MEASURING THE DEGREE OF FULFILLMENT OF THE LAW OF ONE PRICE. APPLICATIONS TO FINANCIAL MARKETS INTEGRATION

Alejandro Balbás and María José Muñoz [*]

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Many empirical papers analyze well-Known arbitrage strategies. Our measures present an important advantage over this approach, since we globally focus on the market to find its arbitrage opportunities, without studying special strategies.

The developed theory is also applied to markets with frictions, and to study the integration of different financial markets. Our measures are continuous with respect to previous measures in the literature, and seem to be better than them since they compute how much money the agents can win due to the arbitrage opportunities in a financial market, or among different ones.

*BALBAS, Departamento de Economía de la Empresa de la Universidad Carlos III; MUÑOZ, Departamento de Economía de la Universidad Carlos III

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Alejandro BALBÁS* and María José MUÑOZ BOUZO[†]

Abstract

This paper gives two measures of the degree of fulfillment of the Law of One Price. These measures are characterized by means of saddle point conditions, and are therefore easy to compute in practical situations.

Many empirical papers analyze well-known arbitrage strategies. Our measures present an important advantage over this approach, since we globally focus on the market to find its arbitrage opportunities, without studying special strategies.

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Key Words and Phrases. Law of One Price. Arbitrage. State Prices. Relative Measure. Financial Markets Integration.

Introduction.

The fulfillment of the Law of One Price (LOP) or the absence of arbitrage opportunities in financial markets may be characterized by the existence of state prices with appropriate properties (Ingersoll (1987) or Chamberlain and Rothschild(1983)). Chen and Knez (1995) characterize the absence of arbitrage opportunities among different financial markets by means of the vanishing of a certain integration measure. Furthermore, their measure has the important property of not depending on the dynamic assumptions applied to price assets. In their empirical analysis of the markets NYSE and NASDAQ , they compute that, based on the data, the two markets seem to violate the strong-form integration (i.e., cross-market arbitrage may be possible). They also check that the measure depends crucially on the frictionless market assumption and consequently they suggest the convenience of extending their discussion to economies with trading frictions and transaction costs.

When the LOP is characterized by the existence of state prices, we may only know whether the LOP holds or does not hold. We follow Chen and Knez(1995)'s idea of introducing a measure to

1

characterize the fulfillment of the LOP. This represented a breakthrough in quantifying pricing discrepancy between markets. The measure we define tries to avoid some of the problems encountered when using the measure proposed by Chen and Knez(1995). First, it is not necessary to deal with more than one market. Second, our measure also vanishes if and only if the LOP holds, and can be positive and small enough whenever the market is very close to the LOP. This makes the models much more flexible and useful in practical situations, since they are less sensitive to measurement errors. Furthermore, they can accurately predict whether the nonfulfillment of the LOP is due to market frictions.

The present paper analyzes first the level of fulfillment of the LOP in a single financial market. Like Chen and Knez (1995), we introduce a measure that does not depend on the dynamic applied to price the assets. Nevertheless, our measure is different from theirs, since ours tests the total amount of money an investor can win when the LOP does not hold. It is well known that any portfolio can be replicated with arbitrary price if the LOP fails. Therefore, we must look for relative measures if we are interested in measuring the level of fulfillment of the LOP in monetary terms. Thus we compute how much money an investor can win from an arbitrage portfolio with respect to the total price of all the exchanged (sold or purchased) assets.

Many empirical papers test the efficiency of financial markets, or even some well-known arbitrage strategies, usually concluding that these strategies could be implemented if the transaction costs were small enough, (Hudson, Dempsey and Keasey (1996) or Kamara and Miller (1995)). The measure we introduce below analyzes the market globally, since we are looking for all the possible ways of replicating portfolios to obtain monetary profits, and therefore it does not only take into account specific strategies. Furthermore, when we compute the measure, (the maximum relative profit), we can discount the transaction costs and guarantee if there still are arbitrage opportunities. This is not possible when the LOP is characterized by the existence of state prices, unless the frictions are assumed to be linear. Hence, the theory we develop here could be considered an alternative to the results of Prisman(1986) for a market with frictions.

As already said, the measure we propose gives the quotient between the total amount of money an agent can win and the price of the assets he/she has to exchange to replicate his/her portfolios. From a mathematical point of view, this measure is obtained from solving the optimization program (10). However, it is not easy to study this program since some of the constraints in (10) are given by strict inequalities. Hence, the existence of a solution to (10) is not guaranteed. Furthermore, even assuming the existence of a solution, the usual analytic techniques for solving (10) do not apply, since the objective is a non-differentiable function.

To avoid these difficulties, we organize the paper as follows. The first section is devoted to present the basic assumptions, the notations and some classic and important results which will frequently be applied. The measure of the degree of fulfillment of the LOP is introduced in the second section. To do it, we begin by assuming that each agent holds a portfolio given by a vector $h = (h_1, h_2, \cdots h_n)$ (which depends on the agent) such that $h_i \geq 0$, where h_i is the number of units on the i-th asset. We also assume that the agents cannot replicate their portfolios by selling more than h_i units in each asset i (an agent cannot sell what he/she does not have). Then program (1_h) gives us the best way for each agent to replicate his/her portfolio. By solving this program we introduce the function $\phi(h)$, which is identically zero if and only if the LOP holds. The fact that ϕ is a homogeneous function of degree one guarantees that program (5) leads to the maximum profit an investor can obtain by replicating the portfolios h relative to the price of h. The optimal value

2

of this program, which, again, vanishes if and only if the LOP holds, will be our measure m of the degree of fulfillment of the LOP.

The third section is devoted to prove a Saddle Point Theorem (Theorem 8), which will be useful to compute the measure m, the portfolio h^* solution of (5) and the portfolio x^* solution of (1_{h^*}). We also show some intuitive interpretations of the Saddle Point Theorem.

Short-selling restrictions disappear in the fourth section: we allow the investors to hold initial portfolios h with short positions in each asset. We also assume there is no limit on the short-selling restrictions that can be taken to replicate portfolios. Under these assumptions, we prove Theorem 11, one of the most important results of the present paper, since it guarantees that after absolutely relaxing the restrictions imposed to the short positions, we obtain the same value for the level of fulfillment (or violation) of the LOP. Hence, m is also the optimal value of program (8), that is, the maximum quotient between the profit an agent can win by replicating (without restrictions) his/her arbitrary portfolio and the price of the sold assets. Furthermore, the best way to replicate is still given by the portfolio x^* already computed. If we are interested in measuring the profit relative to the price of the exchanged assets, Theorem 12 shows that x^* is again the best way to replicate any portfolio. In fact, it is enough to substitute m by $l = \frac{m}{2-m}$ to obtain the optimal value of the relation we are interested in. As we show in this section, both measures m and l have similar properties.

Note that the ideas in the fourth section have important consequences, which are far from evident: although in Section 2 we imposed short-selling restrictions to avoid mathematical difficulties, our results in Section 4 show that the measures m and l do not depend on these restrictions. From the most constrained conditions (no initial short positions are permitted, and the agents cannot sell what they do not have) to the most relaxed ones (no limit on the short positions of the initial portfolio h to be replicated, and no limit on the number of assets to be sold) we obtain the same measure m and the same optimal portfolio x^* to implement the arbitrage. In the first case not all the investors can win the maximum relative profits given by m (an agent needs an initial portfolio proportional to h^*), while in the second case this level of maximum relative profit is available to any agent. Of course, we would obtain the same values for m, l and x^* if we worked under assumptions not so restrictive or not so relaxed.

In the fifth section, we measure the degree of integration of two or more financial markets. To do it, we work in a global market which contains all the assets of the different ones, and we compute m on this global market as a measure of their integration. This represents an alternative to the results in Chen and Knez(1995), where they measure how the markets jointly verify this law by computing the distances among the families of state prices of each market. However, in our work we do not need to impose the assumption that the LOP holds separately on each market. Theorem 13 shows that our measure is continuous with respect to the Chen and Knez(1995) measure (hereafter denoted by g). Therefore, controlling the latter we also control the first. The opposite is false in general: for instance, m can take small values while g remains constant. Two simple examples illustrate this fact and show that g is not always sensitive to the profits the investors can win due to violation of the LOP across the two markets. Their measure, in both examples, remains constant in cases when the LOP fails, which means a serious continuity problem. This also makes the measure very sensitive with respect to eventual errors committed in the measurement process. These difficulties disappear when working with the measure m, as we show after the examples. The first example also illustrates that assuming that the markets do not strictly verify the LOP, the

3

way in which each market is divided into sub-markets has a strong effect on the final value of g, while again m and l avoid these problems.

The last section summarizes the most important conclusions of this paper. All our results are focused on studying the degree of fulfillment (or violation) of the LOP in a financial market or among different ones, but an analogous analysis may be extended to measure the level of arbitrage opportunities (in the strong form). To do this, we only have to change the equality constraint by an inequality, namely,

$$\sum_{i=1}^{n} x_i \alpha_i(k) = 0 \quad \text{by} \quad \sum_{i=1}^{n} x_i \alpha_i(k) \geq 0.$$

1 Preliminaries

Consider an economy endowed with a Hausdorff compact topological space K, on which the linear space $C(K)$ of all continuous functions over $\mathbb{R}$ is defined. When equipped with the norm $\|\alpha\| = \sup\{|\alpha(k)| \mid k \in K\}$ for any $\alpha \in C(K)$, the space $M(K)$ of Radon measures over K is known to be the dual space of $C(K)$ (Riesz representation Theorem). Here we are assuming that K is the set of outcome states and for some $\alpha \in C(K)$, $\alpha(k)$ represents the payoff of a portfolio in the state of nature k for every $k \in K$. This restriction to continuous contingent claims is made for expositional and mathematical ease. In many papers (Harrison and Kreps(1979), Chamberlain amd Rothschild(1983),Chen and Knez (1995)$\cdots$), Hilbert space methods are used to represent pricing functions and to characterize the absence of arbitrage across a market. Thus, much of their analysis leads them to consider an economy endowed with a probability space on which the space of all square-integrable functions is defined. In dealing with a finite number of states of nature both models coincide with the classical theory, and in most other cases it is possible to deduce one model from the other.

Let the number of assets be finite and indexed by $\{1, \cdots n\}$. The current prices of the assets are $p = (p_1, p_2, \cdots, p_n)$, and the total payoff on the ith asset is $\alpha_i \in C(K)$. We denote such a market by $\mathcal{M}_{p,\alpha}$ where $p = (p_1, p_2, \cdots, p_n)$ and $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_n)$, and we assume that $p_i > 0$ for every $i \in I$ and $\alpha_1(k) > 0$ for every $k \in K$. For a portfolio $x = (x_1, x_2, \cdots, x_n) \in \mathbb{R}^n$, the sum $\sum_{i=1}^{n} x_i \alpha_i$ is its total payoff and $\sum_{i=1}^{n} x_i p_i$ is its current price.

Definition 1 *The law of one price (LOP) holds on the market* $\mathcal{M}_{p,\alpha}$ *if any two portfolios generating the same future payoff have the same price.*

The following result is adopted from Chamberlain and Rothschild (1983).

Lemma 2 *The LOP holds on the market* $\mathcal{M}_{p,\alpha}$ *if and only if there exists* $\mu \in M(K)$ *such that* $\int_K \alpha_i \, d\mu = p_i$ *for every* $i = 1, \cdots n$.

2 Measurement of the degree of fulfillment of the LOP.

In order to define a measure indicating the degree of fulfillment (or violation) of the LOP accross $\mathcal{M}_{p,\alpha}$, consider for every $h = (h_1, \cdots h_n) \in \mathbb{R}^n_+$ the following optimization problem

$$\text{Maximize} \quad -\sum_{i=1}^{n} x_i p_i$$

$$(1_h) \qquad \text{subject to} \quad \left\{ \begin{array}{ll} \sum_{i=1}^{n} x_i \alpha_i(k) = 0 & \text{for every } k \in K \\ x_i \geq -h_i & i = 1, \cdots n \end{array} \right.$$

4

The problem (1_h) describes the process of identifying the portfolio (constrained by the bounds in a short position $h_i \geq 0$) which minimizes the initial investment needed to purchase a portfolio that generates a zero payoff in every state of nature. Thus, a solution to the problem (1_h) represents the maximum profit obtained by an agent replicating his/her portfolio in such a way that he/she cannot sell more than h_i units in each asset i. When the LOP holds on the market, the optimal value in (1_h) is obviously zero.

Lemma 3 (1_h) *is solvable for every* $h \in \mathbb{R}^n_+$.

The proof of the above lemma relies on the fact that if the maximum is achieved for some feasible $x^* = (x_1^*, \cdots, x_n^*)$ then

$$(2) \qquad x_j^* \leq \frac{\displaystyle\sum_{i \neq j} p_i h_i}{p_j} = \beta_j \quad \text{for every } j$$

and the original problem is equivalent to

$$(3_h) \qquad \text{Maximize} \quad -\sum_{i=1}^n x_i p_i$$
$$\text{subject to} \quad \left\{ \begin{array}{ll} \displaystyle\sum_{i=1}^n x_i \alpha_i(k) = 0 & \text{for every } k \in K \\ \beta_i \geq x_i \geq -h_i & i = 1, \cdots n \end{array} \right.$$

The feasible set of (3_h) is non void and compact. Hence, there exists an optimal solution to (3_h) and, consequently, to (1_h).

Denote by F_h the feasible set of (1_h) and by $\phi(h)$ the optimal value in (1_h) , that is,

$$\phi(h) = \max\{-\sum_{i=1}^n x_i p_i \mid x \in F_h\}.$$

It is easily verified that

$$\phi(h + h') \geq \phi(h) + \phi(h') \quad \text{and} \quad \phi(\alpha h) = \alpha \phi(h)$$

for every $h, h' \in \mathbb{R}^n_+$ and $\alpha > 0$, so ϕ is a concave function. In order to prove that ϕ is continuous, we introduce the dual problem for (1_h):

$$(4_h) \qquad \text{Minimize} \quad \sum_{i=1}^n h_i \lambda_i$$
$$\text{subject to} \quad \left\{ \begin{array}{l} \displaystyle\int_K \alpha_i \, d\mu + \lambda_i = p_i \\ \mu \in M(K), \lambda_i \geq 0 \quad \text{for every } i = 1, \cdots n \end{array} \right.$$

Lemma 4 *There is strong duality for* (1_h) *(i.e.,(1_h) and (4_h) are both solvable and there is no duality gap for* (1_h) *and* (4_h)*).*

The above lemma is the key to prove the following result.

Lemma 5 ϕ *is the minimum of some finite number of linear functions. Therefore ϕ is a continous piecewise linear function in* $\mathbb{R}^n_+$.

We can now define a measure as the maximum profit obtained by an investor among all the priced one portfolios h of the bounds in a short position. In these terms the problem is to find $h = (h_1, \cdots h_n) \in \mathbb{R}^n$ so as to

$$
\text{Maximize} \quad \phi(h)
$$
$$
(5) \quad \text{subject to} \quad \left\{ \begin{array}{ll} \sum_{i=1}^{n} h_i p_i = 1 \\ h_i \geq 0 & i = 1, \cdots n \end{array} \right.
$$

Since the feasible set

$$
H = \left\{ h \in \mathbb{R}^n \mid \sum_{i=1}^{n} h_i p_i = 1, \, h_i \geq 0, \, i = 1, \cdots n \right\}
$$

is compact and ϕ is continuous, the maximum is achieved for some $h^* \in H$. This leads to the following definition.

Definition 6 *For a market* $\mathcal{M}_{p,\alpha}$ *satisfying* $p_i > 0$, $i = 1, \cdots n$ *and* $\alpha_1(k) > 0$ *for every* $k \in K$ *the disagreement measure is given by*

$$
m = \phi(h^*) = \max\{\phi(h) \mid h \in H\}.
$$

One can check that, with this definition, m verifies the first requirement to be a measure of non fulfillment of the LOP:

Theorem 7 *The LOP holds on the market* $\mathcal{M}_{p,\alpha}$ *if and only if* $m = 0$.

The requirement of the LOP is thus made testable by estimating m directly. The lower the value of m, the closer the market is to the LOP (i.e., the lower the maximum quotient between the profit and the total price of short-selling restrictions.)

As we will see in Section 5, such a test is also valid for a measurement of market integration: for two or more not perfectly integrated markets (i.e., markets which do not assign the same price to the same future payoff) treated as parts of one combined market, m also indicates the degree of market integration. It is important to point out that m does not depend on the way in which the combined market is divided into smaller groups so that the LOP holds on each one. Only the resultant combined market matters.

3 A saddle point characterization.

Suppose now that the maximum is achieved for $h^* \in H$ and let $x^* \in F_{h^*}$ such that

$$
(6) \qquad m = \phi(h^*) = -\sum_{i=1}^{n} x_i^* p_i
$$

From $x_i^* \geq -h_i^*$ one obtains that

$$
-\sum_{i=1}^{n} x_i^* p_i \leq \sum_{i=1}^{n} h_i^* p_i = 1,
$$

and then

$$
0 \leq m \leq 1.
$$

6

The measure m only depends on the current prices p_i and on the prices over the states of nature α. We denote it by $m_{p,\alpha}$ when prices are not fixed. It is easy to check that $m_{p,k\alpha} = m_{p,\alpha}$ for every $k > 0$, so the disagreement measure is current prices relative.

Lemma 4 ensures that $\phi(h)$, the optimal value of (1_h), can be obtained by solving (4_h), that is,

$$\phi(h) = \min_{\lambda \in \Lambda} \sum_{i=1}^{n} h_i \lambda_i \,,$$

where

$$\Lambda = \left\{ \lambda \in \mathbb{R}_+^n \mid \int_K \alpha_i \, d\mu + \lambda_i = p_i, \; i = 1, \cdots n, \; \mu \in M(K) \right\}.$$

Hence, the problem of finding m can be expressed by a max-min problem

$$m = \max_{h \in H} \min_{\lambda \in \Lambda} U(\lambda, h) \,,$$

where U is defined by $U(\lambda, h) = \sum_{i=1}^{n} h_i \lambda_i$.

Theorem 8 *For the convex-concave function $U(\lambda, h)$ defined above there exists a saddle point (λ^*, h^*) and*

$$m = \max_{h \in H} \min_{\lambda \in \Lambda} U(\lambda, h) = U(\lambda^*, h^*) = \min_{\lambda \in \Lambda} \max_{h \in H} U(\lambda, h).$$

In particular, the LOP holds on the market $\mathcal{M}_{p,\alpha}$ if and only if the function $U(\lambda, h)$ possesses a saddle point at $(0, h)$ for every $h \in H$.

In game theoretic terminology the equality in the above Theorem expresses a two-person zero-sum game of the investor against the "market". Since $\lambda_i = p_i - \int_K \alpha_i \, d\mu$ could be interpreted as the error commited by the "market" in the price of each asset for the state prices μ, the sum $\sum_{i=1}^{n} h_i \lambda_i$ would be the payment from the "market" to the investor due to h and λ. Thus, the investor chooses a priced one portfolio of short-selling bounds in such a way that it maximizes the minimal payment desired by the "market" and solves $\max_{h \in H} \min_{\lambda \in \Lambda} U(\lambda, h)$. The problem, $\min_{\lambda \in \Lambda} \max_{h \in H} U(\lambda, h)$ describes the process by which the "market" counteracts the goal of the investor by choosing the feasible λ (i.e., the feasible implicit state prices μ) which minimizes the maximal payment desired by the investor.

Proposition 9 yields another optimization problem to find out the value of m in practical situations.

Proposition 9 *Suppose the LOP does not hold on the market and consider the following optimization problem*

$$(7) \qquad \begin{array}{c} \textit{Minimize} \; \sum_{i=1}^{n} h_i p_i \\[1mm] \textit{subject to} \; \left\{ \begin{array}{l} \phi(h) \geq 1 \\ h_i \geq 0 \qquad i = 1, \cdots n \end{array} \right. \end{array}$$

Then

i) The optimal value in (7) is $\frac{1}{m}$.

ii) The minimum in (7) is achieved for h if and only if the maximum in (5) is achieved for mh.

Note that Lemma 5 ensures that ϕ is the minimum of some finite number of linear functions. Therefore, the inequality constraint $\phi(h) \geq 1$ may be replaced by a finite number of linear inequality constraints and, Problem (7) is just a finite-dimensional linear program which can be solved by the classical optimization techniques.

4 Other representations of the disagreement measure.

In the previous sections we defined the disagreement measure to be the maximum achieved in the optimization problem (5), which represents the maximum profit with short-selling restrictions, stocks for example, with total current price one. With the aid of some lemmas, we can also express the disagreement measure in two alternative ways. First, the measure represents the maximum profit obtained from a portfolio relative to the price of the sold assets. Second, we establish a relation between the disagreement measure and the maximum profit obtained from a portfolio relative to the price of all exchanged (sold and purchased) assets. This relation could be useful to study arbitrage opportunities in economies with frictions. Proceeding in a similar way to Prisman it might be possible to establish conditions to detect the fulfillment of the LOP in economies with frictions and readapt the disagreement measure to consider these frictions.

Lemma 10 *Given a market $\mathcal{M}_{p,\alpha}$, let $h^* \in H$ and $x^* \in F_{h^*}$ verifying (6), and suppose $\phi(h^*) > 0$. Then*

i) $x_j^* = -h_j^*$ or $h_j^* = 0$ (or both) for every $j = 1, \cdots n$.

ii) In particular, if $x_j^* > -h_j^*$ then $x_j^* > 0$.

Lemma 10 ii) says basically that the portfolio where the maximum profit is achieved either sells all the stock or purchases in each asset.

For a portfolio $x \in \mathbb{R}^n$, denote by S_x the set of indices of the sold assets and by L_x the set of indices of the purchased assets, that is, $S_x = \{i = 1, \cdots n \mid x_i < 0\}$ and $L_x = \{i = 1, \cdots n \mid x_i \geq 0\}$.

Theorem 11 *Assume the existence of a portfolio $x \in \mathbb{R}^n$ such that*

$$\sum_{i=1}^n x_i p_i < 0 \quad and \quad \sum_{i=1}^n x_i \alpha_i(k) = 0$$

for every $k \in K$. Then

i) There exists $i \in \{1, 2, \cdots n\}$ such that $x_i < 0$.

ii) $\dfrac{\sum_{i=1}^n x_i p_i}{\sum_{i \in S_x} x_i p_i} \leq \phi(h^*)$. In particular $\phi(h^*) > 0$.

iii) If $x = x^*$ then $\dfrac{\sum_{i=1}^n x_i p_i}{\sum_{i \in S_x} x_i p_i} = \phi(h^*)$.

Theorem 11 proves that when LOP fails x^* solves the following optimization problem

(8)

$$\text{Maximize} \quad \frac{-\sum_{i=1}^n x_i p_i}{-\sum_{i \in S_x} x_i p_i}$$

$$\text{subject to} \quad \begin{cases} \sum_{i=1}^n x_i \alpha_i(k) = 0 & \text{for every } k \in K \\ \sum_{i=1}^n x_i p_i < 0 \end{cases}$$

8

and $\phi(h^*)$ is then the objective optimal value.

This result is mathematically interesting by itself, because it solves an optimization problem with strict inequality constraints and a non differentiable objective. Furthermore, it gives another representation of the disagreement measure in a market without short-selling restrictions as the maximum profit obtained from a portfolio with total sold assets price one. (Observe that

$$f(x) = \frac{-\sum_{i=1}^{n} x_i p_i}{-\sum_{i \in S_x} x_i p_i}$$

is a homogeneous function of degree zero and for every feasible x, kx is also feasible for every $k > 0$.)

Theorem 11 yields some important properties. The complete relaxation of the constraints imposed to the short positions leads to the same measure m. Hence, an agent can replicate his/her portfolio in an arbitrary way and m still tests the maximum relative profit over the total price of the sold assets. Choosing a portfolio x where this maximum profit is achieved, we can deduce from Lemma 10 how must one choose h^* and take into account short-selling restrictions. That is,

$$h_i^* = 0 \quad \text{if} \quad x_i \geq 0 \quad \text{and} \quad h_i^* = \frac{x_i}{\sum_{i \in S_x} x_i p_i} \quad \text{if} \quad x_i < 0.$$

Consider now for every portfolio $x \neq 0$ the function

$$(9) \qquad g(x) = \frac{-\sum_{i=1}^{n} x_i p_i}{-\sum_{i \in S_x} x_i p_i + \sum_{i \in L_x} x_i p_i} = \frac{-\sum_{i=1}^{n} x_i p_i}{\sum_{i=1}^{n} |x_i| p_i}.$$

The function $g(x)$ is the quotient between the profit generated by the zero payoff x and the price of all interchanged assets. Manipulating $g(x)$, it is easy to prove that

$$g(x) = \frac{f(x)}{2 - f(x)} \quad \text{for every} \quad x \in \mathbb{R}^n \quad \text{such that} \quad \sum_{i \in S_x} x_i p_i \neq 0.$$

These observations and the fact that $\frac{t}{2-t}$ is a increasing continuous function in $[0, 1]$ ($0 \leq m \leq 1$) lead to the following theorem.

Theorem 12 *Let $m > 0$ and consider $h^* \in H$ and x^* such that*

$$m = \frac{\sum_{i=1}^{n} x_i^* p_i}{\sum_{i \in S_{x^*}} x_i^* p_i} = \phi(h^*)$$

Then, x^* solves the following optimization problem

$$\text{Maximize} \quad g(x)$$

$$(10) \qquad \text{subject to} \quad \left\{ \begin{array}{l} \sum_{i=1}^{n} x_i \alpha_i(k) = 0 \quad \text{for every } k \in K \\ x \neq 0 \end{array} \right.$$

9

We have then proved that problem (10) is solvable when the LOP does not hold on the market. Obviously, if the LOP holds on the market and (10) is feasible then $g(x) = 0$ for every feasible x in (10) and, (10) is also solvable (the optimal value is then zero.) Denoting by l the optimal solution in (10), we get from (9)

$$(11) \qquad l = g(x^*) = \frac{f(x^*)}{2 - f(x^*)} = \frac{m}{2 - m}.$$

The relation $l = \frac{m}{2-m}$ remains true when the LOP holds on the market. Thus, l is also a measure of the degree of discrepancy of prices. It is also a relative measure which takes values in $[0, 1]$ and such that $l \leq m$, since $0 \leq m \leq 1$.

In this case the measure l computes the maximum profit relative to the total price of the exchanged assets. Thus, it could be tested if the market frictions affect to the fulfillment of the LOP. In fact, the transaction costs can be discounted and it is possible to verify if there still are arbitrage opportunities.

The results in this section guarantee that the measures m and l do not depend on the short-selling restrictions imposed in order to avoid mathematical difficulties. In fact, from the most constrained conditions, where no initial short positions are permitted, and the agents cannot sell what they do not have, to the most relaxed ones where there are no restrictions in each asset to be sold, the same portfolio x^* implements the relative arbitrage profits. Note that in the first case, an agent needs an initial portfolio proportional to h^* in order to win the relative profit m, while this relative profit is available to any agent in the second case. For intermediate situations (short-selling restrictions for some assets but no short-selling restrictions for the remaining ones), the same values for m, l and x^* would be obtained, although in that case not all the investors would be able to win these maximum relative profits.

5   Applications to financial market integration

Chen and Knez (1995) develop a measurement theory of market integration for two markets whenever there exist discrepancies in pricing common asset payoffs or, equivalently, when the LOP is violated across them. They assume the LOP holds separately on each market and use a model slightly different to ours: they consider the linear space of square-integrable random variables L^2 over a probality space (Ω, F, P_R) instead of $C(K)$. Readapting it to the mathematical setting of this paper, they define what they call the weak-integration measure $g(\mathcal{M}_1, \mathcal{M}_2)$ as the minimum distance between the sets of state prices

$$D_1 = \left\{ \mu \in M(K) \mid \int_K \alpha_i \, d\mu = p_i \qquad i = 1, \cdots q \right\}$$

and

$$D_2 = \left\{ \mu \in M(K) \mid \int_K \alpha_i \, d\mu = p_i \qquad i = q + 1, \cdots n \right\}$$

where $\mathcal{M}_1 = \mathcal{M}_{p^1, \alpha^1}$, $\mathcal{M}_2 = \mathcal{M}_{p^2, \alpha^2}$, $p^1 = (p_1, \cdots p_q)$, $\alpha^1 = (\alpha_1, \cdots \alpha_q)$ $p^2 = (p_{q+1}, \cdots p_n)$ and $\alpha^2 = (\alpha_{q+1}, \cdots \alpha_n)$. Thus,

$$g(\mathcal{M}_1, \mathcal{M}_2) = \inf \{ \|\mu_1 - \mu_2\| \mid \mu_1 \in D_1, \mu_2 \in D_2 \} .$$

Chen and Knez (1995) also prove that the weak integration measure equals the maximum difference between the respective prices assigned by both markets to any unit-norm payoff.

The measure m is an alternative to the weak-integration measure. Treating both markets $\mathcal{M}_1$ and $\mathcal{M}_2$ as parts of the combined market $\mathcal{M}_{p,\alpha}$, where $p = (p_1, \cdots p_q, p_{q+1} \cdots p_n)$ and $\alpha =$

$(\alpha_1, \cdots \alpha_q, \alpha_{q+1} \cdots \alpha_n)$, we compute m on this global market. Thus, Theorem 13 reveals that m is continuous with respect to $g(\mathcal{M}_1, \mathcal{M}_2)$, and gives an upper bound for m which depends only on the returns of the different assets available on each market.

Theorem 13 *The following inequalities hold.*[1]

$$i) \qquad l \leq m \leq \sum_{j=1}^{q} \frac{\|\alpha_j\|}{p_j} g(\mathcal{M}_1, \mathcal{M}_2)$$

$$ii) \qquad l \leq m \leq \sum_{j=q+1}^{n} \frac{\|\alpha_j\|}{p_j} g(\mathcal{M}_1, \mathcal{M}_2)$$

$$iii) \qquad l \leq m \leq \frac{1}{2} \sum_{j=1}^{n} \frac{\|\alpha_j\|}{p_j} g(\mathcal{M}_1, \mathcal{M}_2)$$

Theorem 13 iii) remains true for any division of the combined market into two markets in such a way that the assets in each group satisfy the LOP. Note that m can be computed even if both markets do not separately verify the LOP. Hence, m reflects a degree of integration of two or more markets verifying the LOP or not. Chen and Knez (1995) propose to divide those markets violating the LOP into smaller groups, so that the LOP holds separately on each group and to measure among them. But the value of the weak-integration measure depends on the way in which each market violating the LOP is divided. The examples below illustrate this fact.

Example 1. Consider the case where there are two possible states of nature, s_1 and s_2. Suppose one asset A_1 paying \$ 1 in s_1 and \$ 0 in s_2 with a current price of \$ 1, another asset A_2 paying \$ 0 in s_1 and \$ 1 in s_2, with a current price of \$ 1, and a third one A_3 paying \$ 1 in s_1 and \$ α in s_2 with a current price of \$ p $(p > 0)$.
Direct calculations solving (5) or (7) show that for $\mathcal{M} = \{A_1, A_2, A_3\}$ the disagreement measure is given by

$$m = \frac{|1 + \alpha - p|}{\max(1 + \alpha, p, p - \alpha)}$$

We compute g for some of the different ways of dividing the market $\mathcal{M}$ into two markets so that the LOP holds separately on each market.

    i) For $\mathcal{M}_1 = \{A_1, A_2\}$ and $\mathcal{M}_2 = \{A_3\}$ we get $g(\mathcal{M}_1, \mathcal{M}_2) = \frac{|1+\alpha-p|}{\sqrt{1+\alpha^2}}$.

    ii) For $\mathcal{S}_1 = \{A_1\}$ and $\mathcal{S}_2 = \{A_2, A_3\}$ we get $g(\mathcal{S}_1, \mathcal{S}_2) = |1 + \alpha - p|$.

    iii) For $\mathcal{N}_1 = \{A_2\}$ and $\mathcal{N}_2 = \{A_1, A_3\}$ (note that we need $\alpha \neq 0$, or $p = 1$ if $\alpha = 0$) we get $g(\mathcal{N}_1, \mathcal{N}_2) = \frac{|1+\alpha-p|}{|\alpha|}$ if $\alpha \neq 0$ and $g(\mathcal{N}_1, \mathcal{N}_2) = 0$ otherwise.

    iv) For $\mathcal{H}_1 = \{A_1, A_3\}$ and $\mathcal{H}_2 = \{A_2, A_3\}$ and the same assumptions for α and p as in iii) we get $g(\mathcal{H}_1, \mathcal{H}_2) = \frac{|1+\alpha-p|}{|\alpha|}(\sqrt{1+\alpha^2})$ if $\alpha \neq 0$ and $g(\mathcal{H}_1, \mathcal{H}_2) = 0$ otherwise.

This shows that the weak-integration measure g depends on the way in which the market $\mathcal{M}$ is divided. If we compare the results obtained in ii) and iii) for a fixed price p, we observe that

$$\lim_{\alpha \to \infty} g(\mathcal{S}_1, \mathcal{S}_2) = \infty, \quad \text{although} \quad \lim_{\alpha \to \infty} g(\mathcal{N}_1, \mathcal{N}_2) = 1.$$

This is due to the fact that the weak-integration measure computes the differences between the respective prices assigned to any unit-norm payoff in both markets, that is, the maximum profit obtained among all the portfolios with a zero payoff which can be expressed by the difference

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[1] Readapting our model to Chen and Knez's model, the same inequalities would be obtained writing $\|\alpha_j\|_2$ instead of $\|\alpha_j\|$ for every $j = 1, \cdots n$ where $\|\alpha_j\|_2^2 = \int_\Omega \alpha_j^2 \, dP_R$.

between two unit norm common payoffs.

Thus, the measure m presents some advantages with respect to g. First, m is not refered to the unit-norm payoff, but to the portfolios with sold assets total price one, a fact which seems to be more interesting for the investor. Furthermore, in computing m it is necessary to consider all the available portfolios in the combined market and not only the ones expressed by the difference between two unit-norm common payoffs in both markets.

Example 2. Consider the case where there are two possible states of nature, s_1 and s_2. Suppose one asset A_1 paying \$1 in both states and with current price of \$1, another asset A_2 paying $\$1 + \alpha$ in s_1 and $\$1 - \alpha$ in s_2 with a current price of \$1, and a third one A_3 paying $\$1 + 2\alpha$ in s_1 and $\$1 - \alpha$ in s_2 with a current price of \$1.

Dividing the market $\mathcal{M}$ into the markets $\mathcal{J}_1 = \{A_1, A_2\}$ and $\mathcal{J}_2 = \{A_1, A_3\}$ we get

$$g(\mathcal{J}_1, \mathcal{J}_2) = \frac{\sqrt{2}}{6} \quad \text{if} \quad \alpha \neq 0$$

$$\text{and} \quad g(\mathcal{J}_1, \mathcal{J}_2) = 0 \quad \text{if} \quad \alpha = 0$$

First, note that both markets verify the LOP, and also that no arbitrage opportunity exists in each market, in the sense that there are no positive payoffs with negative or zero prices, since for each market there exists a positive state price. Second, the weak-integration measure is not continuous in α. Intuitively, for both markets $\mathcal{J}_1$ and $\mathcal{J}_2$, the closer to 0 the value of α, the more closely integrated the two markets are (that is, the lower the minimum initial investment needed across the combined market to purchase a portfolio with total price of the sold assets one and generating a zero payoff in every state of nature). Direct calculations show that for $\mathcal{M} = \{A_1, A_2, A_3\}$ and $\alpha \in [0, 1)$ the value of m is $\frac{\alpha}{3}$, which is continuous in α. As already said, m is the maximum profit relative to the market price of all the sold assets an investor can obtain. For instance, if we take $\alpha = 0.3$, then $m = 0.1$, which shows that the maximum profit obtained from all the possible portfolios is the 10% of the total price of the sold assets.

We conclude from the two examples above that m avoids some of the problems encountered when using the measure proposed by Chen and Knez(1995). First, imagine in Example 2 that an error in the measurement process provides α very small, but strictly positive, instead of $\alpha = 0$. The weak-integration measure reflects that the maximum squared difference between the respective prices assigned by both markets to any unit-norm common payoff is $\frac{\sqrt{2}}{6}$. Nevertheless, if $\alpha = 0$ the LOP holds across both markets and this difference is zero. Thus, the discontinuity of the weak-integration measure g makes it very sensitive with respect to measurement errors in empirical applications. This problem disappears when working with m. Clearly, m is very small if α is small enough.

Second, the weak-integration measure can not be applied if one of the two markets does not verify the LOP. Chen and Knez(1995) suggest in such a case to divide the market violating the LOP in sub-markets, so that the LOP holds on each group. However, this presents serious problems, since, as we have seen in Example 1, their measure is also very sensitive to the way in which each market is divided into sub-markets. Again, m solves this problem because it is computed in the global market and so, it does not depend on the initial markets and only on the combined one.

Finally, and maybe most importantly, m gives an information in monetary terms, namely either the maximum profits relative to the market price of the portfolio h of short-selling restrictions under the most constrained conditions, or maximum profits relative to the market price of the sold assets if there are no restrictions at all.

12

6 Conclusions

This paper presents two measures (m and l) of the degree of fulfillment of the Law of One Price (LOP) in a financial market. These measures are related by (11) and vanish if and only the LOP holds on the market. When the LOP does not hold on the market, the measures are strictly positive, and they increase if the level of violation of the LOP increases, that is, when there is an increase in the profit that an investor can obtain relative to the total amount of money he/she has to exchange replicating his/her portfolio and implementing the arbitrage. The maximum value of these measures is one, and it is attained in extreme situations in which the agents can replicate their portfolios in such a way that they obtain a profit equal to the price of the sold assets. This is a limiting case which will never appear in practical situations.

The measures do not depend on the short-selling restrictions in the model. We may assume either that the agents cannot replicate by selling the assets they do not have, or the opposite, that is, there is no limit on the short positions the agents can hold. In both cases the same values for both measures are obtained.

The LOP is usually characterized by the existence of state prices, but this is a very specific criterium which only determines the accomplishment or not of the LOP. A measure able to reflect the degree of fulfillment, that is, the discrepancy in pricing the assets, makes the model more flexible, and therefore, more realistic. If the measure is strictly positive and small enough, we are most likely to have an efficient market, although either the trading frictions and transaction costs, or the measurement errors may lead to this positive value.

Our measures also allow us to analyze markets with frictions. In fact, since they quantify the degree of fulfillment of the LOP in (relative) monetary terms, once we have computed them we can discount the transaction costs and, therefore, we may know if the agents can implement the arbitrage opportunities.

Many empirical papers analyze some well-known specific arbitrage strategies. Our measures have the important advantage that in computing them, all the assets in the market are selected and, therefore, all the arbitrage opportunities are considered.

The measures here introduced do not depend on the dynamic assumptions applied to price the assets.

In considering several financial markets, one can work in a global market collecting all the available assets from the (sub)markets. Then m and l (computed on the combined market) may be taken to measure the integration of all them. These new market integration measures are continuous with respect to the Chen and Knez measure g, so estimating and controlling their measure, our measures are also controlled. The opposite is in general false, and there are situations in which m takes small values while g remains large. This is an important fact, once one realizes that m is qualitatively different from g, since it measures in monetary terms (relative profits available by the agents). Furthermore, m seems to be more suited to estimate the degree of multimarket integration: first, it is never insensitive to the arbitrage opportunities (see Example 2). Second, it does not need that each market verifies the LOP. Finally, the value of m does not depend upon the

way in which each market is divided into submarkets.

The theory developed above analyzes the degree of fulfillment or violation of the LOP in a financial market or across different ones, but it may also be extended to reflect the level of arbitrage opportunities in the strong form (or cross-market arbitrage).

Appendix

Proof of Lemma 2. See Chamberlain and Rothschild(1983).

Proof of Lemma 3. Taking into account the remarks after Lemma 3, it only remains to prove that (2) holds. In fact, since $x^* \in F_h$, we have $x_j^* \geq -h_j$ for every $j = 1, \cdots n$. Besides, the optimal value in (1_h) is greater than 0, since the zero vector is in F_h. Combining these two facts, (2) is easily derived.

Proof of Lemma 4. In the inequality -constrained program (1_h), $x \in \mathbb{R}^n$ and the associated positive cone P is $\mathbb{R}^n$, while the inequality constraints take values in $C(K) \times \mathbb{R}^n$, where the associated positive cone Q is $\{0\} \times \mathbb{R}^n$.
P is a cone with compact sole (since $B = \{x \in \mathbb{R}^n \mid \|x\| = 1\}$ is a compact set in P such that 0 is not in B and B spans P.) Besides, if $x \in \mathbb{R}^n$ is such that $(-\sum_{i=1}^{n} x_i \alpha_i, x) \in Q$ and $\sum_{i=1}^{n} x_i p_i = 0$ then $x = 0$.
These two facts allow us to ensure that

$$D' = \left\{ \left(-\sum_{i=1}^{n} x_i \alpha_i, \, x - y, \, \sum_{i=1}^{n} x_i p_i \right) \mid x \in \mathbb{R}^n, \, y \in \mathbb{R}^n_+ \right\}$$

is a closed set (see Theorem 3.19 of Anderson and Nash(1987)). Now, from Theorems 3.10 and 3.22 (Anderson and Nash(1987)), Lemma 3 is deduced.

Proof of Lemma 5. Lemma 4 ensures that $\phi(h)$ is the optimal value of (4_h). Denoting by T the linear map $T(\mu) = (\int_K \alpha_i \, d\mu)_{i=1}^{n}$ from $\mathcal{M}(K)$ to $\mathbb{R}^n$, it follows that $T(\mathcal{M}(K))$ is a vector space in $\mathbb{R}^n$. So, the feasible set in (4_h) is the intersection between $p - T(\mathcal{M}(K))$ and $\mathbb{R}^n_+$. Therefore, there exists only a finite number of extreme points $\lambda^1, \lambda^2 \cdots \lambda^r$ for this feasible set which do not depend on h, since the feasible set does not depend on h. Then

$$\phi(h) = \min \left(\sum_{i=1}^{n} h_i \lambda_i^1, \sum_{i=1}^{n} h_i \lambda_i^2, \cdots, \sum_{i=1}^{n} h_i \lambda_i^r \right)$$

and the proof is concluded.

Proof of Theorem 7. Just observe that the LOP holds on the market if and only if $\sum_{i=1}^{n} x_i p_i = 0$ whenever $\sum_{i=1}^{n} x_i \alpha_i(k) = 0$ for every $k \in K$ or, equivalently, if $\phi(h) = 0$ for every $h \in \mathbb{R}^n_+$. But taking into account that ϕ is homogeneous of degree one, the latter is equivalent to $m = 0$.

Proof of Theorem 8. For every $\lambda \in \Lambda$, denote

$$f(\lambda) = \max_{h \in H} \sum_{i=1}^{n} h_i \lambda_i.$$

14

Since H is a compact set, such a maximum exists. Moreover, the maximum is achieved for some extreme point $A_i(0, \cdots, \frac{1}{p_i}, \cdots 0)$, and then $f(\lambda) = \max(\frac{\lambda_1}{p_1}, \frac{\lambda_2}{p_2} \cdots \frac{\lambda_n}{p_n})$. Hence f is a continuous function. Since $f(\lambda) \geq \phi(h)$ for every $\lambda \in \Lambda$ and $h \in H$, the function ϕ is a bounded. Thus, there exists $\beta = \inf\{f(\lambda) \mid \lambda \in \Lambda\}$.

Note that H and Λ are convex subsets, H is a compact set, $U(\lambda, .)$ is quasiconcave and upper-semicontinuous for every $\lambda \in \Lambda$, and $U(., h)$ is quasiconvex and below-semicontinuous for every $h \in H$. Hence Sion's theorem (Moulin(1979)) yields

$$\beta = \inf\{f(\lambda) \mid \lambda \in \Lambda\} = \max\{\phi(h) \mid h \in H\}.$$

It only remains to prove that such an infimum is achieved for some $\lambda^* \in \Lambda$. To do this, take a sequence $(\lambda^k)_{k=1}^\infty$ in Λ such that $f(\lambda^k) < \beta + \frac{1}{k}$ for every k. Then $f(\lambda^k) < \beta + 1$ and since $f(\lambda^k) = \max(\frac{\lambda_1^k}{p_1}, \cdots, \frac{\lambda_n^k}{p_n})$ it follows that $\lambda_i^k < p_i (\beta + 1)$ for every $k \in \mathbb{N}$ and $i \in \{1, 2 \cdots n\}$. Thus, $(\lambda^k)_{k=1}^\infty$ is a bounded sequence in $\mathbb{R}_+^n$ and, consequently, there exists a subsequence converging to λ^*, which we denote also by $(\lambda^k)_{k=1}^\infty$. Since Λ is a closed set, we have $\lambda^* \in \Lambda$. From the continuity of f and the choice of the sequence $(\lambda^k)_{k=1}^\infty$, the inequality $f(\lambda^*) \leq \beta$ is derived and, then, $f(\lambda^*) = \beta$. Taking now $h^* \in H$ such that $\phi(h^*) = f(\lambda^*)$, it follows that (λ^*, h^*) is a saddle point for U and, therefore, Theorem 8 is already proved.

Proof of Lemma 10. ii) is easily deduced from i). Let us prove i).
In order to simplify the notation and without loss of generality, we will prove i) for $j = n$.
Proceeding by contradiction, suppose $x_n^* > -h_n^*$, and $h_n^* > 0$.

First, assume $x_n^* \geq 0$, and let $h_0 = (h_1^*, \cdots, h_{n-1}^*, 0)$ and $\gamma = \sum_{i=1}^{n-1} p_i h_i^*$. It is easily proved that $0 < \gamma < 1$. Besides $\phi(h_0) \leq \phi(h^*)$, since the feasible sets F_{h_0} and F_{h^*} are such that $F_{h_0} \subseteq F_{h^*}$. From $x^* \in F_{h_0}$ we get $\phi(h^*) = \phi(h_0)$. Finally, we have

$$(A_1) \qquad \phi(h^*) = \phi(h_0) = \gamma\phi(\frac{1}{\gamma}h_0) < \phi(\frac{1}{\gamma}h_0) \leq \phi(h^*),$$

where the last inequality follows from the fact that $\frac{1}{\gamma}h_0 \in H$. Since (A_1) leads to a contradiction, the proof is concluded whenever $x_n^* \geq 0$.

Assume now $-h_n^* < x_n^* < 0$ and let $h^0 = (h_1^*, \cdots, h_{n-1}^*, |x_n^*|)$ and $\delta = \sum_{i=1}^{n-1} p_i h_i^* + p_n |x_n^*|$. As above, we get $0 < \delta < 1$ and

$$\phi(h^*) = \phi(h^0) = \delta\phi(\frac{1}{\delta}h^0) < \phi(\frac{1}{\delta}h^0) \leq \phi(h^*),$$

which concludes the proof of Lemma 10.

Proof of Theorem 11 i) It follows from the fact that $\sum_{i=1}^n x_i p_i \geq 0$ whenever $x_i \geq 0$ for every $i \in \{1, 2 \cdots n\}$.

ii) Let $h' = (h_1', h_2' \cdots h_n')$, where $h_i' = \max(-x_i, 0)$ for every $i = 1, \cdots n$, and $\varepsilon = \sum_{i=1}^n p_i h_i' = -\sum_{i \in S_x} x_i p_i > 0$.

Since $\frac{1}{\varepsilon}h' \in H$ we get $\phi(\frac{1}{\varepsilon}h') = \frac{1}{\varepsilon}\phi(h') \leq \phi(h^*)$.

Besides, from $x \in F_{h'}$ we get $-\sum_{i=1}^n p_i x_i \leq \phi(h')$.

15

Thus, combining both inequalities, ii) is proved.

iii) Assume $x = x^*$. Then $\phi(h^*) = -\sum_{i=1}^{n} x_i^* p_i$. From Lemma 10 we get $x_i^* = -h_i$ whenever $i \in S_{x^*}$ and $h_i^* = 0$ otherwise. Thus,

$$-\sum_{i \in S_{x^*}} x_i^* p_i = \sum_{i \in S_{x^*}} h_i^* p_i = \sum_{i=1}^{n} h_i^* p_i = 1$$

and the proof of iii) is concluded.

Proof of Theorem 13. Proofs of i) and ii) are similar, and iii) is an easy consequence of i) and ii). So, we will just prove i).

Let $h^* \in H$, $x^* \in F_{h^*}$ such that

$$m = \phi(h^*) = -\sum_{i=1}^{n} x_i^* p_i .$$

From Lemma 3, x^* is also a feasible optimal solution of

$$(3_{h^*}) \qquad \begin{array}{l} \text{Maximize} \quad -\sum_{i=1}^{n} x_i p_i \\[2mm] \text{subject to} \quad \left\{ \begin{array}{ll} \sum_{i=1}^{n} x_i \alpha_i(k) = 0 & \text{for every } k \in K \\[2mm] \beta_i \geq x_i \geq -h_i^* & i = 1, \cdots n \end{array} \right. \end{array}$$

where

$$\beta_i = \frac{\sum_{j \neq i} p_j h_j^*}{p_i} = \frac{1 - p_i h_i^*}{p_i} = \frac{1}{p_i} - h_i^* .$$

The dual problem of (3_{h^*}) is given by

$$(A_{2h^*}) \qquad \begin{array}{l} \text{Minimize} \quad \sum_{i=1}^{n} (\frac{1}{p_i} - h_i^*)\lambda_i^- + \sum_{i=1}^{n} h_i^* \lambda_i^+ \\[2mm] \text{subject to} \quad \left\{ \begin{array}{l} \int_K \alpha_i \, d\mu - \lambda_i^- + \lambda_i^+ = p_i \\[2mm] \lambda_i^+ \geq 0, \lambda_i^- \geq 0 \quad i = 1, \cdots n \text{ and } \mu \in \mathcal{M}(K) \end{array} \right. \end{array}$$

Denoting

$$I^+ = \{i \in \{1, 2, \cdots n\} \mid p_i - \int_K \alpha_i \, d\mu \geq 0\}$$

and

$$I^- = \{i \in \{1, 2 \cdots n\} \mid p_i - \int_K \alpha_i \, d\mu < 0\} ,$$

problem (A_{2h^*}) can be reformulated as

$$(A_{3h^*}) \qquad \begin{array}{l} \text{Minimize} \quad \sum_{i \in I^-} \left(\frac{1}{p_i} - h_i^*\right) \left(\int_K \alpha_i \, d\mu - p_i\right) + \sum_{i \in I^+} h_i^* \left(p_i - \int_K \alpha_i \, d\mu\right) \\[2mm] \text{subject to} \quad \mu \in \mathcal{M}(K) \end{array}$$

and then,

$$(A_4) \qquad m \leq \sum_{i \in I^-} \left(\frac{1}{p_i} - h_i^*\right) \left(\int_K \alpha_i \, d\mu - p_i\right) + \sum_{i \in I^+} h_i^* \left(p_i - \int_K \alpha_i \, d\mu\right) .$$

16

Now, set $g = g(\mathcal{M}_1, \mathcal{M}_2)$ and let $\varepsilon > 0$. Choose $\mu_1 \in D_1$ and $\mu_2 \in D_2$ such that

$$g \geq \|\mu_1 - \mu_2\| - \varepsilon = \sup\left\{\left|\int_K \alpha \, d(\mu_1 - \mu_2)\right| \mid \alpha \in \mathcal{C}(K), \|\alpha\| \leq 1\right\} - \varepsilon.$$

Then,

$$g \geq \frac{1}{\|\alpha_i\|}\left|\int_K \alpha_i \, d\mu_1 - \int_k \alpha_i \, d\mu_2\right| - \varepsilon$$

for every $i = 1, \cdots n$. Since $\int_K \alpha_j \, d\mu_1 = p_j$ for $j = 1, \cdots q$, we then have

$$(A_5) \qquad \|\alpha_j\| g \geq \left|\int_K \alpha_j \, d\mu_2 - p_j\right| - \|\alpha_j\| \varepsilon$$

for every $j = 1, \cdots q$. Denoting

$$J^+ = \{j \in \{1, 2, \cdots q\} \mid p_j - \int_K \alpha_j \, d\mu_2 \geq 0\}$$

and

$$J^- = \{j \in \{1, 2 \cdots q\} \mid p_j - \int_K \alpha_j \, d\mu_2 < 0\},$$

we multiply (A_5) by h_j^* for every $j \in J^+$, and by $\frac{1}{p_j} - h_j^*$ for every $j \in J^-$. Adding up all the obtained inequalities, we get

$$(A_6) \qquad Ag \geq \sum_{j \in J^+} h_j^*\left(p_j - \int_K \alpha_j \, d\mu_2\right) + \sum_{j \in J^-} (\frac{1}{p_j} - h_j^*)\left(\int_K \alpha_j \, d\mu_2 - p_j\right) - A\varepsilon,$$

where $A = \sum_{j \in J^+} \|\alpha_j\| h_j^* + \sum_{j \in J^-} \|\alpha_j\|(\frac{1}{p_j} - h_j^*)$. Since $h_i^* \geq 0$ and $\frac{1}{p_i} - h_i \geq 0$ for every $i = 1, 2 \cdots q$, it follows that

$$(A_7) \qquad A \leq \sum_{i=1}^q \|\alpha_i\| h_i^* + \sum_{i=1}^q \|\alpha_i\|(\frac{1}{p_i} - h_i^*) = \sum_{i=1}^q \frac{\|\alpha_i\|}{p_i}.$$

Besides, from the fact that $\int_K \alpha_i \, d\mu_2 = p_i$ for every $i = q + 1, \cdots n$, we obtain that

$$(A_8) \qquad \sum_{j \in J^+} h_j^*\left(p_j - \int_K \alpha_j \, d\mu_2\right) + \sum_{j \in J^-} (\frac{1}{p_j} - h_j^*)\left(\int_K \alpha_j \, d\mu_2 - p_j\right)$$

$$= \sum_{i \in I^+} h_i^*\left(p_i - \int_K \alpha_i \, d\mu_2\right) + \sum_{i \in I^-} (\frac{1}{p_i} - h_i^*)\left(\int_K \alpha_i \, d\mu_2 - p_i\right).$$

From (A_4), (A_6), (A_7), and (A_8) it follows that

$$\sum_{i=1}^q \frac{\|\alpha_i\|}{p_i} \, g \geq m - \varepsilon \sum_{i=1}^q \frac{\|\alpha_i\|}{p_i}$$

and then, Theorem 11 i) is proved.

17

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