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INTERIM EFFICIENT ALLOCATION MECHANISMS **

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Abstract

In this paper, I characterize the set of *interim incentive efficient allocation mechanisms* for a broad class of problems with private information, which includes those associated with the provision of public goods (with or without exclusion) as well as the allocation of one or more units of a private good.

Keywords: Incomplete information, interim efficiency, Pooling of types.

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1 Introduction

This paper focuses on general allocation problems in which a group of individuals, each of them having incomplete information about the others' characteristics, has to decide the provision of a certain good to each member of the group. Provision is costly, and the group of individuals has to decide also how to distribute the costs of provision among the members of the group. Individuals' preferences are linear and are determined by a real number that represents the individual's valuation (in terms of a *numeraire*) of one unit of the good. Each individual's valuation is private information. Agents' beliefs about a particular agent's valuation are common knowledge and can be represented by a continuous distribution function. Different specifications of the set of provision alternatives available and their costs determine the specific nature of the problem. Thus, the framework admits pure public goods problems with or without exclusion, trading private goods, cost sharing problems, etc.

The main objective of the present paper is to characterize the set of feasible allocation mechanisms available in this framework, and to provide a welfare analysis of these mechanisms. The notion of feasibility relevant in this context corresponds to that of *incentive feasibility,* see Holmström and Myerson (1983), that takes into account both the technological (or classical) constraints arising because provision of the good is costly, and the incentive constraints arising due to the informational structure of the problem. That is, feasible mechanisms are *Bayesian incentive compatible* and *budget balanced* (or *budget balanced in expectations).* In order to undertake a welfare analysis of mechanisms, I characterize the set of *interim efficient mechanisms.* The term interim refers to that stage in the decision process at which the individuals know their private information but does not know the others' private information. A Bayesian incentive compatible, budget balanced mechanism is interim efficient if there does not exist another Bayesian incentive compatible, budget balanced mechanism that, in the interim stage, makes some agents better off without making other agents worse off. Notice that the notion of feasibility underlying the notion of interim efficiency takes into account both incentive compatibility and classical feasibility constraints.

The early literature (see e.g., d'Aspremont and Gerard-Varet (1979)) identified Bayesian Incentive Compatible, budget balanced mechanisms that are efficient *ex-post*; i.e., such that in every state of the individuals' information they provide an allocation that is Pareto optimal in that state. Due to the linearity of preferences, determining whether or not an allocation is Pareto optimal in a given state of the individuals' preferences depends only on the provision decision, and does not depend on the distribution of costs. Thus, whether a mechanism is ex-post efficient is independent of how it distributes the cost of provision among the individuals. Recently, Makowski and Mezzetti (1994) have characterized Bayesian incentive compatible, ex-post efficient mechanisms as *(balanced) Groves mechanisms in expectations.*

The fact that Groves mechanism in expectations select Pareto optimal allocations

in every state of the individuals' information does not imply that every Bayesian incentive compatible, budget balanced mechanism is Pareto dominated (in the interim stage) by some Groves mechanism in expectations. In fact, it can be shown that the class of Groves mechanisms in expectations form a proper subset of the set of interim incentive efficient mechanisms. The interest of characterizing the whole set of interim efficient mechanisms (and not just a particular class of mechanisms in this set) arises from both normative and positive considerations.

One the normative side, there might be situations in which the set of mechanisms available is restricted by voluntary participation constraints, imposing that a mechanism provides all agents with at least the same expected interim utility they obtain with a given allocation. In many problems (see e.g., Myerson and Satterthwaite (1983), Cramton, Gibbons and Klemperer (1985), or Dearden (1997)), Groves mechanism in expectations fail to satisfy voluntary participation constraints. Therefore, exploring other mechanisms satisfying certain social objectives is interesting. Note that, if a group of individuals unanimously agree on altering a *statu quo* and use a new mechanism instead, selecting an interim efficient mechanism exhausts the possibilities for further improvements.

On the positive side, one might be interested in characterizing the whole set of interim efficient mechanism to explore the properties of mechanisms that arise in practice. If all decisions (including whether to use a particular mechanism) are taken at the interim stage, then the set of interim efficient mechanisms consists on those incentive compatible mechanisms for which there is no other mechanism that generates unanimous improvement. Thus, we would expect that if an agreement on a particular mechanism is reached, then that mechanism should be interim efficient.

Holmstrom and Myerson (1982) have characterized the set of interim incentive efficient mechanisms in the context of general Bayesian collective choice problems. According to this characterization, a mechanism is interim incentive efficient if it maximizes a weighted average of the agents' interim utilities (subject to Bayesian incentive compatibility and budget balance constraints), where this average is evaluated with respect to a probability (or welfare) distribution defined on the set of types. These results have been particularized to bilateral trading problems by Myerson, and to pure public good problems by Ledyard and Palfrey (1999).

These two articles characterize the provision rules[1] solving a welfare optimization problem in the class mentioned above, and show that such provision rules maximize a linear functional among all provision rules satisfying a qualifying constraint. This constraint is needed to guarantee that the expected probability of provision corresponding to an arbitrary agent is a non-decreasing function of the agent's types, a requirement that the provision rule of any Bayesian incentive compatible mechanism must satisfy.

According to this characterization, optimal provision rules define implicitly a type

[1]The provision rule of a mechanism (see section 3) is the rule selecting a provision alternative as a function of the agents' reported types.

dependent number for each individual, called *virtual valuation* (see Myerson, 1981), that can be regarded as the social value of providing the good to that individual. Virtual valuations depend also on the welfare distribution with respect to which the average interim utility of the mechanism is evaluated. If, for a given welfare distribution, virtual valuations are increasing functions of the agents' types, then the qualifying constraint will never be binding for the optimal provision rule associated to the welfare distribution and a simple closed form characterization of the set of optimal provision rules can be provided. Nothing is said, however, on how to find optimal provision rules associated to welfare distributions that yield virtual valuations that are decreasing functions of the agent's types.

In the present paper I extend these results to other problems involving provision of public or private goods, and provide a procedure to obtain optimal provision rules when virtual valuations are not monotone in type. Such procedure modifies virtual valuations in order to obtain non decreasing functions. The set of optimal provision rules is obtained then by maximizing the sum of these modified virtual valuations associated to a provision decision. As a result, optimal provision rules exhibit *pooling of types,* i.e., there exists an region in the set of possible types of one individual such that the expected probability of provision is constant for all types within such region.

In some specific contexts, pooling types might be deemed desirable. For example, pooling types might yield interesting properties in contexts in which acquiring and processing information is costly, such as those studied in Green and Laffont (1979) or Bobo (1998). These authors propose that these costs could be reduced by using a (random) sampling mechanism. However, a (non random) sampling mechanism in which people participating in the sample are self-selected (through a participation fee, for example) might also reduce these costs. Under these mechanisms, all types who decide not to participate in the sample are pooled by the mechanism. Such decision, however, may provide the designer with some relevant information. In the context of private goods, examples of mechanisms exhibiting pooling of types are double second price auctions, studied by Yoon (1996,1997).

A second objective of the paper is to analyze how the characterizations provided vary under different notions of incentive compatibility and budget balance imposed on the set of mechanisms available. I show that for every mechanism m that maximizes the average welfare (with respect to a given welfare distribution), among those Bayesian incentive compatible, budget balanced mechanisms, one can construct a dominant strategy incentive compatible, budget balanced (in expectations) mechanism m' with the same provision rule as that of m that provides every type of agent with the same interim utility he obtains with m. Thus, the characterization of optimal provision rules is not altered by different requirements on the set of mechanism available. This result suggests that it might be possible to obtain interim incentive mechanisms that are *robust* to prior information assumptions, following the lines suggested by Mookherjee and Reichelstein (1992) and Makowski and Mezzetti (1994). Although several definitions have been provided, a robust mechanism can be defined

as a mechanism whose properties are invariant to changes in the distribution determining the agents' valuations.

The paper is organized as follows:

First (Section 2), I describe the framework and show how different specifications of the set of provision alternatives and their costs determine the specific nature of the problem. I also describe mechanisms and discuss alternative notions of incentive compatibility (Bayesian and dominant strategy incentive compatibility) and budget balance (*ex-ante* and *ex-post*) of a mechanism in this framework.

In Section 3, I characterize the set of Bayesian incentive compatible and (ex-ante) budget balanced mechanisms. I also explore how different notions of incentive compatibility and feasibility of a mechanism restrict the set of mechanisms available in this general framework. To be more precise, I show that for every Bayesian incentive compatible, ex-ante budget balanced mechanism such that its provision rule is a non decreasing function of the agents' types, one can construct a dominant strategy incentive compatible, ex-ante budget balance mechanism without altering the agents' interim utilities.

In Section 4, I characterize the set of interim efficient mechanisms as solutions to a constrained welfare optimization problem. I characterize the provision rules solving a welfare optimization problem in the class mentioned above. Contrary to the approach followed by Myerson (1985) or Ledyard and Palfrey, no restrictions are placed on the welfare distribution with respect to which the average welfare of a mechanism is evaluated. Therefore computing the provision rules that maximize the average welfare may require a procedure that modifies virtual valuations in order to obtain non decreasing functions. Finally, I use explore the properties of interim efficient mechanisms in the context of specific problems within the general framework. It is shown that some particular mechanisms, such as sampling mechanisms (in the context of a public good problem) and double second price auctions (in the context of multilateral trading problems) can be justified as interim efficient.

2 The Problem

The class of allocation problems studied in this paper are those in which a group of individuals has to decide to provide with a consumption good to each member of the group, as well as how to distribute costs of provision among the members of the group. An allocation problem in this class is represented by a set of individuals $I = \{1, 2, 3, ..., N\}$, a compact, convex set of provision alternatives $A \subseteq [0,1]^N$, a continuous cost function $C : A \to \Re$ determining the costs of provision alternatives in terms of a *numeraire,* and a vector $V = \{V_i : i \in I\}$ of continuous and independent random variables generating the individuals' *valuations* for one unit of the good. For each $i \in I$, V_i is distributed according to a distribution $F_i : \Re \to [0,1]$ with positive density f_i on a support $S_i \subseteq \Re_i$. Also, it is assumed that each random variable V_i is

absolutely integrable, that is,

$$E(|V_i|) = \int_{S_i} |x_i| \, dF_i(x_i) < \infty.$$

Given an allocation problem $[I, A, C, V]$, an allocation is a pair

$$(a, t) \equiv (a_1, a_2, ..., a_N, t_1, t_2, ..., t_N) \in A \times \Re^N,$$

where, for each $(a, t) \in A \times \Re^N$ and each $i \in I$,

- a_i determines the amount of the consumption good that is provided to agent i;
- t_i is the tax, measured in terms of the *numeraire*, paid by agent i to finance the costs associated to the provision decision.

An allocation $(a, t) \in A \times \Re^N$ is feasible if

$$\sum_{i \in I} t_i \geq C(a).$$

Each individual $i \in I$ is assumed to have preferences representable by utility functions $u_i : A \times \Re^N \to \Re$ defined, for all $(a; t) \in A \times \Re^N$, by

$$u_i(a, t) = v_i a_i - t_i,$$

where v_i is a realization of V_i and represents the individual's *valuation* for one unit of the good.[2] For each $i \in I$, individual i's valuation is private information. For each $i \in I$, $v_i \in S_i$, the number v_i will be also referred to as *type* v_i of individual i. Let $S = \prod_{i=1}^N S_i$ and let $F : S \to [0, 1]$ denote the joint distribution of valuations. Since it is assumed that agents' valuations are independent one has, for all $v \in S$, $F(v) = \prod_{i=1}^N F_i(v_i)$.

This general framework allows one to study a wide variety of problems, including:

(i) Provision of public goods

These problems, studied by Green and Laffont (1979), d'Aspremont and Gerard-Varet (1979), Rob (1989), Mailath and Postlewaite (1989), Ledyard and Palfrey (1999), and many others, are allocation problem in which the good, if produced, is provided to all agents. Thus, the set of alternatives available is given by

$$A = \left\{ a \in [0, 1]^N : a_i = a_j \ \forall i, j \in I \right\}.$$

[2]Although it has been assumed that the set A of provision alternatives is convex, the framework can be adapted to the case in which the set of alternatives is a discrete set $D \subset \{0, 1\}^N$, and the cost function is arbitrary, provided decisions are interpreted in a stochastic fashion. In this case, a provision alternative a determines, for each $i \in I$, the expected provision to individual i. For a discussion on this, see the Appendix 2.1.

Also, it is commonly assumed that the cost function satisfies, for each $a \in A$,

$$C(a) = ca_1,$$

for some $c \geq 0$.

In some problems, as those studied by Dearden (1997) it is possible to exclude the individuals from the use of the public good. These problems are represented by a set of alternatives $A = [0,1]^N$ and a cost function satisfying, for each $a \in A$,

$$C(a) = c \max \{a_i : i \in I\},$$

where $c \geq 0$ represents the costs of production of one unit the public good.

(ii) Trading private goods

This class of problems consists in allocating $M \leq N$ units of a divisible[3] private good among N agents. The set of alternatives is therefore given by

$$A \equiv \left\{ a \in [0,1]^N : \sum_{i=1}^{N} a_i \leq M \right\},$$

and the cost function C satisfies, for each $a \in A$,

$$C(a) = 0.$$

Examples of problems in this class are the allocation of a single object (if $M = 1$), as in Myerson (1981) or Cramton, Gibbons and Klemperer (1985); bilateral trading problems (if $M = 1$ and $N = 2$) as in Myerson (1981) or Myerson and Satterthwaite (1983); and double auctions (if $M > 2$), as in Wilson (1985) or Yoon (1997).

(iii) Other cost sharing problems

The framework can be used to study situations in which (1) the cost depends only on the number of people for which the good is provided, and (2) providing the good for more people is costly. The set of alternatives is $A = [0,1]^N$ and the cost function is described by a function of the form

$$C(a) = f(\sum_{i=1}^{N} a_i),$$

where $f : \Re_+ \to \Re$ satisfies $f(0) = 0$ and $f'(0) > 0$. Examples include highway tolls, user fees for public parks, television, etc. Other cost functions might also arise in situations in which costs of provision depend on the specific identity of the individuals for which the good is provided. For example, the cost of providing a TV signal to different regions might be different.

[3]If the good is divisible, then for each $i \in I$, a_i can be interpreted as the probability that individual i gets one unit of the good.

2.1 Mechanisms

In order to select an allocation, a mechanism is devised. Here I study direct revelation mechanisms.[4] A mechanism in this class m is a pair of functions

$$m = (\rho, \tau) : S \to A \times \Re^N.$$

For each profile of reported valuations $\widehat{v} \in S$, $\rho(\widehat{v})$ determines the provision alternative selected by the mechanism, and $\tau(\widehat{v}) = (\tau_1(\widehat{v}), \tau_2(\widehat{v}), ..., \tau_N(\widehat{v}))$ gives the tax paid by each individual. The functions $\rho(\cdot)$ and $\tau(\cdot)$ in the definition of a mechanism m will be called, respectively, the *provision rule* and the *tax function* associated to m.

In order to define formally the relevant properties of mechanisms, some additional notation is now introduced. For each $v \in S$ and each $i \in I$, the vector specifying the valuations of all agents except i will be denoted by v_{-i}. Thus, any vector $v \in S$ can be equivalently written as $v = (v_i, v_{-i})$, where i represents an arbitrary individual. Also, for each $i \in I$, let V_{-i} denote the random vector determining the valuations of all agents except i. Finally, for any function $Y : S \to \Re$, let $E[Y(V)]$ denote the expectation of the random variable $Y(V)$. In particular, for any $i \in I$ and any function $Y_{-i} : S_{-i} \to \Re$, the expectation of the random variable $Y_{-i}(V_{-i})$ will be denoted by $E[Y_{-i}(V_{-i})]$.

Since agents' valuations are private information, mechanisms must induce agents to report their true valuations. In the literature, two alternative notions of incentive compatibility have been used. A notion of incentive compatibility, called *Dominant Strategy Incentive Compatibility* ($DSIC$), requires that reporting the true valuation be a dominant strategy for each individual in the revelation game induced by a mechanism m. A mechanism $m \equiv (\rho, \tau)$ satisfies $DSIC$ if for all $i \in I, v_i, \widehat{v}_i \in S_i, v_{-i} \in S_{-i}$ one has

$$v_i\rho_i(v_i, v_{-i}) - \tau_i(v_i, v_{-i}) \geq v_i\rho_i(\widehat{v}_i, v_{-i}) - \tau_i(\widehat{v}_i, v_{-i}). \qquad (DSIC)$$

A weaker notion of incentive compatibility, called *Bayesian Incentive Compatibility* (BIC), requires that reporting the true valuation be a Bayesian Equilibrium of the revelation game induced by m. A mechanism m satisfies BIC if for all $i \in I, v_i, \widehat{v}_i \in S_i$ one has

$$E\{v_i\rho_i(v_i, V_{-i}) - \tau_i(v_i, v_{-i})\} \geq E\{v_i\rho_i(\widehat{v}_i, V_{-i}) - \tau_i(\widehat{v}_i, V_{-i})\}. \qquad (BIC)$$

Note that nothing in the definition of a mechanism guarantees that a mechanism is feasible, in the sense that taxes paid by all agents be sufficient to cover the costs of provision. There are two possible feasibility conditions. The first condition, called *ex post feasibility,* is relevant if the group of agents involved in the allocation problem does not have access to outside resources, and therefore taxes must cover costs of

[4]Restriction to direct revelation mechanisms can be justified by the Revelation Principle (see e.g. Myerson, 1981).

provision in every state of nature. More precisely, a mechanism m is feasible *ex post* if for all $\widehat{v} \in S$ one has

$$\sum_{i \in I} \tau_i(\widehat{v}) \geq C(\rho(\widehat{v})). \tag{EXPF}$$

A second condition, called *ex ante feasibility*, is relevant if agents have access to an outside insurance market. A mechanism m is feasible *ex ante* if

$$E\left(\sum_{i \in I} \tau_i(V) - C(\rho(V))\right) \geq 0. \tag{EXAF}$$

In addition to feasibility, it might be required that a mechanism be *non-wasteful,* in the sense that taxes paid by agents do not exceed costs of provision. A mechanism that is feasible and non-wasteful will be referred to as budget balanced. A mechanism m satisfies *ex-post budget balance* if for all $\widehat{v} \in S$ one has

$$\sum_{i \in I} \tau_i(\widehat{v}) = C(\rho(\widehat{v})). \tag{EXPBB}$$

A mechanism m satisfies *ex ante budget balance* if

$$E\left(\sum_{i \in I} \tau_i(V) - C(\rho(V))\right) = 0. \tag{EXABB}$$

It should be noticed that condition $EXABB$ might be meaningless when applied to mechanisms for which BIC is not satisfied, since there is no reason to presume that the expression in the left hand of $EXABB$ corresponds to the actual expected taxes of such mechanisms. Notice also that a risk neutral external agent that acts as a broker for the agents using a mechanism satisfying BIC and $EXABB$, subsidizing agents when a deficit arises and collecting any surplus, would break even in expectations.

3 Incentive compatible, budget balanced mechanisms

This section discusses how different notions of incentive compatibility and feasibility restrict the set of mechanisms available in a given problem $[I, A, C, F]$. First, the set of mechanisms satisfying BIC is characterized. It is then shown that for any mechanism $m = (\rho, \tau)$ satisfying BIC and $EXABB$, one can construct a mechanism $m' = (\rho, \tau')$ satisfying BIC and $EXPBB$ that provides each agent with the same interim expected utility he obtains with m. Then, it is shown that if the provision rule of a mechanism $m = (\rho, \tau)$ satisfying BIC is a non-decreasing function of the agents' reported valuations, then one can construct a mechanism $m'' = (\rho, \tau'')$ satisfying $DSIC$ and $EXABB$ that provides each agent with the same interim expected utility he obtains with m.

Bayesian incentive compatible mechanisms

Given a provision rule $\rho : S \to A$, for each $i \in I$, $\widehat{v}_i \in S_i$, let $\overline{a}_i(\widehat{v}_i; \rho)$ denote the expected level of provision obtained by i when he reports $\widehat{v}_i$ and all other individuals report their true valuations, that is

$$\overline{a}_i(\widehat{v}_i; \rho) = E\left[\rho_i(\widehat{v}_i, V_{-i})\right].$$

Also, given a tax function $\tau : S \to \Re^N$, let $\overline{t}_i(\widehat{v}_i; \tau)$ denote the expected tax paid by i when he reports $\widehat{v}_i$ and all other individuals report their true valuations, that is

$$\overline{t}_i(\widehat{v}_i; \tau) = E\left[t_i(\widehat{v}_i, V_{-i})\right].$$

Finally, write $\overline{U}_i(v_i; m)$ for the interim expected utility obtained by an arbitrary agent i whose valuation is v_i when he reports his true valuation and all other agents report also their true valuations, that is

$$\overline{U}_i(v_i; m) = v_i \overline{a}_i(v_i; \rho) - \overline{t}_i(v_i; \tau).$$

Proposition 1 shows that Bayesian incentive compatibility imposes certain conditions on the interim expected utility corresponding to each type of agent participating in the mechanism. These conditions are also sufficient to guarantee that a mechanism satisfies BIC.

Proposition 1. *A mechanism $m \equiv (\rho, \tau)$ satisfies BIC if and only if for all $i \in I$:*

(P.1.1) *the function $\overline{a}_i(\cdot; m)$ is non decreasing on S_i,*

and

(P.1.2) *for all $v_i, \widehat{v}_i \in S_i$,*

$$\overline{U}_i(v_i; m) = \overline{U}_i(\widehat{v}_i; m) + \int_{\widehat{v}_i}^{v_i} \overline{a}_i(x_i; \rho) dx_i.$$

Thus, for any mechanism $m \equiv (\rho, \tau)$ satisfying BIC and for each $i \in I$, the expected level of provision obtained by i with the mechanism m must be a non-decreasing function of the individual's reported valuation. Also, an agent's interim expected utility (and interim expected taxes) is determined (except for a term whose expected value is independent of reported valuations) by the provision rule of the mechanism.

Condition $(P.1.2)$ can be restated in terms of the agents' expected interim utilities. Let m be a mechanism satisfying $(P.1.2)$. By setting $\widehat{v}_i = 0$ one has, for $v_i \in S_i$

$$0 \leq \left|\overline{U}_i(v_i; m)\right| \leq \left|\overline{U}_i(0; m) + v_i \overline{a}_i(v_i; \rho)\right| \leq \left|\overline{U}_i(0; m)\right| + |v_i|.$$

Thus, $\left|\overline{U}(\cdot;m)\right|$ is bounded by two integrable functions. Therefore $\overline{U}_i(\cdot;m)$ is integrable on S_i with respect to F_i, and hence $\bar{t}_i(\cdot;m)$ is also integrable on S_i.

Fix $i \in I, \widehat{v}_i \in S_i$ arbitrarily, and observe that $(P.1.2)$ is satisfied for every $v_i \in S_i$. Integrating both sides of $(P.1.2)$ with respect to F_i on S_i yields

$$\int_{S_i} s_i \overline{a}_i(s_i;\rho) dF_i(s_i) - E\left(t_i(V_i;\tau)\right) = \overline{U}_i(\widehat{v}_i;m) + \int_{S_i}\int_{\widehat{v}_i}^{s_i} \overline{a}_i(x_i;\rho) dx_i dF_i(s_i).$$

The double integral in the right side of this equation is equivalent to

$$\begin{aligned}\int_{S_i}\int_{v_i}^{s_i} \overline{a}_i(x_i;\rho) dx_i dF_i(s_i) \;&=\; \int_{s_i \le \widehat{v}_i}\int_{\widehat{v}_i}^{s_i} \overline{a}_i(x_i;\rho) dx_i dF_i(s_i) \\ &\quad + \int_{s_i > \widehat{v}_i}\int_{\widehat{v}_i}^{s_i} \overline{a}_i(x_i;\rho) dx_i dF_i(s_i).\end{aligned}$$

Since each double integral above is invariant to changes in the order of integration (see e.g., Theorem 12.19 in Royden, 1968) one obtains, by reversing the order of integration,

$$\begin{aligned}\int_{S_i}\int_{v_i}^{s_i} \overline{a}_i(x_i;\rho) dx_i dF_i(s_i) \;&=\; -\int_{x_i \le v_i} F_i(x_i)\overline{a}_i(x_i;\rho) dx_i \\ &\quad + \int_{x_i > v_i} \left(1 - F_i(x_i)\right) \overline{a}_i(x_i;\rho) dx_i.\end{aligned}$$

Therefore,

$$\begin{aligned}\overline{U}_i(\widehat{v}_i;m) \;&=\; \int_{x_i \le \widehat{v}_i} \left(x_i + \frac{F_i(x_i)}{f_i(x_i)}\right) \overline{a}_i(x_i;\rho) dF_i(x_i) - E\left(\bar{t}_i(V_i;\tau)\right) \\ &\quad + \int_{x_i > \widehat{v}_i} \left(x_i - \frac{1 - F_i(x_i)}{f_i(x_i)}\right) \overline{a}_i(x_i;\rho) dF_i(x_i). \qquad (3.1)\end{aligned}$$

Note that since $i \in I$, $\widehat{v}_i \in S_i$ have been selected arbitrarily, (3.1) is satisfied, for each $i \in I, \widehat{v}_i \in S_i$, by any mechanism m satisfying $(P.1.2)$.

Given a provision rule $\rho : S \to A$, for each $i \in I$ and $v_i \in S_i$, write

$$\begin{aligned}\overline{U}_i^{\rho}(v_i;\rho) \;&=\; \int_{x_i \le v_i} \left(x_i + \frac{F_i(x_i)}{f_i(x_i)}\right) \overline{a}_i(x_i;\rho) dF_i(x_i) \\ &\quad + \int_{x_i > v_i} \left(x_i - \frac{1 - F_i(x_i)}{f_i(x_i)}\right) \overline{a}_i(x_i;\rho) dF_i(x_i).\end{aligned}$$

Thus, for any mechanism $m \equiv (\rho, \tau)$ satisfying $(P.1.2)$, for each $i \in I$, $v_i \in S_i$ one has

$$\overline{U}_i(v_i;m) = \overline{U}_i^{\rho}(v_i;\rho) - E\left(\bar{t}_i(V_i;\tau)\right). \qquad (P.1.2')$$

It is straightforward to check that the converse is also satisfied, that is, $(P.1.2)$ is satisfied by any mechanism $m \equiv (\rho, \tau)$ satisfying $(P.1.2')$. Note that for any mechanism $m \equiv (\rho, \tau)$ satisfying BIC, $\overline{U}_i^{\rho}(\cdot;\rho)$ gives agent $i's$ interim expected utility in the absence of any lump-sum taxes.

Denote by P the set of all provision rules $\rho : S \to A$ satisfying $(P.1.1)$. A direct consequence of Proposition 1 is that it is always possible to obtain Bayesian incentive compatible mechanisms from any provision rule $\rho \in P$. Furthermore, given a provision rule $\rho \in P$, one can construct a tax function τ in such a way that the mechanism $m \equiv (\rho, \tau)$ satisfying BIC for which the vector specifying the ex-ante expected taxes paid by all individuals is equal to a prespecified $\overline{T} = \left\{\overline{T}_i : i \in I\right\} \in \Re^N$. Given $\rho \in P$, a particular construction is as follows: let $\tau : S \to \Re^N$ be defined, for each $v \in S$, $i \in I$, by

$$\tau_i(v) = v_i \overline{a}_i(v_i; \rho) - \overline{U}_i^g(v_i; \rho) + \overline{T}_i.$$

Note that the mechanism (ρ, τ) satisfies $(P.1.2')$. Hence it satisfies BIC. Since $(P.1.2')$ is satisfied one obtains for all $i \in I$, $v_i \in S_i$,

$$E\left(\overline{t}_i(v_i; \tau)\right) = \overline{T}_i,$$

and therefore the ex-ante expected taxes paid by individual i are equal to the prespecified $\overline{T}_i$. In particular, if the vector $\overline{T}$ is selected in such a way that

$$\sum_{i \in I} \overline{T}_i = E\left(C(\rho(V)\right),$$

then the mechanism $m \equiv (\rho, \tau)$ constructed as above satisfies $EXABB$.

Proposition 2 states this result formally.

Proposition 2. *Let $\overline{T} \in \Re^N$ be arbitrary. For every ρ satisfying $(P.1.1)$, there exists a tax function τ such that the mechanism $m \equiv (\rho, \tau)$ satisfies BIC, and for each $i \in I$* one has $E\left\{\tau_i(V)\right\} = \overline{T}_i$.

In what follows, the set of mechanisms satisfying BIC and $EXAF$ will be denoted by $\mathcal{M}$, that is

$$\mathcal{M} \equiv \left\{m : S \to A \times \Re^N : m \text{ satisfies } BIC \text{ and } EXAF\right\}.$$

Proposition 3 establishes that the tax function of any mechanism satisfying BIC and $EXAF$ can be modified in such a way that $EXPF$ is satisfied without altering the agents' interim utilities.

Proposition 3. *For any mechanism $m \equiv (\rho, \tau)$ satisfying BIC* and $EXAF$, *there exists a tax function τ' such that $m' \equiv (\rho, \tau')$ satisfies BIC and $EXPF$, and for every $v \in S$, $i \in I$ one has*

$$\overline{U}_i(v_i; m') = \overline{U}_i(v_i; m).$$

Thus, according to Proposition 3, using mechanisms satisfying BIC and $EXAF$ but not $EXPF$ does not introduce any new possibilities in terms of interim utilities obtained by agents. Also, by Proposition 2 that for every ρ satisfying $(P.1.1)$ there exists a tax function τ such that the mechanism $m \equiv (\rho, \tau)$ satisfies BIC and $EXPBB$. Hence, if no further requirements are imposed on the class of mechanisms to be considered available, one can restrict attention to mechanisms satisfying BIC and $EXPBB$.

Dominant strategy incentive compatible mechanisms

The notion of dominant strategy incentive compatibility is very attractive because it does not require any knowledge of the agents' beliefs over the other agents' types (see e.g., Ledyard, 1986). In this section, I provide a sufficient condition that guarantees that every mechanism satisfying BIC and $EXAF$ can be modified to obtain a mechanism satisfying $DSIC$ and $EXAF$ without altering the agents' interim utilities. This condition imposes that the provision rule of such mechanism must be a non-decreasing function of the agents' reported valuations.

Proposition 4. *Let $m \equiv (\rho, \tau)$ be a mechanism satisfying BIC and EXAF and suppose that, for each $i \in I$ and $v_{-i} \in S_{-i}$, $\rho_i(\cdot, v_{-i})$ is non-decreasing on S_{-i}. Then there exists a tax function τ' such that the mechanism $m' \equiv (\rho, \tau')$ satisfies DSIC and EXAF, and for all $v \in S, i \in I$ one has*

$$\overline{U}_i(v_i; m') = \overline{U}_i(v_i; m).$$

In the proof of Proposition 4, given in the Appendix, it is constructed a tax function such that the resulting mechanisms may fail to satisfy $EXPF$. Of course, this is consistent with the well known result that dominant strategy incentive compatible mechanisms may fail to satisfy $EXPF$ (see e.g., Green and Laffont, 1979). In these cases the cost paid to obtain a mechanism with stronger incentive properties is a weaker feasibility requirement.

To summarize the results obtained in this section, it has been shown that for each mechanism satisfying BIC, the interim expected utility obtained by any individual of any type is completely described (except for taxes that are lump sum in expectations) by the provision rule of the mechanism. In other words, in order to determine the interim utility obtained by any individual of any type with a mechanism satisfying BIC it is sufficient to specify the expected value of these taxes (which for mechanisms satisfying $EXABB$ must add up to zero). Condition $(P.1.2')$, which is satisfied by any mechanism satisfying BIC, provides an explicit formula to compute interim utilities corresponding to any mechanism satisfying BIC. These taxes can be constructed in such a way that $EXPBB$ is also satisfied. In addition, if the provision rule is a non decreasing function of the agents' reported valuations, then these taxes can be constructed in such a way that $DSIC$ is satisfied.

4 Interim efficient mechanisms

This section explores the efficiency properties of incentive feasible mechanisms. If a group of individuals discuss the possibility of altering a *statu quo* mechanism and use a new mechanism, it seems plausible that the new mechanism exhausts the possibilities for further improvements. The decision whether a mechanism is to be changed is made in the interim stage. Following Holmström and Myerson (1983), mechanisms that exhaust the possibilities for further improvements are referred to as *interim incentive efficient mechanisms (*or simply *interim efficient mechanisms).* A mechanism $m \in \mathcal{M}$ is interim efficient (IE) if there does not exist any other mechanism $m' \in$ M such that

$(E.1)$ for all $i \in I, v_i \in S_i$ one has

$$\overline{U}_i(v_i; m') \geq \overline{U}_i(v_i; m),$$

and

$(E.2)$ there exists $j \in I, v_j \in S_j$ such that

$$\overline{U}_j(v_j; m') > \overline{U}_j(v_j; m).$$

A second, seemingly weaker notion of interim efficiency may be considered. A mechanism $m \in$M is *weakly interim efficient* (WIE) if there does not exist $m' \in$ M such that for all $i \in I$ and $v_i \in S_i$ one has

$$\overline{U}_i(v_i; m') > \overline{U}_i(v_i; m).$$

The following Lemma establishes that the two notions of interim incentive efficiency are equivalent.

Lemma 1. *Every WIE mechanism is IE.*

Proposition 5 establishes that every IE mechanism is a solution to a welfare maximization problem. Let $\mathcal{G}$ denote the set of all distribution functions $G : S \to [0, 1]$ such that

$$\int_S \sum_{i \in I} |x_i| \, dG(x) < \infty.$$

A distribution $G \in \mathcal{G}$ will be regarded as a welfare distribution on the set S of all possible profiles of valuations. Let $\mathcal{U} : \mathcal{M} \times \mathcal{G} \to \Re$ be defined, for each $(m, G) \in \mathcal{M} \times \mathcal{G}$, by

$$\mathcal{U}(m; G) = \int_S \left(\sum_{i \in I} \overline{U}_i \left(x_i; m \right) \right) dG(x).$$

For each (m, G), the number $\mathcal{U}(m; G)$ is interpreted as the *aggregate interim welfare* of the mechanism m (with respect to G). The aggregate interim welfare is a weighted average, evaluated with respect to a distribution function G, of the agents' interim expected utilities.

Theorem 1. *A mechanism $m^* \equiv (\rho^*, \tau^*) \in \mathcal{M}$ is interim efficient if and only if there exists a welfare distribution $G \in \mathcal{G}$ such that m^* solves*

$$\max_{m \in \mathcal{M}} \mathcal{U}(m, G). \qquad (PR1)$$

Recall that by Proposition 1 for every mechanism $m \equiv (\rho, \tau) \in \mathcal{M}$, the individuals' interim utilities are determined, except for terms that are lump sum in expectations, by the provision rule ρ. Also, Proposition 2 establishes that one can always obtain a mechanism $m \in \mathcal{M}$ from any provision rule $\rho \in P$. Propositions 1 and 2 are used now to obtain an equivalent expression for the average interim welfare of a mechanism.

For each $G \in \mathcal{G}$ and each $i \in I$, let $G_i : S_i \to [0, 1]$ be defined, for all $v_i \in S_i$, by

$$G_i(v_i) = \int_{S_{-i}} dG(v_i, x_{-i}),$$

that is, G_i determines individual i's marginal welfare distribution corresponding to G.

Also, given an arbitrary welfare distribution $G \in \mathcal{G}$ and for each $i \in I$, $v_i \in S_i$, let

$$W_i(v_i; G) = v_i + \frac{F_i(v_i) - G_i(v_i)}{f_i(v_i)}.$$

Finally, let $\mathcal{W} : P \times \mathcal{G} \to \Re$ be defined, for each $(\rho, G) \in P \times \mathcal{G}$, by

$$\mathcal{W}(\rho, G) = \int_S \left[\sum_{i \in I} W_i(v_i; G)\, \rho_i(v) - C(\rho(v)) \right] dF(v).$$

Recall that by Proposition 1 each $m \in$M satisfies BIC. Hence for each $i \in I, v_i \in S_i$, $\overline{U}_i(v_i; m)$ satisfies $(P.1.2')$. Integrating both sides of $(P.1.2')$ with respect to G_i one has, for all $m \in$M and all $i \in I$,

$$\begin{aligned} \int_{S_i} \overline{U}_i(x_i; m) dG_i(x_i) &= \int_{S_i} \left(x_i + \frac{F_i(x_i)}{f_i(x_i)} \right) \overline{a}_i(x_i; \rho) dF_i(x_i) - E\left(\overline{t}_i(V_i; \tau)\right) \\ &\quad - \int_{S_i} \left(\int_{s_i \geq x_i} \overline{a}_i(x_i; \rho) dx_i \right) dG_i(x_i). \end{aligned}$$

By reversing the order of integration of each iterated integral in the above expression one obtains

$$\begin{aligned} \int_{S_i} \overline{U}_i(x_i; m) dG_i(x_i) &= \int_{S_i} \left(x_i + \frac{F_i(x_i)}{f_i(x_i)} \right) \overline{a}_i(x_i; \rho) dF_i(x_i) - E\left(\overline{t}_i(V_i; \tau)\right) \\ &\quad - \int_{S_i} \overline{a}_i(x_i; \rho) G_i(x_i) dx_i; \end{aligned}$$

that is,

$$\int_{S_i} \overline{U}_i(x_i;m)dG_i(x_i) = \int_{S_i} \left(x_i + \frac{F_i(x_i) - G_i(x_i)}{f_i(x_i)} \right) \overline{a}_i(x_i;\rho)dF_i(x_i) - E\left(\overline{t}_i(V_i;\tau)\right).$$

Summing up for $i \in I$ and taking into account that $EXABB$ is satisfied yields

$$\sum_{i \in I} \int_{S_i} \overline{U}_i(x_i;m)dG_i(x_i) = \mathcal{W}(\rho, G).$$

Therefore,

$$\mathcal{U}(m, G) = \mathcal{W}(\rho, G).$$

This result is established formally in Theorem 2.

Theorem 2 . *For every $G \in \mathcal{G}$, a mechanism $m = (\rho^*, \tau^*) \in \mathcal{M}$ solves $(PR1)$ if and only if ρ^* solves*

$$\max_{\rho \in P} \mathcal{W}(\rho, G). \qquad (PR2)$$

In order to characterize the solutions to $PR2$ for a given welfare distribution $G \in \mathcal{G}$, it is useful to distinguish between two cases, depending on the behavior of the functions $\{W_i(\cdot; G) : i \in I\}$. For each $i \in I$, the function $W_i(\cdot; G)$ will be referred to as individual $i's$ virtual valuation function (with respect to the distribution G).[5]

Case 1. *Monotonic virtual valuation functions*

Suppose that for each $i \in I$, the function $W_i(\cdot; G)$ is non-decreasing on S_i. Notice that if a provision rule $\rho^* : S \to A$ satisfies for each $v \in S$ and each $a \in A$,

$$\sum_{i \in I} W_i(v_i; G)\, \rho_i^*(v) - C(\rho^*(v)) \geq \sum_{i \in I} W_i(v_i; G)\, a_i - C(a), \qquad (4.1)$$

then it maximizes the functional $\mathcal{W}(\rho, G)$ among all provision rules $\rho : S \to A$. Since for each $i \in I$ the function $W_i(\cdot; G)$ is non-decreasing on S_i, then each function ρ_i^* is non-decreasing on S_i and therefore $\rho^* \in P$. Thus, if each function $W_i(\cdot; G)$ is non-decreasing on S_i, then a provision rule ρ^* satisfying (4.1) solves $PR2$.

Given a welfare distribution G, for each $i \in I$ and $v_i \in S_i$, the number $W_i(v_i; G)$ can be regarded as the social value of providing the good to individual i when v_i is reported. Individual $i's$ virtual valuation is obtained by adding to his reported type v_i the correction term $(F_i(v_i) - G_i(v_i))/f_i(v_i)$. In order to understand why this correction takes place, it might be useful to rewrite $i's$ virtual valuation associated to v_i and G as

$$W_i(v_i, G) = v_i + \frac{(1 - G_i(v_i)) - (1 - F_i(v_i))}{f_i(v_i)}.$$

[5]See e.g., Myerson (1981)

Consider now the problem faced by a social planner who is seeking to maximize the average of the agents' interim expected utilities (where this average is taken with respect the distribution G). A natural way to proceed is to increase the level of provision whenever someone reports a positive valuation and to decrease it when someone reports a negative one. However, increasing provision when an arbitrary individual $i \in I$ reports a positive valuation v_i might have negative effects on expected taxes that can be collected from i if his valuation were higher than v_i. If taxes paid by individual i when he reports a valuation $v_i' > v_i$ are too high, the individual might have incentives to underreport his valuation, since reporting v_i leads to a relatively high level of provision. Thus, the value that the planner assigns to valuation v_i is corrected by,

- a term that increases with the cumulative welfare corresponding to types with valuation higher than v_i, $(1 - G_i(v_i))$,
- a term that decreases with the cumulative objective probability corresponding to types with valuation higher than v_i, $(1 - F_i(v_i))$

and

- a term that decreases with the density associated to valuation v_i, $f_i(v_i)$ increases.

Finally, it should be noticed that for each $G \in \mathcal{G}$ and each $i \in I$ one has

$$\int_{S_i} W_i(x_i; G)\, dF_i(x_i) = \int_{S_i} x_i dG_i(x_i).$$

That is, the ex-ante expected virtual valuation of individual i equals the expectation of a random variable X_i which is distributed on S_i according to the distribution function G_i.

In a recent paper, Ledyard and Palfrey (1999)[6] provide a version of Theorem 1 in a pure public goods framework with linear costs, that is, in a framework described

[6]In Ledyard and Palfrey's characterization, interim efficient mechanisms maximize

$$\sum_{i \in I} \int_{S_i} \overline{U}_i(v_i; m) \lambda_i(v_i) dF_i(v_i).$$

where $\{\lambda_i : S_i \to \Re : i \in I\}$ is a system of welfare weights satisfying, for each $i \in I$,

$$\int_{S_i} \lambda_i(v_i) dF_i(v_i) = 1.$$

Notice that such functional is equivalent to $U(m; G)$ for a distribution G satisfying

$$G_i(v_i) = \int_{x_i \leq v_i} \lambda_i(x_i) dF_i(x_i).$$

by a set of provision alternatives

$$A \equiv \left\{ a \in [0,1]^N : \ a_i = a_j \text{ for every } i,j \in I \right\},$$

and a cost function defined, for each $a \in A$, by

$$C(a) = Ka_1.$$

for some $K \geq 0$. Note that, in pure public goods problems, the provision rule of a mechanism is described by a function $q : S \to [0,1]$ determining the (unique) level of provision corresponding to each profile of reported valuations (that is, the function verifying $q \equiv \rho_i \equiv \rho_j$ for each $i,j \in I$). If for a given welfare distribution G, all the individuals' virtual valuation functions are non-decreasing, then a provision rule $q^* : S \to [0,1]$ solves $PR2$ if it satisfies, for each $v \in S$,[7]

$$q^*(v) = \begin{cases} 0, & \text{whenever } \sum_{i \in I} W_i(v_i;G) < K, \\ 1, & \text{whenever } \sum_{i \in I} W_i(v_i;G) > K. \end{cases}$$

Notice that the characterization provided in Theorem 1 is somewhat incomplete, since it does not indicate how to find a solution to $PR2$ for the case in which some of the individuals' virtual valuation functions are strictly decreasing on some intervals in S. This case is analyzed bellow.

Case 2. *Decreasing virtual valuation functions.*

Note that a provision rule ρ^*:$S \to A$ satisfying (4.1) might not be in the set P and therefore such provision rules are not a solution to $(PR2)$. It can be shown, however, that the set of provision rules solving $(PR2)$ for some $G \in \mathcal{G}$ can always be associated to a collection of *generalized virtual valuation functions*.

Denote by Γ the class of all open intervals in the real line. An arbitrary interval in Γ will be represented by the symbol (a,b), with $a,b \in \Re \cup \{-\infty, \infty\}$. Given $G \in \mathcal{G}$ and $i \in I$, a function $W_i^*(\cdot;G) : S_i \to \Re$ is a *generalized virtual valuation function with respect to* G if (a) it is continuous and non decreasing on S_i, and (b) there exists a (possibly empty) countable collection $\{C_k^i : k \in K\}$ of disjoint intervals in Γ such that, for each $v_i \in S_i$ one has

$$W_i^*(v_i;G) = \begin{cases} \dfrac{\int_{C_k^i} W_i(x_i;G)dF_i(x_i)}{\int_{C_k^i} dF_i(x_i)} & \text{if } v_i \in C_k^i \text{ for some } k \in K, \\ W_i(v_i;G), & \text{otherwise ;} \end{cases} \qquad (b.1)$$

and

$$\int_{x_i \leq v_i} W_i^*(x_i,G)\, dF_i(x_i) \leq \int_{x_i \leq v_i} W_i(x_i;G)\, dF(x_i). \qquad (b.2)$$

[7] A condition which corresponds to Ledyard and Palfrey's *virtual cost benefit criterium.*

Condition $(b.1)$ implies for every $k \in K$,

$$\int_{C_k^i} W_i^*(x_i; G)\, dF(x_i) = \int_{C_k^i} W_i(x_i; G)\, dF(x_i).$$

Therefore,

$$E[W_i^*(V_i; G)] = \int_{S_i} W_i^*(x_i; G)\, dF(x_i) = \int_{S_i} W_i(x_i; G)\, dF(x_i) = \int_{S_i} x_i dG_i(x_i).$$

Summarizing, a generalized virtual valuation is obtained by flattening $W_i(\cdot; G)$ on those regions on which $W_i(\cdot; G)$ is decreasing in such a way that the expected value of $W_i(V_i; G)$ is preserved. Note also that a non-decreasing function $W_i(\cdot; G)$ is a generalized virtual valuation function (with respect to G) since the pair

$$[W_i^*(\cdot; G), C_i] = [W_i(\cdot; G), \emptyset]$$

satisfies $(b.1)$ and $(b.2)$.

Lemma 2. *For every $G \in \mathcal{G}$* and every $i \in I$, *a generalized virtual valuation function with respect to G always exists.*

The proof of Lemma 2 proceeds by constructing a generalized virtual valuation function from an arbitrary $G \in \mathcal{G}$. The procedure by which a virtual valuation function is modified to obtain a generalized virtual valuation function is closely related to a procedure applied by Maskin and Riley (1989) in the context of multiunit auctions.[8].

Theorem 3. *Let $G \in \mathcal{G}$ be arbitrary and let ρ^* be a provision rule satisfying, for each $v \in S$* and *$a \in A$*

$$\sum_{i \in I} W_i^*(v_i; G)\rho_i^*(v) - C(\rho^*(v)) \geq \sum_{i \in I} W_i(v_i; G)\, a_i - C(a), \qquad (T.3)$$

for some collection $\{W_i^*(\cdot; G) : i \in I\}$ of generalized virtual valuation functions with respect to G. Then ρ^* solves $PR2$.

I conclude this section with some examples of interim efficient mechanisms in different contexts.

[8]Their procedure, however, is applied to a specific welfare optimization problem, namely, the maximization of the seller's revenue.

5 Examples

5.1 The role of exclusion in public good problems

Consider a public good problem with linear costs, that is, a problem described by a set of provision alternatives

$$A \subseteq [0,1]^N,$$

and a cost function defined, for each $a \in A$, by

$$C(a) = c\max\{a_i : i \in I\};$$

where $0 < K < N$.

Ledyard and Palfrey (1999) have explored the properties of several provision rules solving $PR2$ in this environment, under the assumption that the public good is non-excludable (i.e., assuming $A \equiv \left\{a \in [0,1]^N : a_i = a_j \forall i,j \in I\right\}$). Since the results obtained above allow for different specifications of the set of alternatives, one can study the effects of introducing exclusion by comparing the solutions to $PR2$ when exclusion is allowed (i.e., if $A \equiv [0,1]^N$) with those studied by Ledyard and Palfrey.

Note that if exclusion is not possible, then solutions to $PR2$ satisfy

$$\rho_i^*(v) = \begin{cases} 0, & \text{if } \sum_{i\in I} W_i^*(v_i;G) < K \\ 1, & \text{if } \sum_{i\in I} W_i^*(v_i;G) \geq K, \end{cases} \tag{5.1}$$

for some collection of generalized virtual valuation functions $\{W_i^*(\cdot;G) : i \in I\}$. If exclusion is possible, then it satisfies

$$\rho_i^*(v) = \begin{cases} 1 & \text{if } W_i^*(v_i;G) \geq 0 \text{ and } \sum_{j\in I: W_i^*(v_i;G)\geq 0} W_j^*(v_j;G) \geq K \\ 0 & \text{otherwise.} \end{cases} \tag{5.2}$$

Thus, *per capita* expected provision of the public good, that is,

$$\frac{E\left(\sum_{i\in I}\rho_i^*(V)\right)}{N}$$

is higher if exclusion is possible. The intuition behind this result is clear: exclusion plays a role in ameliorating the free rider problem, since by underreporting their true valuations the agents might be excluded from the use of the public good.[9]

[9]The role of exclusion in amelliorating the free rider problem has been recently discussed by Dearden (1997). His main argument, however, relies on the assumption that the agents select the mechanism that maximizes the agent's *ex-ante* utilities, subject to individual rationality constraints.

5.2 Mechanisms exhibiting pooling of types

5.2.1 Interim efficient Stochastic Mechanisms

Consider a pure public good problem with linear costs as the one described in the above example. Assume also that, for each $i \in I, V_i$ is distributed uniformly on $[0,1]$ and $0 < K < N$. An interesting class of interim efficient mechanisms arising in this framework are those associated to welfare functions that "jump" at a given point in the interior of its support $S \equiv [0,1]^N$. For example, let G be given for all $v \in S$ and $i \in I$ by

$$G_i(v_i) = \begin{cases} 0, & \text{whenever } v_i < \frac{K}{N}, \\ 1, & \text{whenever } v_i \geq \frac{K}{N}. \end{cases}$$

Then for each $i \in I$, the function

$$W_i^*(v_i; G) = \begin{cases} 2v_i & \text{if } v_i < \frac{K}{2N}, \\ \frac{1}{2}, & \text{if } \frac{K}{2N} \leq v_i \leq \frac{K}{2N} + \frac{1}{2}, \\ 2v_i - 1 & \text{if } v_i > \frac{K}{2N} + \frac{1}{2}, \end{cases}$$

is a generalized virtual valuation function. By Theorem 3, a provision rule ρ^* such that

$$\rho_i^*(v) = \begin{cases} 0, & \text{if } \sum_i W_i^*(v_i; G) < K, \\ 1, & \text{if } \sum_i W_i^*(v_i; G) > K, \end{cases}$$

solves $(PR2)$.

Note that the random variable $\sum_i W_i^*(V_i; G)$ is not continuous, and therefore ρ^* might select an arbitrary alternative $a \in A$ with probability

$$\Pr\left[\sum_i W_i^*(V_i) = K\right] = \prod_{i \in I} \Pr\left[\frac{K}{2N} \leq V_i \leq \frac{K}{2N} + 1\right] = \left(\frac{1}{2}\right)^N.$$

Recall that if the set A represents the set of all stochastic decisions on a discrete set $D \subseteq \{0,1\}^N$, then a provision alternative a is interpreted as a stochastic decision. In particular, if $a \notin \{0,1\}$, then more than one alternative in D can be selected with positive probability.

5.2.2 Sampling mechanisms

Consider again the same public good problem and let G be given for all $v \in S$ and $i \in I$ by

$$G_i(v_i) = \begin{cases} 0, & \text{whenever } v_i < \frac{1}{4}, \\ 1, & \text{whenever } v_i \geq \frac{1}{4}. \end{cases}$$

Then for each $i \in I$,

$$W_i(v_i; G) = \begin{cases} 2v_i & \text{if } v_i < \frac{1}{4}, \\ v_i & \text{if } v_i \geq \frac{1}{4}. \end{cases}$$

Thus, virtual valuations are strictly decreasing on some interval in S_i. For each $i \in I$, the function

$$W_i^*(v_i; G) = \begin{cases} 0 & \text{if } v_i < \frac{1}{2}, \\ 2v_i - 1 & \text{if } v_i \geq \frac{1}{2}, \end{cases}$$

is a generalized virtual valuation function with respect to G. For each $v \in S$, let

$$n(v) = \#\left\{i \in I : v_i > \frac{1}{2}\right\}.$$

Also, Then a provision rule ρ^* such that

$$\rho_i^*(v) = \begin{cases} 0, & \text{if } \sum_{i:v_i > \frac{1}{2}} v_i \leq \frac{K+n(v)}{2}, \\ 1, & \text{if } \sum_{i:v_i > \frac{1}{2}} v_i > \frac{K+n(v)}{2}, \end{cases}$$

solves $(PR2)$. Observe that an interim efficient mechanism (associated to the distribution G) can be implemented by a (non random) sampling mechanism (with a sample size given by $n(v)$) such that people participating in the sample (*i.e.*, those whose valuation is higher than $\frac{1}{2}$) are (self-)selected through a participation tax. Sampling mechanisms may be deemed desirable in contexts in which acquiring and processing information is costly, such as those studied in Green and Laffont (1979) or Bobo (1998). These authors propose that these costs could be reduced by using a (random) sampling mechanism. However, a (non random) sampling mechanism in which people participating in the sample are (self-)selected (through a participation fee, for example) may also reduce these costs, and the decision whether to participate or not may provide the designer with some relevant information.

5.2.3 A second-price double auction with participation charges

Another example of mechanisms exhibiting pooling of types with other interesting properties arises in the context of private goods. Yoon (1997), has studied the properties of double second price auctions (with participation charges) in multilateral trading problems. Recall that a multilateral trading problem consists in allocating $M \leq N$ units of a private good among N agents. The set of alternatives is therefore given by

$$A \equiv \left\{a \in [0,1]^N : \sum_{i=1}^{N} a_i \leq M\right\},$$

and the cost function C satisfies, for each $a \in A$,

$$C(a) = 0.$$

Suppose also that the set of agents is formed by a set J of M selleres and a set $B \sqsubseteq I$ of $N - M$ buyers. The sellers have property rights over the goods that are traded. To

simplify things, assume that the individuals' valuations are independent, identically distributed on an interval $[0, \overline{v}]$.

A double second-price auction (with participation charges) is a two price mechanism such that:

-an external agent, acting as a mediator, announces a participation charge δ_B for the buyers ann δ_J for the sellers, that allows the agents to participate in the mechanism. A seller who decides not to participate retains the good he owns;

-those agents who decide to paticipate report their valuations to the mediator, who orders these reported valuations in an list $v^1, v^2, ..., v^K, v^{K+1}, ..., v^{K+L}$, where K is the number of sellers that decide to participate and L is the number of buyers who decide to participate;

-those agents reporting the K highest valuations are provided with one unit of the good;

-all those buyers who actually buy the good pay a price $p_B = v^K$; while those sellers who actually sell the good receive a price $p_J = v^{K+1}$.

Thus, a double second-price is an extension of Vickrey (1961) second-price auction to markets with more than one seller and participation charges. Yoon (1997a, 1997b) has demonstrated that for an appropiate selection of the participation charges, a double second-price auction satisfies $EXABB$ and induces voluntary participation (in the interim stage) from both the buyers and the sellers. In addition, for all agents who participate, reporting the true valuation is a dominant strategy, and unrealized gains from trade converge to zero as the number of individual becomes large.

Clearly, a double second-price auction with participation charges exhibits pooling of types. More precisely, let $v_b(\delta_B)$ and $v_j(\delta_J)$ be the lowest type of buyer and, respectively, the highest type of a seller who decide to participate. Taking into account that the $S_i = S_j$ for any two agents $i, j \in I$, it follows that the provision rule of a second price double auction satisfies, for each $v \in S$,

$$\sum_{i \in I} W^*(v_i)\rho_i(v) = \max_{a \in A} \left\{ \sum_{i \in I} W^*(v_i)a_i \right\},$$

where W^* is an arbitrary non-decreasing function $W^* : [0, \overline{v}] \to \Re_+$ such that

$$W^*(\cdot) \text{ is strictly increasing on } [v_b(\delta_B), v_j(\delta_J)]; \tag{5.3}$$

$$W^*(v_b(\delta_B)) = W^*(0); \tag{5.4}$$

and

$$W^*(v_j(\delta_J)) = W^*(\overline{v}). \tag{5.5}$$

Thus, in order to check whether a double second-price auction satisfying $EXABB$ is interim efficient it is sufficient to show that there exists a function $W^* : [0,\overline{v}] \to \Re$ satisfying (5.3), (5.4) and (5.5) and a distribution $G^* : [0,\overline{v}]^N \to [0,1]$ such that W^* is a generalized virtual valuation function with respect to G.

Consider, for example, a double second-price auction satisfying $EXABB$ and such that $\delta_J = 0$ and $\delta_B > 0$ (that is, only buyers pay participation charges). Note that for such participation charges one has $v_j(\delta_J) = \overline{v}$ and $v_b(\delta_B) > 0$. In order to obtain a generalized virtual valuation function W^* satisfying (5.3), (5.4) and (5.5), let $\alpha \in [0,1]$ be arbitrary and let G^α be a distribution function satisfying, for each $i \in I, v_i \in S_i$,

$$G_i^\alpha(v_i) = \alpha + (1-\alpha)\, F_i(v_i).$$

Note that for each $i \in I, v_i \in S_i$ one has

$$W_i(v_i; G^\alpha) = v_i - \alpha \frac{1 - F_i(v_i)}{f_i(v_i)}.$$

Observe also that there exists α^* such that

$$W_i(v_i; G^{\alpha^*}) = 0.$$

Let now G^* be defined, for each $i \in I, v_i \in S_i$,

$$G_i^*(v_i) = \begin{cases} 0, & \text{if } v_i < v_i^*, \\ G^\alpha(v_i) & \text{if } v_i \geq v_i^*; \end{cases}$$

where v_i^* is selected in such a way that

$$\int_0^{v_b(\delta_B)} W_i(x_i; G^*) dF_i(x_i) = 0$$

is satisfied. It is straightforward to check that such distribution G^* always exists. Furthermore, if $F_i = F_j$ for any two $i, j \in I$, and each function $W_i(\cdot; G^1)$ is strictly increasing on $[0,\overline{v}]$, then every generalized virtual valuation function $W_i(\cdot; G^*)$ satisfies (5.3), (5.4) and (5.5). Therefore a budget balanced double second-price auction such that $\delta_J = 0$ is interim efficient.

Note that this example shows that a given (budget balanced) double second-price auction can be justified as interim efficient for a wide range of distributions generating the agents' valuations (to be more precise, for all those distributions that yield equal marginal distributions for any two individuals and such that each function $W_i(\cdot; G^1)$ is strictly increasing on $[0,\overline{v}]$). Thus, interim efficiency of some double second price auctions is robust to changes in the specification of the distribution generating the agents' valuations.[10]. Although Wilson (1985) has shown that other

[10]It should be noticed that the participation charges required to achieve budget balance are still dependent on the prior distributions, and therefore one might argue that double auctions are not robust to prior information assumptions. Note, however, that prior information only determines the participation charges and does not affect the rule determining the volume and terms of trade.

trading mechanisms, such as $k-$double auctions (See also Chaterjee and Samuelson (1983)), achieve interim efficiency when when the number of traders is sufficiently large, a $k-$double auction have two unsatisfactory features. First of all, a $k-$double auction imposes on participants a severe informational and computational burden. Second of all, a $k-$double auction might have infinitely many different equilibria, as shown by Leninger, Linhart and Radner (1989).

6 Concluding Remarks

Throughout the paper, no restrictions are imposed on the welfare distributions associated to the set of interim incentive efficient mechanisms. Thus, the whole range of interim efficient mechanisms, including those that exhibit pooling of types, is characterized. As suggested in the examples in Section 5, there might be situations in which mechanisms exhibiting pooling of types yield other interesting properties from both normative and positive considerations.

A possible extension of this work is to analyze the robustness of interim incentive mechanisms to prior information assumptions, following the lines suggested by Mookherjee and Reichelstein (1992) and Makowski and Mezzetti (1994). Although several definitions have been provided, a robust mechanism can be defined as a mechanism whose properties are invariant to changes in the distribution determining the agents' valuations. Note that, since it has been shown that any interim efficient mechanism can be constructed in such a way that it satisfies $DSIC$, incentive compatibility of any interim efficient mechanism does not depend on the distribution determining the agents' valuations.

In spite of this, determining whether a given mechanism is interim efficient requires also that its provision rule maximizes the sum of the agent´s virtual valuations, which in general depends on the prior information held by agents. As suggested in the examples, second price double auctions constitute an exception that might be worth exploring, since they maximize the sum of the agents ´virtual valuations for a wide range of specifications of the distribution generating the agents' valuations.

Other extensions, such as relaxing the assumption of independent types or allowing for non linear utility functions appear to be more difficult open questions.

Appendix A. Stochastic allocations in discrete problems

Consider a problem described by $[I, D, C_D, V]$, where I and V are defined as in section 1; $D \subseteq \{0,1\}^N$ is an arbitrary discrete set of alternatives; and C_D is an arbitrary function $C_D : D \to \Re$. Analogously to the class of problems described in section 1, a feasible allocation is defined as pair $(d,t) \in D \times \Re^N$ such that $\sum_{i \in I} t_i \geq C_D(d)$. For each $i \in I$, utility obtained by i at an allocation $(d,t) \in D \times \Re$ is given by $v_i d_i - t_i$, where v_i is a realization of V_i. Denote by $\Delta(D)$ the set of probability distributions over the set D. Note that since D is discrete, the set $\Delta(D)$ is represented by the simplex in $\Re^{|D|}$, where $|D|$ denotes the cardinality of D, that is,

$$\Delta(D) \equiv \left\{ Q : D \to [0,1] : \sum_{d \in D} Q(d) = 1 \right\}.$$

Suppose that allocations are selected in the following fashion. First, a provision $d \in D$ is selected randomly from a distribution $Q \in \Delta(D)$. Then, each individual $i \in I$ pays a tax $T_i(d)$, where d is the selected alternative. A stochastic decision is therefore represented by a pair of functions

$$(Q,T) : D \to [0,1] \times \Re^N$$

such that

$$\sum_{d \in D} Q(d) = 1.$$

Note that once a provision alternative is selected, taxes are selected in a deterministic fashion; due to linearity of preferences, allowing for stochastic taxes does not introduce any new possibilities in terms of the individuals' expected utilities. A stochastic decision (Q,T) is feasible in $[I, D, C_D, V]$ if for all $d \in D$ such that $Q(d) > 0$ one has

$$\sum_{i \in I} T_i(d) \geq C_D(d).$$

For each $i \in I$, the expected provision obtained by i with a stochastic decision (Q,T) is given by

$$a_i(Q) = \sum_{d \in D} Q(d) d_i.$$

Also, let $t_i(Q;T)$ the expected tax paid by i with (Q,T), that is,

$$t_i(Q;T) = \sum_{d \in D} Q(d) T_i(d)$$

Thus, for each type $v_i \in S_i$ of individual i, the individual's expected utility when a stochastic decision (Q,T) is taken equals,

$$\sum_{d \in D} Q(d) \left[v_i d_i - T_i(d) \right] = v_i a_i(Q) - t_i(Q;T)$$

Given $Q \in \Delta(D)$, let $a(Q) = \{a_i(Q) : i \in I\}$. Also, let $A \subseteq [0,1]^N$ and $C : A \to \Re$ be defined, respectively, by

$$A \equiv \left\{a \in [0,1]^N : \exists\, Q \in \Delta(D) : a = a(Q)\right\}$$

and

$$C(a) = \min_{Q \in \Delta(D)} \left\{ \sum_{d \in D} Q(d) C_D(d) : a = a(Q) \right\}$$

Clearly, A is compact and convex and C is continuous. Thus, given a profile of valuations $v \in S$ and for each stochastic decision (Q,T) that is feasible in $[I, D, C_D, V]$, there exists an allocation $(a,t) \in A \times \Re$ that is feasible in $[I, A, C, V]$ and yields the same utility to all individuals in I. It is now shown that the converse is also true, that is, for each (a,t) that is feasible in $[I, A, C, V]$, there exists a stochastic decision (Q,T) that is feasible in $[I, D, C_D, V]$ and yields the same utility to all individuals in I. To prove this statement is satisfied, let (a,t) be an arbitrary feasible allocation in $[I, A, C, V]$. Then select $Q^* \in \Delta(D)$ such that Q^* solves

$$\min_{Q \in \Delta(D)} \left\{ \sum_{d \in D} Q(d) C_D(d) : a(Q') = a \right\}.$$

For each $i \in I$, let

$$c_i = \frac{1}{N-1} \sum_{j \in I \setminus \{i\}} t_j - \frac{1}{N} \sum_{d \in D} Q^*(d) C(d)$$

Also, let $T : D \to \Re^N$ be defined, for each $d \in D$ and each $i \in I$, by

$$T_i(d) = \frac{C(d)}{N} + c_i + t_i - \frac{1}{N-1} \left(\sum_{j \in I \setminus \{i\}} t_j \right).$$

Summing across I one obtains, for each $d \in D$,

$$\sum_{i \in I} T_i(d) = C(d) + \sum_{i \in I} c_i.$$

Note that

$$\sum_{i \in I} c_i = \sum_{i \in I} t_i - \sum_{d \in D} Q^*(d)\,(C(d)) = \sum_{i \in I} t_i - C(a).$$

Therefore, the fact that (a,t) is feasible in $[I, A, C, V]$ implies $\sum_{i \in I} c_i > 0$. It follows that for each $d \in D$ one has $\sum_{i \in I} T_i(d) > C(d)$, therefore establishing that (Q^*, T) is feasible in $[I, D, C_D, V]$. To show utilities obtained by the individuals with (a,t) equal the expected utilities obtained with (Q^*, T), note from the definition of T that for each $i \in I$ one has

$$t_i(Q^*, T) = \sum_{d \in D} Q^*(d) T_i(d) = t_i.$$

Therefore one has, for each $v \in S$, $i \in I$,

$$v_i a - t_i = v_i a(Q^*) - t_i(Q^*, T).$$

Thus, both allocation problems are equivalent in terms of utilities obtained by agents.

Appendix B. Proofs

Throughout the appendix, the following notation is used. Let $\mathcal{B}$ be the class of all Borel sets in $\Re^N$. For any increasing, right continuous, bounded function $G : \Re^N \to \Re$, let $\mu : \mathcal{B} \to \Re_+$ be the unique measure[11] satisfying, for all $(a, b] \in \mathcal{B}$,

$$\mu(a, b] = G(b) - G(a).$$

For any function $f : \Re \to \Re$ and any Borel set $B \in \mathcal{B}$, the Lebesgue integral $\int_B f d\mu$ will be denoted by $\int_B f(x) dG(x)$. For intervals of the form $(a, b] \subset \Re^N$ the Lebesgue integral $\int_{(a,b]} f(x) dG(x)$ will also be denoted by $\int_a^b f(x) dG(x)$.

The linear space formed by all bounded, Borel measurable functions $f : \Re^N \to \Re$ will be denoted by $L^\infty[G]$. For any $p : 1 \leq p < \infty$, the linear space formed by all Borel measurable functions $f : \Re^N \to \Re$ such that

$$\left(\int_{\Re^N} |f(x)|^a \, dG(x) \right)^{\frac{1}{a}} < \infty$$

will be denoted by $L^p[G]$.

Proposition 1. *A mechanism $m \equiv (\rho, \tau)$ satisfies BIC if and only if for all $i \in I$:*

(P.1.1) *the function $\overline{a}_i(\cdot; m)$ is non decreasing on S_i; and*

(P.1.2) *for all $v_i, \widehat{v}_i \in S_i$,*

$$\overline{U}_i(v_i; m) = \overline{U}_i(\widehat{v}_i; m) + \int_{\widehat{v}_i}^{v_i} \overline{a}_i(x_i; \rho) dx_i.$$

Proof. Sufficiency is proved first. Consider a mechanism $m \equiv (a, t)$ satisfying $(P.1.1)$ and $(P.1.2.)$. Since $(P.1.2)$ is satisfied one has, for any $i \in I$ and any two valuations $v_i, \widehat{v}_i \in S_i$,

$$\overline{U}_i(v_i; m) - \overline{U}_i(\widehat{v}_i; m) = \int_{\widehat{v}_i}^{v_i} \overline{a}_i(x_i; \rho) dx_i;$$

therefore,

$$v_i \overline{a}_i(v_i; \rho) - \overline{t}_i(v_i; \tau) - (\widehat{v}_i \overline{a}_i(\widehat{v}_i; \rho) - \overline{t}_i(\widehat{v}_i; \tau)) = \int_{\widehat{v}_i}^{v_i} \overline{a}_i(x_i; \rho) dx_i. \qquad (B.1)$$

[11] See Proposition 3.12 in Royden (1968, p.301).

By adding $(\widehat{v}_i - v_i)\,\overline{a}_i(\widehat{v}_i;\rho)$ to both sides of $(B.1)$ one obtains,

$$v_i\overline{a}_i(v_i;\rho) - \overline{t}_i(v_i;\tau) - [v_i\overline{a}_i(\widehat{v}_i;\rho) - \overline{t}_i(\widehat{v}_i;\tau)] = \int_{\widehat{v}_i}^{v_i} \overline{a}_i(x_i;\rho)dx_i - (v_i - \widehat{v}_i)\,\overline{a}_i(\widehat{v}_i;\rho).$$

Taking into account that $\overline{a}_i(\cdot;\rho)$ is non decreasing on S_i yields

$$\int_{\widehat{v}_i}^{v_i} \overline{a}_i(x_i;\rho)dx_i - (v_i - \widehat{v}_i)\,\overline{a}_i(\widehat{v}_i;\rho) \geq 0.$$

Hence,

$$v_i\overline{a}_i(v_i;\rho) - \overline{t}_i(v_i;\tau) \geq v_i\overline{a}_i(\widehat{v}_i;\rho) - \overline{t}_i(\widehat{v}_i;\tau),$$

therefore establishing that m satisfies BIC.

To prove necessity, it is shown that $(P.1.1)$ and $(P.1.2)$ are satisfied for any mechanism m satisfying BIC. Let $m \equiv (a,t)$ be a mechanism satisfying BIC and consider an arbitrary agent $i \in I$ and any two valuations $v_i, \widehat{v}_i \in S_i$. Since m satisfies BIC one has

$$v_i\overline{a}_i(v_i;\rho) - \overline{t}_i(v_i;\tau) \geq v_i\overline{a}_i(\widehat{v}_i;\rho) - \overline{t}_i(\widehat{v}_i;\tau) \qquad (B.2)$$

and

$$\widehat{v}_i\overline{a}_i(v_i;\rho) - \overline{t}_i(\widehat{v}_i;\tau) \geq \widehat{v}_i\overline{a}_i(v_i;\rho) - \overline{t}_i(v_i;\tau). \qquad (B.3)$$

Adding terms in each side of the inequalities yields

$$(v_i - \widehat{v}_i)\,(\overline{a}_i(v_i;\rho) - \overline{a}_i(\widehat{v}_i;\rho)) \geq 0.$$

Thus, for any two valuations $v_i, \widehat{v}_i \in S$ one has $\overline{a}_i(v_i;\rho) \geq \overline{a}_i(\widehat{v}_i;\rho)$ whenever $v_i \geq \widehat{v}_i$, and therefore $\overline{a}_i(\cdot;\rho)$ is a non-decreasing function, which establishes $(P.1.1)$.

To prove $(P.1.2)$ is satisfied, use again the inequalities $(P.2)$ and $(P.3)$ to obtain

$$\overline{a}_i(\widehat{v}_i;\rho)\,(v_i - \widehat{v}_i) \leq \overline{U}_i(v_i;m) - \overline{U}_i(\widehat{v}_i;m) \leq \overline{a}_i(v_i;\rho)\,(v_i - \widehat{v}_i)\,. \qquad (B.4)$$

Therefore, $\overline{U}_i(\cdot;m)$ is non-decreasing on S_i since $\overline{a}_i(\cdot;\rho)$ is non-negative on S_i. Also, since $(B.4)$ is satisfied for any two $v_i, \widehat{v}_i \in S_i$ one has, for any $\widehat{v}_i \in S_i$

$$\overline{U}_i(v_i;m) = \sup_{v_i < \widehat{v}_i} \overline{U}_i(v_i;m) = \inf_{x_i > \widehat{v}_i} \overline{U}_i(v_i;m).$$

Hence, $\overline{U}_i(\cdot;m)$ is continuous on S_i.

Given $v_i \in S_i$, denote by $\overline{U}_i'(v_i+;m)$ and $\overline{U}_i'(v_i-;m)$ the right-hand and left-hand derivatives of $\overline{U}_i(\cdot;m)$ at v_i. Note that $(B.4)$ implies

$$\overline{U}_i'(v_i+;m) = \inf_{x_i > v_i} \left\{ \frac{\overline{U}_i(x_i;m) - \overline{U}_i(v_i;m)}{(x_i - v_i)} \right\} = \inf_{x_i > v_i} \overline{a}_i(v_i;\rho),$$

and

$$\overline{U}_i'(v_i-;m) = \inf_{x_i<v_i}\left\{\frac{\overline{U}_i(v_i;m)-\overline{U}_i(x_i;m)}{(v_i-x_i)}\right\} = \sup_{x_i<v_i}\overline{a}_i(v_i;\rho).$$

Thus, if $\overline{a}_i(\cdot;\rho)$ is continuous at a point $v_i \in S_i$, then $\overline{U}_i(\cdot;m)$ is differentiable at x_i and

$$\overline{U}_i'(x_i;m) = \overline{a}_i(x_i;\rho).$$

Since $\overline{a}_i(\cdot;\rho)$ is non-decreasing, it is continuous almost everywhere (with respect to Lebesgue measure) and therefore one has, for almost all $v_i \in S_i$,

$$\overline{U}_i'(v_i;m) = \overline{a}_i(v_i;\rho).$$

Since $\overline{U}_i(\cdot;m)$ is continuous and its right-hand derivative is non-decreasing on S_i, it is convex and hence absolutely continuous on S_i (Propositions 5.17 and 5.18 in Royden, 1968). Furthermore, every absolutely continuous function is the indefinite integral of its derivative (Theorem 5.14 in Royden, 1968) and therefore one has, for any two $v_i, \widehat{v}_i \in S_i$,

$$\int_{\widehat{v}_i}^{v_i}\overline{a}_i(x_i;\rho)dx_i = \overline{U}_i(v_i;m)-\overline{U}_i(\widehat{v}_i;m). \qquad (B.5)$$

which establishes $(P.1.2)$ and therefore completes the proof of Proposition 1.‖

Proposition 3. *For any mechanism $m \equiv (\rho,\tau)$ satisfying BIC* and *$EXABB$, there exists a tax function τ' such that $m' \equiv (\rho,\tau')$ satisfies BIC and $EXPBB$, and for every $v \in S$, $i \in I$ one has*

$$\overline{U}_i(v_i;m') = \overline{U}_i(v_i;m). \qquad (B.6)$$

Proof. Let $m \equiv (\rho,\tau)$ be a mechanism satisfying BIC and $EXABB$. It is first shown that there exists a tax function τ' such that the mechanism $m' \equiv (\rho,\tau')$ satisfies BIC, $EXPBB$ and $(B.6)$. The proof proceeds by constructing τ' as follows: for each $i \in I$, let

$$c_i = E\left[\left(\frac{1}{N-1}\right)\left(\sum_{j\in I\setminus\{i\}}\overline{t}_j(V_j;\tau)\right)-\frac{[C(\rho(V))]}{N}\right],$$

and let $\tau' : S \to \Re^N$ be defined, for each $\widehat{v} \in S$ and each $i \in I$, by

$$\begin{aligned}\tau_i'(\widehat{v}) &= \overline{t}_i(\widehat{v}_i;\tau)+\frac{C(\rho(\widehat{v}))-E\left[C(\rho(\widehat{v}_i,V_{-i}))\right]}{N}+c_i\\ &\quad -\frac{1}{N-1}\left[\sum_{j\in I\setminus\{i\}}\left(\overline{t}_j(\widehat{v}_j;\tau)-\frac{E\left[C(\rho(\widehat{v}_j,V_{-j}))\right]}{N}\right)\right].\end{aligned}$$

It is shown that $m' \equiv (\rho, \tau')$ satisfies $(B.6)$ and BIC. From above one has, for any $i \in I, \hat{v}_i \in S_i$

$$\begin{aligned} \bar{t}_i(\hat{v}_i; \tau') &= t_i(\hat{v}_i; \tau) + c_i - \left(\frac{1}{N-1}\right) E\left(\sum_{j \in I \setminus \{i\}} \bar{t}_j(V_j; \tau^*)\right) + \frac{1}{N} E\left[C(\rho(V))\right] \\ &= t_i(\hat{v}_i; \tau) \end{aligned}$$

Thus, m' satisfies for any $i \in I, \hat{v}_i \in S_i$,

$$\overline{U}_i(\hat{v}_i; m') = \overline{U}_i(\hat{v}_i; m).$$

Hence by Proposition 1, m' satisfies BIC provided m satisfies BIC. To show m' satisfies $EXPBB$, note that for each $v \in S$ one has

$$\sum_{i \in I} \tau_i'(v) = C(\rho(v)) + \sum_{i \in I} c_i.$$

On the other hand, from the definition of $c = (c_1, c_2, ..., c_N)$ and that m satisfies $EXABB$

$$\sum_{i \in I} c_i = E\left[\left(\sum_{i \in I} \bar{t}_i(V_i; \tau)\right) - C(\rho(V))\right] = 0,$$

which implies

$$\sum_{i \in I} \tau_i'(v) = C(\rho(v)).$$

Therefore m' satisfies $EXPBB$. This completes the proof of Proposition 3.||

Proposition 4. *Let $m \equiv (\rho, \tau)$ be a mechanism satisfying BIC and $EXABB$ and suppose that for each $i \in I$ and $v_{-i} \in S_{-i}$, $\rho_i(\cdot, v_{-i})$ is non-decreasing on S_{-i}. Then there exists a tax function τ' such that the mechanism $m' \equiv (\rho, \tau')$ satisfies $DSIC$ and $EXABB$, and for all $v \in S, i \in I$ one has*

$$\overline{U}_i(v_i; m') = \overline{U}_i(v_i; m).$$

Proof

Let $m \equiv (\rho, \tau)$ be a mechanism satisfying BIC and $EXABB$ and suppose that, for each $i \in I, v_{-i} \in S_{-i}$, $\rho_i(\cdot, v_{-i})$ is non-decreasing. A tax function τ' such that $m' \equiv (\rho, \tau')$ satisfies $DSIC$ and $EXABB$ is constructed as follows. For each $i \in I$, let $T_i : S_{-i} \to \Re$ be an arbitrary integrable function (with respect to F), and let $v^* \in S$ be arbitrary. Let now $\tau' : S \to \Re^N$ be defined, for each $v \in S$, $i \in I$, by

$$\tau_i'(v) = v_i \rho_i(v) - \int_{v_i^*}^{v_i} \rho_i(x_i, v_{-i}) dx_i - T_i(v_{-i}).$$

Note that for every $i \in I, v_i, \widehat{v}_i \in S_i, v_{-i} \in S_{-i}$ one has

$$v_i\rho_i(v) - \tau_i'(v) - [v_i\rho_i(\widehat{v}_i, v_{-i}) - \tau_i'(\widehat{v}_i, v_{-i})] = \int_{\widehat{v}_i}^{v_i} \rho_i(x_i, v_{-i})dx_i - (v_i - \widehat{v}_i)\,\rho_i(\widehat{v}_i, v_{-i}).$$

Since for each $i \in I, v_{-i} \in S_{-i}$, $\rho_i(\cdot, v_{-i})$ is non-decreasing one has

$$\int_{\widehat{v}_i}^{v_i} \rho_i(x_i, v_{-i})dx_i - (v_i - \widehat{v}_i)\,\rho_i(\widehat{v}_i, v_{-i}) \geq 0.$$

Thus,

$$v_i\rho_i(v) - \tau_i'(v) - [v_i\rho_i(\widehat{v}_i, v_{-i}) - \tau_i'(\widehat{v}_i, v_{-i})] \geq 0 \qquad (B.7)$$

Therefore the mechanism $m' \equiv (\rho, \tau')$ satisfies $DSIC$. Note also that $(B.7)$ holds for any arbitrary selection of the function $T_i(\cdot)$. Thus, there is no loss of generality in assuming that each function $T_i(\cdot)$ in the definition of τ' is selected to satisfy

$$E(\bar{t}_i(V_i; \tau')) = E(\bar{t}_i(V_i; \tau)).$$

Thus, m' satisfies $EXABB$. Since both m and m' satisfy BIC, they both satisfy $(P.1.2')$. Taking into account that both mechanisms have the same provision rule one obtains, for each $i \in I$, $v_i \in S_i$,

$$\overline{U}_i(v_i; m') = \overline{U}_i(v_i; m),$$

which establishes that m and m' yields the same interim expected utilities to all agents. This completes the proof of proposition 4.||

Lemma 1. *Every $WIIE$ mechanism is IIE.*

Proof. Consider an arbitrary $m \in$M. It is shown that if there exists $m' \in$M satisfying $(E.1)$ and $(E.2)$, then there exists m'' satisfying, for all $v \in S$ and $i \in I$,

$$\overline{U}_i(v_i; m'') > \overline{U}_i(v_i; m). \qquad (B.8)$$

which establishes *Lemma* 1.
(1) Note that to obtain m'' is trivial if there exists $j \in I$, $\epsilon > 0$ such that, for each $v_j \in S_j$ one has

$$\overline{U}_j(v_j; m') > \overline{U}_j(v_j; m) + \epsilon. \qquad (B.9)$$

To see why, suppose such $j \in I, \epsilon > 0$ exist and let $\overline{T} \in \Re^N$ be defined, for all $i \in I \setminus \{j\}$, by

$$\overline{T}_i = E\bar{t}_i(V_i; \tau') - \frac{\varepsilon}{N-1}$$

and, for $i = j$, by

$$\overline{T}_j = E\bar{t}_j(V_j; \tau') + \varepsilon.$$

Using Propositions 1 and 2, there exists a mechanism $m'' \in$M with the same provision rule as that of m' whose tax function satisfies, for all $i \in I$,

$$E\tilde{t}_i(V_i;\tau') = \overline{T}_i.$$

Note that such mechanism m' yields, for all $i \in I\backslash\{j\}$,

$$\overline{U}_i(v_i;m'') = \overline{U}_i(v_i;m') + \frac{\varepsilon}{N-1} > \overline{U}_i(v_i;m')$$

and, for $i = j$,

$$\overline{U}_j(v_j;m'') = \overline{U}_j(v_j;m') - \varepsilon > \overline{U}_i(v_j;m).$$

Therefore m'' satisfies $(B.9)$.

(2) It is now that a mechanism m'' satisfying $(E.1), (E.2)$ and $(B.9)$ can be constructed from any m' satisfying $(E.1)$ and $(E.2)$ but not $(B.9)$. Construction is as follows. Let $j \in I$, $v_j, \widehat{v}_j \in S_j, \epsilon > 0$ be such that

$$\overline{U}_j(\widehat{v}_j;m') - \overline{U}_j(\widehat{v}_j;m) > \epsilon$$

and

$$0 \leq \overline{U}_j(v_j;m') - \overline{U}_j(v_j;m) < \epsilon$$

Note that such $j \in I$, $v_j, \widehat{v}_j \in S_j, \epsilon > 0$ exists since m' satisfies $(E.1)$ and $(E.2)$ but not $(B.9)$. Construction of m'' depends on whether $v_j > \widehat{v}_j$ or $\widehat{v}_j > v_j$. Suppose first that $v_j > \widehat{v}_j$. From $(P.1.2)$ one obtains

$$\int_{\widehat{v}_j}^{v_j} [\overline{a}_j(x_j;\rho') - \overline{a}_j(x_j;\rho)]\, dx_j < 0,$$

which taking into account that both ρ and ρ' are non-decreasing functions implies

$$\overline{a}_j(v_j;\rho) > \overline{a}_j(\widehat{v}_j;\rho). \tag{B.10}$$

Let a be such that

$$\overline{a}(v_j - \widehat{v}_j) = \int_{\widehat{v}_j}^{v_j} \overline{a}_j(x_i;\rho)\, dx_j, \tag{B.11}$$

which exists provided $\overline{U}_j(\cdot;m)$ is continuous. Since $(B.10)$ is satisfied, there exists $v_j^* > \widehat{v}_j$ such that, for every $v_j \in \left(\widehat{v}_j, v_j^*\right)$ one has,

$$\overline{a}(v_j - \widehat{v}_j) > \int_{\widehat{v}_j}^{v_j} \overline{a}_j(x_j;\rho)\, dx_j. \tag{B.12}$$

Let now let $\rho^* : S \to A$ be defined, for each $v \in S$ and $i \in I\backslash\{j\}$, by

$$\rho_i^*(v) = \rho_i'(v),$$

and for each $v \in S$ and $i = j$, by

$$\rho_j^*(v) = \rho_j'(v) + I_{\left[\widehat{v}_j, v_j^*\right]}(v_j)\left[\overline{a} - \rho_j'(v)\right];$$

where $I_{\left[v_j, v_j^*\right]} : S_j \to \{0,1\}$ is the characteristic function of the interval $\left[\widehat{v}_j, v_j^*\right]$. Such provision rule yields, for each $i \in I \backslash \{j\}$ and each $v_i \in S_i$

$$\overline{a}_i(x_i; \rho^*) = \overline{a}_i(x_i; \rho');$$

and for each $v_j \in S_j$,

$$\overline{a}_j(v_j; \rho^*) = \begin{cases} \overline{a}_j(v_j; \rho'), & \text{if } v_j \notin \left[\widehat{v}_j, v_j^*\right], \\ \overline{a} & \text{if } v_j \in \left[\widehat{v}_j, v_j^{*\prime}\right]. \end{cases}.$$

Since $\overline{a} \in \left[\overline{a}_j\left(\widehat{v}_j; \rho\right), \overline{a}_j\left(v_j^*; \rho\right)\right]$, the function $\overline{a}_j(\cdot; \rho)$ is increasing on S_j. By Proposition 2, there exists a tax function τ^* such that the mechanism $m^* \equiv (\rho^*, \tau^*) \in \mathcal{M}$ satisfies, for each $i \in I$,

$$E\overline{t}_i(V_i; \tau^*) = E\overline{t}_i(V_i; \tau').$$

By Proposition 1 such mechanism satisfies $(P.1.2)$ and therefore one obtains, for each $i \in I \backslash \{j\}$and $v_i \in S_i$,

$$\overline{U}_i(v_i; m^*) = \overline{U}_i(v_i; m'),$$

and for any $v_j \in S_j$

$$\overline{U}_j(v_j; m^*) = \begin{cases} \overline{U}_j(v_j; m'), & \text{if } v_j \notin \left[\widehat{v}_j, v_j^*\right], \\ \overline{U}_j(\widehat{v}_j; m') + \overline{a}\left(v_j - \widehat{v}_j\right) & \text{if } v_j \in \left[\widehat{v}_j, v_j^*\right]. \end{cases}.$$

Since $(B.12)$ is satisfied one has, for all $v_j \in \left[\widehat{v}_j, v_j^*\right]$

$$\overline{U}_j(v_j; m^*) - \overline{U}_j(v_j; m) > \epsilon.$$

Thus, m^* satisfies $(B.9)$ for any $v_j \in \left[\widehat{v}_j, v_j''\right]$. The case for which $v_j' < \widehat{v}_j$ is entirely analogous. Proceeding recursively, m^* can be constructed in such a way that $(B.9)$ is satisfied for any arbitrary $v_j \in S_j$. Then, proceeding as in stage (1), a mechanism m^* that yields higher interim utilities to all agents and all possible valuations can be obtained. This completes the proof of Lemma 1.‖

Proposition 4. *A mechanism $m^* \equiv (\rho^*, \tau^*)$ is interim incentive efficient if and only if there exists a welfare distribution $G \in \mathcal{G}$ such that m^* solves*

$$\max_{m \in \mathcal{M}} \mathcal{U}(m, G). \tag{PR1}$$

Proof. To prove necessity, let $m^* \equiv (\rho^*, \tau^*)$ be an arbitrary interim efficient mechanism and consider the sets

$$\mathcal{H} \equiv \left\{ s : S \to \Re \in L^1[F] : \; s(v) > \sum_{i \in I} \overline{U}_i(v_i; m^*), \; \forall v \in S \right\},$$

and

$$\mathcal{U} \equiv \left\{ s : S \to \Re \in L^1[F] : \; \exists \, m \in \mathcal{M} : \; s(v) = \sum_{i \in I} \overline{U}_i(v_i; m), \; \forall \, v \in S \right\}.$$

Note that since m^* is interim efficient, $\mathcal{H}$ and $\mathcal{U}$ are two disjoint, convex subsets of the linear space $L^1[F]$, with both sets having a non-empty interior. By the separating hyperplane theorem (Theorem 10.20 in Royden, 1968), there exists a non zero bounded linear functional $l : L^1[F] \to \Re$ separating $\mathcal{H}$ and $\mathcal{U}$, that is, there exists a bounded linear functional l defined on $L^1[F]$ and a real number r such that

$$l(s^h) \geq r$$

for all $s^h \in \mathcal{H}$, and

$$l(s^m) \leq r$$

for all $s^m \in \mathcal{U}$. The function $s^* \equiv \sum_i \overline{U}_i(\cdot; m^*)$ belongs to $\mathcal{U}$. Therefore, for each $s^h \in \mathcal{H}$ one has

$$l(s^h) \geq l(s^*). \tag{B.13}$$

Moreover, since m^* is interim efficient one has, for each $s^m \in \mathcal{U}$,

$$l(s^*) \geq l(s^m) \tag{B.14}$$

By Riesz's Representation Theorem (Theorem 11.29 in Royden, 1968) for every bounded linear functional $l : L^1[F] \to \Re$ there exists a function $g : S \to \Re^N \in L^\infty[F]$ such that for all $s \in L^1[F]$ one has

$$l(s) = \int_S g(x) s(x) dF(x).$$

Hence from $(B.13)$ and $(B.14)$, g satisfies, for each $s^h \in \mathcal{H}$ and $s^m \in \mathcal{U}$,

$$\int_S g(x) s^h(x) dF(x) \geq \int_S g(x) s^*(x) dF(x) \geq \int_S g(x) s^m(x) dF(x). \tag{B.15}$$

It follows that for each $s^h \in \mathcal{H}$ and $m \in \mathcal{M}$ one has

$$\int_S g(x) s^h(x) dF(x) \geq \int_S g(x) \left(\sum_{i \in I} \overline{U}_i(x_i; m^*) \right) dF(x) \geq \int_S g(x) \left(\sum_{i \in I} \overline{U}_i(x_i; m) \right) dF(x). \tag{B.16}$$

Since the mechanism $m_0 \equiv (0,0) \in \mathcal{M}$, the above inequality implies

$$\int_S g(x)s^h(x)dF(x) \geq \int_S g(x)\left(\sum_{i\in I}\overline{U}_i(x_i;m^*)\right)dF(x) \geq 0.$$

I show now that g is non-negative almost everywhere on S, that is

$$\int_{\{x\in S:g(x)<0\}} dF(x) = 0. \tag{B.17}$$

To see this, suppose not, and consider a function $s \in \mathcal{H}$ defined, for all $v \in S$, by

$$s(v) = s^*(v) + \alpha I_{\{g(v)<0\}}(v),$$

where $\alpha > 0$ is selected in such a way that

$$\int_S g(x)s(v)dF(x) < 0.$$

Note such function s exists provided $(B.17)$ is not satisfied. Thus $s \notin \mathcal{H}$, a contradiction that establishes $(B.17)$.

Let $G : S \to [0,1]$ be defined, for all $v \in S$, by

$$G(v) = \frac{1}{\int_S g(x)dF(x)}\left(\int_{x\leq v} g(x)dF(x)\right).$$

Taking into account that g is bounded (and hence integrable with respect to F), it follows that G is a nonnegative, absolutely continuous, non-decreasing function on S, and satisfies $\int_S dG(x) = 1$. Note that, for each $i \in I$ one has

$$\int_S\left(\sum_i |x_i|\right)dG(x) \leq \frac{\sup\{g(x) : x \in S\}}{\int_S g(x)dF(x)}E\left(\sum_{i\in I}|V_i|\right) < \infty$$

provided $E\left(\sum_{i\in I}|V_i|\right) \leq \infty$. Hence, $G \in \mathcal{G}$. Finally, from the inequalities in $(B.16)$ one has, for any $m \in \mathcal{M}$,

$$\int_S\left(\sum_{i\in I}\overline{U}_i(x_i;m^*)\right)dG(x) \geq \int_S\left(\sum_{i\in I}\overline{U}_i(x_i;m)\right)dG(x),$$

which establishes that m^* maximizes $\mathcal{U}(\cdot,G)$ on $\mathcal{M}$, therefore completing the proof of the *Only if* statement in Theorem 1.

To prove sufficiency, let m^* be a solution to $PR1$ for some $G \in \mathcal{G}$. To prove it is interim efficient, suppose not. By *Lemma* 1, there exists a mechanism $m \in \mathcal{M}$ such that

$$\overline{U}_i(v_i;m) > \overline{U}_i(v_i;m^*) \text{ for all } i \in I, v_i \in S_i$$

Therefore

$$\int_S \left(\sum_{i\in I} \overline{U}_i(x_i; m')\right) dG(x) \geq \int_S \left(\sum_{i\in I} \overline{U}_i(x_i; m^*)\right) dG(x),$$

which contradicts that m^* solves $PR1$. This completes the proof of Theorem 1.‖

Lemma 2. *For every $i \in I$ and every $G \in \mathcal{G}$, a generalized virtual valuation function with respect to G* always exists.

Proof. Recall that a real valued function $W_i^*(\cdot; G)$ defined on S_i is a generalized virtual valuation function with respect to G) if (a) it is continuous and non decreasing on S_i, and (b) there exists a (possibly empty) countable collection $\{C_k^i : k \in K\}$ of disjoint intervals in Γ such that, for each $v_i \in S_i$ one has

$$W_i^*(v_i; G) = \begin{cases} \frac{\int_{C_k^i} W_i(x_i;G)dF_i(x_i)}{\int_{C_k^i} dF_i(x_i)} & \text{if } v_i \in C_k^i \text{ for some } k \in K. \\ W_i(v_i; G), & \text{otherwise ;} \end{cases} \tag{b.1}$$

and

$$\int_{x_i \leq v_i} W_i^*(x_i, G)\, dF_i(x_i) \leq \int_{x_i \leq v_i} W_i(x_i; G)\, dF(x_i). \tag{b.2}$$

To show *Lemma* 2 is satisfied, let $i \in I, G \in \mathcal{G}$ be arbitrary. It is now shown that one can always find a collection $C^i = \{C_k^i : k \in K\}$ of intervals in Γ and a function $W_i^*(\cdot; G)$ such that (a) and (b) are satisfied.

Some previous definitions are needed. Let $v_i^* \in S_i$ be such that

$$W_i(v_i^*; G) = \int_{S_i} W_i(x_i; G)\, dF_i(x_i).$$

Also, let $h^1 : \Gamma \to \Re$ and $h^2 : \Gamma \to \Re$ be defined, for all $(a, b) \in \Gamma$, by

$$h^1(a, b) = \int_a^b (W_i(b, G) - W_i(x_i, G))\, dF_i(x_i),$$

and

$$h^2(a, b) = \int_a^b (W_i(a, G) - W_i(x_i, G))\, dF_i(x_i),$$

Let

$$C^1 \equiv \left\{\left(a_k^1, b_k^1\right) \in \Gamma : k \in K^1\right\}$$

be a (possibly empty) collection of intervals in the set $S_i^1 = \{x \in S_i : x \leq v_i^*\}$ constructed as follows. Let $(a_1^1, b_1^1) = \emptyset$ if $W_i(\cdot; G)$ is non-decreasing on S_i^1; otherwise, let

$$b_1^1 = \sup\left\{b < v_i^* : h^1(a, b) < 0 \text{ for some } a < b\right\}$$

and

$$a_1^1 = \begin{cases} \inf\{a \in S_i : \ h^1(a, b_1^1) < 0\}, & \text{if } \{a \in S_i : \ h^1(a, b_1^1) < 0\} \text{ is bounded,} \\ -\infty, & \text{otherwise.} \end{cases}$$

To complete the construction of C^1 for $k \geq 2$, let $(a_k^1, b_k^1) = \emptyset$ if either one of the following is satisfied:

$$\left(a_{k-1}^1, b_{k-1}^1\right) = \emptyset,$$

$$a_{k-1}^1 = -\infty,$$

or

$$h^1(a, b) > 0 \text{ for all } (a, b) \in \Gamma : a < a_{k-1}^1;$$

otherwise, let

$$b_k^1 = \sup\left\{b < a_{k-1} : h^1(a, b) < 0 \text{ for some } a < b\right\},$$

and

$$a_k^1 = \begin{cases} \inf\{a \in S_i : h^1(a, b_k^1) < 0\}, & \text{if } \{a \in S_i : h^1(a, b_k^1) < 0\} \text{ is bounded,} \\ -\infty, & \text{otherwise.} \end{cases}$$

Let now

$$C^2 \equiv \left\{\left(a_k^2, b_k^2\right) \in \Gamma : k \in K^2\right\}$$

be a (possibly empty) collection of intervals defined as follows. For $k = 1$, let $(a_1^2, b_1^2) = \emptyset$ if $W_i(\cdot, G)$ is non-decreasing on the set $S_2 = \{x \in S_i : x \geq v_i^*\}$;otherwise, let (a_1^2, b_1^2) be defined by

$$a_1^2 = \inf\left\{a > v_i^* : h^2(a, b) < 0 \text{ for some } b > a\right\}$$

and

$$b_1^2 = \begin{cases} \sup\{b \in S_i : \ h^2(a_1^2, b) < 0\}, & \text{if } \{b \in S_i : \ h^2(a_1^2, b) < 0\} \text{ is bounded,} \\ \infty, & \text{otherwise.} \end{cases}$$

To complete the definition of C^2 for $k \geq 2$, let $(a_k^2, b_k^2) = \emptyset$ if either one of the following is satisfied:

$$\left(a_{n-1}^2, b_{n-1}^2\right) = \emptyset,$$

$$a_{k-1}^2 = +\infty,$$

or

$$h^2(a, b) > 0 \text{ for all } (a, b) \in \Gamma : a > b_{k-1}^2;$$

otherwise, let

$$a_k^2 = \inf\left\{a > b_{k-1}^2 : h^2(a, b) < 0 \text{ for some } b > a\right\},$$

and

$$b_k^2 = \begin{cases} \sup\{b \in S_i : h^2(a_k^2, b) < 0\}, & \text{if } \{b \in S_i : h^2(a_k^2, b) < 0\} \text{ is bounded,} \\ +\infty, & \text{otherwise.} \end{cases}.$$

Let now

$$C^i \equiv \{(a_k, b_k) \in \Gamma : k \in K\}$$

be a collection of intervals constructed as follows. If either $b_1^1 < v_i^*$ or $v_i^* < a_1^2$ is satisfied, then let

$$C^i = C^1 \cup C^2;$$

and if $b_1^1 = v_i^* = a_1^2$, then let C^i be the collection formed by the interval (a_1^1, b_1^2) and all intervals in $C^1 \cup C^2$ except the intervals (a_1^1, b_1^1) and (a_1^2, b_1^2).

Observe that C^i has been constructed in such a way that for each $k \in K$ such that (a_k, b_k) is non-empty one has

- $$W_i(a_k; G) = W_i(b_k; G) = \frac{\int_{a_k}^{b_k} W_i(x_i; G)\, dF_i(x_i)}{\int_{a_k}^{b_k} dF_i(x_i)} \text{ whenever } a_k > -\infty \text{ and } a_k < \infty; \tag{B.18}$$

- $$W_i(a_k; G) = \frac{\int_{a_k}^{b_k} W_i(x_i; G)\, dF_i(x_i)}{\int_{a_k}^{b_k} dF_i(x_i)} \text{ whenever } a_k > -\infty \text{ and } b_k = \infty; \tag{B.19}$$

- $$W_i(b_k; G) = \frac{\int_{a_k}^{b_k} W_i(x_i; G)\, dF_i(x_i)}{\int_{a_k}^{b_k} dF_i(x_i)} \text{ whenever } a_k = -\infty \text{ and } b_k < \infty; \tag{B.20}$$

and

$$W_i(v_i^*; G) = \frac{\int_{-\infty}^{\infty} W_i(x_i; G)\, dF_i(x_i)}{\int_{-\infty}^{\infty} dF_i(x_i)} \text{ whenever } a_k = -\infty \text{ and } b_k = \infty. \tag{B.20}$$

With this observation in mind, let now $W_i^*(\cdot; G) : S_i \to \Re$ be defined, for all $v_i \in S_i$, by

$$W_i^*(v_i; G) = \begin{cases} \frac{\int_{C_k^i} W_i(x_i; G) dF_i(x_i)}{\int_{C_k^i} dF_i(x_i)} & \text{if } v_i \in C_k^i \text{ for some } k \in K. \\ W_i(v_i; G), & \text{otherwise ;} \end{cases} \tag{B.21}$$

Clearly, $W_i^*(\cdot, G)$ satisfies $(b.1)$.

I show now that $W_i^*(\cdot, G)$ is continuous and non-decreasing on S_i. To show this, observe first that C^i has been constructed in such a way that, for each $k \in K$,

$W_i^*(\cdot;G)$ is non-decreasing on the interval (b_k, a_{k+1}). Since G_i is non-decreasing and right continuous, this implies that G_i is continuous on (b_k, a_{k+1}). Therefore $W_i(\cdot;G)$ (and hence, $W_i^*(\cdot;G)$) is continuous and non-decreasing on (b_k, a_{k+1}). Also, for each $k \in K$, $W_i^*(\cdot;G)$ is constant through (a_k, b_k). Thus, in order to show $W_i^*(\cdot,G)$ is continuous and non-decreasing on S_i, it suffices to show that $W_i^*(\cdot,G)$ is continuous at every v_i such that there exists $k \in K$ for which either $v_i = a_k$ or $v_i = b_k$ is satisfied. Using $(B.1)$ and $(B.18)$ to $(B.21)$ one obtains, for any such v_i,

$$\lim_{x_i \to v_i} W_i^*(v_i;G) = W_i^*(v_i;G) = W_i(v_i;G), \tag{B.22}$$

which establishes $W_i^*(\cdot,G)$ is continuous and non-decreasing on S_i.

I show now that $W_i^*(\cdot,G)$ satisfies $(b.2)$. Since $W_i^*(\cdot,G)$ satisfies $(b.1)$, it is sufficient to show that for each $k \in K$ and each $v_i \in (a_k, b_k)$ one has

$$\int_{a_k}^{v_i} [W_i^*(x_i;G) - W_i(x_i;G)]\, dF_i(x_i) \leq 0,$$

that is

$$\int_{v_i}^{b_k} [W_i^*(x_i;G) - W_i(x_i;G)]\, dF_i(x_i) \geq 0, \tag{B.23}$$

In order to show $(b.2)$ is satisfied, suppose not. Then there exists $k \in K$ and $v_i \in (a_k, b_k)$ such that

$$\int_{v_i}^{b_k} [W_i^*(x_i;G) - W_i(x_i;G)]\, dF_i(x_i) < 0. \tag{B.24}$$

Note that $(B.24)$ implies there exists $b > b_k$ such that

$$\int_{a}^{b} [W_i(b;G) - W_i(x_i;G)]\, dF_i(x_i) < 0, \tag{B.25}$$

By construction b_k is the largest b for which $(B.25)$ is satisfied, which contradicts that $b > b_k$. This establishes that $W_i^*(\cdot;G)$ satisfies $(b.2)$. This completes the proof of *Lemma* 2.||

Theorem 3. *Let $G \in \mathcal{G}$ be arbitrary and let ρ^* be a provision rule satisfying, for each $v \in S, a \in A$,*

$$\sum_{i \in I} W_i^*(v_i;G)\rho_i^*(v) - C(\rho^*(v)) \geq \sum_{i \in I} W_i(v_i;G)\, a_i - C(a) \tag{T.3}$$

for some collection $\{W_i^*(\cdot;G) : i \in I\}$ of generalized virtual valuation functions with respect to G. Then ρ^* solves PR2.

Proof. Let $G \in \mathcal{G}$ be arbitrary and let ρ^* be a provision rule satisfying $(T.3)$ for some collection $\{W_i^*(\cdot;G) : i \in I\}$ of generalized virtual valuation functions with respect to G. For each $i \in I$, let $h_i : S_i \to \Re$ be defined, for each $v_i \in S_i$, by

$$l_i(v_i) = W_i^*(v_i;G) - W_i(v_i;G).$$

Let now $\mathcal{H}:P \to \Re$ be defined, for each $\rho \in P$, by

$$\begin{aligned}\mathcal{H}(\rho) &= \int_S \left[\sum_{i\in I} \rho_i(x) W_i^*(x_i;G) - C(\rho(x))\right] dF(x) \\ &= \int_S \left[\sum_{i\in I} \rho_i(x) \left(W_i(x_i,G) + h_i(x_i)\right) - C(\rho(x))\right] dF(x),\end{aligned}$$

that is

$$\mathcal{H}(\rho) = \mathcal{W}(\rho,G) + \sum_{i\in I} \int_{S_i} h_i(x_i)\overline{a}_i(x_i;\rho)dF_i(x_i).$$

Clearly, ρ^* maximizes $\mathcal{H}(\cdot)$ on P. To show such provision rule maximizes $\mathcal{W}(\cdot,G)$ on P, recall first that for any provision rule $\rho \in P$, each function $\overline{a}_i(\cdot,\rho)$ is non-decreasing on S_i. For each $i \in I$, let $C^i \equiv \{C_k^i : k \in K\}$ be a collection of intervals such that the pair $[W_i^*(\cdot;G), C^i]$ satisfies (a) and (b). Such collection exists provided $W_i^*(\cdot;G)$ is a generalized virtual valuation function. Note also for each $i \in I$, the function h_i satisfies, for each B such that $B \notin \cup_{k\in K} C_k^i$ and each $v_i \in B$,

$$h_i(x_i) = 0.$$

Therefore the term

$$\int_{S_i} h_i(x_i)\overline{a}_i(x_i;\rho)dF_i(x_i)$$

can be written equivalently as

$$\int_{S_i} h_i(x_i)\overline{a}_i(x_i;\rho)dF_i(x_i) = \sum_{k\in K} \int_{C_k^i} h_i(x_i)\overline{a}_i(x_i;\rho)dF_i(x_i).$$

For each $i \in I$, let $H_i : S_i \to \Re_-$ be defined, for each $v_i \in S_i$, by

$$H_i(v_i) = \int_{x_i \le v_i} h_i(x_i)dF_i(x_i).$$

Taking into account that the function $W_i^*(.;G)$ satisfies (a) and (b) one obtains, for each $k \in K$ and each $v_i \in (a_k, b_k)$

$$H_i(v_i) \le 0 \text{ and } H(b_k) = 0.$$

Also, note that

$$\int_{a_k}^{b_k} h_i(x_i)\overline{a}_i(x_i;\rho)dF_i(x_i) = \int_{a_k}^{b_k} \overline{a}_i(x_i;\rho)dH_i(x_i),$$

which, integrating by parts and taking into account that $\overline{a}_i(\cdot;\rho)$ is non-decreasing yields

$$\int_{a_k}^{b_k} \overline{a}_i(x_i;\rho)dH_i(x_i) \ge H_i(b_k)\overline{a}_i(b_k;\rho) - H_i(a_k)\overline{a}_i(a_k;\rho) \ge 0.$$

Also,

$$\overline{a}_i(v_i;\rho^*) = \overline{a}_i(a_k;\rho^*) = \overline{a}_i(b_k;\rho^*).$$

Then,

$$\int_{S_i} h_i(x_i)\overline{a}_i(x_i;\rho^*)dF_i(x_i) = \sum_{k\in K} \overline{a}_i(a;\rho^*) \int_{a_k}^{b_k} h_i(x_i)dF_i(x_i) = 0.$$

Since i is arbitrary one has, for any provision rule $\rho \in P$,

$$\mathcal{H}(\rho^*) = \mathcal{W}(\rho^*, G) \geq \mathcal{W}(\rho, G) + \sum_{i\in I}\sum_{k\in K} \int_{a_k}^{b_k} h_i(x_i)\overline{a}_i(x_i;\rho)dF_i(x_i) \geq \mathcal{W}(\rho, G),$$

which establishes that ρ maximizes $\mathcal{W}(\cdot, G)$ on P. This completes the proof of Theorem 2.||

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