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TESTING NONLINEARITY: DECISION RULES FOR SELECTING BETWEEN LOGISTIC AND EXPONENTIAL STAR MODELS

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Abstract

This paper introduces new LM specification procedures to choose between Logistic and Exponential Smooth Transition Autoregressive (STAR) models and to improve testing for nonlinearities. The selection procedures introduced here are simpler and have better consistency and power properties than those previously available in the literature. These improvements result from analyzing the properties of the Taylor approximations to the transition function. Monte-Carlo and empirical evidence are provided in support of the new procedures.

Keywords:
Smooth transition nonlinear models, linearity test, logistic versus exponential models, power and size of the tests, industrial production index.

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1 Introduction

This paper improves the specification procedures proposed in Teräsvirta (1994) for testing Smooth Transition Autoregressive (STAR) models and provides useful suggestions for the empirical practitioner. STAR models are a general class of state-dependent, non-linear, time-series models in which the transition between states is generally, endogenously generated.[1] Together with Hamilton's regime-switching model[2] (where the transition between states is exogenously determined by a Markov Chain), state-dependent models are reduced-form models that allow for different dynamic responses that depend on the "state." These models are therefore, particularly well suited to accommodate the asymmetric behavior of economic fluctuations that has been reported in a variety of recent studies.[3]

We find two main results in this paper, which rely on a Taylor series approximation of the transition function (between states) around the scale parameter.[4] First, we introduce an alternative specification strategy for the choice between the logistic and exponential transition functions. This alternative strategy has higher correct-selection frequencies of the right model, avoids some of the pitfalls of the rules proposed in Teräsvirta (1994) and is much simpler to apply. Second, we suggest that in some scenarios, tests of the null hypothesis of linearity against STAR-type nonlinearity[5] should include up to fourth order terms to gain power against alternatives in which an Exponential STAR is involved. These results are supported by Monte Carlo evidence and are implemented using the data in Teräsvirta and

[1] In addition, STAR models encompass other popular families of non-linear time-series models such as the Threshold Autoregressive (TAR) and the Exponential Autoregressive (EAR). See Haggan & Ozaki (1981), Tsay (1989) and Granger and Teräsvirta(1993).

[2] See Hamilton (1989).

[3] See Neftçi (1984), Rothman (1991), and Teräsvirta and Anderson (1992) for example.

[4] Luukkonen, Saikkonen and Teräsvirta (1988a), based on Davies (1977), introduced this solution for STAR models.

[5] Note: this alternative is an assumption imposed by the practitioner. This allows one to narrow down general nonlinearity into a workable family of general non-linear models – STAR models.

Anderson (1992).

The paper is organized as follows: Section 2 briefly reviews STAR models and nonlinearity testing; Section 3 discusses the decision rule proposed by Teräsvirta (1994) and then introduces our alternative; Section 4 reports the Monte-Carlo study for the new alternative procedures; Section 5 presents an empirical application; and Section 6 concludes.

2 STAR Models and Nonlinearity Testing

2.1 Overview

Consider the following STAR model:

$$y_t = \pi' x_t + \Theta' x_t F\left(z_{t-d}, \gamma, c\right) + u_t \tag{1}$$

where y_t is a scalar; $x_t = (1, y_{t-1}, ..., y_{t-p})' = (1, \widetilde{x}_t')'; \pi' = (\pi_0, \pi_1, ..., \pi_p) = (\pi_0, \widetilde{\pi}')$; $\Theta' = (\Theta_0, \Theta_1, ..., \Theta_p) = \left(\Theta_0, \widetilde{\Theta}'\right)$ and $1 \leqslant d \leqslant p$. z_{t-d} is usually y_{t-d} itself, (although it could be any exogenous variable), u_t is a martingale difference sequence with constant variance[6] and y_t is assumed to be stationary and ergodic. The function $F\left(z_{t-d}, \gamma, c\right)$ is at least fourth order continuously differentiable with respect to the scale parameter γ.

The exponential STAR model (ESTAR) has a transition function F, defined by:

$$F\left(z_{t-d}, \gamma, c\right) = \left[1 - \exp\left\{-\gamma \left(z_{t-d} - c\right)^2\right\}\right] \tag{2}$$

Note that when $\Theta_0 = c = 0$ and $z_{t-d} = y_{t-d}$, we have the exponential autoregressive model (EAR) of Haggan and Ozaki (1981) – a particular case of the ESTAR.

The logistic STAR model (LSTAR) has as transition function:[7]

[6] This assumption is useful to derive the LM tests below. See White (1984).

[7] The term $\frac{1}{2}$ is added here merely for convenience and does not affect the results.

$$F\left(z_{t-d},\gamma,c\right)=\left[\left\{1+\exp\left(-\gamma\left(z_{t-d}-c\right)\right)\right\}^{-1}-\frac{1}{2}\right] \tag{3}$$

Testing linearity against STAR-type nonlinearity implies testing the null hypothesis, H_0 : $\Theta' = \underline{0}$ in Equation 1. However, under the null, the parameters γ, and c are not identified. Alternatively, we could choose $H_0^* : \gamma = 0$ as our null hypothesis in which case, neither c nor Θ' would be identified. Davies (1977) first showed that conventional maximum likelihood theory is not directly applicable to this problem. A solution, (proposed in Luukkonen et al. (1988a) and adopted in Teräsvirta (1994)), is to replace $F\left(z_{t-d},\gamma,c\right)$ with a suitable Taylor series approximation. Under the null of linearity, the LM test is shown to possess asymptotically, the usual χ^2 distribution.[8]

In practice, the test is performed by constructing the following auxiliary regression:

$$y_t = \pi' x_t + \left[\Theta' x_t \gamma F_\gamma\left(z_{t-d},\gamma=0,c\right)\right] + v_{1t} \tag{4}$$

where $F_\gamma(.)$ indicates the first derivative of $F(z_{t-d},\gamma=0,c)$ with respect to γ — $\gamma F_\gamma(.)$ is obviously the first term of the Taylor approximation of F around γ. Substituting the value of $F_\gamma(.)$ into 4 gives:

$$y_t = \delta_0 + \delta_1' \widetilde{x}_t + \beta_1' \widetilde{x}_t z_{t-d} + \beta_2' \widetilde{x}_t z_{t-d}^2 + v_{1t} \tag{5}$$

where the null hypothesis of linearity becomes $H_0' : \beta_1' = \beta_2' = \underline{0}$. Call this test NL2. Note that Equation 5 is explosive and generally not a meaningful time series model (see Granger and Andersen (1978)).[9] Luukkonen et al. (1988a) realized that this test would have low power against alternatives where $\widetilde{\Theta}'$ is "small" and Θ_0 is "large" in absolute value if the

[8] The delay parameter d is usually unknown. Based on Tsay (1989), Teräsvirta (1994) proposes choosing d that minimizes the p-value of the nonlinearity test.

[9] Also note that the alternative hypothesis will include models other than the STAR.

model is LSTAR. To overcome this difficulty, they proposed to include up to third order powers. The final version of their test, therefore becomes:

$$y_t = \delta_0 + \delta_1' \tilde{x}_t + \beta_1' \tilde{x}_t z_{t-d} + \beta_2' \tilde{x}_t z_{t-d}^2 + \beta_3' \tilde{x}_t z_{t-d}^3 + v_{3t} \tag{6}$$

where the null hypothesis is $H_0'' : \beta_1' = \beta_2' = \beta_3' = \underline{0}$. The LM test based on running the auxiliary regression 6 (which we will call NL3) is the test adopted by Teräsvirta (1994) — the basis of our study.

2.2 Properties of the Taylor Approximation

The transition function of a STAR model exhibits two important features. First, the logistic function (see Equation 3) has a single inflection point, while the exponential function (see Equation 2) has two inflection points. Second, the even powers of the Taylor expansion of a logistic function are all zero, while the odd powers of the Taylor expansion of an exponential function are all zero.[10] The first feature suggests ways to improve the nonlinearity test NL3. In the next section we will show how use the second feature to propose an alternative selection rule.

The immediate consequence resulting from the difference in shape between the logistic and the exponential functions is that we need a *second order Taylor expansion* in order to capture the two inflection points of the exponential function. This means that the auxiliary regression 6 would need to be expanded with fourth order terms. In particular:

$$F(z_{t-d}, \gamma, c) \simeq F_\gamma(z_{t-d}, \gamma = 0, c)\gamma + \frac{1}{2} F_{\gamma\gamma}(z_{t-d}, \gamma = 0, c)\gamma^2 \tag{7}$$

which for the exponential becomes:

[10] This will become clear in Section 3.

$$F(z_{t-d},\gamma,c) \simeq (z_{t-d}-c)^2\left(\gamma+\gamma^2\right)-\frac{1}{2}(z_{t-d}-c)^4\gamma^2 \tag{8}$$

(Note that the second term of a Taylor expansion vanishes for the logistic). Hence, after combining terms, the auxiliary regression (Equation 6) proposed in Luukkonen et al. (1988a) should be:

$$y_t = \delta_0 + \delta_1'\tilde{x}_t + \beta_1'\tilde{x}_t z_{t-d} + \beta_2'\tilde{x}_t z_{t-d}^2 + \beta_3'\tilde{x}_t z_{t-d}^3 + \beta_4'\tilde{x}_t z_{t-d}^4 + v_{4t} \tag{9}$$

where the null hypothesis that we want to test now is $H_0''' : \beta_1' = \beta_2' = \beta_3' = \beta_4' = \underline{0}$. We call this test NL4.[11] Recall that under the null, some of the parameters of the original model are unidentified. As we mentioned, Davies (1977) first pointed out that this computational problem can be solved by deriving an LM test while keeping the unidentified parameters, Θ, fixed and then selecting the value of the statistic corresponding to sup_Θ *LM*(Θ). Saikkonen and Luukkonen (1988), Teräsvirta et al. (1994), and Teräsvirta (1994) suggest the following procedure to avoid this problem. First, estimate Equation 9 under the null hypothesis by OLS and calculate the sum of squared residuals, SSR_0. Second, using the residuals from the previous step, estimate a model that contains the regressors in Equation 9 to compute the sum of squared residuals SSR_1. Third, the statistic $T(SSR_0\text{-}SSR_1)/SSR_0$ will have a χ^2 distribution with degrees of freedom given by the number of auxiliary regressors. In practice, it is recommended to use the approximation given by the $F-distribution$ because of the good size and power properties of the test in small samples. An alternative approach is to use the Wald test of Hansen (1996). This procedure approximates the unknown limiting distributions by generating p-values based on simulation methods.[12]

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[11] When $z_{t-d} = y_{t-d}$, this test is similar to a high order RESET test.

[12] See Pesaran and Potter (1997) for an interesting application of this technique.

Empirical application of the test involves several important steps, such as choice of lag length of the AR model, choice of delay parameter d, and others which are all well documented in Teräsvirta (1994). However, notice that NL4 requires p extra regressors for the auxiliary regression. This lack of parsimony becomes troublesome with small sample sizes and/or when the order of the AR polynomial (p) is high. Luukkonen et al. (1988a) (who use Equation 6 as the auxiliary regression) recognized this lack of parsimony and suggested an *augmented first order procedure* based on Equation 5 — the spirit of which can be applied to NL4 for particular cases.[13]

3 Choosing between LSTAR or ESTAR

3.1 Teräsvirta's (1994) Decision Rule

Upon rejecting the null hypothesis of linearity (with any of NL2, NL3 or NL4), one might consider using a STAR model as a useful non-linear alternative. Teräsvirta (1994) introduces the following model selection procedure (which we will denominate TP for short) based on Equation 6:

1. Test the null: $H_{03} : \beta_3' = \underline{0}$ (versus the alternative $H_{13} : \beta_3' \neq \underline{0}$) with an F-test ($F_3$). According to Teräsvirta, rejection of this null would imply rejection of the ESTAR specification since cubic powers of z_{t-d} in a first order approximation of $F(z_{t-d}, \gamma, c)$ are 0.
2. Test the null: $H_{02} : \beta_2' = \underline{0} | \beta_3' = \underline{0}$ with an F-test (F_2). Teräsvirta's reasoning is that z_{t-d}^2 terms of a first order approximation to a logistic function are zero when $c = \Theta_0 = 0$ (see Equation 1). However, these terms will be nonzero in the ESTAR case (except in the unlikely case when $\widetilde{\Theta}' = \underline{0}$). Failure to reject this null is taken

[13] The equivalent augmented version of NL4 would consist of equation 5 expanded with the terms: z_{t-d}^3, z_{t-d}^4 and z_{t-d}^5. See Escribano and Jordá (1997).

as evidence in favor of a LSTAR model. Nevertheless, rejection of H_{02} is not very informative one way or the other.

3. Test the null: $H_{01} : \beta_1' = \underline{0} \mid \beta_2' = \beta_3' = \underline{0}$ with an F-test (F_1). Following Teräsvirta, failing to reject H_{01} after rejecting H_{02} points to an ESTAR model. On the other hand, rejecting H_{01} after failing to reject H_{02} supports the choice of LSTAR.

4. Note which hypotheses are rejected and compare the relative strengths of the rejections. If the model is LSTAR, typically H_{01} and H_{03} are rejected more strongly than H_{02}. Therefore, if the p-value of F_2 is the smallest of F_1, F_2, F_3 select a ESTAR specification, otherwise select LSTAR.

To analyze the problems with TP, recall the terms of the Taylor approximations to each nonlinear state in Equation 1:

$$\Theta'\widetilde{x}_t \left[\{1 + \exp\left(-\gamma\left(z_{t-d} - c\right)\right)\}^{-1} - \frac{1}{2} \right] \simeq \Psi_1'\widetilde{x}_t(z_{t-d} - c) + \Psi_3'\widetilde{x}_t(z_{t-d} - c)^3 \tag{10}$$

for the logistic third order expansion, and

$$\Theta'\widetilde{x}_t \left[1 - \exp\left\{-\gamma\left(z_{t-d} - c\right)^2\right\} \right] \simeq \Psi_2'\widetilde{x}_t(z_{t-d} - c)^2 + \Psi_4'\widetilde{x}_t(z_{t-d} - c)^4 \tag{11}$$

for the exponential second order expansion (all zero terms omitted in both equations).[14]

TP has several problems: First, whenever $c \neq 0$, the expansion of the fourth order power in Equation 11 will include non-zero $\widetilde{x}_t z_{t-d}^3$ terms, thus invalidating the reasoning behind H_{03}. This pitfall is particularly problematic when the DGP is an ESTAR model with $c \neq 0$, and/or nonzero constants (see Equation 1), namely π_0 and Θ_0. In addition, when the variance of the error term is "large," the distribution of the data into each state around the threshold

[14] As a reminder, these approximations were used in developing the auxiliary regression for NL4 in Equation 9.

c is asymmetric. As a result, $H_{02} : \beta_2' = \underline{0}|\beta_3' = \underline{0}$ does not allow one to discriminate between a LSTAR with $c \neq 0$ and an ESTAR in general.

The second source of problems lies in the design of the rule itself: The three F-tests are nested. This feature proves to be problematic when $c \neq 0$. For example, if the true model is LSTAR, it is unclear that by conditioning on the cubic terms to be zero (that is, restricting $\beta_3' = 0$), the joint significance of the square terms, $\widetilde{x}_t z_{t-d}^2$ (which are non-zero since $c \neq 0$, see Equation 10) will be also zero since these terms are left to approximate the transition function (an approximation that the cubic terms presumably were successfully capturing).

3.2 A New Alternative Selection Procedure

Consider the following example. Assume that $c = 0$. It is clear then that (based on Equation 10) if the model is LSTAR, the terms $\widetilde{x}_t z_{t-d}^j$ for $j = 2, 4, 6, ...$ are exactly zero (i.e. $\beta_2' = \beta_4' = \underline{0}$ in Equation 9). Alternatively, if the model is ESTAR, (based on Equation 11) the terms $\widetilde{x}_t z_{t-d}^j$ for $j = 1, 3, 5, ...$ are exactly zero (i.e. $\beta_1' = \beta_3' = \underline{0}$ in Equation 9). This suggests the following alternative selection procedure (which we will call **EJP** for short) based on Equation 9 (again, conditional on rejecting linearity with any of NL2, NL3, NL4):

1. Test the null: $H_{0E} : \beta_2' = \beta_4' = \underline{0}$ (against the alternative, $H_{1E} : \beta_2' \neq \underline{0}$, $\beta_4' \neq \underline{0}$) with an F-test ($F_E$).

2. Test the null: $H_{0L} : \beta_1' = \beta_3' = \underline{0}$ (against the alternative, $H_{1L} : \beta_1' \neq \underline{0}$, $\beta_3' \neq \underline{0}$) with an F-test ($F_L$).

3. If the minimum p-value corresponds to F_L, select LSTAR, otherwise, if it corresponds to F_E, select ESTAR.

Note that when $c \neq 0$, the test is still effective since we are relying on testing the joint significance of linear and cubic terms relative to the joint significance of quadratic and fourth order terms without conditioning. In addition, EJP provides information about non-zero thresholds, c. Linear and cubic terms are exactly zero when $c = 0$ and the model is ESTAR. Quadratic and fourth order terms are exactly zero when $c = 0$ and the model is LSTAR. Therefore: Rejecting H_{0L} and failing to reject H_{0E} suggest an LSTAR model with $c = 0$. Rejecting H_{0E} and failing to reject H_{0L} suggest a ESTAR model with $c = 0$. This feature is useful to specify $c = 0$ as a good starting value in the estimation stage.

4 Monte Carlo Experiments

This section examines the properties of EJP versus TP for choosing between a LSTAR or ESTAR specification. In addition, we report simulations regarding the power properties of NL4 when compared to NL3. The models in this study are taken from Luukkonen et al. (1988a,b) and Teräsvirta (1994). Each experiment is replicated 1,000 times. The first 100 observations of each series are disregarded to avoid initialization problems.[15]

4.1 Selection Frequencies of the EJ Selection Procedure

In addition to having a much higher success rate than TP in correctly selecting the type of STAR model, EJP's success rate *always* increases as the sample size increases (a highly desirable "consistency" feature — the result of the design of the procedure). TP on the other hand, lacks consistency. For example, consider $\mu = 1$ in Table 4.1.1. TP's correct selection frequency is 12.9%, 9.5% and 3.9% for sample sizes of 50, 100 and 200 observations, respectively. The numbers for EJP in the same example are 62.4%, 70.4% and 76.5%, respectively.

Teräsvirta (1994) recognized that TP works well when the LSTAR and ESTAR models are not close substitutes. However, TP is less effective when the two models are close substitutes

[15] For a detailed explanation of how the experiments were constructed and additional results with other models, see Escribano and Jorda (1997).

and the true model is ESTAR. It is remarkable that the most impressive gains of using the alternative EJP occur precisely in this situation. The results are fairly conclusive. EJP outperforms TP in general; it is simpler to implement (requiring only two simple F-tests and a straight forward choice); and is "consistent" in the sense mentioned above.

4.2 Power Properties of the NL4 Test

The key question is whether the gains in power from adding the terms $\widetilde{x}_t z_{t-d}^4$ in NL4 outweigh the losses from including additional regressors in equation 9. It is clear that if the DGP is a LSTAR model, we will loose power because from including redundant regressors. If the model is an ESTAR we should expect to perform well whenever $c = 0$. If $c \neq 0$, the benefits of including extra regressors will depend on each particular case. Table 4.2 reports the experiments based on Teräsvirta (1994).

The results of the simulations indicate that with large sample sizes (in our case 300 observations), there is little to no gain or loss from including the extra terms $\widetilde{x}_t z_{t-d}^4$. Either NL3 or NL4 detect non-linearity appropriately, with the power approximating 1 in most cases. However, for smaller sample sizes, (in our study 100 observations), while NL4 performs better when the true model is ESTAR (in particular when the variance of the error term is high and/or c and Θ_0 are nonzero) there are no significant losses of power when the true model is LSTAR. In view of these results (with the disclaimer of their limited generality), we conclude that the NL4 test is probably most useful in small samples, when the AR lag length is short, and parsimony in the auxiliary regression is not an issue.

5 Teräsvirta and Anderson (1992) Revisited

Teräsvirta and Anderson (1992) analyze the dynamic properties of industrial production indices of thirteen OECD countries and a European aggregate using STAR models. The data is quarterly, seasonally unadjusted,[16] and spans from 1960:I to 1986:IV.[17] We will replicate nonlinearity testing and model selection by applying the techniques developed above and comparing them to their results. Table 5.1 reports p-values of nonlinearity tests, delay parameter choice, and STAR model selection for those countries in which nonlinearities were detected (by either NL3 or NL4).[18]

NL3 and NL4 obtain their minimum p-values for the same choice of delay parameter, d, except in the case of the U.S.A. While the results of both tests are similar, NL4 fails to reject linearity at the usual 5% level for 3 countries.[19] With regard to EJP, the same models are selected as with TP except for Austria and Sweden. In the case of Japan, Teräsvirta and Anderson (1992) report that choosing between models (LSTAR or ESTAR) was hard with TP and hence estimated both specifications. After estimation, the preferred model was a ESTAR — a choice that EJP selects unequivocally.

Unfortunately, Teräsvirta and Anderson (1992) did not report their estimates for Austria and Sweden. Consequently, we estimated both LSTAR and ESTAR specifications for these countries. The basic statistics of the preferred models for each specification are reported in Table 5.2. The estimates for Austria are harder to compare since the final models have a different number of parameters — Schwartz's information criterion (SIC) favors the ESTAR specification while Akaike's (AIC) favors the LSTAR specification. However, in the case of Sweden, the final models have the same number of parameters. The preferred specification is the ESTAR (which was selected by EJP but not by TP) with a better fit overall than its LSTAR counterpart. Of course, the true model is unknown, hence the value of this exercise

[16] They make the series approximately stationary by fourth lag differencing (x_t-x_{t-4}).

[17] Source: *OECD Main Economic Indicators.*

[18] Additionally, French and Italian indices were adjusted for strikes and other anomalies and therefore not considered here.

[19] A sample of 104 observations and the extra regressors required by NL4 probably justify this result.

is in terms of checking what specification seemed to work better and what specification test led us to it.

6 Conclusion

This paper provides a new selection procedure to choose between a logistic and an exponential specification when the alternative to linearity considered is a STAR model. Along the way, we have also provided practical guidelines regarding nonlinearity testing and a way to obtain initial guesses for zero thresholds. This new procedure, EJP, is simpler, more intuitive, and has better power and consistency properties than its predecessor: TP. In addition, all the tests developed here can be easily generalized for use in Smooth Transition Regression and multivariate models.[20]

Empirical application of EJP is contingent on prior rejection of the null hypothesis of linearity with a suitable nonlinearity test: Any of NL2, NL3 or NL4. Each particular application, given sample size and the length of the AR polynomial, will determine the best choice . A good strategy would be to begin with NL2 and only use a more complicated test (NL3 and then NL4) if at each stage linearity is not rejected. For more sophisticated users, this paper will be a helpful tool to understand the inner workings of specification and testing in the context of STAR models.

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Table 4.1.1

Relative frequencies of correct specification of STAR model. Table 4, pg. 172, Luukkonen et al. (1988). DGP: ESTAR. (1 = 100% accuracy selecting the correct model, 0 = 0% accuracy).

Sample Size	μ	TP	EJP	Power NL4
50	0	0.632	0.792	0.106
	0.3	0.472	0.736	0.125
	1	0.129	0.624	0.210
100	0	0.805	0.898	0.256
	0.3	0.552	0.830	0.317
	1	0.095	0.704	0.493
200	0	0.899	0.963	0.616
	0.3	0.659	0.923	0.692
	1	0.039	0.765	0.881

Table 4.1.2

Relative frequencies of correct specification of STAR model. Fig. 2, pg. 496, Luukkonen et al. (1988). DGP: LSTAR

	$\pi_1 = -0.5$				$\pi_1 = 0.5$			
Sample Size	θ_1	TP	EJP	Power NL4	θ_1	TP	EJP	Power NL4
50	-0.4	0.500	0.594	0.064	-1.4	0.568	0.947	0.322
	0	.	.	0.039	-1	0.872	0.872	0.203
	0.5	0.736	0.736	0.072	-0.5	0.802	0.630	0.081
	1	0.853	0.871	0.170	0	.	.	0.047
	1.5	0.904	0.936	0.467	0.5	0.841	0.690	0.113
100	-0.4	0.459	0.811	0.122	-1.4	0.594	0.978	0.744
	0	.	.	0.043	-1	0.941	0.935	0.491
	0.5	0.811	0.724	0.127	-0.5	0.898	0.814	0.118
	1	0.952	0.936	0.498	0	.	.	0.037
	1.5	0.963	0.978	0.883	0.5	0.919	0.860	0.272

Table 4.2

Power simulations. Data generated from models 4.1-4.2 and 4.6, pg. 210-211, in Teräsvirta (1994).

	ESTAR		LSTAR		ESTAR		LSTAR	
Model	NL3	NL4	NL3	NL4	NL3	NL4	NL3	NL4
$\pi_{20} = c = 0$	0.612	0.722	0.962	0.951	0.825	0.835	1.000	1.000
$\pi_{20} = 0.02$; $c = 0$	0.983	0.997	0.691	0.656	1.000	1.000	0.993	0.993
$\pi_{20} = 0.04$; $c = 0.02$	0.611	0.623	0.157	0.139	0.984	0.992	0.378	0.373
	Sample Size = 100				Sample Size = 300			

Table 5.1

Linearity testing, determining the delay parameter and selecting between LSTAR and ESTAR models

Country	Max. Lag (AIC)	P - value NL3	P – value NL4	Delay Parameter	TP Choice	EJP Choice
Austria	5	0.010	0.033	1	LSTAR	ESTAR
Belgium	5	0.050	0.259	1	LSTAR	LSTAR
Japan	5	0.000	0.000	1	?	ESTAR
Norway	8	0.031	0.200	5	LSTAR	LSTAR
Sweden	5	0.015	0.040	3	LSTAR	ESTAR
U.K.	8	0.047	0.192	4	ESTAR	ESTAR
U.S.A.	6	0.006	0.054/0.016*	3/5*	LSTAR	LSTAR
EUR	9	0.015	0.043	3	ESTAR	ESTAR

Note: For U.S.A. NL4 minimum p-value was for d = 5.

Table 5.2

Summary Statistics for STAR model estimation: Austria and Sweden.

	AUSTRIA			SWEDEN	
Summary Statistics	**LSTAR**	**ESTAR**	**Summary Statistics**	**LSTAR**	**ESTAR**
R-Squared	0.7079	0.6778	*R-Squared*	0.7251	0.7311
Adj. R^2	0.6819	0.6607	*Adj.* R^2	0.7039	0.7104
SSR	0.0503	0.0554	*SSR*	0.0538	0.0526
AIC	-7.4036	-7.3770	*AIC*	-7.3560	-7.3781
SIC	-7.1677	-7.2207	*SIC*	-7.1463	-7.1684
Durbin-Watson	2.3350	2.0845	*Durbin-Watson*	1.8476	2.0132
No. of params.	9	6	*No. of params.*	8	8