemming from his model is what he calls the *owner's curse*. According to this phenomenon, the higher aggressiveness of the toeholder is so that in the second-price auction he is (rationally) willing to bid more than his valuation. Since this overbidding behavior is absent in the first-price auction due to the traditional trade-off present in this mechanism, the non-equivalence between both standard auctions emerges.[4] Bulow, Huang and Klemperer (1999) also study a two-bidder takeover contest, but under a common value set-up.[5] They compare the sealed-bid first-price and the ascending-price (equivalent to the second-price one) auctions in both the symmetric and the asymmetric cases. They show that with symmetric toeholds, the ascending auction performs better than the first-price auction in terms of the expected selling price per share. In contrast, when analyzing the asymmetric case, they find the opposite result whenever toeholds are very asymmetric and sufficiently small.[6]

The current paper also deals with the issue of how to run a takeover battle. But in sharp contrast with the previous literature, our work is, to the best of our knowledge,

[2] In the context of the Europistas case, it is possible to conjecture about the source of the large price difference observed between the two offers. It seems plausible to argue that this gap reflected not only a higher valuation from the *toehold bidder* (the consortium headed by Sacyr), but also a more aggressive bidding behavior than that exhibited by the *outside bidder* (Isolux).

[3] The price difference of both tender offers (147 millions of euros) represented about eight times the annual net profits of Cintra.

[4] Ettinger (2002) confirms the dominance of the second-price auction over the first-price auction in terms of the expected sale price when buyers have *symmetric* stakes in the seller's surplus.

[5] They study takeovers among financial bidders for which, as the authors point out, the common values environment seems more appropiate.

[6] These contrasting findings rest on two facts. First, the negative effect of the winner's curse on bidders' aggressiveness is larger in asymmetric ownership structures. Second, the first-price auction involves an allocation rule that is less sensitive to the distortions caused by the presence of toeholds.

pioneering in that it adopts a *normative* approach rather than a positive one. That is, instead of taking a particular auction format as given for exogenous reasons, we analyze how the maximizing target price mechanism should be and how it could be implemented. To this end, our methodology follows the mechanism design approach, introduced by Myerson (1981), within an independent private values framework.

Two main features of our model are the possibility of asymmetry among bidders' toeholds and the existence of a bidder without toeholds (outside bidder). The analysis performed here is in close connection with Loyola (2007), a companion paper that characterizes the optimal mechanism in the presence of horizontal crossholdings, i.e., toeholds in other bidders' profits. In contrast with this case, we find that in the presence of vertical toeholds, the optimal allocation rule imposes no bias against any bidder as the presence of vertical toeholds only links the bidders' *payments*, but *not* the bidders' *valuations*. As a consequence, a maximizing revenue seller prefers a symmetric equilibrium even though buyers hold asymmetric stakes. It is shown however that this optimal rule needs to be implemented by a non-standard auction. In particular, we prove that the implementation is possible through a second-price auction augmented with a reserve price and a scheme of asymmetric payments. The latter includes a penalty against the winner (with respect to the non-toehold case) and a payment by the loser whenever he is a toehold bidder. The reason for this apparent contradiction between the allocation rule and the scheme of payments is the same as that behind the failure of the Revenue Equivalence Principle. That is, the presence of toeholds implies the impossibility of fully characterizing the revenues based only on the allocation rule and the payment made by the lowest-type bidder. With toeholds, the *entire* system of transfers plays a role when it becomes to characterizing revenues.

Our discriminatory policy has the following rationale. By imposing a heavier bias against the toehold bidder, the optimal mechanism extracts more surplus from the strongest player in the game. In the context of takeovers, this advantaged player corresponds to the raider who bids more aggressively due to his larger stake in the target. As a result, the discriminatory rule pays the seller, as we show that the expected selling price is increasing in both the common toehold (the symmetric case) and in the degree of asymmetry in these stakes (the asymmetric case).[7] In addition, we show that a sequential negotiation procedure replicates the main properties of the optimal mechanism. This negotiation-based procedure sets an agenda of take-it-or-leave-it offers that gives priority to the more aggressive bidder, i.e., the toeholder, and thus yields a higher expected sale price than both the first-price and the second-price auctions.[8]

[7] This revenue-increasing property of an optimal discriminatory policy has also been found in contests with asymmetric informed buyers (see Povel and Singh 2004, Povel and Singh 2006).

[8] In light of this finding, the Europistas case provides then a clear example of how things should

The last result is in line with the established superiority of sequential mechanisms which give priority to stronger bidders. Povel and Singh (2006), for instance, analyze takeover contests under a general value setting that allows both private and common value environments. They characterize the optimal selling procedure that a target company should design when it faces two outside bidders (without toeholds) who are asymmetrically informed. Interestingly, Povel and Singh also conclude about the optimality of imposing a heavier bias against the strongest bidder (the better-informed one) by means of a two-stage procedure. Similarly, Dasgupta and Tsui (2003) examine in an interdependent value setting the properties of the "matching auction", a sequential procedure where the first mover is also the strong bidder. In their model, the strong player can be either the larger-toehold bidder or the better-informed one. As with our sequential procedure, Dasgupta and Tsui also find that the matching auction allows the target's seller to obtain a higher expected transaction price than with the standard auctions, but only when asymmetry is sufficiently large. An important difference between the last two papers and ours, apart from the valuation environment adopted, lies in the mechanism itself, which implies bidders' participation strategies of different nature. Povel and Singh (2006) propose a hybrid sequential procedure that combines standard auctions and exclusive deals. Similarly, in the auction-based mechanism studied by Dasgupta and Tsui (2003), the bidder moving first *actively* follows a bid strategy, whereas the one moving second only decides whether or not match this bid. In contrast, our procedure is based upon a scheme of take-it or leave-it offers made by the seller so that all bidders are in some sense *passive* players.

This paper proceeds as follows. Section 2.2 sets up a model of takeover contests in the presence of toeholds. In Section 2.3, the optimal selling mechanism is characterized and its main properties are established. In Section 2.4, we propose a simple negotiation procedure that replicates most of these properties. The next section compares this negotiation-based mechanism with the auction formats commonly used in practice. Finally, in Section 2.6 we conclude and stress some policy implications. All the proofs are collected in the Appendix.

## 2.2 The Model

The nonbidding shareholders of a target company (the *seller*), represented by the board of directors or a special committee, face a takeover threat from two possible risk-

*not* be done when selling a target firm in which one of the shareholders could become a bidder. Of course, in this case the nonbidding shareholder (Cintra) chose incorrectly to negotiate and close a deal first with the outside bidder (Isolux) instead of doing it previously with the toehold bidder (the consortium). In this paper we show that an appropriate sequential negotiation mechanism should take the opposed order of negotiations.

neutral buyers (the *bidders*). The value of the target to bidder $i$ is $t_i$, which is private information, but it is common knowledge that it is independently and identically drawn according to c.d.f. $F$ with support $[\underline{t}, \overline{t}]$, density $f$ and hazard rate $H(t_i) = f(t_i)/(1 - F(t_i))$.[9] We denote the value that the initial shareholders assign to the target company by $t_0$, which is common knowledge and is here normalized to zero.[10]

A toehold of bidder $i$ is defined as a partial participation of this bidder in the seller's surplus, or, equivalently, a partial participation in the ownership of the target company. We assume that bidder 1 has a larger initial stake in the seller's surplus than bidder 2. The parameter $\phi_i$ represents the share of bidder $i$ in the seller's surplus. Thus, $(1 - \phi_1 - \phi_2)$ represents the participation of the seller in her surplus. Toeholds are assumed common knowledge, with $1/2 > \phi_1 \geq \phi_2 \geq 0$.[11]

We will also refer to the players as follows: a bidder with toehold as a *bidding shareholder* (or *toehold bidder*), a bidder without toehold as an *outside bidder* (or *non-toehold bidder*) and the seller as the *nonbidding shareholder.* Given this ownership structure, we interpret $t_0$ as the common value that all shareholders assign to the firm when they own it partially. In other words, $t_0$ represents the value that all shareholders assign to the firm under the current management, i.e., either before the takeover takes place or when this process is finally unsuccessful. In contrast, we understand $t_i$ to be the private value that bidder $i$ assigns to the target when he owns it fully. In consequence $t_i$ can be interpreted as a private synergy that bidder $i$ can exploit when he wins the contest and obtains absolute control of the company. It is also called the value "to run the firm".[12] Implicit in this interpretation is the assumption that the takeovers modeled in the present paper are not partial. That is, all shareholders must sell their stakes to the winning contestant (and he must buy it) according to the price stated by the contest's rules.

## 2.3 The Optimal Mechanism

Due to to the revelation principle, we only need to focus on direct revelation mechanisms. We denote the vector of signal realizations of all bidders by $t$, i.e., $t = (t_1, t_2)$,

[9] As it is standard in auction theory, we concentrate on the regular case, that is, increasing hazard rates.

[10] As we will see below, the seller may not be an exclusive initial owner.

[11] Notice that this formulation allows the presence of an outside bidder (non-toeholder), which is precisely the case analyzed in Section 2.5, given its predominance in actual takeovers. Bradley et al. (1988) find that 66% of the bidders in their sample of 236 successful tender offers have zero toeholds, while Betton and Eckbo (2000) find that 47% of initial bidders in their sample of over 1,300 tender offers (including failed ones) have zero toeholds (see Goldman and Qian 2005).

[12] Alternatively, since we have normalized $t_0 = 0$, $t_i$ can be interpreted as an incremental cash flow generated by the new control and management under bidder $i$ (See Singh 1998).

and similarly, denote by $t_{-i}$ the vector of signal realizations of all bidders except bidder $i$. Let $T$ and $T_{-i}$ be the support of $t$ and $t_{-i}$, respectively.[13] Let us define $p_i(t)$ as the probability with which the optimal mechanism allocates the target company to bidder $i$, conditional on the vector of reported signal realizations $t$, and, define $x_i(t)$ as the transfer from bidder $i$ to the seller, conditional on the same vector. Let $Q_i(t_i)$ be bidder $i$'s conditional probability of winning given that his type is $t_i$, i.e., $Q_i(t_i) \equiv \int_{T_{-i}} p_i(t_i, t_{-i}) f(t_{-i}) dt_{-i}$. Bidder $i$'s expected payoff, conditional on signal $t_i$ and announcement $\widehat{t_i}$, is then given by[14]

$$U_i(\widehat{t_i}/t_i) \equiv \int_{T_{-i}} \left[ (t_i p_i - (1-\phi_i) x_i) + \phi_i x_j \right] f(t_{-i}) dt_{-i}$$

for all $t_i, \widehat{t_i} \in \left[\underline{t}, \overline{t}\right]$ and for $i, j = 1, 2, i \neq j$. We define bidder $i$'s truthtelling payoff as $V_i(t_i) \equiv U_i(t_i/t_i)$ and the seller's expected revenue when all bidders report their true type as follows[15]

$$U_0 \equiv \sum_{i=1}^{2} \int_T (1 - \phi_1 - \phi_2) x_i(t) f(t) dt. \tag{2.1}$$

Let us define $c_i(t_i)$, bidder $i$'s marginal revenue,[16] as

$$c_i(t_i) \equiv t_i - \frac{1}{H(t_i)} \text{ for all } i.$$

Following Myerson (1981) (see more details in the Appendix), it can be shown that the optimal mechanism solves the following problem:[17]

$$\max_{p_i, V_i(\underline{t})} \sum_{i=1}^{2} \left[ -V_i(\underline{t}) + \int_T c_i(t_i) p_i(t) f(t) dt \right] \tag{2.2}$$

$$s.t.$$

$$V_i(\underline{t}) \geq 0, \text{ for all } i. \tag{2.3}$$

$$Q_i'(t_i) \geq 0 \text{ for all } t_i \in \left[\underline{t}, \overline{t}\right] \text{ and for all } i. \tag{2.4}$$

$$\sum_{i=1}^{2} p_i(t) \leq 1 \text{ and } p_i(t) \geq 0, \text{ for all } i \text{ and for all } t \in T, \tag{2.5}$$

[13] In our set-up $t_{-i}$ is just $t_j$. We have opted for the notation $t_{-i}$ since the characterization of the optimal mechanism can be easily extended to the case of more than two bidders. For the three-bidder case (two asymmetric toeholders and one outside bidder) the characterization can be obtained from the author upon request.

[14] For the sake of presentation, we have omitted the arguments of $p_i$ and $x_i$, but it should be clear that $p_i = p_i(\widehat{t_i}, t_{-i})$ and $x_i = x_i(\widehat{t_i}, t_{-i})$, for all $i$.

[15] This function is similar to that defined for the nonbidding shareholders by Bulow, Huang and Klemperer (1999) in the context of a takeover contest with *common* values.

[16] Bulow and Roberts (1989) provide an interpretation of $c_i(t_i)$ as the bidder $i$'s *marginal revenue*, instead of the bidder $i$'s *virtual valuation* concept defined by Myerson (1981).

[17] Notice that this problem is identical to the optimization program in Myerson (1981), who does *not* consider the presence of toeholds.

where (2.3) is a sufficient condition for bidder $i$'s participation constraint to hold, (2.4) is a sufficient condition for the incentive compatibility constraints of the bidders to hold and (2.5) corresponds to the feasibility constraints.

### 2.3.1 Optimal allocation rule

**Lemma 11** *The optimal mechanism sets* $V_i(\underline{t}) = 0$ *and*

$$p_i(t) = \begin{cases} 1 & \text{if } c_i(t_i) > \max\{0, \max_{j\neq i} c_j(t_j)\} \\ 0 & \text{otherwise} \end{cases}$$

*for all* $i$*, and for all* $t \in T$*.*

Note that bidder $i$'s marginal revenue is larger than bidder $j$'s if and only if $t_i > z_{ij}(t_j) \equiv c_i^{-1}(c_j(t_j))$ for all $i \neq j$. In addition, let us define $t_i^* \equiv c_i^{-1}(0)$ as the threshold signal for which bidder $i$'s marginal revenue is larger than the seller's. Since $c_i$ is well-behaved, so it is its inverse function, and thus it is equivalent to say that the optimal mechanism sets $V_i(\underline{t}) = 0$ and

$$p_i(t) = \begin{cases} 1 & \text{if } t_i > \max\{t_i^*, \max_{j\neq i} z_{ij}(t_j)\} \\ 0 & \text{otherwise} \end{cases} \tag{2.6}$$

for all $i$, and for all $t \in T$.

Lemma 11 establishes that, in the presence of vertical toeholds, the optimal allocation rule is *not* a discriminatory one as the policy function satisfies that $z_{ij}(t_j) = t_j$ as $c_i(.) = c(.)$ for all bidders.[18] This implies that even though bidders possess asymmetric toeholds, it is revenue maximizing for the nonbidding shareholders to offer them the same chances of winning whenever they report the same signal value. This result is surprising because one would expect that, since a vertical toehold induces a more aggressive bidding behavior, the seller should take it into account to design the optimal rule. Our interpretation is that, as opposed to horizontal crossholdings (see Loyola 2006), vertical toeholds only impose links between the bidders' *payments*, but *not* between the bidders' *valuations*. Consequently, in the terminology of Bulow and Roberts (1989), the marginal revenue function (which depends only on valuations) is the same for all bidders. This implies that the seller perceives all bidders as symmetric players, and hence, it is optimal to impose no bias and to attain a symmetric equilibrium.

However, as we will see in the next subsection, this optimal symmetric equilibrium requires the seller to introduce an asymmetry into the payment scheme. The

[18] In the terminology introduced by Bulow and Roberts (1989), all bidders exhibit the same marginal revenue function for the seller, who is interpreted as a monopolist.

underlying rationale for this apparent contradiction between the allocation rule and the scheme of transfers is the same as the one behind the break-down of the Revenue Equivalence Principle. That is, when toeholds exist, revenues *do* depend on the entire payment scheme, not only on the transfers made by the lowest type bidder. As a result, it does not suffice to examine only the allocation rule to state the properties of the optimal mechanism. In fact, one needs to characterize the payment scheme fully as this is crucial in order to recognize the non-standard and discriminatory nature of the optimal selling procedure.

### 2.3.2 Implementation

Since all bidders provide the same marginal revenue, the implementation of the optimal allocation rule requires a scheme of payments that induce an efficient allocation, that is, one which guarantees that the target firm be awarded to the bidder who values it the most. Since we have assumed that players are asymmetric in their toeholds, and thus in their expected payoff functions, the only way to attain an efficient allocation is to design a scheme of "*personalized*" payments. This implies that we must rule out any standard auction, as it imposes symmetric payments on the players and thus results in an asymmetric and inefficient equilibrium. This fact is formalized in the next corollary.

**Corollary 12** *A standard auction cannot implement the optimal selling mechanism.*

From the incentive compatible constraint, we show next that the optimal allocation rule can be implemented by a selling mechanism with an asymmetric scheme of transfers.

**Proposition 13** *In the presence of toeholds, the optimal mechanism can be implemented by a modified second price auction with a reserve price and a scheme of payments that includes a penalty against the winner and a payment by the loser. The scheme is the following one:*

$$x_i(t) = \begin{cases} z_i(t_{-i}) + [\delta_i - 1]\, z_i(t_{-i}) & \text{if } p_i(t) = 1 \\ \pi_i z_j(t_{-j}) & \text{if } p_i(t) = 0 \text{ and } p_j(t) = 1 \\ 0 & \text{otherwise} \end{cases}$$

*for all* $i, j = 1, 2$, $i \neq j$, *and for all* $t \in T$, *where*

$$\delta_i \equiv \frac{1 - \phi_j}{(1 - \phi_i - \phi_j)}, \quad \pi_i \equiv \frac{\phi_i}{(1 - \phi_i - \phi_j)},$$

*and* $z_i(t_{-i}) = \inf \{s_i : c_i(s_i) \geq 0 \text{ and } c_i(s_i) \geq c_j(t_j)\}$.

This scheme of payments has the following properties.

*Discriminatory policy with winning penalties and losing payments.* First, since $z_i(t_{-i}) > 0$ and $\delta_i \geq 1$, this implies that when the winner is a bidder with toeholds, his payment has a *penalty* when compared to the payment he would make in case of holding no toeholds. This penalty is given by $[\delta_i - 1]\, z_i(t_{-i})$. Second, since $z_j(t_{-j}) > 0$, and $\pi_i \geq 0$, this means that when the loser is a bidder with an initial stake, his payment is *positive.* Third, from $\phi_1 > \phi_2$, it follows that $\delta_1 > \delta_2$ and $\pi_1 > \pi_2$. Thus, it is clear that the scheme of transfers proposed imposes a discriminatory policy with a bias against the bidder with the largest initial stake.[19]

*Truthtelling and efficient mechanism.* The discriminatory scheme of winning penalties and losing payments implies that the payoff of bidders 1 and 2 simplifies to

$$\pi_i(t_i) = \begin{cases} t_i - z_i(t_{-i}) & \textit{if } p_i(t) = 1 \\ 0 & \textit{otherwise} \end{cases}$$

The scheme of transfers therefore induces symmetric objective functions for all bidders, as in the standard problem when there are no toeholds (see Myerson 1981).

*Average sale price increasing with common toeholds and asymmetry.* First, let $\Pi_0^*$ be the seller's expected revenue under the optimal mechanism, and hence, define $\rho_0^* \equiv \Pi_0^*/(1 - \phi_1 - \phi_2)$, the average sale price under the same procedure. From (2.1) and Proposition 13, it follows directly that $\Pi_0^*$, and thus $\rho_0^*$, are increasing with both the winning penalty and the losing payment. Second, consider the symmetric toeholds case (i.e. $\phi_1 = \phi_2 = \phi > 0$). In this case, both the winner's penalty and the loser's payment are increasing in the common toehold, as it is easy to check that $\partial\delta_i/\partial\phi > 0$ and $\partial\pi_i/\partial\phi > 0$ for all $i$. All of this implies that, at the optimal mechanism, the seller's expected revenue (and thereby, the average sale price) is *increasing* with the size of common toeholds. Finally, consider the asymmetric toeholds case (i.e. $\phi_1 > \phi_2 > 0$). Let us define $\varepsilon \equiv \phi_1 - \phi_2$ so that the parameters of the winning penalty and the losing payment can be rewritten as

$$\begin{aligned} \delta_1 &= \frac{1 - \phi_2}{1 - 2\phi_2 - \varepsilon},\ \delta_2 = \frac{1 - \phi_2 - \varepsilon}{1 - 2\phi_2 - \varepsilon} \\ \pi_1 &= \frac{\phi_2 + \varepsilon}{1 - 2\phi_2 - \varepsilon},\ \pi_2 = \frac{\phi_2}{1 - 2\phi_2 - \varepsilon} \end{aligned}$$

Hence, it is easy to verify that $\partial\delta_i/\partial\varepsilon > 0$ and $\partial\pi_i/\partial\varepsilon > 0$ for all $i$. Therefore, the optimal mechanism is such that the seller's expected revenue (and thus, the average sale price) is *increasing* with the degree of asymmetry in toeholds. All of this implies that a discriminatory policy pays to the seller.

[19] Moreover, this discriminatory policy gets exacerbated with the degree of asymmetry, as the gaps of both winning penalties and losing payments are increasing with the difference in toeholds.

## 2.4 A sequential negotiation procedure

In this section we show that a simple sequential negotiation procedure replicates the main properties of the optimal mechanism. The negotiation procedure works as follows:

**Stage I**

**I.1**. The seller makes a take-it-or-leave-it offer $\rho_1$ to bidder 1, where the offer $\rho_i$ is the price to be paid by bidder $i$ for the target shares.

**I.2.** Bidder 1 accepts or rejects this offer. If he accepts, the target is sold to him and the game is over. If bidder 1 rejects the exclusive deal, negotiation moves to the next round.

**Stage II**

**II.1**. The seller makes a new take-it-or-leave-it offer $\rho_2$ to bidder 2.

**II.2.** Bidder 2 accepts or rejects this offer. If he accepts, the target is sold to him. Otherwise, the target company remains under the current ownership structure and management.

The next proposition illustrates the discrimination policy resulting from the negotiation procedure for the uniformly distributed valuations case.[20]

**Proposition 14** *Suppose that $t_i$ is uniformly distributed in the interval $[0,1]$ for all $i = 1, 2$. At the Subgame Perfect Nash equilibrium of the game induced by the sequential negotiation procedure, it is optimal for the seller to set $\rho_1^* > \rho_2^*$ for all $\phi_1 \geq \phi_2 \geq 0$.*

With sequential negotiations the sale price charged to the first bidder is higher than the one charged to the second bidder. As the first-mover is the buyer with the highest toehold, the sequential mechanism discriminates against him. Moreover, the degree of this bias *increases* with the degree of asymmetry in the toeholds. More precisely, if we define the degree of asymmetry by $\varepsilon \equiv \phi_1 - \phi_2$, then the difference in prices offered by the seller, i.e., $\Delta\rho(\phi_2, \varepsilon) \equiv \rho_1^* - \rho_2^*$, is increasing in $\varepsilon$. To see this note that

$$\Delta\rho(\phi_2, \varepsilon) \equiv \rho_1^* - \rho_2^* = \frac{1 - 2\phi_2 + 4\varepsilon}{8(1 - (\phi_2 + \varepsilon))(1 - \phi_2)} \text{ so that } \partial\Delta\rho(\phi_2, \varepsilon)/\partial\varepsilon > 0.$$

Note also that $\Delta\rho(\phi_2, \varepsilon)$ is strictly increasing in $\phi_1$ and strictly decreasing in $\phi_2$, with $\Delta\rho(\phi_2, \varepsilon)$ strictly increasing in $\phi_2$ for fixed and given $\varepsilon$. Hence, the negotiation procedure highlights the importance of establishing an asymmetric scheme of payments, as the price charged to the high-toehold bidder exceeds that of the low-toehold one, and this bias is larger when the ownership stakes become more asymmetric.

[20] For simplicity and wihout loss of generality, all the results in the paper are henceforth stated assuming uniformly distributed valuations on the unitary interval.

To analyze whether this price discrimination policy pays to the seller we must look at the average sale price delivered by the equilibrium of the sequential procedure. Let $\Pi_0^{SN}$ be the seller's expected revenue under the sequential procedure, and consequently, define $\rho_0^{SN} \equiv \Pi_0^{SN}/(1-\phi_1-\phi_2)$, the average sale price under the same mechanism.[21] Rewriting $\rho_0^{SN}$ in terms of $\varepsilon = \phi_1 - \phi_2$, it follows that

$$\rho_0^{SN} = \frac{1}{16(1-\phi_2)^2}\left[\frac{(5-6\phi_2)^2}{4(1-\phi_2-\varepsilon)} + \phi_2 + \varepsilon\right].$$

It is easy to verify that $\partial\rho_0^{SN}/\partial\varepsilon > 0$ for all $\phi_2, \varepsilon \in (0, 1/2)$ so that the average sale price is *increasing* in the *degree of asymmetry.* This result is displayed in Figure 1.

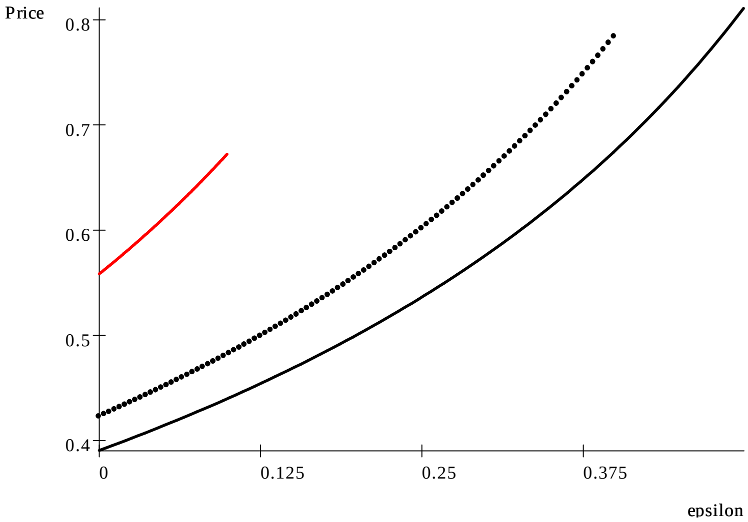


Figure 1. Average sale price from the sequential negotiation mechanism with two bidders and $\phi_1 > \phi_2 \geq 0$, for $\phi_2 = 0$ (solid line), $\phi_2 = .1$ (dotted line) and $\phi_2 = .4$ (dash line).

Furthermore, similarly to the optimal mechanism in the *symmetric* case, the aforedefined sequential procedure yields an average sale price which is also *increasing* in the *common toehold*. In fact, when $\phi_1 = \phi_2 = \phi$, it is possible to check that $\partial\rho_0^{SN}/\partial\phi > 0$ for all $\phi \in (0, 1/2)$, as it is illustrated in Figure 2.

[21] See the Appendix (Proof of Proposition 14) for details on how this average price is computed.

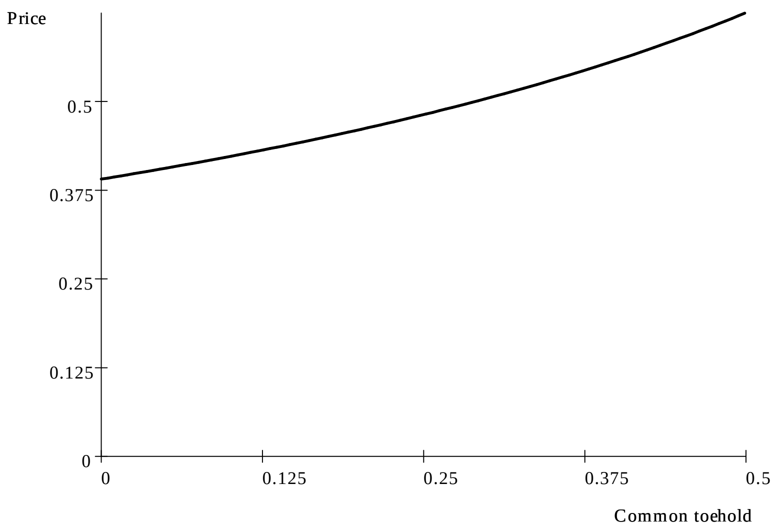


Figure 2. Average sale price from the sequential negotiation mechanism with two bidders for $\phi_1 = \phi_2 = \phi \geq 0$.

Notice however, that unlike the optimal mechanism described in the previous section, the sequential procedure always discriminates against the bidder moving first, even if the toeholds are symmetric or zero. In fact, as the proof of Proposition 14 establishes, the prices charged to both players in the symmetric case (i.e., $1/2 > \phi_1 = \phi_2 = \phi \geq 0$) satisfy the following inequality

$$\rho_1^* = \frac{5-6\phi}{8(1-\phi)^2} > \frac{1}{2(1-\phi)} = \rho_2^*$$

In addition, the different priorities given by the negotiation timetable to different buyers implies that, unlike the optimal procedure, the sequential mechanism may be ex post *inefficient.*

In sum, and despite these differences, our sequential procedure replicates the two most important properties of the optimal mechanism: the expected selling price is increasing in both the common toehold and the degree of asymmetry in the initial stakes held by bidders.

## 2.5 Sequential procedure vs. auctions

Although there is not a specific practice to sell a company, sometimes the legal framework implicitely induces the board of directors to conduct an auction among the raiders.[22] The underlying rationale behind this recommendation is the idea that an

[22] For instance, the Delaware law in the US establishes that the board must act as "*auctioneers* charged with getting the best price for the stock-holders at a sale of the company". See also Cramton and Schwartz (1991).

auction run with several bidders at once offers a more competitive environment than a negotiation held with a single buyer at each round. Nevertheless, and despite this idea, the coexistence of both types of mechanisms in real world takeover processes has been widely documented.[23] In this Section we compare the sequential procedure to the auction formats commonly used in practice from the nonbidding shareholders' point of view. We show that the nonbidding shareholders benefit from the discrimination policy to the extent that the sequential procedure generates a higher expected selling price than both the first-price and the second-price auctions.

We analyze here two ownership structures in which this result holds: (i) the symmetric case, i.e. $\phi_1 = \phi_2 = \phi \geq 0$, and (ii) a particular asymmetric case in which there are two classes of bidders: one toeholder and one outsider, i.e., $\phi_1 > \phi_2 = 0$.[24] For both of these ownership structures, the literature provides a ranking between the first and second price formats. In the second-price auction, and for both ownership environments, the toehold bidder exhibits the *owner's curse,* an overbidding behavior according to which the equilibrium bid exceeds his valuation. This overbidding phenomenon is however not present in the case of the outside raider, as bidding his true valuation continues to be a dominant strategy for him. In contrast, given the traditional bidding trade-off present in the first-price auction, the owner's curse is absent in this selling format. Because of this, the second-price auction outperforms the first-price auction in terms of revenue, in both the symmetric and asymmetric structures.[25] As a result, it suffices to compare the selling price generated by the sequential mechanism with that generated by the second-price auction.

The following auxiliary result characterizes the expected selling price in the second-price auction.

**Lemma 15** *Let $\rho_0^{SPA}$ be the average sale price resulting from the second-price auction. Then,*
*(1) In the symmetric case, this price is given by*

$$\rho_0^{SPA} = \frac{(1+2\phi)(1-\phi)}{(1-2\phi)(1+\phi)} - \frac{2}{3(1-2\phi)}$$

*(2) In the asymmetric case, it corresponds to*

$$\rho_0^{SPA} = \frac{1}{1-\phi_1}\left[\frac{\phi_1}{\phi_1+1} - \frac{5}{6}\phi_1 - \frac{1}{2\phi_1+2} + \frac{2}{3\phi_1+3} + \frac{1}{6}\right].$$

[23] See the evidence provided by Boone and Mulherin (2003), Boone and Mulherin (2004), Povel and Singh (2006), and Bulow and Klemperer (2007).

[24] As the evidence presented by Bradley et al. (1988), Betton and Eckbo (2000), and Betton Eckbo and Thorburn (2005) suggests, the presence of an outside bidder is very common in actual takeovers.

[25] Ettinger (2002) performs this comparison for the symmetric case, and Ettinger (2005) does it for the specific asymmetric environment analyzed here.

Now, we establish the predominance of our sequential mechanism over the auction formats commonly used in practice, irrespective of the degree of symmetry in toeholds.

**Proposition 16** *The sequential procedure generates a higher average sale price than both the first-price and the second-price auctions, no matter the degree of asymmetry.*

As mentioned in the previous section, the sequential procedure always discriminates against the first-mover bidder. This fact implies that it yields a larger expected sale price than both auction formats in the symmetric case, even when there are no toeholds at all. The average sale price comparison for the symmetric case between the second-price auction and our sequential mechanism is depicted in Figure 3. Note from the figure that the second-price auction induces a *concave* average sale price whereas the negotiation procedure exhibits a *convex* one. As a result, the price gap between both mechanisms is larger when the toehold becomes sufficiently low or sufficiently high. The difference attains its minimum for values around .25.

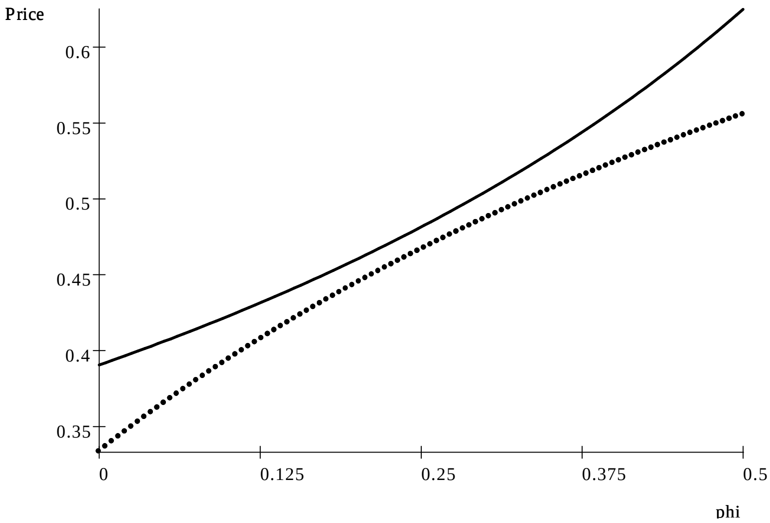


Figure 3. Average sale price from the sequential negotiation mechanism (solid line) and the SPA (dotted line) with two bidders for $\phi_1 = \phi_2 = \phi \geq 0$.

Furthermore, the superiority of our sequential mechanism over auctions is exacerbated in the asymmetric case, as the discriminatory policy involves a sequence of negotiations with a pecking order consistent with the aggressiveness of each buyer (see Figure 4).

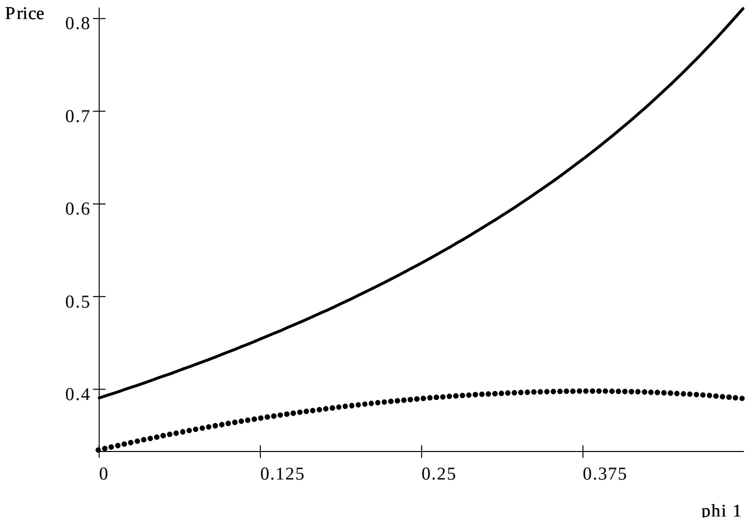


Figure 4. Average sale price from the sequential negotiation mechanism (solid line) and the SPA (dotted line) with two bidders for $\phi_1 > \phi_2 = 0$.

Notice that the clear advantage of the sequential mechanism becomes larger when the degree of asymmetry (represented in this case by $\phi_1$) increases. This is a consequence of the fact that whereas $\rho_0^{SN}$ is always an increasing and convex function in $\phi_1$, $\rho_0^{SPA}$ is a concave function and an increasing one *only* for a sufficiently low degree of asymmetry (for all $\phi_1 < .38$).

This last result is formalized in the following statement.

**Corollary 17** *The larger the degree of asymmetry, the better the sequential procedure when compared with both the first-price and the second-price auctions.*

Finally, let us mention that our results here are in line with the well-established supremacy of sequential mechanisms which give higher priority to stronger bidders.[26] Accordingly, and in contrast with the standard auction formats, the particular order of negotiations involved in our procedure allows for an exploitation of the higher aggressiveness of raiders with larger stakes in the target.

## 2.6 Concluding Remarks

We have characterized how a target firm should be sold when bidders possess prior stakes in its ownership. This optimal mechanism corresponds to a non-standard auction with a scheme of asymmetric payments that imposes a bias against toeholders. The rationale of such a discriminatory policy is the fact that a standard mechanism is unable to induce a symmetric and efficient allocation rule, as it preserves the initial

[26] See Povel and Singh (2006) and Dasgupta and Tsui (2003).

advantage of toehold bidders. In contrast, a scheme of asymmetric winning penalties and losing payments allows both to take advantage of the higher aggressiveness of toeholders and to go back to a symmetric environment.

The presence of losing payments in the optimal procedure is in line with similar results found in the literature devoted to characterizing optimal auctions when externalities exist. For instance, Goeree et al. (2005) show that the positive externalities present in fund-raising activities lead to discarding winner-pay auctions in favor of all-pay formats. In a result reminiscent of ours, they establish the optimality of an auction with a reserve price and payments by the losers - a mix between participation fees and an all-pay auction run in a subsequent stage-, which depend on the degree of the externality. Moreover, Goeree et al. (2005) emphasize that some characteristics of this optimal procedure are present in the procedures used for raising funds in the real world. As a consequence, the characteristics of our non-standard auction in the takeover case are not far from those exhibited by the optimal procedure in other contests with externalities.

We have also demonstrated that the nonbidding shareholders benefit from the discriminatory mechanism, as the target average sale price is increasing in both the common toehold and the degree of asymmetry in these stakes. The latter finding is in sharp contrast with the properties of standard auction formats in takeover battles, which then lead to opposite policy implications. For instance, Bulow, Huang and Klemperer (1999) show that in general the asymmetry in toeholds lowers prices in common-value ascending auctions. As a result, they recommend the "level the playing field" practice, according to which it may be revenue increasing to sell toeholds very cheaply to the buyer with the smaller stake in the target. On the contrary, our normative approach suggests that the seller should follow strategies with the aim of preserving this asymmetry. Accordingly, the board of directors should block or discourage the entrance of new shareholders suspected of becoming competitors against the incumbent toeholder in a future takeover battle.

As an alternative to the optimal non-standard auction-based mechanism, we have proposed a simpler and realistic negotiation procedure that replicates the main properties of the first one. This mechanism contains a timetable that gives priority to the higher-toehold bidders, but charges higher prices to them. Such a negotiation-based procedure shares some features of other selling procedures already considered in the literature. In particular, it balances out properly the trade-off between creation and extraction of value caused by the implicit *threats* involved in the sequential nature of the negotiation process. This characteristic is also present in the posted-price rule discussed by Campbell and Levin (2006) in an environment with interdependent valuations. These authors find conditions under which a hybrid mechanism of a posted-price rule and a random rationing may outperform standard auctions. This fact occurs essentially when the increase of all buyers' willingness to pay offsets the losses stemming from ex post inefficient allocations. Similarly, in the context of our paper, the individual and sequential feature of the negotiation scheme imposes costs and benefits on nonbidding shareholders. On the one hand, the expected target price decreases due to both less competition and less efficiency. On the other hand, the higher priority given to the high-toehold bidder increases his willingness to pay, as the opportunity of winning the contest emerges even though his value may be lower than the small-toehold bidder's one. We have proved that the last effect dominates the shortcomings, therefore keeping open the ongoing debate on auctions versus negotiations in takeover wars.

## 2.7 Appendix

**Appendix A. The optimal mechanism problem.**

The optimal mechanism solves the following problem:

$$\max_{x_i \in \mathbb{R},\ p_i \in [0,1]} \ U_0 \tag{2.7}$$

$$s.t.$$

$$V_i(t_i) \ \geq 0 \quad \forall t_i \in \left[\underline{t}, \overline{t}\right], \ i = 1,2 \tag{2.8}$$

$$V_i(t_i) \ \geq U_i(\widehat{t_i}/t_i) \quad \forall t_i, \widehat{t_i} \in \left[\underline{t}, \overline{t}\right], \ i = 1,2 \tag{2.9}$$

$$\sum_{i=1}^{2} p_i(t) \leq 1 \text{ and } p_i(t) \geq 0, \ i = 1,2, \forall t \in T \tag{2.10}$$

where (2.7) is the seller's expected revenue, (2.8) is bidder $i$'s participation constraint, (2.9) represents the incentive compatibility constraints of the bidders and (2.10) corresponds to the feasibility constraints.[27] From Myerson (1981), standard substitutions and computations lead to state the equivalence between the incentive compatibility constraints and the following two consitions:

(i) $\frac{\partial V_i(t_i)}{\partial t_i} = Q_i(t_i)$

(ii) $\frac{\partial Q_i(t_i)}{\partial t_i} \geq 0$

These conditions allow to replace (2.9) by (ii) and

$$V_i(t_i) \ = V_i(\underline{t}) + \int_{\underline{t}}^{t_i} Q_i(s_i) ds_i. \tag{2.11}$$

[27] Following Jehiel, Moldovanu and Stachetti (1996) and (1999), it is possible to show that the optimal threat for the non-participating bidder is that the target remains under the current management and control. As a result, the outside utility for the lowest-type bidder is the same for all buyers (toeholders and outsiders), and so, it can be normalized to zero (see Loyola 2007, Section 3).

Similarly, (2.8) is guaranteed to hold if $V_i(\underline{t}) \geq 0$ for all $i$. Hence, straightforward computations allow us to rewrite the seller's expected payoff and to simplify the maximization problem as presented in Section 2.3.

**Appendix B. Proofs.**

*Proof of Lemma 11* From (2.2), it is in the seller's interest to make $V_i(\underline{t}) = 0$ for all $i$ because $V_i(\underline{t}) > 0$ is suboptimal and setting $V_i(\underline{t}) < 0$ violates the Participation Constraint. Moreover, $H'(t_i) > 0$ implies that $c_i'(t_i) > 0$ and thereby $\partial p_i(t)/\partial t_i \geq 0$, so that $Q_i'(t_i) \geq 0$ is satisfied for all $i$. Finally, since $t_0 = 0$, the optimal allocation rule is found by comparing for a given $t = (t_1, t_2)$ the terms $c_i(t_i)$, whenever they are positive. The solution sets then $p_i(t) = 1$ iff $c_i(t_i) > \max\{0, \max_{j \neq i} c_j(t_j)\}$ .□

*Proof of Proposition 13* For any vector $t_{-i}$ consider

$$z_i(t_{-i}) = \inf\{s_i : c_i(s_i) \geq 0 \text{ and } c_i(s_i) \geq c_j(t_j) \text{ for all } j \neq i\}$$

for all $i$, *i.e.*, the infimum of all winning values for $i$ against $t_{-i}$. Then, in equilibrium

$$p_i(s_i, t_{-i}) = \begin{cases} 1 & \text{if } s_i > z_i(t_{-i}) \\ 0 & \text{if } s_i < z_i(t_{-i}) \end{cases} \tag{2.12}$$

and

$$\int_{\underline{t}}^{t_i} p_i(s_i, t_{-i}) ds_i = \begin{cases} t_i - z_i(t_{-i}) & \text{if } t_i \geq z_i(t_{-i}) \\ 0 & \text{if } t_i < z_i(t_{-i}) \end{cases} \tag{2.13}$$

for all $i$. Substitute $Q_i(s_i)$ into (2.11), change the order of integration and substitute $V_i(t_i)$. After rearranging, we obtain that the truthtelling payoff of the bidder with the lowest signal can be written as

$$V_i(\underline{t}) = \int_{T_{-i}} \{t_i p_i(t) - [1-\phi_i]\, x_i(t) + \phi_i \sum_{j\neq i} x_j(t) - \int_{\underline{t}}^{t_i} p_i(s_i, t_{-i}) ds_i\} f(t_{-i}) dt_{-i} \tag{2.14}$$

for all $i$ and $t_i \in [\underline{t}, \overline{t}]$. Since it is optimal $V_i(\underline{t}) = 0$ for all $i$, then sufficient conditions for (2.14) to hold are:

$$t_i p_i(t) - [1-\phi_i]\, x_i(t) + \phi_i \sum_{j\neq i} x_j(t) = \int_{\underline{t}}^{t_i} p_i(s_i, t_{-i}) ds_i$$

for all $i$ and for all state $t = (t_i, t_{-i})$. If we fix a particular state $t = (t_i, t_{-i})$, three cases are possible: (i) a winning bidder exists different from bidder 3, (ii) bidder 3 is the winner, and (iii) the object is not awarded to any bidder. Applying (2.12) and (2.13), the solution of this system of equations for the three cases yields the desired scheme of asymmetric payments.□

*Proof of Proposition 14* Using backward induction, we first characterize the Nash equilibrium resulting from Stage II. In this stage, bidder 2 accepts the offer if $t_2 - (1-\phi_2)\rho_2 > 0$, i.e., if $t_2 > (1-\phi_2)\rho_2$, and rejects otherwise. The seller's problem is hence

$$\max_{\rho_2} \left[(1-\phi_1-\phi_2)\rho_2\right]\left[1-(1-\phi_2)\rho_2\right],$$

whose solution is given by $\rho_2^* = 1/2(1-\phi_2)$. The optimal seller's expected revenue from this stage is equal to $(1-\phi_1-\phi_2)/4(1-\phi_2)$.

In stage I.2, bidder 1 accepts any seller's offer if his expected payoff is larger than the expected payoff at the equilibrium of stage II. That is, if $t_1 - (1-\phi_1)\rho_1 > E_{t_2}\left[\phi_1\rho_2^*)\right] = \phi_1/4(1-\phi_2)$, which is equivalent to the condition $t_1 > (\phi_1/4(1-\phi_2)) + (1-\phi_1)\rho_1$. Thus, the seller's optimal offer is characterized by

$$\begin{aligned}\rho_1^* &= \arg\max_{\rho_1}(1-\phi_1-\phi_2)\rho_1\left[1-\frac{\phi_1}{4(1-\phi_2)}-(1-\phi_1)\rho_1\right] \\ &\quad + \frac{1-\phi_1-\phi_2}{4(1-\phi_2)}\left[\frac{\phi_1}{4(1-\phi_2)}+(1-\phi_1)\rho_1\right].\end{aligned}$$

The solution is given by $\rho_1^* = (5-6\phi_2)/8(1-\phi_1)(1-\phi_2)$, which yields an optimal seller's expected revenue equal to

$$\Pi_0^{SN} = \frac{(1-\phi_1-\phi_2)}{16(1-\phi_2)^2}\left[\frac{(5-6\phi_2)^2}{4(1-\phi_1)}+\phi_1\right],$$

and an average sale price equal to

$$\rho_0^{SN} \equiv \Pi_0^{SN}/(1-\phi_1-\phi_2) = \frac{1}{16(1-\phi_2)^2}\left[\frac{(5-6\phi_2)^2}{4(1-\phi_1)}+\phi_1\right]. \qquad (2.15)$$

Since $1/2 > \phi_1 \geq \phi_2 \geq 0$, it is simple to verify that

$$\rho_1^* = \frac{5-6\phi_2}{8(1-\phi_1)(1-\phi_2)} \geq \frac{5-6\phi_2}{8(1-\phi_2)^2} > \frac{1}{2(1-\phi_2)} = \rho_2^*$$

which proves the statement of the proposition.□

*Proof of Lemma 15* Since that the *asymmetric* case is the most general one, we first prove the second part of the proposition. In the second-price auction, bidder 2's payoff function, when his signals is $t_2$ and he behaves as if it were $\widehat{t}_2$, is given by

$$\pi_2(t_2,\widehat{t}_2) = \max_{\widehat{t}_2}\int_0^{b_1^{-1}(b_2(\widehat{t}_2))}(t_2-b_1(t))dt, \qquad (2.16)$$

that is, the traditional payoff function in a second-price auction without toeholds. Consequently, it follows that $b_2(t_2) = t_2$. Given the bid strategies $b_1(.)$ and $b_2(t_2) = t_2$, bidder 1's optimal choice of $\widehat{t_1}$ when he observes $t_1$ is obtained by maximizing his expected profits

$$\pi_1(t_1, \widehat{t_1}) = \max_{\widehat{t_1}} \int_0^{b_1(\widehat{t_1})} (t_1 - (1-\phi_1)t)dt + \phi_1 \int_{b_1(\widehat{t_1})}^1 b_1(\widehat{t_1})dt. \tag{2.17}$$

From Ettinger (2005), bidder 1's equilibrium bid is given by

$$b_1(t_1) = \frac{\phi_1}{1+\phi_1} + \frac{t_1}{1+\phi_1}.$$

Now, in order to compute the seller's revenues, let us define $\psi_j(t_i)$, the equilibrium correspondence function, such that $b_i(t_i) = b_j(\psi_j(t_i))$ for all $i, j = 1, 2$. Applying the definition of $\psi_j(.)$ to the equilibrium bid strategies yields

$$\psi_2(t_1) = \frac{\phi_1}{1+\phi_1} + \frac{t_1}{1+\phi_1}, \tag{2.18}$$

$$\psi_1(t_2) = -\phi_1 + t_2(1+\phi_1). \tag{2.19}$$

Appealing to the Envelope Theorem, and using the fact that $\psi_2(.) = b_1(.)$ and $\psi_1(.) = b_1^{-1}(b_2(.))$, it can be verified that $\frac{d\pi_i(t_1,\widehat{t_i})}{dt_i} = \psi_j(t_i)$, which implies

$$\pi_i(t_i) = \pi_i(1) - \int_{t_i}^1 \psi_j(t)dt \tag{2.20}$$

for all $i, j = 1, 2$. Evaluating $t_i = 1$ in (2.16) and (2.17), and using the fact that in equilibrium $\psi_j(\widehat{t_i}) = \psi_j(t_i)$ and $\psi_j(1) = 1$, it can be shown that

$$\pi_1(1) = 1 - \frac{1-\phi_1}{2} \tag{2.21}$$

$$\pi_2(1) = \frac{1}{2(1+\phi_1)}. \tag{2.22}$$

Substituting (2.18), (2.19), and the results (2.21) and (2.22) into (2.20), bidder $i$'s interim payoff becomes

$$\pi_1(t_1) = 1 - \frac{(1-\phi_1)}{2} - \frac{(1-t_i^2)}{2(1+\phi_1)} - \frac{\phi_1(1-t_1)}{(1+\phi_1)}$$

$$\pi_2(t_2) = \frac{1}{2(1+\phi_1)} - 1 + \frac{1+\phi_1}{2} + \frac{(1+\phi_1)t_2^2}{2} - \phi_1 t_2.$$

After taking expectations, bidder $i$'s ex-ante payoff is given by

$$\begin{aligned} \Pi_1 &= 1 - \frac{(1-\phi_1)}{2} - \frac{1}{3(1+\phi_1)} - \frac{\phi_1}{2(1+\phi_1)} \\ \Pi_2 &= \frac{1}{2(1+\phi_1)} + \frac{(1+\phi_1)}{6} - \frac{1}{2} \end{aligned}$$

The nonbidding shareholders' expected revenues are then given by

$$\begin{aligned}
\Pi_0^{SPA} &= \left[\int_0^1 t_1 \int_0^{\psi_2(t_1)} dt_2 dt_1 + \int_0^1 t_2 \int_0^{\psi_1(t_2)} dt_1 dt_2\right] - \Pi_1 - \Pi_2 \\
&= \left[\int_0^1 t_1\psi_2(t_1)dt_1 + \int_0^1 t_2\psi_1(t_2)dt_2\right] - \Pi_1 - \Pi_2 \\
&= \frac{\phi_1}{\phi_1+1} - \frac{5}{6}\phi_1 - \frac{1}{2\phi_1+2} + \frac{2}{3\phi_1+3} + \frac{1}{6}
\end{aligned}$$

and the average selling price is

$$\rho_0^{SPA} \equiv \Pi_0^{SPA}/(1-\phi_1) = \frac{1}{1-\phi_1}\left[\frac{\phi_1}{\phi_1+1} - \frac{5}{6}\phi_1 - \frac{1}{2\phi_1+2} + \frac{2}{3\phi_1+3} + \frac{1}{6}\right].$$

We now turn to demonstrate the statement for the *symmetric* case. From Proposition 1 in Ettinger (2002), the second-price auction equilibrium bid is given by

$$b_i(t_i) = \frac{\phi}{1+\phi} + \frac{t_i}{1+\phi}$$

for all $i$. Hence, $\psi_2(t) = \psi_1(t) = t$ for all $t$. Applying the same line of reasoning used in the asymmetric case, it can be verified that the seller's expected revenues are given by

$$\begin{aligned}
\Pi_0^{SPA} &= \left[\int_0^1 t_1^2 dt_1 + \int_0^1 t_2^2 dt_2\right] - 2\left[\frac{2}{3} - \frac{(1+2\phi)(1-\phi)}{2(1+\phi)}\right] \\
&= \frac{(1+2\phi)(1-\phi)}{(1+\phi)} - \frac{2}{3}
\end{aligned}$$

and the corresponding average sale price becomes

$$\rho_0^{SPA} = \frac{(1+2\phi)(1-\phi)}{(1-2\phi)(1+\phi)} - \frac{2}{3(1-2\phi)}$$

which completes the proof.□

*Proof of Proposition 16* Consider the symmetric case. Substituting $\phi_1 = \phi_2 = \phi$ into (2.15), and using Lemma 15, we can state that

$$\rho_0^{SN} = \frac{32\phi^2 - 56\phi + 25}{64(1-\phi)^3} > \rho_0^{SPA} \geq \rho_0^{FPA}$$

where the second inequality is strict when $\phi > 0$, and follows from Proposition 3 in Ettinger (2002).

Consider now the asymmetric case. Lemma 15 and the substitution of $\phi_1 > \phi_2 = 0$ into (2.15) yields

$$\rho_0^{SN} = \frac{1}{16}\left[\frac{25}{4(1-\phi_1)} + \phi_1\right] > \rho_0^{SPA} > \rho_0^{FPA}$$

where the last inequality holds as overbidding is not present in the first-price auction.□

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# Chapter 3

# On Bidding Markets: The Role of Competition

**Abstract.** This paper analyzes the effects of industrial concentration on bidding behavior and hence, on the seller's expected proceeds. These effects are studied under the CIPI model, an affiliated value set-up that nests a variety of valuation and information environments. We formally decompose the revenue effects coming from less competition into four types: a *competition effect,* an *inference effect*, a *winner's curse effect* and a *sampling effect*. The properties of these effects are discussed and conditions for (non)monotonicity of both the equilibrium bid and revenue are stated. Our results suggest that it is more likely that the seller benefits from less competition in markets with more complete valuation and information structures.

*Keywords*: auctions, competition, affiliation, inference
*JEL Classification*: C62, D44, D82, L41

## 3.1 Introduction

The typical concern about any illegal collusion practice (cartels) or legal collusion arrangement (mergers or consortia) is that these practices reduce the number of participants in the market and hence, lessen competition, negatively affecting both the price and the bid-taker's revenue. Nevertheless, in the context of auctions and bidding markets, this conventional wisdom applies only to the case of independent private value settings, as it has been modelled theoretically and empirically supported by abundant literature.[1] The simplicity of the valuation and information environments analyzed by this literature makes collusion practices negatively affect the intensity of competition, what has been called the *competition effect*.[2]

However, under a common value and/or affiliated signals model, the higher concentration provoked by joint bidding leads to other effects that may counteract the competition effect, and induce a more aggressive bidding behavior. These effects can be grouped into three classes: a *winner's curse effect*, an *inference effect* and an *information pooling effect*. First, the reduction in the number of bidders in a common value environment permits alleviation of the winner's curse, because now defeating fewer bidders makes the ex post overoptimism less likely. This implies that a higher industrial concentration increases the expected value of the item conditional on winning the auction, and in consequence, bidders are less conservative.[3] The inference effect may arise from some affiliated information structures, and can be present in both private and common value environments.[4] In this case, the reduction in the number of participants may increase the aggressiveness of the bidding behavior. The reason for this is that, although winning is interpreted as information that the intensity of the competition is lower than before the auction starts, this perception is weakened when the winner faces fewer rivals. Finally, the information pooling effect improves

[1] For theoretical works on (legal) joint bidding under the independent private value setting, see Waehrer [39], Waehrer and Perry [40], Froeb, Tschantz and Crooke [9], [37], and [10], and Dalkir, Logan and Masson [6]. Theoretical analysis on bidding rings with private values are provided by Robinson [35], Mailath and Zemsky [21], McAfee and McMillan [25], Marshall et al. [22], and Pesendorfer [18]. Finally, most empirical literature on illegal collusion derives its estimation models from a theoretical set-up with private values as well. Some papers along these lines are Hewitt, McClave and Sibley [14], Porter and Zona [33], Pesendorfer [18], Lanzillotti [19], Scott [36], Porter and Zona [32], Bajari and Ye [2], and Baldwin, Marshall and Richard [3].

[2] Given some properties of bidding rings (efficiency and the possibility of side payments), illegal collusion and mergers have the same anticompetitive effects on auction markets if values are private (see McAfee [24]).

[3] Theoretical approaches that characterize the winner's curse effect include Bulow and Klemperer [1] and Hendricks, Pinkse and Porter [5]. On the other hand, a number of recent papers provide empirical evidence of this effect in several auction markets such as Hong and Shum [8], Hendricks, Pinkse and Porter [5], and Athias and Nuñez [1].

[4] The previous literature refers to this effect as the *affiliation effect*; see Pinkse and Tan [19], Hong and Shum [8], and Hendricks, Pinkse and Porter [5].

the precision of the bidder's value estimate because a coalition of bidders can observe either a new signal or a larger amount of signals with better stochastic properties than an individual bidder. This effect also allows the winner's curse correction on bids to mitigate, leading to more aggressive bidding behavior.[5]

Therefore, all these effects go in the same direction and encourage more aggressive bids when an auction market becomes more concentrated because of mergers or other joint bidding arrangements. As these effects dominate the competition effect plus the statistic effect produced by the overall reduction in the number of participants (a *sampling effect*), the possibility for increasing the bid-taker's expected revenue remains open. As a result, the standard viewpoint that less competition is always undesirable can clearly become challenged.[6]

All of this underlines the importance of analyzing, in a valuation and information setting which is as complete as possible, the effects of (legal) joint bidding practices. As a starting point for this general objective, this paper studies the effects of a change in the number of bidders on both the equilibrium bid strategy and the seller's proceeds.[7] Consequently, we abstract away from any information pooling type effect. This implies that one can infer the other effects from the hypothetical exercise in which bidders merger but the acquired bidder's information is not used by the acquiring one. We then make this exercise equivalent, from a methodological point of view, to the case in which the number of bidders decrease because some of them do not attend some particular auction or because they leave the industry.

From the previous literature, a good point of departure for our analysis is provided by Pinkse and Tan [19], who examine conditions under which the equilibrium bid is monotonic increasing with respect to the number of bidders $n$ in affiliated private-value models of first-price auctions. In particular, they show the existence of a large class of such models in which the equilibrium bid function is indeed *not* strictly increasing in $n$. Furthermore, they propose a decomposition of the bidding effects into two parts: a competition effect and an affiliation effect. This latter effect is precisely the source of the surprising finding of Pinkse and Tan in a private value environment, and it can also be present in a common value set-up. They illustrate their results with the conditionally independent private value (CIPV) model, a special case of the affiliated private value (APV) model in which bidders' valuations are affiliated

[5] See DeBrock and Smith [3], Hendricks and Porter [6], Krishna and Morgan [13]. Mares and Shor [23] show that indeed this information pooling effect works unambiguously for second-price auctions, but for first-price auctions it induces more aggressive bids only for signals that are sufficiently *low*.

[6] In addition, it has been argued that joint bidding has other pros such as facilitating entry of wealth-constrained bidders and improving risk diversification (see DeBrock and Smith [3]).

[7] We do not examine the welfare effects of competition. For an analysis of such issues, see, for instance, Compte and Jehiel [5].

through a common random component, but they are independently distributed given a realization of this common component. In this environment , the winner never regrets its winning so that the winner curse effect has no bite.

Accordingly, it is clear that in order to also examine the winner's curse effect, we need to consider a more general framework than that provided by the APV model - and in particular by the CIPV model -, as this effect cannot emerge from the valuation structure characterized by these settings. One way to do this is by means of the conditionally independent private information (CIPI) model, a special class of the general affiliated value (AV) model which encompasses both the CIPV and the pure common value setups as polar cases. In the CIPI model, the bidders' signals (private information) are affiliated through a common variable (which can also be the ex post common value of the object), but they are independently distributed conditional on a realization of this common variable. As a consequence, this framework provides an environment rich enough to evaluate all the revenue effects.

We group the effects on revenue coming from more competition into two classes: (i) those that affect *bidding behavior* and (ii) a pure *sampling effect*. On the one hand, changes in the number of buyers influence the equilibrium bid. As discussed above, in environments with interdependent valuations and dependent information, bidding behavior can become more or less aggressive with more competition. The final sign of these influences on bids, as well as on revenues, is therefore ambiguous, and depends on the relative magnitudes of the bidding-based effects considered. A more in-depth characterization of these bidding effects can then become worthwhile for a seller interested in adopting revenue-enhancing instruments in the face of mergers or any joint bidding practice. Consequently, we propose a decomposition of this bidding effect that allows us to isolate and formally evaluate the *winner's curse*, the *competition* and the *inference* effects. The properties of all these effects are established, and conditions for the (non)monotonicity of the equilibrium bid are stated.

On the other hand, the sampling effect reflects the upward impact on the seller's proceeds due to the fact that more competition implies a winning signal's distribution with better stochastic properties. We then combine both the bidding effect and the sampling effect, providing conditions for the (non)monotonicity of revenues. In particular, the paper shows that the seller's expected proceeds can be decreasing in the number of buyers as a negative and sufficiently large bidding effect dominates the sampling effect. The main implication is that in the CIPI model, in contrast to the CIPV setting, the conditions that allow the seller to benefit from less competition are less stringent. The rationale of this finding is the presence of the winner's curse effect, absent in affiliated private value environments. In fact, as the winner's curse constitutes an additional force for bids decreasing in the number of buyers, it makes the conditions for nonmontonic revenue to hold more likely. In a broader sense, this work highlights therefore the role played by the valuation and information structure assumed to be satisfied in a particular auction-based market. Accordingly, our results suggest that, by analyzing a more complete auction environment, the traditional idea that more concentration is always undesirable may no longer hold.

Our model accounts for existing empirical evidence that, in auction markets in which the winner's curse seems to be particularly strong, the bid-taker may be better off when the number of bidders decreases. For instance, DeBrock and Smith [3] study offshore oil lease auctions under a framework with values and signals that are log-normally distributed. They show that the joint bidding increases the total social value of the lease offering and, in some cases, *increases* the fraction of this value appropriated by the seller (the government). Similarly, Hong and Shum [8] construct a model of a low-bid procurement auction with common value and affiliated signals. The bidder's cost of completing a project is given by a log-additive formulation that includes both a private (or idiosyncratic) and a common cost component, which are independently log-normal distributed. They find that, for a large subset of construction procurement auction contracts, the median cost *rises* as the number of participants increases.

It is noteworthy that while the evidence presented by these works is derived starting from a framework that assumes specific functional forms for valuations and/or distributions, our model yields these predictions without such restrictive assumptions. What is even more interesting it is likely that the available evidence *against* the non-monotonicity of revenue with the number of bidders is based largely on these specific assumptions as well. For instance, Mares and Shor [23] develop a model with pure common value and independent signals, where the value of the item is the average of all bidders' signals. Their findings, corroborated by experimental exercises, suggest that the seller's expected revenue decreases with less competition mainly because of the sampling effect.[8] Nevertheless, since their valuation structure depends precisely upon the number of participants, some of the effects described could be absent if other valuation functions were assumed.

The results of this paper have a scope of applicability that goes beyond a mere academic interest, as they concern antitrust issues which are currently widely discussed. In a recent policy-oriented article, Klemperer [12] analyzes the characteristics that the competition policy on bidding markets should possess. His general conclusion is that, although the markets organized as auctions do have some special features such as common values behavior, a tendency to overemphasize the importance of these features has erroneously lead to positions in favor of a more lenient antitrust policy.

[8] Since their model assumes independence and symmetry, the revenue equivalence theorem implies that this result holds for both first-price and second-price auctions.

In what concerns the role played by the winner's curse, the arguments provided by Klemperer rest on two examples under the pure common value environment in which less competition would unambiguously hurt the seller.[9] However, similar to Mares and Shor's results, the conclusion of Klemperer may strongly depend on the particular valuation structures considered in his examples. In contrast, our main insight, derived without such specific assumptions, suggests the need for an antitrust policy that scrutinizes mergers more carefully or other joint bidding arrangements in bidding markets in which more sophisticated valuation and information environments are present.

This paper is organized as follows. Section 3.2 summarizes the CIPI model, noting how the CIPV and the pure common value models can be derived from this as a special cases. Section 3.3 studies the relationship between competition and bidding behavior in a first-price auction under the CIPI setting. As a consequence, we provide conditions for the (non)monotonicity of the equilibrium bid strategy and propose a new three-part decomposition of the bidding-type effects. In Section 3.4, we examine the conditions that guarantee the (non)monotonicity of the seller's revenue with respect to $n$. Finally, Section 3.5 concludes. All the proofs are collected in the Appendix.

## 3.2 The CIPI model

Consider a seller who wants to auction off a single object among $n$ bidders, using a first-price auction with a possible reserve price $r \geq 0$. Each bidder observes a signal $x_i \in [\underline{x}, \overline{x}]$, $\underline{x} > 0$, which is private information to him. Bidder $i$'s utility (valuation) is represented by the function $U(v, x_i)$, where $v \in [\underline{v}, \overline{v}]$, $\underline{v} > 0$, denotes an unknown random variable common to all bidders with c.d.f. $F_v$ and p.d.f. $f_v$. Let $\mathbf{z} = (x_1, ...x_n, v)$ be a random vector distributed according to the c.d.f. $F$ and the p.d.f. $f$, with $F$ *affiliated* and symmetric in its first $n$ arguments. All players are risk-neutral.

Whenever the signals $x_i$'s are affiliated through the common random component $v$, but they are independently and identically distributed given a realization of this common random variable, such a model belongs to the conditionally independent private information family (CIPI, for short). As a consequence, the signals $x_i|v$ are i.i.d. according to the c.d.f. $F_{x|v}(t|s) = \Pr(x_i \leq t|v = s)$ and the p.d.f. $f_{x|v}$ with support $[\underline{x}, \overline{x}]$, $\underline{x} > 0$. Notice that since this statistical structure requires $x_i$ and $v$ to be affiliated, we adopt the equivalent assumption that $F_{x|v}$ satisfies the (strict) MLRP. [10] The CIPI model can then be interpreted as a special case of the more general affiliated

[9] These examples are the *wallet game* (in which valuation corresponds to the sum of all bidders' signals) and the *maximum game* (in which valuation is the maximum among all bidder's signals).

[10] Assuming that the p.d.f. of the signals conditional on $v$, $f(x_1, ...x_n|v)$, is twice continuously differentiable, affiliation among the signals is equivalent to the following two conditions: (i)

value model (AV, for short) described above, as it can be verified that the joint distribution $F$ satisfies affiliation and symmetry in its first $n$ arguments from the following expression:

$$\begin{aligned} f(x_1, ...x_n, v) &= f_v(v) f(x_1, ...x_n|v) \\ &= f_v(v) \prod_{i=1}^{n} f_{x|v}(x_i|v) \end{aligned}$$

Imposing particular functional forms on bidder's valuations (utilities), two polar cases can be derived from the CIPI model.

***The CIPV model.*** Consider the case in which bidder $i$'s utility (valuation) is given by the function $U(v, x_i) = x_i$. Since the valuation to each bidder is given entirely by his own information, we are in the private value setting as each bidder fully knows his valuation ex ante. The only remaining uncertainty is hence about the other bidders' valuations. In particular, since now each bidder's value is equal to his signal, the model corresponds to the conditionally independent private value (CIPV, hereafter) model.[11] An economic interpretation of this model is as follows. While the random variable $v$ is interpreted as the ex post value that the *average* bidder assigns to the object for sale, the difference between each bidder's valuation and this average value, i.e., $(x_i - v)$, represents a bidder's *specific* characteristic such as productive efficiency, opportunity cost or idiosyncratic preference.[12] Note that the CIPV model is a special case of the affiliated private value setting, and also a polar case of the CIPI model.

***The CIPI-CV model.*** Consider now the case in which bidder $i$'s utility (valuation) is given by the function $U(v, x_i) = v$. Since all bidders share the same ex post valuation, and only observe an estimate of this value, we are in the pure common value setting. In consequence, no preference heterogeneity is considered. A traditional economic interpretation of this setting is the so called *mineral rights model.* All bidders exhibit the same ex post value for a tract given by $v$, derived from its exact mineral content. Nevertheless, at the time of the auction, they only observe a noisy signal of this content, $x_i$. We will refer to this polar case of the CIPI family as the CIPI-CV model. Finally, notice that this pure common value setting also constitutes a special case of the general affiliated value model.

**Model's Choice.** As we shall see in the next section, the impact of concentration on bidding behavior can be decomposed into three effects: the *competition effect*, the

$\frac{\partial^2 \log f(x_1,...x_n|v)}{\partial x_i \partial x_j} \geq 0$, and (ii) $\frac{\partial^2 \log f(x_1,...x_n|v)}{\partial x_i \partial v} \geq 0$, for all $i, j$ [see Topkis [38], p. 310]. As de Castro [8] discusses, the conditional independence models only guarantee the first condition. To obtain the second condition, one must assume explicitely that $x_i$ and $v$ are affiliated.

[11] This is the one studied by Pinkse and Tan [19].

[12] This interpreation is taken from Li et al [20].

*inference effect* and *the winner's curse effect.* While the first effect comes from the competitive environment involved in an auction mechanism no matter the valuation and information structure, the last two effects arise in environments with common value and dependence among signals, respectively.

This suggests that a good starting point for our analysis is provided by Pinkse and Tan [19]. They examine conditions under which the equilibrium bid is strictly increasing with respect to the number of bidders in first-price auctions under the CIPV model. From this, a first matter of interest concerning the model's choice is related to the valuation and information structure to be studied. It is clear that in order to examine the winner's curse effect as well, we need to widen our analysis to a more general setting than that provided by affiliated private value environments, and in particular by the CIPV model. We argue that the natural candidate which could have bite is the CIPI setting. As discussed above, this family of models is a special class of the general affiliated value model that encompasses the CIPV and the pure common value (CIPI-CV) setups as polar cases.[13] It is noteworthy that for our purpose, it suffices to focus only on the CIPI-CV case since it constitutes the simplest setting with an environment that is sufficiently rich to evaluate all the effects aforementioned.[14]

A second choice concerning our modelling strategy is given by the auction format to be examined. The bidding trade-off present in the first-price auction implies that the competition effect is more severe in this mechanism than in the second-price auction. Furthermore, as long as we assume any kind of dependence among the signals, the Revenue Equivalence Theorem no longer holds. As the classical linkage principle stated by Milgrom and Weber [17] points out, in such an environment the second-price outperforms the first-price auction. All of this suggests an important reason for preferring the latter format to study the effects of concentration in bidding markets: by analyzing the first-price auction, one does indeed consider the worst scenario for the seller. Hence, if we are able to show that under this mechanism concentration may increase revenues, we can directly extend this conclusion to the second-price auction.[15]

[13] The CIPI model was first studied by Li et al. [20], who tested their results in OCS wildcat auctions.

[14] The results derived in this paper can be particularly relevant for wildcat lease auctions. For instance, Hendricks, Pinkse and Porter [5] provide evidence that the bidding behavior for oil and gas auctions is consistent with a first-price auction under a symmetric pure common value environment with conditionally independent private signals, i.e., the CIPI-CV model.

[15] Moreover, by choosing the first-price sealed-bid auction, the conclusions of our work concern an auction format that is more frequently used than the second-price auction in the real world, as stated in Paarsch and Hong [28] (p. 22). In addition, it is likely that first-price sealed-bid auctions account for the bulk of transaction by value since procurements are often conducted via *low-price, sealed-bid tenders.*

## 3.3 Competition and bidding

In this section, we study the relationship between competition and bidding behavior in a first-price auction under the CIPI-CV model. We then provide conditions for (non)monotonicity of the equilibrium bids with respect to the number of buyers and propose a three-effect decomposition of the impact of concentration on bidding.

### 3.3.1 (Non)monotonicity of the equilibrium bid

Since our main purpose is to analyze the role played by the number of bidders, in what follows we adopt the (uncommon) notation according to which some functions of the model (bids, distributions, reverse hazard) depend on two arguments: $x$ and $n$.

Define $y_{1:n-1} = \max_{j=1,..n,\ j\neq i} x_j$, the first-order statistic of all bidders' signals except bidder $i$'s, and denote its c.d.f. and p.d.f. conditional on $x_i = x$ by $F_{y|x}(.|x)$ and $f_{y|x}(.|x)$, respectively. Let $\lambda(x;n) = f_{y|x}(x|x)/F_{y|x}(x|x)$ be its associated reverse hazard rate when the signals of the $(n-1)$ bidder $i$ 's rivals are smaller than or equal to $x$, given that its signal realization is $x$.[16]

We also assume that the seller can set a reserve price $r \geq 0$. Under a symmetric equilibrium, the expected payoff to bidder $i$ when he observes $x_i = x$ and bids $b$ in a first-price auction is then given by

$$\begin{aligned} \pi(b,x) &= E\left[(v-b)1_{\{\max\{B(y_{1:n-1};n),r\}\leq b\}}|x_i = x\right] \\ &= \int_{\underline{x}}^{B^{-1}(b;n)} \left[v(x,s;n) - b\right] f_{y|x}(s|x)ds \end{aligned}$$

where $B(.;n)$ is the equilibrium bidding strategy followed by all bidders except $i$ when facing $n$ rivals and $v(x,y;n) = E(v|x_i = x, y_{1:n-1} = y)$. If $B$ forms part of a symmetric equilibrium, then it must satisfy the following first-order differential equation

$$B_x(x;n) = \left[v(x,x;n) - B(x;n)\right]\lambda(x;n) \tag{3.1}$$

where $B_x(x;n)$ denotes $\partial B(x;n)/\partial x$;[17] and the appropriate boundary condition given by $B(a;n) = r$, where $a = a(r;n)$ is defined as follows

$$a = \{\inf x | E(v|x_i = x, y_{1:n-1} \leq x) \geq r\}$$

Solving the differential equation, bidder $i$'s equilibrium strategy is given by

$$B(x;n) = rL(a|x) + \int_a^x v(s,s;n)dL(s|x)ds \tag{3.2}$$

[16] In other words, $\lambda(x;n)$ corresponds to the reverse hazard rate of the second-order statistic conditional on $x_i = x$ being the first-order stastistic.

[17] For the functions $B$ and $v$, we use the subscripts $x$ and $n$ throughout the paper to denote their partial derivatives w.r.t. these variables.

for all $x \in [a, \overline{x}]$, where $L(s|x) = \exp(-\int_s^x \lambda(u; n)du)$.

Pinkse and Tan have shown that, in the CIPV model, if the reverse hazard rate is increasing in $n$ then bids are strictly increasing in the number of buyers. Nevertheless, as the following example illustrates, in the CIPI model properties for the reverse hazard rate no longer suffice for such bid monotonicity.

**Example 1** (from Wilson [41]). Consider the pure common value model $U(v, x_i) = v$ for all $i$. Suppose that $v$ is distributed according to the Pareto distribution such that $F_v(v) = 1 - v^{-\alpha}$ for $v \geq 1$ and $\alpha > 2$. Suppose also that the signals $x_i$'s are i.i.d conditional on $v$, so that $F_{x|v}(x|v) = (x/v)^\beta$ for $0 \leq x \leq v$.

It can be verified that $\lambda(x; n) = (n-1)\beta/x$, and that the equilibrium bid is given by

$$B(x; n) = \left[ \frac{(n-1)\beta + \max\{x, 1\}^{-(n-1)\beta - 1}}{(n-1)\beta + 1} \right] v(x, x; n) \tag{3.3}$$

Notice that $B(x; n)$ is *not* strictly increasing in $n$. Figure 1 (see Appendix C) displays the case in which $\alpha = 2.5$ and $\beta = .5$, showing that the equilibrium bidding function is indeed *decreasing* for signals that are sufficiently low when the number of bidders increases from $n = 2$ to $n = 3$. Interestingly, this example shows therefore that a nonmonotonicity of bids can be observed even though the reverse hazard is strictly increasing in $n$, as $\partial\lambda(x; n)/\partial n = \beta/x > 0$.

We can then conclude that the presence of an additional winner's curse-based effect in the CIPI setting requires more demanding conditions to guarantee the monotonicity of bids in the number of buyers. Equivalently, this also means that the set of conditions under which bids decreasing in $n$ can be observed becomes richer.

We begin characterizing a condition that ensures monotonic equilibrium bids with respect to the number of buyers.

**Proposition 18** *Let $\overline{b} = \max_n B(\overline{x}; n)$. Suppose that for all $x \in (\underline{x}, \overline{x})$,*

$$\frac{v(x, x; n+1) - \overline{b}}{v(x, x; n) - \overline{b}} > \frac{\lambda(x; n)}{\lambda(x; n+1)}$$

*Then for all $r < \overline{b}$ and $x \in (a, \overline{x})$, $B(x; n)$ is strictly increasing in $n$.*

A possible interpretation for this result is as follows. Since $\overline{b}$ constitutes the maximum possible bid to be made in the game, let us define $\underline{\pi}(x; n) \equiv v(x, x; n) - \overline{b}$, the *minimum benefits* that a bidder with signal $x$ can get conditional on defeating $(n-1)$ rivals. Thus, $\underline{\pi}$ can be seen as a lower bound of the winning bidder's benefits in the hypothetical case in which he were forced to participate in an auction with a reserve price equal to $\overline{b}$.[18] As is stated in the next section, while the winner's

[18] Of course, $\underline{\pi}$ can be negative.

curse effect ensures that $\underline{\pi}(x;n)$ is decreasing in $n$, the mixed effect coming from more competition and better inference on the degree of this competition may cause $\lambda(x;n)$ to be increasing in $n$. Consequently, Proposition 18 establishes that in the CIPI model, bids will be strictly increasing in the number of buyers as long as the negative effects on the minimum benefits stemming from the winner's curse be overcome (proportionally) by the (possible) positive competition-driven effects.

Since in the CIPV setting we have $v(x,x;n)=x$ for all $n$, the next result follows directly from Proposition 18.

**Corollary 19** *In the CIPV model, a reverse hazard function strictly increasing in $n$ suffices for Proposition 18.*

Therefore, in contrast to the CIPV model, in the CIPI setting the fact that the reverse hazard is strictly increasing in $n$ constitutes only a necessary condition, but *not* a sufficient condition for the equilibrium bid to be strictly increasing as well.[19] The intuition behind this result is the presence of the winner's curse effect in the CIPI model, absent in affiliated private value frameworks such as the CIPV setting. As a consequence, more restrictive conditions are needed for guaranteeing the monotonicity of bidding behavior in environments with interdependent values.

In order to establish conditions for the nonmonotonicity of the equilibrium bid, notice that our assumption of strict MLRP for $F_{x|v}$ guarantees that $\partial v(x,x;n)/\partial n \leq 0$ for all $x \in [\underline{x}, \overline{x}]$ (see Milgrom [27]). This property allows us to characterize the sufficient conditions for bids to be decreasing when signals are sufficiently low as follows.

**Proposition 20** *Consider the two following situations:*[20]
***(1)*** *Suppose that for some values of $n$ and $r$, it is verified either (A1) or (A2) with:*

$$\lambda(a(n+1);n+1) < \lambda(a(n+1);n) \tag{A1}$$

$$\frac{v(a(n+1),a(n+1);n+1)-r}{v(a(n+1),a(n+1);n)-r} < \frac{\lambda(a(n+1);n)}{\lambda(a(n+1);n+1)}. \tag{A2}$$

*Then, $B(x;n+1) < B(x;n)$ must hold for some $x > a(n+1) \geq a(n)$.*
***(2)*** *Suppose that there is no reserve price and, for some value of $n$, it is verified (A3) with:*

$$v(\underline{x},\underline{x};n) > v(\underline{x},\underline{x};n+1). \tag{A3}$$

*Then, $B(x;n+1) < B(x;n)$ must hold for some $x > \underline{x}$.*

[19] This is because such a condition guarantees that $\lambda(x;n)/\lambda(x;n+1) <$ *1*, which is also satisfied by the ratio $(v(x,x;n+1)-\overline{b})/(v(x,x;n)-\overline{b})$.

[20] Recall that $a$ depends on two arguments so that $a=a(r;n)$. For the sake of presentation, we have omitted $r$.

Condition (A1) emphasizes that the existence of a winner's curse-based effect in the CIPI model means that a reverse hazard decreasing in $n$ for signals that are low enough suffices for the nonmonotonicity of bids. As in the affiliated private value settings the winner's curse phenomenon is absent, the same condition on the reverse hazard also ensures the nonmonotonicity of bids in the CIPV model studied by Pinkse and Tan [19].

At the same time however, the additional presence of the winner's curse effect in the CIPI setting implies that other sufficient conditions for such a nonmonotonicity can be stated even when the reverse hazard is strictly increasing. One of these conditions is characterized in Proposition 20 by (A2), which constitutes a sort of reverse of Proposition 18. An interpretation for this condition can be provided following a similar line of reasoning as before. Accordingly, let us define $\overline{\pi}(a(n+1); n+1) \equiv v(a(n+1), a(n+1); n+1) - r$, the *maximum benefits of the marginal bidder* (the one indifferent between participating or not) conditional on defeating $n$ rivals. Then, $\overline{\pi}$ can be thought of as an upper bound of the winning marginal bidder's benefits when participating in an auction with a reserve price $r$.[21] Note that whereas a decrease in the number of bidders exerts an upward influence on $\overline{\pi}$ due to a reduced winner's curse, it may also induce an downward effect on the reverse hazard. As a result, condition (A2) states that if the first effect dominates (proportionally) the second one for the marginal bidder, then, at least for signals that are sufficiently low, *less* competition will bring *more* aggressive bids.

Furthermore, when there is no reserve price, condition (A3) guarantees the nonmonotonicity of the equilibrium bid irrespective of the properties exhibited by the reverse hazard. Such a sufficient condition is that of $v$ being strictly decreasing in $n$ for the lowest type. Note that Example 1 satisfies this condition, as can be verified that $v(x, x; n) = \max\{x, 1\}(\alpha + n\beta)/(\alpha + n\beta - 1)$ (see details in the Appendix). Hence, we have that $v(x, x; n) = (\alpha + n\beta)/(\alpha + n\beta - 1) > (\alpha + \beta(n+1)) / (\alpha + \beta(n+1) - 1) = v(x, x; n+1)$ for all $0 \leq x \leq 1$ and for all $n$. Thus, condition (A3) holds and thereby, the nonmonotonicity of the equilibrium bid with respect to $n$ follows.[22]

### 3.3.2 The bidding effect: A multiplicative decomposition

The previous subsection characterized the circumstances under which the participation of one more bidder can increase or decrease the bid aggressiveness. The ambiguity of this relationship highlights the importance of studying the sources of this bidding effect. In fact, identifying what forces affect positively or negatively the bidding

[21] Notice that $\overline{\pi}$ can be strictly positive as $v(a(n), a(n); n) = E(v|x_i = a(n), y_{1:n-1} = a(n)) \geq E(v|x_i = a(n), y_{1:n-1} \leq a(n)) \geq r$.

[22] See Figure 1 in Appendix C.

behavior would allow the seller to improve her decisions on auction formats. Accordingly, in this subsection we propose a decomposition of the bidding effect into three effects, a decomposition that we have named *multiplicative* decomposition.[23]

For simplicity, we assume throughout this subsection that there is no reserve price. As a result, the equilibrium bid becomes

$$B(x;n) = v(x,x;n) - \int_{\underline{x}}^{x} v_x(s,s;n)L(s|x)ds \tag{3.4}$$

Taking derivative on (3.4) w.r.t. $n$, we get that

$$B_n(x;n) = \left[v_n(x,x;n) - \int_{\underline{x}}^{x} L(s|x)v_{xn}(s,s;n)ds\right] - \int_{\underline{x}}^{x} L_n(s|x)v_x(s,s;n)ds \tag{3.5}$$

where $v_{xn}(x,x;n) = \partial v_x(x,x;n)/\partial n$ and $L_n(s|x) = \partial L(s|x)/\partial n$.

Let $\mathcal{W}$ be the event in which bidder $i$ wins the auction, i.e., $\mathcal{W} \equiv \{x > \max_{j\neq i} x_j\}$. Hence, denote $\rho(v|\mathcal{W},x)$ as the posterior density function of $v$ conditional on a bidder of type $x$ winning the auction.

Then, the reverse hazard can also be written as[24]

$$\lambda(x;n) = \int_{\underline{v}}^{\overline{v}} \lambda(x;n,v)\rho(v|\mathcal{W},x)dv \tag{3.6}$$

where $\lambda(x;n,v) \equiv (n-1)f_{x|v}(x|v)/F_{x|v}(x|v)$ corresponds to the reverse hazard associated to the situation in which $(n-1)$ rivals of a bidder $i$ of type $x$ draw their signals *independently* from the c.d.f. $F_{x|v}(x|v)$. The reverse hazard $\lambda(x;n)$ can thus be written as an average of $\lambda(x;n,v)$ in which the posterior density $\rho(v|\mathcal{W},x)$ are the weights.[25] Then, taking derivative on (3.6) w.r.t. $n$, under the assumption that the product inside the integral is twice continuously differentiable, we obtain that

$$\lambda_n(x;n) = \int_{\underline{v}}^{\overline{v}} \lambda_n(x;n,v)\rho(v|\mathcal{W},x)dv + \int_{\underline{v}}^{\overline{v}} \lambda(x;n,v)\rho_n(v|\mathcal{W},x)dv \tag{3.7}$$

[23] This decomposition is refered to as *multiplicative* as an alternative to the *additive* version performed by Pinkse and Tan [19] in the context of the CIPV model. We argue that our decomposition, as opposed to that of Pinkse and Tan, works even when the MLRP assumption only holds weakly (see Appendix B for an example).

[24] See Pinke and Tan [19].

[25] We derive the name the *multiplicative* decomposition proposed in this subsection from the *product* $\lambda(x;n,v)\rho(v|\mathcal{W},x)$.

Substituting (3.7) into (3.5), we get the following decomposition

$$\begin{aligned} B_n(x;n) &= \left[v_n(x,x;n) - \int_{\underline{x}}^{x} L(s|x)v_{xn}(s,s;n)ds\right] + \\ & \int_{\underline{x}}^{x} L(s|x)v_x(s,s;n)\left(\int_s^x \int_{\underline{v}}^{\overline{v}} \lambda_n(x;n,v)\rho(v|\mathcal{W},x)dvdu\right)ds + \\ & \int_{\underline{x}}^{x} L(s|x)v_x(s,s;n)\left(\int_s^x \int_{\underline{v}}^{\overline{v}} \lambda(x;n,v)\rho_n(v|\mathcal{W},x)dvdu\right)ds \end{aligned} \tag{3.8}$$

The change in the equilibrium bid strategy due to changes in the number of bidders can then be written as a sum of three components. The first term of the R.H.S. of equation (3.8) represents the effect coming from the winner's curse phenomenon associated with the common value environment. In fact, as long as we are in the private value setting - the CIPV model, for instance -, the fact that $v(x,x;n) = x$ implies that this effect disappears. Consequently, we refer to this effect as the *winner's curse effect* (WCE).[26]

The second term depends on $\lambda_n(x;n,v)$. Note that by the definition of $\lambda(x;n,v)$, its derivative w.r.t. $n$ is related to how $B(x;n)$ changes with $n$ in a setting with *independence* between the signals. Since under this environment one can associate any change of this class *only* to the traditional bidding trade-off existing in a first-price auction mechanism, this effect corresponds to the so-called *competition effect* (CE).

Finally, the third term depends on the partial derivative $\rho_n(v|\mathcal{W},x)$, which under affiliated information structures is negative (positive) for a large (small) enough $v$, and for a given $x$ and $n$.[27] In consequence, this term allows an inverse relationship between the bids and the number of buyers based on an inference-type effect generated by a positive dependency among the signals. Because of this, we will refer to this effect as the *inference effect* (IE).

In sum, we have identified three effects on the equilibrium bid strategy coming from changes in $n$ : the winner's curse effect (WCE), the competition effect (CE) and the inference effect (IE).[28] As we are interested in the nature of these effects, the next proposition formally states their signs.

**Proposition 21** *Suppose that in a CIPI-CV model, it is verified that (A1) $F_{x|v}$ satisfies the MLRP, and (A2) $|v_n(x,x;n)| \geq |v_n(\underline{x},\underline{x};n)|$ for all $x > \underline{x}$. Then, using the Multiplicative Decomposition, for all $x \in (\underline{x},\overline{x})$ and $n \geq 2$, it is verified that:*
*(i) The winner's curse effect (WCE) is negative*

[26] Krishna and Morgan [13], and Mares and Shor [23] study a similar winner's curse-based effect in the context of consortia, but they call it *inference effect* and *competition effect*, respectively. Notice that we use these terms to name two other effects of a different nature.

[27] See Proof of Proposition 21 in the Appendix.

[28] Notice that this decomposition nests indeed a variety of affiliated value models within the CIPI set-up, with the CIPV and the CIPI-CV models as polar cases.

*(ii) The competition effect (CE) is positive*
*(iii) The inference effect (IE) is ambiguous.*

The intuition behind the signs of these effects is as follows. First, the CE comes from the *less* aggressiveness that bidders exhibit when the chances of winning increase because the number of rivals decreases. Thus, this effect can be associated to the traditional negative consequences attributed to the industrial concentration in ordinary markets. Second, more concentration allows the winner's curse to be mitigated because defeating fewer bidders reduces the probability of such an overbidding phenomenon. As a consequence, bidders carry out a lower winner's curse downward correction in bids, and thereby, the WCE takes a negative sign. Finally, the IE stems from the affiliation among signals (or valuations).[29] The affiliation may also cause that a larger concentration results in a lower conservatism in bids. This may occur because, although winning is interpreted by the winner as information of a less degree of competition, this perception is weakened when he faces fewer rivals.[30]

So, whereas the inference effect can exacerbate the negative influence of the winner's curse adjustment, the competition effect always goes in the opposite direction. In consequence, as long as the combination of the first two effects dominate the latter, the equilibrium bid may be decreasing in $n$ as established in the previous subsection.

The signs of these effects are verified for Example 1. For instance, with $\alpha = 2.5$, $\beta = 0.5$, $n = 2$ and $x = 1.4$, the Multiplicative Decomposition yields

| *Effect* | *Magnitude* |
|---|---|
| $WCE$ | $-8.2408 \times 10^{-2} < 0$ |
| $CE$ | $4.0895 \times 10^{-2} > 0$ |
| $IE$ | $-1.4172 \times 10^{-7} < 0$ |

with a final effect given by

$$\begin{aligned} B_n(1.4;2) &= WCE + CE + AE \\ &= -0.041513 < 0 \end{aligned}$$

Note that in this example, although a negative inference exists, its magnitude is smaller than the positive one coming from the competition effect. The winner's curse effect is therefore crucial for the equilibrium bid to be a non-monotonic function in $n$ for signals that are sufficiently low.

[29] Pinkse and Tan [19] also examine an inference-type effect that they call the *affiliation effect*.
[30] A more detailed analysis of the inference effect is provided in the next subsection.

### 3.3.3 The inference effect: An illustrative example

In order to obtain a better intuition of the inference effect, let us analyze, in the context of Example 1, the source of the non-monotonicity of bids coming from the affiliated structure of the signals.

Similar to $\rho(v|\mathcal{W}, x)$, define $p(v|x)$ as the posterior density function of $v$ conditional on a bidder of type $x$. From Bayes' Theorem, it is easy to check that

$$p(v|x) = \frac{f_{x|v}(x|v)f_v(v)}{\int_{\underline{v}}^{\overline{v}} f_{x|v}(x|v)f_v(v)dv}$$

In Example 1, these two posterior density functions are given by $\rho(v|\mathcal{W}, x) = (\alpha + n\beta)\max\{1, x\}^{n\beta+\alpha} / v^{\alpha+n\beta+1}$ and $p(v|x) = 1/(\alpha+\beta)$. We can therefore state that for a given $x$ and $n$, the two following properties hold:

$$\rho(v|\mathcal{W}, x) > p(v|x) \tag{3.9}$$

$$\rho_n(v|\mathcal{W}, x) > 0 \tag{3.10}$$

for a small enough $v$, and the reverse inequality is verified otherwise. For instance, with $\alpha = 2.5$, $\beta = 0.5$, $n = 2$ and $x = 1.4$, we get that $\rho(v|\mathcal{W}, x) = 11.364/v^{4.5} > p(v|x) = 0.33$ for $v < 1.8201$; otherwise, the reverse inequality is satisfied (see Figure 2). Moreover, notice that

$$\begin{aligned}\rho_n(v|\mathcal{W}, x) &= \beta\frac{\max(x,1)^{(\alpha+n\beta)}}{v^{\alpha+n\beta+1}} - \left(\ln v \max(x,1)^{(\alpha+n\beta)}\right)\frac{\alpha\beta+n\beta^2}{v^{\alpha+n\beta+1}} \\ &\quad + \left(\ln(\max(x,1))\max(x,1)^{(\alpha+n\beta)}\right)\frac{\alpha\beta+n\beta^2}{v^{\alpha+n\beta+1}}\end{aligned}$$

Hence, for the same parameter values considered above, we get that $\rho_n(v|\mathcal{W}, x) = (3.535\,1/v^{4.5}) - (5.681\,8/v^{4.5})\ln v > 0$ for $v < \exp(0.622\,19)$, and the opposite result otherwise (see Figure 3).

The intuition behind these two conditions is the following. The former means that the event of winning the auction indeed represents *bad news* because the probability of small (high) realizations of $v$ increases (decreases) for a type $x$ bidder after knowing that his signal is the largest one. Additionally, the second condition points out that this bad news is reinforced by the increase in the number of bidders, as the posterior probability of small (high) values of $v$ increases (decreases) when $n$ becomes larger.

These conditions then provide a clear source for the non-monotonicity of the reverse hazard and thus, for the non-monotonicity of the equilibrium bid. For instance, in the CIPI-CV model, condition (3.9) implies that conditional on winning, bidder $i$ of type $x$ will estimate more likely that the ex post common value $v$ is smaller. As a result of the affiliation assumption on $F_{x|v}$, he will estimate more likely that his rivals' signals are smaller as well. Since the symmetric equilibrium bid strategy is increasing in the signals, it will lead finally to a perception of a *less* intense competition from the bidder $i$'s point of view. Given that this analysis is performed fixing the event of winning the auction, bidder $i$ should react following a *less* aggressive bidding behavior, which we call the *inference phenomenon*. However, as condition (3.10) means that this perception of lower competition is *counteracted* when $n$ decreases, we will eventually observe a lower conservatism in bids when a concentration process takes place. As a result, the *inference effect*, i.e. a possible shade in the inference-based downward adjustement in bids due to the reduction in the number of participants, can finally lead to an inverse relationship between revenue and $n$.

Note that, in contrast to the winner‘s curse, this inference phenomenon is of a *strategic* nature, as it emerges as a reaction of a rational player who, focusing only on this phenomenon, is able to reduce his bid without decreasing his probability of winning. That is, the winner's curse provokes a decrease in bids because the estimation of his/her *own* object's valuation is shaved. In contrast, in the case of the inference phenomenon, this greater conservatism is caused by a shade in the estimation of the *rivals*' bidding strategies.

## 3.4 (Non)monotonicity of revenues

In this section, we examine the conditions that guarantee the (non)monotonicity of the seller's revenue with respect to $n$ under the CIPI-CV model.

In the first-price auction, the expected revenue is given by[31]

$$R(n) = E(B(x_{1:n}; n)) = \int_{\underline{x}}^{\overline{x}} B(\alpha; n) f_{x_1}(\alpha; n) d\alpha \tag{3.11}$$

where $f_{x_1}$ and $F_{x_1}$ are the p.d.f. and c.d.f. of the maximum signal $x_{1:n} = \max_{i=1,..n} x_i$, respectively. Denote by $G(b; n)$ the distribution function of $B(x_{1:n}; n)$. From (3.11), it is clear that a first (and natural) condition that guarantees $R$ to be monotonically increasing in $n$ is that $G(b; n+1)$ first-order stochastically dominate $G(b; n)$ for all $n$. In order to gain an insight into the conditions that allow this stochastic dominance to hold, we need to invest in some additional concepts and notations. Let us define both $MRS_B(x; n)$, the *marginal rate of substitution in bids*, and $MRS_F(x; n)$, the *marginal rate of substitution in the winning signal distribution,* as follows

$$MRS_B(x; n) \equiv \frac{B_n(x; n)}{B_x(x; n)}$$

[31] For simplicity, we assume throughout this section that there is no reserve price.

and

$$MRS_F(x,n) \equiv \frac{F_{x_{1n}}(x;n)}{F_{x_{1x}}(x;n)}$$

where the subscripts $x$ and $n$ in $B$ and $F_{x_1}$ denote the partial derivative of these functions w.r.t. the respective variable.[32] In the context of auctions, the meaning of these marginal rates of substitution is as follows. Suppose that a marginal increase in the number of buyers occurs. In that case, $MRS_B(x;n)$ points out how and how much the change in $x$ needed to keep constant the level of the equilibrium bid is. Similarly, $MRS_F(x,n)$ represents the characteristics of the change in $x$ needed to keep the accumulated probability of the winning signal constant.

On the bid side, this required change in $x$ may be either an increase or a decrease, depending on the sign of $B_n(x;n)$. As discussed in previous sections, this partial derivative can be positive or negative, according to the magnitudes of the winner's curse, the inference and the competition effects. In contrast, the partial derivative $B_x(x;n)$ is always positive, as bids are strictly increasing in signals. All of this implies that, if bids are increasing (decreasing) in $n$, this larger competition will indeed require a decrease (increase) in signal values to preserve the equilibrium bid's level. As a consequence, the marginal rate of substitution in bids, $MRS_B(x;n)$, may take either a positive or a negative sign.

In contrast, on the side of the winning signal's distribution, the change in $x$ needed to preserve the accumulated probability of $x_{1:n}$ will always be an increase. This non ambiguity follows directly from the fact that $MRS_F(x,n)$ accounts for a fourth effect arising from changes in the number of buyers, which is not present when focusing on bids. This is the so-called *sampling effect*: an additional bidder means an additional draw from the signal distribution. Because of the properties of the first order statistics, the distribution of $x_{1:n+1}$ first-order stochastically dominates the distribution of $x_{1:n}$.[33] As a result, the partial derivative $F_{x_{1n}}(x;n)$ takes a negative sign unambiguously, and thus, the marginal rate of substitution in the winning signal distribution, $MRS_F(x,n)$, is always negative. Furthermore, the first stochastic dominance induced by the sampling effect on $F_{x_1}$ due to more competition translates eventually into higher seller's expected revenue. Notice that from equation (3.11), this point is very clear as the equilibrium bid is an increasing function in signals.[34]

In sum, when concluding as to the final effect of competition on revenues, we have to examine the properties of both the *bidding effect* and the *sampling effect* by

[32] For instance, $F_{x_{1n}}(x;n) \equiv \partial F_{x_1}(x;n)/\partial n$.

[33] In fact, one additional draw from the signal distribution implies that the highest signal is greater with probability $1/(n+1)$ and equal with probability $n/(n+1)$.

[34] Alternatively, we can interpret the sampling effect as an effect contributing positively to inducing a first stochastic dominance property in the winning bid distribution (see Proposition 22).

means of $MRS_B(x;n)$ and $MRS_F(x,n)$, respectively. This relationship between both marginal rates of substitution can be summarized defining the following term

$$\Delta(x;n) \equiv MRS_B(x;n) - MRS_F(x,n) \tag{3.12}$$

According to the previous analysis, three cases can emerge:

**Case 1.** A positive bidding effect: $MRS_B(x;n) \geq 0$, and hence, $\Delta(x;n) \geq 0$.

**Case 2.** A negative and dominated bidding effect: $MRS_B(x;n) \leq 0$ and $|MRS_B(x;n)| \leq |MRS_F(x,n)|$. Thus, $\Delta(x;n) \geq 0$.

**Case 3.** A negative and dominant bidding effect: $MRS_B(x;n) \leq 0$ and $|MRS_B(x;n)| \geq |MRS_F(x,n)|$. Thus, $\Delta(x;n) \leq 0$.

Equipped with these concepts and notation, we can go back to characterize the circumstances under which the seller may be better off or worse off with more competition. We start with the next proposition, which provides a sufficient condition to ensure revenues that are strictly increasing in the number of bidders.

**Proposition 22** $\Delta(x;n) \geq 0$ *for all* $x$ *if and only if* $G_n(b;n) \leq 0$ *for all* $b$*. Furthermore,* $G_n(b;n) \leq 0$ *for all* $b$ *implies that* $R(n)$ *is increasing in* $n$*.*

This result has the following implications. First, it means that as long as the effect of an increase in $n$ on bids is positive for all signals the final effect on revenue will be positive as well. In terms of our previous analysis, this means that as long as the bidding effect is *positive* (i.e. Case 1), the seller will benefit from more competition. This conclusion is true because, as explained before, the sampling effect always induces an increase in proceeds. Consequently, combining the result concerning bids (Proposition 18) with Proposition 22, we can state the next result.

**Corollary 23** *Suppose that for all* $x \in (\underline{x}, \overline{x})$*,*

$$\frac{\underline{\pi}(x;n+1)}{\underline{\pi}(x;n)} > \frac{\lambda(x;n)}{\lambda(x;n+1)}$$

*Then for all* $r < \overline{b}$*,* $R(n)$ *is strictly increasing in* $n$*.*

Moreover, Proposition 22 also implies that even though more concentration may cause a more aggressive bidding behavior, it may bring a reduction of revenue if the sampling effect is sufficiently large. This can occur when we are in Case 2, i.e. when a negative, but *dominated* bidding effect exists. Nevertheless, and in contrast to the affiliated private value model studied by Pinkse and Tan [19], in a CIPI-CV setting the last property is more difficult to be fulfilled. This is because the winner's curse effect, absent in the private value environments, demands a higher sampling effect to offset the inverse influences arising from bids. A direct consequence of this fact is that in the CIPI-CV model, the family of exponential distributions analyzed by Pinkse and Tan does not necessarily satisfy one of the sufficient conditions for monotonic revenues. This result is formalized in the following statement.

**Proposition 24** *Consider the CIPI-CV model. Suppose that for some function $\psi$, we can write $F_{x/v}(x/v) = \exp(\psi(x)v)$ for all $x$ and $v$. Then, the sign of $\Delta(x;n)$ is ambiguous.*

Note that Pinkse and Tan [19] show that for this class of distributions, $\Delta(x;n) \geq 0$ for all $x$ in the CIPV model. Consequently, Proposition 22 allows us to rule out the presence of such a polar case in the CIPI framework as long as an inverse relationship between revenue and number of bidders is observed.

**Corollary 25** *Consider the CIPI model. Suppose that for some function $\psi$, we can write $F_{x/v}(x/v) = \exp(\psi(x)v)$ for all $x$ and $v$. Then, if for some $n$, $R(n+1) < R(n)$, the valuation environment cannot be that of the CIPV model.*

Finally, note that the reverse of the first part of Proposition 22 provides a necessary and sufficient condition for the distribution of the winning bid with $n$ bidders to first-order stochastically dominate the distribution with $n+1$ bidders.

**Proposition 26** *For a given $n$, $\Delta(x;n) \leq 0$ for all $x$ if and only if $G(b;n) \leq G(b;n+1)$ for all $b$. Furthermore, $G(b;n) \leq G(b;n+1)$ for all $b$ implies that $R(n+1) < R(n)$.*

Notice that the last result indeed constitutes a sufficient condition for revenue to be non-monotonically increasing in the number of bidders. Using the analysis performed before, note that such a property of revenues holds as long as we are in Case 3, i.e. when there is a *negative* and *dominant* bidding effect.

Hence, and based on the results stated for bids in the previous section, we can establish the next statement on the nonmonotonicity of the seller's proceeds.

**Proposition 27** *Consider the two following situations:*
***(1)*** *Suppose that for some values of $n$ and $r$, it is verified that either (A1) or (A2) hold with:*

$$\lambda(a(n+1);n+1) < \lambda(a(n+1);n) \tag{A1}$$

$$\frac{\overline{\pi}(a(n+1);n+1)}{\overline{\pi}(a(n+1);n)} < \frac{\lambda(a(n+1);n)}{\lambda(a(n+1);n+1)}. \tag{A2}$$

*If* $|MRS_B(x;n)| > |MRS_F(x;n)|$ *then* $R(n+1) < R(n)$.
***(2)*** *Suppose that there is no reserve price and it is verified (A3) with:*

$$v(x,x;n) > v(x,x;n+1). \tag{A3}$$

*for all* $n$ *and* $x$. *If* $|MRS_B(x;n)| > |MRS_F(x;n)|$ *then* $R(n+1) < R(n)$.

In this statement, both (A1) and (A2) constitute sufficient conditions for the bidding effect to be negative. Additionally, the superiority (in absolute value) of the marginal rate of substitution in bids over that of the winning signal's distribution ensures that the bidding effect dominates the sampling effect.

Moreover, condition (A3) characterizes another situation allowing revenues to be decreasing in the number of bidders, but in a framework without a reserve price. In such a case, this revenue's property requires $v$ to be strictly decreasing in $n$.

In sum, the last proposition states that as long as bidding behavior becomes more aggressive with concentration (i.e., a negative bidding effect), the seller may indeed benefit from a reduction in competition. This phenomenon could occur if the mixed influence exerted by the winner's curse and the inference effects more than compensates the sampling effect.

This nonmonotonicity of seller's proceeds is illustrated by the following two examples.[35] First, in the case of Example 1, the expected revenue can be analytically computed, and is given by

$$R(n) = \left[1 - \frac{n\beta}{(\alpha + n\beta - \beta)(\alpha + n\beta - 1)}\right] \frac{\alpha}{\alpha - 1}.$$

Notice that

$$R_n(n) = \frac{\left(\alpha - \beta + \alpha\beta - \alpha^2 + n^2\beta^2\right)\alpha\beta}{(\alpha - \beta + n\beta)^2\,(\alpha + n\beta - 1)^2\,(\alpha - 1)}$$

where $R_n \equiv \partial R(n)/\partial n$. Hence, $sign(R_n(n)) = sign(\ \alpha - \beta + \alpha\beta - \alpha^2 + n^2\beta^2)$. In particular, for $\alpha = 2.5$ and $\beta = 0.5$, we have that

$$sign(R_n(n)) = \begin{cases} < 0 & \text{for all } n \in [0, 3.4641] \\ > 0 & \text{otherwise} \end{cases}$$

This case is depicted in Figure 4, showing that in the presence of the winner's curse and the inference effects, the expected revenue may be nonmonotonic with $n$. In particular, this example illustrates the fact that the seller is better off when a concentration

[35] Pinkse and Tan [30] emphasize that in the CIPV model the dominance of the bidding effects over the sampling effect requires too extreme distributional assumptions. As a result, they are unable to provide an example in which revenue is nonmonotonic with the number of buyers. In contrast, our more general CIPI setting permits us to attain this nonmonotonicity result without these extreme assumptions, as the next two examples show.

process takes place in a very concentrated market than when it does so in a competitive one.[36]

Second, a seller who benefits from less competition under the CIPI framework is also illustrated by Example 2, which describes a mineral right model that previous literature has showed (numerically) to yield nonmonotonic revenue in the number of bidders.

**Example 2** *The lognormal model* (from Reece [34] and DeBrock and Smith [3]). Consider the auction of a single offshore oil tract lease. The gross value of the petroleum reserve is given by $v$, a random variable distributed according to a *lognormal* probability density function represented by $f_v(v|\mu_v, \sigma_v)$, where $\mu_v$ and $\sigma_v$ correspond to the mean and standard deviation of log $v$, respectively. The net value of the tract is given by $V = v - c$, where $c$ is a known constant that represents the cost of postsale exploratory drilling. Each bidder observes $x_i$, an estimate of gross tract value that is, conditional on $v$, drawn from an independent and identically distributed *lognormal* distribution represented by the p.d.f. $f_{x|v}(x|\mu_x(v), \sigma_x(v))$ and c.d.f. $F_{x|v}(x|v)$.[37]

Note that in this model it is not possible to obtain an analytical solution for the equilibrium bid strategies starting from the first-order conditions of the bidder's maximization problem. However, DeBrock and Smith [3] find numerical solutions using specified values of parameters (means, standard deviations and number of bidders) consistent with real-world conditions of offshore oil leasing. Interestingly, their results suggest that the share of the social value of the tract captured by the seller (the government) can increase when joint bidding is allowed at a moderate level.

## 3.5 Conclusions

This paper has examined the revenue effect of having one more bidder at the auction stage. To this end, we considered the first-price auction format under the CIPI model, an environment that encompasses a wide variety of valuation and information settings. We decomposed the revenue effect coming from more competition into two general sources: (i) the bidding effect, and (ii) the sampling effect.

[36] A result consistent with this is theoretically stated by Hendricks et al. [7], who analyze the mineral rights model. This paper shows that bidders have more incentives to form rings when the number of potential participants in the auction is sufficiently large. Interestingly, they also confirm this prediction empirically for the offshore oil and gas lease auctions run by the U.S. government. However, we do not consider any information pooling effect as Hendricks et al. do. A similar result emerges in both papers notwithstanding, because of the presence of the winner's curse effect: whereas in Hendricks et al. the winner's curse affects the information precision of the bidding ring, in our paper this phenomenon influences directly the individual bidder's behavior.

[37] Notice that $F_{x|v}$ belongs to the normal distribution family, and thus, it satisfies the (strict) MLRP. As discussed before, this property is equivalent to assuming that $x_i$ and $v$ are affiliated. Although irrelevant for fitting the CIPI framework, this class of models additionally assumes that $E(x_i|v) = v$.

The former includes all the effects on bidding behavior, which in turn, we have grouped into three classes: the competition, the inference and the winner's curse effects. The first effect corresponds to the traditional positive consequences on bid aggressiveness due to the fact that higher competition intensity reduces the bidders' probability of winning in environments with independent signals. On the contrary, the inference effect stems from the affiliation among signals. In such environment, the participation of one more bidder induces more bid conservatism as the perception that winning conveys information of less rivalry -the *inference phenomenon*- becomes exacerbated when the number of bidders increases. Finally, the winner's curse effect arises in common value settings, and induces unambiguously less bid aggressiveness since more competition reinforces such an overbidding phenomenon. As a consequence, the sign of the bidding effect on revenue is ambiguous and depends on the relative magnitudes of their three subeffects.

In contrast, as the participation of one more bidder improves the stochastic properties of the winning signal, the sampling effect is always revenue-increasing because of more competition.

Our main result points out that situations exist in which the participation of an additional buyer can lower the seller's expected proceeds. Consequently, from the seller's point of view, more competition is not always desirable, as it may deteriorate revenue. Equivalently, the industrial concentration need not be negative for bid-takers. The results derived in this paper suggest therefore how inconvenient it can be to advise the seller regarding a policy that always either promotes more bidder participation or discourage mergers or any joint bidding practice.[38,39]

This work shows that the situations in which more competition can be revenue-decreasing are characterized by a negative and sufficiently large bidding effect that dominate the sampling effect. Our analysis identifies two cases in which the last condition is met. First, a bidding effect with these features can emerge if there is a negative and large enough inference effect that overcome the traditional competition effect. This condition, represented by a reverse hazard not strictly increasing in $n$, can be present in all settings nested by the CIPI environment, including the affiliated private value case given by the CIPV model. Second, we state that, in the CIPI model,

[38] Note that if the information pooling effect induces more aggressive bids, the situations in which the seller benefits from less competition would constitute a lower bound of the revenue-increasing cases caused by joint bidding arrangements.

[39] Policymakers have de facto adopted a more tolerant position in markets with these characteristics. For instance, in the U.S. offshore oil lease auctions. Before 1976, no restrictions were imposed on joint bidding ventures, and since 1976, these arrangements have been permitted for firms which are small enough. Similarly, bidding consortia in takeover battles is generally accepted as a legal practice; see, for example, the recent bidding takeover processes won by the consortia Enel-Acciona and RBS-Banco Santander-Fortis for the control of Endesa and ABN Amro, respectively.

an additional condition suffices for a negative and dominant bidding effect, and thereby, for revenue loss in the face of more competition. This extra condition arises from the winner's curse phenomenon, absent in the affiliated private value environments. Accordingly, we show that the seller may also benefit from concentration as long as the winner's curse effect is sufficiently large.[40]

Therefore, we conclude that in the CIPI setting, and thus in the general affiliated value model, the conditions that allow a nonmonotonic revenue in the number of bidders are less stringent than in affiliated private value frameworks. As a result, situations in which the seller is better off with less competition should be more frequent in environments with not only dependent information, but also interdependent valuations. Interestingly, the available empirical evidence supports this prediction, especially that related to bidding markets in which the winner's curse seems to play an important role such as wildcat auctions.

## 3.6 Appendix

**Appendix A: Proofs.**

*Proof of Proposition 18* We prove this statement by contradiction. Suppose that, for some $n$, $r$ and some $\widetilde{x}$, $B(\widetilde{x};n) \geq B(\widetilde{x};n+1)$. From boundary condition, we know that $B(a(n);n) = B(a(n+1);n+1) = r$. Substituting this into the differential equation given by (3.1), it is verified at $x = a(n)$ that

$$B_x(a(n);n) = [v(a(n), a(n);n) - r]\,\lambda(a(n);n) \tag{3.13}$$

Since by assumption

$$\frac{v(x,x;n+1) - r}{v(x,x;n) - r} > \frac{v(x,x;n+1) - \bar{b}}{v(x,x;n) - \bar{b}} > \frac{\lambda(x;n)}{\lambda(x;n+1)} \tag{3.14}$$

it follows from (3.13) that $B_x(a(n);n+1) > B_x(a(n);n)$. It must therefore be true that for some $x^* \in (a(n), \widetilde{x})$

$$B(x^*;n) = B(x^*;n+1) \equiv b^* \tag{3.15}$$

and

$$B_x(x^*;n) > B_x(x^*;n+1) \tag{3.16}$$

[40] Athias and Nuñez [1] argue that a strong winner's curse effect may be weakened as the perspective of renegotiation increases. They show evidence of that this phenomenon can be particularly relevant in toll road concession contract auctions. Thus, our analysis concerning the role of competition in auction markets should be extended to consider the impact of not only the *ex ante*, but also the *ex post* conditions on bidding behavior.

Notice however that (3.16) violates (3.1) because according to this differential equation it can be established the opposite condition

$$\begin{aligned} B_x(x^*;n) &= [v(x^*,x^*;n)-b^*]\,\lambda(x^*;n) \\ &< [v(x^*,x^*;n+1)-b^*]\,\lambda(x^*;n+1) = B_x(x^*;n+1) \end{aligned} \tag{3.17}$$

applying the same logic of (3.14) for $b^*$ instead of $r$.□

*Proof of Proposition 20* (1) We prove the first part of this statement by construction. First, because the boundary condition, $B(a(n);n) = B(a(n+1);n+1) = r$ for all $n$. Hence, evaluating the differential equation given by (3.1) at $x = a(n+1)$, it follows that

$$\begin{aligned} B_x(a(n+1);n) &= [v(a(n+1),a(n+1);n)-r]\,\lambda(a(n+1);n) \\ &\geq [v(a(n+1),a(n+1);n+1)-r]\,\lambda(a(n+1);n) \\ &> [v(a(n+1),a(n+1);n+1)-r]\,\lambda(a(n+1);n+1) \\ &= B_x(a(n+1);n+1) \end{aligned}$$

where the first inequality holds because $v_n(x,x;n) \leq 0$ for all $x \in [\underline{x},\overline{x}]$ as $F_{x|v}$ satisfies the (strict) MLRP (see Milgrom [27], Proposition 4 and Section 6), and the second one does since our assumption that the reverse hazard is strictly decreasing at $x = a(n+1)$. All of this implies that $B(x;n+1) < B(x;n)$ for all $x \in (a(n+1), a(n+1)+\delta)$ and $\delta > 0$.

We now show the second part of the first statement. Since boundary condition, $B(a(n);n) = B(a(n+1);n+1) = r$ for all $n$. Hence, and after evaluating the differential equation (3.1) at $x = a(n+1)$, it is straightforward to verify that assumption (A2) ensures that, given some $n$ and $r$, $B_x(a(n+1);n) > B_x(a(n+1);n+1)$. All of this implies finally that the equilibrium bid satisfies the desired property.

(2) Both the boundary condition (without reserve price) and the assumptions of the statement imply that

$$B(\underline{x};n) = v(\underline{x},\underline{x};n) > v(\underline{x},\underline{x};n+1) = B(\underline{x};n+1)$$

for some $n$. From this, it follows that $B(x;n) > B(x;n+1)$ for some $x \in (\underline{x}, \underline{x}+\delta)$ and $\delta > 0$.□

*Proof of Proposition 21* (i) First, our assumption of MLRP for $F_{x|v}$ guarantees that $v_n(x,x;n) \leq 0$ for all $x \in [\underline{x},\overline{x}]$ (see Milgrom [27], Proposition 4 and Section 6). So, if we are able to show that irrespective of the sign of $v_{xn}(s,s;n)$, its magnitude is smaller (in terms of absolute value) than $v_n(x,x;n)$, the desired result is attained. This is indeed true as

$$\begin{aligned} |v_n(x,x;n)| &\geq |v_n(x,x;n)| - |v_n(\underline{x},\underline{x};n)| \\ &= \left|\frac{\partial}{\partial n}\left[v(x,x;n) - v(\underline{x},\underline{x};n)\right]\right| \\ &= \left|\frac{\partial}{\partial n}\int_{\underline{x}}^{x} v_x(s,s;n)ds\right| \end{aligned}$$

where the first equality follows from assumption (A2). Moreover, since $v$ is twice continuously differentiable, it is verified that

$$\begin{aligned} \left|\frac{\partial}{\partial n}\int_{\underline{x}}^{x} v_x(s,s;n)ds\right| &= \left|\int_{\underline{x}}^{x} v_{xn}(s,s;n)ds\right| \\ &\geq \left|\int_{\underline{x}}^{x} L(s|x)v_x(s,s;n)ds\right| \end{aligned}$$

for all $x \in [\underline{x}, \overline{x}]$ (with the strict inequality for all $x \in (\underline{x}, \overline{x}]$) because of $0 \leq L(s|x) \leq 1$ for all $s$, $x \in [\underline{x}, \overline{x}]$ as $L(.|.)$ satisfies the properties of a c.d.f.

(ii) Note that $\lambda_n(x,n;v) = f_{x|v}(x|v)/F_{x|v}(x|v) > 0$, as we have assumed that $F_{x|v}(x|v)$ does not depend on $n$. Furthermore, affiliation ensures that $v_x(s,s;n) > 0$, and $L(s|x)$ and $\rho(v|\mathcal{W},x)$ are positive for all $s$, $x \in (\underline{x}, \overline{x}]$. As a result, the desired property holds.

(iii) First, we need to prove the next auxiliary result.

**Lemma 28** *In the CIPI model, for a given $x$ and $n$, it holds that*

$$\rho_n(v|\mathcal{W},x) > 0 \tag{3.18}$$

*for a $v$ small enough, and the reverse inequality is verified otherwise.*

*Proof of Lemma 28* Using the Bayes' Theorem, it is possible to verify that

$$\rho(v|\mathcal{W},x) = \frac{F_{x|v}^{n-1}(x|v)f_{x|v}(x|v)f_v(v)}{\int_{\underline{v}}^{\overline{v}} F_{x|v}^{n-1}(x|v)f_{x|v}(x|v)f_v(v)dv}$$

Hence, it is easy to state that for a given $x$ and $n$, the desired property holds.□

Then, rewrite $\int_{\underline{v}}^{\overline{v}} \lambda(x,n;v)\rho_n(v|\mathcal{W},x)dv$ as follows

$$\begin{aligned} \int_{\underline{v}}^{\overline{v}} \lambda(x,n;v)\rho_n(v|\mathcal{W},x)dv &= \int_{\underline{v}}^{\widehat{v}} \lambda(x,n;v)\rho_n(v|\mathcal{W},x)dv \\ &+ \int_{\widehat{v}}^{\overline{v}} \lambda(x,n;v)\rho_n(v|\mathcal{W},x)dv \end{aligned}$$

The sign of the last effect depends then on the magnitude of the areas delimited by the cut-off value $\widehat{v}$, from which according to Lemma 28, the partial derivative $\rho_n(v|\mathcal{W}, x)$ can take a negative sign for a given $x$ and $n$.□

*Proof of Proposition 22* Define $B^{-1}$ so that $B^{-1}(B(x;n);n) = x$ for all $x$ and $n$. Note that since $B$ is increasing in $x$, it holds that

$$\begin{aligned} G(b;n) &= \Pr(B(x_{1:n};n) \le b) \\ &= \Pr(x_{1:n} \le B^{-1}(b;n)) \\ &= F_{x_1}(B^{-1}(b;n);n) \end{aligned}$$

For short, denote $B^{-1}(b;n)$ by $t$. Then,[41]

$$\begin{aligned} G_n(b;n) &= F_{x_{1n}}(t;n) + F_{x_{1x}}(t;n) B_n^{-1}(b;n) \\ &= F_{x_{1n}}(t;n) - F_{x_{1x}}(t;n) \frac{B_n(t;n)}{B_x(t;n)} \\ &= -F_{x_{1x}}(t;n)\Delta(t;n) \end{aligned}$$

which yields the first desired result. Finally, from (3.11), it is clear that the first-order stochastic dominance induced by an increase in $n$ on $G(b;n)$ guarantees $R$ to be monotonically increasing in $n$.□

*Proof of Proposition 24* Recall from (3.5) that

$$\begin{aligned} B_n(x;n) &= \left[ v_n(x,x;n) - \int_{\underline{x}}^{x} L(s|x) v_{xn}(s,s;n) ds \right] - \int_{\underline{x}}^{x} L_n(s|x) v_x(s,s;n) ds \\ &= \left[ v_n(x,x;n) - \int_{\underline{x}}^{x} L(s|x) v_{xn}(s,s;n) ds \right] + \int_{s}^{x} \lambda_n(u;n) du \int_{\underline{x}}^{x} L(s|x) v_x(s,s;n) ds \end{aligned}$$

where the equality holds because $L(s|x) = \exp(-\int_s^x \lambda(u;n)du)$. Let us define

$$A(x;n) \equiv \lambda(x;n) MRS_F(x,n)$$

Pinkse and Tan [19] shows that

$$\int_{s}^{x} \lambda_n(u;n) du \ge A(x;n) \tag{3.19}$$

for the CIPV model. Notice however that $A(x;n)$, by definition, considers the source of two effects on revenue coming from more competition: (i) the sampling effect, through $MRS_F(x,n)$, and (ii) the bidding effect, but with the exception of the winner‘s curse

[41] Recall that the subscripts $n$ and $x$ denote the partial derivative of the respective function with respect to these variables.

effect, through $\lambda(x;n)$. Consequently, the inequality (3.19) also holds for the CIPI model. All of this implies therefore that

$$B_n(x;n) \geq \left[v_n(x,x;n) - \int_{\underline{x}}^{x} L(s|x)v_{xn}(s,s;n)ds\right] + A(x;n)\int_{\underline{x}}^{x} L(s|x)v_x(s,s;n)ds \tag{3.20}$$

Furthermore, using the equilibrium bid function as stated in (3.2), it is easy to see that

$$B_x(x;n) = \lambda(x;n)\int_{\underline{x}}^{x} L(s|x)v_x(s,s;n)ds$$

Hence, and by the definition of $A(x;n)$, the inequality (3.20) becomes

$$B_n(x;n) \geq \left[v_n(x,x;n) - \int_{\underline{x}}^{x} L(s|x)v_{xn}(s,s;n)ds\right] + B_x(x;n)MRS_F(x,n)$$

from which, rearranging and using the definition of $\Delta(x;n)$, it follows directly that

$$\Delta(x;n) \geq \frac{v_n(x,x;n) - \int_{\underline{x}}^{x} L(s|x)v_{xn}(s,s;n)ds}{B_x(x;n)}$$

Note that, according to (3.21), the numerator of the R.H.S. of the last inequality corresponds to the winner's curse effect. Since this effect is always negative and $B$ is increasing in $x$, the sign ambiguity of $\Delta(x;n)$ holds.□

*Proof of Proposition 26* It follows directly from Proposition 22 and the properties of the first-order stochastic dominance.□

*Proof of Proposition 27* From Proposition 20, either condition (A1) or (A2) implies that $B(x;n+1) < B(x;n)$ for some $x > a(n+1)$. As a result, $MRS_B(x,n)$ is negative, which constitutes a necessary condition for the nonmonotonicity of revenue. This condition and the fact that $|MRS_B(x,n)| > |MRS_F(x,n)|$ ensure then that $\Delta(x;n) \leq 0$, which according to Proposition 26, provides a sufficient condition for $R(n+1) < R(n)$.□

**Appendix B. The Additive Decomposition: A counter-example.**

Following Pinkse and Tan [19], the reverse hazard can be decomposed additively as $\lambda(x;n) = \lambda^Q(x;n) + \Delta\lambda(x;n)$. The first term corresponds to the reverse hazard consistent with the case in which $(n-1)$ bidder $i$'s rivals draw their signals *independently* and identically from the c.d.f. $Q(x|x)$, where $Q(t|x) = \Pr(x_j \leq t|x_i = x)$ and $q(t|x)$ it is its associated p.d.f. Hence, $\lambda^Q(x;n) = (n-1)q(x|x)/Q(x|x)$. The second term, i.e., $\Delta\lambda(x;n)$, is defined residually as it corresponds to the difference $\lambda(x;n) - \lambda^Q(x;n)$.

It is easy to verify that applying this additive decomposition to the CIPI-CV model, we have that

$$\begin{aligned} B_n(x;n) &= \left[ v_n(x,x;n) - \int_{\underline{x}}^{x} L(s|x) v_{xn}(s,s;n) ds \right] + \\ & \left[ \int_{\underline{x}}^{x} L(s|x) v_x(s,s;n) \left( \int_{s}^{x} \lambda_n^Q(u;n) du \right) ds \right] + \\ & \left[ \int_{\underline{x}}^{x} L(s|x) v_x(s,s;n) \left( \int_{s}^{x} \Delta\lambda_n(u;n) du \right) ds \right] \end{aligned} \tag{3.21}$$

By construction, it follows directly that the R.H.S. of equation (3.21) represents the sum of three bidding-type effects: the *winner's curse effect* (WCE), the *competition effect* (CE), and the *affiliation effect* (AE).

Consider now the pure common value model illustrated by Example 1. First, since

$$v_n(x,x;n) = \frac{-\beta \max\{1,x\}}{(\alpha + n\beta - 1)^2} < 0$$

and

$$v_{xn}(x,x;n) = \begin{cases} \frac{-\beta}{(\alpha+n\beta-1)^2} < 0 & \text{if } x > 1 \\ 0 & \text{otherwise} \end{cases}$$

the winner's curse effect is then given by

$$WCE = \frac{-\left(\max\{x,1\}^{\beta} + \beta \max\{x,1\}^{1+n\beta}(n-1)\right)\beta}{(\alpha+n\beta-1)^2 (n\beta - \beta + 1)\left(\max\{x,1\}^{n\beta}\right)} < 0$$

which confirms the sign attributed to this effect.[42] Second, we decompose the competition effect and the affiliation effect based on the Pinkse and Tan's approach. Notice however that given that $F_{x|v}$ does not satisfies the strict MLRP assumption, this decomposition does not work as $\lambda(x;n) = (n-1)\beta/x$ does not depend on $v$ and it is strictly increasing in $n$.[43] As a result, $\lambda^Q(x;n) = (n-1)\beta/x$ and hence the competition effect is given by

$$CE = \frac{\alpha + n\beta}{\alpha + n\beta - 1} \int_1^{\max\{x,1\}} \left( (\frac{\max\{x,1\}}{s})^{-(n-1)\beta} \right) \left( \ln(\frac{\max\{x,1\}}{s})^{\beta} \right) ds > 0$$

which also corroborates the expected sign. Nevertheless, since $\Delta\lambda(x;n) = \lambda(x;n) - \lambda^Q(x;n) = 0$, the affiliation effect becomes *null*. Thus, the additive decomposition proposed by Pinke and Tan does not capture in this case the inference-type effect that

[42] In particular, since $v_n(x,x;n) = -\beta \max\{1,x\} / (\alpha + n\beta - 1)^2$, the negativeness of the WCE is ensured by $v_n(x,x;n) < 0$ and $|v_n(x,x;n)| \geq |v_n(\underline{x},\underline{x};n)|$ for all $x > \underline{x}$ (see Proposition 21)

[43] That is, strict affiliation does not hold as $F_{x|v}$ satisfies only the weak MLRP.

arises from the statistic structure of the bidders' information assumed in this example. This provides us with the rationale for proposing an alternative *multiplicative* decomposition that identify an inference effect even though the MLRP assumption be weakly satisfied.

## Appendix C: Figures.

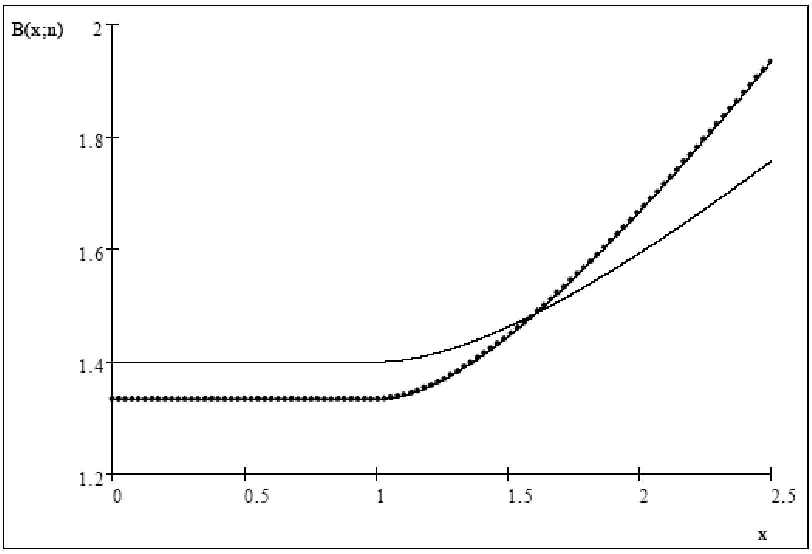


Figure 1. Equilibrium bid of Example 1 with $\alpha = 2.5$ and $\beta = .5$ for $n = 2$ (solid line) and $n = 3$ (dot line).

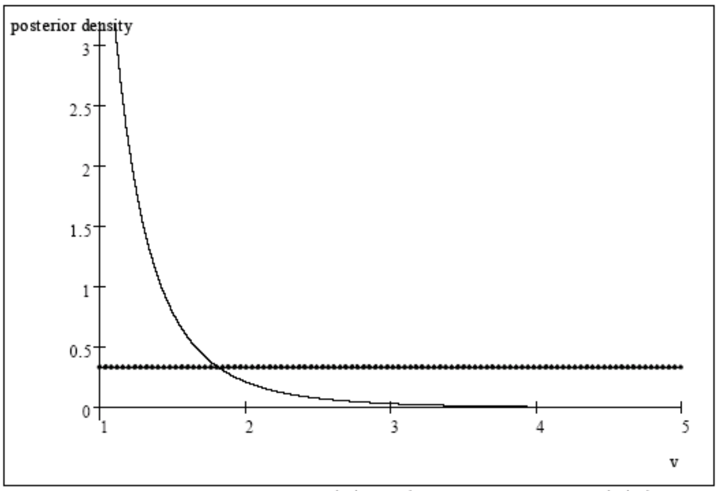


Figure 2. Posterior density functions $\rho(v|W,x)$ (solid line) and $p(v|x)$ (dot line) of type $x = 1.4$ for Example 1 with $\alpha = 2.5$, $\beta = .5$ and $n = 2$.

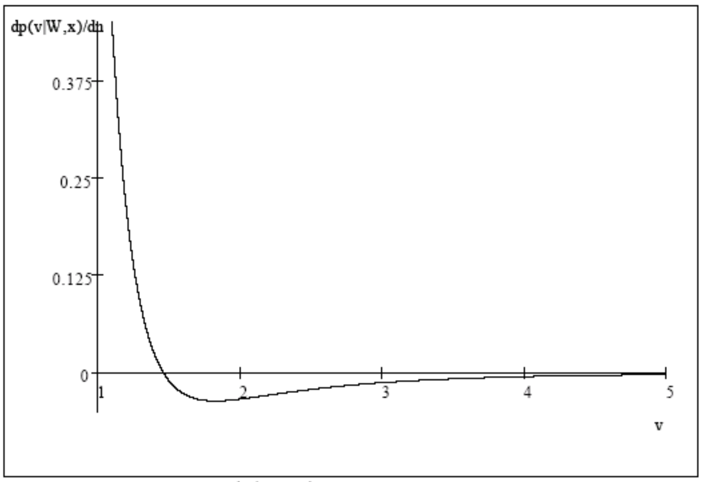


Figure 3. Partial derivative $\rho_n(v|W,x)$ of type $x = 1.4$ for Example 1 with $\alpha = 2.5$, $\beta = .5$ and $n = 2$.

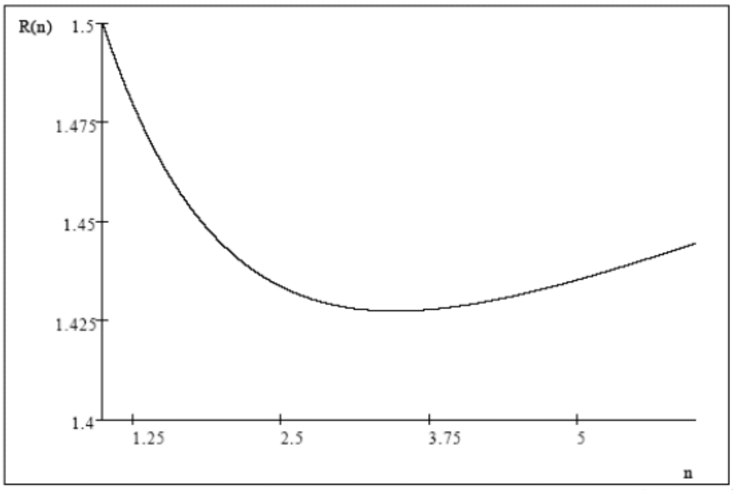


Figure 4. Revenue and number of bidders for Example 1 with $\alpha = 2.5$ and $\beta = .5$.

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# Chapter 4

# On Bidding Consortia: The Role of Information

**Abstract.** This draft is a first step to study the information effects of legal joint bidding practices (consortia and mergers) on bidding behavior and hence, on the seller's expected revenue. These effects are analyzed under an additive formulation of an interdependent values model with independent signals. We find that the extent to which the information sharing inside bidding coalitions induces more aggressive bids depends on three elements: (i) the valuation environment, (ii) the auction format, and (ii) the degree of bidders' risk-aversion on *information*. Nevertheless, we conclude that in general the seller benefits from the informational effects driven by joint bidding arrangements. In particular, it is shown that the expected revenue increase under both the first-price and the second-price auctions.

*Keywords*: auctions, joint bidding, information pooling, risk aversion on information

*JEL Classification*: C62, D44, D82, L41

## 4.1 Introduction

Bidding coalitions have become commonplace in many auction markets during the last decade. Although they are not a new phenomenon at all, especially in some wildcat auctions, the presence of bidding consortia has remarkably increased in procurement auctions, takeover battles, public works concessions and privatization processes.

A primary reason for such legal joint bidding arrangements to take place comes from the benefits that they may derive for its members as they lessen competition. Nevertheless, accepting this motivation as true and unique implies that bid-takers, in the absence of efficiency gains, are worse off when these practices are adopted. Following this line of reasoning, it would be however paradoxical that bidding coalitions be accepted, sometimes even encouraged, by the auctioneer and admitted as legal practices by the antitrust authorities. To sort out this apparent paradox, some additional motivations for the consortia formation have been provided.

One of the arguments in favor of legal bidding coalitions rests on the information sharing process carried out by their members.[1] The general intuition is the fact that this process allows the consortium to have access to *more* and *better* information. This would be especially useful in a common value environment, in which the rivals' signals are relevant for estimating the true object's valuation and uncertainty about this value is large. Thus, if better information is thought of as more precise one, this may induce the auction participants to bid more aggressively and thereby, benefit the seller.

This paper explores the validity of this argument by studying the informational effects of joint bidding practices on bidding behavior, and hence, on the seller's expected revenues. As a first step for this general objective, this draft focus on a model with interdependent values and independent signals. In particular, we assume that each bidder's valuation for the item corresponds to an additive combination of functions which depend on all bidders' signals. In this environment, independent signals and symmetric bidding coalitions imply that: (i) *more* information can always be interpreted as *better* one from the seller's point of view, and (ii) the revenue equivalence principle holds. As a result, information pooling increases the bid-taker's proceeds, and the auction format plays no role on these revenue effects. However, this simple setting provides us with a *benchmark* for comparing these effects under more sophisticated environments in which more information may not benefit the seller and the revenue equivalence breaks down. Examples of these possible extensions consider both statistical dependence among signals and asymmetric consortia formation.

[1]Other arguments in favor of these coaltions are that they allow to enhance both bidders' participation (by reducing financial constraints) and bidder's diversification of risk (See DeBrock and Smith [3]).

Despite the simplicity of our framework, it is sufficiently general to nest a wide range of valuation structures. In particular, it allows us to examine two polar cases extensively studied in the auction literature: the pure common value (CV) and the independent private values (IPV) environments. On the one side, the presence of a common value component permits the analysis of the impact a larger informational precision has on the winner's curse-driven bid shaving carried out in such valuation environment. As pointed out above, a primary intuition suggests that the information pooling should be especially important in a framework in which a buyer's valuation depends not only on his own private signal but also on his rivals' information. The access to better information should therefore improve the precision of his value estimation, and hence, falling prey to the winner's curse should be less likely. Our analysis allows us to state that, although this intuition is correct in the second-price auction, extra conditions are needed to guarantee that indeed unambiguously more aggressive bids emerge in the first-price format. These additional conditions concern the notion of risk aversion on *information*, that is, bidders' attitude towards more disperse (risky) information structures. Accordingly, we also get a monotone comparative statics in terms of bids in the first-price auction when buyers are risk averse on information. Despite these differences, the choice of one of these two auction mechanisms is irrelevant from the seller's point of view, as the expected revenues increase by the same magnitude in both of them.

On the other side, our analysis of the IPV environment reveals that bidding and revenue effects of the information pooling do *not* constitute an exclusive concern of the common value set-up. In fact, we show that under certain suitable assumptions, the informational effect is also positive for the auctioneer in a private values environment. Again, the seller remains indifferent with respect to both auction formats as the revenue effects of the information pooling are the same. However, it is demonstrated that the auction format matters for the magnitude of the informational effects on bids, but in the exactly opposite direction to that established under the interdependent values environment. That is, while there is no bidding effects in the second-price auction, bidders become uniformly more aggressive in the first-price format.

These contrasting results hinge decisively on the different informativeness criteria adopted to evaluate the informational improvements in each valuation environment. When considering a common value component, we focus on a criterion of informativeness based on the larger precision the consortium's signal distribution exhibits as compared with the solo bidder's one. In contrast, when studyng the IPV environment, we focus on a criterion of informativeness based on a stronger notion of stochastic improvement than that adopted in the interdependent values framework. Roughly speaking, while joint bidding introduces an informational improvement in terms of less "dispersion" in the latter valuation environment, this improvement is in terms of a higher "level" in the IPV setting.[2]

The main caveat on our results concerns the fact that this paper concentrates on the informational effects of joint bidding arragements, but it does not address the consequences coming from the reduction in the number of bidders. Competition-type effects are thus not taken into account. The justification for this restrictive approach stems from the *nonmonotonicity* between revenues and competition theoretically characterized and empirically supported by recent literature.[3] Accordingly, our results may be useful to explain why bid-takers allow the bidding consortia formation when their competition-driven issues seem to be ambiguous or even negative.

This paper is related to at least three strands of the auction literature. First, it extends the results obtained by Mares and Shor [15] and Krishna and Morgan [13], who also study the information pooling effect due to joint bidding practices. While these two articles work with the average value model, we generalize the valuation structure to admit the notion of risk aversion on information. This allows us to gain an insight into the conditions that effectively ensure a monotone result in terms of bidding behavior and revenue, especially in the first-price auction. It is worthy to note that, in contrast to all this previous literature, we also examine the private values environment. This extension is particularly relevant since it permits to stress that in the IPV setting the effects of information sharing may become as important as in the common value framework.

Our work can also be related to some papers on the optimal auctioneer's incentives to provide bidders with more precise information structures, such as Bergemann and Pesendorfer [2] and Ganuza and Penalva [4]. In fact, since in general it is the seller who may accept or reject bidding consortia formation, this decision can be restated as a decision about providing better information. Nevertheless, these works differ from ours in two key aspects: their informativeness criterion to order information structures and their focus on the private values setting. Finally, this paper is in connection with the literature that studies how changes in information according to different statistical orderings affect the auction outcomes, such as Hopkins and Kornienko [9] and [10]. For instance, the first of these articles examine the effects of a decrease in the dispersion of signals, in the sense of second order stochastic dominance, on bids and revenues in the common value environment. Interestingly, it shows that this information change yields comparative statics results very similar to those characterized in the current

[2] More specifically, in the IPV setting the informational improvement is in a sense that constitutes a refinement of the first-order stochastic dominance. In the interdependent values environment such improvement is, however, based on a refinement of the second-order stochastic dominance.

[3] For theoretical approachs, see Bulow and Klemperer [1], Pinkse and Tan [19], and Loyola [14]. For empirical works, see Hong and Shum [8], and Hendricks, Pinkse and Porter [5].

paper for the first-price and the second-price auctions.

This draft proceeds as follows. Section 4.2 describes our general additive values model with independent signals. In Section 4.3, we study the informational effects of joint bidding practices in the first-price and the second-price auctions when values are interdependent. The next section performs the same comparative statics analysis in the context of the IPV environment. Finally, Section 4.5 discusses some directions of the future research. All proofs are relegated to the Appendix.

## 4.2 The Model: Valuation and information structure

Consider a seller who wants to auction off a single item among $n$ bidders. Each bidder $i$ privately observes a value estimate that is independently and identically distributed according to the c.d.f. $F_x$. The associated p.d.f. $f_x$ is assumed to be a log-concave function that takes always a positive value in its support $[\underline{x}, \overline{x}]$. Let $v_i$ be the value of the object to bidder $i$. Let us assume it takes the following additive formulation:[4]

$$v_i = \psi(x_i) + \sum_{j \neq i} \varphi(x_j) \tag{4.1}$$

for all $i = 1, ...n$. The functions $\psi(.)$ and $\varphi(.)$ are non-decreasing functions that play the role of *weighting functions* on the information held by each bidder, with $\psi(\underline{x}) \geq 0$ and $\varphi(\underline{x}) \geq 0$. These weighting functions ensure a *symmetric* treatment on different rivals' signals when forming the valuation for the object, which is consistent with our assumed symmetric informational structure.[5] Furthermore, they will allow us to study the notion of *risk-aversion on information* and its impact on bidders degree of aggressiveness when they hold more precise signals. This concept is measured by the degree of concavity of $\psi$ and $\varphi$ so that while $\psi'' = \varphi'' = 0$ represents a bidder risk-neutral on information, $\psi'' < 0$ and $\varphi'' < 0$ is consistent with an information risk-averse individual. As a consequence, although we assume that all players are risk-neutral on wealth, bidders may be risk-averse in another sense.

Our choice of a model with interdependent values but independent signals is justified on different grounds. First, we intend to investigate how a more precise information may mitigate the winner's curse, and hence, increase bidders aggressiveness. Accordingly, we need a valuation structure that includes a common value component.[6]

[4] This form of interdependent value auction is a generalization of the one discussed in Klemperer [11] (Appendix D).

[5] Note that this framework can be extended to allowe for *asymmetric* treatments of the information coming from different bidders when forming the common valuation for the object. An asymmetric weighting scheme can emerge, for instance, if the source of information of some particular bidder is in some sense more *informative* or *precise*, which should be consistent with asymmetric signal's distributions.

[6] It is possible to formalize this idea assuming that $\psi(x_i) = \varphi(x_i) + \gamma(x_i)$ for all $i$.

Second, our valuation setting is sufficiently general to encompass two important polar cases: the pure common value (CV) and the independent private values (IPV) models. The pure CV environment can be attained by assuming that $\psi(x_i) = \varphi(x_i)$ so that $v_i = v \equiv \sum_{j=1}^{n} \varphi(x_j)$ for all $i$.[7] Furthermore, the IPV model can be characterized by assuming that $\psi(x_i) = x_i$ and $\varphi(x_j) = 0$ for all $i \neq j$ so that $v_i = x_i$, as we will actually do in Section 4.4. Third, we want to isolate the interdependence between *valuations* from the statistical dependence among *signals*. This is particularly useful as any (positive) statistical dependence between consortium members' value estimates brings an additional source of variability when signals are pooled. As a consequence, we here adopt an environment with signals independently distributed, and the extension to an affiliated information structure is left for future research.

## 4.3 Interdependent Values

In this section we examine the informational effects of joint bidding practices on bidding behavior and revenues when values are interdependent. To this end, we perform a comparative statics analysis between one situation with individual bidders and another one with bidding consortia.

### 4.3.1 Informativeness criterion and consortium's signal

In this subsection we characterize the signal observed by bidding coalitions under the interdependent values setting. The choice of this signal is directly related to the notion of the "better" information the consortium may access when pooling the pieces of information held by its members. To this end, we firstly need to define some criterion of informativeness (in particular a precision notion) of the signals observed by the participants in the auction. Since, as it will be seen below, we need to deal with the comparison of conditional distributions (and conditional expectations), we adopt here the notion of precision defined by Whitt [21].

**Definition 29** *Suppose that two signals $x$ and $y$ are distributed according to the c.d.f.'s $F$ and $G$, respectively, with associated p.d.f.'s $f$ and $g$, respectively. We say then that $x$ is less dispersed than $y$ in the uniform conditional variability ordering, denoted by $x \prec_{UV} y$ and $f \prec_{UV} g$, if supp($f$) $\subseteq$supp($g$), the ratio of densities $f/g$ is unimodal in the support of $g$ and both variables cannot be stochastically ordered.*[8]

Notice that given that *supp($f$)* $\subseteq$supp($g$), two conditions are necessary and sufficient for unimodality: (i) that the number of changes in sign of $(f - g)$ be 2, and

[7] A particular example of this pure CV model is described in Example 1.

[8] That is, we cannot state that either $F(s) \leq G(s)$ or $F(s) \geq G(s)$ hold for all $s$.

(ii) that the ratio $f/g$ be a log-concave function.[9] The second condition is particularly appealing because it constitutes in general an easier condition to be checked, and unlike the uniform conditional variability ordering, it introduces a *transitive* ordering.

**Definition 30** *Let us define* $r_f$*, the index of log-concavity, as follows:* $r_f(s_1, s_2, t_1, t_2) \equiv \frac{f(t_2-s_2)f(t_1-s_1)}{f(t_2-s_1)f(t_1-s_2)}$. *The density* $f$ *is log-concave relative to another* $g$*, i.e.* $f \preceq_{lc} g$*, if the support of* $f$ *is a subset of the support of* $g$ *and* $r_f(s_1, s_2, t_1, t_2) \geq r_g(s_1, s_2, t_1, t_2)$ *for all* $s_1 < s_2$ *and* $t_1 < t_2$.

Let us now characterize the consortium's signal that better allows to study the precision improvement in the sense of the uniform conditional variability ordering. We assume that there exist $l$ symmetric bidding consortia, each of one formed by $m$ members which, before the constitution of the coalition, participated as individual bidders in the auction. Thus, the information to which each bidding consortium has access is given by the vector $(x_1, ...x_m)$. We suppose that consortium $i$ summarizes such information through the signal $z_i$, which is defined as follows

$$z_i \equiv \frac{\sum_{k=1}^{m} x_k}{m}$$

and drawn according to the c.d.f. $G_z$ and the density function $g_z$. The choice of this signal for the consortium can be justified on different grounds. First, the average of the signals constitutes a sufficient statistic. Second, since all members of the coalition are symmetric ex ante (in particular because their signals are identically distributed), it seems natural to use a statistic that assign the same weight to each member's signal. Third, since our analysis concentrates on additive formulations of the object's value, the average of the signals preserves this additive manner of combining the information stemming from bidders. Fourth, in an interdependent values context, the main goal regarding informational effects of joint bidding practices is to analyze the higher *precision* on value estimation. Thus, we want to focus on the relative dispersion between both individual and consortium bidders. One way to do this is to abstract from the first moments of the distributions, so that we cannot compare them according to the traditional stochastic ordering, but we can state a comparison in terms of variability. In fact, notice that the definition of $z_i$ means that supp$(g)$ =supp$(f)$ and $E(z_i) = E(x_i)$. These facts and the assumption that $f_x$ is a log-concave function guarantee that $z_i \prec_{UC} x_i$.[10] Consequently, since the average is the simplest statistic that satisfies all these properties, the choice of $z_i$ seems to us appropriate.

[9] Formally, let $S(g - f)$ be the number of changes of the function $g - f$. Therefore, the first condition requires that $f$ be in some sense less variable than $g$ if $S(g - f) = 2$ with sign sequence $+, -, +$.

[10] See Proof of Proposition 33 and Mares and Shor [15].

### 4.3.2 Individual's and consortium's bidding

Define $y_{1:n-1} = \max_{j=1,..n,\ j\neq i} x_j$, the first-order statistic of all individual bidders' signals except bidder $i$'s, and denote its c.d.f. and p.d.f. conditional on $x_i = x$ by $F_{y|x}(.|x)$ and $f_{y|x}(.|x)$, respectively. Let $\lambda(x;n) = f_{y|x}(x|x)/F_{y|x}(x|x)$ be its associated reverse hazard rate when the signals of the $(n-1)$ bidder $i$ 's rivals are smaller or equal than $x$, given that its signal realization is $x$.[11] Moreover, define $v(x,x;n) \equiv E(v_i|x_i = x, y_{1:n-1} = x)$.

Given that all bidders' signals are identically distributed, we here concentrate on symmetric equilibria. Consequently, let $B_x^{FPA}(x_i;n)$ and $B_x^{SPA}(x_i;n)$ denote the *individual* bidder's equilibrium bids in the first-price and the second-price auctions, respectively.[12] Following Milgrom and Weber [17], the next lemma provides expressions for these equilibrium strategies under our additive interdependent values setting with independent signals.

**Lemma 31** *Consider the additive structure of values described by (4.1). Then, the first-price and the second-price auction equilibrium bids individual bidder i are given by*

$$B_x^{FPA}(x;n) = E(\psi(y_{1:n-1})|y_{1:n-1} \leq x) + (n-1)E(\varphi(x_j)|x_j \leq x) \tag{4.2}$$

*and*

$$B_x^{SPA}(x;n) = v(x,x;n) = \psi(x) + \varphi(x) + (n-2)E(\varphi(x_j)|x_j \leq x) \tag{4.3}$$

*for all $x \in [\underline{x}, \overline{x}]$ and for all $i \neq j$.*

Similarly, for bidding consortia, define $w_{1:l-1} = \max_{j=1,..l,\ j\neq i} z_j$. Denote its c.d.f. and p.d.f. conditional on $z_i = z$ by $G_{w|z}(.|z)$ and $g_{w|z}(.|x)$, respectively. Let $\lambda(z;l) = g_{w|z}(z|z)/G_{w|z}(z|z)$ be its associated reverse hazard when the signals of the $(l-1)$ consortium $i$ 's rivals are smaller than or equal to $z$, given that its signal realization is $z$. Moreover, define $\phi(z,z;l) \equiv E(v_i|z_i = z, w_{1:l-1} = z)$.

Let $B_z^{FPA}(z;l)$ and $B_z^{SPA}(z;l)$ denote the *consortium*'s equilibrium bid in the first-price and the second-price auction, respectively. From Lemma 31, we can directly characterize these equilibrium functions, as the next corollary formally states.

**Corollary 32** *Suppose that valuations are additively separable such that $v_i = \psi(z_i) + \sum_{j\neq i}\varphi(z_j)$ for all $i = 1,...l$. Then, the first-price and the second-price auction equilibrium bids of the consortium i are given by*

$$B_z^{FPA}(z;l) = E(\psi(w_{1:l-1})|w_{1:l-1} \leq z) + (l-1)E(\varphi(z_j)|z_j \leq z) \tag{4.4}$$

[11] In other words, $\lambda(x;n)$ corresponds to the reverse hazard rate of the second-order statistic conditional on $x_i = x$ being the first-order stastistic.

[12] The subscript in the bid function indicates the signal from which bidder observes a realization. Then, while individual bidders observe $x$, we shall see in the next subsection that bidding consortia observe a different signal $z$.

*and*

$$B_z^{SPA}(z;l) = \phi(z,z;l) = \psi(z) + \varphi(z) + (l-2)E(\varphi(z_j)|z_j \leq z) \tag{4.5}$$

*for all* $z \in [\underline{x}, \overline{x}]$ *and for all* $i \neq j$.

### 4.3.3 Comparative Statics Analysis

This subsection examines the effects generated by the information pooling phenomenon present in the consortia formation. In order to isolate these effects from those provoked by the reduction in the number of bidders, we perform an artificial exercise according to which the number of participants remains constant. As a consequence, we compare the equilibrium strategies of both the individual bidder and the bidding consortium, while fixing the number of buyers. That is, we suppose that $l = n$.

The larger or smaller aggressiveness resulting from the information pooling can be analyzed by comparing equations (4.2) and (4.4) (for the first-price auction) and equations (4.3) and (4.5) (for the second-price auction) under the assumptions that: (i) both classes of bidders observe the same realization, i.e., $x_i = z_i = t$, and (ii) the number of individual bidders is identical to the number of bidding coalitions, i.e., $l = n$.

For the first-price auction, the next three propositions perform this comparison depending on the degree of bidders' risk-aversion on information, or equivalently, the degree of concavity of the weighting functions. We begin with the simplest case in which these functions are linear.

**Proposition 33** *Suppose that* $\psi(t) = \alpha t$ *and* $\varphi(t) = \beta t$, *with* $\alpha, \beta \geq 0$. *Then, there exists* $t_1, t_2$ *with* $t_1 \leq t_2$ *such that*
*(i)* $B_z^{FPA}(t;n) \geq B_x^{FPA}(t;n)$ *for all* $t \in [\underline{x}, t_1]$
*(ii)* $B_z^{FPA}(t;n) \leq B_x^{FPA}(t;n)$ *for all* $t \in [t_2, \overline{x}]$.

This result stresses that, although the larger informational precision permits to reduce the winner's curse-based bid shading, this is not sufficient for getting an unambiguously higher aggressiveness. In fact, Proposition 33 points out that the rules of the first-price auction counteracts the mitigation of the winner's curse at the extent that there exist two cut-off signals.[13] Below the first threshold, the bidding consortium will be *more* aggressive than the individual bidder because of the information pooling effect. However, there exists also a region of signals sufficiently high so that the coalition becomes indeed more conservative.

This ambiguous result comes from the fact that, in the first-price auction, both equilibrium bids (the individual and the consortium) depends on the second-highest

[13] Of course, these cut-off signals can coincide.

signal. Although unimodality is inherited by the ratio of the order statistics' densities, the equality of their first moments is not preserved.[14] Consequently, it is not possible to state a comparison between $w_{1:n-1}$ and $y_{1:n-1}$ in the uniform conditional variability ordering, and so, a less aggressive behavior by a bidding consortium with a very high signal cannot be discarded. The intuition behind this result is as follows. Because of the less dispersion of the consortium's signal, a very high value is perceived as less likely than in the individual bidder case. Accordingly, the consortium estimates the event of being defeated by other bidders as less likely than when participating as an individual bidder. Given the natural trade-off present in the first-price auction mechanism, this implies that the coalition will eventually bid more conservatively as it is possible to get a larger profit without decreasing substantially the chances of winning.

Notice that this non-monotone result in terms of bid aggressiveness takes place in an environment in which risk-aversion on information plays no role, as information risk-neutral bidders do not have a preference-based inclination for less dispersed information structures. That is, the less conservatism exhibited by buyers with low signals is uniquely driven by *statistical* considerations.

Let us turn to the case in which the weighting functions are *convex.*

**Proposition 34** *Suppose that* $\psi$ *and* $\varphi$ *are strictly convex functions. Then, there exists a value* $t_2 \in [\underline{x}, \overline{x})$ *such that* $B_x^{FPA}(t;n) \geq B_z^{FPA}(t;n)$ *for all* $t \in [t_2, \overline{x}]$.

This statement indicates that when the item's valuation corresponds to the sum of convex functions of the signals, bidders tend to be more conservative. In fact, we are only able to ensure that for signal realizations high enough consortium's bidding will be less agressive than individual's one. Nevertheless, we cannot now guarantee the existence of a region of values sufficiently low in which the information pooling phenomenon generates a larger bid aggressiveness. The intuition of this result is that bidders risk-lover on information are worse off when the signal distribution becomes less dispersed.

Lastly, we examine the case in which the weighting functions are strictly *concave.*

**Proposition 35** *Suppose that* $\psi$ *and* $\varphi$ *are strictly concave functions. Then, the informational effect of consortia formation induces an unambiguously larger bid aggressiveness.*

This result stresses how the role played by the less dispersion of the consortium's signal is magnified when the weighting functions in the additive formulation of the

[14] This is due to $y_{1:n-1} = \max(x_1, ...x_{n-1})$ and $w_{1:n-1} = \max(z_1, ...z_{n-1})$. Since max is a convex function, the more variability of $x_i$ over $z_i$ in the convex ordering sense then implies that $E(y_{1:n-1}) \geq E(w_{1:n-1})$ (for more details, see Proof of Proposition 33).

value are concave. This occurs because bidders risk-averse on information are better off when signals are more precise. In this case, the information pooling implies that the consortium's bidding strategy shifts out so that a larger aggressiveness emerges unambiguously.

In sum, without imposing additional conditions, even in our very special additive valuation structure with independent signals, one cannot conclude that information pooling results in more aggressive bids when a first-price auction is run. In fact, in this auction format, bidders' attitude towards information risk can either exacerbate, counterbalance, or even overcome the effects by the less variability of the consortium's signal.

Let us consider now the second-price auction. From Lemma 31 and Corollary 32, it follows that the comparison between $v(t, t; n)$ and $\phi(t, t; n)$ for all $t$ suffices to determine the nature of the informational effect on bidding behavior. The following proposition establishes a monotone result for bids based on this comparison.

**Proposition 36** *Suppose that $\psi$ and $\varphi$ are weakly concave functions. Then, in the second-price auction, the informational effect of consortia formation leads to uniformly higher bids.*

It is worthy to remark that the last result implies that in the second-price auction the information pooling effect results in a *uniformly* more aggressive behavior, even when bidders observe high signals or when they are risk-neutral on information. The strength of this monotone result is based on the fact that the trade-off between benefits from winning and chances of winning, present in the first-price auction, is absent in the second-price mechanism. This means that the larger precision has only an effect on the winner's curse-based bid shading, leading eventually to a larger bid aggressiveness irrespective of the signal observed by bidders.

In the second-price auction the seller's expected revenue are equal to the expectation of the bid submitted by the bidder with the second-highest signal. Hence, and from the monotone result established in Proposition 36, one can get easily the next statement.

**Proposition 37** *In a second-price auction under the interdependent values and independent signals setting, the larger informational precision induced by consortia formation implies higher revenues.*

Note that in an environment with interdependent values and independent signals, the revenue equivalence principle holds as long as bidders are symmetric. Since we analyze joint bidding practices that preserve such symmetry, revenue equivalence remains after the consortia formation takes place. Hence, one can infer directly from the last statement that informational effects of legal joint bidding arrangements also benefit the auctioneer in the first-price mechanism.

**Corollary 38** *In a first-price auction under the interdependent values and independent signals setting, the larger informational precision induced by consortia formation leads to higher revenue.*

As an illustration of our results, consider the following auction model that fits well with the environment adopted in this paper.

**Example 1.** *The average value auction.* The most natural illustration of the results derived in this section is perhaps the average value auction model. This example corresponds to one of the polar cases of the interdependent values environment: the so called pure common value model. In this setting, the unknown and common value of the object is equal to the average of the signals observed by all bidders, i.e. $v_i = v \equiv \sum_{j=1}^{n}(x_j/n)$ and $v_i = v \equiv \sum_{j=1}^{n}(z_j/n)$ for all $i$. These signals are independently distributed according to a log-concave density function $f_x$. By adopting $\alpha = \beta = 1/n$, the statement concerning bidders risk-neutral on information in the first-price auction holds (Proposition 33). Furthermore, an uniformly larger bid aggressiveness in the second-price auction is also satisfied (Proposition 36). Finally, the seller's revenue-enhancing result due to the larger informational precision induced by joint bidding holds in both the first-price auction (Proposition 37) and the second-price format (Corollary 38).

## 4.4 Private values model

In this section we analyze the informational effects driven by the consortia formation when bidders' valuations are private and independently distributed. As before, we compare the equilibrium bids of both individual bidders and bidding coalitions under the assumption that the number of participants in the auction remains identical. Finally, results on seller's expected revenue are derived.

### 4.4.1 Valuation and information structure

Consider the special case of the general valuation structure described in Section 4.2 in which $\psi(x_i) = x_i$ and $\varphi(x_j) = 0$ for all $i \neq j$. The value of the item to bidder $i$ is thus $v_i = x_i$, so that we are in the simple independent private values (IPV) environment as the only relevant information for each bidder to form his valuation is his own signal. We also abstract from any concern about risk aversion on information, which, as we shall see later, does not alter qualitatively our results.

The examination of the IPV model is interesting for two reasons. First, the IPV model allows us to isolate the informational effects on the bid shading due to the first-price auction rules, as in this valuation environment the bid shading driven by the winner's curse is absent. Our analysis on the IPV setting suggests that the rules of the auction format do matter for the magnitude of the informational effects on bids: while a larger bid aggressiveness results in the first-price auction, there is no effect at all on bids in the second-price format.

Second, at first glance, the information sharing phenomenon taking place inside bidding consortia should be more valuable as long as more bidders' signals are useful to form the object's valuation. Following this line of reasoning, the information pooling should induce more bid aggressiveness in the interdependent value structure than in the pure private values one. As a consequence, the seller should benefit more from informational effects due to joint bidding arrangements in the former environment. Nevertheless, we show in this section that, at least for the symmetric consortia formation, there is no such clear-cut result in terms of either more bid aggressiveness or larger revenue in favor of structures with a common value component. This is because the comparison of both valuation environments in terms of bidding behavior is sensitive to the auction format analyzed. That is, in the second-price auction, the informational effects on bidding are clearly less intense in the private values framework than the interdependent values one. However, in the first-price auction, a uniformly larger aggressiveness from information risk-neutral bidders only emerges in the IPV setting. Furthermore, we prove that the information pooling leads to a revenue increase in both valuations environments, and not only when individual values depend on their opponents' signals.

### 4.4.2 Consortium's signal and informativeness criterion

We assume that in the IPV model the information observed by each consortium can be summarized through the signal $z_i$, which is defined as follows

$$z_i \equiv \max_{j=1,\ldots m} \{x_j\}$$

for all $i = 1, ..l$. Hence, $z_i$ is distributed according to the c.d.f. $G_z(z) = F_x^m(z)$ and p.d.f. $g_z(z) = mF_x^{m-1}(z)f_x(z)$. The key assumption underlying this signal choice is the fact that the consortium is an *efficient* coalition, i.e. it selects the *highest* value firm among its members as the representative bidder.[15] Accordingly, we suppose that the consortium considers the maximum valuation among its members as its relevant

[15] See Pesendorfer [18] for a discussion on efficient bidding coalitions in the context of procurement auctions.

signal, and thus, forms its joint bid based on such information.[16]

As discussed in previous sections, the choice of a consortium's signal is closely related to the choice of a criterion of informativeness, that is, a notion of the sense in which information pooling provides the coalition with a "better" signal. In the IPV model, the maximum signal also implies that the consortium bases its bid in a signal with better stochastic properties than the solo bidder. However, it is worthy to remark that the nature of such stochastic dominance is *stronger* than that induced by the average signal in the interdependent value structure. To understand this point, we previously need to state some notions of informativeness.

**Definition 39** *Suppose that two signals $x$ and $y$ are distributed according to the c.d.f.'s $F$ and $G$, respectively, and the associated p.d.f.'s correspond to $f$ and $g$, respectively. We say then that $x$ dominates $y$ in the monotone likelihood ratio (MLR) ordering, denoted by $x \prec_{MLR} y$ and $F \prec_{MLR} G$, if supp($f$) $\subseteq$supp($g$) and the ratio of densities $f/g$ is strictly increasing in the support of $g$.*

Recall that we require a criterion of informativeness that allows us to compare conditional distributions of the signals observed by the individual bidder and the bidding consortium. This is because in the IPV setting, the comparative statistics in terms of bidding behavior in the first-price auction depends crucially on the stochastic ordering stated between both signals. To this end, consider the following ratio ordering.

**Definition 40** *Suppose that two signals $x$ and $y$ are distributed according to the c.d.f.'s $F$ and $G$, respectively. We say then that $x$ dominates $y$ in the monotone probability ratio (MPR) ordering, denoted by $x \prec_{MPR} y$ and $F \prec_{MPR} G$, if supp($f$) $\subseteq$supp($g$) and the ratio of distributions $F/G$ is strictly increasing in the support of $g$.*

This definition means that for all $s < t$

$$\frac{F(s)}{G(s)} < \frac{F(t)}{G(t)}. \tag{4.6}$$

Rearranging (4.6), Bayes' Theorem implies that for all $s < t$

$$\Pr(x < s|x < t) = \frac{F(s)}{F(t)} < \frac{G(s)}{G(t)} = \Pr(y < s|y < t). \tag{4.7}$$

From the last expression, it follows that the MPR ordering implies the concept of "conditional stochastic dominance" (CSD), which is formalized in the next lemma.[17]

[16] We assume that if there is an opportunity for getting benefits from joint bidding, the coalition will be able to design a stable mechanism to share these profits. In that sense, we abstract from any mechanism design issue inside the bidding consortium.

[17] Maskin and Riley [16] define this class of dominance in a more general way, allowing for the possibility of different supports and atoms at the lower bound.

**Lemma 41** $F \prec_{MPR} G \Longrightarrow F \prec_{CSD} G$.

The following lemma establishes that the MLR ordering is a sufficiently strong notion of informativeness so that it implies the MPR ordering, and thus, the concept of conditional stochastic dominance as well.

**Lemma 42** $F \prec_{MLR} G \Longrightarrow F \prec_{MPR} G \Longrightarrow F \prec_{CSD} G$.

Notice that the conditional stochastic dominance guarantees a sort of conditional *first-order* stochastic dominance, which is crucial for getting a *monotone* comparative statics for the first-price auction in Subsection 4.4.4. This is true because the signal observed by the consortium (the maximum signal among its members) dominates the individual bidder's one in the MLR ordering, as the next result states.[18]

**Lemma 43** *Let $x_i$ and $z_i \equiv \max_{j=1,\ldots m}\{x_j\}$ be the signals observed by the individual bidder and the sole consortium bidder, respectively. Then, $z_i \prec_{MLR} x_i$.*

### 4.4.3 Individual's and consortium's bidding

As discussed above, the IPV setting is a special case of the general interdependent values environment. Hence, the simple substitution of weighting functions $\psi$ and $\varphi$ into Lemma 31 allows to characterize the equilibrium bids of both the solo bidder and the consortium. This is the content of the following corollary.

**Corollary 44** *Consider the IPV environment so that $\psi(x_i) = x_i$, $\varphi(x_j) = 0$ for all $i \neq j$, and thus, $v_i = x_i$. Then,*
*(i) The equilibrium bids of the individual bidder are given by*

$$B_x^{FPA}(x; n) = E(y_{1:n-1} | y_{1:n-1} \leq x) \tag{4.8}$$

*and*

$$B_x^{SPA}(x; n) = x \tag{4.9}$$

*for all $x \in [\underline{x}, \overline{x}]$.*
*(ii) The equilibrium bids of the bidding consortium are given by*

$$B_z^{FPA}(z; l) = E(w_{1:l-1} | w_{1:l-1} \leq z) \tag{4.10}$$

*and*

$$B_z^{SPA}(z; l) = z \tag{4.11}$$

*for all $z \in [\underline{x}, \overline{x}]$.*

[18] In the IPV model, this class of stochastic dominance imposes no restrictions on the distribution of the individual bidder's signal. Notice that, in contrast, the uniform conditional variabiliy ordering discussed in Section 4.3 for the interdependent values environment requieres the log-concavity of $f_x$.

### 4.4.4 Comparative Statics

Again, in order to isolate the pooling information effect from any competition effect, we compare the individual bidder's and the consortium's equilibrium bids assuming that $x_i = z_i = t$ and $l = n$.

Let us start with the second-price auction. Given the allocation rules of this mechanism, there not exists an auction's rule-based bid shading as in the first-price auction. In addition, in the private values setting, there neither exists a winner's curse-based shading as opponents' signals no matter to form the own value estimation. Therefore, telling the truth is a (weakly) dominant strategy as long as we are in a symmetric environment. Since we study joint bidding practices that preserve an initial symmetric industrial structure, this implies finally that by accessing to more and better information has *no* effects on bid aggressiveness. This idea is formalized in the next proposition.

**Proposition 45** *Consider the IPV environment. Then, in the second-price auction, the information pooling of consortia formation has no effects on bidding behavior.*

This result is diametrically different from that obtained for the second-price auction under the interdependent values framework, in which the information pooling leads to a uniformly larger aggressiveness from bidders.[19] The reason for these contrasting results lies in the fact in the IPV setting the equilibrium is in dominant strategies.

Let us consider now the first-price auction mechanism. In this case, the bidding trade-off inherent to the rules of this auction format induces a shading on bids. Equilibrium bids are then equal to the expectation of the second order statistic conditional on winning. This order statistic inherits the better stochastic properties of the consortium's signal $z_i$ by comparison with the individual bidder's one $x_i$. That is, the second-highest consortium's value dominates, in the conditional stochastic sense, the second-highest individual bidder's one. As a consequence, this induces a monotone result in terms of bid aggressiveness in the IPV framework. This is the content of the following proposition.

**Proposition 46** *Consider the IPV environment. Then, in the first-price auction, the informational effect of consortia formation leads to uniformly higher bids.*

Thus, in the first-price auction under the IPV setting the larger bid aggressiveness induced by the information pooling phenomenon is unambiguous. This result differs

[19] This finding holds true for bidders either risk-neutral or risk-averse on information (see Proposition 36).

from that obtained under the interdependent values structure, in which a less conservatism emerges only when either: (i) information risk-neutral bidders observe signals sufficiently low, or (ii) bidders are sufficiently risk-averse on information. The reason for getting a stronger comparative statics in the IPV environment is again the absence of the winner's curse. To clarify this point, consider the case in which bidders are risk neutral on information. While in the interdependent values setting the winner's curse gets exacerbated for high signals, in the IPV environment such overoptimism phenomenon is not present.

Finally, we analyze the revenue effects of the information sharing in the IPV environment. Since this paper studies the formation of bidding coalitions with the same number of members, the symmetry among bidders is always preserved. This symmetry and the independence of signals ensures that the revenue equivalence theorem holds. Thus, if the seller's expected revenue increase in the second-price auction, they do so in the first-price format. The next result states that the auctioneer indeed benefits from informational effects of consortia formation in both auction mechanisms.

**Proposition 47** *In the IPV environment the information pooling induced by the consortia formation implies larger seller's expected revenue in both the first-price and the second-price auction.*

## 4.5 Discussion and Future Work

This draft constructs a benchmark setting for assessing the information effects of legal joint bidding arrangements on auction outcomes. Since this framework considers independent signals and symmetric bidding coalitions, *more* information can always be interpreted as *better* one from the seller's point of view. As a consequence, information pooling increases the bid-taker's proceeds under a wide range of valuation structures, including the IPV environment. However, this information sharing process does not induce the same monotone comparative statics in the bidding behavior, as this effect depends on three aspects: (i) the valuation environment, (ii) the auction format, and (iii) the risk aversion on information structures.

Some caveats on our results need to be emphasized, which can also be seen as avenues for future research. First, in this draft we only focus on the informational effects of joint bidding practices, but we do not consider the consequences coming from the reduction in the number of auction participants. That is, competition-type effects are not taken into account. To this end, we perform an artificial comparative exercise which assumes that the number of bidders remains constant after joint bidding arragements are attained. Thus, although each consortium has access to more information, we suppose that the number of these bidding coalitions is equal to the original number of individual bidders. Second, we take the consortia formation as an exogenous phenomenon, without exploring neither their motivations nor the characteristics of the process by means of which such arrangements are materialized. Accordingly, we do not model, for instance, decisions related to profit-sharing rules and other mechanism design issues.[20] Third, our comparative statics findings are conditioned for two modelling choices closely related: the consortium's signal and the criterion of informativeness adopted for assessing in what terms this signal exhibits better stochastic properties than the solo bidder's one. In our paper, these modelling issues are crucial to contradict the primary intuition that informational improvements generated by joint bidding practices should be a major concern only in the common value environment. Indeed, the fact that the notion of informativeness resulting in the IPV framework be stronger than in the interdependent values setting hinges decisively on what information for the consortium is considered relevant: while in the former environment it is the maximum signal, in the latter one it is given by the average signal. Fourth, we study symmetric environments so that always all bidders (individual or collective ones) observe the same number of signals identically distributed. In general, this implies that *all* bidders access to more and better information, and thus, the existence of strong and weak players is ruled out. Consequently, there are no additional informational rents that affect negatively the seller's expected revenue.[21,22] At the same time, this symmetry guarantees, together with the independence among signals, that the favorable results for the auctioneer hold across different standard auction formats. Finally, our last caveat is precisely in connection with the total absence of any statistical dependence among the value estimates. This is a point particularly important as any (positive) statistical dependence between consortium members' signals -affiliation or simple correlation- may bring an additional source of variability when signals are pooled through a summary statistic. As a consequence, two interesting questions may came about: (i) more information may no longer mean more estimate precision, and (ii) revenue effects coming from the information pooling may become sensitive to the auction format.

[20] A paper which studies the informational effects of legal bidding coalitions and also models formally these mechanism design issues is that of Hendricks, Porter and Tan [7].

[21] Klemperer [12] discusses the formation of consortia that originates asymmetric industrial structures, and thus, hurts the bid-taker.

[22] From an empirical point of view, the assumption of symmetric environments seems no too far from what occurs in several bidding markets. Some anecdotal evidence suggests that consortia tend to compete mainly between them in wildcat auctions (Hendricks, Pinkse and Porter [5]), and takeover battles (Loyola [14]).

## 4.6 Appendix

*Proof of Lemma 31* Notice that

$$\begin{aligned} v(x,x;n) &\equiv E(v_i|x_i = x, y_{1:n-1} = x) \\ &= E(v_i|x_i = x, y_{1:n-1} = x, y_{2:n-1} \leq x, ..., y_{n-1:n-1} \leq x) \\ &= E(v_i|x_1 = x, x_2 = x, x_3 \leq x, ..., x_n \leq x) \end{aligned}$$

where the second equality is without of loss of generality. Let us define the event $\Im \equiv \{x_1 = x, x_2 = x, x_3 \leq x, ..., x_n \leq x\}$. Since $v_i$ is additively separable, it follows that

$$\begin{aligned} v(x,x;n) &= E(\psi(x_i) + \sum_{j=1,j\neq i}^{n} \varphi(x_j)|\Im) \\ &= E(\psi_1(x_1)|\Im) + \sum_{j=2}^{n} E(\varphi(x_j)|\Im) \\ &= \psi(x) + \varphi(x) + (n-2)E(\varphi(x_j)|x_j \leq x) \end{aligned} \tag{4.12}$$

where the second equality holds because the conditional expectation is a linear operator and the last equality follows from the independence of $x_i$. Similarly, the independence between $x_i$ and $y_{1:n-1}$ implies that $F_{y|x}(x|x) = F_x^{n-1}(x)$. Accordingly, we have that

$$\begin{aligned} L(s|x) &\equiv \exp(-\int_s^x \lambda(u;n)du) \\ &= \left(\frac{F_x(s)}{F_x(x)}\right)^{n-1} \end{aligned}$$

Hence,

$$\begin{aligned} l(s|x) &= \frac{\partial L(s|x)}{\partial s} \\ &= \frac{\partial F_x^{n-1}(s)}{\partial s} / F_x^{n-1}(x). \end{aligned}$$

Following Milgrom and Weber [17], we can then rewrite $B_x^{FPA}(x;n)$ as follows

$$\begin{aligned} B_x^{FPA}(x;n) &= \int_{\underline{x}}^{x} v(s,s;n)dL(s|x) \\ &= \int_{\underline{x}}^{x} l(s|x)\left[\psi(s) + \varphi(s) + (n-2)E(\varphi(x_j)|x_j \leq s)\right] ds \\ &= \frac{1}{F_x^{n-1}(x)} \int_{\underline{x}}^{x} \left[\psi(s) + \varphi(s) + (n-2)E(\varphi(x_j)|x_j \leq s)\right] dF_x^{n-1}(s) \end{aligned}$$

Since

$$
\begin{aligned}
E\left[\varphi(x_j)|x_j \leq s)\right] &= \frac{\int_{\underline{x}}^{s} \varphi(\alpha) dF_x(\alpha)}{F_x(s)} \\
&= \varphi(s) - \frac{\int_{\underline{x}}^{s} \varphi'(\alpha) F_x(\alpha) d\alpha}{F_x(s)} \qquad (4.13)
\end{aligned}
$$

where the last equality follows from integration by parts, then

$$
\begin{aligned}
B_x^{FPA}(x;n) = {} & \frac{1}{F_x^{n-1}(x)} \int_{\underline{x}}^{x} (\psi(s) + (n-1)\varphi(s)) dF_x^{n-1}(s) \\
& - \frac{1}{F_x^{n-1}(x)} \int_{\underline{x}}^{x} \frac{(n-2) \int_{\underline{x}}^{s} \varphi'(\alpha) F_x(\alpha) d\alpha}{F_x(s)} dF_x^{n-1}(s) \qquad (4.14)
\end{aligned}
$$

The first term of the r.h.s. of equation (4.14) corresponds to

$$
\frac{\int_{\underline{x}}^{x} (\psi(s) + (n-1)\varphi(s)) dF_x^{n-1}(s)}{F_x^{n-1}(x)} = E(\psi(y_{1:n-1})|y_{1:n-1} \leq x) + (n-1) E(\varphi(y_{1:n-1})|y_{1:n-1} \leq x) \qquad (4.15)
$$

and the second term can be written as follows

$$
\begin{aligned}
& \frac{1}{F_x^{n-1}(x)} \int_{\underline{x}}^{x} \frac{(n-2) \int_{\underline{x}}^{s} \varphi'(\alpha) F_x(\alpha) d\alpha}{F_x(s)} (n-1) F_x^{n-2}(s) f_x(s) ds \\
= {} & \frac{(n-1)}{F_x^{n-1}(x)} \int_{\underline{x}}^{x} \left( \int_{\underline{x}}^{s} \varphi'(\alpha) F_x(\alpha) d\alpha \right) dF_x^{n-2}(s)
\end{aligned}
$$

After integrating by parts the last expression becomes

$$
\begin{aligned}
& \frac{(n-1)}{F_x^{n-1}(x)} \left[ F_x^{n-2}(x) \int_{\underline{x}}^{x} \varphi'(\alpha) F_x(\alpha) d\alpha - \int_{\underline{x}}^{x} \varphi'(\alpha) F_x^{n-1}(s) ds \right] \\
= {} & (n-1) \left[ \frac{\int_{\underline{x}}^{x} \varphi'(\alpha) F_x(s) ds}{F_x(x)} - \frac{\int_{\underline{x}}^{x} \varphi'(\alpha) F_x^{n-1}(s) ds}{F_x^{n-1}(x)} \right] \\
= {} & (n-1) \left[ E(\varphi(y_{1:n-1})|y_{1:n-1} \leq x) - E(\varphi(x_j)|x_j \leq x) \right] \qquad (4.16)
\end{aligned}
$$

where the last equality holds because of (4.13). Substituting (4.15) and (4.16) into (4.14) yields

$$
B_x^{FPA}(x;n) = E(\psi(y_{1:n-1})|y_{1:n-1} \leq x) + (n-1) E(\varphi(x_j)|x_j \leq x)
$$

which proves the first part of the statement.

For the second-price auction, from Milgrom and Weber [17] it holds that $B_x^{SPA}(x;n) = v(x,x;n)$. Moreover, notice that from (4.12), $v(x,x;n) = \psi(x) + \varphi(x) + (n-2) E(\varphi(x_j)|x_j \leq x)$, which completes the proof.□

*Proof of Corollary 32* Applying a similar line of reasoning that in Lemma 31 for the case of $z_i$, both results follow directly.□

*Proof of Proposition 33* From Mares and Shor [15], log-concavity of $f_x$ guarantees (A1)-(A3), where

(A1) $z_i \prec_{UV} x_i$,

(A2) $E(x_i|x_i \leq t) \leq E(z_i|z_i \leq t)$ for all $t$, and

(A3) $E(y_{1:n-1}|y_{1:n-1} \leq t) \leq E(w_{1:n-1}|w_{1:n-1} \leq t)$ for all $t \in [\underline{x}, t_0]$ where $t_0 < \overline{x}$.

In addition, we need to state the following auxiliary result.

**Claim 48** *$E(\varphi(x_j)|x_j \leq t) \leq E(\varphi(z_j)|z_j \leq t)$ for all $t$ and $j$.*

*Proof of Claim 48* By the linearity of the expectation operator and (A2), it follows that

$$\begin{aligned} E(\varphi(x_j)|x_j &\leq t) = E(\beta x_j|x_j \leq t) \\ &= \beta E(x_j|x_j \leq t) \\ &\leq \beta E(z_j|z_j \leq t) \\ &= E(\beta z_j|z_j \leq t) \\ &= E(\varphi(z_j)|z_j \leq t) \end{aligned}$$

for all $t$ and $j$.□

Consider (4.2) and (4.4) evaluated at $\psi(t) = \alpha t$, $\varphi(t) = \beta t$, $x = z = t$ and $l = n$. Rearranging the first term in the r.h.s. of these both equations, it holds that

$$\alpha E(y_{1:n-1}|y_{1:n-1} \leq t) \leq \alpha E(w_{1:n-1}|w_{1:n-1} \leq t) \tag{4.17}$$

for all $t \in [\underline{x}, t_0]$, because $\alpha \geq 0$ preserves the inequality stated by (A3). Then, applying Claim 48 together with inequality (4.17) to (4.2) and (4.4), allows us to prove the first part of the lemma.

In order to demonstrate the second part of this lemma, we previously need to establish two auxiliary results.

**Claim 49** *$E(\varphi(x_j)|x_j \leq \overline{x}) = E(\varphi(z_j)|z_j \leq \overline{x})$ for all $j$.*

*Proof of Claim 49* In fact,

$$\begin{aligned} E(\varphi(x_j)|x_j &\leq \overline{x}) = E(\varphi(x_j)) \\ &= \beta E(x_j) \\ &= \beta E(z_j) \\ &= E(\varphi(z_j)|z_j \leq \overline{x}) \end{aligned} \tag{4.18}$$

for all $j$.□

**Claim 50** $E(y_{1:n-1}) \geq E(w_{1:n-1})$.

*Proof of Claim 50* Condition (A1) and the fact that $E(x_i) = E(z_i)$ and $\text{supp}(f) = \text{supp}\,(g)$ imply that the uniform variability ordering induces the convex order so that $z_i \prec_{CX} x_i$ or, equivalently, $E(\phi(x_i)) \geq E(\phi(z_i))$ for any convex function $\phi$ (see Shaked and Shanthikumar [20]). Accordingly, as $y_{1:n-1} = \max(x_1, ...x_{n-1})$ and $w_{1:n-1} = \max(z_1, ...z_{n-1})$ and max(.) is a convex function, it holds that $E(y_{1:n-1}) \geq E(w_{1:n-1})$.□

Thus, Claim 49 and 50 mean that $B_x^{FPA}(\overline{x}; n) \geq B_z^{FPA}(\overline{x}; n)$. Since the equilibrium bids are increasing functions, this last result and the first part of the proposition guarantee the existence of $t_2 \geq t_1$, and so, proves the second part of the statement.□

*Proof of Proposition 34* Applying the same line of reasoning of Claim 50 in Proposition 33 based on the convex order, it must be that $E(\varphi(x_j)) \geq E(\varphi(z_j))$ for all $j$. Moreover, define the function $m = \psi \circ \max$ so that $m(x_1, ...x_{n-1}) = \psi(\max(x_1, ...x_{n-1})) = \psi(y_{1:n-1})$. It is easy to check that $m$ is a non-decreasing and convex function in all of its arguments. Thus, as before we can apply the same property of the convex order and thereby, it holds that $E(\psi(y_{1:n-1})) \geq E(\psi(w_{1:n-1}))$. Since $B_x^{FPA}$ and $B_z^{FPA}$ are increasing functions, using all of this in (4.2) and (4.4) allows us to establish the existence of $t_2 \in [\underline{x}, \overline{x})$ with the desired property.□

*Proof of Proposition 35* (A1) and (A2) allow us to apply Corollary 1 in Whitt [21], as the fact that $\varphi$ is a non-decreasing and concave function implies that $E(\varphi(z_j)|z_j \leq t) \geq E(\varphi(x_j)|x_j \leq t)$ for all $t$ and $j$. Similarly, since concavity of $\psi$ ensures that $m$ is also a non-decreasing and concave function, it follows that $E(\psi(w_{1:n-1})|w_{1:n-1} \leq t) \geq E(\psi(y_{1:n-1})|y_{1:n-1} \leq t)$ for all $t$. These two facts together permit from (4.2) and (4.4) to prove the statement.□

*Proof of Proposition 36* From Lemma 31 and Lemma 32, notice that

$$v(t, t; n) = \psi(t) + \varphi(t) + (n-2)E(\varphi(x_j)|x_j \leq t) \tag{4.19}$$

and

$$\phi(t,t;n) = \psi(t) + \varphi(t) + (n-2)E(\varphi(z_j)|z_j \leq t) \tag{4.20}$$

Conditions (A1) and (A2) allow us to apply Corollary 1 in Whitt [21], as the fact that $\varphi$ is a non-decreasing and concave function ensures that $E(\varphi(z_j)|z_j \leq t) \geq E(\varphi(x_j)|x_j \leq t)$ for all $t$ and $j$. From (4.19) and (4.20), it follows directly that $\phi(t,t;n) > v(t,t;n)$. Since in the second-price auction the equilibrium bids are so that $B_x^{SPA}(t;n) = v(t,t;n)$ and $B_z^{SPA}(t;n) = \phi(t,t;n)$, the last inequality yields the desired result.□

*Proof of Proposition 37* The expected revenue with individual bidders in the second-price auction corresponds to the expected bid by the bidder with the second-highest signal, i.e., $R_x^{SPA} = E(B_x^{SPA}(y_{1:n-1};n))$. Since $B_x^{SPA}(t;n) = v(t,t;n)$, it follows that $R_x^{SPA} = \int_{\underline{x}}^{\overline{x}} v(t,t;n)dF_y(t)$. Hence, the expected revenue with bidding consortia is given by $R_z^{SPA} = \int_{\underline{x}}^{\overline{x}} \phi(t,t;n)dF_w(t)$. From Proposition 36, $\phi(t,t;n) > v(t,t;n)$ for all $t \in (\underline{x}, \overline{x})$, which ensures that $R_z^{SPA} > R_x^{SPA}$.□

*Proof of Corollary 38* The result constitutes a direct application of the revenue equivalence principle.□

*Proof of Lemma 39* This result is stated in the body of the paper.□

*Proof of Lemma 40* Let $x$ and $y$ be two variables with the common support $[\underline{x}, \overline{x}]$. Define $L_{x,y}(t)$ as the ratio of densities of these variables, i.e. $L_{x,y}(t) = f(t)/g(t)$. Hence, after integrating on $(\underline{x}, s)$ we get that $\int_{\underline{x}}^{s} dF(t) = \int_{\underline{x}}^{s} L_{x,y}(t)dG(t)$, or equivalently,

$$F(s) = \int_{\underline{x}}^{s} L_{x,y}(t)dG(t) \tag{4.21}$$

Let us assume then that $x \prec_{MLR} y$, which is equivalent to say that

$$L_{x,y}(s_2) \geq L_{x,y}(s_1) \tag{A4}$$

for all $s_2 > s_1 \geq \underline{x}$. Similarly, let $P_{x,y}(t) = F(t)/G(t)$ be the ratio of c.d.f.'s of the variables $x$ and $y$, respectively. From (4.21), it follows that

$$P_{x,y}(s) = \frac{\int_{\underline{x}}^{s} L_{x,y}(t)dG(t)}{G(s)}. \tag{4.22}$$

Thus, we must prove that $P_{x,y}(s_2) \geq P_{x,y}(s_1)$ for all $s_2 > s_1 \geq \underline{x}$. From (4.22), we have that

$$\begin{aligned} P_{x,y}(s_2) &= \frac{\int_{\underline{x}}^{s_1} L_{x,y}(t)dG(t) + \int_{s_1}^{s_2} L_{x,y}(t)dG(t)}{G(s_2)} \\ &= \frac{P_{x,y}(s_1)G(s_1) + \int_{s_1}^{s_2} L_{x,y}(t)dG(t)}{G(s_2)}. \end{aligned}$$

Hence, and by assumption (A4), note that

$$P_{x,y}(s_2) \geq \frac{P_{x,y}(s_1)G(s_1) + L_{x,y}(s_1)(G(s_2) - G(s_1))}{G(s_2)}.$$

From this, $P_{x,y}(s_2) - P_{x,y}(s_1) \geq 0$ if and only if

$$(L_{x,y}(s_1) - P_{x,y}(s_1)) \left[\frac{G(s_2) - G(s_1)}{G(s_2)}\right] \geq 0.$$

Notice that the term inside the square bracket is nonnegative as $s_2 > s_1 \geq \underline{x}$. By (4.22), the fact that the term inside the first bracket be also nonnegative is equivalent to say that

$$L_{x,y}(s_1) \geq \frac{\int_{\underline{x}}^{s_1} L_{x,y}(t)dG(t)}{G(s_1)}.$$

This is true because assumption (A4) ensures that $L_{x,y}(s_1)G(s_1) = \int_{\underline{x}}^{s_1} L_{x,y}(s_1)dG(t) \geq \int_{\underline{x}}^{s_1} L_{x,y}(t)dG(t)$, which completes the proof of the first part of the lemma. Finally, the second part of the statement follows immediately from Lemma 39.□

*Proof of Lemma 41* Let $L_{z,x}(t) = g_z(t)/f_x(t)$ be the ratio of densities of the signals observed by the consortium and the solo bidder, respectively. It is easy to check that $L_{z,x}(t) = mF_x^{m-1}(t)$, and hence, $\partial L_{z,x}/\partial t = m(m-1)F_x^{m-2}(t)f_x(t) > 0$ for all $t \in (\underline{x}, \overline{x}]$ and $m > 1$, which completes the proof.□

*Proof of Proposition 42* It follows directly from (4.9) and (4.11) when $x_i = z_i = t$ and $l = n$.□

*Proof of Proposition 43* We previously need to state the following auxiliary results.

**Claim 51** $w_{1:n-1} \prec_{MLR} y_{1:n-1}$.

*Proof of Claim 51* When $l = n$, independence among bidders' signals implies that $F_y(t) = F_x^{n-1}(t)$ and $G_w(t) = G_z^{n-1}(t) = F_x^{m(n-1)}(t)$. Hence, $f_y(t) = (n-1)F_x^{n-2}(t)f_x(t)$ and $g_w(t) = m(n-1)F_x^{m(n-1)-1}(t)f_x(t)$. Accordingly, the ratio of densities $L_{w,y}(t) = g_w(t)/f_y(t)$ becomes $m^{n-2}L_{z,x}^{n-1}(t)$. Since from Lemma 5.3, $\partial L_{z,x}/\partial t > 0$ for all $t \in (\underline{x}, \overline{x}]$, the desired result follows.□

**Claim 52** $w_{1:n-1} \prec_{CSD} y_{1:n-1}$.

*Proof of Claim 52* From Lemma 40, Claim 51 implies directly this result.□

Finally, applying Claim 52 to (4.8) and (4.10) when $l = n$ and $x = z = t$ for all

$t \in [\underline{x}, \overline{x}]$ allows us to show the statement.□

*Proof of Proposition 44* The expected revenue with individual bidders in the second-price auction under the IPV setting corresponds to expectation of the second-highest value, i.e., $R_x^{SPA} = E(y_{1:n-1})$. Similarly, the expected revenue with bidding consortia is given by $R_z^{SPA} = E(w_{1:n-1})$. From Claim 52 in Proof of Proposition 43, we know that $w_{1:n-1} \prec_{CSD} y_{1:n-1}$. Since CSD implies first-order stochastic dominance (FOSD), it follows that $w_{1:n-1} \prec_{FOSD} y_{1:n-1}$ and thereby, $R_z^{SPA} = E(w_{1:n-1}) > E(y_{1:n-1}) = R_x^{SPA}$. Lastly, the straightforward application of the revenue equivalence theorem ensures that the same inequality holds true for the first-price auction, i.e., $R_z^{FPA} > R_x^{FPA}$.□

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# Chapter 5

# Summary (in Spanish)

**Ensayos en Teoría de Subastas**

Esta tesis consiste en cuatro artículos que estudian, desde una perspectiva analítica y formal, temas de amplia discusión hoy en día en los mercados organizados como subastas.

El primer artículo, "*How to sell to buyers with crossholdings*", caracteriza el mecanismo de venta óptimo ante la presencia de relaciones de propiedad entre los postores de una subasta (*crossholdings*). La motivación para este ejercicio proviene del hecho que la literatura anterior ha mostrado que un principio clásico en Teoría de Subastas, como el Teorema de Equivalencia de Ingresos, no se cumple cuando existen estos crossholdings. Por tanto, conocer cuál es el procedimiento de venta que debiese usar un subastador en estos casos resulta ser un ejercicio interesante desde una perspectiva teórica y aplicada.

La caracterización de este mecanismo maximizador de ingresos está basada en tres vertientes de la literatura: el enfoque de diseño de mecanismos (Myerson 1981), el enfoque del ingreso marginal (Bulow y Roberts 1989), y la Teoría de Subastas.

El principal resultado es que el mecanismo óptimo impone una política discriminatoria en contra de los postores más fuertes, de tal modo que los ingresos esperados del subastador son crecientes en: (i) el tamaño de un *crossholding* común (caso simétrico), y (ii) el grado de asimetría de estas participaciones de propiedad (caso asimétrico). Además, establecemos que este procedimiento óptimo puede ser implementado mediante un mecanismo secuencial que incluye un esquema de precio-preferencia y un posible acuerdo exclusivo con el postor más débil de la subasta. Adicionalmente, mostramos que un procedimiento de negociación secuencial bastante simple, aunque subóptimo, rinde ingresos esperados para el vendedor mayores que una subasta al primer precio y una al segundo precio.

El segundo trabajo, "*Optimal takeover contests with toeholds*", caracteriza cómo debería ser vendida una firma que es objeto de una OPA por parte de eventuales compradores que ya poseen participaciones de propiedad en ésta (*toeholds*). El objeto de este artículo tiene dos motivaciones principales: una de carácter teórica y otra de naturaleza práctica. En primer lugar, estudios previos han establecido la falta de indiferencia de parte del vendedor entre distintos mecanismos de venta tradicionales en presencia de *toeholds*. En segundo lugar, la frecuente presencia de estas participaciones de propiedad en los procesos de toma de control en el mundo real proporciona una fuerte justificación práctica para la realización de este ejercicio teórico. La caracterización del procedimiento óptimo de venta se basa en el diseño de mecanismos. Adicionalmente, el análisis de procedimientos alternativos (secuenciales) constituye una aplicación de las técnicas de la Teoría de Subastas y la Teoría de Juegos, especialmente aquellas referidas a juegos dinámicos con información incompleta.

El artículo establece formalmente que el mecanismo que maximiza el precio de la compañía bajo estas circunstancias debe ser implementado por una subasta no estándar, que imponga un sesgo contra los postores con *toeholds* más grandes. Este procedimiento discriminatorio es tal que el precio de venta promedio de las acciones es creciente tanto en el tamaño de un *toehold* común (caso simétrico), como en el grado de asimetría de esas participaciones de propiedad (caso asimétrico). Finalmente, demostramos que un mecanismo basado en una ronda de negociaciones que dé prioridad a los postores más fuertes (aquellos con *toeholds* más grandes), replica las principales propiedades del procedimiento óptimo. Como resultado de esto último, establecemos que este mecanismo de negociación secuencial domina, en términos de precio de venta de las acciones, a las subastas tradicionalmente utilizadas en los procesos de toma de control corporativo.

El tercer artículo, "*On bidding markets: the role of competition*", analiza los efectos de la concentración industrial sobre las pujas de equilibrio y por ende, sobre los ingresos esperados del subastador. Estos efectos son estudiados bajo el modelo CIPI (*conditionally independent private information*), un marco de análisis que incluye ambientes con diferentes valoraciones y estructuras de información. El uso de este modelo es especialmente innovador debido a que permite extender la frontera del conocimiento respecto a las consecuencias de la concentración en mercados de subastas con valoraciones comunes e información dependiente (señales afiliadas).

La aplicación de la Teoría de Subastas a este tipo de ambientes (Milgrom y Weber 1982) nos permite descomponer formalmente el impacto sobre los ingresos, debido a menor competencia, en cuatro clases de efectos: un efecto competencia, un efecto inferencia, un efecto maldición del ganador y un efecto tamaño de muestra. Discutimos las propiedades de cada uno de estos efectos, y caracterizamos las condiciones que garantizan la (no)monotonicidad de la puja de equilibrio y los ingresos.

Nuestros resultados sugieren que es más probable que el subastador se beneficie de menos competencia en mercados con estructuras de valoraciones e información más completas. Un hallazgo que es particularmente interesante es el hecho que las condiciones bajo las cuales la mayor concentración industrial podría ser beneficiosa son menos exigentes que aquellas encontradas por la literatura previa (Pinkse y Tan 2005).

El último artículo, "*On bidding consortia: the role of information*", constituye un primer paso para estudiar los efectos de información de las prácticas de puja conjunta (consorcios y fusiones) sobre las pujas de equilibrio y los ingresos esperados del subastador. Estos efectos son analizados bajo una formulación aditiva del modelo de valoraciones interdependiente y señales independientes. Este marco de análisis permite examinar dos casos polares: el paradigma de valoraciones comunes puras y el paradigma de valoraciones privadas puras. Asimismo, provee a futuras investigaciones de un modelo de referencia para comparar ambientes más completos respecto de valoraciones e información, y grados de simetría en la formación de consorcios.

Mediante las técnicas de la Teoría de Subastas, establecemos que el grado en el que el proceso de intercambio de información en una coalición de postores lleva a pujas más agresivas depende de tres elementos: (i) el ambiente de valoraciones, (ii) el formato de la subasta, y (iii) el grado de aversión al riesgo en la información.

No obstante lo anterior, concluimos que en general el subastador se beneficia de los efectos de información producto de las prácticas de puja conjunta. En particular, el trabajo demuestra que los ingresos esperados aumentan cuando, tanto una subasta al primer precio como una al segundo precio, son utilizadas bajo un amplio rango de ambientes de valoraciones.