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TESIS DOCTORAL

ESSAYS ON THE ROLE OF INFORMATION

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Abstract

My Ph.D. thesis consists of three chapters on Information Economics in which I explore the consequences of the lack of information in the decision-making process of the agents. In the first Chapter, I present a monopoly model of two periods with risk-averse consumers. The quality of the product is uncertain for all the agents in the first period and the switching costs arise endogenously because of the faced risk. When the consumers learn the quality in the second period, two effects take place: first, the uncertainty disappears and the willingness to pay increases; and second, the true quality is revealed and the willingness to pay reacts consequently (it increases for high qualities and decreases for low qualities). When the consumers do not learn the quality in the second period, the uncertainty keeps being penalized. The model predicts that the bargain-then ripoff pattern of prices can be reversed for sufficiently low realizations of the quality, and that the first-period price set in the presence of switching costs is always lower than the first-period price set in a market without switching costs.

In the second Chapter, I extend the previous set-up to a duopoly market. One of the firms is known to offer a product of quality zero, whereas the quality of the product supplied by the other firm is uncertain but larger than zero in expected terms. The two effects described in the monopoly case when the consumers learn the quality still take place. However, in this model the consumers have two pieces of information (the prices of the two firms) to infer the uncertain quality in the second period if they did not gather the information through direct experience. The predictions of the model differ from those of the monopoly case. First, the bargain-then-ripoff pattern of prices can be reversed but not for the two firms simultaneously. Second, the most relevant result is that the average price in the first period can be larger in the presence of switching costs than in their absence, contrary to the conventional wisdom. This result happens because, given the expected quality, the firm that offers the riskless product has a competitive advantage in the first period when consumers are sufficiently risk averse that does not exist when consumers are risk neutral. It may happen that its rival cannot compensate completely this advantage through a price decrease because negative prices are not allowed.

In the third Chapter, I construct a model to analyze the effect of the piracy in the music industry in a situation with initial copyrights. I assume a for-profit platform and an open platform, two consumers who are heterogeneous in their willingness to pay for the tracks, and two artists that may be heterogeneous in their popularity (famous or unknown). The artists obtain their profits from both the sale of tracks and the attendance to concerts. These two businesses are related in a very particular way if the artist is unknown: a consumer will decide about attending or not to the concert only if she has learned about the existence of the artist in advance by listening to his tracks. Then, the introduction of piracy has two effects: the prices charged by the for-profit platform decrease, but the unknown artists always obtain the maximal degree of diffusion (the

famous artists had it without piracy too). I find that the total surplus does not decrease with piracy if the revenues from concerts are included in the negotiation process between the artist and the for-profit platform, but it may decrease if the parties only share the revenues from the tracks. Also, the for-profit platform and the famous artists are always damaged by piracy, although the consumers and the unknown artists may be better off.

Resumen

Mi tesis doctoral consta de tres capítulos sobre la Economía de la Información en los que exploro las consecuencias de la falta de información en el proceso de toma de decisiones de los agentes. En el primer Capítulo, presento un modelo de monopolio de dos periodos con consumidores aversos al riesgo. La calidad del producto es incierta para todos los agentes en el primer periodo y los costes de cambio surgen endógenamente por al riesgo asociado. Cuando los consumidores aprenden la calidad en el segundo periodo, dos efectos tienen lugar: primero, la incertidumbre desaparece y la disposición a pagar aumenta; y segundo, se revela la auténtica calidad y la disposición a pagar reacciona en consecuencia (aumenta para calidades altas y disminuye para las bajas). Cuando los consumidores no aprenden la calidad en el segundo periodo, la incertidumbre se sigue penalizando. El modelo predice que el patrón de precios ganga-timo se puede revertir para realizaciones de la calidad suficientemente bajas, y que el precio de equilibrio del primer periodo con costes de cambio es siempre más bajo que sin costes de cambio.

En el segundo Capítulo, extiendo el marco anterior a un mercado de duopolio. Se sabe que una de las empresas ofrece un producto de calidad cero, mientras que la calidad del producto ofrecido por el rival es incierta pero mayor que cero en términos esperados. Los dos efectos descritos anteriormente cuando los consumidores aprenden la calidad también tienen lugar aquí. Sin embargo, en este modelo los consumidores tienen dos piezas de información (los precios de las empresas) para inferir la calidad en el segundo periodo si no compraron previamente. Las predicciones del modelo difieren de las del caso de monopolio. Primero, el patrón de precios ganga-timo se puede revertir pero no para las dos empresas simultáneamente. Segundo, el resultado más relevante es que el precio medio en el primer periodo puede ser mayor en presencia de costes de cambio que en su ausencia, al contrario de lo predicho por los modelos tradicionales. Este resultado sucede porque, dada la calidad esperada, la empresa que ofrece el producto sin riesgo tiene una ventaja competitiva en el primer periodo cuando los consumidores son aversos al riesgo que no existe cuando los consumidores son neutrales al riesgo. Es posible que el rival no pueda compensar totalmente esta ventaja mediante una bajada de precios ya que los precios negativos no se permiten.

En el tercer Capítulo, construyo un modelo para analizar el efecto de la piratería en la industria musical partiendo de una situación inicial con copyright. Asumo una plataforma con fines de lucro y una plataforma abierta, dos consumidores heterogéneos en su disposición a pagar por las pistas de música, y dos artistas que pueden ser heterogéneos en su popularidad (famosos o desconocidos). Los artistas obtienen sus beneficios tanto de la venta de pistas como de la asistencia a los conciertos. Estos dos negocios se relacionan de un modo muy particular si el artista es desconocido: un consumidor decidirá si asistir o no al concierto solo si sabe previamente de la existencia del artista gracias a haber escuchado sus pistas. Así, la piratería tiene dos efectos:

los precios que puede cargar la plataforma con fines de lucro disminuyen, pero los artistas desconocidos siempre obtienen el grado máximo de difusión (los artistas famosos ya lo tenían sin piratería). Encuentro que el bienestar total no disminuye con la introducción de la piratería si los ingresos por conciertos se incluyen en el proceso de negociación entre el artista y la plataforma con fines de lucro, pero puede disminuir si solo se reparten los ingresos de las pistas. Del mismo modo, la plataforma con fines de lucro y los artistas famosos siempre salen perjudicados por la piratería, aunque los consumidores y los artistas desconocidos pueden mejorar.

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Nunca te entregues ni te apartes
junto al camino, nunca digas
no puedo más y aquí me quedo
—Palabras para Julia (J.A. Goytisolo)—

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Chapter 1

Switching Costs derived from Uncertainty: the Monopoly Case

1.1 Introduction

In many markets, consumers who have acquired the product offered by a certain firm face costs of switching to a competitor's product. This implies that products that were ex-ante homogeneous become ex-post heterogeneous. The idea can be easily extended to a monopolistic market when the outside option of not buying is allowed: in the second period, consumers are divided into those who bought the product in the first period and those who did not. Assuming that the good is not storable and that price discrimination is not possible, a consumer who buys the good in the second period has a larger utility if she also acquired it in the first period because of the switching cost.

In two-period models with switching costs, the equilibrium prices usually display a bargain-then-ripoff pattern. In the second period, the firm takes advantage of the captive consumers and sets a high price (this is commonly known as the harvesting effect). Anticipating the large profit of the second period, the firm has incentives to decrease the price in the first period (investment effect). In this regard, most of the research has focused on determining which effect dominates in terms of the average market price: conventional wisdom states that the harvesting effect dominates (Klemperer (1987b); Farrell and Klemperer (2007)), but this result has been challenged several times (Doganoglu (2010); Cabral (2012)).

Different reasons for the existence of switching costs have been established: a need for compatibility with existing equipment, transaction costs, learning costs, uncertainty about the product quality, and discount coupons (Klemperer (1995)). Nevertheless, not much attention has been

given to determine whether the firm actually has incentives to follow the bargain-then-ripoff pattern of prices in the presence of all possible types of switching costs. One of the scarce examples of this is Nilssen (1990): he distinguishes transaction costs and learning costs, such that transaction costs are incurred by a consumer at every switch between suppliers and learning costs are incurred only at a switch to a supplier that is new to the consumer. By keeping constant the total switching cost faced by a consumer, the author examines the effect of changing the proportion of each kind of switching cost in a market open for an infinite number of periods. Assuming that price discrimination is possible, he finds that for a fixed proportion of learning and transaction costs, there exists an equilibrium in which all prices in the steady state are below the monopoly price. Even more, these prices are ordered such that the price charged by the incumbent in the second period to a loyal customer is higher than the price charged to a disloyal customer, higher than the price set by the rival firm, and higher than the single first-period price. Also, the author shows that an increase in the relative weight of the transaction costs increases the price offered to loyal consumers, whereas decreases the first-period prices and the total welfare.
His approach differs from mine in several ways. First, in my framework of switching costs derived from uncertainty I do not allow for price discrimination, so I will not have to differentiate between several equilibrium prices in the second period. Second, and most important, this type of switching costs can be spread to all consumers in the market through a credible signal about the quality (price): if the uncertainty is eliminated in the second period through a credible price, then all the risk-averse consumers benefit from the lack of uncertainty in the second period and increase their willingness to pay in the same amount, independently of having acquired the good in the previous period or not.

As can be inferred from the previous paragraph, my proposal is designed to point out that, in principle, it is not necessary to acquire the good to learn its quality: this information could be transmitted truthfully through a signal. Therefore, consumers who tested the good and consumers who did not can have the same increase of utility in the second period due to the reduction of the uncertainty about the product quality (notice that, for this increase to take place, consumers have to be risk-averse). I use a mean-variance analysis to differentiate in a natural way between two effects when additional information is gathered: if consumers are risk-averse, more information means less uncertainty and higher utility (source of the endogenous switching cost[1]); also, consumers' willingness to pay has to react according to the disclosed quality of the product (the higher the quality, the larger the willingness to pay). The single element that enters heterogeneity across consumers in the first period is their location on the unit interval (horizontal differentiation).

A critical aspect of the model is to determine under what conditions the signal is credible, in which case the information acquired through direct experience is exactly the same as the one

[1] A different alternative to model endogenous switching costs can be found in Villas-Boas (2006)

revealed by the firm through second-period prices. Thus, in addition to the literature of switching costs, the research is related to the signaling theory. A close paper is Judd and Riordan (1994): the authors consider a dynamic monopoly problem of two periods in which consumer's valuation is uncertain because nobody knows the level of the quality. In the first period, nature chooses the quality level but nobody observes it and the monopolist announces the first-period price. After purchasing, a consumer observes his/her individual experience, that is a signal of the quality, and the firm observes a different signal also correlated to the quality level before setting the second-period price. Then, each consumer's information set will consist of his/her own experience and the commonly observed price. A high second-period price has two effects: the direct effect of a decrease in the demand, and the indirect effect of an improvement in the consumers' inference about product quality. They analyze the Nash-Bayes equilibrium under several assumptions, and find that the equilibrium price of the second period is larger than the monopoly price when the quality level is common knowledge; that is, for the signal to be a valid statistic, it has to exceed the full-information price. Optimal fist-period price is just the monopoly level.

There is an important difference between Judd and Riordan (1994) approach and mine. Although in both models the quality cannot be observed until consumers have purchased the good, they assume that the signal observed by the monopolist is different than the experience obtained by consumers, and in equilibrium the two can be informative. On the contrary, I assume that the signal observed by the monopolist is the same as the one derived from direct experience. Also, I consider that the quality is a random variable that follows a normal distribution and that all production costs are normalized to zero, independently of the quality. Under these assumptions, it is possible to show that a separating equilibrium does not exist if the market share of the firm in the first period was positive but not all the consumers tested the product: all the qualities below a certain threshold have incentives to mimic the prices of other superior qualities.

Other related studies are Bagwell and Riordan (1991); and Wolinsky (1983). Bagwell and Riordan (1991) assume a multi-period problem in which the firm has to introduce a new product which can be of high or low quality. The quality level is known by the firm before setting the first-period price, but not by consumers. They show that there exists a separating equilibrium such that the high quality product will be sold at a price higher than the full-information monopoly price in the first period. The high introductory price signals high quality because a firm facing high costs is more willing to restrict the volume of sales than a firm facing low costs. As information about product quality diffuses and more consumers become informed, it becomes easier for the firm to signal a high quality; in other words, the mark-up between the market price and the full-information price does not need to be so large. Consequently, the price declines until reaching the full-information level. The dynamic pattern of prices is explained because the firm knows its true quality before setting the price in the first period and because prices are not fully informative: instead, after observing the price, consumers believe the product to be of high quality with probability $b \in (0, 1)$. These two features differ in my approach: as in Judd and Riordan (1994), I consider that the firm does not know its quality when setting the first-period

prices; as it was mentioned above, a separating equilibrium cannot exist in the second period unless all consumers tested the product in the first period: the qualities which earn zero profits in the full-information equilibrium will always want to mimic the prices of superior qualities if there is a chance of deceiving a positive fraction of consumers, independently of the price distortion used by the superior qualities. Nevertheless, we both work with the hypothesis that consumers who acquire the product learn its true quality.
In Wolinsky (1983), the product quality is not random: firms can select any quality they like, but higher qualities are more costly to produce. Consumers do not know the quality chosen by a firm, but prices will convey some information about it. The author shows the existence of a Bayesian equilibrium in which each quality level is signaled by a different price. As before, equilibrium prices are higher than the full-information prices. Further, the poorer the information received by consumers, the larger the mark-up.

In this paper, I present a monopoly model[2] to answer two questions: first, if the usually predicted bargain-then-ripoff pattern of prices can be reversed when we analyze the case of switching costs which arise endogenously due to the lack of information; second, if the first-period price in the presence of switching costs can be higher than the first-period price in the same model without the switching cost (this is, in a model with risk-neutral consumers who only care about the quality of the product).
Let me provide here the intuitive answers to the previous questions. In the first period, there may be three outcomes: everybody tests the product, nobody tests the product, and only some consumers test the product. The realization of one of the three outcomes will determine the equilibrium price of the second period: in the first outcome, the information is disclosed and the monopolist charges the full-information price; in the second outcome, information cannot be disclosed and the monopolist has to compensate for the uncertainty still faced by consumers; and in the third outcome, there is no separating equilibrium. The result of the third outcome is not important for the first-period equilibrium price, since under the assumptions maintained in the model, the probability of this event does not depend on the first-period prices. Then, only the first two outcomes matter. If the first outcome takes place in the first period, the producer does not have to compensate for the uncertainty in the second period because everybody tested the product and gathered the information directly; nevertheless, the willingness to pay reacts to the quality (and remember that the support of a random variable normally distributed is $(-\infty, +\infty)$. If the second outcome takes place in the first period, uncertainty still has to be compensated in the second period but consumers take the expected quality in the ex-ante utility. In addition, consumers also have to be compensated for the uncertainty in the first period. The first-period objective function of the monopolist then considers the expected profit in the first period plus the expected profit in the second period, and the first-period equilibrium price is the solution to the maximization problem. The bargain-then-ripoff pattern of prices can be reversed if information is disclosed and the realization of the quality is low enough: in the problem of the first period every possible realization of the quality is weighted by its probability, but a low

[2] In a monopoly model, the outside option is seen as a riskless alternative acquired at zero cost.

realization in the second period when everybody tested the product leads to a low price, that can be lower than the first-period price when the quality is bad enough. However, the price charged in the first period with risk-averse consumers is always lower than the price charged in the first period with risk-neutral consumers.
About the second question, notice the following: the expected profits in the second period when information is disclosed are the same when consumers are risk-averse and when they are risk-neutral: if information is disclosed, there is no uncertainty. However, the expected profits in the second period if information cannot be disclosed are larger when consumers are risk-neutral than when they are risk-averse, because in the first case the uncertainty does not have to be compensated. Then, the incentive of the monopolist to induce the consumers to acquire the good in the first period is stronger when consumers are risk-averse: therefore, it charges a lower first-period price in the presence of switching costs.

The rest of the paper is organized as follows: the model is described in Section 2. The equilibrium of the second period is derived in Section 3 and the equilibrium of the first period is derived in Section 4. In Section 5 I solve the problem without switching costs and compare the outcomes, and finally in Section 6 I present the conclusions.

1.2 The Model

Consider a model of two periods with a monopolist located at point 0 of the unit interval. It sells a product or a service that is not storable, lives during the entire game and its discount factor is equal to 1 (the monopolist is not myopic). The charged prices can be different across periods, but price-discrimination among consumers who live in the same period is not allowed. All the production costs are normalized to zero.

There is a mass of consumers uniformly distributed along the unit interval in each one of the two periods; that is, consumers of the first period are different from consumers of the second period. Nevertheless, a consumer located at point x_i in the second period inherits the information gathered by the consumer located at her position in the first period. It is reasonable to assume this because the evidence states that people learn from the experience of others[3].

In the first period, the quality of the product is unknown and is denoted by q. $\bar{q}$ denotes its realization. When the quality is unknown, all the agents assume the following:

[3] Although this assumption is technically equivalent to considering myopic consumers, the interpretation is very different.

Assumption 1. The quality is a random variable normally distributed with mean $\mu > 0$ and variance σ^2:

$$q \sim N(\mu, \sigma)$$

The timing of the game is as follows:

Period 1

- The quality, q, is realized and remains the same for the rest of the game. At this stage, nobody observes the realization $\bar{q}$
- The monopolist sets the first-period price, p_1
- The consumers make their consumption choices
- The monopolist observes the first period market share, $\hat{x}_1$. Both the monopolist and the consumers who acquired the product observe the true quality $\bar{q}$ and the agents receive their payoffs

Period 2

- The consumers of the second period observe the first-period market share
- The consumer located at x_i in the second period inherits the information gathered by the consumer located at her position in the first period, $\forall x_i \in [0, 1]$. The consumers are then divided into two groups: informed (those who have information about the quality of the good through the previous experience of the individuals located at their position) and uninformed (those who do not)
- The monopolist sets the second-period price, p_2
- The uninformed consumers use the second-period price and the first-period market share to update their beliefs
- The consumers make their consumption choices, agents receive their payoffs and the game finishes.

It can seem weird that the monopolist does not know the quality of its product at the beginning of the game. The term *quality* should be understood as something slightly more sophisticated: it is the perception of the consumers about the product. Then, it makes sense that this perception can only be learned after the consumption choices of the first period. Concretely, consumers who tested the product get this information directly, and the monopolist undertakes some market research at a negligible cost to learn it. Communication among consumers is not allowed, and I consider implicitly that the consumers do not have the resources owned by the monopolist to undertake the market research[4].

[4] Similar considerations are assumed in Judd and Riordan (1994).

Let me now explain how consumers make their consumption decisions. The consumers can acquire one unit of the product or stay out of the market. If they decide not to acquire it, the utility of the outside option is normalized to 0 in both periods:

$$u_{o,1} = u_{o,2} = u_o = 0 \tag{1.1}$$

The utility function is exponential

$$u_i(q) = -e^{-\rho(q-K_{i,t})} \tag{1.2}$$

where $\rho > 0$ is the risk-aversion coefficient and $K_{i,t}$ is a set of other parameters that can differ across periods and consumers.

When the true realization is unknown, the quality is assumed to be normally distributed. Then, maximize the expected utility of the consumer is equivalent to maximize the certainty equivalent[5]:

$$\mu - \frac{1}{2}(\rho\sigma^2) - K_{i,t} \tag{1.3}$$

In other words, the mean-variance analysis is optimal.

In the first period,

$$K_{1,i} = p_1 + x_i - z \tag{1.4}$$

Thus, the certainty equivalent of the consumer located at x_i in the first period is

$$\mu - \frac{1}{2}(\rho\sigma^2) - p_1 - x_i + z \tag{1.5}$$

where p_1 is the first-period price, x_i is the location[6] of the individual i and z is private information of the consumers.

Notice that the private information of the consumers, z, is an element of vertical differentiation. Also is quality, but all the agents are uninformed about its true value in the first period, so they use the value of the expected quality μ. There is a single element of horizontal differentiation: the location of the consumers.

The marginal consumer of the first period comes from equalizing the equations (1.1) and (1.5):

$$\hat{x}_1 = \frac{1}{2}\left(2\mu - \rho\sigma^2 - 2p_1 + 2z\right) \tag{1.6}$$

[5] A formal proof of this statement can be found in the Appendix.

[6] The transportation costs are normalized to 1: $\tau = 1$.

Since the firm does not know the value of z, it considers that it is a random variable uniformly distributed:

Assumption 2. The monopolist takes z as a random variable uniformly distributed with zero mean:

$$z \sim U(-Z, Z)$$

with Z large enough.

Notice that for z large enough, $\hat{x}_1 \geq 1$; since the size of the market is 1, a marginal consumer greater than 1 means that all the consumers acquire the good and obtain a strictly positive surplus. For z small enough, $\hat{x}_1 \leq 0$; since negative demands are not allowed, a marginal consumer lower than 0 means that all the consumers choose the outside option.

In the second period,

$$K_{2,i} = p_2 + x_i \tag{1.7}$$

Remember that in the second period the consumers are divided in two groups: informed and uninformed. If the consumer is informed, she knows the true quality and the uncertainty completely disappears. Then, her utility function is

$$\bar{q} - p_2 - x_i \tag{1.8}$$

At this point, it is worth noting that becoming informed through direct experience has two effects. First, the uncertainty is eliminated and the utility increases in $\rho\sigma^2/2$ with respect to the case of no information. Second, the true quality is learned and it substitutes the expected quality μ: then, the utility changes in $\bar{q} - \mu$ (increases with respect to the case of no information if $\bar{q} > \mu$ and decreases otherwise).

The marginal consumer in the second period among the informed consumers is obtained by equalizing the equations (1.1) and (1.8):

$$x_2^I = \bar{q} - p_2 \tag{1.9}$$

In contrast to the informed consumers, the uninformed consumers do not have direct information about the true quality of the product and they use the second-period price and the first-period market share to update their beliefs:

$$E[q|(p_2, \hat{x}_1)] - \frac{1}{2}(\rho Var[q|(p_2, \hat{x}_1)]) - p_2 - x_i \tag{1.10}$$

The marginal consumer in the second period among the uninformed consumers is obtained by equalizing the equations (1.1) and (1.10):

$$x_2^U = E[q|(p_2, \hat{x}_1)] - \frac{1}{2}(\rho Var[q|(p_2, \hat{x}_1)]) - p_2 \tag{1.11}$$

Although there is a different mass of consumers in each period, the individuals living in the first period communicate information to the individuals living in the second period. As it was explained before, this fact affects the inference (and the utility) of the individuals of the second period. Since the monopolist is forward-looking, the game has to be solved by backward induction. The equilibrium concept used is the Perfect Bayesian Nash Equilibrium (PBNE).

1.3 Equilibrium of the Second Period

There may be three possible situations in the first period, depending on the value of z: all consumers tested the product ($\hat{x}_1 \geq 1$), all consumers chose the outside option ($\hat{x}_1 \leq 0$) and only some consumers tested the product ($\hat{x}_1 \in (0,1)$). The optimal pricing strategy in the second period depends on the outcome of the first period.

1.3.1 Corner Equilibria

Corner A

This corner takes place when all consumers tested the good in the first period, that is, when

$$\hat{x}_1 \geq 1 \Leftrightarrow z \geq 1 - \mu + \frac{1}{2}\rho\sigma^2 + p_1 \tag{1.12}$$

In this case, all consumers are informed and prices are not informative signals. The equilibrium prices and market shares are stated in the next Proposition:

Proposition 1. The equilibrium prices and market shares in the case of full information are as follows:

$$p_2^I = \begin{cases} 0 & \bar{q} \leq 0 \\ \bar{q}/2 & \bar{q} \in (0,2) \\ \bar{q} - 1 & \bar{q} \geq 2 \end{cases}$$

$$x_2^I = \begin{cases} 0 & \bar{q} \leq 0 \\ \bar{q}/2 & \bar{q} \in (0,2) \\ 1 & \bar{q} \geq 2 \end{cases}$$

As expected, the market share increases with the quality when the information is complete, but the corner solutions have to be taken into account: the market share is zero for negative qualities and one for qualities larger than 2. Notice that these thresholds are "arbitrary": we could re-scale the utility function by adding a positive term R: then, some negative qualities would have a positive market share due simply to the intrinsic utility of consuming the good (the threshold 2 will also change). However, for a finite R there will always be a set of qualities that will supply a null amount of the product and a set of qualities that will supply to the entire market, charging the corresponding corner prices.
It is also important to remark that negative prices in the second period are dominated: the firm prefers to make zero profit rather than negative profits.

Corner B

In this corner, nobody tested the good in the first period. Since the demand was zero, the monopolist cannot undertake the market research to discover how consumers perceive its product; then, I will say that the information cannot be disclosed.
Technically, this corner takes place when

$$\hat{x}_1 \leq 0 \Leftrightarrow z \leq -\mu + \frac{1}{2}(\rho\sigma^2) + p_1 \tag{1.13}$$

Since consumers observe the first-period market share, they correctly infer that the second-period price conveys no information in this corner. Then, making $E[q|(p_2, \hat{x}_1)] = \mu$ and $Var[q|(p_2, \hat{x}_1)] = \sigma^2$ in equation (1.11), the marginal consumer in this corner is

$$x_2^U = \mu - \frac{1}{2}(\rho\sigma^2) - p_2 \tag{1.14}$$

The next Proposition establishes the equilibrium prices and market shares when information cannot be disclosed:

Proposition 2. The equilibrium prices and market shares when the information cannot be disclosed are as follows:

$$p_2^U = \begin{cases} 0 & \rho \geq 2\mu/\sigma^2 \\ (2\mu - \rho\sigma^2)/4 & \mu \leq 2 \quad \text{and} \quad \rho < 2\mu/\sigma^2; \quad \text{or} \\ & \mu > 2 \quad \text{and} \quad (-4 + 2\mu)/\sigma^2 < \rho < 2\mu/\sigma^2 \\ (-2 + 2\mu - \rho\sigma^2)/2 & \mu > 2 \quad \text{and} \quad \rho \leq (-4 + 2\mu)/\sigma^2 \end{cases}$$

$$x_2^U = \begin{cases} 0 & \rho \geq 2\mu/\sigma^2 \\ (2\mu - \rho\sigma^2)/4 & \mu \leq 2 \quad \text{and} \quad \rho < 2\mu/\sigma^2; \quad \text{or} \\ & \mu > 2 \quad \text{and} \quad (-4+2\mu)/\sigma^2 < \rho < 2\mu/\sigma^2 \\ 1 & \mu > 2 \quad \text{and} \quad \rho \leq (-4+2\mu)/\sigma^2 \end{cases}$$

Notice that the consumers have to be compensated for the faced risk; or in other words, the price is negatively related to the risk-aversion coefficient. In the extreme case of sufficiently high risk-aversion, the consumers are simply better off by choosing the outside option and the good is not traded. As happened with the equilibrium of the previous corner, the qualitative properties of this equilibrium would not change if we re-scale the utility function: only the thresholds would be different.
As before, negative prices are dominated.

1.3.2 Interior Equilibrium

In the interior equilibrium, only some consumers tested the good in the first period. This means that there is a fraction of consumers who do not take the price as a signal of the quality, but the rest of the consumers do (and actually, they are susceptible of being deceived).

In the signaling games, there may be three different types of equilibria: separating, pooling and semi-pooling (also known as semi-separating). In this paper, I check the existence or not of a separating equilibrium. In a separating equilibrium, each type of sender optimally chooses a different message (in this context, each quality would charge a different price) so that the consumers can infer his type with no error.

As it was mentioned in the previous section, consumers are divided in two groups: informed and uninformed consumers. The informed consumers do not take the prices as a signal because they know the true quality of the product. Then, the demand from this sector can be expressed as

$$D_2^I = \min\{\hat{x}_1, \max\{0, \bar{q} - p_2\}\} \tag{1.15}$$

The operators *min* and *max* come from the corners of the problem: the marginal consumer among the informed consumers cannot be part of the uninformed group, and negative demands are not allowed.

The uninformed consumers take the price as a signal of the quality. In a separating equilibrium, the consumers perfectly infer the quality given the price, so the uncertainty is not an element of the utility function anymore. Then, the demand from the uninformed sector can be expressed as

$$D_2^U = \max\{0, \min\{1, \tilde{q} - p_2\} - \hat{x}_1\} \tag{1.16}$$

where $\tilde{q}$ is the inferred quality.
As before, the operators *min* and *max* come from the corners of the problem: the marginal consumer among the uninformed consumers cannot be part of the informed group, and the demand is constrained by the size of the market.

The trade-off faced by quality q when it pretends to be quality $q_F > q$ is clear: by increasing the price, it increases the expectation of the uninformed consumers but it also loses part of the demand derived from the informed consumers.

Proposition 3. A separating equilibrium does not exist.

A separating equilibrium does not exist because the non-positive qualities make exactly zero profits in the full-information equilibrium: it is always profitable to try to deceive to the uninformed consumers because, in the worst case of no deceiving them, the monopolist also makes zero profits. This result holds even when the high qualities distort their prices for the same reason.
It is also worth noting that, for positive qualities, the qualities that are profitable to mimic are determined by the fraction of informed consumers. Consider for instance that $\hat{x}_1 = 0.3$ and the realized quality $\bar{q} = 3$: here, it is not profitable to mimic $\tilde{q} = 4$ ($\pi_{mimic} = 1.8 < 2 = \pi_{inf}$), but it is profitable to mimic $\tilde{q} = 5$ ($\pi_{mimic} = 2.4 > 2 = \pi_{inf}$). The example illustrates why it is statistically impossible to avoid mimicking among the positive qualities: for low fractions of informed people is not profitable to mimic similar qualities, but it is profitable to mimic qualities that are much higher. The fraction of informed consumers necessary for making unprofitable to mimic a very high quality tends to 1. Therefore, the problem is not the infinitesimal distance between the different qualities: the problem is that very high qualities actually exist.

1.4 Equilibrium of the First Period

The objective function of the monopolist in the first period is the expected profit[7] obtained in period 1 plus the expected profit obtained in period 2.

[7] The profit of the first period is also expected because the market share is a random variable for the monopolist.

I have shown that there is no separating equilibrium in the interior case, which in principle could be a problem to find the expected profit of the second period. However, the next Lemma states that this equilibrium is irrelevant for the intertemporal maximization problem:

Lemma 1. The probability of being in the interior case in the second period is independent of the first-period price.

Therefore, in mathematical terms, the equilibrium of the interior case would be like adding a constant term to the first-period objective function and it would not affect to the solution of the problem.

The expected profit obtained in the Corner A depends on the parameters of the distribution of the product quality, μ and σ:

$$\begin{aligned}\Pi_I(\mu,\sigma) &= Pr(q \leq 0)E(0|q \leq 0) + Pr(q \geq 2)E(q-1|q \geq 2) + Pr(0 < q < 2)E\left(\frac{1}{4}q^2 \middle| 0 < q < 2\right)\\ &= Pr(q \geq 2)E(q-1|q \geq 2) + Pr(0 < q < 2)E\left(\frac{1}{4}q^2 \middle| 0 < q < 2\right)\\ &> 0\end{aligned} \tag{1.17}$$

The profit obtained in the Corner B is defined by the following piecewise function:

$$\Pi_U(\mu,\rho,\sigma) = \begin{cases} 0 & \rho \geq 2\mu/\sigma^2 \\ (2\mu-\rho\sigma^2)^2/16 & \mu \leq 2 \quad \text{and} \quad \rho < 2\mu/\sigma^2; \quad \text{or} \\ & \mu > 2 \quad \text{and} \quad (-4+2\mu)/\sigma^2 < \rho < 2\mu/\sigma^2 \\ (-2+2\mu-\rho\sigma^2)/2 & \mu > 2 \quad \text{and} \quad \rho \leq (-4+2\mu)/\sigma^2 \end{cases} \tag{1.18}$$

The probability of being in the Corner A is

$$\begin{aligned}PCA(p_1) &= Pr(z \geq 1-\mu+\frac{1}{2}\rho\sigma^2+p_1)\\ &= \frac{-1-p_1+Z+\mu-\frac{1}{2}(\rho\sigma^2)}{2Z}\end{aligned} \tag{1.19}$$

and the probability of being in the Corner B is

$$\begin{aligned} PCB(p_1) &= Pr(z \leq -\mu + \frac{1}{2}(\rho\sigma^2) + p_1) \\ &= \frac{p_1 + Z - \mu + \frac{1}{2}(\rho\sigma^2)}{2Z} \end{aligned} \tag{1.20}$$

Therefore, the objective function of the first period is

$$\Pi = p_1 \hat{x}_1^E(p_1) + PCA(p_1)\Pi_I(\mu,\sigma) + PCB(p_1)\Pi_U(\mu,\rho,\sigma) \tag{1.21}$$

where $\hat{x}_1^E(p_1) = E[\hat{x}_1] = \frac{1}{2}\left(2\mu - \rho\sigma^2 - 2p_1\right)$.

The next Proposition states the solution to the intertemporal problem:

Proposition 4. The first-period equilibrium price is

$$p_1^* = \frac{2Z\mu - Z\rho\sigma^2 - \Pi_I(\mu,\sigma) + \Pi_U(\mu,\rho,\sigma)}{4Z}$$

In principle, negative first-period prices are allowed: the firm maximizes the overall profit without imposing the non-negativity of the profits in each period. It is worth noting that the first-period equilibrium price is decreasing in the risk-aversion coefficient: directly because of the effect of ρ on the expected market share of the first period, and indirectly because of its effect on the profits when the information cannot be disclosed.

Previously, I mentioned that Z had to be large enough. At this stage, I can be more concrete: Z has to be such that both the probability of being in the Corner A and the probability of being in the Corner B belong to the interval $(0,1)$. The two figures below represent the overall profit Π as a function of p_1.

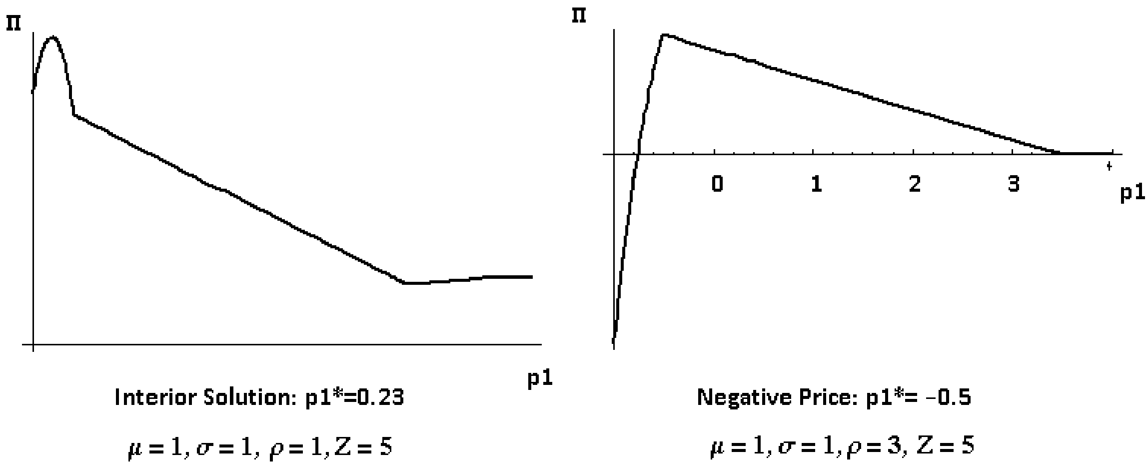

Figure 1.1: First-period equilibrium price

To conclude the section, it is interesting to check if the bargain-then-ripoff pattern of prices can be reversed when information is disclosed: in this case, the switching cost disappears because the consumers face no uncertainty anymore; however, the quality can be low enough to more than offset this positive effect on the second-period price. Notice that for the pattern of prices to be reversed, a necessary condition is $p_1^* > 0$: if the price is negative in the first period, the pattern cannot be reversed because in the worst case the monopolist charges a price equal to zero in the second period. The next Proposition states the thresholds of the qualities for this reversion to take place:

Proposition 5. The bargain-then-ripoff pattern of prices is reversed when information is disclosed in the following cases:

5.1 If $0 < p_1^* < 1$, when $\bar{q} < 2p_1^*$

5.2 If $p_1^* \geq 1$, when $\bar{q} < p_1^* + 1$.

The result of the reversion of the bargain-then-ripoff pattern is natural in this framework: the first-period price is based on the weighted average of all the future possibilities, good and bad outcomes. However, once the outcome is realized and everybody gathers the information through direct experience, two different effects happen: first, the uncertainty disappears and the consumers' willingness to pay increases; second, the true quality is revealed. If the quality is high enough, both effects work in the same direction and the model predicts the classical bargain-then-ripoff pattern of prices. Nevertheless, if the quality is low, the consumers' willingness to

pay decreases; furthermore, if the quality is sufficiently low, this effect more than offsets the positive effect on the willingness to pay of the lack of uncertainty. Thus, the bargain-then-ripoff pattern does not hold anymore in these cases.
Notice also that, for the reversion to take place, we need strictly positive first-period prices, because in the second period the negative prices are dominated.

The reversion of the classical pattern of prices does not imply that the price set in the first period in a market with switching costs is higher compared to the first-period price set in market without switching costs. The next section addresses this question.

1.5 The Model Without Risk Aversion

The model without risk aversion is totally analogous to the one solved in the two previous sections: simply, we have to fix $\rho = 0$. Therefore,

1. The Lemma 1 also holds
2. The probability of the Corner A is $(-1-p_1+\mu+Z)/2Z$ and the probability of the Corner B is $(p_1-\mu+Z)/2Z$
3. The prices and the market shares in the full information case are identical to those stated in Proposition 1
4. The prices and the market shares when information cannot be disclosed are

$$p_2^U = \begin{cases} \mu/2 & \mu < 2 \\ \mu - 1 & \mu \geq 2 \end{cases}$$

$$x_2^U = \begin{cases} \mu/2 & \mu < 2 \\ 1 & \mu \geq 2 \end{cases}$$

Notice that the prices and the market shares when the information cannot be disclosed and the consumers are risk-neutral are always higher than the corresponding prices and market shares when the information cannot be disclosed but the consumers are risk-averse: simply, the risk does not have to be compensated. Then, the profits obtained in this corner are also higher when $\rho = 0$ than when $\rho > 0$. The profits in the full-information case does not change with respect to the previous sections.

The next Proposition establishes the first-period equilibrium price when the consumers are risk-neutral and its relationship with the first-period equilibrium price when the consumers are risk-averse:

Proposition 6. When the consumers are risk-neutral, it holds that

6.1 The first-period equilibrium price is

$$p^*_{1,rn} = \frac{2Z\mu - \Pi_I(\mu,\sigma) + \Pi_U^{rn}(\mu)}{4Z}$$

where $\Pi_U^{rn}(\mu)$ is $\mu^2/4$ when $\mu < 2$ and $\mu - 1$ otherwise, and $\Pi_I(\mu,\sigma)$ is defined as in (1.17).

6.2 The first-period equilibrium price is always larger than the first-period equilibrium price when the consumers are risk-averse

The first part of the Proposition is simply the solution to the intertemporal problem of the firm when consumers are risk-neutral. The second part states that, in the monopoly case, the classical intuition about the behavior of the firm also holds when the switching costs derive from the uncertainty about the quality of the product: the monopolist has incentives to set a lower price in the first period in the presence of switching costs. The intuition here is that, in the first place, consumers do not have to be compensated for the risk in the first period when they are risk-neutral, so only the expected quality matters; in the second place, because of the same reason, the profits obtained when the information cannot be disclosed in the second period are always higher when the consumers are risk-neutral. Therefore, in equilibrium the monopolist sets a higher price when the consumers are risk-neutral than when they are risk-averse.
The previous reasoning is reflected in the mathematical expressions of the first-period equilibrium prices. First, observe that the risk-aversion parameter ρ affects negatively to the price in Proposition 4 but that it does not appear in the price in Proposition 6. Second, the price depends positively on the profits when the information cannot be disclosed (and these profits also depend negatively on the risk-aversion parameter).

1.6 Conclusion and Discussion

I have presented a monopolistic model of two periods in which the consumers are risk-averse and switching costs arise endogenously due to the lack of information about the quality of the traded product. When information is revealed, we can differentiate between two effects: first, the uncertainty disappears and the consumers' willingness to pay increases; second, the true quality of the product is revealed and the consumers' willingness to pay increases only for high qualities (and decreases otherwise).

Depending on the realization of a random variable in the first period, the information of the quality can be disclosed to the entire market, disclosed only to a share of the market, or not disclosed at all. Of course, the strategies adopted by the monopolist in the second period are different for each one of the outcomes. Concretely, the firm charges a price increasing in the quality when the information is disclosed to the entire market, it charges a price decreasing in the risk-aversion coefficient when the information was not disclosed, and there is no separating equilibrium when the information is disclosed only to a share of the market. The equilibrium price of the first period is calculated by backward induction.

The model answers two questions. First, if the classical bargain-then-ripoff pattern of prices predicted by most of the models of two periods with switching costs can be reversed with this type of switching costs. Second, if the first-period price set in a market with switching costs is lower than the first-period price set in a market without switching costs.

The answer to the first question is affirmative, and actually it is natural in this environment because the first-period price is calculated using the expectation of the future outcomes (that is, good and bad outcomes are weighted by their probabilities). When the information is disclosed to the entire market, the two effects described in the previous paragraph take place. If the quality of the product is low enough, this effect can be sufficiently strong to lead the monopolist to set a price in the second period below the first-period equilibrium price.

The answer to the second question is affirmative too: although the classical pattern of prices can be reversed, the first-period price charged in the presence of switching costs is still lower than the first-period price charged in a market without switching cost. In the classical models, this result happens because the firm correctly anticipates the higher profits of the second period when consumers face switching costs and it has an incentive to compete fiercely in the first period. In this case, the logic is slightly different: if information is disclosed to the entire market, the expected profits are the same with and without switching costs (with switching costs uncertainty does not exist, and without switching costs the uncertainty is not penalized); however, if the information cannot be disclosed at all, the profits when the consumers are risk-averse are smaller than when they are risk-neutral, precisely because of the penalization of the uncertainty. In other words, when consumers are risk-neutral, the alternative of not disclosing the information is ”not as bad” as it is when consumers are risk-averse. Then, the monopolist has a stronger incentive to disclose information when the consumers are risk-averse, and it increases the probability of this event by decreasing the price in the first period.

Let me discuss two assumptions of the model.

First, it sounds weird that in a model of switching costs a single firm supplies to the entire market: at the end, if there is no other firm, how could the consumers switch their supplier? In this model, the consumers have unitary demands, but they also have the option of not acquiring the good. In this sense, the outside option of not buying the good works as a riskless and costless alternative: the consumers know that they will obtain a utility of zero if they do not buy, but they face the risk of the uncertainty, and this is penalized in their utility levels: that is why the demand is zero if consumers are very risk-averse. A particular feature of this "competition" is that the outside option do not charge any price, so those consumers who did not gathered the information through direct experience only have one signal (price) in the second period to infer the quality of the product.

Second, it is important to remark that the mean-variance analysis is optimal when the consumers have exponential preferences and the random variable (in this case, the quality) is normally distributed. The assumption does not create big problems when the information is disclosed to the entire market and when the information cannot be disclosed to any consumer. However, it is the reason of the non-existence of a separating equilibrium when only some consumers have direct information: the infinitesimal distance between qualities is not the problem, the problem is that the support of the quality is $(-\infty, +\infty)$ and it is always profitable to mimic the strategy of a sufficiently large quality unless all consumers are perfectly informed. This result contrasts with the classical result obtained when there are only two qualities, high and low: in this case, there exists a separating equilibrium and prices are not distorted when the fraction of directly informed consumers is large enough (although it is not 1). However, if we assume only two qualities the mean-variance analysis is not optimal anymore and the endogenous switching costs would not appear. Nevertheless, the non-existence of the separating equilibrium is not a big deal when solving the intertemporal problem, because the probability of the interior event does not depend on the first-period price (thus, with or without equilibrium in the interior case, the optimal first-period price would not change).

Chapter 2

Better the Devil you Know: A Dynamic Duopoly Model with Switching Costs

2.1 Introduction

Switching costs are simply the costs faced by a consumer when switching from her current supplier to a new one. Classic examples of markets with switching costs are airlines, software, car insurances, cell phones and bank deposits[1].

In a market with switching costs, products that were ex-ante homogeneous become ex-post heterogeneous. Because of this, the consumer is partially forced to continue using the product she initially selected. However, the definition of switching costs is pretty general and does not provide any information about the origin of the switching cost. According to Klemperer (1995), "a switching cost results from a consumer's desire for compatibility between her current purchase and a previous investment". Therefore, there are switching costs related to compatibility with the existing equipment, transaction costs, the cost of learning and the uncertainty about the quality of untested brands. Notice that the switching costs are not always expressed in monetary terms: if a consumer wants to move her savings from her old bank to a new one, she will probably have to pay a cancellation fee (transaction cost measured in money); but if a consumer wants to use a different operating system in her computer, she has to spend some time in understanding how it works (learning cost measured in time).

Regardless of whether it is a monetary cost or not, the switching costs make the alternative offered by the initial supplier relatively less costly than the alternative offered by rivals. Therefore, its existence affects the market equilibrium prices. It is very well-known that, in models of two periods with switching costs, firms have incentives to decrease prices in the first period

[1] The interested reader can check Honka (2013) for an empirical analysis of the switching costs in the U.S. auto insurance industry, and Shy (2002) for data about the switching costs of the Israeli cellular phone market and the Finnish market for bank deposits.

(investment effect) and to increase prices in the second period (harvesting effect). It is said that prices display a bargain-then-ripoff pattern: in the second period, the firms take advantage of the captive consumers and set a high price; but, anticipating the large profit of the second period, firms compete fiercely in the first period by decreasing their prices in order to get as many captive consumers as possible. These two effects also appear in models of more than two periods, although not so clearly separated: when firms compete over a long time horizon, at every period they want to attract new customers at the same time that they exploit the old captive customers.

The literature has studied which of the two effects dominates; in other words, whether switching costs make markets more or less competitive in the average. The conventional wisdom states that the harvesting effect dominates and, therefore, the switching costs increase the average price (Klemperer (1987b); Farrell and Klemperer (2007)). However, this result has been challenged several times. For instance, Doganoglu (2010) shows that, if price discrimination among consumers is not allowed, in a duopoly market with competition in an infinite horizon with experience goods the steady state equilibrium prices are lower with small switching costs than with no switching costs. Cabral (2012) also assumes a duopoly in an infinite-horizon model, but he allows for price discrimination between locked-in and not locked-in consumers. Contrary to *Doganoglu*, *Cabral*'s analysis goes beyond the case of small switching costs. He provides conditions for switching costs to be pro-competitive or anti-competitive. In concrete, he shows that switching costs decrease equilibrium prices in markets where both firms have approximately the same chance of attracting a consumer. In the same spirit, Rhodes (2013) analyzes a wide range of switching costs. In addition, he differentiates between the effects of the switching costs in the short-run and in the long-run in an infinite-horizon model with discrete time. His main finding is that, although switching cost can be pro-competitive or anti-competitive in the long-run, they are more likely to be anti-competitive in the short-run. Fabra and Garcia (2012) also differentiate between the short-run and the long-run, but they analyze a problem with continuous time in which only a random fraction of consumers considers switching in an infinitesimal amount of time. They show that the effect of the switching costs in equilibrium depends critically on the asymmetries of the market shares: if they are asymmetric enough, larger switching costs lead to higher prices; as market shares become symmetric, price competition turns fiercer and the overall effect is pro-competitive. Arie and Grieco (2012) relax the duopoly assumption and identify an additional effect for small switching costs to be pro-competitive in the short-term: if consumers switch in equilibrium, firms may want to decrease prices to partially offset the cost of consumers who are switching (compensation effect). Pearcy (2011) also considers the general case of n firms and concludes that, with a sufficiently large number of firms, the equilibrium prices are lower in the presence of switching costs.

All these papers have something in common: the switching cost is an additive term in the utility function that appears after consuming the product offered by a certain brand. Therefore, they are in line with the wide definition of switching costs discussed previously. My proposal consists

in focusing on switching costs derived from the uncertainty about the quality of untested brands. I construct a duopoly model of two periods in which the firms live during the entire game. Firm A is located at point 0 of the unit interval, and the quality of its product is a random variable normally distributed with mean larger than zero. Firm B is located at point 1 of the unit interval, and it is common knowledge during the entire game that its product is of quality zero. Firms compete simultaneously in prices in each period; furthermore, price commitment is not possible and price discrimination across consumers in the same period is not allowed. Neither firms nor consumers can observe the quality of the product supplied by A before setting the first-period prices. In the first period, there is a mass 1 of consumers uniformly distributed along the unit interval. The utility function is positively related to the expected quality of the product, but negatively related to the uncertainty about that quality because consumers are assumed to be risk-averse. Therefore, in the first period we can differentiate between two effects: expected quality effect and switching cost effect. These two effects are natural and reasonable when we give a deeper structure to the switching costs: information reduces uncertainty, but also reveals the quality of the product. The expected quality effect works in the first period in favor of firm A, because its expected quality is larger than zero; however, the switching cost effect works in the first period in favor of firm B, because its quality is common knowledge and there is no uncertainty when buying its product (uncertainty that exists when buying the product of A).
If the firm A supplied (at least partially) to the market in the first period, both firms observe the true quality $\bar{q}_A$ after consumption choices. In the second period, there is a new mass 1 of consumers uniformly distributed along the unit interval. All consumers can observe the past market shares. The individual located at point x_i in the second period inherits the information gathered by the inhabitant located at x_i in the first period: if the previous inhabitant acquired the product supplied by A, she learns the true quality $\bar{q}_A$. Nevertheless, also both firms know the true quality of the product offered by A and the prices are signals used to infer the quality by those consumers who did not gather information through direct experience in the second period[2], eliminating the faced switching cost. The consumers who have information from direct experience do not consider the prices as signals because they cannot be deceived.

Depending on the market shares of the first period, I can distinguish three situations: (a) the firm A was the single supplier, (b) the firm B was the single supplier, and (c) both firms A and B supplied to a positive share of the market. If the firm A was the single supplier in the first period, all the agents in the second period are fully informed; equilibrium prices do not depend on the risk-aversion coefficient and are not distorted. If the firm B was the single supplier in the first period, $\bar{q}_A$ remains unknown for all the agents; the equilibrium price of B is increasing on the risk-aversion coefficient whereas the price of A is decreasing on it and, furthermore, the price charged by B is higher than the price charged by A if the risk-aversion coefficient is above a certain threshold. If both firms supplied to a positive share in the first period, a separating

[2]Prices of the first period do not convey any relevant information about the quality q_A because its true realization is unknown for all the agents when first-period prices are fixed.

equilibrium does not exist. Compared with the situation without switching costs, the equilibrium of the situation (a) is exactly the same, and again a separating equilibrium does not exist in the situation (c). However, prices would not depend on the risk-aversion coefficient in the equilibrium of the situation (b), and the price charged by the firm A would always be larger than the price charged by the firm B.
In the first period, I show that the price set by firm A is decreasing in the risk-aversion coefficient, whereas the price set by firm B is increasing in this parameter. Even more, I show that the (expected) average first-period price in the problem with switching cost can be larger than in the problem without switching costs. This contrasts with the classic result of lower first-period prices in the presence of switching costs, but it is due to the nature of the switching costs considered here. This result happens because, given the expected quality, the firm that offers the riskless product has a competitive advantage in the first period when consumers are sufficiently risk averse that does not exist when consumers are risk neutral. It may happen that its rival cannot compensate completely this advantage through a price decrease because negative prices are dominated.

As can be noticed from the previous discussion, the switching costs arise endogenously from the consumer's history of consumption. A relevant study which also considers endogenous switching costs is Villas-Boas (2006). A key difference between our approaches is that signaling plays no role in his model: instead of being uncertain about the quality, consumers are uncertain about their valuation of a certain good. This information about their own preferences can only be gathered after experiencing a certain product. After testing a product, a consumer has more information about it than about other products, and this is translated into an informational advantage that the firms can exploit. Of course, the larger the market share today, the larger the informational advantage tomorrow. The author considers this kind of competition in an infinite-horizon framework in a duopoly market with overlapping generations of consumers. Two effects arise: forward-looking consumers realize that current consumption choices will affect future prices; forward-looking firms realize that they gain in the future from having a greater market share today. The first effect results in higher equilibrium prices, whereas the second effect results in lower equilibrium prices. When discount factors are similar for consumers and firms, the former effect dominates and prices increase in the informational advantages.

An interesting feature of the switching cost that derives from the lack of information is that they could be eliminated through credible signals. For instance, a software firm cannot reduce the time that a consumer has to invest to learn how its program works most of the times; but if the cost derives from the uncertainty about the quality of the product, the firm can save this cost to consumers by sending a credible signal. Therefore, this paper also contributes to the literature on signaling. Bagwell and Riordan (1991) study the signaling problem of a monopolist in a dynamic environment. The authors claim that high prices are efficient means of signaling, because the derived loss of profits is more harmful for low-quality products. Their main finding is that, as time goes by and the amount of informed consumers increases, the signaling distortion decreases. In my model, that result does not hold because of the continuum of qualities: distortion is not profitable and it can be shown that, for some sets of qualities, at least one of the firms wants to deviate when some consumers do not have full information. Hertzendorf and Overgaard (2001) study a signaling static problem in a vertically differentiated duopoly. The nature randomly assigns one firm to be the high-quality firm and the other to be the low-quality firm. Firms have two signaling tools: advertising levels and prices. The authors conclude that pure price separation is impossible if the vertical differentiation is small: dissipative advertising plays a role and ensures the existence of a separating equilibrium in this case. They also introduce two equilibrium refinements to deal with the multi-sender nature of the game. Bester and Demuth (2011) analyze a duopoly problem with vertical and horizontal differentiation. As it happens in my model, the firms may differ in their qualities; however, they only consider two qualities: high and low. One of the firms is known to be of low quality, whereas the other can be high or low. As I do, they assume that there is a share of consumers who know both qualities, but in their approach these consumers are randomly located in the unit interval: that is why their problem does not show the corners that appear in mine, in which the informed consumers are located in a compact set at the beginning of the unit interval. The consumers who do not have full information take the market prices as signals, and the problem is then characterized by a signaling rivalry. They find that the only sustainable equilibrium is separating, with no distorted prices, and takes place when there is a sufficiently large proportion of consumers informed about the two qualities (this proportion depends on the distance between the high and the low quality). The main difference between their approach and mine is the number of qualities: with two, the deviations are defined and the authors can define the out-of-equilibrium beliefs; in my problem, all prices are part of the equilibrium support. Also, they assume risk-neutral consumers instead of risk-averse consumers, although I find that the result of no existence of my model also holds for risk-neutral consumers. Finally, Janssen and Roy (2010) relax the duopoly assumption and consider competition among n firms in a symmetric Bertrand oligopoly where products can differ only in their quality. A key assumption is that each firm's product quality level is private information (that is, unknown for consumers and rivals). Even in this type of competition, the firms signal their quality through prices. The authors determine that there is no fully revealing equilibrium in pure strategies, but that there exist symmetric fully revealing perfect Bayesian equilibria in which high-quality firms charge a deterministic price and low-quality firms randomize over an interval of prices. Furthermore, there is a unique symmetric fully revealing equilibrium satisfying the D1 criterion.

The rest of the paper is organized as follows: the model is presented in Section 2, and its solution is derived in Sections 3 and 4. In Section 5, I compare the equilibrium prices of the two firms with respect to a situation without switching costs. I conclude in Section 6 by discussing the role of some assumptions and providing some lines for future research.

2.2 The Model

Consider a model of two periods. There are two firms, A and B, located at the extreme points of the unit interval ($x_A = 0$ and $x_B = 1$). They sell a non-storable good in each one of the two periods and all production costs are normalized to zero. The two firms live during the entire game and have a discount factor equal to 1. They compete simultaneously in prices in each period, such that price commitment and price discrimination across consumers living in the same period are not allowed. However, the charged prices in the first and second period can be different.

There is a mass 1 of risk-averse consumers uniformly distributed along the unit interval in each period (horizontal differentiation); that is, consumers of the first period are different from consumers of the second period. Nevertheless, a consumer located at point x_i in the second period inherits the information gathered by the consumer located at her position in the first period. It is reasonable to assume this because the evidence states that people learn from the experience of others. I consider that the market has to be fully covered: each consumer buys either one unit of the product offered by A or one unit of the product offered by B.

The traded good has two dimensions: quality and color. Quality remains constant during the entire game, whereas the color can be changed across periods.
The quality of the product supplied by the firm $F = \{A, B\}$ is denoted by q_F. The color of the product supplied by the firm $F = \{A, B\}$ in period $t = \{1, 2\}$ is denoted by c_{tF}. All consumers prefer the same color, but this information is unknown for the firms in the first period (private information of the consumers). The consumers observe the color offered by each brand before making their consumption choices.

In the same spirit of vertical differentiation, the consumers have monotonic preferences with respect to the quality of the products. It is common knowledge that the quality of the product supplied by the firm B is zero: $q_B = 0$. In the first period, the quality of the product supplied by the firm A is unknown and all the agents assume the following:

Assumption 3. The quality of the product supplied by the firm A is a random variable normally distributed with mean $\mu > 0$ and variance σ^2:

$$q_A \sim N(\mu, \sigma)$$

The timing of the game is as follows:

Period 1

- The quality of the product offered by A, q_A, is realized and stays constant during the entire game. At this stage, it is unknown for all the agents.
- Each firm decides the color of its product.
- Firms set the first-period prices $p_1 = (p_{1A}, p_{1B})$ simultaneously.
- Consumers make their consumption choices: those who acquire the product offered by A observe the realization $\bar{q}_A$. Agents receive their first-period payoffs.

Period 2

- All agents observe the first-period market shares. The two firms undertake some market research to observe $\bar{q}_A$ if the market share of A in the first period was strictly positive.
- Each firm decides the color of its product.
- Consumer located at x_i in the first period communicates her experience to the consumer located at x_i in the second period. Then, the first-period consumer leaves the market.
- Consumers of the second period are divided in two groups: fully informed (denoted by the superscript I) and partially informed (denoted by the superscript P).
- Firms set the second-period prices $p_2 = (p_{2A}, p_{2B})$ simultaneously.
- Partially informed consumers use the vector of prices and the first-period market shares to update their beliefs.
- Consumers make their consumption choices and all agents receive their second-period payoffs.

The framework of a known quality and an uncertain quality reflects a situation in which a new firm enters into the market: if the quality is understood as something more sophisticated, for instance the perception of the product that the consumers have, the experimentation in the first period is necessary to gather this information because nobody tested the product before. Therefore, it also makes sense that this perception cannot be signaled by the firms at the beginning of the game. I implicitly assume that the two firms undertake some market research at a negligible cost for them to learn about the consumers' perception of the product whenever possible (that is, after the first period if the market share of the firm A in that period was strictly positive), and that the consumers do not have enough resources to undertake the same market research. Communication among consumers who live in the same period is not allowed.

Let me explain in detail how the consumers make their consumption choices.
The utility function is exponential. Concretely, the utility of the consumer located at x_i derived from consuming brand F in period t is

$$u_{i,t,F} = -e^{-\rho(q_F - K_{i,t,F})} \tag{2.1}$$

where $\rho > 0$ is the risk-aversion coefficient and $K_{i,t,F}$ is a set of other parameters that can differ across consumers, periods and brands.

Since the quality of the product supplied by A is assumed to be normally distributed when it is uncertain, maximize the expected utility of the consumer is equivalent to maximize the certainty equivalent[5]:

$$u_{i,t,A} = \mu - \frac{1}{2}(\rho\sigma^2) - K_{i,t,A} \tag{2.2}$$

In other words, the mean-variance analysis is optimal.
The quality of the product supplied by B is known to be zero with certainty during the entire game. Then,

$$u_{i,t,B} = -K_{i,t,B} \tag{2.3}$$

In the first period,

$$K_{i,1,A} = p_{1A} + x_i + (\zeta - c_{1A})$$
$$K_{i,1,B} = p_{1B} + (1 - x_i) + (\zeta - c_{1B})$$

where $p_{1F} \geq 0$ is the price of the brand F in the first period, ζ is the favorite color of the consumers and c_{1F} is the color of the brand F in the first period. Then, $K_{i,1,F}$ is interpreted as the sum of all the costs[6] faced by the consumer located at x_i derived from acquiring the brand F in the first period.

Then,

$$u_{i,1,A} = \mu - \frac{1}{2}(\rho\sigma^2) - p_{1A} - x_i - (\zeta - c_{1A}) \tag{2.4}$$

and

$$u_{i,1,B} = -p_{1B} - (1 - x_i) - (\zeta - c_{1B}) \tag{2.5}$$

In the first period, the consumers make their consumption decisions and observe the quality of the tested brand with no error: those who acquire brand B do not learn anything new, but those who acquire A learn the true realization of the quality $\bar{q}_A$. Before leaving the market in the second period, the consumer located at point x_i communicates the information she gathered to the consumer who will be located at x_i in the second period[7]. Notice that this assumption

[5] A formal proof of this statement can be found in the Appendix.
[6] The transportation cost parameter is normalized to 1.
[7] To avoid complexity, I assume that all consumers in the two periods perceive the quality in the same way.

is technically equivalent to consider myopic consumers, but the interpretation is quite different: what I mean here is that consumers learn from the experience of others.
Since consumers of the first period communicate their experience to the consumers of the second period but do not internalize their future utility, the marginal consumer of the first period $\hat{x}_1$ is obtained by equalizing the equations (2.4) and (2.5):

$$\begin{aligned} \hat{x}_1 &= \frac{1}{4}\left(2 - 2p_{1A} + 2p_{1B} + 2\mu - \rho\sigma^2 + 2(c_{1A} - c_{1B})\right) \\ &= \frac{1}{4}\left(2 - 2p_{1A} + 2p_{1B} + 2\mu - \rho\sigma^2 + 2(\Delta c_1)\right) \end{aligned} \tag{2.6}$$

Notice three things from equation (2.6): first, the term $\mu/2$ represents the quality effect and works in favor of firm A since $\mu > 0$; second, the term $-\rho\sigma^2/4$ is the switching cost effect and works in favor of firm B; and third, the marginal consumer of the first period is actually a random variable for the firms. The reason of the third remark is that the term Δc_1 is consumers' private information. Then, firms think about it as a random variable when solving their maximization problems. Since there are infinitely many colors, it is reasonable to assume the following:

Assumption 4. The firms consider Δc_1 as a random variable uniformly distributed with zero mean:

$$\Delta c_1 \sim U(-C, C)$$

with C large enough.

The direct implication of this assumption is that we can have corner solutions in the first period. Concretely, depending on the value of Δc_1, there are three possibilities: all consumers acquire the brand A in the first period and all consumers will be fully informed in the second period; all consumers acquire the brand B in the first period and all consumers will be partially informed in the second period; and only some consumers acquire brand A in the first period and the market will be divided into fully informed consumers and partially informed consumers in the second period.
The right way to interpret the corner solutions is as follows: since the size of the market is limited, a mathematical result of $\hat{x}_1 > 1$ means that all the consumers acquire the brand A and that all of them obtain a strictly positive surplus. Analogously, a mathematical result of $\hat{x}_1 < 0$ means that all the consumers acquire the brand B and that all of them obtain a strictly positive surplus.

I assume that all the agents observe the first-period market shares[8]. This is crucial for two reasons. For firms, the first-period market shares reveal which one of the two offered colors was closer to the favorite color of the consumers, ζ. Then, in the second period the two firms offer the same color $c_{2A} = c_{2B} = c_2$ and the difference between the favorite color and the offered color is not relevant anymore to determine the marginal consumer. For consumers, it is important to know the amount of fully informed people to determine the truthfulness of the prices.
Then, in the second period,

$$K_{i,2,A} = p_{2A} + x_i + (\zeta - c_2)$$
$$K_{i,2,B} = p_{2B} + (1 - x_i) + (\zeta - c_2)$$

In the second period, consumers are divided into two groups: fully informed and partially informed. The fully informed consumers are those who learn the quality of the brand A through the experience of the individuals living in their positions in period 1; the partially informed consumers are those who do not have the information about the quality of A because the individuals living in their positions in period 1 did not acquire that brand and, therefore, use the second-period prices to infer the quality of A.

The utility of a fully informed consumer located at x_i in the second period when she acquires the product offered by A is

$$u^I_{i,2,A} = \bar{q}_A - p_{2A} - x_i - (\zeta - c_2) \tag{2.7}$$

Notice that the uncertainty has disappeared and that the consumer uses the true realization of the quality instead of the expected quality.

Denoting the vector of prices of the second period by $p_2 = (p_{2A}, p_{2B})$, the utility of a partially informed consumer located at x_i in the second period when she acquires the product offered by A is

$$u^P_{i,2,A} = E[q_A|(p_2, \hat{x}_1)] - \frac{1}{2}\rho Var[q_A|(p_2, \hat{x}_1)] - p_{2A} - x_i - (\zeta - c_2) \tag{2.8}$$

The utility of a consumer located at x_i in the second period when she acquires the product offered by B is

$$u_{i,2,B} = -p_{2B} - (1 - x_i) - (\zeta - c_2) \tag{2.9}$$

The utility derived from consuming the brand B does not differ across fully informed and partially informed consumers because its quality is common knowledge.

[8]As it is argued in Caminal and Vives (1996), it is more plausible that the consumers observe the past market shares instead of the vector of past prices.

The marginal consumer in the second period among the fully informed consumers is obtained by equalizing the equations (2.7) and (2.9):

$$x_2^I = \frac{1}{2}\left(1 - p_{2A} + p_{2B} + \bar{q}_A\right) \tag{2.10}$$

And the marginal consumer in the second period among the partially informed consumers is obtained by equalizing the equations (2.8) and (2.9):

$$x_2^P = \frac{1}{2}\left(1 - p_{2A} + p_{2B} + E[q_A|(p_2, \hat{x}_1)] - \frac{1}{2}\rho Var[q_A|(p_2, \hat{x}_1)]\right) \tag{2.11}$$

Because of the communication between the consumers who live in the first period and the consumers who live in the second period, it is not optimal for the firms to solve the problem of each period separately: since they are forward-looking, the game has to be solved by backward induction. The equilibrium concept used is the Perfect Bayesian Nash Equilibrium (PBNE).

2.3 Equilibrium of the Second Period

There may be three possible situations in the first period, depending on the value of Δc_1: all consumers acquire the brand A ($\hat{x}_1 \geq 1$), all consumers acquire the brand B ($\hat{x}_1 \leq 0$) and some consumers acquired the brand A whereas the rest acquire the brand B ($\hat{x}_1 \in (0, 1)$). The optimal pricing strategy in the second period depends on the outcome of the first period.

2.3.1 Corner Equilibria

Corner A

This corner takes place when all consumers acquired the brand A in the first period, that is, when

$$\hat{x}_1 \geq 1 \Leftrightarrow \Delta c_1 \geq 1 + p_{1A} - p_{1B} - \mu + \frac{1}{2}\rho\sigma^2 \tag{2.12}$$

In this case, all consumers of the first period tested the brand A and this information is communicated to all the consumers of the second period. Then, all the agents know the true realization of the quality of the product offered by the firm A, $\bar{q}_A$ and we say that the market is fully informed (remember that it is common knowledge that the quality of the product offered by the firm B is zero).

Since no consumer is susceptible to be deceived through the prices, signaling plays no role in this case. Thus, the marginal consumer is the one defined in equation (2.10).

The Proposition below states the equilibrium prices and market shares:

Proposition 7. The equilibrium prices and market shares in the case of full information are as follows:

$$p_{2A}^I = \begin{cases} 0 & \bar{q_A} \leq -3 \\ 1 + \frac{\bar{q_A}}{3} & \bar{q_A} \in (-3, 3) \\ \bar{q_A} - 1 & \bar{q_A} \geq 3 \end{cases}$$

$$p_{2B}^I = \begin{cases} -\bar{q_A} - 1 & \bar{q_A} \leq -3 \\ 1 - \frac{\bar{q_A}}{3} & \bar{q_A} \in (-3, 3) \\ 0 & \bar{q_A} \geq 3 \end{cases}$$

$$x_2^I = \begin{cases} 0 & \bar{q_A} \leq -3 \\ \frac{3+\bar{q_A}}{6} & \bar{q_A} \in (-3, 3) \\ 1 & \bar{q_A} \geq 3 \end{cases}$$

As expected, the price fixed by A and its market share are increasing in the realization of the quality of the product that it supplies. In the same way, the price fixed by B and its market share are decreasing in the realization of the quality of the product supplied by A. Notice that the thresholds are "arbitrary": we could fix the quality of the product supplied by the firm B at a level different from zero and still get the two corners and the interior solution, but in that case the prices would depend on the differential between the qualities of A and B and the thresholds for each outcome would be different.

The corner outcomes mean that the differential among qualities is big enough for one of the firms to supply to the entire market.

It is worth remarking that these prices are not distorted, because all second-period consumers have complete information derived from the experience of the first-period consumers.

Corner B

In this corner, all the consumers acquired the product supplied by the firm B in the first period. Since the sales of the product supplied by A were zero, the firms cannot undertake the market research to discover how consumers perceive the product A; then, I will say that the information about the quality of A cannot be disclosed.

Mathematically, this corner takes place when

$$\hat{x}_1 \leq 0 \Leftrightarrow \Delta c_1 \leq -1 + p_{1A} - p_{1B} - \mu + \frac{1}{2}\rho\sigma^2 \tag{2.13}$$

In this case, nobody bought the product supplied by A in the first period and the true realization $\bar{q}_A$ is unknown for the firms. The consumers observe the past market shares and know that the firms cannot send any signal about the uncertain quality through the prices. Therefore, the consumers do not update their beliefs and simply take the unconditional values for the mean and the variance. Thus, the equation (2.11) describing the marginal consumer becomes into

$$x_2^P = \frac{1}{2}\left(1 - p_{2A} + p_{2B} + \mu - \frac{1}{2}\rho\sigma^2\right) \tag{2.14}$$

The next Proposition establishes the equilibrium prices and market shares when the information about the quality of A cannot be disclosed:

Proposition 8. The equilibrium prices and market shares when the information about the quality of A cannot be disclosed are as follows:

$$p_{2A}^P = \begin{cases} 0 & \rho \geq (6+2\mu)/\sigma^2 \\ 1 + \frac{\mu}{3} - \frac{\rho\sigma^2}{6} & \mu \leq 3 \quad \text{and} \quad \rho < (6+2\mu)/\sigma^2; \quad \text{or} \\ & \mu > 3 \quad \text{and} \quad (-6+2\mu)/\sigma^2 < \rho < (6+2\mu)/\sigma^2 \\ \frac{1}{2}(-2+2\mu-\rho\sigma^2) & \mu > 3 \quad \text{and} \quad \rho \leq (-6+2\mu)/\sigma^2 \end{cases}$$

$$p_{2B}^P = \begin{cases} \frac{1}{2}(-2-2\mu+\rho\sigma^2) & \rho \geq (6+2\mu)/\sigma^2 \\ 1 - \frac{\mu}{3} + \frac{\rho\sigma^2}{6} & \mu \leq 3 \quad \text{and} \quad \rho < (6+2\mu)/\sigma^2; \quad \text{or} \\ & \mu > 3 \quad \text{and} \quad (-6+2\mu)/\sigma^2 < \rho < (6+2\mu)/\sigma^2 \\ 0 & \mu > 3 \quad \text{and} \quad \rho \leq (-6+2\mu)/\sigma^2 \end{cases}$$

$$x_2^P = \begin{cases} 0 & \rho \geq (6+2\mu)/\sigma^2 \\ \frac{1}{12}(6+2\mu-\rho\sigma^2) & \mu \leq 3 \quad \text{and} \quad \rho < (6+2\mu)/\sigma^2; \quad \text{or} \\ & \mu > 3 \quad \text{and} \quad (-6+2\mu)/\sigma^2 < \rho < (6+2\mu)/\sigma^2 \\ 1 & \mu > 3 \quad \text{and} \quad \rho \leq (-6+2\mu)/\sigma^2 \end{cases}$$

This equilibrium shows interesting features. Since all agents in the market are partially informed, the firm B takes advantage of the switching cost: its equilibrium price is increasing in the risk-aversion coefficient, whereas the equilibrium price of the rival firm A is decreasing in it. Nevertheless, the quality effect is favorable to firm A because $\mu > 0$. These two effects exactly offset each other when $\rho = 2\mu/\sigma^2$: in this case, the two firms charge the same price and the market splits up equally. If the risk-aversion coefficient is above this threshold, the switching cost dominates: despite consumers know that the quality offered by B is zero, they consider too risky to test the product of A without additional information. Therefore, the firm B charges a price larger than its rival in equilibrium and it supplies to a higher share of the market. Exactly the opposite takes place when the risk-aversion coefficient is below the threshold: the quality effect dominates and consumers prefer to take the risk of testing the brand for which they have no information. In this case, the price charged by the firm A is higher than the price charged by B, and A supplies to a higher share of the market. For extreme values of ρ, only one of the two firms supplies to the entire market.

2.3.2 Interior Equilibrium

In the interior equilibrium, only some consumers tested the product supplied by A in the first period. This means that there is a fraction of consumers who do not take the prices as a signals of the quality, but the rest of the consumers do (and actually, they are susceptible of being deceived).
Contrary to the monopoly case, the partially informed consumers have two signals to update their beliefs about the quality of the product supplied by A (the price charged by A and the price charged by B). Then, this game is characterized by the "signaling rivalry".

In the signaling games, there may be three different types of equilibria: separating, pooling and semi-pooling (also known as semi-separating). In this paper, I check the existence or not of a separating equilibrium. In a separating equilibrium with two senders, the consumers can infer with no error the quality of the product A for each vector of prices. This implies that the messages sent by the two firms have to be consistent.
Suppose that the full-information prices can be supported as a signaling equilibrium under additional conditions. If the consumers could only observe one of the prices, the inference could be incomplete. For instance, if the firm A charges a zero price, the partially informed consumers can only infer that $\bar{q}_A \leq -3$. The information would be complete with the price charged by B: if $p_{2B} = 4$, the partially informed consumers can infer that $\bar{q}_A = -5$. It is worth remarking that the two messages do not conflict, so the consumers can infer the quality of A perfectly. However, if the price vector is $(4, 4)$, the consumers would be confused: the price charged by A signals $\bar{q}_A = 5$, whereas the price charged by B signals $\bar{q}_A = -5$. At most, the consumers would infer that the true quality belongs to the interval $[-5, 5]$, but they do not know if A, B or both are deviating from the equilibrium price.

The informed consumers do not take the prices as a signal because they know the true quality of the product. Then, the demands from this sector can be expressed as

$$\begin{aligned} D^I_{2A} &= \min\left\{\hat{x}_1, \max\left\{0, \frac{1}{2}(1 - p_{2A} + p_{2B} + \bar{q}_A)\right\}\right\} \\ D^I_{2B} &= \hat{x}_1 - D^I_{2A} \end{aligned} \tag{2.15}$$

The operators *min* and *max* come from the corners of the problem: the marginal consumer among the informed consumers cannot be part of the uninformed group, and negative demands are not allowed.

The partially informed consumers use the market shares of the period 1 and the vector of prices of the period 2 to update their beliefs. Then, the demands from this sector can be expressed as

$$\begin{aligned} D_{2A}^{P} &= \max\left\{0, \min\left\{1, \frac{1}{2}\left(1 - p_{2A} + p_{2B} + \breve{q}_A - \frac{1}{2}\rho s^2\right)\right\} - \hat{x}_1\right\} \\ D_{2B}^{P} &= \max\left\{0, 1 - \max\left\{\hat{x}_1, \frac{1}{2}\left(1 - p_{2A} + p_{2B} + \breve{q}_A - \frac{1}{2}\rho s^2\right)\right\}\right\} \end{aligned} \tag{2.16}$$

where $\sigma^2 > s^2 \geq 0$ represents the updated uncertainty and $\breve{q}_A$ represents the updated quality when the observed vector of prices is (p_{2A}, p_{2B}) and the market share of the firm A in the first period was $\hat{x}_1 > 0$. $s^2 = 0$ when the two prices would signal the same quality $\bar{q}_A$ and, in general, $\breve{q}_A$ would be a linear combination of the qualities signaled by each one of the prices.

The profits of each firm are

$$\begin{aligned} \pi_{2A} &= p_{2A}\left(D_{2A}^{I} + D_{2A}^{P}\right) \\ \pi_{2B} &= p_{2B}\left(D_{2B}^{I} + D_{2B}^{P}\right) \end{aligned} \tag{2.17}$$

Notice that the firms face a trade-off when they pretend to signal a quality that is not the true one: by increasing the price, the updated quality and uncertainty of the partially informed consumers are affected and this strategy may generate extra profits; however, the demand in the informed sector decreases.

Proposition 9. A separating equilibrium may not exist.

Consider $\bar{q}_A = 4$. If the information is disclosed, the firm B makes exactly zero profits. Then, it is at least as good as if, for instance, it would signal a quality of $\bar{q}_A = -4$ through a positive price independently of the consumers beliefs (the profit has a lower bound in zero). In the same way, if $\bar{q}_A = -4$ and the information is disclosed, the firm A makes exactly zero profits. Then, it is as least as good as if it would signal a quality of $\bar{q}_A = 4$ through a positive price. It is easy to see that, if the vector of prices is such that the price of A signals a quality of 4 and the price of B signals a quality of -4, the consumers could not determine which firm is deviating and they could not infer the true quality. In other words, the separating equilibrium would not exist.

In principle, the result of non-existence could make impossible to solve the intertemporal maximization problem. However, I show in the next section that, under the assumptions of the model, this is not the case.

2.4 Equilibrium of the First Period

As it was mentioned in Section 2, the two firms live during the entire game, are forward-looking and have a discount factor equal to 1. Then, the objective function of the first period is composed of the first-period profits plus the expected second-period profits.

I have shown that a separating equilibrium may not exist in the interior case, which in principle could be a problem to find the expected profit of the second period. However, the next Lemma states that this equilibrium is irrelevant when solving the intertemporal maximization problem:

Lemma 2. The probability of being in the interior case in the second period is independent of the first-period price.

Therefore, in mathematical terms, the equilibrium of the interior case would be like adding a constant term to the first-period objective function and it would not affect to the solution of the problem.

The expected profit obtained by each firm in the Corner A depends on the parameters of the distribution of the product quality, μ and σ:

$$\begin{aligned}
\Pi^I_{2A}(\mu,\sigma) &= Pr(\bar{q}_A \leq -3)E(0|\bar{q}_A \leq -3) + Pr(\bar{q}_A \geq 3)E(\bar{q}_A - 1|\bar{q}_A \geq 3)+ \\
&+ Pr(-3 < \bar{q}_A < 3)E\left(\frac{1}{18}(3+\bar{q}_A)^2 \middle| -3 < \bar{q}_A < 3\right) \\
&= Pr(\bar{q}_A \geq 3)E(\bar{q}_A - 1|\bar{q}_A \geq 3) + Pr(-3 < \bar{q}_A < 3)E\left(\frac{1}{18}(3+\bar{q}_A)^2 \middle| -3 < \bar{q}_A < 3\right) \\
&> 0
\end{aligned} \tag{2.18}$$

$$\begin{aligned}
\Pi^I_{2B}(\mu,\sigma) &= Pr(\bar{q}_A \leq -3)E(-\bar{q}_A - 1|\bar{q}_A \leq -3) + Pr(\bar{q}_A \geq 3)E(0|\bar{q}_A \geq 3)+ \\
&+ Pr(-3 < \bar{q}_A < 3)E\left(\frac{1}{18}(3-\bar{q}_A)^2 \middle| -3 < \bar{q}_A < 3\right) \\
&= Pr(\bar{q}_A \leq -3)E(-\bar{q}_A - 1|\bar{q}_A \leq -3) + Pr(-3 < \bar{q}_A < 3)E\left(\frac{1}{18}(3-\bar{q}_A)^2 \middle| -3 < \bar{q}_A < 3\right) \\
&> 0
\end{aligned} \tag{2.19}$$

And the profit obtained by each firm in the Corner B is defined by the following piecewise functions:

$$\Pi_{2A}^{P}(\mu,\rho,\sigma)=\begin{cases} 0 & \rho \geq (6+2\mu)/\sigma^2 \\ \frac{1}{72}(6+2\mu-\rho\sigma^2)^2 & \mu \leq 3 \quad \text{and} \quad \rho < (6+2\mu)/\sigma^2; \quad \text{or} \\ & \mu > 3 \quad \text{and} \quad (-6+2\mu)/\sigma^2 < \rho < (6+2\mu)/\sigma^2 \\ \frac{1}{2}(-2+2\mu-\rho\sigma^2) & \mu > 3 \quad \text{and} \quad \rho \leq (-6+2\mu)/\sigma^2 \end{cases} \tag{2.20}$$

$$\Pi_{2B}^{P}(\mu,\rho,\sigma)=\begin{cases} \frac{1}{2}(-2-2\mu+\rho\sigma^2) & \rho \geq (6+2\mu)/\sigma^2 \\ \frac{1}{72}(6-2\mu+\rho\sigma^2)^2 & \mu \leq 3 \quad \text{and} \quad \rho < (6+2\mu)/\sigma^2; \quad \text{or} \\ & \mu > 3 \quad \text{and} \quad (-6+2\mu)/\sigma^2 < \rho < (6+2\mu)/\sigma^2 \\ 0 & \mu > 3 \quad \text{and} \quad \rho \leq (-6+2\mu)/\sigma^2 \end{cases} \tag{2.21}$$

Being $p_1 = (p_{1A}, p_{1B})$ the price vector of the first period, the probability of being in the Corner A is

$$\begin{aligned} PCA(p_1) &= Pr(\Delta c_1 \geq 1-\mu+\frac{1}{2}\rho\sigma^2+p_{1A}-p_{1B}) \\ &= \frac{-1-p_{1A}+p_{1B}+\mu-\frac{1}{2}(\rho\sigma^2)+C}{2C} \end{aligned} \tag{2.22}$$

and the probability of being in the Corner B is

$$\begin{aligned} PCB(p_1) &= Pr(\Delta c_1 \leq -1-\mu+\frac{1}{2}\rho\sigma^2+p_{1A}-p_{1B}) \\ &= \frac{-1+p_{1A}-p_{1B}-\mu+\frac{1}{2}(\rho\sigma^2)+C}{2C} \end{aligned} \tag{2.23}$$

Therefore, the objective functions of the first period are

$$\begin{aligned} \Pi_A &= p_{1A}\hat{x}_1^E(p_1) + PCA(p_1)\Pi_{2A}^I(\mu,\sigma) + PCB(p_1)\Pi_{2A}^P(\mu,\rho,\sigma) \\ \Pi_B &= p_{1B}(1-\hat{x}_1^E(p_1)) + PCA(p_1)\Pi_{2B}^I(\mu,\sigma) + PCB(p_1)\Pi_{2B}^P(\mu,\rho,\sigma) \end{aligned} \tag{2.24}$$

where $\hat{x}_1^E(p_1) = E[\hat{x}_1] = (2-2p_{1A}+2p_{1B}+2\mu-\rho\sigma^2)/4$.

The next Proposition states the solution to the intertemporal problem:

Proposition 10. The first-period equilibrium prices are

$$p^*_{1A} = \begin{cases} 0 & \text{if } 0 > \dfrac{C(6+2\mu-\rho\sigma^2)+2\tilde{\Pi}}{12C} \\[2em] \dfrac{-4\Pi^I_{2A}+4\Pi^P_{2A}+2\Pi^I_{2B}-2\Pi^P_{2B}+C(6+2\mu-\rho\sigma^2)}{6C} & \text{if } 0 < \dfrac{C(6+2\mu-\rho\sigma^2)+2\tilde{\Pi}}{12C} < 1 \\[1em] \dfrac{-2+2\mu-\rho\sigma^2}{2} & \text{if } 1 < \dfrac{C(6+2\mu-\rho\sigma^2)+2\tilde{\Pi}}{12C} \end{cases}$$

$$p^*_{1B} = \begin{cases} \dfrac{-2-2\mu+\rho\sigma^2}{2} & \text{if } 0 > \dfrac{C(6+2\mu-\rho\sigma^2)+2\tilde{\Pi}}{12C} \\[2em] \dfrac{-2\Pi^I_{2A}+2\Pi^P_{2A}+4\Pi^I_{2B}-4\Pi^P_{2B}+C(6-2\mu+\rho\sigma^2)}{6C} & \text{if } 0 < \dfrac{C(6+2\mu-\rho\sigma^2)+2\tilde{\Pi}}{12C} < 1 \\[1em] 0 & \text{if } 1 < \dfrac{C(6+2\mu-\rho\sigma^2)+2\tilde{\Pi}}{12C} \end{cases}$$

with $\tilde{\Pi} = \Pi^I_{2A} - \Pi^P_{2A} + \Pi^I_{2B} - \Pi^P_{2B}$

It was mentioned previously that C had to be large enough. Concretely,

Assumption 5. C has to be such that

$$0 < PCA(p^*_1) < 1$$
$$0 < PCB(p^*_1) < 1$$
and
$$C > 2$$

Under the previous assumption, it holds that the first-period price charged by A is decreasing in the risk-aversion coefficient, whereas the first-period price charged by B is increasing in it.

It is important to remark that negative prices are weakly dominated in the first period, contrary to what happened in the monopoly case. Take the corner $0 > (C(6+2\mu-\rho\sigma^2)+2\tilde{\Pi})/12C$ (the reasoning is analogous for the other corner). In this case, the firm B optimally plays the strategy $p_{1B} = (-2 + 2p_{1A} - 2\mu + \rho\sigma^2)/2$. This means that the firm A will not get additional profits when charging a negative price: its demand in expected terms is zero in the first period, and the probability of each corner in the second period when the firm B plays the previous strategy is $1/2$.

The figures below represent the overall profit Π_j as a function of p_j, given the equilibrium price p^*_{-j}:

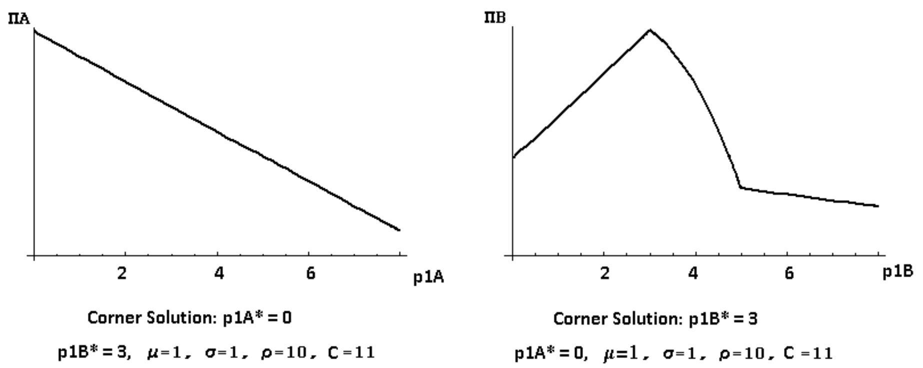

Figure 2.1: First-period equilibrium price, corner solution

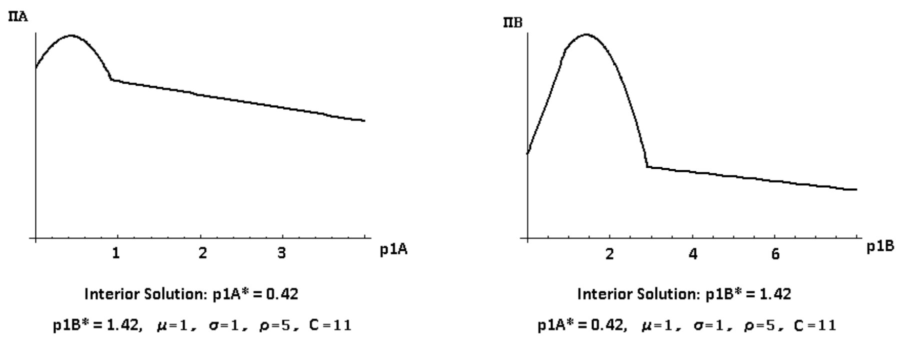

Figure 2.2: First-period equilibrium price, interior solution

To conclude the section, it is interesting to check if the bargain-then-ripoff pattern of prices can be reversed when information is disclosed for the two firms simultaneously. From the previous Chapter, we know that the classical pattern of prices can be reversed when there is a single firm in the market, depending on the values of the risk-aversion coefficient and the true quality. In this case, the effect of the price competition is important too. For the pattern of prices to be reversed for the two firms simultaneously, a necessary condition is $p^*_{1A} > 0$ and $p^*_{1B} > 0$: if the price is zero in the first period, the pattern cannot be reversed because negative prices are dominated in the second period. The next Proposition states how the patterns of prices evolve when the information is disclosed:

Proposition 11. If the bargain-then-ripoff pattern is reversed for one of the firms, it cannot be reversed for the other at the same time.

Notice that the Proposition does not state that the bargain-then-ripoff pattern necessarily reverses for one firm: in fact, it may happen that the classical pattern holds for the two firms simultaneously. It only states that the bargain-then-ripoff pattern cannot be reversed for the two firms at the same time. Actually, it makes sense: in the first period, the firm B takes advantage of the risk faced by the consumers. When the information is disclosed, this risk disappears and only the true quality matters. If the quality is sufficiently low (high), the firm B maintains (loses) its advantage in the market and it sets a higher (lower) price than in the first period. However, the firm A sets a price lower (higher) than in the first period to compensate (take advantage of) such a low (high) quality.

2.5 The Model Without Risk Aversion

The model without risk aversion considers risk-neutral consumers; that is, now the risk-aversion parameter ρ is equal to zero instead of being strictly positive. This new assumption does not change the way to solve the model. Then,

1. The Lemma 2 also holds
2. The probability of the Corner A is $(-1 - p_{1A} + p_{1B} + \mu + C)/2C$ and the probability of the Corner B is $(-1 + p_{1A} - p_{1B} - \mu + C)/2C$
3. The prices and the market shares in the full information case are identical to those stated in Proposition 7
4. The prices and the market shares when information cannot be disclosed are

$$p_{2A}^{P,rn} = \begin{cases} 1 + \frac{\mu}{3} & \text{if} \quad \mu \leq 3 \\ (-1 + \mu) & \text{if} \quad \mu > 3 \end{cases}$$

$$p_{2B}^{P,rn} = \begin{cases} 1 - \frac{\mu}{3} & \text{if} \quad \mu \leq 3 \\ 0 & \text{if} \quad \mu > 3 \end{cases}$$

$$x_2^{P,rn} = \begin{cases} \frac{1}{6}(3+\mu) & \text{if} \quad \mu \leq 3 \\ 1 & \text{if} \quad \mu > 3 \end{cases}$$

From these equilibrium prices, it is important to remark two things: first, the prices charged by the firm A are always higher with risk-neutral consumers than with risk-averse consumers (the opposite happens for B); second, B never charges a price higher than the price charged by A, contrary to what happened in the case of risk-averse consumers. Furthermore, one of the corners has disappeared: it cannot happen that the firm B supplies to the entire market in equilibrium. Since the risk does not have to be compensated anymore, the competitive advantage works in favor of A simply because its expected quality is larger than zero.

The next Proposition states the first-period prices charged by the firms when the consumers are risk-neutral, and compares the average first-period market price when the consumers are risk-neutral and when the consumers are risk-averse:

Proposition 12. When the consumers are risk-neutral, it holds that

12.1 The first-period equilibrium prices are

$$p_{1A}^* = \begin{cases} \dfrac{-2\Pi_{2A}^I + 2\Pi_{2A}^{P,rn} + \Pi_{2B}^I - \Pi_{2B}^{P,rn} + C(3+\mu)}{3C} & \text{if } 0 < \dfrac{C(3+\mu)+\tilde{\Pi}}{6C} < 1 \\ \\ -1+\mu & \text{if } 1 < \dfrac{C(3+\mu)+\tilde{\Pi}}{6C} < 1 \end{cases}$$

$$p_{1B}^* = \begin{cases} \dfrac{-\Pi_{2A}^I + \Pi_{2A}^{P,rn} + 2\Pi_{2B}^I - 2\Pi_{2B}^{P,rn} + C(3-\mu)}{3C} & \text{if } 0 < \dfrac{C(3+\mu)+\tilde{\Pi}}{6C} < 1 \\ \\ 0 & \text{if } 1 < \dfrac{C(3+\mu)+\tilde{\Pi}}{6C} < 1 \end{cases}$$

where Π_{2A}^I is defined as in equation (2.18),

Π^{I}_{2B} is defined as in equation (2.19),

$\Pi^{P,rn}_{2j}$ comes from multiplying the corresponding price times the market share as defined at the beginning of this section,

and $\tilde{\Pi} = \Pi^{I}_{2A} - \Pi^{P,rn}_{2A} + \Pi^{I}_{2B} - \Pi^{P,rn}_{2B}$

12.2 The average first-period market price, measured as $p_{1A}x_1^E + p_{1B}(1 - x_1^E)$, can be smaller than when the consumers are risk-averse.

The first part of the Proposition is simply the solution to the intertemporal problem of the firm when consumers are risk-neutral. About the second part, it is contrary to what happened in the monopoly case, and it is due to the competition forces. The result takes place because with risk-aversion, the firm B has an advantage that disappears when consumers are risk-neutral. Concretely, the advantage can be so large that the firm B optimally charges the same price in the first period and in the second period when the information cannot be disclosed. This possibility is ruled out with no risk-aversion.
Consider the next example: $\mu = 1$, $\sigma = 1$, $C = 11$ and $\rho = 10$. In the equilibrium of the first period, $p_{1A} = 0$, $p_{1B} = 3$ and $x_1^E = 0$. Then, the average first-period market price is 3.
Suppose now the same values, but $\rho = 0$. In the equilibrium of the first period, $p_{1A} = 1.33$, $p_{1B} = 0.668$ and $x_1^E = 0.668$. Then, the average first-period price is $1.11 < 3$.

2.6 Conclusion and Discussion

I have presented a model of endogenous switching costs derived from the lack of information. There are two firms who live during two periods: A and B. The product supplied by firm B is known to be of quality equal to zero, whereas the quality of the product supplied by firm A is unknown in the first period: it is a continuous random variable normally distributed. All consumers are assumed to be risk-averse. Those who live in the first period maximize their instantaneous utility and communicate the information they gathered through direct experience to the consumers who will live in the second period. Two effects appear due to the lack of information in the first period: first, the expected quality of the product supplied by A is higher than the quality of the product supplied by B; second, consumers (who are risk-averse) consider to test the product offered by A as a risky choice. The decision of the consumers who live in the first period is affected by their location, the risk-aversion coefficient, the set prices and a private information component. Then, there are three possible outcomes: all consumers test the brand A and learn its quality, nobody tests the brand A and its quality remains unknown, and the market splits up such that only a positive share of consumers test A and learn its quality. Notice that the switching costs are endogenous because they depend on the purchasing histories.

In the second period, consumers also maximize their instantaneous utility. Nevertheless, the private information component is not present anymore, but information across consumers may differ at the beginning of the period.
If all consumers tested A in the first period, in the second period all the agents are fully informed because they know the two qualities with certainty. Then, nobody faces a switching cost and equilibrium prices are not distorted. The prices will be equal if the product supplied by A is of quality zero, but this event happens with zero probability. A will charge a price higher than B if its product is of high quality and viceversa.
If all consumers tested B in the first period, the quality of A remains unknown for all the agents in the second period and cannot be signaled through distorted prices. In this case, all consumers face a switching cost but the quality effect is also present. Then, the firm B charges an equilibrium price higher than its rival if the switching cost effect dominates, that is, if the risk-aversion coefficient is sufficiently large. The opposite happens if the quality effect dominates, that is, if the risk-aversion coefficient is low enough. In other words, the model predict that B can be the dominant firm in the market if consumers are sufficiently risk-averse and the quality of its rival cannot be signaled.
If the market split up in the first period, consumers are initially divided in two different groups in the second period: those who know the quality of A and those who do not. Those who know the quality of A is due to the previous experience of the consumers of the first period, and they cannot be deceived. Those who do not know the quality of A take the second-period prices as informative signals. The game is characterized by signaling rivalry between the two firms, and I have showed that a separating equilibrium may not exist.

Since the two firms are forward-looking, the objective function of the first period is composed of the addition of the first-period profits and the expected second-period profits. The solution states that the first-period equilibrium prices are different for the two firms: it is a reasonable result because the firms are not homogeneous in the first period, as it was explained at the beginning of the section. Furthermore, the model predicts that B will charge a first-period price higher than the price of its rival if the risk-aversion coefficient is large enough.

Interesting results are obtained when analyzing the pattern of equilibrium prices. In the standard models of two periods with switching costs, the prices display a bargain-then-ripoff pattern: firms compete fiercely in the first period to attract as many consumers as possible and make them captive; then, they exploit the captive consumers in the second period by setting high prices. In this model, prices do not display that pattern in all the cases. Actually, when the information about the quality of A is disclosed in the first period, it may happen that only one of the firms harvests after having invested in the first period. Concretely, if the quality of A

is high enough, the firm A charges an equilibrium price in the second period larger than its equilibrium price in the first period, whereas the rival firm B has to fix a price lower than the price it set in the first period. The opposite happens when the quality of A is low enough.

Compared to a situation without switching costs, I find that the average price of the first period may be higher with than without switching costs if the risk-aversion coefficient is sufficiently large. This contrasts with the classic result of lower first-period prices in the presence of switching costs, but it is due to the nature of the switching costs considered here. The result happens because, given the expected quality, the firm that offers the riskless product has a competitive advantage in the first period when consumers are sufficiently risk averse that does not exist when consumers are risk neutral. It may happen that its rival cannot compensate completely this advantage through a price decrease because negative prices are weakly dominated.

These results are due to the nature of the switching costs considered here and to the initial heterogeneity of the firms. When the information is disclosed, two effects take place. First, consumers do not face the switching cost anymore: consequently, the firm B loses its advantage and the firm A does not have to compensate for that cost anymore. Second, the consumers' willingness to pay for the brand A reacts according to the learned true quality. Furthermore, the firm B can start taking advantage of the switching cost effect in the first period (instead of waiting until the second) if the risk-aversion coefficient is sufficiently large. When consumers are risk-neutral ($\rho = 0$), the switching cost effect plays no role, but the quality effect still appears. The different values of the risk-aversion coefficient and the true quality of the product supplied by A determine which effect dominates and leads to the patterns of prices described before.

To conclude the section, I proceed to discuss the role of some assumptions.
The presence of the random component in the consumers' utility allows for corner outcomes in the market shares in the first period: this outcome would determine if consumers face the switching cost also in the second period and creates an incentive for firms to compete fiercely in the first period. If I remove this assumption, the solution in the first period will always be interior and a separating equilibrium may not exist.
Two other assumptions are also crucial. The consideration of a different mass of consumers in each period means that people learn from the experience of others, that is a fact supported by evidence. However, this assumption can be removed to consider instead that consumers live during the entire game and that they are forward-looking. In this case, they would be less sensitive to low prices in the first period. The assumption of no price discrimination across consumers who live in the same period can also be removed if we consider that a firm can identify in the second period to those consumers who acquired its product in the first period. These extensions are left to future research.

Chapter 3

Do you want to steal my songs? The importance of diffusion in the music industry

3.1 Introduction

Nowadays, there is a huge debate on online piracy (i.e. books, movies, TV shows, music...). Some artists and labels argue that the existence of websites that allow conumers to engage in piracy leads to a decline of their original profits[1]. Due to the pressure of the supporters of stronger intellectual property enforcement, the Parliaments of many countries have approved different laws oriented to restrict the file sharing, such as SOPA and PIPA in the United States, HADOPI in France (also known as "Loi Oliviennes") and LES in Spain (also known as "Ley Sinde-Wert"). The most recent event related to this fight against online piracy is the shutdown of Megaupload (file-sharing site) by officials[2].

Nevertheless, these laws have some loopholes and, as judges have pointed out, there are several problems regarding the proposed mechanisms that aim to prevent copyright violations. For instance, in United Kingdom, Judge Birss declaired that copyright owners could not accuse people of illegal filesharing by using their IP addresses, because IP addresses identify an internet connection but not the person who is using the connection at a certain moment in time[3]. In Spain,

[1] A report produced by the Institute for Policy Innovation (IPI) in 2007 estimates that music piracy causes $12.5 billion of economic losses every year in the US (although this amount is not only due to online piracy). Nevertheless, other authors maintain that this report has several methodological problems, including double counting.
http://www.ipi.org/docLib/20120515_SoundRecordingPiracy.pdf
http://www.freakonomics.com/2012/01/12/how-much-do-music-and-movie-piracy-really-hurt-the-u-s-economy/

[2] http://www.bbc.co.uk/news/technology-16642369

[3] http://www.guardian.co.uk/technology/2011/feb/08/filesharing-prosecutions-digital-economy?INTCMP=SRCH
http://arstechnica.com/tech-policy/2011/02/court-confirms-ip-addresses-arent-people-and-p2p-lawyers-know-it/

the software developer Pablo Soto was found not guilty in the judgment against Warner, Universal, EMI, Sony and Promusicae, in which the labels demanded 13 million from Soto in terms of damages. The Court estimated that "Soto has developed a technological device that allows for direct file sharing among consumers, peer to peer, being its technical function completely neutral"[4].

The argument used by the labels is not new in economics: creation of intellectual property requires a strong initial investment (sunk cost), but the cost of each additional copy of the product is very small (in software products, it even goes to zero). Then, when the planner has to maximize the welfare, he faces a trade-off between creation and usage: once the good is in the market, fixing a price equal to the marginal cost of the copy is efficient because the number of people who have access to the good is maximized, and the producer covers its variable costs. Nevertheless, with this policy the creator is not able to recover his initial investment, and he is better off simply not producing the good which leads to a decrease of welfare for the society. Therefore, in order to compensate for the sunk cost, the sale prices have to be above the marginal cost in equilibrium.
On the other hand, we find the defenders of the free culture. Their main arguments are as follows:

1. The mark-up is excessive: defenders of the free culture sustain that, with lower prices, many consumers would renounce to download the contents illegally and would acquire legal copies instead, and creators would still make positive profits (after covering the sunk cost). Even more so, many cultural activities obtain public subsidies, so consumers who buy legal copies are actually paying twice, apart from the classical inefficiencies generated by subsidies and taxes.

2. Free knowledge allows for the creation of new inventions which increase the welfare level of the society. Boldrin and Levine (2010) explain in the first chapters of their book how "after the expiration of Watt's patents, not only was there an explosion in the production and efficiency of engines, but steam power came into its own as the driving force of the industrial revolution"[5].

3. Simply, it is not feasible to stop the file sharing. So, it would be much better for creators to stop wasting resources in legal wars trying to keep the monopoly profits that they get with the current model and try to adapt to the new conditions in order to obtain positive profits and not to be driven out of the market.

It is possible to find several approaches in the literature showing that the existence of piracy can be profitable both for artists and for consumers. A more in depth analysis will be provided in the next section, but it is useful to mention here that these approaches mainly rely on the

[4] http://estaticos.elmundo.es/documentos/2011/12/19/sentencia_pablo_soto.pdf
[5] http://levine.sscnet.ucla.edu/papers/imbookfinalall.pdf

complementarity of the businesses of the artists (tracks and live performances), or on the effect of sampling when a consumer tries to learn who is his/her favorite artist.

To the best of my knowledge, this is the first approach in considering the co-existence of two heterogeneous platforms. The open platform can be used by the artists to set their tracks to the consumers for free in a situation with copyright, whereas the for-profit platform charges a positive price for the tracks. Artists have two revenue sources, concerts and tracks. These two businesses are related in a very particular way: if the artist is unknown, consumers attend to his/her concerts only if they have listened his/her tracks before; this externality does not work for the famous artist, because his/her style is already known. Furthermore, higher attendance to concerts does not imply a higher sale of tracks. In other words, the externality works only in one direction[6].

With this framework, is very simple to see the trade-off for an artist who is not famous in a situation with copyright. He/she can either set his/her tracks in the for-profit platform or in the open platform. If he/she uses the for-profit platform, some consumers would not know about his/her existence unless the for-profit platform sells the tracks at a low price. If he/she uses the open platform, all the consumers will know about him/her, but he/she will not earn anything from the sale of tracks. The negotiation process with copyright is modeled as a Nash bargaining game: first, the for-profit platform proposes a price. If the artist accepts, the two parties negotiate on the jointly generated surplus until reaching the Nash solution. If the artist rejects, he/she uses the open platform. If the artist is famous, he/she will always negotiate. If the artist is unknown, he/she will optimally use the open platform in some parametric regions.

When we introduce piracy, all the tracks would be available in the open platform independently of the desires of the artists. The immediate consequences are a decrease in the prices of the tracks and the achievement of the maximal degree of diffusion for all the artists. I consider two definitions for the jointly generated surplus with piracy. If the jointly generated surplus includes the same elements as in the situation without piracy (of course taking into account the decrease in prices), the total welfare does not decrease when moving from a situation with copyright. If the jointly generated surplus only includes the sale of tracks, total welfare can actually decrease with piracy.

With respect to the surpluses of the agents, piracy always implies a redistribution of welfare. I find that the for-profit platform and the famous artists are always damaged, whereas consumers and unknown artists may be better off.

The rest of the paper is organized as follows: Section 2 is devoted to a review of the literature. Section 3 explains the main features of the model, Section 4 provides the solution with copyrights and Section 5 explains the differences between the two specifications of the piracy. Section 6 analyzes how the equilibrium prices and individual surpluses change with the introduction of piracy. Section 7 studies the changes in total welfare. Section 8 examines the special case in which the quality of the tracks available in the two platforms is almost the same and Section 9 concludes.

[6] If the activities were complementary, more attendance to concerts would imply higher sales of tracks.

3.2 Related literature

Borrowing the definition from Belleflamme and Peitz (2010a)[7], I define piracy as the "unauthorized reproduction, use, or diffusion of a copyrighted work". Concretely, I present a model of end-consumer piracy. Contrary to the commercial (or for-profit) piracy, the open platform does not make any profits in my framework: it is simply a platform in which the consumers share their files; even more, this action cannot be legally punished[8].

Nevertheless, "unauthorized" does not mean "unprofitable", and several models have pointed out the positive effects of piracy in the final profits obtained by the artists and/or the music industry.
Peitz and Waelbroeck (2006b) present a model with consumer sampling. The authors consider a multi-product monopolist who offers N products equidistantly located in the Salop circle and consumers with a total mass of 1, that have an ideal variety and who buy at most one product. The consumers make two choices: downloading (yes/no) and buying (yes/no). If consumers download, they learn the exact location of the product that provides the best fit to their preferences; if they do not, they can only buy randomly. When downloading is allowed, there will be some consumers who would not buy the version for sale anymore (causing a decrease in the profits of the monopolist), but the consumers who buy after downloading have a higher willingness to pay because they buy exactly their ideal variety (then, the monopolist can charge a higher price and increase its profits). The authors show that the possibility of downloading yields higher profits if there is enough taste heterogeneity and enough product diversity. Although in my model piracy may be profitable too, our approaches have two main differences: first, I consider the artists and the platforms as different agents with different incentives and that is the reason behind the result that piracy may be profitable for the artists, but that it will always be damaging for the for-profit platform; and second, I consider two different artists instead of a multi-product monopolist and I allow the consumers to acquire the tracks and concerts of both artists that is why I can describe a demand-creation effect. Nevertheless, the sampling effect has a similar flavor to the externalities that I describe: in Peitz and Waelbroeck (2006b), consumers learn about their ideal variety after downloading; in my model, consumers learn about the existence of certain varieties they may want to consume after downloading.

Minniti and Vergari (2010) analyze how the presence of a private, small-scale file-sharing community affects the pricing behavior and profitability of producers of digital goods. The authors assume two producers *à la Hotelling* and three possible types of consumers: consumers who buy one good, consumers who buy one good and download the other, and consumers who do not use any of the goods. The file-sharing community considered has a very particular feature: to gain access, the consumers have to buy and share a digital good with other members (that is why

[7] This is an excellent survey that the reader can check to get a general picture of the current literature about end-consumer piracy.

[8] The reader can think for instance of u-Torrent.

we cannot observe consumers who only download). By comparing fully covered and partially covered markets, the model states that firms benefit from piracy in emerging markets since it allows to reach a higher share of customers who otherwise will not use any of the two goods. Nevertheless, this effect of demand creation is absent in mature markets and piracy is therefore detrimental for firms in this context. I find a similar conclusion in my model: piracy is always harmful for famous artists (or "mature") because it does not yield the demand-creation effect, whereas it does for unknown artists (or "emerging"). But in my framework, the file-sharing community is represented by an open platform: consumers do not need to buy anything in advance to use it. Instead, I consider the existence of externalities between the profits derived by two different activities: sold tracks and concerts. An additional difference is that Minniti and Vergari (2010) do not include the figure of the intermediaries, whereas I differentiate between the incentives of the artists ("producers") and the for-profit platform.

A relevant paper is one by Gayer and Shy (2006). Similar to my findings, the authors also point out the conflict of interests between the artists and their "publishers" (for-profit platform in my case): the two parties share the profits derived from selling the copyrighted tracks, but the artists also earn profits from other market activities, like the concerts, that are not always shared with the publishers. Gayer and Shy (2006) assume that the consumers benefit from network externalities both in recordings and in live performances, and that these two goods are complements. Because of these network externalities in the two markets, the authors find that piracy can be profitable at the same time for publishers and for artists, although this is not necessarily the case (depending on the parameters, it may be that the piracy is harmful for the artist and the publisher, or profitable for the artist whereas harmful for the publisher). Since my externality only works in one direction, in my model the existence of piracy is always detrimental for the for-profit platform, although it may be profitable for the artist under certain conditions. In addition, I endogenize the share in profits for the agents by entering a Nash bargaining process, instead of assuming it as given and independent of the existence of piracy. As expected, the share of profits derived from the bargaining process differs with and without piracy.

All the previously discussed papers follow a theoretical approach. The interested reader can consult Waldfogel (2012a) for a quantitative study about the quality of the new recorded music since Napster. The paper concludes that there is no evidence of a reduction in the quality of music released since Napster, and actually the study suggests an increase in quality since 1999.

3.3 The model

Consider the music industry. There are three groups of agents: artists (producers of music), consumers (consumers of music) and platforms (intermediaries).

There are two different platforms: one of them is an open platform and the other is a for-profit platform. The open platform supplies music to the consumers without charging any price or membership and, consequently, makes zero profits. A real example of this kind of platform is u-Torrent. The existence of intermediaries who earn no profits has been justified in the literature because of individual preferences, long-run cooperation, and networking. The for-profit platform supplies music at a positive price to all consumers who are willing to pay for it, and it does not discriminate in prices. A real example of this kind of platform is i-Tunes. Notice that the for-profit platform does not create any music; therefore, it has to sign a contract with at least one artist to have some tracks to supply. The optimal terms of these contracts will be specified later.

There are two artists. Each one can be famous or unknown, and production costs are normalized to zero for the two types[9]. The industry can be configured by two famous artists, one famous and one unknown, or two unknown artists. Each artist has two revenue sources: his corresponding share when his tracks are sold in the for-profit platform, and the attendance to his concerts. The artists are single-homing without piracy, that is, the artist who accepts the contract offered by the for-profit platform knows that his tracks will be supplied only by that platform in the same vein, he can use the open platform if he finds the offered contract inconvenient. With piracy, tracks will be available at the open platform even if the artists accept the contract offered by the for-profit platform.

Finally, there are two consumers. They are homogeneous in their valuation of concerts, w, but heterogeneous in their valuation of tracks: these assumptions are made to focus on the role of the diffusion in the music industry. consumers are multi-homing with and without piracy, that is, they can acquire the tracks wherever available.

I make the next assumption about consumers' valuation of tracks:

Assumption 6. Each consumer values the tracks of the two artists equally, but these valuations differ across consumers. Denoting artists by a and b and consumers by i and j, without loss of generality:

$$v_a^i = v_b^i = v > 0$$

$$v_a^j = v_b^j = \Omega v, \text{ with } \Omega \in (0, 1)$$

The, we can define utilities as follows:

[9]This assumption will be discussed in the Conclusions.

Definition 1. The utility of consumer i of purchasing a track in the for-profit platform is

$$u_P^i = v - p \tag{3.1}$$

and the utility when she uses the open platform instead is

$$u_O^i = \alpha v \tag{3.2}$$

with $\alpha \in (0,1)$. In the same way, the utility of consumer j of purchasing a track in the for-profit platform is

$$u_P^j = \Omega v - p \tag{3.3}$$

and the utility when she uses the open platform instead is

$$u_O^j = \alpha \Omega v \tag{3.4}$$

If the track is not available in the open platform and the consumer does not acquire it in the for-profit platform, her utility is normalized to zero

$$u_{NC}^i = u_{NC}^j = 0 \tag{3.5}$$

Notice that the price is not an element of the utility derived from using the open platform, because tracks are supplied for free. Nevertheless, the fact that α belongs to the interval $(0,1)$ captures the idea that the two platforms are not perfect substitutes in terms of gross utility. This parameter α can be justified by thinking that the qualities of the tracks are different in the two platforms, or that consumers waste some time surfing the internet until they find the correct file that they want to download.

This a standard way to model piracy in the literature. Nevertheless, other alternatives can be used as long as there exists a positive price such that consumers prefer to use the open platform for prices above that threshold instead of using the for-profit platform.

The previous assumption is made to have a finite set of prices that the for-profit platform can use in the contracts. When the for-profit platform has to select a high price or a low price when offering the contract to the artist, we can see intuitively the conflict of interests between the players: both consumers acquire the tracks in the for-profit platform with the low price, whereas only one of the two consumers acquires the tracks in the for-profit platform with the high price. The attendance to concerts is directly related to the number of people who know the artist. Therefore, an unknown artist prefers the platform to charge a low price when there is no piracy, but the profits of the platform can be maximized by charging a high price. The trade-off also exists with a continuum of consumers and prices, but is easier to understand with a discrete number of prices.

Now, I proceed to explain how the two businesses of the artists, concerts and tracks, are related. As it was mentioned above, I assume that all consumers have the same willingness to pay for a concert, $w > v$, and that this is in fact the price of a ticket. By doing so, I reduce the number of choices that the artist has to make: with this framework, his single decision is to accept or reject the contract offered by the for-profit platform. If I would add the decision of the optimal ticket price, the problem would be more complex and it would only serve as a distraction from the role of the diffusion.

It is reasonable to think that the probability of attendance to the concert of a famous artist does not depend on the consumers having listened his most recent tracks. The artist' style is already known by everyone, and someone who likes it would attend to the concert in any case (and she would not attend if she dislikes). Therefore, the number of sold tracks is not related to the revenues by concerts if the artist is famous, that is, diffusion means nothing to him.

On the contrary, diffusion is crucial for an unknown artist: if a consumer does not know about the existence of an artist, the probability of attending to a concert is zero. Nevertheless, the consumer will attend to the concert if she listen his tracks and likes them (and will not attend if she dislikes the tracks). Because of this, unknown artists have incentives to care about diffusion. The previous reasoning is formalized in this assumption:

Assumption 7. Let us denote the probability of attendance to a concert by q if the consumer has listened the tracks and by $\tilde{q}$ if she did not. Then:

If the artist $c = \{a, b\}$ is famous, the probability that the consumer $k = \{i, j\}$ attends to his concert does not depend on having listened his last tracks:

$$1 \geq q^k_{Fc} = \tilde{q}^k_{Fc} \geq 0$$

If the artist $k = \{i, j\}$ is unknown, the probability that the consumer $c = \{a, b\}$ attends to his concert is zero if she does not listen before his tracks. The probability of attendance after listening his tracks may be positive:

$$1 \geq q^k_{Uc} \geq \tilde{q}^k_{Uc} = 0$$

Furthermore, all the probabilities are common knowledge.

The previous assumption can be interpreted in terms of externalities. higher diffusion implies (weakly) higher revenues from concerts, but the effect does not work in the other direction, that is, higher attendance to a concert does not imply a higher number of sold tracks. The reason is that I consider that nobody is willing to pay the price of a ticket w without having confirmed if she likes the style of the artist. This claim is justified if we agree that the price of a ticket is much higher than the price of a track. Then, the acquisition of a track happens before attending to the concert, and not the other way around. Therefore, I say that the externality only goes in one direction from tracks to concerts.

3.4 The situation with copyright

In a situation with copyright, the artist is sure that his tracks will be supplied only by the platform he wants. If the artist signs a contract to put his tracks in the for-profit platform, two possible prices can be charged to consumers: a high price p_H that comes from equalizing equations (3.1) and (3.5), and a low price p_L that comes from equalizing equations (3.3) and (3.5). Concretely,

$$p_H = v \tag{3.6}$$

and

$$p_L = v\Omega. \tag{3.7}$$

Notice that, since copyright is not violated and the for-profit platform does not discriminate in prices, the high price extracts all the surplus of the consumer with the largest willingness to pay[10]; and the low price extracts all the surplus of the consumer with the lowest willingness to pay and leaves positive rents for the other consumer.
In the contracting process, the for-profit platform sets the price and the artist says "yes" or "no". If the answer is "yes", then the two parties (artist and for-profit platform) negotiate. The generated surplus is shared according to the Nash bargaining solution.

Observe that the surplus generated by the agreement differs depending on the artist type. If the artist is famous, the generated surplus is the number of sold tracks times the price: revenues from concerts cannot be included, since the consumers already know him. If the artist is unknown, the generated surplus is the number of sold tracks times the price plus the revenues from concerts: in this case, the for-profit platform has a relevant role in the number of consumers who learn about the existence of the unknown artist, and it is key for the size of his revenues from concerts. Of course, we need the revenue from concerts to be verifiable (otherwise, they cannot be included in the Nash function).
If agreement between the two parties does not happen, the for-profit platform earns zero, whereas the artist c earns $w(q_c^i + q_c^j)$ because he can use the open platform to set his tracks[11].
Let me denote by r the share of the generated surplus belonging to the artist. In maths, the Nash bargaining program when the artist is famous and the platform sets the high price is

$$\underset{r_H}{\text{maximize}} \quad [(1 - r_H)v - 0][(r_H)v - (wq_F^i + wq_F^j)]$$

The equilibrium share is

$$r^*_{H;F} = \frac{v + wq_F^i + wq_F^j}{2v} \tag{3.8}$$

[10] The consumer with the lowest willingness to pay does not buy the track at this price.
[11] Remember that tracks are sold at zero price at the open platform. Then, the artist gets the maximal degree of diffusion.

In the same way, if the platform sets the low price:

$$\underset{r_L}{\text{maximize}} \quad [(1-r_L)2v\Omega - 0][(r_L)2v\Omega - (wq_F^i + wq_F^j)]$$

The equilibrium share is

$$r^*_{L;F} = \frac{2v\Omega + wq_F^i + wq_F^j}{4v\Omega} \tag{3.9}$$

Notice that the two equilibrium shares are strictly positive. Further, they can be higher than one as well: in this case, we will be in a corner solution and the artist would receive all the generated surplus. Of course, a famous artist will always choose the for-profit platform to set his tracks since he always get a positive proportion of a surplus that is not generated if he chooses the open platform.

Proceeding analogously for the unknown artist, the Nash bargaining program when the for-profit platform sets the high price is

$$\underset{r_H}{\text{maximize}} \quad [(1-r_H)(v + wq_U^i) - 0][(r_H)(v + wq_U^i) - (wq_U^i + wq_U^j]$$

The equilibrium share is

$$r^*_{H;U} = \frac{v + 2wq_F^i + wq_F^j}{2v + 2wq_F^i} \tag{3.10}$$

In the same way, if the platform sets the low price:

$$\underset{r_L}{\text{maximize}} \quad [(1-r_L)(2v\Omega + wq_U^i + wq_U^j) - 0][(r_L)(2v\Omega + wq_U^i + wq_U^j) - (wq_U^i + wq_U^j)]$$

The equilibrium share is

$$r^*_{L;U} = \frac{v\Omega + wq_U^i + wq_U^j}{2v\Omega + wq_U^i + wq_U^j} \tag{3.11}$$

Observe that the two equilibrium shares are strictly positive. Furthermore, the equilibrium share when the for-profit platform sets the low price is strictly less than one. In this case, the unknown artist accepts the contract because it is always more profitable than choosing the open platform. The situation is slightly more complicated if the for-profit platform sets the high price: if $r^*_{H;U} \geq 1$, the artist gets all the generated profit, that is the price of one track and the revenue from concerts when one consumer attends. Nevertheless, whenever $r^*_{H;U} \geq 1$ it holds that the artist is actually better off by putting his tracks in the open platform and obtaining the maximal degree of diffusion with associated profits $wq_U^i + wq_U^j$. Concretely, $r^*_{H;U} \geq 1$ if and only if $q_U^j \geq v/w$.

Then, the unknown artist says "no" to the negotiation in this case. On the contrary, if $q_U^j < v/w$, the unknown artist optimally says "yes" to the negotiation with the for-profit platform.

To conclude the section, let me remark that the equilibrium shares from the Nash bargaining have an intuitive interpretation. Each part receives half of the excess generated surplus (surplus generated by the agreement minus the sum of the two outside options) plus its outside option, everything normalized by the size of the generated surplus. For instance, the equilibrium share when the artist is famous and the for-profit platform sets the high price for the tracks is

$$r^*_{H;F} = \frac{1}{v}\left(\frac{v - wq_F^i - wq_F^j}{2} + wq_F^i + wq_F^j\right) = \frac{v + wq_F^i + wq_F^j}{2v}$$

3.5 The situation with piracy: two specifications

In a situation with piracy, the artist knows that his tracks will be supplied by the open platform even if he accepts the contract offered by the for-profit platform. With piracy, a dominant strategy for any artist is to sign the contract offered by the for-profit platform: he is going to obtain the maximal degree of diffusion anyways, and accepting the agreement with the for-profit platform provides an extra profit derived from the sale of tracks.
When the artist is unknown, we can model piracy in two different ways. In the first way, the revenues from concerts are still included in the Nash bargaining program: despite maximal degree of diffusion is guaranteed, some consumers know about the existence of the artist thanks to the for-profit platform. In the second way, the revenues derived from concerts are not included in the Nash bargaining program: the generation of these revenues do not depend on the agreement between the artist and the for-profit platform, because the tracks are supplied by the open platform as well.
Again, two prices can be charged for acquiring the tracks in the for-profit platform: high and low. But, when copyright is violated, these prices do not extract all the surplus of the marginal consumer. Concretely, the high price $\hat{p}_H$ comes from equalizing the equations (3.1) and (3.2); and the low price $\hat{p}_L$ comes from equalizing the equations (3.3) and (3.4):

$$\hat{p}_H = v(1 - \alpha) \tag{3.12}$$

and

$$\hat{p}_L = v\Omega(1 - \alpha). \tag{3.13}$$

Of course, the previous distinction makes no sense if the artist is famous: revenues from concerts are never part of the Nash bargaining program. Then, the Nash bargaining program if the for-profit platform sets the high price $\hat{p}_H$ is:

$$\underset{r_H}{\text{maximize}} \quad [(1 - r_H)(v(1-\alpha)) - 0][(r_H)(v(1-\alpha)) - (wq_F^i + wq_F^j)]$$

The equilibrium share is

$$\hat{r}_{H;F} = \frac{v(1-\alpha) + wq_F^i + wq_F^j}{2v(1-\alpha)} \tag{3.14}$$

Similarly, the Nash bargaining program if the for-profit platform sets the low price $\hat{p}_L$ is

$$\underset{r_L}{\text{maximize}} \quad [(1 - r_L)(2v\Omega(1-\alpha)) - 0][(r_L)(2v\Omega(1-\alpha)) - (wq_F^i + wq_F^j)]$$

The equilibrium share is

$$\hat{r}_{L;F} = \frac{2v\Omega(1-\alpha) + wq_F^i + wq_F^j}{4v\Omega(1-\alpha)} \tag{3.15}$$

Notice that $r^*_{H;F} < \hat{r}_{H;F}$ and that $r^*_{L;F} < \hat{r}_{L;F}$: the surplus generated by the agreement is lower with piracy because of the decrease in the prices, but the outside options do not change. Then, the famous artist is relatively "stronger" and his equilibrium share is higher.

3.5.1 Specification 1

As it was commented previously, the revenues from concerts are included in the Nash bargaining program in this specification. Then, when the artist is unknown and the for-profit platform sets the high price $\hat{p}_H$:

$$\underset{r_H}{\text{maximize}} \quad [(1 - r_H)(v(1-\alpha) + wq_U^i) - 0][(r_H)(v(1-\alpha) + wq_U^i) - (wq_U^i + wq_U^j]$$

The equilibrium share is

$$\hat{r}^*_{H;U} = \frac{v(1-\alpha) + 2wq_F^i + wq_F^j}{2v(1-\alpha) + 2wq_F^i} \tag{3.16}$$

Analogously, when the artist is unknown and the for-profit platform sets the low price $\hat{p}_L$:

$$\underset{r_L}{\text{maximize}} \quad [(1 - r_L)(2v\Omega(1-\alpha) + wq_U^i + wq_U^j) - 0][(r_L)(2v\Omega(1-\alpha) + wq_U^i + wq_U^j) - (wq_U^i + wq_U^j)]$$

The equilibrium share is

$$\hat{r}^*_{L;U} = \frac{v\Omega(1-\alpha) + wq^i_U + wq^j_U}{2v\Omega(1-\alpha) + wq^i_U + wq^j_U} \tag{3.17}$$

It is worth observing that $r^*_{H;U} < \hat{r}^*_{H;U}$ and that $r^*_{L;U} < \hat{r}^*_{L;U}$. The same logic as before applies: since the outside options do not change and the total generated surplus is lower with piracy due to the price decrease, the unknown artist is relatively "stronger" and his equilibrium share is higher.

3.5.2 Specification 2

We proceed in the same way, but keeping in mind that the revenues from concerts are not included in the Nash bargaining programs.
when the artist is unknown and the for-profit platform sets the high price $\hat{p}_H$:

$$\underset{r_H}{\text{maximize}} \quad [(1-r_H)(v(1-\alpha)) - 0][(r_H)(v(1-\alpha)) - (wq^i_U + wq^j_U]$$

The equilibrium share is

$$\hat{r}^{**}_{H;U} = \frac{v(1-\alpha) + wq^i_F + wq^j_F}{2v(1-\alpha)} \tag{3.18}$$

Analogously, when the artist is unknown and the for-profit platform sets the low price $\hat{p}_L$:

$$\underset{r_L}{\text{maximize}} \quad [(1-r_L)(2v\Omega(1-\alpha)) - 0][(r_L)(2v\Omega(1-\alpha)) - (wq^i_U + wq^j_U)]$$

The equilibrium share is

$$\hat{r}^{**}_{L;U} = \frac{2v\Omega(1-\alpha) + wq^i_U + wq^j_U}{4v\Omega(1-\alpha) + wq^i_U + wq^j_U} \tag{3.19}$$

Again, it holds that $r^*_{H;U} < \hat{r}^{**}_{H;U}$ and that $r^*_{L;U} < \hat{r}^{**}_{L;U}$ because the surplus generated by the agreement is lower with piracy and the outside options do not change.

Furthermore, the same idea works across the two specifications: $\hat{r}^*_{H;U} < \hat{r}^{**}_{H;U}$ and $\hat{r}^*_{L;U} < \hat{r}^{**}_{L;U}$ because the surplus generated by the agreement is higher in the first specification and the outside options do not change; thus, the unknown artist is relatively "stronger" in the second specification and it is reflected in a higher equilibrium share.

3.6 Configurations of the music industry

In this section, I calculate the players' surpluses and state some results about who wins and who loses if we move from a situation with copyright to a situation with piracy.

3.6.1 Two famous artists

Without loss of generality, I denote by Fa to the famous artist a and by Fb to the famous artist b.
In a situation with copyright, the surpluses of the agents when the for-profit platform sets p_H are

$$\begin{aligned} CS^*_{H;FaFb} &= 0 \\ \pi^*_{H;FaFb} &= (1 - r^*_{H;Fa})v + (1 - r^*_{H;Fb})v \\ \pi^*_{H;Fa} &= r^*_{H;Fa}v + wq^i_{Fa} + wq^j_{Fa} \\ \pi^*_{H;Fb} &= r^*_{H;Fb}v + wq^i_{Fb} + wq^j_{Fb} \end{aligned} \tag{3.20}$$

The corresponding surpluses when the for-profit platform sets p_L are

$$\begin{aligned} CS^*_{L;FaFb} &= 2v(1 - \Omega) \\ \pi^*_{L;FaFb} &= (1 - r^*_{L;Fa})2v\Omega + (1 - r^*_{L;Fb})2v\Omega \\ \pi^*_{L;Fa} &= r^*_{L;Fa}2v\Omega + wq^i_{Fa} + wq^j_{Fa} \\ \pi^*_{L;Fb} &= r^*_{L;Fb}2v\Omega + wq^i_{Fb} + wq^j_{Fb} \end{aligned} \tag{3.21}$$

In a situation with piracy, the surpluses of the agents when the for-profit platform sets $\hat{p}_H$ are

$$\begin{aligned} \hat{CS}_{H;FaFb} &= 2v\alpha(1 + \Omega) \\ \hat{\pi}_{H;FaFb} &= (1 - \hat{r}_{H;Fa})v(1 - \alpha) + (1 - \hat{r}_{H;Fb})v(1 - \alpha) \\ \hat{\pi}_{H;Fa} &= \hat{r}_{H;Fa}v(1 - \alpha) + wq^i_{Fa} + wq^j_{Fa} \\ \hat{\pi}_{H;Fb} &= \hat{r}_{H;Fb}v(1 - \alpha) + wq^i_{Fb} + wq^j_{Fb} \end{aligned} \tag{3.22}$$

The corresponding surpluses when the for-profit platform sets $\hat{p}_L$ are

$$\begin{aligned}
\hat{CS}_{L;FaFb} &= 2v(1-\Omega) + 4v\Omega\alpha \\
\hat{\pi}_{L;FaFb} &= (1-\hat{r}_{L;Fa})2v\Omega(1-\alpha) + (1-\hat{r}_{L;Fb})2v\Omega(1-\alpha) \\
\hat{\pi}_{L;Fa} &= \hat{r}_{L;Fa}2v\Omega(1-\alpha) + wq^i_{Fa} + wq^j_{Fa} \\
\hat{\pi}_{L;Fb} &= \hat{r}_{L;Fb}2v\Omega(1-\alpha) + wq^i_{Fb} + wq^j_{Fb}
\end{aligned} \tag{3.23}$$

The next proposition states who wins and who loses if we move from copyright to piracy:

Proposition 13. If the industry is configured by two famous artists and we move from a situation with copyright to a situation with piracy, the consumers are better off whereas the for-profit platform and the two artists are worse off.

The intuition of the result is as follows: since the criteria to set a high or a low price is the same with and without piracy, the only effect of the piracy is to decrease the prices and therefore the size of the surplus generated by the agreement. Then, the consumers' utility increases whereas the sellers' profits decrease.

3.6.2 A famous artist and an unknown artist

Without loss of generality, I consider that the artist a is famous and that the artist b is unknown. In a situation with copyright, the famous artist always signs the contract independently of the set price. The unknown artist always signs if the set price is low, but only accepts the contract with the high price if $q^j_U < v/w$.
I will denote by the subindex U that the unknown artist negotiates with the for-profit platform, and by the subindex $\bar{U}$ that he does not negotiate.
Then, the total surpluses of the agents when the for-profit platform sets p_H and $q^j_U < v/w$ are:

$$\begin{aligned}
CS^*_{H;FU} &= 0 \\
\pi^*_{H;FU} &= (1-r^*_{H;F})v + (1-r^*_{H;U})(v + wq^i_U) \\
\pi^*_{H;F} &= r^*_{H;F}v + wq^i_F + wq^j_F \\
\pi^*_{H;U} &= r^*_{H;U}(v + wq^i_U)
\end{aligned} \tag{3.24}$$

The total surpluses of the agents when the for-profit platform sets p_H and $q_U^j \geq v/w$ are:

$$\begin{aligned} CS^*_{H;F\bar{U}} &= v\alpha(1+\Omega) \\ \pi^*_{H;F\bar{U}} &= (1-r^*_{H;F})v \\ \pi^*_{H;F} &= r^*_{H;F}v + wq_F^i + wq_F^j \\ \pi^*_{H;\bar{U}} &= wq_U^i + wq_U^j \end{aligned} \tag{3.25}$$

And the total surpluses of the agents when the for-profit platform sets p_L are

$$\begin{aligned} CS^*_{L;FU} &= 2v(1-\Omega) \\ \pi^*_{L;FU} &= (1-r^*_{L;F})2v\Omega + (1-r^*_{L;U})(2v\Omega + wq_U^i + wq_U^j) \\ \pi^*_{L;F} &= r^*_{L;F}2v\Omega + wq_F^i + wq_F^j \\ \pi^*_{L;U} &= r^*_{L;U}(2v\Omega + wq_U^i + wq_U^j) \end{aligned} \tag{3.26}$$

With piracy, both the famous artist and the unknown artist accept the contract offered by the for-profit platform independently of the price.
Consider the specification 1. The total surpluses of the agents when the for-profit platform sets $\hat{p}_H$ are

$$\begin{aligned} \hat{CS}^*_{H;FU} &= 2v\alpha(1+\Omega) \\ \hat{\pi}^*_{H;FU} &= (1-\hat{r}_{H;F})(v(1-\alpha)) + (1-\hat{r}^*_{H;U})(v(1-\alpha) + wq_U^i) \\ \hat{\pi}^*_{H;F} &= \hat{r}_{H;F}v(1-\alpha) + wq_F^i + wq_F^j \\ \hat{\pi}^*_{H;U} &= \hat{r}^*_{H;U}(v(1-\alpha) + wq_U^i) + wq_U^j \end{aligned} \tag{3.27}$$

The corresponding surpluses when the for-profit platform sets $\hat{p}_L$ are

$$\begin{aligned} \hat{CS}^*_{L;FU} &= 2v(1-\Omega) + 4v\Omega\alpha \\ \hat{\pi}^*_{L;FU} &= (1-\hat{r}_{L;F})2v\Omega(1-\alpha) + (1-\hat{r}^*_{L;U})(2v\Omega(1-\alpha) + wq_U^i + wq_U^j) \\ \hat{\pi}^*_{L;F} &= \hat{r}_{L;F}2v\Omega(1-\alpha) + wq_F^i + wq_F^j \\ \hat{\pi}^*_{L;U} &= \hat{r}^*_{L;U}(2v\Omega(1-\alpha) + wq_U^i + wq_U^j) \end{aligned} \tag{3.28}$$

Consider the specification 2. The total surpluses of the agents when the for-profit platform sets $\hat{p}_H$ are

$$\begin{aligned} \hat{CS}^{**}_{H;FU} &= 2v\alpha(1+\Omega) \\ \hat{\pi}^{**}_{H;FU} &= (1-\hat{r}_{H;F})(v(1-\alpha)) + (1-\hat{r}^{**}_{H;U})(v(1-\alpha)) \\ \hat{\pi}^{**}_{H;F} &= \hat{r}_{H;F}v(1-\alpha) + wq^i_F + wq^j_F \\ \hat{\pi}^{**}_{H;U} &= \hat{r}^{**}_{H;U}v(1-\alpha) + wq^i_U + wq^j_U \end{aligned} \tag{3.29}$$

And the total surpluses of the agents when the for-profit platform sets $\hat{p}_L$ are

$$\begin{aligned} \hat{CS}^{**}_{L;FU} &= 2v(1-\Omega) + 4v\Omega\alpha \\ \hat{\pi}^{**}_{L;FU} &= (1-\hat{r}_{L;F})2v\Omega(1-\alpha) + (1-\hat{r}^{**}_{L;U})(2v\Omega(1-\alpha)) \\ \hat{\pi}^{**}_{L;F} &= \hat{r}_{L;F}2v\Omega(1-\alpha) + wq^i_F + wq^j_F \\ \hat{\pi}^{**}_{L;U} &= \hat{r}^{**}_{L;U}2v\Omega(1-\alpha) + wq^i_U + wq^j_U \end{aligned} \tag{3.30}$$

The next proposition states who wins and who loses when moving from a situation with copyright to a situation with piracy.

Proposition 14. Consider an industry configured by a famous artist and an unknown artist. If piracy is defined according to specification 1,

- Only consumers are better off with piracy if $\Omega \geq 1/2$
- If $\Omega < 1/2$ and the for-profit platform sets p_H with copyright, when we enter piracy consumers are better off, the for-profit platform and the famous artist are worse off, and the unknown artist may be better off.
- If $\Omega < 1/2$ and the for-profit platform sets p_L with copyright, when we enter piracy only consumers are better off.

If piracy is defined according to specification 2,

- If $\Omega \geq 1/2$, when we enter piracy consumers are better off, the for-profit platform and the famous artist are worse off, and the unknown artist may be better off.
- If $\Omega < 1/2$ and the for-profit platform sets p_H with copyright, when we enter piracy consumers are better off, the for-profit platform and the famous artist are worse off, and the unknown artist may be better off.
- If $\Omega < 1/2$ and the for-profit platform sets p_L with copyright, when we enter piracy it is possible to find examples in which only the unknown artist is better off.

Although the formal proof is quite complicated, let me provide the main intuition of the results. It is pretty obvious that the for-profit platform and the famous artist are worse off when piracy is allowed[12]. Since the surpluses derived from agreements are lower and the equilibrium shares for the artists are higher, the profits of the for-profit platform always decrease. The higher equilibrium share corresponding to the famous artist is not enough to compensate the smaller size of the surplus generated by the agreement[13].

The intuitions for to the consumers and the unknown artist are not so evident.

It can be proved that, with specification 1, if price p_H (p_L) is set with copyright, price $\hat{p}_H$ ($\hat{p}_L$) will be set with piracy. Nevertheless, with specification 2 it can happen that the for-profit platform sets price p_L with copyright and price $\hat{p}_H$ with piracy. The logic behind this is that with specification 1, the surpluses included in the Nash programs have the same elements with and without piracy: the sale of tracks for the famous artist, and the sale of tracks and the corresponding concerts for the unknown artist. Nevertheless, this is not true for the specification 2.

If $\Omega \geq 1/2$, the for-profit platform sets the low price with and without piracy, and the two artists sign the contract. Both consumers acquire the two songs with and without piracy, but with piracy they pay a lower price and their utility increases. If we consider the specification 1 for the piracy, the unknown artist shares all the surplus with the for-profit platform: the higher equilibrium share corresponding to the artist is not enough to compensate the smaller size of the generated surplus. Nevertheless, with the specification 2, the artist does not share the revenues derived from concerts: depending on the attendance probabilities of the consumers, it may be better off or worse off.

If $\Omega < 1/2$ and the for-profit platform sets p_L without piracy, only consumers are better off. The reason is the same explained in the previous paragraph. Nevertheless if the for-profit platform sets p_H, in addition to consumers, the unknown artist may be better off: the reason is that, with piracy, consumer j learns about his existence in the open platform and can go to his concert, contrary to the case without piracy. To be better off, this positive earning has to compensate the decrease in the profits due to the smaller size of the generated surplus.

To illustrate the last point, let me consider a numerical example. Parameter values are as follows: $v = 1/2$, $w = 1$, $\alpha = 1/4$, $\Omega = 3/8$, $q_a^i = 1/64$, $q_a^j = 1/4$, $q_b^i = 1/128$ and $q_b^i = 17/64$. The subindex a refers to the famous artist and the subindex b refers to the unknown artist. The equilibrium shares without piracy are $r^*_{H,a} = 49/64$ and $r^*_{H,b} = 10/13$ when the platform sets the high price p_H, and $r^*_{L,a} = 41/48$ and $r^*_{L,b} = 59/83$ when it sets the low price p_L. Then, profits of the platform are 15/64 if the high price is charged and 31/128 if the low price is charged instead. Since 31/128¿15/64, the platform sets the low price in equilibrium without piracy. The consumer surplus is 5/8 and the profit of the unknown artist is 59/128.

Since $\Omega < 1/2$, the for-profit platform sets the high price $\hat{p}_H$ with piracy (specification 2). The profit of the for-profit platform is 27/256, the consumer surplus is 11/32 and the profit of the unknown artist is 153/256.

Notice that the for-profit platform sets the price to maximize its profits in each situation (with

[12] The famous artist is never better off with piracy, but for some parameter values his profits keep unchanged.

[13] Remember that the earnings derived from concerts are never included in the Nash bargaining program, so they are not crucial when comparing the profits of the famous artist with and without piracy.

or without piracy), and it does not take into account the welfare of the other agents. It is clear that the unknown artist is worse off because, without piracy, he had to share all the generated surplus with the for-profit platform, whereas with piracy he only shares the revenues from the sale of tracks. The decrease in the consumer surplus may look more surprising, but actually it is not: with the low price with copyright, the consumer with the highest willingness to pay enjoys positive rents and the other consumer do not buy any song. With piracy and the high price, both of them have a utility equal to the utility derived from listening the tracks in the open platform. Of course, the utility of consumer j is higher with piracy, but the utility of consumer i is lower and this negative effect dominates the overall when comparing with the situation without piracy.

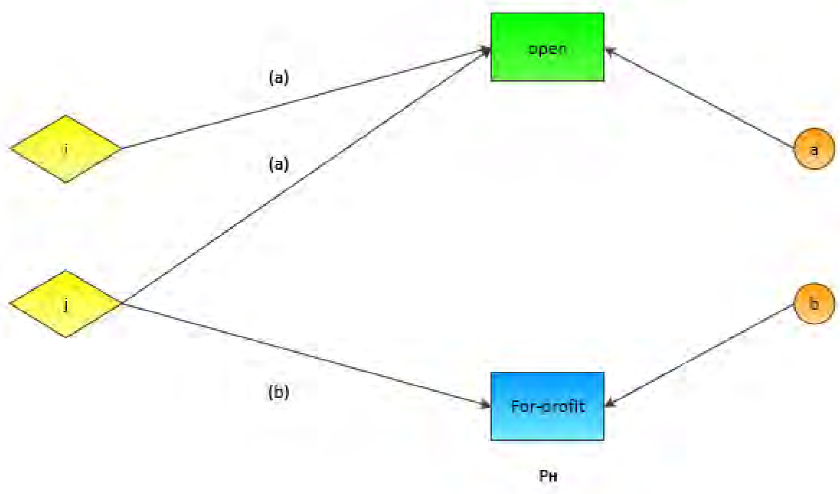

Figure 3.1: Equilibrium with copyright

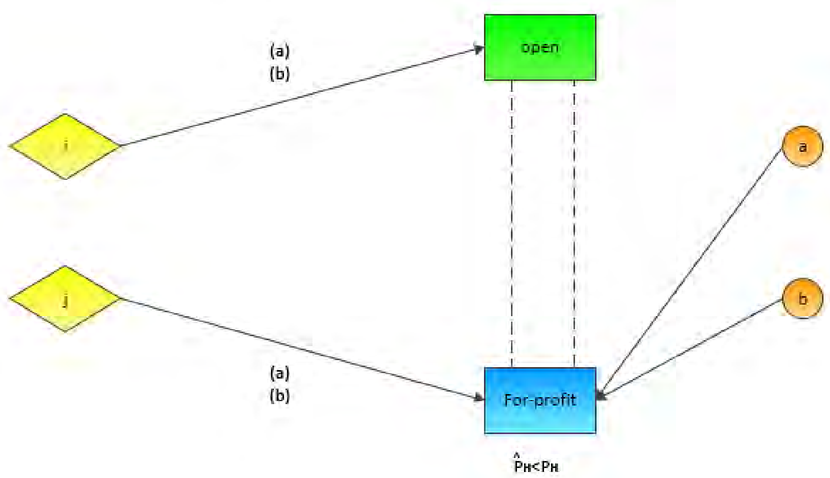

Figure 3.2: Equilibrium with piracy

3.6.3 Two unknown artists

With copyright, the unknown artist c negotiates with the for-profit platform when it sets the high price p_H if $q_c^j < v/w$, and always negotiates when the for-profit platform sets the low price p_L.
I will denote by the subindex Uc that the unknown artist c accepts the contract offered by the for-profit platform, and by the subindex $\bar{U}c$ that he does not negotiate.
The total surpluses of the agents when the for-profit platform sets p_H if $q_{Ua}^j < v/w$ and $q_{Ub}^j < v/w$ are

$$\begin{aligned} CS^*_{H;UaUb} &= 0 \\ \pi^*_{H;UaUb} &= (1 - r^*_{H;Ua})(v + wq^i_{Ua}) + (1 - r^*_{H;Ub})(v + wq^i_{Ub}) \\ \pi^*_{H;Ua} &= r^*_{H;Ua}(v + wq^i_{Ua}) \\ \pi^*_{H;Ub} &= r^*_{H;Ub}(v + wq^i_{Ub}) \end{aligned} \tag{3.31}$$

The surpluses when the for-profit platform sets p_H if $q_{Ua}^j < v/w$ and $q_{Ub}^j > v/w$ are

$$\begin{aligned} CS^*_{H;Ua\bar{U}b} &= \alpha v(1 + \Omega) \\ \pi^*_{H;Ua\bar{U}b} &= (1 - r^*_{H;Ua})(v + wq^i_{Ua}) \\ \pi^*_{H;Ua} &= r^*_{H;Ua}(v + wq^i_{Ua}) \\ \pi^*_{H;\bar{U}b} &= w(q^i_{Ub} + q^j_{Ub}) \end{aligned} \tag{3.32}$$

The surpluses when the for-profit platform sets p_H if $q_{Ua}^j > v/w$ and $q_{Ub}^j < v/w$ are

$$\begin{aligned} CS^*_{H;\bar{U}aUb} &= \alpha v(1 + \Omega) \\ \pi^*_{H;\bar{U}aUb} &= (1 - r^*_{H;Ub})(v + wq^i_{Ub}) \\ \pi^*_{H;\bar{U}a} &= w(q^i_{Ua} + q^j_{Ua}) \\ \pi^*_{H;Ub} &= r^*_{H;Ub}(v + wq^i_{Ub}) \end{aligned} \tag{3.33}$$

Notice that, if $q_{Ua}^j > v/w$ and $q_{Ub}^j > v/w$, the firm optimally sets p_L (otherwise, it makes zero profits).

The surpluses when the for-profit sets p_L are

$$\begin{aligned} CS^*_{L;UaUb} &= 2v(1-\Omega) \\ \pi^*_{L;UaUb} &= (1-r^*_{L;Ua})(2v\Omega + wq^i_{Ua} + wq^j_{Ua}) + (1-r^*_{L;Ub})(2v\Omega + wq^i_{Ub} + wq^j_{Ub}) \\ \pi^*_{L;Ua} &= r^*_{L;Ua}(2v\Omega + wq^i_{Ua} + wq^j_{Ua}) \\ \pi^*_{L;Ub} &= r^*_{L;Ub}(2v\Omega + wq^i_{Ub} + wq^j_{Ub}) \end{aligned} \tag{3.34}$$

With piracy, both artists accept the contract offered by the for-profit platform independently of the price.
Consider the specification 1. The total surpluses of the agents when the for-profit platform sets $\hat{p}_H$ are

$$\begin{aligned} \hat{CS}^*_{H;UaUb} &= 2v\alpha(1+\Omega) \\ \hat{\pi}^*_{H;UaUb} &= (1-\hat{r}^*_{H;Ua})(v(1-\alpha) + wq^i_{Ua}) + (1-\hat{r}^*_{H;Ub})(v(1-\alpha) + wq^i_{Ub}) \\ \hat{\pi}^*_{H;Ua} &= \hat{r}^*_{H;Ua}(v(1-\alpha) + wq^i_{Ua}) + wq^j_{Ua} \\ \hat{\pi}^*_{H;Ub} &= \hat{r}^*_{H;Ub}(v(1-\alpha) + wq^i_{Ub}) + wq^j_{Ub} \end{aligned} \tag{3.35}$$

The corresponding surpluses when the for-profit platform sets $\hat{p}_L$ are

$$\begin{aligned} \hat{CS}^*_{L;UaUb} &= 2v(1-\Omega) + 4v\Omega\alpha \\ \hat{\pi}^*_{L;UaUb} &= (1-\hat{r}^*_{L;Ua})(2v\Omega(1-\alpha) + wq^i_{Ua} + wq^j_{Ua}) + (1-\hat{r}^*_{L;Ub})(2v\Omega(1-\alpha) + wq^i_{Ub} + wq^j_{Ub}) \\ \hat{\pi}^*_{L;Ua} &= \hat{r}^*_{L;Ua}(2v\Omega(1-\alpha) + wq^i_{Ua} + wq^j_{Ua}) \\ \hat{\pi}^*_{L;Ub} &= \hat{r}^*_{L;Ub}(2v\Omega(1-\alpha) + wq^i_{Ub} + wq^j_{Ub}) \end{aligned} \tag{3.36}$$

Consider the specification 2. The surpluses of the agents when the for-profit platform sets $\hat{p}_H$ are

$$\begin{aligned} \hat{CS}^{**}_{H;UaUb} &= 2v\alpha(1+\Omega) \\ \hat{\pi}^{**}_{H;UaUb} &= (1-\hat{r}^{**}_{H;Ua})(v(1-\alpha)) + (1-\hat{r}^{**}_{H;Ub})(v(1-\alpha)) \\ \hat{\pi}^{**}_{H;Ua} &= \hat{r}^{**}_{H;Ua}(v(1-\alpha)) + wq^i_{Ua} + wq^j_{Ua} \\ \hat{\pi}^{**}_{H;Ub} &= \hat{r}^{**}_{H;Ub}(v(1-\alpha)) + wq^i_{Ub} + wq^j_{Ub} \end{aligned} \tag{3.37}$$

The next proposition states who wins and who loses when moving from a situation with copyright to a situation with piracy.

Proposition 15. Consider an industry configured by two unknown artists.
If piracy is defined according to specification 1,

- Only consumers are better off with piracy if $\Omega \geq 1/2$
- If $\Omega < 1/2$ and the for-profit platform sets p_H with copyright, when we enter piracy consumers are better off, the for-profit platform is worse off, and the artists may be better off.
- If $\Omega < 1/2$ and the for-profit platform sets p_L with copyright, when we enter piracy only consumers are better off.

If piracy is defined according to specification 2,

- If $\Omega \geq 1/2$, when we enter piracy consumers are better off, the for-profit platform is worse off, and the artists may be better off.
- If $\Omega < 1/2$ and the for-profit platform sets p_H with copyright, when we enter piracy consumers are better off, the for-profit platform is worse off, and the artists may be better off.
- If $\Omega < 1/2$ and the for-profit platform sets p_L with copyright, when we enter piracy it is possible to find examples in which only the artists are better off.

The logic behind those results is the same one explained in the previous subsection. To illustrate the last point, consider the following values: $v = 1/2$, $w = 1$, $\Omega = 11/32$, $\alpha = 1/4$, $q^i_{Ua} = 1/512$, $q^j_{Ua} = 65/256$, $q^i_{Ub} = 1/8$ and $q^j_{Ub} = 1/16$. The profits of the platform are 175/512 if it sets the high price without piracy, and 11/32 if it sets the low price. Clearly, the for-profit platform sets the low price with copyright. The consumer surplus in this case is 21/32, the artist a earns 219/512 and the artist b earns 23/64.
Since $\Omega < 1/2$, the for-profit platform sets the high price $\hat{p}_H$ with piracy (specification 2). Its profits in this case are 157/1024. Consumer surplus is 43/128, the artist a earns 585/1024 and the artist b earns 15/32.

3.7 Welfare Analysis

To continue with the previous logic, it is useful to state the conclusions related to the total welfare according to the configuration of the music industry.

3.7.1 Two famous artists

As it was pointed out in the subsection 6.1, it makes no sense to differentiate between the two specifications of the piracy if the two artists are famous: the revenue from concerts is independent of the price fixed by the for-profit platform, so it cannot be part of the Nash bargaining program either with piracy or without it. Furthermore, a famous artist always signs the contract offered by the for-profit platform, given the equilibrium shares.
To calculate the total surpluses, we just need to add up the agents' surpluses found out in subsection 6.1. Then, the total welfare[14] with copyright when the for-profit platform optimally sets the high price is

$$TS^{*}_{H;FF} = 2v + w(q^i_a + q^j_a) + w(q^i_b + q^j_b) \tag{3.38}$$

and the total welfare with copyright when the for-profit platform optimally sets the low price is

$$TS^{*}_{L;FF} = 2v(1+\Omega) + w(q^i_a + q^j_a) + w(q^i_b + q^j_b). \tag{3.39}$$

The total welfare with piracy when the for-profit platform optimally sets the high price is

$$\hat{TS}^{**}_{\hat{H};FF} = \hat{TS}^{*}_{\hat{H};FF} = 2v(1+\alpha\Omega) + w(q^i_a + q^j_a) + w(q^i_b + q^j_b) \tag{3.40}$$

and the total welfare with piracy when the for-profit platform optimally sets the low price is

$$\hat{TS}^{**}_{\hat{L};FF} = \hat{TS}^{*}_{\hat{L};FF} = 2v(1+\Omega) + w(q^i_a + q^j_a) + w(q^i_b + q^j_b). \tag{3.41}$$

We know that in a situation with copyright, the for-profit platform sets the high price p_H in equilibrium if $\Omega < 1/2$, and the low price otherwise. Also that in a situation with piracy, the for-profit platform sets the high price $\hat{p_H}$ in equilibrium if $\Omega < 1/2$, and the low price otherwise. The next proposition establishes the changes on total welfare when we move from a situation with copyright to a situation of piracy in an industry configured by two famous artists.

Proposition 16. Consider an industry configured by two famous artists.

- If $\Omega < 1/2$, the total surplus is higher with piracy than with copyright. Concretely, the increase in the consumer surplus more than offsets the decrease in the profits of the for-profit platform and the two artists.
- If $\Omega \geq 1/2$, the total surplus is the same with and without piracy. In particular, the for-profit platform and the artists make a welfare transfer to the consumers.

It is straightforward to check that the expression of the total welfare when the for-profit platform sets the low price with copyright, p_L, is identical to the expression of the total welfare when the platform sets the low price with piracy, $\hat{p_L}$. But obviously, the distribution of this total welfare

[14] Notice that the total surplus does not depend on the value of the equilibrium shares, r. Then, it is not necessary to distinguish among interior and corner values.

is different in the two cases: with piracy, both artists and the for-profit platform are worse off whereas the consumers are better off.
On the contrary, the total welfare when the for-profit platform sets the high price with copyright, p_H, is lower than the total surplus when it sets the high price with piracy, $\hat{p_H}$. As before, both artists and the for-profit platform are worse off whereas the consumers are better off. The difference with the previous paragraph is that, with low price and copyright, the two consumers buy two tracks. Then, the decrease in prices yielded by piracy does not mean more people buying more products. However, with high price and copyright, only one consumer acquires the tracks whereas the other stays out of the market. With piracy and high price, the consumer who buys the tracks enjoys a higher surplus because of the decrease of the price, but the other consumer that was previously excluded now has access to the tracks through the open platform. Revenues derived from concerts are the same in every case and they are not relevant when making welfare comparisons.

3.7.2 A famous artist and an unknown artist

Contrary to the previous subsection, it is necessary to differentiate between the two specifications of piracy when the industry is configured by a famous artist and an unknown artist. The revenues from concerts of the famous artist would never be part of the Nash bargaining program, but the incentives of the for-profit platform to set one price or another depend on whether the profits from concerts of the unknown artist are part of the bargaining process.
Without loss of generality, I will say that the famous artist is a and the unknown artist is b. Remember that, with copyright, the famous artist will always sign the contract with the for-profit platform. The unknown artist will sign if the for-profit platform sets the low price p_L, or if the for-profit platform sets the high price p_H and the probability of consumer j to attend to his concert after listening his tracks is $q^j_{Ub} < v/w$. If $q^j_{Ub} \geq v/w$ and the for-profit platform sets the high price p_H, the unknown artist prefers to use the open platform.
By adding up the agents' surpluses, it is automatic to check that: The total welfare with copyright when the for-profit platform optimally sets the high price and $q^j_{Ub} < v/w$ is

$$TS^*_{H;F\bar{U}} = 2v + w(q^i_a + q^j_a) + w(q^i_b); \tag{3.42}$$

the total welfare with copyright when the for-profit platform optimally sets the high price and $q^j_{Ub} \geq v/w$ is

$$TS^*_{H;FU} = v(1 + \alpha + \alpha\Omega) + w(q^i_a + q^j_a) + w(q^i_b + q^j_b) \tag{3.43}$$

and the total welfare with copyright when the for-profit platform optimally sets the high price is

$$TS^*_{L;FU} = 2v(1 + \Omega) + w(q^i_a + q^j_a) + w(q^i_b + q^j_b). \tag{3.44}$$

Notice that consumer j cannot attend to the concert of the unknown artist in a situation with copyright when the for-profit platform sets the high price and the unknown artist signs the contract with it: since the track of the unknown artist is supplied by the for-profit platform at a high price, consumer j does not acquire it and does not attend to the concert simply because she does not know about the existence of the unknown artist.

Again, to sign the contract offered by the for-profit platform in a situation with piracy is always a dominant strategy for the two types of artists. Then, the total welfare with piracy when the for-profit platform optimally sets the high price is

$$\hat{TS}^{**}_{\hat{H};FU} = \hat{TS}^{*}_{\hat{H};FU} = 2v(1+\alpha\Omega) + w(q^i_a + q^j_a) + w(q^i_b + q^j_b) \tag{3.45}$$

and the total welfare with piracy when the for-profit platform optimally sets the low price is

$$\hat{TS}^{**}_{\hat{L};FU} = \hat{TS}^{*}_{\hat{L};FU} = 2v(1+\Omega) + w(q^i_a + q^j_a) + w(q^i_b + q^j_b). \tag{3.46}$$

An important detail to remark is that, although the total surpluses coincide for both specifications of piracy, the allocation of the total welfare across players is different among them. This is the key to understand the next proposition:

Proposition 17. Consider an industry configured by a famous artist and an unknown artist. Total welfare does not decrease when we move from a situation with copyright to a situation with piracy in which surpluses share according to specification 1. Nevertheless, this result does not hold in every case for specification 2.

Consider the same example analyzed in the Subsection 6.2. The total welfare with piracy is 209/128, and the total welfare with copyright is 245/128, clearly higher than 209/128. As it was showed in the Subsection 6.2, only the unknown artist is better off in the presence of piracy with specification 2, and this positive effect is not enough to compensate the lower welfare of the rest of the agents.

3.7.3 Two unknown artists

As in the previous subsection, we have to differentiate between the two specifications of the piracy.

By adding the agents' surpluses calculated in the Subsection 6.3, I can find the total welfare for each case. The total welfare with copyright if the for-profit platform optimally charges p_H when $q^j_{Ua} < v/w$ and $q^j_{Ub} < v/w$ is

$$TS^{*}_{H;UaUb} = 2v + wq^i_a + wq^i_b \tag{3.47}$$

The total welfare with copyright if the for-profit platform optimally charges p_H when $q^j_{Ua} < v/w$ and $q^j_{Ub} > v/w$ is

$$TS^{*}_{H;Ua\bar{U}b} = v(1+\alpha+\alpha\Omega) + wq^i_a + w(q^i_b + q^j_b) \tag{3.48}$$

The total welfare with copyright if the for-profit platform optimally charges p_H when $q^j_{Ua} > v/w$ and $q^j_{Ub} < v/w$ is

$$TS^*_{H;\bar{U}aUb} = v(1 + \alpha + \alpha\Omega) + w(q^i_a + q^j_a) + wq^i_b \tag{3.49}$$

The total welfare with copyright if the for-profit platform optimally charges p_L is

$$TS^*_{L;UaUb} = 2v(1 + \Omega) + w(q^i_a + q^j_a) + w(q^i_b + q^j_b) \tag{3.50}$$

Again, to accept the contract offered by the for-profit platform is a dominant strategy for both artists in the presence of piracy. Then, the total welfare with piracy when the for-profit platform optimally sets the high price is

$$\hat{TS}^{**}_{\hat{H};UaUb} = \hat{TS}^*_{\hat{H};UaUb} = 2v(1 + \alpha\Omega) + w(q^i_a + q^j_a) + w(q^i_b + q^j_b) \tag{3.51}$$

and the total welfare with piracy when the for-profit platform optimally sets the low price is

$$\hat{TS}^{**}_{\hat{L};UaUb} = \hat{TS}^*_{\hat{L};UaUb} = 2v(1 + \Omega) + w(q^i_a + q^j_a) + w(q^i_b + q^j_b). \tag{3.52}$$

As in 7.2, although the total surpluses coincide for both specifications of piracy, the allocation of the total welfare across players is different among them. This is the key to understand the next proposition:

Proposition 18. Consider an industry configured by a famous artist and an unknown artist. Total welfare does not decrease when we move from a situation with copyright to a situation with piracy in which surpluses share according to specification 1. Nevertheless, this result does not hold in every case for specification 2.

Go back to the example analyzed in the Subsection 6.3. We saw that, without piracy, the for-profit platform optimally sets p_L, whereas with the second specification of piracy it sets $\hat{p}_H$. The total welfare without piracy is 915/512, clearly higher than the total surplus with piracy, 783/512.

3.8 A serious threat: α goes to one

When α goes to one in the presence of piracy, the quality of the tracks available in the open platform is practically identical to the quality of the tracks supplied by the for-profit platform. This implies that the two prices that the for-profit platform can charge go to zero.

In addition, independently of the specification of piracy and independently of the type of the artist, the share of the profits belonging to the artist tends to infinity when α approaches to one. It means that the corner solution (artist's share greater or equal to one and, consequently,

profits of the platform equal to zero) can be reached for a value of α which is relatively far away from the limiting value of one, contrary to what is stated by the conventional wisdom.
These two things together imply that when α goes to one:

1. the for-profit platform makes exactly zero profits
2. the joint profit of the artists goes to $w(q_a^i + q_a^j) + w(q_b^i + q_b^j)$
3. the total consumer surplus tends to $2v(1 + \Omega)$

We can state the following with respect to the distribution of the welfare:

Corollary 1. When α tends to one, the total welfare is the same as the welfare obtained when the for-profit platform sets the low price. Nevertheless, the distribution of the welfare when α goes to one collapses to the distribution of welfare if the for-profit platform would leave the market: since only the open platform would work, transactions would happen at zero price. Because of the competition among platforms and the almost perfect quality, consumers get the maximal surplus and, consequently, artists obtain the minimal.

The intuition is very simple: when α goes to one, the tracks supplied by the open platform are almost perfect substitutes of the tracks offered by the for-profit platform. Then, the prices tend to zero and, consequently, the consumers' utility increases and the profits of the artists decrease.

3.9 Conclusions and Discussion

I have constructed a model to analyze the effect of the piracy in the music industry in a situation with initial copyrights. To do so, I consider two artists that can be heterogeneous in their degree of popularity (famous or unknown), two consumers that are heterogeneous in their willingness to pay for tracks but homogeneous in their valuations of concerts, and two platforms: a for-profit platform and an open platform.
Artists have two revenue sources: sale of tracks and concerts. If the artist is unknown, a consumer cannot attend to his concert without having listened to his tracks before. This is not true for a famous artist: his style is already known, and a consumer does not need to listen to his songs in advance to check if she likes him or not. Therefore, diffusion is crucial for the unknown artists.
With copyright, the artists know that their tracks will be supplied only by the platform they want. If the artists accept the price set by the for-profit platform, the two parties negotiate and share the surplus generated by the agreement according tho the Nash bargaining solution.

The for-profit platform sets a non-zero price for the tracks, so for the high price the unknown artist does not obtain the maximal degree of diffusion. The open platform supplies the tracks at zero price, so the unknown artist gets the maximal degree of diffusion but he does not receive anything from the sale of tracks. Because of this, the unknown artist can choose to set his tracks in the open platform in equilibrium. The famous artist always sets his tracks in the for-profit platform.

With piracy, the tracks of all the artists will be available in the open platform, independently of having accepted the contract of the for-profit platform. Then, piracy decreases the prices that the for-profit platform can charge for the tracks and allows all the artists to get the maximal degree of diffusion. With piracy, the dominant strategy for all the artists is to set their songs in the for-profit platform.

I consider two possible specifications for piracy. In the first specification, the revenues from concerts are included in the Nash bargaining between the unknown artist and the for-profit firm. In the second specification, parties only negotiate about how to share the revenues derived from the sale of tracks.

I find that with the first specification, the total surplus never decreases: it can remains the same or increase. But total welfare is always redistributed, such that consumers are always better off with piracy, the for-profit platform and the famous artist are worse off, and the unknown artist may be better off in some cases.

With the second specification, the total surplus can decrease. It happens when the for-profit platform sets a low price with copyright and a high price with piracy. In this case, it is possible to find examples in which even the consumers lose welfare with respect to the situation with copyright. Further, the for-profit platform and the famous artist are always damaged also with this second specification, whereas the unknown artist can earn higher profits.

As a policy recommendation, we can say that the specification 1 is more convenient from a social point of view.

Let me discuss some key assumptions maintained during the analysis.

The most controversial assumption is to have costs normalized to zero. Precisely, one of the arguments used against piracy is that creators would not be able to recover their initial investments and, consequently, they would prefer not to compose the tracks. With this model, I can determine the upper bound of the fixed costs such that the artist c finds profitable to make the investment. Concretely, if the fixed cost is lower than $w(q_c^i + q_c^j)$, the artist creates the song.

To gain clarity in the exposition and see more easily the trade-off faced by the unknown artist between revenues from tracks and diffusion, I made discrete the number of consumers. Their willingness to pay were chosen such that the demand function was downward sloping. Nevertheless, the qualitative results should not change if we consider a continuum of consumers instead.

An important assumption is the direction of the externality. Concretely, I state that consumers cannot attend to the concert of the unknown artist unless they have listened to his songs before. However, I do not consider the other direction: if someone goes to a concert and likes the music, she will buy the tracks of the artist. This approach is used in other papers in the literature and they find that, with this additional effect, the piracy can be profitable even for the for-profit platform. In Section 3 I discussed why my approach could seem more realistic. Furthermore, in the Introduction I have provided evidence of a passionate defense of the intellectual property from the labels and famous artists, but also of the incentives of the unknown artists to allow consumers to download their tracks from their websites. This suggests that, if it exists, the effect of acquiring the tracks after attending to a concert is relatively small compared to the direction of the externality that I considered in the analysis.

It is also worth noting that the model has the classical limitations of any static model. I had to assume the degree of popularity of the artists exogenously, and I did not take into account the entrance of new artists and consumers into the market. By including some dynamic considerations, it would be possible to determine if piracy actually increases the number of artists in the industry and if it is easier to become into a famous artist.

An implicit strong assumption was that consumers were aware of any new file available in the open platform immediately. Evidence shows that actually this behavior is not feasible, so an interesting extension would be to consider the role of the worth of mouth in the diffusion across a network of consumers.

Finally, the model could be enriched by considering two-sided markets. In two-sided markets, consumers' utility increases in the number of artists who set their tracks in the same platform. This could be justified because of the reduction in time looking for songs in different platforms. Then, they would be willing to pay a higher price for the tracks and the for-profit platform could use this extra money to convince the artists for being the only supplier of the tracks.

Appendix A

Appendix Chapter 1

Suppose a consumer that exhibits an exponential utility function[8]

$$u(q) = -e^{-\rho(q-p-x+z)}$$

where $q \gtrless 0$ means quality, $\rho > 0$ is the risk-aversion coefficient, $p \geq 0$ is the price, $x \geq 0$ is the location of the consumer and $z \gtrless 0$ is an element of vertical differentiation different from quality.
If q is a continuous random variable, the expected utility can be expressed as

$$E(u) = \int f(q)u(q)\mathrm{d}q$$

If the quality is normally distributed, the density function is

$$f(q) = \frac{1}{(2\pi\sigma^2)^{1/2}} e^{-\frac{(q-\mu)^2}{2\sigma^2}}$$

where μ is the expectation of the quality and σ is the variance.

Then, the expected utility is

[8] This is also known as CARA (Constant Absolute Risk Aversion) utility function

$$\begin{aligned}
E(u) &= \int \left(\frac{1}{(2\pi\sigma^2)^{1/2}} e^{-\frac{(q-\mu)^2}{2\sigma^2}} \right) \left(-e^{-\rho(q-p-x+z)} \right) \mathrm{d}q \\
&= \int \left(\frac{1}{(2\pi\sigma^2)^{1/2}} e^{-\frac{(q-\mu)^2}{2\sigma^2}} \right) \left(\left(-e^{-\rho q} \right) \left(-e^{\rho(p+x-z)} \right) \right) \mathrm{d}q \\
&= \left(-\frac{1}{(2\pi\sigma^2)^{1/2}} \int e^{\frac{-q^2-\mu^2+2\mu q-2\rho q\sigma^2}{2\sigma^2}} \mathrm{d}q \right) \left(-e^{\rho(p+x-z)} \int \frac{1}{(2\pi\sigma^2)^{1/2}} e^{-\frac{(q-\mu)^2}{2\sigma^2}} \mathrm{d}q \right) \\
&= \left(-\frac{1}{(2\pi\sigma^2)^{1/2}} \int e^{\frac{-(q+(\rho\sigma^2-\mu))^2+\rho^2\sigma^4-2\rho\sigma^2\mu}{2\sigma^2}} \mathrm{d}q \right) \left(-e^{\rho(p+x-z)} \right) \\
&= -e^{\frac{\rho^2\sigma^2}{2}-\rho\mu} \left(\frac{1}{(2\pi\sigma^2)^{1/2}} \int e^{\frac{-(q+(\rho\sigma^2-\mu))^2}{2\sigma^2}} \mathrm{d}q \right) \left(-e^{\rho(p+x-z)} \right) \\
&= -e^{\frac{\rho^2\sigma^2}{2}-\rho\mu+\rho(p+x-z)}
\end{aligned}$$

We can take a monotonic transformation of the expected utility to obtain

$$E(u) = \mu - \frac{\rho\sigma^2}{2} - p - x + z$$

Proof of Proposition 1:

Proof. The monopolist solves the below optimization problem:

$$\max_{p_2} \quad p_2 x_2^I$$

The optimal interior price under full information is

$$p_2^I = \frac{\bar{q}}{2}$$

Of course, prices must be greater or equal to zero: otherwise, the firm will be paying to the consumers for acquiring the good and its profits would be negative; consequently, the monopolist would prefer to be out of the market. Notice that the equilibrium price found in the previous equation is below zero for any non-positive quality. Therefore, all qualities $\bar{q} \leq 0$ charge a price equal to zero under full information. For any $\bar{q} \leq 0$, the utility of the consumer located at point x_i given the equilibrium price is

$$\bar{q} - 0 - x_i \leq 0 \Rightarrow x_2^I = 0$$

Consider now $\bar{q} > 0$. The equilibrium market share is

$$x_2^I = \frac{\bar{q}}{2}$$

Since the size of the market is 1, the market share would be interior $x_2^I \in (0,1)\ \forall \bar{q} \in (0,2)$. For the rest of positive qualities, the market share is the corner solution $x_2^I = 1$ and the maximizing price is

$$\bar{q} - p - 1 = 0 \Rightarrow p_2^I = \bar{q} - 1$$

□

Proof of Proposition 2:

Proof. The monopolist solves the below optimization problem:

$$\max_{p_2} \quad p_2 x_2^U$$

with x_2^U defined as in equation (1.14).

The optimal interior price when information cannot be disclosed is

$$p_2^U = \frac{1}{4}(2\mu - \rho\sigma^2)$$

and the interior market share is

$$x_2^U = \frac{1}{4}(2\mu - \rho\sigma^2)$$

Again, the market share is constrained to the size of the market and negative demands are not allowed. The case of no demand is $x_2^U = 0$

$$x_2^U \leq 0 \Leftrightarrow \rho \geq \frac{2\mu}{\sigma^2}$$

And the charged price is zero: the risk-aversion coefficient is so large, that the monopolist would have to pay to the consumers in order to convince them to buy the product.

The monopolist supplies to the entire market when $x_2^U = 1$

$$x_2^U \geq 1 \Leftrightarrow \mu > 2 \quad \text{and} \quad \rho \leq \frac{-4 + 2\mu}{\sigma^2}$$

And the charged price is

$$\mu - \frac{1}{2}(\rho\sigma^2) - p_2^U - 1 = 0 \Rightarrow p_2^U = \frac{1}{2}(-2 + 2\mu - \rho\sigma^2) \geq 1$$

In the rest of the cases, the solution is interior □

Proof of Proposition 3:

Proof. In a separating equilibrium, each type of sender optimally chooses a different message (in this context, each quality would charge a different price) so that the consumers can infer his type with no error. Then, I have to check if there are some types who have incentives to mimic the price set by other types.

I start checking if the full-information prices can be a separating equilibrium[7]. It is worth remarking that there is no restriction about the quality q_F that the quality q wants to mimic. Then, there may be five possibilities: a non-positive quality mimics the price of a quality between 0 and 2, a non-positive quality mimics the price of a quality larger than 2, a quality between 0 and 2 mimics the price of another quality between 0 and 2, a quality between 0 and 2 mimics the price of a quality larger than 2, and a quality larger than 2 mimics the price of another quality larger than 2.

All the programs are very similar: we just need to adjust the price mimicked according to Proposition 1.

Suppose that the quality $2 > q > 0$ wants to mimic the strategy of $q_F > 2$. Given $\hat{x}_1 \in (0,1)$, the separating equilibrium with the full-information prices is sustainable if

$$\frac{q^2}{4} \geq (q_F - 1)(\max\{0, \min\{q - (q_F - 1), \hat{x}_1\}\} + \max\{1 - \hat{x}_1, 0\})$$

The solution states that q does not want to mimic the price of q_F if the proportion of informed people is sufficiently large. Similar results are obtained from the other programs for any $q > 0$.

Suppose that the quality $q \leq 0$ wants to mimic the strategy of $q_F > 2$. Given $\hat{x}_1 \in (0,1)$, the separating equilibrium with the full-information prices is sustainable if

$$0 \geq (q_F - 1)(0 + \max\{1 - \hat{x}_1, 0\})$$

The solution states that any non-positive quality always have incentives to mimic the full-information price of a quality larger than 2 ($\forall \hat{x}_1 \in (0,1)$). Then, consumers would never be able to infer that a quality is non-positive.

Given that the full-information prices cannot be sustained as a separating equilibrium, we have to check if an equilibrium in which the high qualities distort their prices can be sustained. Concretely, I check if there exists a pair of prices (p, p_F), $p \geq 0$ charged by the non-positive qualities and $p_F \geq q_F - 1$ charged by the qualities larger than 2, such that:

$$0 > p_F(0 + \max\{q_F - p_F - \hat{x}_1, 0\})$$

and

$$p_F(\max\{0, q_F - p_F\}) > p(\max\{0, \min\{q_F - p, \hat{x}_1\}\} + \max\{q_F - p_F - \hat{x}_1, 0\})$$

[7] Notice that in the full information equilibrium, all the non-positive qualities charge a price equal to zero. However, consumers would not need more information, because their optimal strategy is not to buy any non-positive quality, it does not matter if it is -1 or -100.

The previous conditions state that the non-positive quality should not have incentives to mimic the distorted price, and that to apply such a distortion should not be excessively costly for the high quality.
However, it does not exist any pair of prices (p, p_F) for which the previous conditions hold at the same time. Then, a separating equilibrium does not exist. □

Proof of Lemma 1:

Proof. Take any subinterval $(a, b) \subseteq (0, 1)$. The probability of the event $\hat{x}_1 \in (a, b)$ is

$$\begin{aligned}
Pr(a < \hat{x}_1 < b) &= Pr\left(a < \frac{1}{2}(2\mu - \rho\sigma^2 - 2p_1 + 2z) < b\right) \\
&= Pr\left(\frac{2a + 2p_1 - 2\mu + \rho\sigma^2}{2} < z < \frac{2b + 2p_1 - 2\mu + \rho\sigma^2}{2}\right) \\
&= \frac{2b + 2p_1 - 2\mu + \rho\sigma^2 + 2Z}{4Z} - \frac{2a + 2p_1 - 2\mu + \rho\sigma^2 + 2Z}{4Z} \\
&= \frac{b-a}{2Z}
\end{aligned}$$

□

Proof of Proposition 4:

Proof. It comes from direct maximization with respect to p_1.
When Z is large enough, we can ensure that the probabilities of being in each one of the corners belong to the interval $(0, 1)$. Furthermore, when Z is large enough, the monopolist does not want to deviate. □

Proof of Proposition 5:

Proof. From Proposition 1, we know the prices for the case of full information. If the first-period equilibrium price p_1^* is less than 1 and greater than 0, the relevant constraint happens when $\bar{q} < 2$. Then, the pattern of prices is reversed if

$$p_1^* > p_2^I \Leftrightarrow p_1^* > \frac{\bar{q}}{2} \Leftrightarrow \bar{q} < 2p_1^*$$

If the first-period equilibrium price $p_1^* \geq 1$, the relevant constraint happens when $\bar{q} \geq 2$. Then, the pattern of prices is reversed if

$$p_1^* > p_2^I \Leftrightarrow p_1^* > \bar{q} - 1 \Leftrightarrow \bar{q} < p_1^* + 1$$

□

Proof of Proposition 6:

Proof. The first part of the Proposition comes from direct maximization.
The second part comes from the comparison between the previous result and the equilibrium price stated in Proposition 4: the first-period equilibrium price when the consumers are risk-neutral is simply the expression of the first-period price stated in Proposition 4 evaluated at $\rho = 0$. Since the first-period price depends negatively on ρ, the result is proved. □

Appendix B

Appendix Chapter 2

Suppose a consumer that exhibits an exponential utility function[8]

$$u(q) = -e^{-\rho(q-p-x-(\zeta-c))}$$

where $q \gtrless 0$ means quality, $\rho > 0$ is the risk-aversion coefficient, $p \geq 0$ is the price, $x \geq 0$ is the location of the consumer and $(\zeta - c) \gtrless 0$ is the difference between the favorite color of the consumers and the color offered by the brand.
If q is a continuous random variable, the expected utility can be expressed as

$$E(u) = \int f(q)u(q)\mathrm{d}q$$

If the quality is normally distributed, the density function is

$$f(q) = \frac{1}{(2\pi\sigma^2)^{1/2}} e^{-\frac{(q-\mu)^2}{2\sigma^2}}$$

where μ is the expectation of the quality and σ is the variance.

Then, the expected utility is

[8] This is also known as CARA (Constant Absolute Risk Aversion) utility function

$$\begin{aligned}
E(u) &= \int \left(\frac{1}{(2\pi\sigma^2)^{1/2}} e^{-\frac{(q-\mu)^2}{2\sigma^2}} \right) \left(-e^{-\rho(q-p-x-(\zeta-c))} \right) \mathrm{d}q \\
&= \int \left(\frac{1}{(2\pi\sigma^2)^{1/2}} e^{-\frac{(q-\mu)^2}{2\sigma^2}} \right) \left(\left(-e^{-\rho q}\right) \left(-e^{\rho(p+x+(\zeta-c))} \right) \right) \mathrm{d}q \\
&= \left(-\frac{1}{(2\pi\sigma^2)^{1/2}} \int e^{\frac{-q^2-\mu^2+2\mu q-2\rho q\sigma^2}{2\sigma^2}} \mathrm{d}q \right) \left(-e^{\rho(p+x+(\zeta-c))} \int \frac{1}{(2\pi\sigma^2)^{1/2}} e^{-\frac{(q-\mu)^2}{2\sigma^2}} \mathrm{d}q \right) \\
&= \left(-\frac{1}{(2\pi\sigma^2)^{1/2}} \int e^{\frac{-(q+(\rho\sigma^2-\mu))^2+\rho^2\sigma^4-2\rho\sigma^2\mu}{2\sigma^2}} \mathrm{d}q \right) \left(-e^{\rho(p+x+(\zeta-c))} \right) \\
&= -e^{\frac{\rho^2\sigma^2}{2}-\rho\mu} \left(\frac{1}{(2\pi\sigma^2)^{1/2}} \int e^{\frac{-(q+(\rho\sigma^2-\mu))^2}{2\sigma^2}} \mathrm{d}q \right) \left(-e^{\rho(p+x+(\zeta-c))} \right) \\
&= -e^{\frac{\rho^2\sigma^2}{2}-\rho\mu+\rho(p+x+(\zeta-c))}
\end{aligned}$$

We can take a monotonic transformation of the expected utility to obtain

$$E(u) = \mu - \frac{\rho\sigma^2}{2} - p - x - (\zeta - c)$$

Proof of Proposition 7

Proof. When $\bar{q}_A$ is known for all the agents, the optimization problems for firms A and B are, respectively:

$$\max_{p_{2A}} \quad p_{2A} x_2^I$$
$$\max_{p_{2B}} \quad p_{2B} \left(1 - x_2^I\right)$$

Finding the reaction functions and solving the system, the equilibrium prices are

$$p_{2A}^I = 1 + \frac{\bar{q}_A}{3}; \quad p_{2B}^I = 1 - \frac{\bar{q}_A}{3}$$

Plugging these prices into the equation (2.10) to find the equilibrium market share:

$$x_2^I = \frac{3 + \bar{q}_A}{6}$$

Since negative prices are not allowed, this interior solution is valid for $\bar{q}_A \in (-3, 3)$. The marginal consumer is also interior in this support.

When $\bar{q}_A \leq -3$, the firm A charges a zero price and the firm B supplies to the entire market. The price that it optimally charges come from the below equation:

$$\bar{q}_A = -p_{2B} - 1 \Rightarrow p_{2B}^I = -\bar{q}_A - 1$$

In a similar way, when $\bar{q}_A \geq 3$, the firm B charges a zero price and the firm A supplies to the entire market. The price that it optimally charges come from the below equation:

$$\bar{q}_A - p_{2A} - 1 = 0 \Rightarrow p_{2A}^I = \bar{q}_A - 1$$

□

Proof of Proposition 8:

Proof. When $\bar{q}_A$ cannot be disclosed, the optimization problems for firms A and B are, respectively:

$$\max_{p_{2A}} \quad p_{2A} x_2^P$$
$$\max_{p_{2B}} \quad p_{2B} \left(1 - x_2^P\right)$$

Finding the reaction functions and solving the system, the equilibrium prices are

$$p_{2A}^P = 1 + \frac{\mu}{3} - \frac{\rho\sigma^2}{6}; \quad p_{2B}^P = 1 - \frac{\mu}{3} + \frac{\rho\sigma^2}{6}$$

Plugging these prices into the equation (2.14) to find the equilibrium market share:

$$x_2^I = \frac{3 + \bar{q}_A}{6}$$

Since negative prices are not allowed, this interior solution is valid when $\mu \leq 3$ and $\rho < (6 + 2\mu)/\sigma^2$; or when $\mu > 3$ and $(-6 + 2\mu)/\sigma^2 < \rho < (6 + 2\mu)/\sigma^2$. The marginal consumer is also interior under the previous constraints.

When $\rho > (6 + 2\mu)/\sigma^2$, the firm A charges a zero price and the firm B supplies to the entire market. The price that it optimally charges come from the below equation:

$$\mu - \frac{1}{2}(\rho\sigma^2) = -p_{2B} - 1 \Rightarrow p_{2B}^P = \frac{1}{2}\left(-2 - 2\mu + \rho\sigma^2\right)$$

In a similar way, when $\mu > 3$ and $\rho \leq (-6 + 2\mu)/\sigma^2$, the firm B charges a zero price and the firm A supplies to the entire market. The price that it optimally charges come from the below equation:

$$\mu - \frac{1}{2}(\rho\sigma^2) - p_{2A} - 1 = 0 \Rightarrow p_{2A}^P = \frac{1}{2}\left(-2 + 2\mu - \rho\sigma^2\right)$$

□

Proof of Proposition 9:

Proof. To show that the separating equilibrium may not exist, is enough to provide a counterexample. Just consider the one discussed in the main text. □

Proof of Lemma 2:

Proof. Take any subinterval $(a, b) \subseteq (0, 1)$. The probability of the event $\hat{x}_1 \in (a, b)$ is

$$\begin{aligned} Pr(a < \hat{x}_1 < b) &= Pr\left(a < \frac{1}{4}\left(2 - 2p_{1A} + 2p_{1B} + 2\mu - \rho\sigma^2 + 2(\Delta c_1)\right) < b\right) \\ &= Pr\left(\frac{-2 + 4a + 2p_{1A} - 2p_{1B} - 2\mu + \rho\sigma^2}{2} < \Delta c_1 < \frac{-2 + 4b + 2p_{1A} - 2p_{1B} - 2\mu + \rho\sigma^2}{2}\right) \\ &= \frac{-2 + 4b + 2p_{1A} - 2p_{1B} - 2\mu + \rho\sigma^2 + 2C}{4C} - \frac{-2 + 4a + 2p_{1A} - 2p_{1B} - 2\mu + \rho\sigma^2 + 2C}{4C} \\ &= \frac{b - a}{C} \end{aligned}$$

□

Proof of Proposition 10:

Proof. The interior part is simply the result of the regular maximization, with the constraint meaning that both firms supply to a positive fraction of the market. The corner solutions state that, when one of the firms optimally supplies to the entire market, its optimal strategy is to charge a price such that the most distant consumer gains no surplus, given the price of the rival. As it was explained in the main text, in this situation the negative prices are weakly dominated for the rival, because its profits are fixed: it supplies to nobody in the first period in expected terms, and the probability of each one of the corners is 1/2. □

Proof of Proposition 11:

Proof. Remember that, for the two patterns to be reversed simultaneously, it is necessary $p^*_{1A} > 0$ and $p^*_{1B} > 0$. Start considering that $\bar{q}_A \in (-3, 3)$. Then,

$$p^I_{2A} < p^*_{1A} \Leftrightarrow \bar{q}_A < 3(p^*_{1A} - 1)$$

and

$$p^I_{2B} < p^*_{2A} \Leftrightarrow \bar{q}_A > 3(1 - p^*_{1B})$$

For these two things to happen simultaneously,

$$2 < p^*_{1A} + p^*_{1B} \Leftrightarrow 2 < \frac{-\Pi^I_{2A} + \Pi^P_{2A} + \Pi^I_{2B} - \Pi^P_{2B} + 2C}{C} \Leftrightarrow -\Pi^I_{2A} + \Pi^P_{2A} + \Pi^I_{2B} - \Pi^P_{2B} > 0$$

Several numerical simulations show that, in fact, $-\Pi^I_{2A} + \Pi^P_{2A} + \Pi^I_{2B} - \Pi^P_{2B} < 0$, so contradiction. Notice that, by a monotonicity argument, the reversion of the two pattern of prices cannot happen either when $\bar{q}_A \leq -3$ or when $\bar{q}_A \geq 3$, because the equilibrium prices of the second period are even more extreme. □

Proof of Proposition 12:

Proof. The first part of the proposition is simply the solution to the model with risk-neutral consumers. For the second part, consider the example provided in the main text. □

Appendix C

Appendix Chapter 3

Proof of Proposition 13

Proof. First, by comparison of profits, it is automatic to check that the for-profit platform sets the high price with copyright if $\Omega < 1/2$ and the low price with copyright if $\Omega \geq 1/2$. In the same way, it sets the high price with piracy if $\Omega < 1/2$ and the low price with piracy if $\Omega \geq 1/2$. Then, we have to compare the sets of equations (3.20) and (3.22) when $\Omega < 1/2$ and the sets of equations (3.21) and (3.23) when $\Omega \geq 1/2$. Simple algebra provides the result when equilibrium shares are plugged. $\square$

Proof of Proposition 14

Proof. The proof is based on the comparison between profits. Because of the complexity derived from checking all the possible parametric regions, including the corners, the proof was done with the help of the software *Mathematica*. Files are available upon request. $\square$

Proof of Proposition 15

Proof. The proof is based on the comparison between profits. Because of the complexity derived from checking all the possible parametric regions, including the corners, the proof was done with the help of the software *Mathematica*. Files are available upon request. $\square$

Proof of Proposition 16

Proof. The proof requires the expressions of surpluses found out in subsection 6.1.
Let me start with the case of $\Omega < 1/2$. The for-profit platform optimally sets the price p_H with copyright and $\hat{p}_H$ with piracy. Since $\alpha\Omega > 0$, we conclude that the total surplus with piracy is higher than the total surplus with copyright by comparing equations (3.38) and (3.40). From Proposition , we know that the profits of the artists and the for-profit platform decrease, whereas the consumer surplus increases. Therefore, we can conclude that the increase in the consumer surplus more than offsets the losses of the other agents.
Now consider $\Omega \geq 1/2$. The for-profit platform optimally sets the price p_L with copyright and $\hat{p}_L$ with piracy. We conclude that the total surplus with piracy is identical to the total surplus with copyright by comparing equations (3.39) and (3.41). From Proposition 13, we know that the profits of the artists and the for-profit platform decrease, whereas the consumer surplus increases. Therefore, we can conclude that there is simply a redistribution of welfare from the for-profit platform and the artists to the consumers. □

Proof of Proposition 17

Proof. The proof is based on the comparison between profits. Because of the complexity derived from checking all the possible parametric regions, including the corners, the proof was done with the help of the software *Mathematica*. Files are available upon request. □

Proof of Proposition 18

Proof. The proof is based on the comparison between profits. Because of the complexity derived from checking all the possible parametric regions, including the corners, the proof was done with the help of the software *Mathematica*. Files are available upon request. □

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