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A MODEL FOR FINANCIAL INTERMEDIATION AND PUBLIC INTERVENTION

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Abstract

Based on Chari and Jagannathan (1988), this paper models information-induced and "pure-panic" runs in an environment of risk-averse agents. In this framework, deposits are needed to provide insurance against investors' unexpected demand for liquidity and therefore, a role for a financial intermediary is justified. A welfare analysis of two traditional devices to prevent runs (namely, suspension of convertibility versus deposit insurance), is presented.

Key Words and Phrases
Banking, Deposit Contracts, Deposit Insurance, Suspension of convertibility.

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Contents

Introduction

Due to all advances in information economics and financial innovations, the theory of banking has been reconfigured in the last fifteen years.

One of the basic issues analyzed in modern financial intermediation is the problem of bank failures and the study of the different intervention mechanisms in order to prevent them.

Banking panics were a recurrent phenomenon in the United States until the 1930s. They have reemerged as a source of public concern and much theoretical research recently.

Bank failures are important, not primarily in their own right but because of their indirect effect. They are a mechanism through which a drastic decline is produced in the stock of money and this last feature plays an important role in economic development.

In the recent past, a stream of literature has begun to reexamine and extend the theses in prior research. Examples of such work include Kareken and Wallace [19], Bryant [8], Bhattacharya [4], Diamond and Dybvig [12], Diamond and Dybvig [13], Bhattacharya and Gale [3], Chari and Jagannathan [9], Bhattacharya and Jacklin [2].

An important contribution in this area is the paper by Diamond and Dybvig [12]. They model bank runs as one of the two possible equilibria that are obtained in a two stage game of complete but imperfect information.

Diamond and Dybvig give a first explicit analysis of the demand for liquidity and the transformation service provided by banks. This transformation of illiquid assets into more liquid liabilities through the demand deposit contract gives a rationale for the existence of banks and at the same time for their vulnerability to runs. In this role, banks can be viewed as providing insurance for individuals subject to preference shocks.

The demand deposit contract can improve on a competitive market by providing better risk-sharing among people who need to consume at different random times. If confidence is maintained there will be efficient risk-sharing; in this (Pareto-dominant) equilibrium only those agents who have a genuine preference for early consumption will make an early withdrawal. However, if agents panic, incentives are distorted and the bank-run equilibrium will occur.

In the bank run equilibrium all individuals withdraw at date 1, fearing withdrawals by others; this equilibrium is socially inefficient: it provides allocations that are worse for all agents than those they would have obtained if they had waited until date 2 to withdraw and the bank is forced to liquidate its long-term technology at a loss.

Bhattacharya and Gale [3] consider a variation of the Diamond and Dybvig model in which there are many intermediaries that are subject to privately observed liquidity shocks. They show that unconstrained walrasian trading among intermediaries would lead to underinvestment in the liquid asset and therefore demonstrate the welfare gains from setting up an institution such as a central bank, offering borrowing and lending opportunities at a subsidized rate.

In a very similar framework, Bhattacharya and Jacklin [2] examine the relative degrees of risk-sharing provided by bank deposit and traded equity contracts. They characterize a relationship between the riskiness and information of the stream of returns and the desirability of equity over deposit contracts. (The basic result is that deposit contracts tend to the better for financing low risk assets).

An interesting addition to this literature is the paper by Chari and Jagannathan [9], which models two types of bank runs, information-induced runs, that is, runs that arise due to negative information about the bank's solvency, and pure panic runs, that occur even when no one has any adverse information. They showed that where there is uncertainty about both asset returns and the proportion of early withdrawal seekers, sometimes runs occur even though there is no adverse information held by any agent. The reason is that uninformed individuals condition their beliefs on the size of the withdrawal queue at the bank. If this size is large enough due to many agents wishing to consume early they may infer that there is a possibility that the bank is about to fail and precipitate a bank run. In their model, extra market constraints, such as suspension of convertibility, can prevent bank runs and result in superior allocations.

The main criticism to Chari and Jagannathan's model was that it implied no apparent role for a bank or other financial intermediary; as individuals were risk neutral.

This paper introduces risk-averse preferences in Chari and Jagannathan's model.

A first motivation for this extension is to give a positive role for a financial intermediary in the economy, by providing insurance to individuals subject to preference shocks. As already shown in Diamond Dybvig, it is this important liquidity insurance service what gives a rationale for the existence of banks and for their vulnerability to runs. This extended framework considers this valuable service performed by banks, and shows how coordination problems may arise from deposit contracting.

In the ex-ante period, depositors invest their endowment in the bank and are offered a menu of contracts. In the interim period, individuals select their withdrawal stream (given their liquidity needs or information received concerning bank asset quality). A large withdrawal queue size at the bank may be due to an important liquidity shock or to negative information about asset returns. Uninformed agents cannot distinguish between these two shocks and as a result, in some states of nature all individuals choose to withdraw their deposits from the bank (although sometimes there may be no adverse information held by any agent).

Conditions to assure bank runs are derived without having abstracted from the important issues that banks perform.

A second motivation for this extension is to complete Chari and Jagannathan's welfare analysis, by comparing suspension of convertibility versus deposit insurance, given their relative benefits and costs (of randomization in meeting liquidity needs or deadweight taxation). In this framework, suspension does no longer lead to an improvement of ex-ante utility and is not always the preferred instrument to cope with runs. The level of relative risk-aversion is crucial to determine the choice between the two policies. The numerical results show that if individuals are not very risk-averse, suspension would be welfare superior, for higher levels of risk-aversion deposit insurance would be better, and finally, for very high levels of risk-aversion suspension is again preferred. The range of parameters for which either measure is preferred also depends on the distribution function of the random return, the value of the deadweight tax and the random proportion of type 1 agents in the economy.

The structure of the paper is as follows: The basic framework of the model is presented in section 2, the ex-ante (banker's) contract and the ex-post (depositors') problems are defined in sections 2.1 and 2.2 respectively. A condition to assure a panic run is derived in section 3 and numerical computations are given in section 4. Two traditional devices in order to prevent runs are described in section 5 and a welfare comparison of both measures is presented in section 6. Section 7 ends the paper with some conclusions with respect to the above public measures and suggestions for further research.

1.- A Description of the model

The model can be summarized as follows:

a.- Hypotheses

i.- Three period economy, T=0,1,2

ii.- A single commodity.

iii.- Investment technology: There are two investment technologies on the side of the intermediary:

- A short-term asset at T=0 that yields one unit at T=1
- A long-term asset at T=0 that yields a random return $\tilde{R}$ at T=2. For simplicity, it is assumed that the long-term technology cannot be liquidated early (or equivalently, only at a loss)[1].

The random return is defined as follows:

$$\tilde{R} \in (Rh, Rl) \quad \text{with} \quad p.d.f\ (1-p, p) \quad \text{and} \quad Rl \to 0 \tag{1}$$

It is also assumed that the probability of the low return occurring (p), is sufficiently small.

iv.- Preferences: There is a continuum of ex-ante identical agents at T=0 that maximize expected utility of consumption. They are subject at T=1 to a privately observed uninsurable risk of being of either of two types.

- Type-1 agents derive only utility from consumption in period one.
- Type-2 agents derive utility from consumption in both periods 1 and 2, i.e:

$$\begin{aligned} &\text{Type-1 agents} \quad U^1(c_1, c_2, \rho_1) = \rho_1 U(c_1) + (1-\rho_1) U(c_2) \\ &\text{Type-2 agents} \quad U^2(c_1, c_2, \rho_2) = \rho_2 U(c_1) + (1-\rho_2) U(c_2) \end{aligned} \tag{2}$$

It is assumed that $0 \le \rho_2 < \rho_1 \le 1$ and $\rho_1 \to 1$.

The proportion of type-1 agents is stochastic and defined as:

$$\tilde{t} \in (t_1, t_2) \quad \text{with} \quad p.d.f\ (r_1, r_2) \quad \text{and} \quad \begin{array}{l} r_1 + r_2 = 1 \\ t_1 < t_2 \end{array} \tag{3}$$

v.- Initial endowments: All agents are endowed with one unit of the good at T=0.

vi.- Information: At T=1 a random fraction, $\tilde{\alpha}$, of type-2 individuals receives information about time 2 returns[2].

It is assumed that this information is perfect.

The random variable $\tilde{\alpha}$, is defined as follows:

$$\tilde{\alpha} \in (0, \alpha) \quad \text{with} \quad p.d.f\ (1-q, q) \qquad [4]$$

It is observed that in some states of nature, there will be no informed agents in the model.

As in Chari and Jagannathan, the random proportion of type-1 agents is needed in order to create confusion between a large withdrawal queue size at the bank due to liquidity shocks, t_2 realized, or negative information shocks.

vii.- Parameter restrictions: In order for individuals to have a non-trivial signalling extraction problem, the following parameter restriction is assumed, (it will become clear why this assumption is needed).

$$t_2 = t_1 + \alpha(1 - t_1) \qquad [5]$$

viii.- Utility functions: Considering the agents' preferences hypothesis and in order to get numerical results, the following form for the utility function is assumed:

$$U^i(c_1, c_2, \rho_i) = \rho_i \frac{(k+c_1)^{1-\gamma}}{1-\gamma} + (1-\rho_i)\frac{(k+c_2)^{1-\gamma}}{1-\gamma} \qquad [6]$$

where k is a constant and $i = 1,\ 2$.[3]

b.- Data

The state of nature is described by the vector $\tilde{\theta} = (\tilde{t},\ \tilde{\alpha},\ \tilde{R})$ that contains the 3 random variables that are i.i.d.

In Table II, the expressions for the different states of nature and its associated probabilities are given.

1.1.- The banking contract: The ex-ante program.

As it was mentioned in the introduction, banks perform an important service by transforming highly illiquid assets into more liquid deposits. In this role, banks can be viewed as providing insurance to individuals that are uncertain about their future time preferences or liquidity needs. The demand deposit contract allows agents to consume whenever they need it, as it satisfies a sequential service constraint.

Formally, the demand deposit contract may be defined as a contract that requires an initial investment at T=0 with the intermediary in exchange for the right to withdraw per unit of initial investment (at the discretion of depositor and conditional on the bank's solvency) either:

a.- c_{11} units in period 1 and $\tilde{c}_{21}$ units in period 2
b.- c_{12} units in period 1 and $\tilde{c}_{22}$ units in period 2

As shown by Jacklin [16], the demand deposit contract optimally combines the two types of deposits that banks usually hold, a time deposit and a more typical demand deposit contract.

That is, at T=0 or ex-ante period, individuals deposit their unit of endowment at the bank and are offered a menu of contracts. At T=1 or interim period, depositors select their preferred contract (given their liquidity needs or information received).

A combination of these contracts could also be possible, in this case, it could be seen as depositors being allowed to withdraw at T=1 part of the second period withdrawal stream (subject to some early withdrawal penalty)[4].

It should be observed that the contract between the bank and depositors takes place at T=0, before neither the liquidity nor information shocks are realized. A summary of events is given in Table I.

In mathematical terms, the optimal contract choice for a deposit contract in the absence of interim information can be obtained as the solution to the following problem:

$$\max_{c_{ij},K}\left\{\underset{\tilde{R},\tilde{t}}{E}\left[\tilde{t}\,U^1\left(c_{11},\tilde{c}_{21},\rho_1\right)+(1-\tilde{t})\,U^2\left(c_{12},\tilde{c}_{22},\rho_2\right)\right]\right\} \qquad [7]$$

$$\text{s.t.}\quad \begin{aligned}&\tilde{t}c_{11}+(1-\tilde{t})c_{12}\leq K\\ &\tilde{t}\tilde{c}_{21}+(1-\tilde{t})\tilde{c}_{22}\leq(1-K)\tilde{R}\end{aligned} \qquad [8]$$

$$\underset{\tilde{R}}{E}\left[U^j\left(c_{1j},\tilde{c}_{2j},\rho_j\right)\right]\geq\underset{\tilde{R}}{E}\left[U^j\left(c_{1i},\tilde{c}_{2i},\rho_j\right)\right]\text{ for } i\neq j;\ i,j=1,2 \qquad [9]$$

where:

c_{1j}	Consumption at time T=1 for the type j agent
$\tilde{c}_{2j}$	Consumption at time T=2 for the type j agent dependent on the random return $\tilde{R}$ ($c_{2j}(\tilde{R})$)
K	Investment in the liquid short-lived asset at T=0
$1-K$	Investment in the illiquid long-lived asset at T=0
$\tilde{R}$	Random return of the long-lived asset at T=2.

The first two constraints are resource balance constraints and the last two are incentive compatibility constraints that guarantee that type-1 depositors will prefer their withdrawal stream to the type-2 withdrawal stream and viceversa, that is, the contract is designed so that depositors self-select their type contract.

In the above maximization problem both $\tilde{t}$ (the proportion of type-1 agents) and $\tilde{R}$ (the return on the long-term asset) are random variables.

A first simplification to the problem has been done by substituting $\tilde{t}$ by its expected value $t=t_1r_1+t_2r_2$. The bank solves its ex-ante program for the average proportion of type-1 agents[5].

Taking this into consideration, the maximization problem defined by equations (7), (8) and (9) is approximated as follows:

$$\max_{c_{ij}} \left\{ \underset{\tilde{R}}{E} \left[t\, U^1\left(c_{11}, \tilde{c}_{21}, \rho_1\right) + (1-t)\, U^2\left(c_{12}, \tilde{c}_{22}, \rho_2\right) \right] \right\} \tag{10}$$

$$\text{s.t.} \quad t\left(c_{11} + \frac{\tilde{c}_{21}}{\tilde{R}}\right) + (1-t)\left(c_{12} + \frac{\tilde{c}_{22}}{\tilde{R}}\right) = 1 \tag{11}$$

$$\underset{\tilde{R}}{E}\left[U^j\left(c_{1j}, \tilde{c}_{2j}, \rho_j\right)\right] \geq \underset{\tilde{R}}{E}\left[U^j\left(c_{1i}, \tilde{c}_{2i}, \rho_j\right)\right] \text{ for } i \neq j;\ i,j = 1,2 \tag{12}$$

where the two resource constraints have been substituted by a unique constraint.

The second type of uncertainty reflects the fact that, having invested in a risky technology, the bank may not be able to make its promised second period payments in full. One way to think of this, is that the bank promises an amount (c_{21}, c_{22}) it will be able to pay if $R = Rh$. If $R = Rl$ really occurs, the bank is considered insolvent and depositors get $\frac{Rl}{Rh}$ of their promised payments. It is then assumed that:

$$\tilde{c}_{2j} = c_{2j} \frac{\tilde{R}}{Rh} \quad (j = 1,2) \tag{13}$$

Given this dependence between consumption and returns, the above maximization problem is reformulated as follows[6]:

$$\max_{c_{ij}} \left\{ \begin{aligned} & t\left(\rho_1 \frac{(k+c_{11})^{1-\gamma}}{1-\gamma} + (1-\rho_1) E\left[\frac{(k+c_{21})^{1-\gamma}}{1-\gamma}\right]\right) + \\ & + (1-t)\left(\rho_2 \frac{(k+c_{12})^{1-\gamma}}{1-\gamma} + (1-\rho_2) E\left[\frac{(k+c_{22})^{1-\gamma}}{1-\gamma}\right]\right) \end{aligned} \right\} \tag{14}$$

s.t.

$$\begin{aligned} & t\left(c_{11} + \frac{c_{21}}{Rh}\right) + (1-t)\left(c_{12} + \frac{c_{22}}{Rh}\right) = 1 \\ & \rho_1 \frac{(k+c_{11})^{1-\gamma}}{1-\gamma} + (1-\rho_1) E\left[\frac{(k+c_{21})^{1-\gamma}}{1-\gamma}\right] \geq \rho_1 \frac{(k+c_{12})^{1-\gamma}}{1-\gamma} + (1-\rho_1) E\left[\frac{(k+c_{22})^{1-\gamma}}{1-\gamma}\right] \\ & \rho_2 \frac{(k+c_{12})^{1-\gamma}}{1-\gamma} + (1-\rho_2) E\left[\frac{(k+c_{22})^{1-\gamma}}{1-\gamma}\right] \geq \rho_2 \frac{(k+c_{11})^{1-\gamma}}{1-\gamma} + (1-\rho_2) E\left[\frac{(k+c_{21})^{1-\gamma}}{1-\gamma}\right] \end{aligned} \tag{15}$$

and where λ_1, λ_2, λ_3 are the multipliers associated with the corresponding resource and incentive constraints.

Given that the ex-ante probability of Rl is sufficiently small, the ex-ante contract has been approximated ignoring consumption changes produced by interim signals.

Considering the relationship $\hat{c}_{ij} = c_{ij} + k$ and applying Kunh-Tucker conditions, the optimal solution is obtained.

If the value of γ is such that the following inequation[7] is satisfied:

$$\rho_2\frac{\hat{c}_{12}^{1-\gamma}}{1-\gamma}+(1-\rho_2)(1-p)\frac{\hat{c}_{22}^{1-\gamma}}{1-\gamma}\geq\rho_2\frac{\hat{c}_{11}^{1-\gamma}}{1-\gamma}+(1-\rho_2)(1-p)\frac{\hat{c}_{21}^{1-\gamma}}{1-\gamma} \quad [16]$$

the optimal consumption levels are:

$$\hat{c}_{11}=\frac{1+k+(1-t)\frac{k}{Rh}}{t+(1-t)\left\{\left(\frac{\rho_2}{\rho_1}\right)^{\frac{1}{\gamma}}+Rh^{\frac{1-\gamma}{\gamma}}\left[\frac{(1-\rho_2)(1-p)}{\rho_1}\right]^{\frac{1}{\gamma}}\right\}} \qquad \hat{c}_{12}=\left(\frac{\rho_2}{\rho_1}\right)^{\frac{1}{\gamma}}\hat{c}_{11}$$

$$\hat{c}_{21}=k \qquad \hat{c}_{22}=\left[\frac{(1-\rho_2)(1-p)Rh}{\rho_1}\right]^{\frac{1}{\gamma}}\hat{c}_{11} \quad [17]$$

$$\lambda_1=\hat{c}_{11}^{-\gamma}\rho_1 \qquad \lambda_2=0 \qquad \lambda_3=0 \quad [18]$$

Otherwise, the optimal consumption levels are:

$$\hat{c}_{11}=\left[\frac{D(\hat{c}_{12})B\rho_2\hat{c}_{12}^{-\gamma}}{B\hat{c}_{12}^{-\gamma}\rho_1-\hat{c}_{12}^{-\gamma}+D(\hat{c}_{12})}\right]^{-1/\gamma} \qquad \hat{c}_{21}=k \qquad \hat{c}_{22}=\left[\frac{1-\rho_2}{\rho_2}(1-p)Rh\right]^{\frac{1}{\gamma}}\hat{c}_{12} \quad [19]$$

$$\lambda_1=D(\hat{c}_{12})\rho_2 \qquad \lambda_2=0 \qquad \lambda_3=t\left(\frac{\rho_1}{\rho_2}-\frac{\lambda_1}{\hat{c}_{11}^{-\gamma}\rho_2}\right) \quad [20]$$

and where $\hat{c}_{12}$ is obtained as a solution to the following non-linear equation:

$$\rho_2\hat{c}_{12}^{1-\gamma}+(1-\rho_2)(1-p)\left\{\left[(1-p)Rh\frac{1-\rho_2}{\rho_2}\right]^{\frac{1-\gamma}{\gamma}}\hat{c}_{12}^{1-\gamma}-k^{1-\gamma}\right\}-\rho_2\left[\frac{DB\rho_2\hat{c}_{12}^{-\gamma}}{B\hat{c}_{12}^{-\gamma}\rho_1-\hat{c}_{12}^{-\gamma}+D}\right]^{\frac{\gamma-1}{\gamma}}=0 \quad [21]$$

and,

$$M=1+k+\frac{k-t}{Rh}+\frac{\hat{c}_{12}}{Rh}(t-1)\left\{\left[(1-p)Rh\frac{1-\rho_2}{\rho_2}\right]^{1/\gamma}+Rh\right\}$$

$$B=-\frac{t}{(1-t)\rho_2} \qquad D(\hat{c}_{12})=\frac{\left(\frac{M}{t}\right)^{-\gamma}\hat{c}_{12}^{-\gamma}(B\rho_1-1)}{B\rho_2\hat{c}_{12}^{-\gamma}-\left(\frac{M}{t}\right)^{-\gamma}} \quad [22]$$

Lemma 1.- The optimal demand deposit contract satisfies: $c_{11}>c_{12}$ and $c_{22}>c_{21}=0$

Proof: See Appendix I.A for a detailed resolution of the problem.

1.2.- The ex-post problem

In the interim stage, the liquidity and information shocks are realized, and so every individual learns his type and also some type-2 agents will be informed about the return of the long-term asset at T=2. Individuals will select the contract they prefer (given their liquidity needs or information received) in order to maximize their utility function, (the choices of the individuals are defined by the dimensionless coefficients μ_1, μ_I and μ_2 respectively. See footnote 4).

As will be seen in this subsection, type-1 individuals will withdraw for liquidity reasons while informed type-2 agents withdraw whenever they receive negative information concerning the return of the long-term asset. Finally, uninformed type-2 agents condition their beliefs about the long term return on the size of the withdrawal queue at the bank. This size could be large due to many agents wishing to consume early (liquidity shock), or because some individuals have received a negative information shock. As uninformed agents cannot distinguish between these two shocks, in some states of nature, individuals may end up withdrawing their deposits from the bank, although their is no adverse information held by any agent.

The behaviour of the different agents is formulated as follows:

a).- Type-1 agents

The value of μ_1 is chosen in order to maximize their utility function and subject to their two period constraint; that is:

$$\begin{aligned} \max_{\mu_1} \quad & U^1(c_1,c_2,\rho_1) = \max_{\mu_1}\left\{\rho_1\frac{(k+c_1)^{1-\gamma}}{1-\gamma} + (1-\rho_1)E\left[\frac{(k+c_2)^{1-\gamma}}{1-\gamma}\right]\right\} \\ \text{s.t} \quad & c_1=c_{12}+\mu_1(c_{11}-c_{12}) \\ & c_2=c_{21}+(1-\mu_1)(c_{22}-c_{21}) \\ & \mu_1\leq 1 \end{aligned} \qquad [23]$$

The solution to the type-1 problem is given by Lemma 2:

Lemma 2.- Absent any information, type-1 agents will always select their own contract, that is, the optimal solution to the type-1 problem is $\mu_1^*=1$ (c_{11}, c_{21}).

Proof: See Appendix I.B.

The solution to the type-1 problem is trivial, as these individuals do not care about second period consumption, therefore they will withdraw to consume in the interim period.

b).- Informed type-2 agents.

In each state and conditional on the information about $\tilde{R}$ they solve the following problem:

$$\max_{\mu_I} U^I(c_1,c_2,\rho_2) = \max_{\mu_I}\left\{\rho_2\frac{(k+c_1)^{1-\gamma}}{1-\gamma}+(1-\rho_2)E\left[\frac{(k+c_2)^{1-\gamma}}{1-\gamma}\Big/\tilde{R}\right]\right\} \quad [24]$$

$$\text{with:}\quad \begin{aligned} &c_1=c_{12}+\mu_I(c_{11}-c_{12})\\ &c_2=c_{21}+(1-\mu_I)(c_{22}-c_{21})\\ &\mu_I\leq 1 \end{aligned} \quad [25]$$

There are two different values for μ_I, depending on the information about $\tilde{R}$ received by these agents at T=1.

b1).- If $\tilde{R}=Rh$ is the information received at date 1, then the informed type-2 agents find their consumption by solving the following problem:

$$\max_{\mu_I}\left\{\rho_2\frac{(k+c_1)^{1-\gamma}}{1-\gamma}+(1-\rho_2)E\left[\frac{(k+c_2)^{1-\gamma}}{1-\gamma}\Big/R=Rh\right]\right\} \quad [26]$$

$$\text{with}\quad \begin{aligned} &c_1=c_{12}+\mu_I(c_{11}-c_{12})\\ &c_2=c_{21}+(1-\mu_I)(c_{22}-c_{21})\\ &\mu_I\leq 1 \end{aligned} \quad [27]$$

Lemma 3.- If type-2 agents receive positive information concerning the asset's return, they would choose a combination of the two contracts, that is:

$$0 \nless \mu_I=\frac{B_{2I}(k+c_{22})-(k+c_{12})}{B_{2I}(c_{22}-c_{21})+(c_{11}-c_{12})}<1 \quad \text{with} \quad B_{2I}=\left(\frac{c_{22}-c_{21}}{c_{11}-c_{12}}\,\frac{1-\rho_2}{\rho_2}\right)^{-\frac{1}{\gamma}} \quad [28]$$

Proof: See Appendix I.B.

b2).- If $\tilde{R}=Rl$ is the value of $\tilde{R}$ revealed to type-2 agents, then the level of consumption is obtained in a similar way as above:

$$\max_{\mu_I}\left\{\rho_2\frac{(k+c_1)^{1-\gamma}}{1-\gamma}+(1-\rho_2)E\left[\frac{(k+c_2)^{1-\gamma}}{1-\gamma}\Big/R=Rl\right]\right\} \quad [29]$$

$$\text{with:}\quad \begin{aligned} &c_1=c_{12}+\mu_I(c_{11}-c_{12})\\ &c_2=c_{21}+(1-\mu_I)(c_{22}-c_{21})\\ &\mu_I\leq 1 \end{aligned} \quad [30]$$

The solution to this problem is given by Lemma 4:

Lemma 4.- If type-2 agents receive negative information concerning the asset's return they will always claim the type-1 contract that is, the optimal solution is $\mu_I^*=1$ (c_{11}, c_{21}).

Proof: See Appendix I.B.

c).- Uninformed type-2 agents.

These agents maximize expected utility conditional on the observation of a noisy signal, which is the withdrawal queue size at the bank, or equivalently, the level of aggregate demanded consumption (CT) at T=1. The coefficient $\mu_2=\mu_2(CT)$ is chosen in order to maximize:

$$\max_{\mu_2} U^2(c_1,c_2,\rho_2)=\max_{\mu_2}\left\{\frac{(k+c_1)^{1-\gamma}}{1-\gamma}+E\left[\frac{(k+\tilde{c}_2)^{1-\gamma}}{1-\gamma}\bigg/ CT\right]\right\}$$

$$\text{s.t.}\quad \begin{aligned} &c_1=c_{12}+\mu_2(c_{11}-c_{12})\\ &c_2=c_{21}+(1-\mu_2)(c_{22}-c_{21})\\ &\mu_2\leq 1\end{aligned} \qquad [31]$$

It should be noticed that aggregate demanded consumption could be high either because the proportion of type-1 agents is large or because informed agents have received negative information concerning the asset's return at T=2, this confusion is crucial to the results.

The value of CT depends on each state of nature $\tilde{\theta}=(\tilde{t},\ \tilde{\alpha},\ \tilde{R})$ with probability $p(\tilde{\theta})$ and its expression is:

$$\tilde{CT}=\tilde{t}\,cc_{11}+(1-\tilde{t})[\tilde{\alpha}cc_{12I}+(1-\tilde{\alpha})cc_{12}] \qquad [32]$$

and where cc_{11} represents the consumption at date 1 of type-1 agents (value of c_1 obtained in (a)) and cc_{12I}, cc_{12} that of informed and uninformed type-2 agents respectively (obtained in (b) and the value of c_1 to be found as a result to the above problem).

The values of CT are shown in Table II. It should be observed that there for any $\mu_2(CT)$ or equivalently any consumption level (cc_{12}), uninformed agents may choose, there is always confusion between states 3 and 4 as type-1's consumption at T=1 (cc_{11}) is c_{11} and informed type-2's consumption in the case of a negative information shock (cc_{12L}) is also c_{11}.

More generally, let θ^*_{CT} be defined as the finite set $\theta^*_{1,CT},\theta^*_{2,CT},...\theta^*_{N,CT}$ of all the states of nature that give for the same value of $\mu_2=\mu_2(CT)$ the same value of aggregate consumption. The probability of state $\theta^*_{i,CT}$ is:

$$p(\theta^*_{i,CT}/CT)=\frac{p(\theta^*_{i,CT})}{\sum_{i=1}^{N}p(\theta^*_{i,CT})} \qquad [33]$$

The set of N states of nature can be divided into 2 groups. The first N_1 states correspond to $\tilde{R}$=Rl and the last N_2 to the case $\tilde{R}$=Rh. On the other side, given the independence of the random variables equation (33) can be rewritten:

$$p(\theta^*_{i,CT}/CT)=\frac{p(t^*_{i,CT})p(\alpha^*_{i,CT})}{\sum_{i=1}^{N}p(\theta^*_{i,CT})}p=\pi_i p \qquad \text{if} \quad 0<i<N_1 \tag{34}$$

$$p(\theta^*_{i,CT}/CT)=\frac{p(t^*_{i,CT})p(\alpha^*_{i,CT})}{\sum_{i=1}^{N}p(\theta^*_{i,CT})}(1-p)=\pi_i(1-p) \qquad \text{if} \quad N_1<i<N_2 \tag{35}$$

And so the problem defined by (31) is reformulated as follows:

$$\max_{\mu_2}\left\{\rho_2\frac{(k+c_1)^{1-\gamma}}{1-\gamma}+(1-\rho_2)\left[\frac{(k+c_2)^{1-\gamma}}{1-\gamma}\pi_1^*+\frac{\left(k+c_2\frac{Rl}{Rh}\right)^{1-\gamma}}{1-\gamma}\pi_2^*\right]\right\}$$
$$\text{s.}t. \quad c_1=c_{12}+\mu_2(c_{11}-c_{12})$$
$$c_2=c_{21}+(1-\mu_2)(c_{22}-c_{21})$$
$$\mu_2\leq 1 \tag{36}$$

where:

$$\pi_1^*=(1-p)\sum_{i=1+N_1}^{N}\pi_i \qquad \pi_2^*=p\sum_{i=1}^{N_1}\pi_i \tag{37}$$

with solution:

$$0<\mu_2=\frac{B_{CT}(k+c_{22})-(k+c_{12})}{B_{CT}(c_{22}-c_{21})+(c_{11}-c_{12})}\leq 1 \quad \text{with} \quad B_{CT}=\left(\frac{c_{22}-c_{21}}{c_{11}-c_{12}}\,\frac{1-\rho_2}{\rho_2}\,\pi_1^*\right)^{-\frac{1}{\gamma}} \tag{38}$$

$cc_{11}=c_{12}+\mu_1(c_{11}-c_{12})=c_{11}$ Aggregate consumption of type-1 agent at time T=1

cc_{12} Aggregate consumption of uninformed type-2 agent at time T=1

cc_{12H}, cc_{12L} Aggregate consumption of informed type-2 agent at time T=1 depending on the information: *Rh, Rl.*

If $t_2=t_1+\alpha(1-t_1)$ then there is confusion between states 3 and 4 due to the fact that, $cc_{12L}=c_{12}+(\mu_l=1)(c_{11}-c_{12})=c_{11}$

2.- Condition to assure a panic run

Bank runs occur whenever uninformed type-2 agents start making type-1 withdrawals upon observation of aggregate consumption at T=1.

Conditions for both information-induced and pure panic runs to occur are given by Proposition 1.

Proposition 1.- In the model, bank runs occur as a unique equilibrium, if the following conditions hold:

$$\rho_2\frac{(k+c_{11})^{1-\gamma}}{1-\gamma}+(1-\rho_2)\left[\frac{(k+c_{21})^{1-\gamma}}{1-\gamma}\pi_{1_{\mu=1}}+\frac{\left(k+c_{21}\frac{Rl}{Rh}\right)^{1-\gamma}}{1-\gamma}\pi_{2_{\mu=1}}\right]>$$
$$>\rho_2\frac{(k+c_{12})^{1-\gamma}}{1-\gamma}+(1-\rho_2)\left[\frac{(k+c_{22})^{1-\gamma}}{1-\gamma}\pi_{1_{\mu=0}}+\frac{\left(k+c_{22}\frac{Rl}{Rh}\right)^{1-\gamma}}{1-\gamma}\pi_{2_{\mu=0}}\right] \quad [39]$$

$$\rho_2(k+c_1)^{-\gamma}(c_{11}-c_{12})-(1-\rho_2)(k+c_2)^{-\gamma}\pi_{2_{\mu=1}}>0 \quad [40]$$

where:

$$\pi_{1_{\mu=1}}=\frac{(1-p)(1-q)}{1-q+pq} \qquad \pi_{2_{\mu=1}}=\frac{p}{1-q+pq}$$
$$\pi_{1_{\mu=0}}=\frac{(1-p)r_2}{r_1pq+r_2(1-q)+r_2(1-p)q} \qquad \pi_{2_{\mu=0}}=\frac{p[r_1q+r_2(1-q)]}{r_1pq+r_2(1-q)+r_2(1-p)q} \quad [41]$$

and c_1, c_2 as defined in the uninformed type-2 agents maximization problem.

Proof: See Appendix II.

If these conditions are satisfied there are bank-runs in states 3, 4 and 6 and the levels of aggregate demanded consumption are the ones given by column 5 in Table II. It is observed that aggregate demanded consumption in those states is c_{11}, which exceeds ex-ante planned consumption at T=1 or the investment in the liquid asset K. In fact,

$$c_{11}>tc_{11}+(1-t)c_{12} \Rightarrow (1-t)c_{11}>(1-t)c_{12} \quad [42]$$

and $c_{11}>c_{12}$ by definition of the optimal deposit contract.

Whenever the withdrawal queue size at the bank exceeds the ex-ante investment in the liquid asset, the bank suspends convertibility, as will be seen in subsection IV.A.

In all these cases, bank runs occur as a unique equilibrium, in states 3 and 6 there are information-induced runs as there is a negative information shock, however in state 4 there is a pure panic run as there is no adverse information held by any agent in this state. As already mentioned it may arise due to a confusion in the observation of aggregate consumption at T=1, it models the idea that runs may be a contagion phenomenon, in which individuals observing long lines at the bank, they infer that there is a possibility that the bank is about to fail and precipitate a bank run.

The fact that there is a probability of having informed agents in the model makes this type of runs possible.

3.- An example of application

In order to plot a numerical solution for the just mentioned model an example of application has been developed for the following input data[8].

```
An example of Public intervention in Financial Intermediation

Proportions of type-1 agents                         t1, t2...... 0.30  0.51
Probabilities of the above proportions               r1, r2...... 0.99  0.01
Proportions of informed type-2 agents                0, alpha.... 0.00  0.30
Probabilities of the above proportions               q, 1-q...... 0.90  0.10
Low and high random returns                          Rl, Rh...... 1e-20 1.50
Probabilities of the low and high returns            p, 1-p...... 0.10  0.90
Liquidation cost                                     aa.......... 0.00
Minimum risk-aversion coefficient                    gamma1...... 0.30
Maximum risk-aversion coefficient                    gamma2...... 3.50
Incrementum of the risk-aversion coefficient         dgamma...... 0.10
Intertemporal coefficients of types 1, 2 agents rho1, rho2.. 0.99  0.50
```

The ex-ante optimal demand deposit contract is shown in Figure 1 for different values of the risk-aversion coefficient.

Figure 2 illustrates ex-ante aggregate consumption (planned by the bank) and ex-post aggregate demanded consumption for all states of nature and for $\gamma=1.5$.[9] It can be seen that in states 3, 4 and 6 aggregate demanded consumption exceeds the ex-ante one, in these states there are bank runs as all depositors choose the type-1 withdrawal stream[10] and the bank cannot meet all withdrawals. In states 3 and 6 there are information-induced runs as there is a negative information shock, but in state 4 there is a "pure panic" run as there is no adverse information held by any agent.

It is assumed that a subsidy can be received from the government so that type-1 withdrawals are always feasible (when the highest $\tilde{t}$ is realized).

Figure 3 shows that the conditions of Proposition 1 are satisfied in this example. The expected utility in states 3 and 4 attains its highest value for $\mu=1$.

As it was mentioned before, "panic runs" occur because there is a probability of having informed agents in the model.

Figure 4 gives the threshold level of q (probability of informed agents) above which the conditions of Proposition 1 are satisfied, that is, "panic" runs occur.

This threshold level has been derived by solving inequations (39) and (40) in q (for the exogenously fixed parameters ρ_1, ρ_2, t_1, t_2, r_1, r_2, Rh, Rl, p, γ).

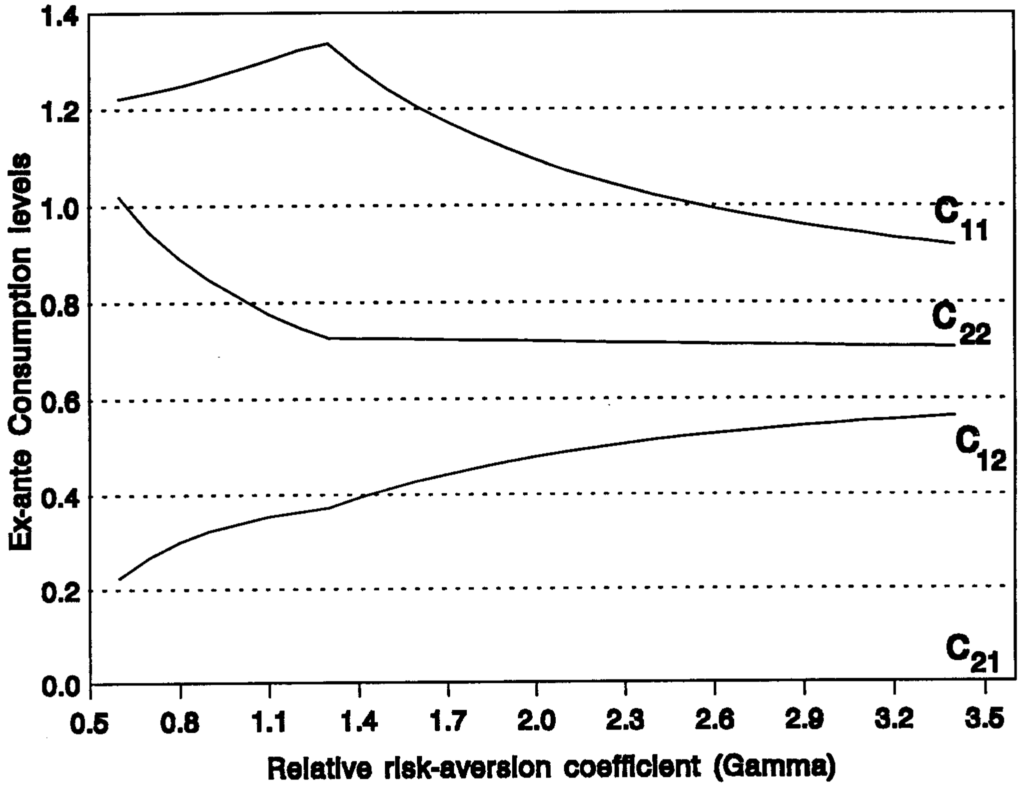

Figure 1.- Optimal demand deposit contract

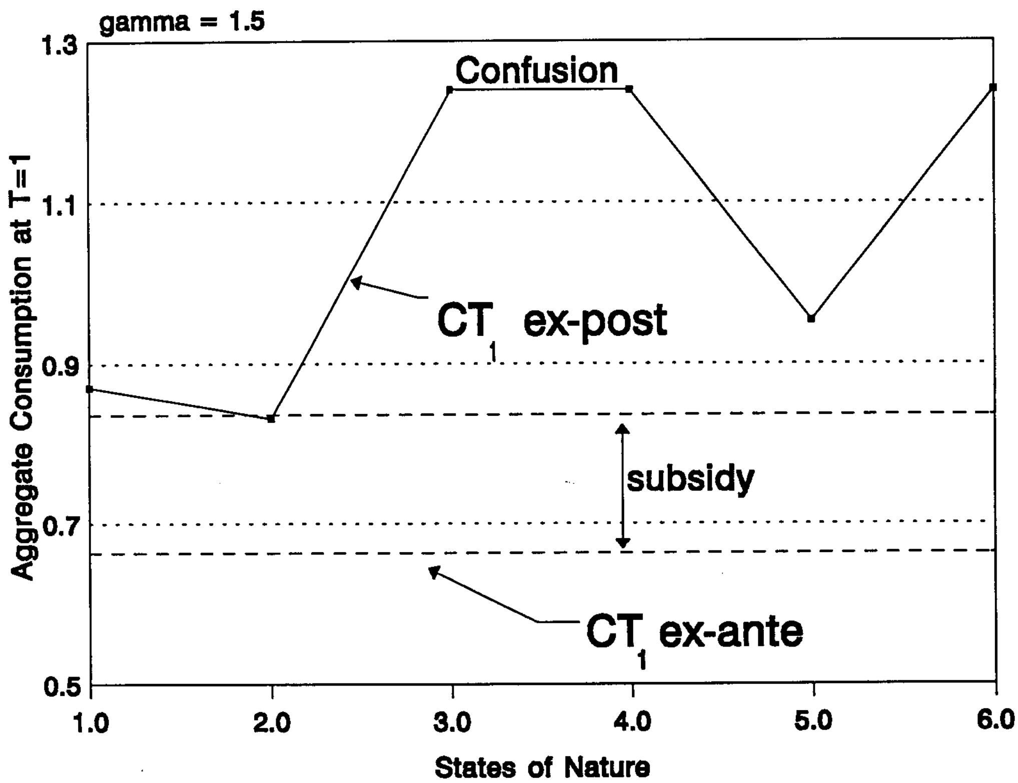

Figure 2.- Aggregate consumption at T=1.

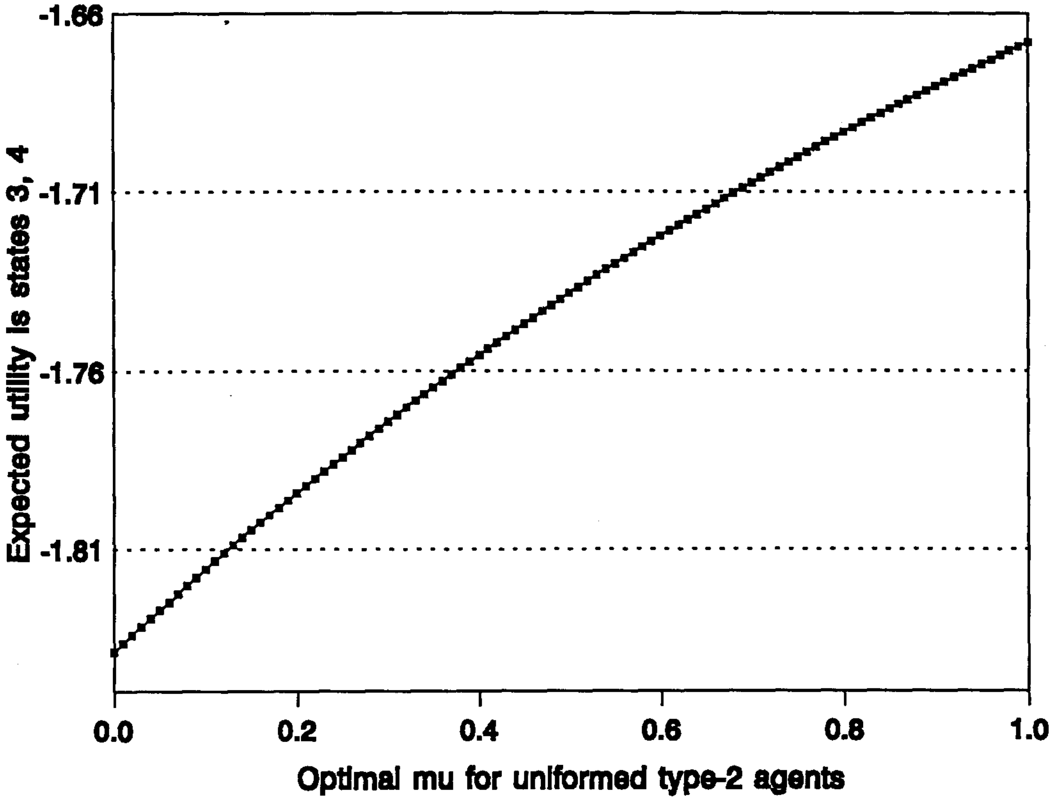

Figure 3.- Optimal mu of uninformed type-2 agents

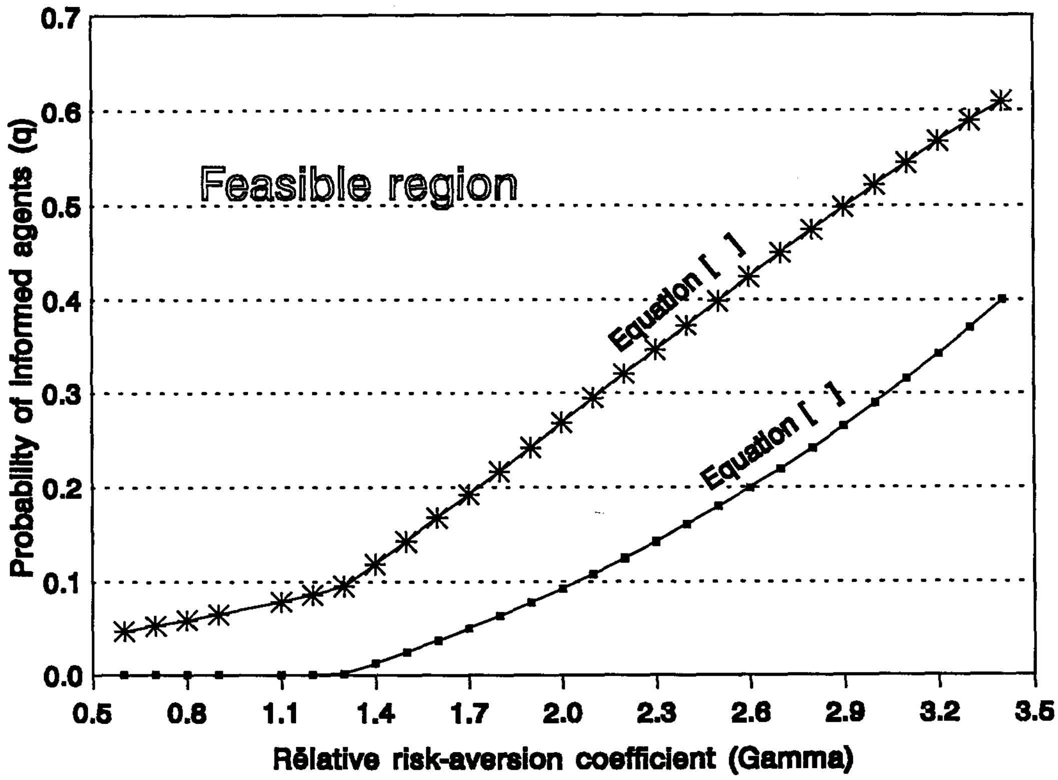

Figure 4.- Feasibility region for Proposition 1.

4.- Public intervention

Banks perform a very valuable function, which is to transform highly illiquid assets into more liquid liability payoffs through the demand deposit contract. This transformation service gives a rationale for the existence of banks and at the same time for their vulnerability to runs.

Coordination problems derived from deposit contracting have been considered important due to the fear of systematic risk. An example may be found in the banking panics that ocurred in the 1930s and the strict regulation that was imposed as a reaction to the crisis.

However, although some of these measures have been effective in preventing bank runs, they have created additional problems, which gives a starting point for banking regulation.

The aim of this section is to analyze two devices that have traditionally been used by banks in order to prevent runs, namely suspension of convertibility (of deposits into currency) and deposit insurance.

The costs of suspension of convertibility (in randomization in meeting liquidity needs) versus deposit insurance (in deadweight taxation) will be compared.

4.1.- Suspension of convertibility

It is a measure that banks have historically used against runs, in the pre-deposit insurance era.

During the 19^{th} and early 20^{th} the American Banking System suspended convertibility eight times. In each case, suspension was the response to a banking panic which was coincident (or nearly so) with a business cycle peak. A curious aspect of suspension is that neither banks, depositors nor the courts opposed to it at any time. Gorton [15] shows that this accommodating behaviour arose because suspension was part of a mutually beneficial arrangement.

In Diamond and Dybvig [12], a policy of suspension of convertibility attains the Pareto dominant equilibrium if there is no aggregate uncertainty with respect to the proportion of early diers; otherwise this measure would not be so effective[11].

In Chari and Jagannathan [9], a suspension clause improves on the ex-ante utility, however it is not a first-best measure as some early diers would not get all their withdrawal stream when there are many of them.

In the following pages, this measure is introduced into the model; suspension of convertibility occurs at the level of the highest proportion of early diers, that is:

$$K + G = t_2 c_{11} + (1 - t_2) c_{12} \qquad [43]$$

where K is the investment planned ex-ante by the bank in the liquid asset and G is a subsidy received from the government in order to cover withdrawals up to the highest proportion of early diers.

However, in the ex-post situation, there may be some states of nature for which the value of aggregate consumption $\tilde{CT}_1$ is greater than the assumed ex-ante one.

The value of the aggregate demanded consumption $\tilde{CT}_1$ in state $\tilde{\theta}=(\tilde{t},\ \tilde{\alpha},\ \tilde{R})$ at $T=1$ is given by the expression:

$$CT_1=\tilde{t}cc_{11}+\tilde{\alpha}(1-\tilde{t})c\tilde{c}_{12I}+(1-\alpha)(1-\tilde{t})cc_{12} \tag{44}$$

Then, the deficit $\Delta\tilde{CT}_1$ is defined as follows:

$$\begin{aligned} &\Delta\tilde{CT}_1=\tilde{CT}_1-(K+G) && \text{if} \quad \tilde{CT}_1\geq K+G \\ &\Delta\tilde{CT}_1=0 && \text{if} \quad \tilde{CT}_1<K+G \end{aligned} \tag{45}$$

This deficit in the consumption has to be shared among the different agents. There exist several ways to carry out this deficit distribution, three alternative mechanisms will be discussed here, namely:

a.- A constant deduction of the consumption among all type-2 agents.

b.- A proportional deduction of the consumption among all type-2 agents.

c.- The consumption deficit is completely supported by the last agents trying to convert their returns.

In the first two alternatives it is assumed that type-1 agents are always first in line to get their withdrawal stream and therefore the consumption deficit will be shared only among type-2 agents. In the third alternative, depositors are treated on a first-come-first-served-basis, and so the available funds are allocated randomly among all depositors.

In the following, the formulation for each alternative is given.

Alternative (a)

In this alternative there is a constant deduction of consumption among all type-2 agents, defined as: $\delta^a=\dfrac{\Delta\tilde{CT}_1}{1-\tilde{t}}$.

The resulting consumption levels of informed (cc^a_{12I}, cc^a_{22I}) and uninformed type-2 agents (cc^a_{12}, cc^a_{22}) are:

$$\begin{aligned} &cc^a_{12I}=cc_{12I}-\tilde{\delta}^a\nless 0 && cc^a_{12}=cc_{12}-\tilde{\delta}^a\nless 0 \\ &cc^a_{22I}=cc_{22I} && cc^a_{22}=cc_{22} \end{aligned} \tag{46}$$

Alternative (b)

In this situation, the consumption levels of type-2 agents are diminished proportionally to their demanded consumption, therefore, the proportion of the total to be divided is:

$$\delta^b=\frac{\Delta\tilde{CT}_1}{\tilde{\alpha}(1-\tilde{t})cc_{12I}+(1-\tilde{\alpha})(1-\tilde{t})cc_{12}} \tag{47}$$

where: informed type-2 support $\delta^I=cc_{12I}\delta^b$ and uninformed type-2 support $\delta^2 = cc_{12}\delta^b$.

The modified consumption levels of informed and uninformed type-2 agents would be:

$$
\begin{array}{ll}
cc^b_{12I} = cc_{12I} - \tilde{\delta}^I & cc^b_{12} = cc_{12} - \tilde{\delta}^2 \\
cc^b_{22I} = cc_{22I} & cc^b_{22} = cc_{22}
\end{array}
\qquad [48]
$$

Alternative (c)

In this alternative the available funds are allocated randomly among depositors. Let $\tilde{\beta}$ be the random proportion of agents of each type that supports the deficit respect to the total number of the same type of agents, these agents will receive what was planned in the ex-ante analysis, i.e.:

$$
\Delta \tilde{CT}_1 = \tilde{\beta}\left\{(1-\tilde{t})\left[\tilde{\alpha}(cc_{12I} - c_{12}) + (1-\tilde{\alpha})(cc_{12} - c_{12})\right] + \tilde{t}(cc_{11} - c_{12})\right\} \qquad [49]
$$

or:

$$
\tilde{\beta} = \frac{\Delta \tilde{CT}_1}{(1-\tilde{t})\left[\tilde{\alpha}(cc_{12I} - c_{12}) + (1-\tilde{\alpha})(cc_{12} - c_{12})\right] + \tilde{t}(cc_{11} - c_{12})} \qquad [50]
$$

It should be noted that $\tilde{\beta} \geq 0$ because μ_1, μ_I and μ_2 are positive numbers and cc_{11}, cc_{12I} and $cc_{12} > c_{12}$.

In this alternative the consumptions are then:

Type-1 agents:

$$
\begin{array}{ll}
cc^{c_1}_{11} = cc_{11} & \text{for the first } (1-\tilde{\beta})\tilde{t} \\
cc^{c_1}_{21} = cc_{21} & \text{type-1 agents} \\
cc^{c_2}_{11} = c_{12} & \text{for the last } \tilde{\beta}\tilde{t} \\
cc^{c_2}_{21} = cc_{21} & \text{type-1 agents}
\end{array}
\qquad [51]
$$

Informed type-2 agents:

$$
\begin{array}{ll}
cc^{c_1}_{12I} = cc_{12I} & \text{for the first } (1-\tilde{\beta})\tilde{\alpha}(1-\tilde{t}) \text{ informed} \\
cc^{c_1}_{22I} = cc_{22I} & \text{type-2 agents} \\
cc^{c_2}_{12I} = c_{12} & \text{for the last } \tilde{\beta}\tilde{\alpha}(1-\tilde{t}) \text{ informed} \\
cc^{c_2}_{22I} = cc_{22I} & \text{type-2 agents}
\end{array}
\qquad [52]
$$

Uninformed type-2 agents:

$$\begin{aligned}
&cc_{12}^{c_1} = cc_{12} \quad \text{for the first} \quad (1-\tilde{\beta})(1-\tilde{\alpha})(1-\tilde{t}) \quad \text{uninformed}\\
&cc_{22}^{c_1} = cc_{22} \quad \text{type-2 agents}\\
&cc_{12}^{c_2} = c_{12} \quad \text{for the last} \quad \tilde{\beta}(1-\tilde{\alpha})(1-\tilde{t}) \quad \text{uninformed}\\
&cc_{22}^{c_2} = cc_{22} \quad \text{type-2 agents}
\end{aligned} \tag{53}$$

Given these modified consumption levels after suspension, the aggregate expected utility, U_T, is defined as follows:

$$\begin{aligned}
EU_T = \sum_\theta U_T p(\theta) = \sum_\theta \Bigg\{ & \left[\rho_1 \frac{\left(k + cc_{11}^{s_1}\right)^{1-\gamma}}{1-\gamma} + (1-\rho_1)\frac{\left(k + \frac{R^\theta}{Rh} cc_{21}^{s_1}\right)^{1-\gamma}}{1-\gamma}\right] t^\theta (1-\beta^\theta) + \\
& \left[\rho_1 \frac{\left(k + cc_{11}^{s_2}\right)^{1-\gamma}}{1-\gamma} + (1-\rho_1)\frac{\left(k + \frac{R^\theta}{Rh} cc_{21}^{s_2}\right)^{1-\gamma}}{1-\gamma}\right] t^\theta \beta^\theta + \\
& \left[\rho_2 \frac{\left(k + cc_{12I}^{s_1}\right)^{1-\gamma}}{1-\gamma} + (1-\rho_2)\frac{\left(k + \frac{R^\theta}{Rh} cc_{22I}^{s_1}\right)^{1-\gamma}}{1-\gamma}\right] (1-t^\theta)\alpha^\theta (1-\beta^\theta) + \\
& \left[\rho_2 \frac{\left(k + cc_{12I}^{s_2}\right)^{1-\gamma}}{1-\gamma} + (1-\rho_2)\frac{\left(k + \frac{R^\theta}{Rh} cc_{22I}^{s_2}\right)^{1-\gamma}}{1-\gamma}\right] (1-t^\theta)\alpha^\theta \beta^\theta + \\
& \left[\rho_2 \frac{\left(k + cc_{12}^{s_1}\right)^{1-\gamma}}{1-\gamma} + (1-\rho_2)\frac{\left(k + \frac{R^\theta}{Rh} cc_{22}^{s_1}\right)^{1-\gamma}}{1-\gamma}\right] (1-t^\theta)(1-\alpha^\theta)(1-\beta^\theta) + \\
& \left[\rho_2 \frac{\left(k + cc_{12}^{s_2}\right)^{1-\gamma}}{1-\gamma} + (1-\rho_2)\frac{\left(k + \frac{R^\theta}{Rh} cc_{22}^{s_2}\right)^{1-\gamma}}{1-\gamma}\right] (1-t^\theta)(1-\alpha^\theta)\beta^\theta \Bigg\} p(\theta)
\end{aligned} \tag{54}$$

where $\theta = 1,..,6$

where $s = a,\ b,\ c$ and $s_1 = s_2$ for alternatives $\boldsymbol{a}$ and $\boldsymbol{b}$.

The welfare measure to be considered in this study will be:

$$\text{cert.equivalent}(EU_T) - G(1+s) \tag{55}$$

The subsidy (G) would be like a deadweight tax on individuals of value s.

4.2.- Deposit insurance

In the U.S, this measure was first introduced in ***the Glass-Steagall Act***, an important banking legislation reaction to the bank-runs that occurred during the Great Depression.

It has been the most successful device in order to prevent runs, however it has created problems of its own. There are two effects associated with deposit insurance: on one side, it encourages banks to take excessive risk and on the other side depositors have less incentives to monitor their bank under a deposit insurance system.

As already shown in Bryant [8], deposit insurance eliminates incentives for agents to seek socially wasteful information in the presence of undiversifiable risk.

Diamond-Dybvig [12] also advocate deposit insurance, but for a different reason, in their model, it adjusts for an aggregate shock, in the proportion of agents wishing to withdraw early. Deposit insurance also eliminates the Pareto-inferior equilibrium (bank-run) because the deposit contract schedule, conditioned of the realized aggregate shock, always has the feature that waiting to withdraw dominates early withdrawal, and so the Pareto-dominant equilibrium can be implemented.

In the model it is assumed that there is government deposit insurance of the bank deposits at T=2.

This removes the incentives of agents to become informed and so information-induced runs will no longer occur. In this case, it is assumed that agents will consume what was planned in the ex-ante contract.

Deposit insurance is introduced into the model as follows:

Whenever the realized benefit at T=2, $\tilde{R}\,(1-K)$ is less than the assumed consumption for that period, that is, $tc_{21}+(1-t)c_{22}$, the difference is always supplied by the government. The expected cost of the insurance would be:

$$Cd=\sum_{\theta}\left[tc_{21}+(1-t)c_{22}-R^{\theta}(1-K)\right]p(\theta) \qquad [56]$$

The expected cost of the insurance may be considered also as a deadweight tax on individuals and so the welfare measure for deposit insurance will be:

$$\text{cert. equivalent}(U\text{exante})-Cd(1+s)-\left[\sum_{\theta}G\,p(\theta)\right](1+s) \qquad \theta=4,\ 5,\ 6 \qquad [57]$$

where G is a subsidy received from the government to cover the liquidity shock, that is, in those states of nature in which t_2 is realized.

5.- Comparison among the different public intervention measures: numerical examples

The two public intervention measures discussed in the preceeding section, have been applied to the numerical example of section III.

The costs of suspension of convertibility (in randomization in meeting liquidity needs) have been compared to those of deposit insurance (in deadweight taxation).

Figure 5 shows that in the numerical example of section III suspension of convertibility yields a higher level of utility than deposit insurance.

With respect to the different suspension measures, suspensions of convertibility a (constant deduction to all type-2 agents) or b (a proportional deduction of consumption to type-2 agents) yield a higher level of utility than c (it applies to the last agents arriving to the bank).

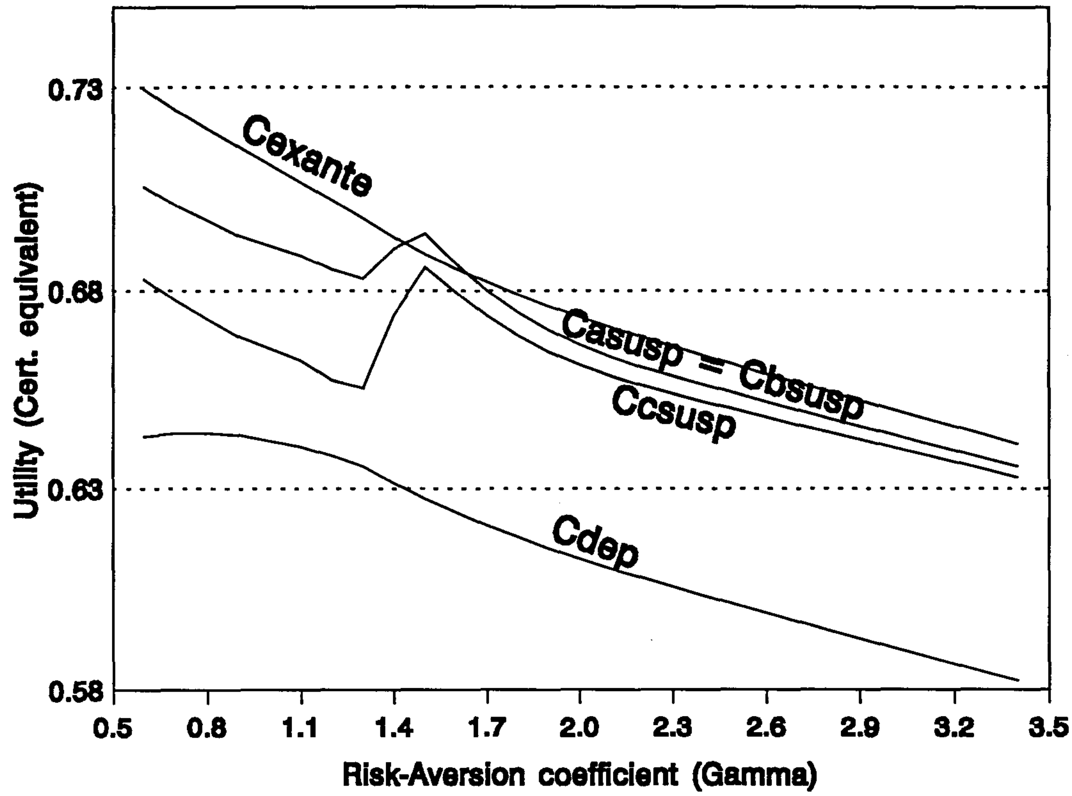

Figure 5.- Suspension of Convertibility versus Deposit Insurance

However, the results are very sensitive to the exogenous parameters of the model.

In the numerical simulations, it has been assumed that with a high probability the lowest proportion of type-1 agents (t_1) is realized ($r_1 = 0.99$). The motivation for this assumption is to create confusion between a large withdrawal queue size at the bank, due to a high liquidity shock (t_2 realized) or a negative information shock. It is also assumed that the probability of the low value of the random return occuring (Rl) is sufficiently small ($p = 0.10$), and this allows to simplify the ex-ante contract maximization problem.

The sensitivity analysis has been carried out with respect to the standard deviation of the random return, $\tilde{R}$, (keeping the mean return constant and increasing the dispersion of $\tilde{R}$)[12].

Figure 6 shows the certainty equivalent of the utility attained with suspension of convertibility (c) minus deposit insurance, as a function of the relative risk aversion coefficient (γ) and for different values of the standard deviation of the random return (σ).

The first line ($\sigma = 0.21$) shows that for low values of γ, suspension would be welfare superior, for intermediate values of γ deposit insurance would be better (up to $\gamma = 1.70$) and from then on, suspension yields again higher

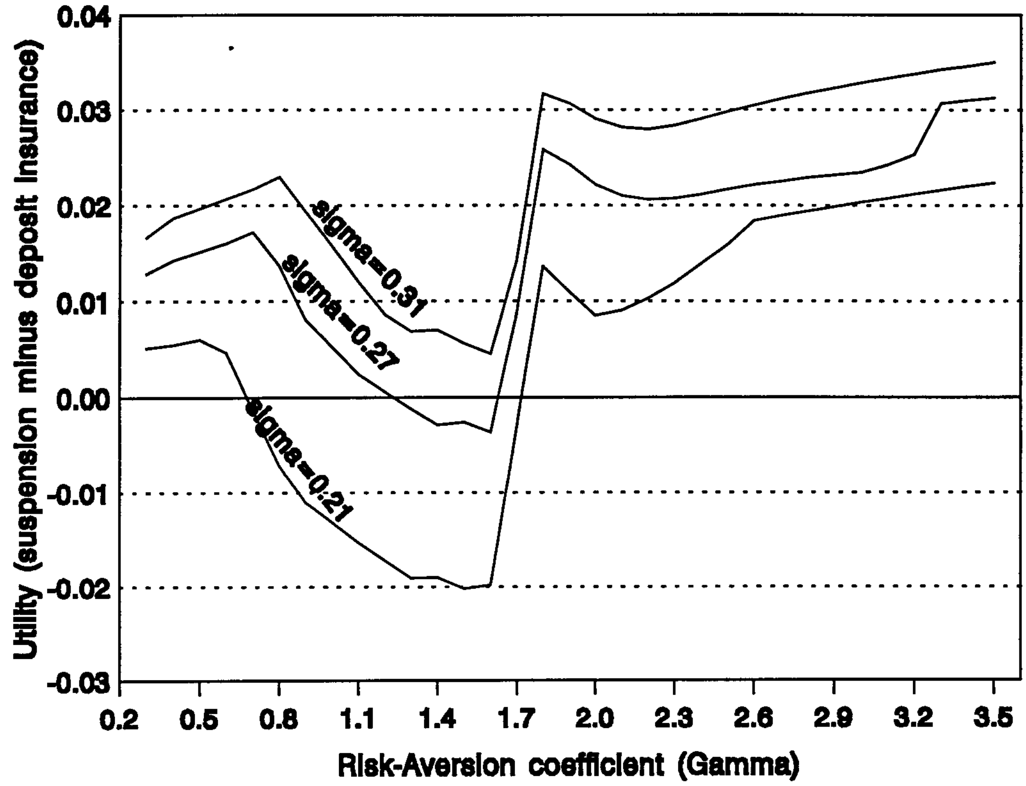

Figure 6.- Suspension of Convertibility versus Deposit Insurance, with s=0.20

utility. As σ increases, suspension is welfare superior for nearly all levels of risk-aversion and for *very risky* investments *suspension would be always the preferred measure.* These results support regulatory proposals that aim to restrict insured liabilities to finance very low risk assets.

The range of parameters for which either measure would be preferred also depends on the exogenous value of the deadweight tax (s) or the random proportion of type-1 agents ($\tilde{t}$).

Table IV summarizes how the welfare analysis (suspension of convertibility c minus deposit insurance) is affected by variations in the exogenous parameters of the model.

6.- Conclusions and suggestions for further research

This paper has introduced risk-averse preferences in Chari and Jagannathan'model. A first motivation for this extension was to give a positive role for a financial intermediary in the economy.

The introduction of risk-aversion in the CH-J model, implies an ex-ante definition of the optimal insurance contract. This transformation service (through the demand deposit contract) is one of the important functions performed by banks.

Once the banking contract or ex-ante program has been designed, all agents solve their maximization problem in the interim period conditional on their information (if any) and they decide on their level of consumption for both periods. Conditions to assure bank-runs (both information-induced and "panic" runs) are derived.

A second motivation for this extension was to complete Chari and Jagannathan's welfare analysis, by comparing suspension of convertibility versus deposit insurance, given their relative benefits and costs (of randomization in meeting liquidity needs or deadweight taxation). Three alternative mechanisms for the suspension measure have been introduced: A constant deduction of the consumption among all type-2 agents, a proportional deduction to the consumption among all type-2 agents and the "random queue" technique where the consumption deficit is supported by the 0last agents trying to convert their returns.

The numerical results have shown that for low and very high levels of risk-aversion supension is welfare superior and in the intermediate cases deposit insurance would be better. Also, in all cases, as the dispersion of the random return increases, suspension of convertibility improves with respect to deposit insurance. For very risky investments, suspension would be the preferred measure to prevent runs.

The range of parameters for which each measure is preferred also depends on the value of the deadweight tax and the proportion of early diers. Obviously, increasing the deadweight tax raises the cost of deposit insurance, and in a limit case, suspension would be welfare superior for all levels of risk aversion. Similarly, increasing the proportion of type-1 agents raises the cost of suspension, and if the proportion of these early withdrawers is very large, then a measure of deposit insurance would be justified in this situation.

However, a rigorous prescription of the optimal public intervention measure is difficult to be done as both measures have their shortcomings. Suspension of convertibility only provides temporary relief until the bank's doors reopen (this measure would only be effective if it could be applied until the bank's assets reach maturity, which does not seem plausible). Concerning deposit insurance, as already shown in earlier papers, is a measure that encourages an excessive risk-taking behaviour on the side of banks. It should be accompanied by additional banking regulation, as imposing restrictions on what banks can do, monitoring the banks continually, capital requirements or risk-sensitive insurance premiums.

In general, the results of this paper support a radical policy proposal that aim to restrict insured liabilities to finance very low risk assets. This proposal, known as "narrow banking", would consist in dividing the banking industry into two types of banks: a "narrow" group of banks, whose deposits would be insured, and who are restricted in their assets' choices and a broad class of banks, with greater flexibility in the use of their uninsured deposits. The narrow banking idea has received support in the literature, among others, by Kareken [18], Boot and Greenbaum [7], Jacklin [17] and Craine [10].

One way in which this model could be extended is to incorporate the linkage between runs on a particular bank and on the banking system as a whole, that is, a model to predict bank panics.

For simplicity, liquidations costs have been neglected, it could be also interesting to see the effect in the results of considering a costly liquidation technology.

Appendix

7.- Maximization problems

7.1.- Ex-ante Contract

The maximization problem is given by:

$$\max_{c_{ij}} \left\{ t\left(\rho_1 \frac{(k+c_{11})^{1-\gamma}}{1-\gamma} + (1-\rho_1) E\left[\frac{(k+c_{21})^{1-\gamma}}{1-\gamma}\right]\right) + \right. \qquad [1]$$
$$\left. + (1-t)\left(\rho_2 \frac{(k+c_{12})^{1-\gamma}}{1-\gamma} + (1-\rho_2) E\left[\frac{(k+c_{22})^{1-\gamma}}{1-\gamma}\right]\right)\right\}$$

s.t.

$$t\left(c_{11} + \frac{c_{21}}{Rh}\right) + (1-t)\left(c_{12} + \frac{c_{22}}{Rh}\right) = 1$$
$$\rho_1 \frac{(k+c_{11})^{1-\gamma}}{1-\gamma} + (1-\rho_1) E\left[\frac{(k+c_{21})^{1-\gamma}}{1-\gamma}\right] \geq \rho_1 \frac{(k+c_{12})^{1-\gamma}}{1-\gamma} + (1-\rho_1) E\left[\frac{(k+c_{22})^{1-\gamma}}{1-\gamma}\right] \qquad [2]$$
$$\rho_2 \frac{(k+c_{12})^{1-\gamma}}{1-\gamma} + (1-\rho_2) E\left[\frac{(k+c_{22})^{1-\gamma}}{1-\gamma}\right] \geq \rho_2 \frac{(k+c_{11})^{1-\gamma}}{1-\gamma} + (1-\rho_2) E\left[\frac{(k+c_{21})^{1-\gamma}}{1-\gamma}\right]$$

In this maximization problem, it can be shown that the incentive constraint for type-1 agents is never binding and that of type-2 agents ***may be*** binding (depending on the exogenous parameters that are considered, as will be seen below).

The F.O.C to the above maximization problem are:

$$\frac{\partial L}{\partial c_{11}}=(k+c_{11})^{-\gamma}(t\rho_1-\rho_2\lambda_3)-t\lambda_1=0 \qquad [a]$$

$$\frac{\partial L}{\partial c_{21}}=\left[(k+c_{21})^{-\gamma}(1-p)+p\left(k+c_{21}\frac{Rl}{Rh}\right)^{-\gamma}\frac{Rl}{Rh}\right]\left[t(1-\rho_1)-(1-\rho_2)\lambda_3\right]-\frac{t}{Rh}\lambda_1=0 \qquad [b]$$

$$\frac{\partial L}{\partial c_{12}}=(k+c_{12})^{-\gamma}\left[(1-t)\rho_2+\rho_2\lambda_3\right]-(1-t)\lambda_1=0 \qquad [c]$$

$$\frac{\partial L}{\partial c_{22}}=\left[(k+c_{22})^{-\gamma}(1-p)+p\left(k+c_{22}\frac{Rl}{Rh}\right)^{-\gamma}\frac{Rl}{Rh}\right]\left[(1-t)(1-\rho_2)+(1-\rho_2)\lambda_3\right]-\frac{(1-t)}{Rh}\lambda_1=0 \qquad [d] \qquad [3]$$

$$\frac{\partial L}{\partial \lambda_1}=1-t\left(c_{11}+\frac{c_{21}}{Rh}\right)-(1-t)\left(c_{12}+\frac{c_{22}}{Rh}\right)=0 \qquad [e]$$

$$\frac{\partial L}{\partial \lambda_3}=\rho_2\frac{(k+c_{12})^{1-\gamma}}{1-\gamma}+(1-\rho_2)E\frac{(k+c_{22})^{1-\gamma}}{1-\gamma}-\rho_2\frac{(k+c_{11})^{1-\gamma}}{1-\gamma}-(1-\rho_2)E\frac{(k+c_{21})^{1-\gamma}}{1-\gamma}=0 \qquad [f]$$

The new unknowns are introduced:

$$\hat{c}_{1j}=c_{1j}+k \qquad\qquad [4]$$
$$\hat{c}_{2j}=c_{2j}+k \qquad \text{where: } j=1,\ 2$$

Given that $\rho_1\to 1$, it is assumed the corner solution $\hat{c}_{21}=k$ (or $c_{21}=0$).

i).- The two incentive constraints are never binding ($\lambda_2=0,\ \lambda_3=0$).

The first order conditions become:

$$\frac{\partial L}{\partial \hat{c}_{11}}=\hat{c}_{11}^{-\gamma}t\rho_1-t\lambda_1=0 \qquad [a]$$

$$\frac{\partial L}{\partial \hat{c}_{21}}=\hat{c}_{21}^{-\gamma}(1-p)t(1-\rho_1)-\frac{t}{Rh}\lambda_1\le 0 \qquad [b]$$

$$\frac{\partial L}{\partial \hat{c}_{12}}=\hat{c}_{12}^{-\gamma}(1-t)\rho_2-(1-t)\lambda_1=0 \qquad [c] \qquad [5]$$

$$\frac{\partial L}{\partial \hat{c}_{22}}=\hat{c}_{22}^{-\gamma}(1-p)(1-t)(1-\rho_2)-\frac{(1-t)}{Rh}\lambda_1=0 \qquad [d]$$

$$\frac{\partial L}{\partial \lambda_1}=1+\left(1+\frac{1}{Rh}\right)k-t\left(\hat{c}_{11}+\frac{\hat{c}_{21}}{Rh}\right)-(1-t)\left(\hat{c}_{12}+\frac{\hat{c}_{22}}{Rh}\right)=0 \qquad [e]$$

From [5][a] and [5][c]:

$$\hat{c}_{12}=\left(\frac{\rho_2}{\rho_1}\right)^{\frac{1}{\gamma}}\hat{c}_{11} \qquad [6]$$

From [5][a] and [5][d]:

$$\hat{c}_{22}=\left[\frac{(1-p)(1-\rho_2)Rh}{\rho_1}\right]^{\frac{1}{\gamma}}\hat{c}_{11} \qquad [7]$$

Substituting [6] and [7] in [5][e] the value of $\hat{c}_{11}$ is obtained.

In this case it is assumed that the incentive constraint for type-2 agents is not binding, that is:

$$\rho_2 \frac{\hat{c}_{12}^{1-\gamma}}{1-\gamma} + (1-\rho_2) E \frac{\hat{c}_{22}^{1-\gamma}}{1-\gamma} - \rho_2 \frac{\hat{c}_{11}^{1-\gamma}}{1-\gamma} - (1-\rho_2) E \frac{\hat{c}_{21}^{1-\gamma}}{1-\gamma} \geq 0 \qquad [8]$$

Substituting the optimal consumption levels in the above expression, a condition on the relative risk-aversion coefficient (γ) for this case to hold is obtained.

ii).- The incentive constraint for type-1 agents is not binding and that of type-2 is binding ($\lambda_2 = 0,\ \lambda_3 > 0$). The first order conditions become:

$$\begin{aligned}
&\frac{\partial L}{\partial \hat{c}_{11}} = \hat{c}_{11}^{-\gamma}(t\rho_1 - \rho_2\lambda_3) - t\lambda_1 = 0 && [a]\\
&\frac{\partial L}{\partial \hat{c}_{21}} = \hat{c}_{21}^{-\gamma}(1-p)\left[t(1-\rho_1) - (1-\rho_2)\lambda_3\right] - \frac{t}{Rh}\lambda_1 \leq 0 && [b]\\
&\frac{\partial L}{\partial \hat{c}_{12}} = \hat{c}_{12}^{-\gamma}\left[(1-t)\rho_2 + \rho_2\lambda_3\right] - (1-t)\lambda_1 = 0 && [c]\\
&\frac{\partial L}{\partial \hat{c}_{22}} = \hat{c}_{22}^{-\gamma}(1-p)\left[(1-t)(1-\rho_2) + (1-\rho_2)\lambda_3\right] - \frac{(1-t)}{Rh}\lambda_1 = 0 && [d]\\
&\frac{\partial L}{\partial \lambda_1} = 1 + \left(1 + \frac{1}{Rh}\right)k - t\left(\hat{c}_{11} + \frac{\hat{c}_{21}}{Rh}\right) - (1-t)\left(\hat{c}_{12} + \frac{\hat{c}_{22}}{Rh}\right) = 0 && [e]\\
&\frac{\partial L}{\partial \lambda_3} = \rho_2 \frac{\hat{c}_{12}^{1-\gamma}}{1-\gamma} + (1-\rho_2) E \frac{\hat{c}_{22}^{1-\gamma}}{1-\gamma} - \rho_2 \frac{\hat{c}_{11}^{1-\gamma}}{1-\gamma} - (1-\rho_2) E \frac{\hat{c}_{21}^{1-\gamma}}{1-\gamma} = 0 && [f]
\end{aligned} \qquad [9]$$

From [9][c]:

$$\lambda_3 = -(1-t)\left[1 - \frac{\lambda_1}{\hat{c}_{12}^{-\gamma}\rho_2}\right] \qquad [10]$$

From [9][d]:

$$\lambda_3 = -(1-t)\left[1 - \frac{\lambda_1}{\hat{c}_{22}^{-\gamma} Rh(1-p)(1-\rho_2)}\right] \qquad [11]$$

From [10] and [11]:

$$\hat{c}_{22} = \left[(1-p)Rh\frac{1-\rho_2}{\rho_2}\right]^{\frac{1}{\gamma}} \hat{c}_{12} \qquad [12]$$

From [9][a]:

$$\lambda_3 = t\left[\frac{\rho_1}{\rho_2} - \frac{\lambda_1}{\hat{c}_{11}^{-\gamma}\rho_2}\right] \qquad [13]$$

From [10] and [13]:

$$\hat{c}_{11}^{-\gamma}=\frac{DB\rho_2\hat{c}_{12}^{-\gamma}}{\hat{c}_{12}^{-\gamma}(B\rho_1-1)+D} \qquad [14]$$

where:

$$M=1+k+\frac{k-t}{Rh}+\frac{\hat{c}_{12}}{Rh}(t-1)\left\{\left[(1-p)Rh\frac{1-\rho_2}{\rho_2}\right]^{1/\gamma}+Rh\right\}$$
$$B=-\frac{t}{(1-t)\rho_2} \qquad D(\hat{c}_{12})=\frac{\lambda_1}{\rho_2} \qquad [15]$$

Substituting in [9][e], [12] and [14]:

$$\lambda_1=D\rho_2=\frac{\left[\frac{M}{t}\right]^{-\gamma}\hat{c}_{12}^{-\gamma}(B\rho_1-1)}{B\rho_2\hat{c}_{12}^{-\gamma}-\left[\frac{M}{t}\right]^{-\gamma}}\rho_2 \qquad [16]$$

Subsituting in [9][f]:

$$(1-\rho_2)(1-p)\left\{\left[(1-p)Rh\frac{1-\rho_2}{\rho_2}\right]^{\frac{1-\gamma}{\gamma}}\hat{c}_{12}^{1-\gamma}-k^{1-\gamma}\right\}-\rho_2\left\{\left[\frac{DB\rho_2\hat{c}_{12}^{-\gamma}}{\hat{c}_{12}^{-\gamma}(B\rho_1-1)+D}\right]^{\frac{\gamma-1}{\gamma}}-\hat{c}_{12}^{1-\gamma}\right\}=0 \qquad [17]$$

The solution to the non linear equation [12] yields a value for $\hat{c}_{12}$, and from it the rest of the unknowns of the problem are obtained.

The ex-ante contract has been solved assuming $Rl\to 0$. The more general case in which $Rl\neq 0$ (sensitivity analysis with respect to $\tilde{R}$) has been solved applying the Newton-Raphson technique to the F.O.C in Equation [3].

7.2.- Ex-Post Problem

7.2.1.- Type-1 agents

The value of μ_1 is chosen in order to maximize their utility function and subject to their two period constraint; that is:

$$\max_{\mu_1} U^1(c_1,c_2)=\max_{\mu_1}\left\{\rho_1\frac{(k+c_1)^{1-\gamma}}{1-\gamma}+(1-\rho_1)\left[(1-p)\frac{(k+c_2)^{1-\gamma}}{1-\gamma}+p\frac{\left(k+c_2\frac{Rl}{Rh}\right)^{1-\gamma}}{1-\gamma}\right]\right\}$$
$$\text{s.t} \quad c_1=c_{12}+\mu_1(c_{11}-c_{12})$$
$$c_2=c_{21}+(1-\mu_1)(c_{22}-c_{21}) \qquad [18]$$
$$\mu_1\leq 1$$

The F.O.C of the problem are:

$$\rho_1(k+c_1)^{-\gamma}(c_{11}-c_{12})-(1-\rho_1)(1-p)(k+c_2)^{-\gamma}(c_{22}-c_{21})-\lambda_1=0 \qquad [19]$$

and given that $\rho_1 \to 1$ the multiplier associated with the constraint is:

$$\lambda_1=\rho_1(k+c_1)^{-\gamma}(c_{11}-c_{12})>0 \qquad \text{as: } c_{11}>c_{12} \qquad [20]$$

and therefore the constraint is always binding, that is, $\mu_1=1$

7.2.2.- Informed type-2 agents

In each state and conditional on the information about $\tilde{R}$ they solve the following problem:

$$\max_{\mu_I}=\rho_2\frac{(k+c_1)^{1-\gamma}}{1-\gamma}+(1-\rho_2)E\left[\frac{(k+c_2)^{1-\gamma}}{1-\gamma}\bigg/\tilde{R}\right] \qquad [21]$$

$$\text{with:} \quad \begin{aligned} &c_1=c_{12}+\mu_I(c_{11}-c_{12})\\ &c_2=c_{21}+(1-\mu_I)(c_{22}-c_{21})\\ &\mu_I\leq 1 \end{aligned} \qquad [22]$$

There are two different values for μ_I, depending on the information about the random return ($\tilde{R}$) received by these agents at T=1.

a.- If $\tilde{R}=Rh$ is the information received at date 1, then the informed type-2 agents find their consumption by solving the following problem:

$$\max_{\mu_I} U^1(c_1,c_2)=\max_{\mu_I}\left\{\rho_2\frac{(k+c_1)^{1-\gamma}}{1-\gamma}+(1-\rho_2)\frac{(k+c_2)^{1-\gamma}}{1-\gamma}\right\} \qquad [23]$$

$$\text{with} \quad \begin{aligned} &c_1=c_{12}+\mu_I(c_{11}-c_{12})\\ &c_2=c_{21}+(1-\mu_I)(c_{22}-c_{21})\\ &\mu_I\leq 1 \end{aligned} \qquad [24]$$

The F.O.C of the problem are:

$$\rho_2(k+c_1)^{-\gamma}(c_{11}-c_{12})-(1-\rho_2)(k+c_2)^{-\gamma}(c_{22}-c_{21})-\lambda_1=0 \qquad [25]$$

with solution:

$$0 \nless \mu_2=\frac{B_I(k+c_{22})-(k+c_{12})}{B_I(c_{22}-c_{21})+(c_{11}-c_{12})}<1 \quad \text{with} \quad B_I=\left(\frac{c_{22}-c_{21}}{c_{11}-c_{12}}\,\frac{1-\rho_2}{\rho_2}\right)^{-\frac{1}{\gamma}} \qquad [26]$$

b.- If $\tilde{R}=Rl$ is the value of $\tilde{R}$ revealed to type-2 agents, then the level of consumption is obtained in a similar way as above:

$$\max_{\mu_I} U^1(c_1,c_2) = \max_{\mu_I}\left\{\rho_2\frac{(k+c_1)^{1-\gamma}}{1-\gamma}+(1-\rho_2)\frac{\left(k+c_2\frac{Rl}{Rh}\right)^{1-\gamma}}{1-\gamma}\right\} \quad [27]$$

$$\text{with:}\quad \begin{aligned} &c_1=c_{12}+\mu_I(c_{11}-c_{12})\\ &\tilde{c}_2=\tilde{c}_{21}+(1-\mu_I)(\tilde{c}_{22}-\tilde{c}_{21})\\ &\mu_I\leq 1\end{aligned} \quad [28]$$

The F.O.C of the problem are:

$$\rho_2(k+c_1)^{-\gamma}(c_{11}-c_{12})-(1-\rho_2)\left(k+c_2\frac{Rl}{Rh}\right)^{-\gamma}\frac{Rl}{Rh}(c_{22}-c_{21})-\lambda_1=0 \quad [29]$$

and therefore the value of λ_1 is:

$$\lambda_1=\rho_2(k+c_1)^{-\gamma}(c_{11}-c_{12})>0 \quad [30]$$

which implies $\mu_I=1$

8.- Proof of Proposition 1

Proposition 1 assures that the optimal choice of uninformed type-2 agents in states 3, 4 and 6 is $\mu_2^*=1$, that is, the choice of the type-1 contract.

First, considering states 3 and 4 and due to the existence of conditional probabilities, three different expressions for the utility function, $OF(\mu_2)$, can be written:

1).- If $\mu_2=1$ then aggregate consumption at date 1 would be the same for states 1, 3, 4 and 6 and so the conditional probabilities would be the ones given by $\pi_{1_{\mu=1}}$ and $\pi_{2_{\mu=1}}$ and the utility function is:

$$OF_{(\mu_2=1)}=\rho_2\frac{(k+c_{11})^{1-\gamma}}{1-\gamma}+(1-\rho_2)\left[\frac{(k+c_{21})^{1-\gamma}}{1-\gamma}\pi_{1_{\mu=1}}+\frac{\left(k+c_{21}\frac{Rl}{Rh}\right)^{1-\gamma}}{1-\gamma}\pi_{2_{\mu=1}}\right] \quad [31]$$

2).- If $0<\mu_2<1$ then, there is only confusion between states 3 and 4 and the conditional probabilities would be given by:

$$\pi_{1_{\mu=1}}=\frac{(1-p)r_2(1-q)}{r_1pq+r_2(1-q)} \qquad \pi_{2_{\mu=1}}=\frac{p[r_1q+r_2(1-q)]}{r_1pq+r_2(1-q)} \quad [32]$$

The expression for the utility function would be:

$$\begin{aligned}&OF_{(0<\mu_2<1)}=\rho_2\frac{(k+c_1)^{1-\gamma}}{1-\gamma}+(1-\rho_2)\left[\frac{(k+c_2)^{1-\gamma}}{1-\gamma}\pi_{1_{\mu=1}}+\frac{\left(k+c_2\frac{Rl}{Rh}\right)^{1-\gamma}}{1-\gamma}\pi_{2_{\mu=1}}\right]\\ &c_1=c_{12}+\mu_2(c_{11}-c_{12})\\ &c_2=c_{21}+(1-\mu_2)(c_{22}-c_{21})\end{aligned} \quad [33]$$

3).- Finally, if $\mu_2=0$, there is confusion among states 3, 4 and 5[13], then the conditional probabilities would be $\pi_{1_{\mu\to 0}}$ and $\pi_{2_{\mu\to 0}}$ and the utility function:

$$OF_{(\mu_2=0)}=\rho_2\frac{(k+c_{12})^{1-\gamma}}{1-\gamma}+(1-\rho_2)\left[\frac{(k+c_{22})^{1-\gamma}}{1-\gamma}\pi_{1_{\mu\to 0}}+\frac{\left(k+c_{22}\frac{Rl}{Rh}\right)^{1-\gamma}}{1-\gamma}\pi_{2_{\mu\to 0}}\right] \quad [34]$$

First remark:

$$\lim_{\mu\to 1} OF_{0<\mu<1}=OF_{\mu=1} \quad [35]$$

that is,

$$\lim_{\mu\to 1} OF_{(0<\mu_2<1)}=\rho_2\frac{(k+c_{11})^{1-\gamma}}{1-\gamma}+\frac{1-\rho_2}{1-\gamma} \quad [36]$$

The utility function is continuous for $\mu_2>0$[14], and hence in order to assure a maximum at $\mu_2^*=1$, the following condition(s) should hold:

$$\begin{aligned}&\rho_2\frac{(k+c_{11})^{1-\gamma}}{1-\gamma}+(1-\rho_2)\left[\frac{(k+c_{21})^{1-\gamma}}{1-\gamma}\pi_{1_{\mu=1}}+\frac{\left(k+c_{21}\frac{Rl}{Rh}\right)^{1-\gamma}}{1-\gamma}\pi_{2_{\mu=1}}\right]>\\&>\rho_2\frac{(k+c_{12})^{1-\gamma}}{1-\gamma}+(1-\rho_2)\left[\frac{(k+c_{22})^{1-\gamma}}{1-\gamma}\pi_{1_{\mu\to 0}}+\frac{\left(k+c_{22}\frac{Rl}{Rh}\right)^{1-\gamma}}{1-\gamma}\pi_{2_{\mu\to 0}}\right]\end{aligned} \quad [37]$$

$$\frac{\partial OF}{\partial\mu_2}=\rho_2(k+c_1)^{-\gamma}(c_{11}-c_{12})-(1-\rho_2)(k+c_2)^{-\gamma}\pi_{2_{\mu=1}}>0 \quad [38]$$

The first of these conditions is only needed when the utility function is discontinous in $\mu_2=0$.

In order to assure that the second condition holds, it is only necessary to look for the extreme value of q for which the following condition is satisfied:

$$\left.\frac{\partial OF}{\partial\mu_2}\right|_{\mu_2^*=1}=\rho_2(k+c_{11})^{-\gamma}(c_{11}-c_{12})-(1-\rho_2)(k+c_{21})^{-\gamma}\pi_{2_{\mu=1}}=0 \quad [39]$$

Finally, with respect to state 6, there are two different expressions for the utility function:

1).- For any $0\leq\mu_2<1$, state 6 is never confounded with any other state and so the uninformed type-2 agents assign probability one to being in state 6. The utility function is given by:

$$\begin{aligned}OF_{(0\leq\mu_2<1)}&=\rho_2\frac{(k+c_1)^{1-\gamma}}{1-\gamma}+(1-\rho_2)\frac{\left(k+c_2\frac{Rl}{Rh}\right)^{1-\gamma}}{1-\gamma}\\c_1&=c_{12}+\mu_2(c_{11}-c_{12})\\c_2&=c_{21}+(1-\mu_2)(c_{22}-c_{21})\end{aligned} \quad [40]$$

If $\mu_2=1$, the utility function coincides with that of states 3 and 4, that is given in equation [31].

It can be shown, as before, the continuity of the utility function, and so we just need to impose that:

$$\frac{\partial OF}{\partial \mu_2}=\rho_2(k+c_1)^{-\gamma}(c_{11}-c_{12})>0 \qquad [41]$$

and this condition is always satisfied.

9.- Numerical Example

Input Data			
Proportions of type-1 agents	t0=0.00	t1=0.30	t2=0.51
Probabilities of the above proportions	r0=0.00	r1=0.99	r2=0.01
Proportions of informed type-2 agents	0=0.00	α=0.30	
Probabilities of the above proportions	1-q=0.10	q=0.90	
Low and high random returns	Rl=0.00	Rh=1.50	
Probabilities of the low and high returns	p=0.10	1-p=0.90	
Liquidation cost	aa=0.00		
Mimimum risk-aversion coefficient	τ1=1.50		
Maximum risk-aversion coefficient	τ2=1.50		
Incrementum of the risk-aversion coefficient	dτ=0.10		
Intertemporal coefficients of types 1, 2 agents	rho1=1.00	rho2=0.50	

Ex-ante contract			
c11	c21	c12	c22
1.24D+00	0.00D+00	4.12D-01	7.25D-01

Ex-post

Type-1 agents			
mu	cc11	cc21	
1.00D+00	1.24D+00	0.00D+00	

Informed type-2 agents			
State	mu	cc12i	cc22i
1	0.00D+00	0.00D+00	0.00D+00
2	2.93D-01	6.55D-01	5.13D-01
3	1.00D+00	1.24D+00	0.00D+00
4	0.00D+00	0.00D+00	0.00D+00
5	2.93D-01	6.55D-01	5.13D-01
6	1.00D+00	1.24D+00	0.00D+00

Uninformed Type-2 agents			
State	mu	cc12	cc22
1	3.60D-01	7.11D-01	4.18D-01
2	2.90D-01	6.53D-01	5.15D-01
3	1.00D+00	1.24D+00	3.17D-27
4	1.00D+00	1.24D+00	4.28D-07
5	2.90D-01	6.53D-01	5.15D-01
6	1.00D+00	1.24D+00	3.17D-27

Suspension A

Type-1 agents			
	cc11	cc21	
	1.24D+00	0.00D+00	

Informed type-2 agents			
State		cc12i	cc22i
1		0.00D+00	0.00D+00
2		6.55D-01	5.13D-01
3		6.61D-01	0.00D+00
4		0.00D+00	0.00D+00
5		4.14D-01	5.13D-01
6		4.12D-01	0.00D+00

Uninformed Type-2 agents			
State		cc12	cc22
1		6.61D-01	4.18D-01
2		6.53D-01	5.15D-01
3		6.61D-01	3.17D-27
4		4.12D-01	4.28D-07
5		4.11D-01	5.15D-01
6		4.12D-01	3.17D-27

Suspension B

Type-1 agents			
	cc11	cc21	
	1.24D+00	0.00D+00	

Informed type-2 agents			
State		cc12i	cc22i
1		0.00D+00	0.00D+00
2		6.55D-01	5.13D-01
3		6.61D-01	0.00D+00
4		0.00D+00	0.00D+00
5		4.13D-01	5.13D-01
6		4.12D-01	0.00D+00

Uninformed Type-2 agents			
State		cc12	cc22
1		6.61D-01	4.18D-01
2		6.53D-01	5.15D-01
3		6.61D-01	3.17D-27
4		4.12D-01	4.28D-07
5		4.12D-01	5.15D-01
6		4.12D-01	3.17D-27

Suspension C

Type-1 agents			
	cc11	cc21	
Quick	1.24D+00	0.00D+00	
Slow	4.12D-01	0.00D+00	

Informed type-2 agents			
State		cc12i	cc22i
1		0.00D+00	0.00D+00
2		6.55D-01	5.13D-01
3		1.24D+00	0.00D+00
4		0.00D+00	0.00D+00
5		6.55D-01	5.13D-01
6		1.24D+00	0.00D+00

Uninformed Type-2 agents			
State		cc12	cc22
1		7.11D-01	4.18D-01
2		6.53D-01	5.15D-01
3		1.24D+00	3.17D-27
4		1.24D+00	4.28D-07
5		6.53D-01	5.15D-01
6		1.24D+00	3.17D-27

Informed type-2 agents (Slow)			
State	Beta %	cc12i	cc22i
1	7.61	4.12D-01	0.00D+00
2	0.00	4.12D-01	5.13D-01
3	49.00	4.12D-01	0.00D+00
4	49.00	4.12D-01	0.00D+00
5	21.84	4.12D-01	5.13D-01
6	49.00	4.12D-01	0.00D+00

Uninformed Type-2 agents (Slow)			
State		cc12	cc22
1		4.12D-01	4.18D-01
2		4.12D-01	5.15D-01
3		4.12D-01	3.17D-27
4		4.12D-01	4.28D-07
5		4.12D-01	5.15D-01
6		4.12D-01	3.17D-27

Ex-ante aggregate consumption			
	CT1	CT2	CT1+Subsidy
	6.63D-01	5.06D-01	8.35D-01

Ex-post aggregate consumption			
State	CT1	CT2	Deficit
1	8.70D-01	2.92D-01	3.48D-02
2	8.30D-01	3.60D-01	0.00D+00
3	1.24D+00	1.55D-27	4.06D-01
4	1.24D+00	2.10D-07	4.06D-01
5	9.53D-01	2.52D-01	1.18D-01
6	1.24D+00	1.09D-27	4.06D-01

Ex-post aggregate consumption with suspension			
State	CT1	CT2	
1	8.35D-01	2.92D-01	
2	8.30D-01	3.60D-01	
3	8.35D-01	1.55D-27	
4	8.35D-01	2.10D-07	
5	8.35D-01	2.52D-01	
6	8.35D-01	1.09D-27	

Welfare measures			
Deposit	Suspen. A	Suspen. B	Suspen. C
6.27D-01	6.94D-01	6.94D-01	6.86D-01

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List of Tables

Table I.- Summary of events

T = 0		T = 1		T = 2	
1 Unit	↑	c_{11} or c_{12}	↓	c_{21} or c_{22}	↓
Intermediary Inflows	↑	Intermediary Outflows			↓
Ex ante identical agents		Preference shock realized Information shock realized		Random return realized	

Table II.- States of Nature

θ_i i	State $\tilde{r}$ $\tilde{\alpha}$ $\tilde{R}$	Prob. $p(\theta_i)$	Aggregate consumption at $T=1$, CT	CT (Prop1 satisfied)
1	t_1 0 $\tilde{R}$	$r_1(1-q)$	$t_1 cc_{11} + (1-t_1) cc_{12}$	-
2	t_1 α Rh	$r_1(1-p)q$	$t_1 cc_{11} + (1-t_1)[\alpha cc_{12H} + (1-\alpha) cc_{12}]$	-
3	t_1 α Rl	$r_1 pq$	$t_1 cc_{11} + (1-t_1)[\alpha cc_{12L} + (1-\alpha) cc_{12}]$	c_{11}
4	t_2 0 $\tilde{R}$	$r_2(1-q)$	$t_2 cc_{11} + (1-t_2) cc_{12}$	c_{11}
5	t_2 α Rh	$r_2(1-p)q$	$t_2 cc_{11} + (1-t_2)[\alpha cc_{12H} + (1-\alpha) cc_{12}]$	-
6	t_2 α Rl	$r_2 pq$	$t_2 cc_{11} + (1-t_2)[\alpha cc_{12L} + (1-\alpha) cc_{12}]$	c_{11}

Table III.- Numerical example

Ex-ante contract						
c_{11}=1.24		c_{21}=0.00		c_{12}=0.41	c_{22}=0.725	
Ex-post problem						
Type-1					μ_1=1	
Informed Type-2					μ_I=0.29 if R=Rh μ_I=1.00 if $\tilde{R}$=Rl	
Uninformed Type-2						
State	1	2	3	4	5	6
μ	0.36	0.29	1.00	1.00	0.29	1.00

Table IV

Variation in the exogenous parameter	Suspension minus deposit
Increase in the standard deviation of the random return (σ)	Increase
Increase in the deadweight tax (s)	Increase
Increase in the proportion of type-1 agents (t_1)	Decrease
Increase in relative risk-aversion (γ)	Decrease (for low γ) Increase (for medium-high γ)

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Notes

1. *One interpretation of this assumption is that the asset represents long-term loans which cannot be "called in" early and for which no secondary market exists perhaps due to a "lemmons" problem, as in Akerlof [1].*

2. *This assumption, as in Bhattacharya and Jacklin [2], is motivated by the fact that, if information were costly, type-2 agents would be more likely to purchase the information, and also if depositors were of different sizes, larger depositors would also be more likely to purchase the information. These unmodeled aspects are taken into account, by considering that a random proportion* α *of type-2 agents becomes informed.*

3. *This function solves the problem of zero consumption having an infinite negative value in terms of utility, when* γ *is greater than one.*

4. *Formally, this represents the choice for the depositor of* $\mu\,(0 \leq \mu \leq 1)$*:*

$\mu = 1$ *implies the choice of the type-1 contract* $(c_{11}, \bar{c}_{21})$

$\mu = 0$ *implies the choice of the type-2 contract* $(c_{12}, \bar{c}_{22})$

$0 < \mu < 1$ *implies a combination of the two contracts*

5. *This simplification is justified by the "first-come-first-served" nature of the deposit contract.*

6. *See Jacklin and Bhattacharya [2] for a proof of this result.*

7. $\hat{c}_{ij}$ *(i, j = 1, 2) are the ones given by equation [17].*

8. *The working procedure is a computer program that has been written in Ms-Dos Qbasic.*

9. *This example is shown in more detail in Appendix 9.*

10. *Note the value of* μ_2 *(in states 3, 4 and 6),* μ_I *and* μ_1 *in Table III*

11. *See deposit insurance in the next Section 4.2.*

12. *The assumption* $Rl \to 0$ *has been relaxed, although sufficiently small values for Rl have been considered so that bad information about asset quality leads always to a run. See Appendix 7.1 for the calculation procedure in this case.*

13. *This confusion only occurs if informed type-2 agents have also chosen* $\mu_I^* = 0$ *in state 5, otherwise, if* $\mu_2 = 0$ *there is only confusion between states 3 and 4 and therefore the conditional probabilities and the utility function would coincide with those expressed in point 2).*

14. *Or the utility function may be continuous in the interval [0,1] whenever points 2) and 3) coincide, as explained in footnote 13.*