UNIVERSIDAD CARLOS III DE MADRID working papers

Working Paper 07-02
Business Economics Series 02
February 2007

Departamento de Economía de la Empresa
Universidad Carlos III de Madrid
Calle Madrid, 126
28903 Getafe (Spain)
Fax (34-91) 6249607

COMMONALITY IN THE LME ALUMINIUM AND COPPER VOLATILITY PROCESSES THROUGH A FIGARCH LENS*

Isabel Figuerola-Ferretti[1] and Christopher L. Gilbert[2]

Abstract

We consider dynamic representation of spot and three month aluminium and copper volatilities. These are the two most important metals traded in the London Metal Exchange (LME). They share common business cycle factors and are traded under identical contract specifications. We apply the bivariate FIGARCH model which allows parsimonious representation of long memory volatility processes. Our results show that spot and three month aluminium and copper volatilities follow long memory processes, that they exhibit a common degree of fractional integration and that the processes are symmetric. However, there is no evidence that the processes are fractionally cointegrated. This high degree of commonality may result from the common LME trading process.

Keywords: volatility, persistence, fractional cointegration, commodity futures
JEL Classification: C32, C12, C13, G12

[1] Figuerola-Ferretti, Departamento de Economía de la Empresa, Universidad Carlos III de Madrid. Spain; email ifgarrig@emp.uc3m.es

[2] Gilbert: Dipartimento di Economia, Università degli Studi di Trento, Via Inama 5, 38100 Trento, Italy; email cgilbert@economia.unitn.it

1. Introduction

Aluminium and copper are the most important metals traded on the London Metal Exchange (LME) in terms of volume and open interest figures. They are traded under identical trading rules in terms of lot size, minimum price movement and delivery dates. LME aluminium and copper options are also identically specified in terms of strike price gradation and tick size. Aluminium and copper also share common demand side fundamentals as they are used across the entire range of manufacturing and particularly in the construction, electrical and transport industries.[1] These related fundamentals and identical trading processes may generate common or highly related volatility processes.

Copper was one of the original LME metals and has been traded on the LME continuously since the reopening of the exchange after the Second World War. The aluminium industry was dominated by a group of producers who set prices on a cost plus basis for much of the post-War period. Trading of aluminium futures on the LME commenced in October 1978, and the LME price became the main world reference price from the mid-nineteen eighties. It has now overtaken copper as the most valuable of the LME's contracts – see Figuerola-Ferretti and Gilbert (2005).[2]

A process x may be described as integrated of degree d, i.e. as $x \sim I(d)$, for integer $d \geq 0$ if the dth difference of x is stationary, i.e. $(1-L)^d x \sim I(0)$, where L is the lag operator. Generalization to fractional values of the differencing parameter d permits greater flexibility in describing persistence. Shocks to I(0) series decay exponentially fast. This implies that many processes require long lag lengths if they are adequately to account for observed persistence patterns. Values of d in the range $0 < d < ½$ imply stationarity but with a slower degree of decay than for I(0) series (Parke, 1999). It is often found that fractional specifications allow more parsimonious specifications of time series than those required using conventional ARIMA (AutoRegressive Integrated Moving Average) representations.

[1] Although aluminium and copper are highly related in the demand side, the supply sides of the market are different. However supply responses are typically slower than demand responses and short term price movements tend to be dominated by demand movements (Ghosh *et al*, 1987; Gilbert, 1995).

[2] Copper and aluminium are also traded on the Comex division of NYMEX and on the Shanghai Metal Exchange. Aluminium is additionally traded on the Tokyo Commodity Exchange. In both cases, the LME price continues to function as the world benchmark priced.

Robinson (2003) surveys the application of long memory processes in economics – see also Teyssiere and Kirman (2006).

A substantial literature supports the view that the volatilities of financial asset prices decay more slowly than standard GARCH (Generalized AutoRegressive Conditional Heteroscedasticity) models suggest and that they may be represented as stationary fractional processes – see in particular Baillie *et al*. (1996) and Ding and Granger (1996). There is evidence that much of the short term variation in financial asset volatility is driven by the trading process – see French and Roll (1986), Amihud and Mendelson (1987, 1991), Stoll and Whaley (1990), Lyons (1996) and Chelley-Steeley (2005). We conjecture that this impact will be similar for contracts traded on the same exchange and often by the same individuals. However, it is not clear how similarity of trading conditions and mechanisms can generate the observed degree of volatility persistence. One possibility is that this arises out of persistence in trading volume (Eps and Eps, 1976; Tauchen and Pitts, 1993; Andersen, 1996).

The second theme of the paper is quantification of the extent of comovement in the volatilities of the two commodities. In the integer case, two I(d) series x and y are said to be cointegrated if there exists some linear combination $x + \delta y$ which is I(d-1). Generalizing to the fractional case, the two series may be said to be fractionally cointegrated if there exists a linear combination which is I(d') where $d' < d$ (Granger, 1986). In this case, shocks to the linear combination are less persistent than shocks to the original series. Cointegration, and by extension fractional cointegration, are important concepts because they show that two series are more closely related in the long than in the short run. Where we observe such cointegration, we need to ask what factors cause this long run association.

We use the bivariate constant correlation FIGARCH (Fractionally Integrated Generalized AutoRegressive Conditional Heteroscedastic) representation introduced by Teyssiere (1996) and Brunetti and Gilbert (2000, henceforth BG). BG apply a bivariate FIGARCH model to a single commodity (crude oil) traded in two different exchanges (the IPE and NYMEX). The bivariate constant correlation FIGARCH may be reparameterized into ECM-FIGARCH (Error Correction Mechanism FIGRACH) which allows testing for fractional cointegration. BG (2000) found a common order of fractional integration for the two crude oil contracts and tested for fractional cointegration, confirming that NYMEX and

IPE volatilities are indeed fractionally cointegrated. This is compatible with the view that, despite short run differences in the volatility movements arsing out of particular regional and market circumstances, the longer evolution of volatility in the two markets largely reflects economic fundamentals and is independent of the exchange on which the commodities are traded.

In this paper we ask to what extent the LME copper and aluminium spot and future volatility processes exhibit common long temporal dependence, and whether they are fractionally cointegrated. These questions raise issues which are similar but not identical to those discussed by BG (2000). The commonality in the market fundamentals implies that the two prices and volatilities are likely to move together and may respond in similar ways to shocks. This will be true of both spot and three month processes, but the spot volatility process will also be substantially affected by the volatility of the convenience yield, less important for three month prices. There is no a priori basis for knowing to what extent the two convenience yield volatility processes are likely to be related or similar but the additional volatility component in the spot is likely to lead to differences between the respective spot and three month volatility processes.

We describe two volatility processes as similar if, within the same representation, they have a common degree of fractional integration. If the degree of fractional cointegration is common, we may ask whether the two processes are also fractionally cointegrated. We consider the following three possibilities[3]

i) Non-identical, non-cointegrated volatility processes: Despite correlation between the two return and volatility processes driven by common factors, the two markets are distinct and independent. There is no reason to expect either a common structure for the volatility processes or cointegration.

ii) Identical, non-cointegrated volatility processes: If information arrival relates primarily to common business cycle factors driven by demand related fundamentals we would expect price movements to be highly related for copper and aluminium, and this might result in the same order of fractional integration in the two processes. Volatilities will be driven by a common information arrival process well mediated through a common trading

[3] The fourth logical possibility of non-identical cointegrated volatility processes is excluded by the fact that fractional cointegration presupposes a common degree of fractional integration – see Robinson and Marinucci (2003).

process. However, in the long run, supply as well as demand fundamentals become important, so we should not expect the volatility processes to be cointegrated

iii) Identical, cointegrated volatility processes: Exchange price movements are determined by speculative and hedging pressures and are not intimately related to economic fundamentals which only become known with a lag. Investors interested in taking a position in non-ferrous metals regard the two metals as close substitutes and trade which ever appears to offer the greatest trading advantage at the time of trade initiation. The same is true for options. In this case, there may be a common, cointegrated, volatility process.

The results support the second of these possibilities, i.e. the LME aluminium and copper volatility processes are identical but not cointegrated.

2. The GARCH and FIGARCH classes of volatility models

Granger and Joyeux (1980) and Hosking (1981) independently introduced the Fractionally Integrated ARMA or ARFIMA process

$$\varphi(L)(1-L)^d(h_t-\mu)=\beta(L)\nu_t \qquad (1)$$

where $\varphi(L)=1-\sum_{j=1}^{q}\varphi_j L^j$ and $\beta(L)=1+\sum_{j=1}^{p}\beta_j L^j$ and ν_t is a white noise error process. In the ARFIMA class of models, the short term behaviour of the time series h_t is captured by the ARMA parameters in the φ and θ lag distributions, while the long run dependence is modelled through the fractional differencing parameter d. This model may be applied to variances by taking the observed h_t series as the price returns $h_t=\varepsilon_t^2=(\Delta \ln p_t)^2$. A variant of this model substitutes the absolute returns $|\varepsilon_t|$ for the squared returns ε_t^2.

In the same way that this generalization of standard ARMA models to the ARFIMA class of models has proved important empirically, a corresponding move has taken place in modelling conditional variances. Baillie *et al* (1996) introduced the Fractionally Integrated GARCH (FIGARCH) class of models generalizing the GARCH and Integrated GARCH specifications. Analogously to the ARFIMA class of models for the conditional mean, a shock to the conditional FIGARCH variance is transitory, meaning that the influence of the future forecast of the conditional variance decays at a slow hyperbolic rate.

Squared returns are not the same as variances both because each squared return is a single realization and because use of the unconditional return supposes weak form market efficiency. In common with the GARCH class, FIGARCH models take the variance h_t as latent. An ARFIMA may be regarded as approximating the corresponding FIGARCH model with volatility approximated by squared or absolute returns. This approximation will be good provided under weak form efficiency when there is little explanatory power in the first order process.

Write the first order return process as

$$\left[1-\gamma\left(L\right)\right]\Delta \ln p_t = \mu + \varepsilon_t \tag{2}$$

where $\gamma(L)$ is of length m. Define the conditional variance of the error term in (2) as $h_t = Var\left[\varepsilon_t \mid \Omega_{t-1}\right]$ where Ω_{t-1} is the information set in period t-1. The FIGARCH(p,d,q) representation introduced by Baillie, *et al* (1996) may be represented as

$$\varphi(L)\left(1-L\right)^d \varepsilon_t^2 = \omega + \left[1-\beta(L)\right] v_t \tag{3}$$

where the roots of $\varphi(L)$ and $\left[1-b(L)\right]$ lie outside the unit circle. The conditional variance of the FIGARCH process may be written as

$$h_t = \frac{\omega}{1-\beta(1)} + \lambda(L) \tag{4}$$

where $\lambda(L) = 1 - \dfrac{\varphi(L)(1-L)^d}{1-\beta(L)}$. A shock to the conditional variance decays at an exponential rate for the covariance stationary GARCH (p,q) model. By contrast, in the FIGARCH (p,d,q) model the effect of shock in the future conditional variance dissipates at a slower hyperbolic rate of decay. The fractional differencing parameter is therefore identified by the decay rate of a shock to the conditional variance (or the decay rate of the autocorrelations), and not by its impact on the forecast for the long run conditional variance – see Bollerslev and Mikkelsen (1996).

In an unrelated development, Bollerslev *et al* (1988) generalized the univariate GARCH to a multivariate framework. As in the univariate case, the conditional variance-covariance matrix of the n-dimensional error term e_t in the multivariate GARCH(p,q) model is conditional on the information available at time t-1. Hence the elements of the covariance matrix follow a vector ARMA process in the squares and cross products of the innovations.

Setting $p = q = 1$ and defining the covariance as $h_{12,t} = \rho\left(h_{11,t}h_{22,t}\right)^{1/2}$, Bollerslev (1990) developed the constant correlation bivariate GARCH(1,1) specification:

$$\begin{aligned} h_{11,t} &= \varphi_{11}\varepsilon_{1,t-1}^2 + \beta_{11}h_{11,t-1} + \omega_1 \\ h_{22,t} &= \varphi_{22}\varepsilon_{2,t-1}^2 + \beta_{22}h_{22,t-1} + \omega_2 \\ h_{12,t} &= \rho\left(h_{11,t}h_{22,t}\right)^{1/2} \end{aligned} \quad (5)$$

Note that the first two lines of equation (5) may be expressed as

$$h_{jj,t} = \lambda_{jj}\varepsilon_{j,t-1}^2 + \frac{\omega_j}{1-\beta_{jj}} \quad (j = 1,2) \quad (6)$$

where $\lambda_{jj} = \dfrac{\varphi_{jj}}{1-\beta_{jj}}$.

The extension of multivariate GARCH model to multivariate FIGARCH is due to Teyssiere (1996) and BG (2000) both of whom used the constant correlation parameterization – see also Dark (2004). The system may be written as

$$\Phi(L)\left(1-L\right)^d \varepsilon_t^2 = \omega + \left[1 - B(L)\right]v_t \quad (7)$$

where ε^2, ω and ν are now vectors. In the bivariate case which we consider, $\Phi(L)$ and $B(L)$ are both 2x2 lag polynomial matrices. In terms of equation (6), the bivariate constant correlation FIGARCH(1,d,1) implies

$$\lambda_{jj} = \frac{\varphi_{jj}\left(L\right)\left(1-L\right)^{d_j}}{1-\beta_{jj}} \quad (j = 1,2) \quad (8)$$

The advantage of this specification is that it is highly parsimonious and positive definitiveness of the variance covariance matrix and stationarity is achieved under weak restrictions. Bollerslev and Mikkelsen (1996) showed that positive definiteness in the bivariate diagonal FIGARCH(1,d,1) model is assured provided

$$\left|\rho\right| < 1 \quad \beta_{jj} - d_j \le \tfrac{1}{3}(2-d_j) \text{ and } d_j\left[\varphi_{jj} - \tfrac{1}{2}(1-d_j)\right] \le \beta_{jj}(\varphi_{jj} - \beta_j + d_j) \quad (9)$$

The conditional variance of this process will be stationary when $0 \le d_j \le 1$.

3. Fractional cointegration

Cointegration relates to the long run stable relationship between two or more variables which allow for short time divergences. Granger (1981, 1983) introduced the concept of fractional cointegration. Two time series $y_{1,t} \sim I(d)$ and $y_{2,t} \sim I(d)$ with identical order of integration

d are said to be cointegrated of order (d, b) if there exists a $\delta_2 \neq 0$ so that $z_t = y_{1,t} + \delta_2 y_{2,t} \sim I(d-b)$ where $b > 0$. In this case, the linear combination represented by z_t has lower order of integration than its components $y_{1,t}$ and $y_{2,t}$. Although there is no reason to believe that two arbitrarily chosen series $y_{1,t}$ and $y_{2,t}$ will be integrated of the same order, a common order of integration is a necessary condition for finding a linear cointegrating relationship (Robinson and Marinucci, 2003). If the two time series are integrated of different orders, a non-trivial linear combination of the two series will be integrated of the higher of the two orders.

While a considerable amount of recent work has emphasised the role of persistence of shocks, most of it has been directed towards testing the presence of unit roots in autoregressive representations of univariate and vector processes. Thus in most cases testing for cointegration involved preliminary unit root test. If the series were found to be I(1) then a cointegration test is implemented. This has been described by Robinson and Marinucci (2003) as the "I(1)/I(0)" paradigm which sets d=b=1. The distinction between I(0) and I(1) can be too restrictive when using high frequency financial data – see Baillie and Bollerslev (1994) and Baillie (1996). Fractionally integrated processes are a halfway house between the I(0) and I(1) paradigms. In contrast to the I(0) models where the correlations decay at an exponential rate, the autocorrelation of an I(d) series dissipate at a slower hyperbolic rate of decay. This permits parsimonious representation of series which exhibit non-zero autocorrelations at high lags.

There are two main approaches to testing for cointegration in the I(0)/I(1) case – the Engle and Granger (1987) and the Johansen (1988) methods. The standard test for fractional cointegration follows the Engle-Granger two stage approach based on inclusion of the lagged residuals from the OLS estimates of the cointegrating equation in the short run adjustment equation. The following steps are required:

1. Determine the orders of integration, d_1 and d_2 of the series under consideration.
2. Test whether $d_1 = d_2 = d$ (say) i.e. whether the series are integrated of a common order.
3. Estimate the candidate cointegrating linear combination of the series and determine the order of integration d' of the residuals.
4. Test whether $d' < d$, i.e. the two series are fractionally cointegrated.

In the FIGARCH context, the initial step is implemented by estimating an unrestricted bivariate FIGARCH model using the specification given in equations (6) - (8). This allows the common degree of integration specification to be directly imposed and tested as the second step. If the test rejects a common order of integration, the investigation terminates. If not, one moves to the third and most difficult step, that of finding the cointegrating vector δ. Here, BG simplify by assuming a unit cointegrating vector. They justify this assumption by stating that it is plausible that volatilities of the same product traded in related markets should move uniformly together. According to the Engle-Granger Representation Theorem (Engle and Granger, 1987), cointegration implies the existence of an error correction representation. Granger (1986) shows that, in the case of bivariate FIGARCH, this may be written as

$$\Phi^*(L)(1-L)^d \varepsilon_t^2 = \omega + (\mathrm{I} - \mathrm{B}(L))v_t - \left[1-(1-L)^b\right](1-L)^{d-b}\xi\delta'\varepsilon_t^2 \tag{10}$$

where $\Phi^*(L) = \Phi(L) + \left[1-(1-L)^{-b}\right]\xi\delta'$. Estimation of equation (10) allows a test of the fractional cointegration hypothesis $b > 0$.[4]

4. The LME aluminium and copper contracts

The LME differs from other futures exchange in trading contracts which mature on every single trading day, rather than concentrating delivery in specific months. On any particular day, market liquidity in the ring focuses on the "prompt" (spot) contract, which requires settlement within two working days, and the three month contract, which will become prompt exactly three months from the day in question. Traders wishing to close out different contracts will do through appropriate borrowing or lending transactions through LME brokers. From the point of view of data analysis, this has the advantage that the LME spot and three month prices are the prices of "continuous futures" by construction.

We use the complete set of daily LME settlement prices for cash and three month copper and aluminium over the period 3 October 1982 to 30 December 2005.[5] Figure A1 in

[4] BG (2000) note that the zero order terms of the lag polynomial $\left[1-(1-L)^{-b}\right]$ are null, it is therefore always possible to find an infinite order polynomial $\Phi^*(L)$ satisfying equation (10) and with $\Phi_0^* = \mathrm{I}$ for any given vectors x and g. The implication is that Φ^*, z and δ, are not identified in the absence of restrictions on $\Phi^*(L)$. Restrictions can be in terms of either order or diagonality or both. This lack of identification implies that finding $b < d$ in (10) is insufficient as a demonstration of fractional cointegration. In practice, estimation requires a restriction on the order of Φ^*(L).

appendix graphs aluminium and copper spot volatilities, measured as monthly standard deviations of daily returns on an annualized basis (futures volatilities are similar). The aluminium price volatility shows an upward trend over the early period when the LME contract was gaining in importance. Both aluminium and copper volatilities were particularly high in the final years of the nineteen eighties as the consequence of tight market conditions.

Figures A2 and A3 graph autocorrelation functions for the absolute values of spot aluminium and copper returns (Figure A2), and those of the three month returns (Figure A3) estimated over the complete set of daily settlement prices from 1984 through 2005. The autocorrelations for the four set of absolute returns remain positive for over one hundred lags and visually appear to exhibit long memory characteristics. In the case of the spot returns (Figure A2), the absolute copper price returns appear less persistent than those of aluminium whereas in the case the autocorrelation functions of the two sets of absolute three month returns (Figure A3) appear very similar.

Overall, Figures A2 and A3 show a high degree of persistence, consistent with a fractional degree of integration, in aluminium and copper spot and three month volatilities. While the autocorrelation functions of the three month volatilities are very similar, those of the spot volatilities differ with aluminium volatilities exhibiting a greater degree of autocorrelation at all lag lengths.

5. ARFIMA models

We first estimate an ARFIMA(1,*d*,1), as specified in equation (1), for both absolute returns and squared returns. [6] The selected optimal models are estimated using the exact Maximum Likelihood (ML) estimator of the ARFIMA process under normality derived by Sowell (1992).[7] Consistency of these estimates depends on the validity of the choice of the lag length. Lobato (1999) has demonstrated using Monte Carlo analysis that estimates of long

---

[5] Source: Brunetti and Gilbert (1995), updated from http://www.ecowin.com. Aluminium has always been traded in US dollars. However, copper was traded in sterling until 1 July 1993. Sterling prices were converted into dollars at the exchange rate of the day.

[6] We also estimated more general specifications – see Figuerola-Ferretti (2002). We report the ARFIMA(1,*d*,1) results for comparability with the from the FIGARCH(1,*d*,1) model which follow.

[7] We use the Ox routine ARFIMA developed by Doornik and Ooms (1999). See also Ooms and Doornik (1998). We also estimated more highly parameterized models. We report results for the AFRFIMA(1,*d*,1) specification as being directly comparable with the FIGARCH(1,*d*,1). See Figuerola-Ferretti (2002) for estimates of higher order models.

memory models are particularly sensitive to the misspecification of the length of the autoregressive polynomial. Estimates are reported in Table 1.

Table 1
ARFIMA(1,*d*,1) Estimates of Fractional Differencing Parameter

	Absolute Returns			Squared Returns		
	Aluminium	Copper	Differences	Aluminium	Copper	Differences
Spot	0.4301	0.4079	0.3041	0.4352	0.4365	0.2329
	(0.0317)	(0.0368)	(0.0362)	(0.0314)	(0.0383)	(0.0252))
3 month	0.3693	0.3993	0.3018	0.3436	0.3769	0.2480
	(0.0325)	(0.0382)	(0.0351)	(0.0339)	(0.0426)	(0.0380)

The table reports the estimated value of the fractional differencing parameter d in the ARFIMA(1,d,1) model, given by setting $p = q = 1$, in equation (1) for absolute and squared daily returns over the sample 4 October 1982 to 30 December 2005 (5866 observations). "Differences" refer to the differences between the squared aluminium returns (or absolute returns) to the corresponding measure for copper. Standard errors are given in parentheses. Full estimates are in appendix Table A1.

The estimated fractional differencing parameter is significantly greater than zero and is less than one half in all cases, although not significantly so for the two spot squared return processes processes. We can therefore conclude that aluminium and copper daily volatility, as proxied by absolute and squared returns exhibit long memory and that the absolute return processes and the squared three month processes are stationary. Taylor (1986), Ding *et al.* (1993), Granger and Ding (1995) and Ding and Granger (1996) report that absolute returns tend to yield a higher estimate of the fractional integration parameter d than do squared returns, a phenomenon which Granger and Ding (1995) refer to as the "Taylor effect". This effect is apparent in our estimates for the three month volatility processes but not for the spot processes.

Table 1 also reports the estimated fractional differencing parameter for the differences in the squared (and absolute) returns for the two metals. These differences remain fractionally integrated but the estimated fractional differencing parameter is substantially lower than in the original series. This motivates our interest in the possibility of fractionally cointegration see section 7.

6. FIGARCH models

We now turn to the univariate FIGARCH(1,d,1) model defined by equations (2), (3) and (4) which we estimate using the methodology suggested by Baillie *et al.* (1996). The mean (first order) process is specified as second order autoregressive. The first two columns of Table 2 report the estimated fractional differencing parameter *d*. The unrestricted estimates (rows 1 and 4) of the fractional parameter are uniformly lower than those implied by the ARFIMA processes, and all four processes are now clearly both fractional and also stationary. The spot volatility processes are seen as being more persistent than the corresponding three month processes, and the copper processes as more persistent than the corresponding aluminium processes. Full parameter estimates are given in appendix Table A2, columns 1 and 4.[8]

Table 2
Univariate FIGARCH Hypothesis Tests

		d_1	d_2	H_1/H_2 , H_0/H_1	H_0/H_2
Spot prices					
H_2	Unrestricted	0.3000 (0.0568)	0.3493 (0.0549)		
H_1	$\beta_{11} = \beta_{22}, \varphi_{11} = \varphi_{22}$	0.3241 (0.0420)	0.3411 (0.0405)	$\chi^2(2) = 0.89$ [63.9%]	
H_0	$\beta_{11} = \beta_{22}, \varphi_{11} = \varphi_{22}, d_1 = d_2$	0.3345 (0.0386)		$\chi^2(1) = 0.40$ [52.5%]	$\chi^2(3) = 1.30$ [72.9%]
3 month prices					
H_2	Unrestricted	0.2171 (0.0372)	0.2942 (0.0466)		
H_1	$\beta_{11} = \beta_{22}, \varphi_{11} = \varphi_{22}$	0.2541 (0.0334)	0.2586 (0.0311)	$\chi^2(2) =2.52$ [28.4%]	
H_0	$\beta_{11} = \beta_{22}, \varphi_{11} = \varphi_{22}, d_1 = d_2$	0.2570 (0.0299)		$\chi^2(1) = 0.04$ [85.0%]	$\chi^2(3) = 2.55$ [43.6%]

The table gives the estimated fractional differencing parameters d_1 and d_2 and the outcomes of the likelihood ratio tests on the bivariate FIGARCH(1,*d*,1) model defined in equations (2) and (3) with $p = q = 1$ and $m = 2$ for aluminium and copper spot and 3 month returns.
Standard errors in “()” parentheses; tail probabilities in “[]” parentheses.
Full estimates are reported in appendix Table A2.

The second and fourth rows of Table 4 impose symmetry in the autoregressive and moving average parameters. The data fail to reject these restrictions With these restrictions imposed, the estimated fractional difference parameters differ little between the two metals, in particular for the three month contracts. The third and sixth rows report estimates in which a common fractional differencing parameter is imposed. The data fail to reject this restriction

[8] Although the fractional differencing parameter *d* should be broadly comparable across the ARFIMA and FIGARCH specifications, the autoregressive and moving average parameters differ markedly.

and also joint imposition of the two sets of restriction. This implies that, within the univariate framework, we can regard the aluminium and copper volatilities as following the same process.

Moving to the bivariate framework which allows for correlations in the two error processes. We confine ourselves to the bivariate FIGARCH(1,*d*,1) defined by equations (6) - (8).[9] We maintain the symmetry hypotheses $\beta_{11} = \beta_{22}$, $\varphi_{11} = \varphi_{22}$, and $d_1 = d_2$ established in the univariate context and reported in section 5.

Table 3
Bivariate FIGARCH Hypothesis Tests

		d	ρ	$\beta_{12}= \beta_{21}$	$\varphi_{12}= \varphi_{21}$	H_0^* / H_1^*
Spot prices						
H_1^*	Unrestricted	0.3510 (0.0267)	0.5370 (0.0091)	-0.0177 (0.0047)	-0.0057 (0.0079)	
H_0^*	Diagonal	0.3814 (0.0274)	0.5317 (0.0091)			33.88 [0.00%]
3 month prices						
H_1^*	Unrestricted	0.3070 (0.0241)	0.5666 (0.0087)	-0.0165 (0.0049)	-0.0135 (0.0078)	
H_0^*	Diagonal	0.3306 (0.0244)	0.5638 (0.0087)			19.70 [0.00%]

The table gives the estimated fractional differencing parameters d_1 and d_2 and the outcomes of the likelihood ratio tests on the bivariate FIGARCH(1,*d*,1) model defined in equations (2), (6), (7) and (8) with $p = q = 1$ and $m = 2$ for aluminium and copper spot and 3 month returns.
Standard errors in "()" parentheses; tail probabilities in "[]" parentheses.
Full estimates are reported in appendix Table A3

The diagonality assumption ($\beta_{12}= \beta_{21}=\varphi_{12}= \varphi_{21}=0$) is fairly standard in this literature – see for example Bollerslev *et al.* (1988) although BG (2000) included off-diagonal terms. Imposition of diagonality implies that although the two scedastic processes may be correlated, they do not interact – effectively, a "seemingly unrelated" FIGARCH model. We start by estimating a non-diagonal model but we impose symmetry in the interaction terms, i.e. $\beta_{12} = \beta_{21}$ and $\varphi_{12} = \varphi_{21}$. We refer to this model as hypothesis H_1^*. The symmetry restrictions on the off-diagonal terms are natural given the other symmetry restrictions we have imposed but perhaps more importantly, attempts to estimate a general model in the

Measurement of volatility by the absolute or squared price changes generates substantial noise which may be autocorrelated. The latent volatility measure in the FIGARCH estimates is much smoother.

bivariate context leads to serious identification problems. We compare the model H_1^* with off-diagonal terms with the restricted "seemingly unrelated" model H_0^* which sets $\beta_{12}=\beta_{21}=0$ and $\varphi_{12}=\varphi_{21}=0$.

Table 3 summarises the estimates (given in full in appendix Table A3). The volatility correlations ρ are precisely determined and are do not vary across specifications. They are above 0.5 for both spot and three month contracts but marginally higher for the latter. The significance of these values demonstrates clearly that the two volatility processes not only share a similar structure but that they are also move closely together. Turning to the off-diagonal interaction terms, there are estimated as small and negative. The autoregressive parameters and φ_{12} and φ_{21} do not differ significantly from zero while the moving average parameters β_{12} and β_{21} are significant, although not strongly so. Despite this lack of strong significance at the level of the individual coefficients, the diagonal model is decisively rejected for both spot and three month contracts by the likelihood ratio tests.

7. Fractional cointegration

If we take the FIGARCH(1,*d*,1) as the valid representation, the failure to reject the null hypothesis of a common fractional parameter suggests that there may be fractional cointegration between aluminium and copper three month volatility processes. We follow BG (2000) in imposing a unit cointegrating vector so that $\delta' = (1 \quad -1)$. We justify use of this procedure by noting that the unconditional volatilities for copper and aluminium are almost identical.[10] Even having imposed this restriction, we found that, although there is evidence of cointegration, identification was poor and that it was not possible to jointly estimate the cointegration parameter b and the two error correction parameters ξ_1 and ξ_2 in equation (10) with any precision. Using these data, it appears necessary to impose a value for at least one of these parameters in order to determine the other two.

The ARFIMA estimates discussed in Table 1 suggest a substantially lower value fractional differencing parameter for the differences in squared returns relative to those obtained from the original squared return series. Based on these estimates, we investigated

[9] Identification in higher order specifications was poor.

[10] The ratio of the two unconditional spot and three month volatilities are 0.88 and 0.90 respectively.

fixing the value of the parameter b to 0.2. This allows us to estimate the two reaction coefficients ξ_1 and ξ_2 conditional on these values for b. Preliminary estimates suggested the restriction $\xi_1 = \xi_2$, in line with the symmetry restrictions we have already imposed on the β and φ coefficients. This restriction is easily satisfied in all specifications – see appendix Table A4. Table 4 reports summary results for both diagonal specifications with this additional restriction imposed.

Table 4
ECM-FIGARCH Hypothesis Tests

		d	ξ	$\beta_{12}=\beta_{21}$	$\varphi_{12}=\varphi_{21}$	H_0^{**}/H_1^{**}	H_1^{*}/H_1^{**}
Spot prices							
H_1^{**}	Unrestricted	0.3748 (0.0357	0.0477 (0.0545)	-0.0103 (0.0085)	-0.0094 (0.0089)		0.37 [54.2%]
H_0^{**}	Diagonal	0.3893 (0.0258)	0.0801 (0.0184)			2.38 [30.4%]	
3 month prices							
H_1^{**}	Unrestricted	0.2594 (0.0360)	0.0608 (0.0293)	-0.0333 (0.0125)	-0.0173 (0.0108)		1.24 [26.5%]
H_0^{**}	Diagonal	0.3383 (0.0238)	0.0423 (0.0153)			5.63 [5.98%]	

The table gives the estimated fractional differencing parameters d_1 and d_2 and the outcomes of the likelihood ratio tests on the bivariate FIGARCH(1,d,1) model defined in equations (2), (6), (7) and (10) with $p = q = 1$, $m = 2$ and $b = 0.2$ for aluminium and copper spot and 3 month returns.
Standard errors in "()" parentheses; tail probabilities in "[]" parentheses.
Full estimates are reported in appendix Table A4.

Consider first the unrestricted estimates (H_1^{**}) which allow off-diagonal β and φ coefficients. The error correction coefficient ξ does not differ significantly from zero for the spot price process and only does so marginally for the three month process. If we relax the restriction of b from 0.2 in this specification, both the spot and three month likelihood functions become very flat but are maximized by allowing b to tend to zero at which point ξ becomes unidentified.[11] These unrestricted estimates therefore offer very little support for fractional error correction.

It is also notable that the estimated off-diagonal β and φ coefficients in the H_1^{**} specification do not differ significantly from zero with the exception of $\beta_{12}=\beta_{21}$ in the three month equation. (This contrasts with the higher levels of statistical significant implied by the estimated off-diagonal models (H_1^*) reported in Table 3). These reduced levels of significance suggest that the fractional error correction mechanism may be performing the same role as the off-diagonal β and φ coefficients in generating interactions between the aluminium and copper scedastic processes. This impress is reinforced by inspection of the estimated specification (H_0^{**}) in which diagonality is imposed and where the error correction mechanism is the only link between the two variance processes. The ξ error correction coefficients do now differ significantly from zero, strongly so in the case of the spot price equation. In this case, unrestricted estimation of b sees it tending towards the boundary of $b = d$.[12] That might be taken to imply that the difference between the two variables is I(0), but we know from the ARFIMA estimates reported in Table 1 that this is not the case. Instead, setting $b = d$ in equation (10) collapses the model back to the FIGARCH model (7). Again, therefore, the estimates fail to support the presence of fractional error correction.

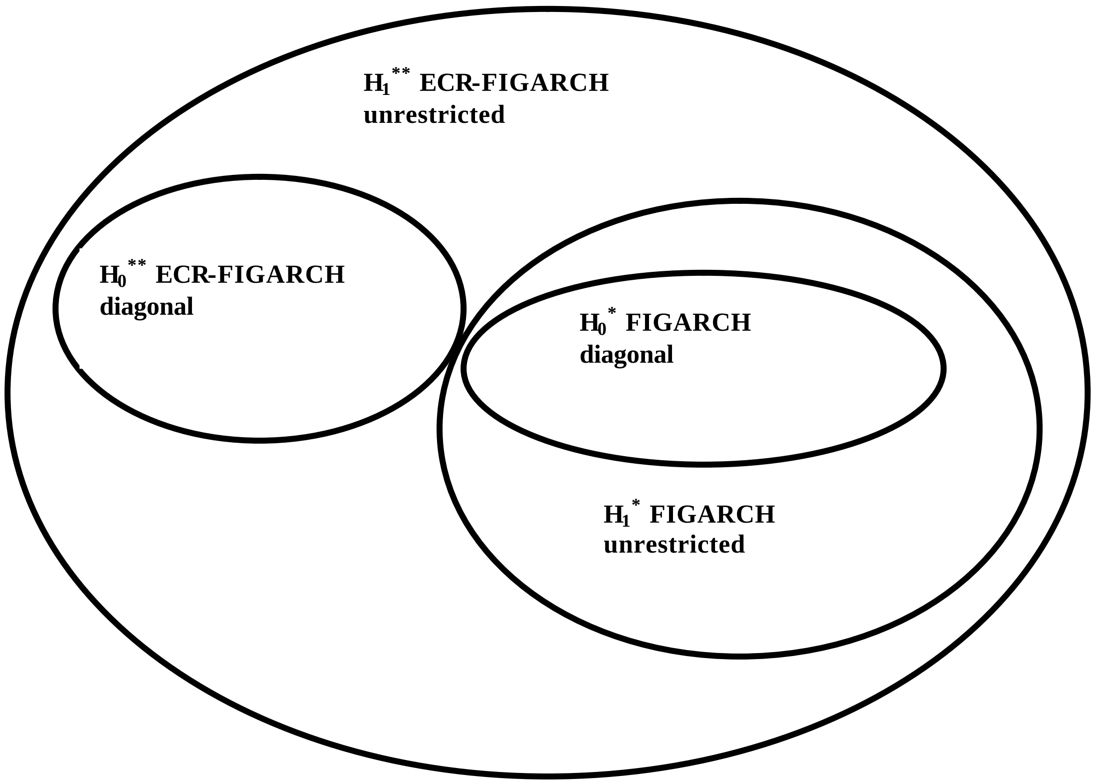

Figure 1: Relationship between Hypotheses

[11] Estimates available from the authors on request.
[12] Estimates available from the authors on request

Figure 1 shows the relationship between the hypotheses under consideration. The diagonal and unrestricted FIGARCH models (H_1^* and H_0^*) discussed in section 6 and the diagonal ECR-FIGARCH (H_0^{**}) are nested within the unrestricted ECR-FIGARCH (H_1^{**}) but have only a single point in common – the point at which $b = \beta_{12} = \beta_{21} = \varphi_{12} = \varphi_{21} = 0$. Clearly H_0^* is nested within H_1^* but the diagonal ECR-FIGARCH (H_0^{**}) and the unrestricted FIGARCH are non-nested – see BG (2000). The tests reported in Table 4 fail to reject restriction of H_1^{**} to either H_0^{**} or H_1^* while those of Table 3 emphatically reject restriction of H_1^* to H_0^*. Despite the fact that H_1^* is associated with a higher log-likelihood than H_0^{**} for both spot and three month prices (see appendix Tables A3 and A4), we are unable to formally test between H_0^{**} and H_1^*. We could adopt a non-nested testing methodology but this seems unnecessary in view of the low significance of the estimated ξ coefficients, and the fact that likelihood is maximized for H_0^{**} by setting $b = 0$, we conclude that the evidence is decisively in favour of a symmetric non-diagonal FIGARCH specification and against the ECM-FIGARCH model.

8. Conclusions

Using long series (eighteen years) of daily price data, we have established that the volatility processes for LME aluminium and copper returns exhibit long memory. Shocks to these volatilities are more persistent than standard low order GARCH models imply. This is true of both spot and three month volatilities. These processes can be parsimoniously represented by fractionally integrated GARCH (FIGARCH) models.

There is considerable evidence of fractional integration across a wide range of financial many volatility series and so it is perhaps not surprising that we find that LME aluminium and copper volatilities conform to this paradigm. More remarkable is the fact that the latent volatility processes exhibit a common degree of fractional integration. Further, within the FIGARCH framework, we are unable to reject the hypothesis that the two scedastic processes are identical except for the intercept term. There would be no reason to expect such a result for the return volatilities of unrelated assets. This common persistence may result either from the shared LME trading process or by the fact that demand for the two metals tends to be driven by the same factors.

We have also found strong evidence that the aluminium and copper scedastic processes interact. From a technical standpoint, this implies that imposition of diagonality in the bivariate FIGARCH model is not sustainable in this context. On the other hand, the evidence for fractional cointegration is weak. Fractional cointegration is a second route through which the scedastic processes might interact, and, in our dataset, in which the fractional cointegration parameter appears poorly identified, it is difficult to accurately distinguish between the long term comovement which fractional cointegration would generate and the short term scedastic interactions resulting from the off-diagonal terms in the FIGARCH process. Despite this, the evidence favours the latter explanation over fractional cointegration.

Our overall conclusion is therefore that the LME aluminium and copper scedastic processes are both highly persistent, that this persistence allows the series to be represented as fractionally integrated and modelled as FIGARCH processes, that the two processes are identical apart from in their intercepts, that they interact (with the implication that diagonality cannot be imposed), but that they are not fractionally cointegrated (so there is no long term tendency form them to move together). The strong symmetry of the two processes suggests that the processes may be the outcome of common market microstructure factors.

References

Amihud, Y. and H. Mendelson (1987), "Trading mechanisms and stock returns: an empirical investigation", *Journal of Finance*, **42**, 533–53.

Amihud, Y. and H. Mendelson (1991), "Volatility, efficiency and trading. Evidence from the Japanese stock market", *Journal of Finance*, **46**, 1765–89.

Andersen, T.G. (1996), "Return volatility and trading volume: an information flow interpretation of stochastic volatility", *Journal of Finance*, **51**, 169-204.

Baillie, T.T. (1996), "Long memory processes and fractional integration in econometrics", *Journal of Econometrics*, **73**, 5-59.

Baillie, R.T, and T. Bollerslev, (1994), "Cointegration fractional cointegration and exchange rate dynamics", *Journal of Finance*, **49**, 737-45.

Baillie, R.T, T. Bollerslev, and H.O. Mikkelsen (1996), "Fractionally integrated generalized autoregressive conditional heteroscedasticity", *Journal of Econometrics*, **74**, 3-30

Bollerslev, T. (1990), "Modelling the coherence in short-run nominal exchange rates: a multivariate generalized ARCH approach", *Review of Economic Studies*, **79**, 489-525.

Bollerslev, T., R.F. Engle and J.M. Wooldridge (1988), "A capital-asset pricing model with time-varying coefficients", *Journal of Political Economy*, **96**, 116-31.

Bollerslev, T and H.O. Mikkelsen (1996), "Modelling and pricing long memory in stock market volatility", *Journal of Econometrics*, **73**, 151-84.

Brunetti, C. and Gilbert, C.L. (2000), "Bivariate FIGARCH and fractional cointegration", *Journal of Empirical Finance*, **7**, 509-530

Chelley-Steeley, P. (2005), "Noise and the trading mechanism: the case of SETS", *European Financial Management*, **11**, 387-424.

Dark, J. (2004), "Bivariate error correction FIGARCH and FIAPARCH models on the Australian All Ordinaries Index and its SPI futures", *Working Paper* 04/04, Department of Econometrics and Business Statistics, Monash University.

Ding, Z., and C.W.J. Granger (1996), "Modelling volatility persistence of speculative returns: a new approach", *Journal of Econometrics*, **73**, 185-215

Ding, Z., C.W.J. Granger, and R.E. Engle (1993), "A long memory property of stock market returns and a new model", *Journal of Empirical Finance* 1, 83-106.

Doornick J., Ooms, M., 1999. A package for estimating, forecasting and simulating ARFIMA models: ARFIMA package 1.0 for Ox, Mimeo, Nuffield College, Oxford.

Engle, R.F., and C.W.J. Granger (1987), "Cointegration and error correction: Representation, estimation and testing", *Econometrica*, **55**, 251-76.

Eps, T.W., and M.L. Eps, (1976), "The stochastic dependence of security price changes and transactions volumes: implications for the mixture-of-distributions hypothesis", *Econometrica*, **44**, 305-321.

Figuerola-Ferretti, I. (2002), *The Economics of Different Marketing Methods in the Non-Ferrous Metals Industry*, Ph.D. thesis, Queen Mary, University of London .

Figuerola-Ferretti, I., and C.L. Gilbert (2005), "Price discovery in the aluminium market", *Journal of Futures Markets*, **25**, 967-88.

French, K.T., and R. Roll (1986), "Stock return variances: the arrival of information and the reaction of traders", *Journal of Financial Economics*, **8**, 55-69.

Ghosh, S., Gilbert, C.L. and Hughes Hallett, A.J. (1987), "Stabilizing Speculative Commodity Markets", Oxford, *Oxford University Press*.

Gilbert, C.L. (1995), "Modelling market fundamentals: a model of the aluminium market", *Journal of Applied Econometrics*, **10**, 385-410.

Granger, C.W.J. (1986), "Developments in the study of cointegrated economic variables", *Oxford Bulletin of Economics and Statistics*, **48**, 213-28.

Johansen, S. (1988), "Statistical analysis of cointegration vectors", *Journal of Economic Dynamics and Control*, **12**, 231-54.

Lyons, R.K. (1996), “Foreign exchange volume: sound and fury signifying nothing?”, in J. Frankel and G. Galli, eds., *The Microstructure of Foreign Exchange Markets*, Chicago, University of Chicago Press.

Parke, W.R. (1999), “What is fractional integration?”, *Review of Economics and Statistics*, **81**, 632-38.

Robinson, P.M. (2003), “Long memory time series”, in P.M. Robinson ed., *Time Series with Long Memory*, Oxford, Oxford University Press.

Robinson P.M. and D. Marinucci, (2003), “Semiparametric frequency domain analysis of fractional cointegration”, in P.M. Robinson ed., *Time Series with Long Memory*, Oxford, Oxford University Press.

Sowel, F., (1992), “Maximum likelihood estimation of stationary univariate fractionally integrated time series models”, *Journal of Econometrics* **53**, 165-188.

Stoll, H. and R. Whaley, (1990), “Stock market structure and volatility”, *Review of Financial Studies*, **3**, 37–71.

Tauchen, G., and M. Pitts (1983), “The price variability-volume relationship in speculative markets”, *Econometrica*, **51**, 485-505.

Taylor, S.J. (1986), *Modelling Financial Time Series*, Chichester, Wiley.

Teyssiere, G., (1996), “Double long memory financial time series”, *Working Paper*, 348, Department of Economics, Queen Mary University of London.

Teyssiere, G., and A. Kirman (2006), *Long Memory in Economics*, Berlin, Springer.

Appendix

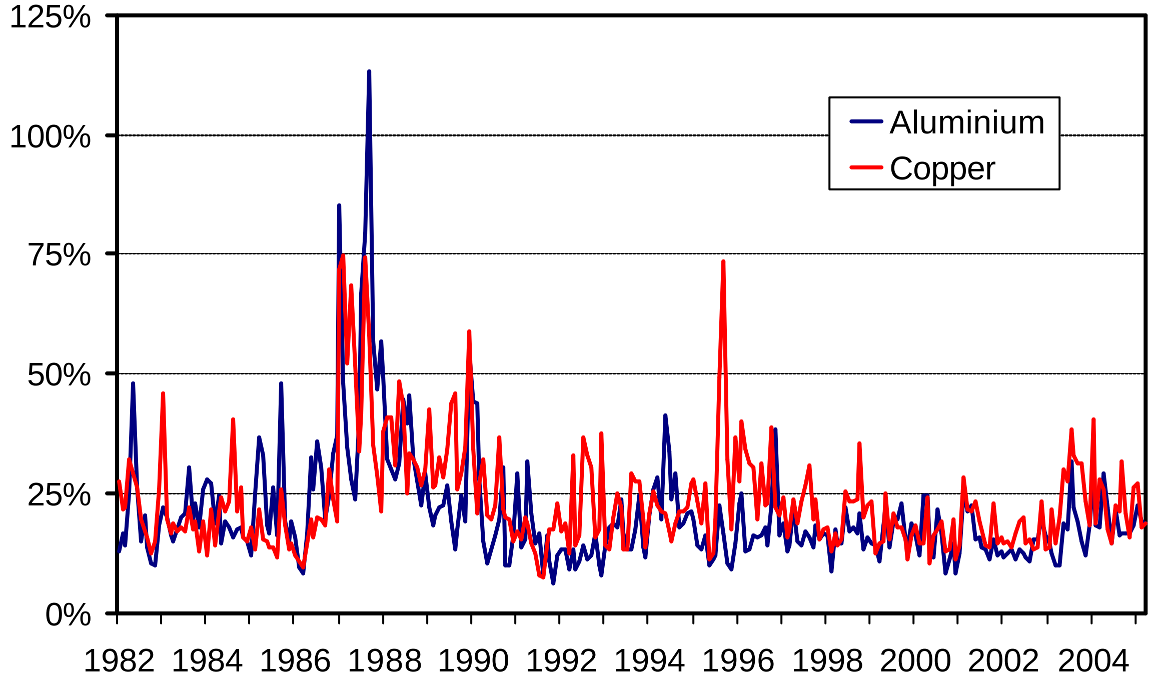

Figure A1: Monthly Spot Volatilities, Aluminium and Copper, 1982-2005

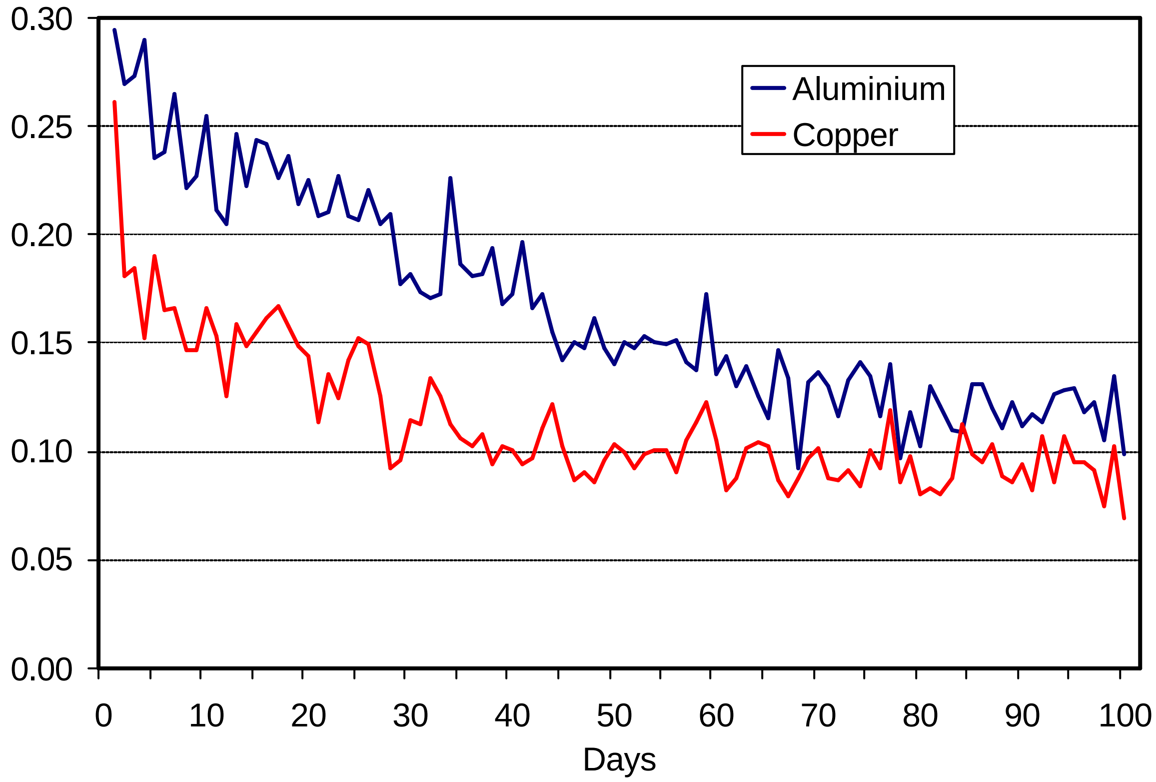

Figure A2: Autocorrelation Function: Aluminium and Copper Absolute Spot Returns

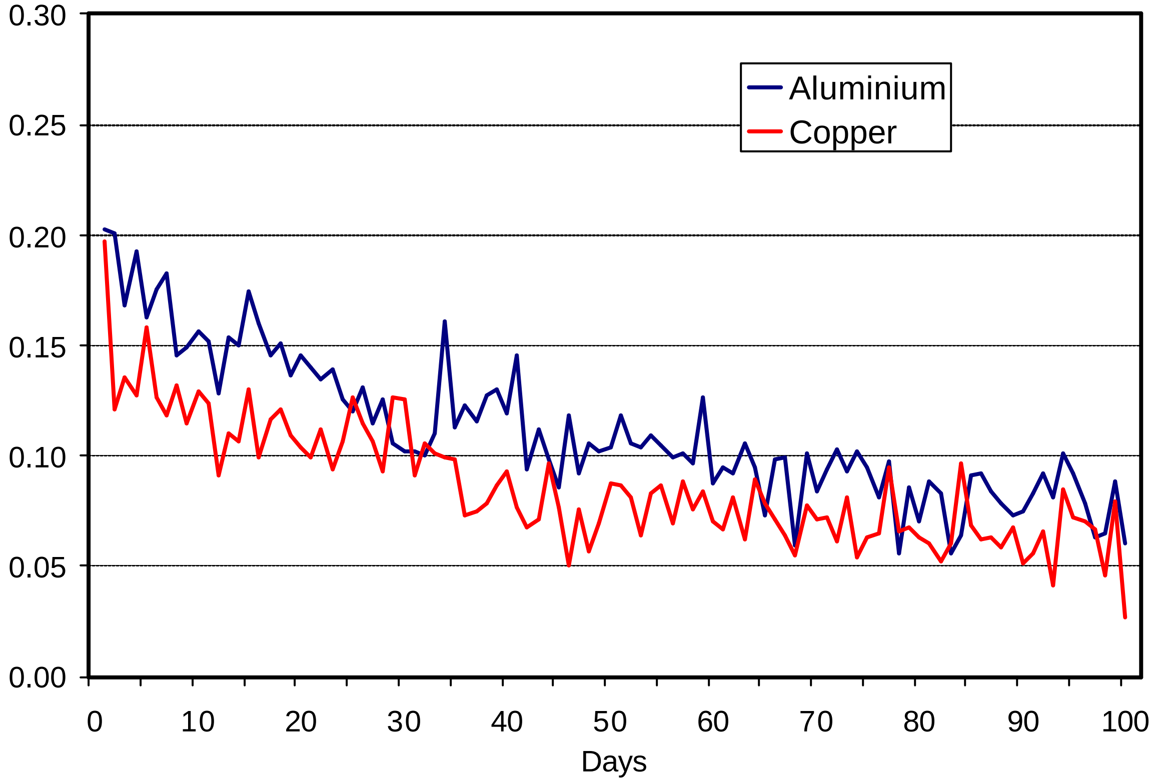

Figure A3: Autocorrelation Function: Aluminium and Copper Absolute 3 Month Returns

Table A1 ARFIMA (1, *d*, 1) Estimates				
	Spot Prices		**3 Month Prices**	
Aluminium	Squared	Absolute	Squared	Absolute
v_1	0.0002 (0.0004)	0.0100 (0.0049)	0.0002 (0.0001)	0.0089 (0.0020)
φ_{11}	0.2720 (0.0287)	0.2794 (0.0367)	0.4067 (0.0386)	0.3079 (0.0429)
β_{11}	-0.6115 (0.0353)	-0.5937 (0.0482)	-0.6308 (0.0441)	-0.5804 (0.0553)
d_1	0.4352 (0.0314)	0.4301 (0.0317)	0.3426 (0.0339)	0.3693 (0.0325)
Log-likelihood	33295.65	18680.46	37007.00	19785.91
Copper	Squared	Absolute	Squared	Absolute
v_2	0.0003 (0.0003)	0.0115 (0.0036)	0.0002 (0.0001)	0.0100 (0.0026)
φ_{22}	0.5928 (0.0327)	0.4753 (0.0335)	0.6079 (0.0357)	0.4470 (0.0319)
β_{22}	-0.8233 (0.0138)	-0.7246 (0.0342)	-0.8232 (0.0227)	-0.7231 (0.0342)
d_2	0.4365 (0.0383)	0.4079 (0.0368)	0.3769 (0.0426)	0.3993 (0.0382)
Log-likelihood	33889.16	18303.10	35713.89	19280.27
Differences	Squared	Absolute	Squared	Absolute
ϖ	0.0000 (0.0000)	-0.0014 (0.0018)	0.0000 (0.0000)	-0.0011 (0.0014)
φ	0.2567 (0.0893)	0.3832 (0.0698)	0.4622 (0.1217)	0.3805 (0.0609)
β	-0.3664 (0.0976)	-0.5663 (0.0858)	-0.5688 (0.1366)	-0.5706 (0.0746)
d	0.2329 (0.0252)	0.3041 (0.0362)	0.2480 (0.0380)	0.3018 (0.0351)
Log-likelihood	32315.30	17252.57	36100.14	18597.32

The table reports estimates of equation (1) with $p = q = 1$. "Differences" refer to the differences between the squared aluminium returns (or absolute returns) to the corresponding measure for copper.
Sample: 4 October 1982 to 30 December 2005 (5866 observations).
Standard errors in parentheses.

Table A2 Univariate FIGARCH (1, *d*, 1) Estimates						
	Spot Prices			**3 Month Prices**		
	H_2 Unrestricted	**H_1 Restricted**	**H_0 Restricted**	**H_2 Unrestricted**	**H_1 Restricted**	**H_0 Restricted**
		$\beta_{11} = \beta_{22}$ $\varphi_{11} = \varphi_{22}$	$\beta_{11} = \beta_{22}$ $\varphi_{11} = \varphi_{22}$ $d_1 = d_2$		$\beta_{11} = \beta_{22}$ $\varphi_{11} = \varphi_{22}$	$\beta_{11} = \beta_{22}$ $\varphi_{11} = \varphi_{22}$ $d_1 = d_2$
Aluminium						
μ_1	-0.0059 (0.0168)	-0.0063 (0.0168)	-0.0069 (0.0168)	-0.0052 (0.0156)	-0.0055 (0.0156)	-0.0057 (0.0156)
g_{11}	0.0129 (0.0172)	0.0125 (0.0174)	0.0126 (0.0174)	0.0106 (0.0175)	0.0104 (0.0175)	0.0105 (0.0176)
g_{12}	-0.0587 (0.0172)	-0.0580 (0.0169)	-0.0586 (0.0169)	-0.0526 (0.0171)	-0.0519 (0.0168)	-0.0520 (0.0168)
v_1	0.1245 (0.0516)	0.0975 (0.0245)	0.0967 (0.0233)	0.1634 (0.0683)	0.1045 (0.0272)	0.1033 (0.0260)
φ_{11}	0.1616 (0.1267)	0.2470 (0.0559)	0.2489 (0.0555)	0.2268 (0.1704)	0.3664 (0.0669)	0.3671 (0.0663)
β_{11}	0.3690 (0.1594)	0.4695 (0.0721)	0.4719 (0.0712)	0.3469 (0.1866)	0.5169 (0.0737)	0.5181 (0.0731)
d_1	0.3000 (0.0568)	0.3241 (0.0420)	0.3345 (0.0386)	0.2171 (0.0372)	0.2541 (0.0334)	0.2570 (0.0299)
Copper						
μ_2	0.0400 (0.0206)	0.0405 (0.0206)	0.0399 (0.0206)	0.0290 (0.0186)	0.0298 (0.0186)	0.0296 (0.0186)
γ_{21}	-0.0915 (0.0172)	-0.0909 (0.0171)	-0.0905 (0.0171)	-0.0827 (0.0174)	-0.0819 (0.0174)	-0.0819 (0.0174)
γ_{22}	-0.0344 (0.0168)	-0.0350 (0.0169)	-0.0352 (0.0168)	-0.0163 (0.0166)	-0.0170 (0.0168)	-0.0170 (0.0168)
v_2	0.1187 (0.0364)	0.1314 (0.0339)	0.1307 (0.0343)	0.0992 (0.0326)	0.1372 (0.0364)	0.1372 (0.0364)
φ_{22}	0.2833 (0.0656)	0.2470 (restricted)	0.2489 (restricted)	0.4180 (0.0709)	0.3664 (restricted)	0.3671 (restricted)
β_{22}	0.5091 (0.0820)	0.4695 (restricted)	0.4719 (restricted)	0.6006 (0.0738)	0.5169 (restricted)	0.5181 (restricted)
d_2	0.3493 (0.0549)	0.3411 (0.0405)	0.3345 (restricted)	0.2942 (0.0466)	0.2586 (0.0311)	0.2570 (restricted)
Log-likelihood H_1/H_2 , H_0/H_1 $\chi^2(2)$, $\chi^2(1)$ H_0/H_2 $\chi^2(3)$	-13235.51	-13235.96 0.89 [63.9%]	-13236.16 0.40 [52.5%] 1.30 [72.9%]	-12396.88	-12398.14 2.52 [28.4%]	-12398.16 0.04 [85.0%] 2.55 [43.6%]

The table reports estimates of equations (2) and (3) with $p = q = 1$ and $m = 2$. The log-likelihood is the sum of the log-likelihood for the two equations.
Sample: 4 October 1982 to 30 December 2005 (5866 observations).
Standard errors in "()" parentheses; tail probabilities in "[]" parentheses.

Table A3
Bivariate FIGARCH (1, *d*, 1) Estimates

	Spot Prices		3 Month Prices	
	H_1^* Unrestricted	H_0^* Diagonal	H_1^* Unrestricted	H_0^* Diagonal
Aluminium				
μ_1	-0.0143 (0.0144)	-0.0155 (0.0144)	-0.0088 (0.0133)	-0.0085 (0.0133)
g_{11}	-0.0236 (0.0131)	-0.0253 (0.0130)	-0.0425 (0.0130)	-0.0442 (0.0129)
g_{12}	-0.0449 (0.0125)	-0.0481 (0.0124)	-0.0324 (0.0121)	-0.0325 (0.0121)
v_1	0.0246 (0.0149)	0.0750 (0.0134)	0.0265 (0.0133)	0.0654 (0.0120)
Copper				
μ_2	0.0025 (0.0166)	0.0000 (0.0165)	0.0010 (0.0152)	0.0005 (0.0151)
γ_{21}	-0.0894 (0.0128)	-0.0912 (0.0127)	-0.0881 (0.0128)	-0.0905 (0.0127)
γ_{22}	-0.0450 (0.0124)	-0.0473 (0.0123)	-0.0256 (0.0121)	-0.0271 (0.0121)
v_2	0.0707 (0.0191)	0.1084 (0.0192)	0.0672 (0.0166)	0.0943 (0.0168)
Common				
$\varphi_{11} = \varphi_{22}$	0.3145 (0.0416)	0.3159 (0.0352)	0.4168 (0.0411)	0.4090 (0.0364)
$\varphi_{12} = \varphi_{21}$	-0.0057 (0.0079)		-0.0135 (0.0078)	
$\beta_{11} = \beta_{22}$	0.5243 (0.0493)	0.5578 (0.0426)	0.5987 (0.0444)	0.6164 (0.0397)
$\beta_{12} = \beta_{21}$	-0.0177 (0.0047)		-0.0165 (0.0049)	
$d_1 = d_2$	0.3510 (0.0267)	0.3814 (0.0274)	0.3070 (0.0241)	0.3306 (0.0244)
ρ	0.5370 (0.0091)	0.5317 (0.0091)	0.5666 (0.0087)	0.5638 (0.0087)
Log-likelihood	-19250.34	-19267.28	-17714.78	-17724.63
H_0^*/H_1^* $\chi^2(2)$		33.88 [0.00%]		19.70 [0.00%]

The table reports estimates of equations (2) and (7) with $p = q = 1$ and $m = 2$.
Sample: 4 October 1982 to 30 December 2005 (5866 observations).
Standard errors in "()" parentheses; tail probabilities in "[]" parentheses.

Table A4 ECM-FIGARCH (1, *d*, 1) Estimates				
	Spot Prices		**3 Month Prices**	
	H_1^{**} Unrestricted	**H_0^{**} Diagonal**	**H_1^{**} Unrestricted**	**H_0^{**} Diagonal**
Aluminium				
μ_1	-0.0140 (0.0145)	-0.0148 (0.0144)	-0.0088 (0.0133)	-0.0089 (0.0133)
g_{11}	-0.0233 (0.0131)	-0.0233 (0.0131)	-0.0431 (0.0130)	-0.0427 (0.0130)
g_{12}	-0.0446 (0.0125)	-0.0446 (0.0124)	-0.0324 (0.0121)	-0.0325 (0.0121)
v_1	0.0382 (0.0126)	0.0643 (0.0120)	0.0741 (0.0156)	0.0583 (0.0111)
Copper				
μ_2	0.0027 (0.0165)	0.0001 (0.0165)	-0.0008 (0.0151)	-0.0005 (0.0151)
γ_{21}	-0.0892 (0.0128)	-0.0911 (0.0128)	-0.0884 (0.0128)	-0.0904 (0.0127)
γ_{22}	-0.0444 (0.0124)	-0.0459 (0.0123)	-0.0261 (0.0122)	-0.0268 (0.0121)
v_2	0.0830 (0.0238)	0.1042 (0.0183)	0.0450 (0.0199)	0.0939 (0.0163)
Common				
$\varphi_{11} = \varphi_{22}$	0.3018 (0.0399)	0.3072 (0.0335)	0.4699 (0.0625)	0.3968 (0.0353)
$\varphi_{12} = \varphi_{21}$	-0.0094 (0.0089)		-0.0173 (0.0108)	
$\beta_{11} = \beta_{22}$	0.5269 (0.0460)	0.5408 (0.0414)	0.6150 (0.0536)	0.6017 (0.0395)
$\beta_{12} = \beta_{21}$	-0.0103 (0.0085)		-0.0333 (0.0125)	
$d_1 = d_2$	0.3748 (0.0357)	0.3893 (0.0258)	0.2594 (0.0360)	0.3383 (0.0238)
$\xi_1 = \xi_2$	0.0477 (0.0545)	0.0801 (0.0184)	0.0608 (0.0293)	0.0423 (0.0153)
ρ	0.5355 (0.0092)	0.5328 (0.0090)	0.5692 (0.0088)	0.5639 (0.0087)
Log-likelihood	-19249.97	-19252.29	-17713.54	-17719.02
$\xi_1 = \xi_2$ $\chi^2(1)$	0.09 [75.8%]	0.04 [85.0%]	0.17 [68.3%]	0.01 [91.0%]
H_1^{*}/H_1^{**} $\chi^2(1)$ H_0^{**}/H_1^{**} $\chi^2(2)$	0.37 [54.2%]	2.38 [30.4%]	1.24 [26.5%]	5.63 [5.98%]
The table reports estimates of equations (2) and (9) with $p = q = 1$, $m = 2$ and $b = 0.2$. Sample: 4 October 1982 to 30 December 2005 (5866 observations). Standard errors in "()" parentheses; tail probabilities in "[]" parentheses.				