IRREVERSIBILITY, UNCERTAINTY AND UNDEREMPLOYMENT EQUILIBRIA

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Abstract

In a competitive overlapping generation model, underutilization of labor and equipment can be due to the combination of irreversibility of human capital, physical capital and technology with idiosyncratic productivity shocks. Irreversibilities and uncertainty generate an inefficient allocation of resources among sectors, which takes the form of underemployment and underutilization of capacities at the aggregate level and affects the equilibrium path of capital. We provide examples in which this missallocation, called structural "mismatch," can be responsible, a.o., for an "inescapable poverty trap," or for periodic orbits generating endogenous fluctuations in underemployment.

Key Words
Underemployment, Underutilization, Irreversibility, Poverty trap, Endogenous Fluctuations

Introduction

The existence of underemployment of both capital and labor is an important stylized fact of actual economies. It is relevant for business cycle analysis as well as for growth, as shown by the Europe experience of a continuous growth over the last twenty years with a high degree of resources underutilization. In this paper we are interested in studying the growth aspect of underemployment, and in particular in pointing out the nature of underemployment at steady-state in an otherwise standard equilibrium model. The aim is not to discuss whether unemployment is voluntary or not but is focused on whether unemployment is efficient or not. Our model will be shown to add a theory of underemployment to the standard Walrasian growth model à la Diamond (1965).

Two possibilities could be explored to account for equilibrium underemployment. The first one relies on the aggregation of separate markets, each of which is frictionless, with the outcome in each separate market being either unemployed workers or unfilled vacancies. This is the approach proposed by Hansen (1970) and Tobin (1972) but which was not followed by others in a neo-classical context.[1] The other possibility is to assume that the process of matching workers and jobs in each market is not instantaneous as, e.g., in Blanchard and Diamond (1989) and Pissarides (1990). This process is formalized by a matching function, which represents a transaction technology which is not infinitely efficient. This technology generates search externalities because searching firms and workers cause congestion for each other during trade. Contrary to the Walrasian view of exchanges, the information about prices is not sufficient to clear the market at each point in time.

In this paper we go back to Hansen's original idea and we show that it may give rise to an interesting interpretation of underemployment relying on the presence of uncertainty and irreversibilities. Malinvaud (1980) has stressed the importance of demand uncertainty in the explanation of underutilized capacities at the firm level. The main assumption imposed by Malinvaud is the existence of technological irreversibilities, such as "time-to-build" and putty-clay technologies. Indeed, in our model, irreversibilities and uncertainty generate an inefficient allocation of resources among sectors. Aggregation over heterogenous markets with underemployment and underutilized capacities (unfilled vacancies), as in Hansen (1970), generates the coexistence of underutilized capital and labor at the macroeconomic equilibrium. This missallocation affects the equilibrium path and we provide examples in which it can be responsible for catastrophes like "inescapable poverty traps" and/or self-driven oscillatory phenomena.

The interpretation of underemployment in our framework is as follows: Underemployment of production factors results from irreversible skill decisions of the households and the irreversible investment decisions of the firms being taken without knowing with cer-

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[1] The "aggregation over micromarkets in desequilibrium" hypothesis proposed by Muellbauer (1978) follows the idea of Hansen. It was developped for empirical proposes by Lambert (1988) and Kooiman (1984).

tainty firms productivity. This implies that some agents have invested their physical and human capital in sectors that are hitten by a negative shock, generating underemployment of labor because of a lack of productive capacities. On the other hand, some other agents have invested in sectors that are hitten by a positive shock, being unable to fully utilize their capacities because of a lack of skilled workers in their sector.

In Section 1 we present the main characteristics of the model and we solve the agents' problems. The aggregates and the equilibrium conditions are derived in Section 2. In particular, Section 2.1 provides the conditions for the coexistence of unemployment and underutilization of capacities. Examples are presented in Section 2.3. Section 3 concludes.

1 The model

The main assumptions under which this model is built are the following.

First, it is a two period overlapping generation model, as in Diamond (1965), where individuals live two periods and markets are competitive. In each period we have two generations: youngs and olds. Let us assume that there is a continuum of youngs in the interval $[0, N_t]$, with N_t growing at the rate n. Each young has one unit of a specific labor endowment, works, consumes and lends savings to firms. Old people only consume. There is only one good in the economy, which can be consumed or accumulated as capital. To produce this good there exist different sectorial technologies, each of them depending on specific labor and capital inputs.[2]

Secondly, technological choices are irreversible (a putty-clay technology). As it is standard in OLG models, the capital stock is decided one period ahead. A given technology is always associated to this capital stock implying that the capital-labor ratio is chosen when the equipment is bought.

Third, the factors of production are firm specific, i.e., the labor markets are segmented and investment is irreversible and cannot be valuable elsewhere. There is a continuum of sectors in the interval $[0, 1]$, with a large number of firms and workers in each sector. Each sector is denoted by i and corresponds to a particular segment of the labor market. The number of workers in each sector is supposed to be equal to N_t.[3] We normalize the

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[2]This economy can be seen as a particular case of a more general economy where there is a continuum of goods, each of them being produced with specific capital and labor inputs. In this particular version all goods are perfect substitutes. Alternatively, we can see this economy as one in which firms are geographically located and sectors represent a particular location; goods are allowed to move costlessly among places, while inputs are not.

[3]We can see this economy as if individuals live three periods. In each period we have three generations: children, youngs and olds. Each child does not consume at all (its consumption is implicitly in the utility function of its parents) and chooses costlessly a specific human capital. At the time of the kids' decision, expected labor incomes are the same for all types of human capital, implying that kids are distributed uniformly over the different sectors of the labor market at equilibrium.

number of firms to N_t,[4] allowing us to work with per- capita variables.

Finally, at the time of the decisions on capital and the related technology, there is some idiosyncratic uncertainty concerning the average productivity of capital. Moreover, after their realization, shocks become public information. Consumption, savings, wages, employment and production take place simultaneously under full-information. Since all uncertainty is firm specific (there is no aggregate uncertainty), the bond market portfolio pays the riskless rate of return. This timing, even if it is relatively standard, is relevant in generating underemployment of production factors.

1.1 The consumer problem

All individuals have identical preferences over consumption when young c_{i1t}, consumption when old c_{i2t+1} and employment l_{it}. These preferences are assumed to be additively separable in consumption and employment. The utility of consumption is represented by a function $U(c_{i1t}, c_{i2t+1})$, which is supposed to be homogeneous of degree one and increasing in its arguments, differentiable and concave in the positive orthant. The disutility of labor is given by the function $v(l_{it})$, which is increasing, differentiable and convex. The representative individual of generation t with specific labor endowment i solves the following problem:

$$\max_{c_{i1t}, c_{i2t+1}, l_{it}} U(c_{i1t}, c_{i2t+1}) - v(l_{it})$$

subject to the intertemporal budget constraint

$$c_{i1t} + \frac{c_{i2t+1}}{1 + r_{t+1}} = w_{it} l_{it} \quad \text{and} \quad l_{it} \leq 1.$$

The real wage w_{it} paid in sector i and the real interest rate r_{t+1} are taken as given by the household.

The corresponding labor supply function, $l_{it}^s \equiv l^s(w_{it}, r_{t+1})$, must verify

$$w_{it} = \xi(r_{t+1}) \, v'(l_{it}^s), \tag{1}$$

where

$$\xi(r) \equiv \frac{1}{U_1 \left(1 - \theta(r), \theta(r)(1 + r)\right)}$$

represents the inverse of U_1, the marginal utility of consumption when young. Equation (1) is standard and states that the real wage must be equal to the marginal disutility of labor.

The corresponding saving function is:

$$s_{it} = \theta(r_{t+1}) \, w_{it} \, l^s(w_{it}, r_{t+1}), \tag{2}$$

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[4]Because the production function has constant returns to scale, the number of firms is undetermined and irrelevant.

where s_{it} represents savings. Given that the utility function is supposed to be linear-homogeneous, individual savings are a proportion of individual labor income, i.e., the function θ represents the "propensity to save" and depends only on the interest rate. Since labor is firm specific, individuals from the same generation could have a different labor endowment and therefore a different labor income. However, they have the same propensity to save, implying that aggregate savings are a proportion θ of aggregate labor income.

1.2   The putty-clay technology

As it is standard in OLG models, the capital employed in production at time t is bought at time $t-1$. The technology is putty-clay and the choice of time t productivities is also done at time $t-1$ (the bought capital incorporates a given factors' productivity). Moreover, each sector has its own technology and employs specific inputs. Let y_i denote the ith sector production and k_i and l_i the ith specific capital and labor inputs respectively. The technology is then different from one segment of the labor market to another, even if for reason of symmetry the production function is assumed to take the same functional form for all sectors. Assuming constant-return-to-scale, average productivities depend on the capital-labor ratio. The average productivities of labor and capital are given by $f(x_{it})$ and $\frac{f(x_{it})}{x_{it}}\mu_{it}$ respectively, where x_{it} is the ex-ante capital-labor ratio and μ_{it} is a stochastic shock, which is defined below. Due to the putty-clay assumption, the capital stock k_{it} and the capital-labor ratio x_{it} are decided at time $t-1$, and production y_{it} and the labor input l_{it} are chosen at time t.

The average productivity of capital is supposed to be affected by some multiplicative stochastic shock, denoted by $\mu_{it} \in R_+$.[5] We assume that μ_{it} is drawn from the distribution $F(\mu_{it})$, the same for all i and t, such that $E(\mu_{it}) = 1$. There is "heterogeneity" in this economy and it is related to the realizations of the idiosyncratic shocks μ_{it}. The distribution over sectors of the realized μ_{it} shocks follows the same distribution $F(\mu_{it})$.

The ex-post production function is Leontief with given productivities for capital and labor. Since capital was chosen in the previous period and the stochastic shock μ_{it} has unit mean, we can define

$$y_{it}^p \equiv \frac{f(x_{it})}{x_{it}} k_{it},$$

where y_{it}^p represents expected productive capacities.

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[5] We assume for simplicity that the productivity of labor is non stochastic.

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1.3 The sectorial labor market equilibrium

In each segment of the labor market wages and employment are determined competitively. Since k_{it} and x_{it} were decided in period $t-1$, labor demand is infinitely elastic until full-capacity is reached and then it becomes infinitely inelastic, as it is shown in Figure 1. Firms are optimally engaged in production if the real wage is at most equal to the average productivity of labor, in which case the ex-post optimal choice of production and labor implies

$$y_{it} = f(x_{it})l_{it} \leq y_{it}^p \, \mu_{it} \, ,$$

when $w_{it} \leq f(x_{it})$. Let us define $l_{it}^p \equiv \frac{k_{it}}{x_{it}}\mu_{it}$ as the employment level needed to produce at full-capacity. Let us call $l_{it}^f = l^s(f(x_{it}), r_{t+1})$ the number of hours optimally supplied by each worker when the equilibrium wage is equal to the average productivity of labor. Since firms would not hire workers if the real wage was greater than the average productivity, l_{it}^f defines an upper bound on employment, which we call the "efficient-employment" level. Without uncertainty the firm optimally chooses k_{it}, such that $l_{it}^p = l_{it}^f$ and the economy is at the efficient-employment equilibrium. Because capacities decided at $t-1$ are hitten by a stochastic shock, they are not always sufficiently high to allow for "efficient-employment". In this case, the equilibrium wage is smaller than the average productivity of labor and workers decide optimally to work less than "efficient-employment". More precisely they will work if $l_{it}^p \mu_{it} < l_{it}^f$.

Let us call $\bar{\mu}_{it}$ the value of μ_{it} such that capacities are just enough to employ the maximum number of hours:

$$\frac{k_{it}}{x_{it}}\bar{\mu}_{it} = l_{it}^f.$$

Since all firms in sector i are identical, they set the same k_{it} and x_{it}. The definition of $\bar{\mu}$ allows us to express the outcome of the competitive equilibrium as a function of the idiosyncratic shock:

$$w_{it}(\mu_{it}) = \begin{cases} \xi(r_{t+1}) \, v'\left(\frac{k_{it}}{x_{it}}\mu_{it}\right) & \text{if} \quad \mu_{it} \leq \bar{\mu}_{it} \\[2mm] f(x_{it}) & \text{otherwise} \end{cases} \tag{3}$$

$$l_{it}(\mu_{it}) = \begin{cases} \frac{k_{it}}{x_{it}}\mu_{it} & \text{if} \quad \mu_{it} \leq \bar{\mu}_{it} \\[2mm] l^s(f(x_{it}), r_{t+1}) & \text{otherwise.} \end{cases} \tag{4}$$

In equation (4) we assume that the rationing scheme is uniform, i.e., total labor supply is allocated uniformely among firms. Notice that, even if rationed firms were interested in increasing labor supply by paying a wage greater than the equilibrium wage, this policy would not be optimal since, with a marginal productivity, such a wage would yield negative profits.

Figure 1a represents the first of these two cases and Figure 1b the second. The realized productivity shock μ_{it} could be: (a) "bad" (i.e. $\mu_{it} \leq \bar{\mu}_{it}$), in which case capacities are so

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Figure 1: The labor market equilibrium

Figure 1a   Figure 1b
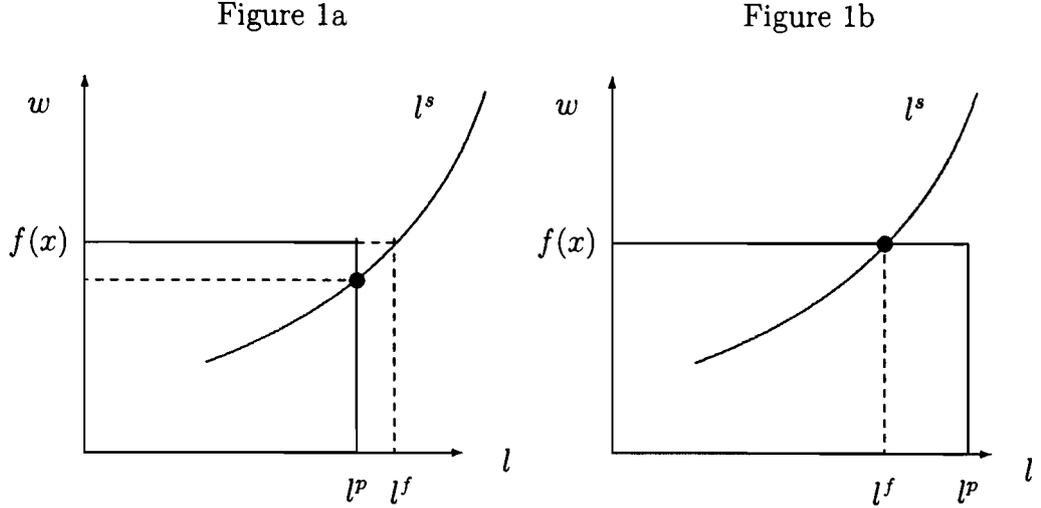
small that the firm produces at full-capacity, and the equilibrium wage is smaller that the average productivity of labor; or (b) "good" (i.e. $\mu_{it} \geq \bar{\mu}_{it}$), in which case the equilibrium wage is given by the average productivity of labor and capacities are underemployed. We label "underemployment equilibrium" the first case (Figure 1a) and "underutilization (of capital) equilibrium" the second case (Figure 1b).

1.4 Firm's capital and technological choices

As stated before, the capital stock and the capital-labor ratio for period t are chosen at $t-1$. Since there is uncertainty about the productivity of capital, the firm h of sector i chooses x_{it}^h and k_{it}^h in order to maximize expected profits, i.e.,

$$\max_{k_{it}^h, x_{it}^h} E_{t-1} \left[(f(x_{it}^h) - w_{it})l_{it}^h - (\delta + r_t)k_{it}^h \right]$$

where

$$l_{it}^h = \min\left[\frac{k_{it}^h}{x_{it}^h} \mu_{it}, \, l_{it}^f \right].$$

The parameter $0 \leq \delta \leq 1$ represents the depreciation rate. Wages are taken as given by the firm and, from (3), are a function of the shock. Expected profits can be written as:

$$\frac{k_{it}^h}{x_{it}^h} \int_0^{\bar{\mu}_{it}^h} (f(x_{it}^h) - w_{it}(\mu_{it})) \, \mu_{it} \, dF(\mu_{it}) + l_{it}^f \int_{\bar{\mu}_{it}^h}^{\infty} (f(x_{it}^h) - w_{it}(\mu_{it})) \, dF(\mu_{it}) - (\delta + r_t)k_{it}^h$$

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where

$$\bar{\mu}_{it}^h = \frac{x_{it}^h \, l_{it}^f}{k_{it}^h}$$

The first order necessary conditions for x_{it}^h and k_{it}^h are respectively:

$$x_{it}^h \, f'(x_{it}^h) \, \mathrm{E}_{t-1}(l_{it}^h) = \frac{k_{it}^h}{x_{it}^h} \int_0^{\bar{\mu}_{it}^h} (f(x_{it}^h) - w_{it}(\mu_{it})) \, \mu_{it} \, \mathrm{dF}(\mu_{it}) \qquad (5)$$

$$\delta + r_t = \frac{1}{x_{it}^h} \int_0^{\bar{\mu}_{it}^h} (f(x_{it}^h) - w_{it}(\mu_{it})) \, \mu_{it} \, \mathrm{dF}(\mu_{it}). \qquad (6)$$

To interpret these conditions let us call d_{it}^h at the ratio of expected production to expected capacities:

$$d_{it}^h = \frac{x_{it}^h \, \mathrm{E}_{t-1}(l_{it}^h)}{k_{it}^h}.$$

Combining conditions (5) and (6) with the definition of d, one has

$$\delta + r_t = f'(x_{it}^h) \, d_{it}^h. \qquad (7)$$

It states that the user cost of capital must be equal to the expected marginal benefit of increasing capital at the optimum. The marginal benefit is equal to the marginal productivity of capital times the ratio of expected production to expected capacities, i.e., marginal productivity is weighted by the probability that the new equipment be effectively utilized.

Since the economy is perfectly competitive and returns to scale are constant, from the optimality conditions we can easily show that expected profits are zero.

2   The aggregate equilibrium

Since all segments are ex-ante identical and the expected value of the shock is one, heterogeneity in the economy can be seen as deviations from the capital average productivity. But this heterogeneity exists only ex-post, after the realization of the idiosynchratic shocks. Since there is no heterogeneity ex-ante, all firms choose the same capital stock and capital-labor ratio. By symmetry, the optimality conditions for x and k are the same for all firms in all sectors and they are given by (5) and (7), i.e.,

$$x_t f'(x_t) \, l_t = \frac{k_t}{x_t} \int_0^{\bar{\mu}_t} \left[f(x_t) - \xi(r_{t+1}) v'\left(\frac{k_t}{x_t}\right) \mu \right] \mu \, \mathrm{dF}(\mu) \qquad (8)$$

$$\delta + r_t = f'(x_t) \frac{x_t \, l_t}{k_t}, \qquad (9)$$

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where

$$\bar{\mu}_t = \frac{l^s(f(x_t), r_{t+1}) \, x_t}{k_t}. \tag{10}$$

Wages have been replaced by their value from (3) and x_t, k_t and l_t represent optimal capital-labor ratio, capital stock and expected employment respectively. Per-capita aggregate employment, which is equal to expected employment, results from the aggregation over μ of individual employment and it can be written as

$$l_t = \frac{k_t}{x_t} \int_0^{\bar{\mu}_t} \mu \, dF(\mu) + l^s(f(x_t), r_{t+1}) \int_{\bar{\mu}_t}^{\infty} dF(\mu). \tag{11}$$

From (3) and (4) aggregate labor income is given by

$$w_t l_t = \xi(r_{t+1}) \frac{k_t}{x_t} \int_0^{\bar{\mu}_t} v'\left(\frac{k_t}{x_t}\mu\right) \mu \, dF(\mu) + f(x_t) \, l^s(f(x_t), r_{t+1}) \int_{\bar{\mu}_t}^{\infty} dF(\mu).$$

Combined with the optimal condition for x (equation (8)), the aggregate labor income becomes:

$$w_t l_t = [f(x_t) - x_t f'(x_t)] \, l_t \tag{12}$$

where l_t is given by (11). Note that the wage index is equal to the ex-ante marginal productivity of labor.

From the optimality conditions for capital and the capital-labor ratio, we know that aggregate pure profits are zero, even if some firms have negative profits and other firms have positive profits. A costless insurance contract taken by the olds is supposed to share the aggregate zero pure profits and to avoid that some firms be unable to repay their debts.

Finally, the equilibrium requires the clearing condition between savings and the capital stock. which from (2) and (12) is

$$\theta(r_{t+1}) \left[f(x_t) - x_t f'(x_t)\right] l_t = k_{t+1}(1 + n), \tag{13}$$

The equilibrium path is described by (8), (9), (10), (11) and (13) with given initial values x_0 and k_0, which verify equation (8). The dynamic behavior of the economy is characterized by the non-linear first-order difference equation (13), after substitution of all other equations. As it is standard in OLG models, different types of equilibria are possible. Some characteristics of the general model are presented in sections 2.1 and 2.2, and a deeper analysis of the macroeconomic effects of uncertainty and heterogeneity is provided in section 2.3 by the mean of examples in which equilibria and their qualitative properties depend on the variance of the idiosyncratic shock.

2.1 Underemployment and capacity utilization

Proposition 1. *Provided that the probability of being in the "bad state" is strictly positive, there is underemployment at equilibrium.*

Proof: if $F(\bar{\mu}) > 0$, then

$$\frac{l_t}{l_t^f} = \int_0^{\bar{\mu}} \frac{\mu}{\bar{\mu}} \, dF(\mu) + \int_{\bar{\mu}}^{\infty} dF(\mu) \quad < \quad \int_0^{\infty} dF(\mu) = 1$$

since $\int_0^{\bar{\mu}} \frac{\mu}{\bar{\mu}} \, dF(\mu) < \int_0^{\bar{\mu}} dF(\mu)$.$\square$

At equilibrium, if a positive measure of firms are in an "underemployment equilibrium", there is underemployment of labor in the aggregate.

Proposition 2. *Provided that the probability of being in the "good state" is strictly positive, there is underutilization of capital at equilibrium.*

Proof: if $1 - F(\bar{\mu}) > 0$ then

$$d_t = \frac{l_t}{l_t^p} = \int_0^{\bar{\mu}} \mu \, dF(\mu) + \int_{\bar{\mu}}^{\infty} \bar{\mu} \, dF(\mu) \quad < \quad \int_0^{\infty} \mu \, dF(\mu) = 1$$

since $\int_{\bar{\mu}}^{\infty} \bar{\mu} \, dF(\mu) < \int_{\bar{\mu}}^{\infty} \mu \, dF(\mu)$.$\square$

At equilibrium, if a positive measure of firms are in an "underutilization equilibrium", there is underutilization of capacities in the aggregate.

The coexistence of underemployment of labor and underutilization needs only that the probability of being in both the "bad" and the "good" states be strictly positive. For any non-degenerate continuous distribution function F defined in $]0, \infty[$, this property is verified if $0 < \bar{\mu} < \infty$.

Underemployment of production factors results from the fact that the irreversible skill decisions of the households and the irreversible investment decisions of the firms are taken without knowing with certainty firms productivity. This implies that some agents have invested their physical and human capital in sectors that are hitten by a negative shock, generating underemployment of labor because there is a lack of productive capacities. On the other hand, some other agents have invested in sectors that are hitten by a positive shock, being unable to fully utilize their capacities because ther e is a lack of skilled workers in their sector. In this economy, uncertainty and irreversibilities preclude an efficient allocation of capital and labor across sectors. This type of inefficiency is known as "structural mismatch" (see Sneessens (1987)).

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2.2 The capital-labor ratio

We have in this framework three different definitions for the capital-labor ratio: x_t represents the optimal capital-labor ratio, which is incorporated in the machines; k_t represents the capital stock per-capita; k_t/l_t represents the effective capital-labor ratio. From the definition of d_t we know that

$$x_t = d_t \left(\frac{k_t}{l_t} \right).$$

The effective capital-labor ratio is larger than the optimal one because some units of capital are not employed at equilibrium. Moreover, from the definitions of d_t and l_t, we know that

$$\left[\frac{k_t}{l_t} \right]^{-1} = [x_t]^{-1} \int_0^{\bar{\mu}} \mu \, dF(\mu) + [k_t]^{-1} \int_{\bar{\mu}}^{\infty} l_t^s \, dF(\mu).$$

This means that the effective capital-labor ratio (the one which is observed at the macroeconomic level) is a weighted average of (i) the ex-ante capital-labor ratio which is the effective ratio prevailing in the firms with a bad productivity shock and (ii) the capital stock per capital which is the capital-labor ratio prevailing in firms with a good productivity shock.

2.3 Examples

To illustrate the effect of uncertainty on the equilibrium path, we impose the following particular assumptions. The utility function $U(c_1, c_2)$ is Cobb-Douglas, which implies that the propensity to save is constant $\theta(r) = \theta$. The disutility of labor is zero, i.e., $v(l_t) = 0$; implying that $l_t^f = 1, \forall t$. The production function is CES,

$$f(x_t) = A(\alpha x_t^{-\gamma} + 1 - \alpha)^{-\frac{1}{\gamma}}$$

where $A > 0$, $\gamma \geq -1$ and $0 < \alpha < 1$. The idiosyncratic shock is lognormal distributed, i.e.,

$$\mu_{it} = \exp\left\{ -\frac{\sigma^2}{2} + \epsilon \right\}$$

where $\epsilon \sim N(0, \sigma^2)$. Following Lambert (1988) equation (11) can be approximated by

$$l_t \approx \left(\left(\frac{x_t}{k_t}\right)^\rho + 1 \right)^{-\frac{1}{\rho}} \quad \text{where} \quad \rho = -1 + \frac{2}{\sigma} \frac{\phi(-2/\sigma)}{\Phi(-2/\sigma)},$$

ϕ represents the standard normal density and Φ the standard cumulative normal distribution. Notice that $\rho > 0$ and it is an inverse function of σ. $1/\rho$ measures the structural mismatch. Additionally, with the same approximation it can be shown that

$$\int_0^{\bar{\mu}} \mu \, dF(\mu) \approx \left(\frac{x_t l_t}{k_t} \right)^{\rho+1}.$$

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Figure 2: The saddle-node bifurcation
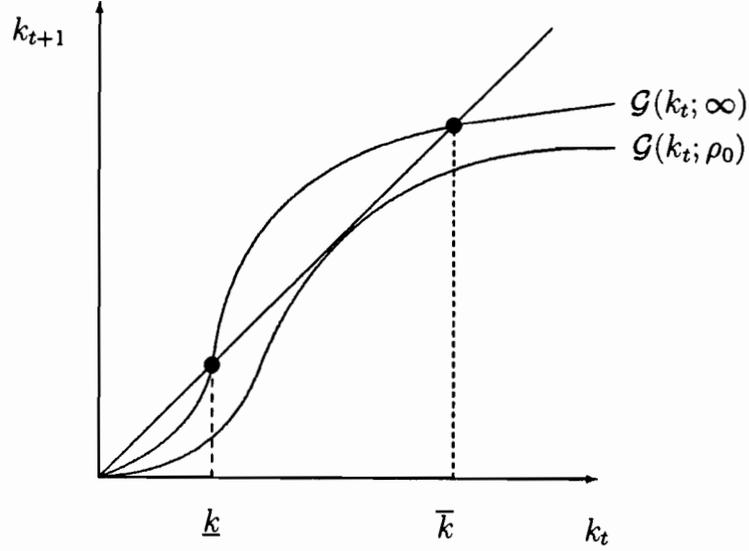
Under these assumptions, from (8) and (11), there is a positive and simple relation between k_t and x_t,

$$x_t = \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\gamma+\rho}} k_t^{\frac{\rho}{\gamma+\rho}}. \tag{14}$$

From equations (8), (11) and (13) the equilibrium condition can be written as a first order difference equation for capital k_t

$$k_{t+1} = \mathcal{G}(k_t; \rho) \equiv \left(\frac{A\,\theta}{1+n}(1-\alpha)\right) \left[\left(\frac{\alpha}{1-\alpha}\right)^{\frac{\rho}{\gamma+\rho}} k_t^{\frac{-\gamma\rho}{\gamma+\rho}} + 1\right]^{-\frac{\gamma+\rho+\gamma\rho}{\gamma\rho}} \tag{15}$$

with given initial value k_0. Under similar assumptions on preferences and technology, the transition function $\mathcal{G}(k_t; \infty)$ corresponds to the standard Diamond economy, that we call, in the sequel, the benchmark case.

Notice first that, in the Cobb-Douglas case ($\gamma = 0$), (14) is linear and (15) ensures that there exists a unique and stable positive steady state value for k, which is monotonically increasing in ρ. The rise in microeconomic uncertainty increases the misallocation of production factors across sectors and the underemployment rates, and decreases the steady state capital stock.

Let us now consider two cases in which the presence of irreversibilty and uncertainty, reflected here in the mismatch parameter ρ, changes the qualitative nature of the equilibrium.

2.3.1 An inescapable poverty trap

When $\gamma > 0$, a positive steady state is almost never unique in the benchmark case.[6] The saving locus is shown in Figure 2. When there exist two positive steady state equilibria $\underline{k}$ and $\overline{k}$, the equilibria $\overline{k}$ and zero are asymptotically stable and $\underline{k}$ can be interpreted as a "poverty trap": If the initial value k_0 is lower than $\underline{k}$ the economy converges to the zero steady state; If k_0 is larger than $\underline{k}$ the economy converges to the high equilibrium $\overline{k}$. When uncertainty increases, implying a decrease in ρ, $\mathcal{G}(k; \rho)$ moves down in the $\{k_t, k_{t+1}\}$ space and both positive equilibria move nearer. There is a "saddle-node bifurcation" at $\rho = \rho_0$, where both positive equilibria become equal.[7] When $\rho < \rho_0$ there is no positive steady-state equilibrium.

In this example, the rise in structural mismatch generates a reduction in revenues and savings, moving down the transition function. The highest steady state value of capital decreases and the poverty trap increases until the bifurcation point is reached, after what the poverty trap becomes inescapable.

2.3.2 A two-period endogenous cycle

When $\gamma = -1$, the production factors are perfectly substitute and $\mathcal{G}(k; \infty)$ is horizontal. When uncertainty appears, $\mathcal{G}(k; \rho)$ becomes negatively sloped and the steady state capital stock decreases. The (local) dynamics of capital is characterised by damped oscillations. There is a "flip-bifurcation" at $\rho = \rho_1$, where the slope of $\mathcal{G}(k; \rho_1)$ is equal to -1. When ρ becomes smaller than ρ_1 the positive steady-state is still unique but becomes unstable. As the steady-state equilibrium looses stability, a stable two-cycle appears. With this self-driven oscillatory phenomenon (see Boldrin and Woodford (1990) for a survey of such fluctuations in equilibrium models), the economy moves from (a) a period in which olds are poor, the technology is labor intensive, unemployment of youngs is low and savings are high, to (b) a period where olds are rich, the technology is capital intensive, unemployment of youngs is high and savings are low. Generations move from rich to poor in the two-period cycle. The amplitude of this cycle increases when ρ diminishes.[8]

In this example, the rise in structural mismatch moves down the transition function, as before, but affects also its slope around the steady-state, making the economy less and less quick to converge. At the flip-bifurcation point, the steady-state is no longer stable and a two-period endogenous cycle appears. This shows that, even in cases where the benchmark model is very poor, the introduction of irreversibility and uncertainty may give rise to interesting dynamics. As the models of Pissarides (1990) and Diamond and Fudenberg (1989), our framework is able to generate unemployment cycles.

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[6]It is a standard example in the OLG literature. See Azariadis (1993).

[7]The bifurcation point must verify $\mathcal{G}(k; \rho_0) = k$ and $\mathcal{G}'(k; \rho_0) = 1$.

[8]Other changes in the qualitative behavior of the dynamic system appear for very small values of ρ. $(\rho \leq 1)$

Figure 3: The flip bifurcation
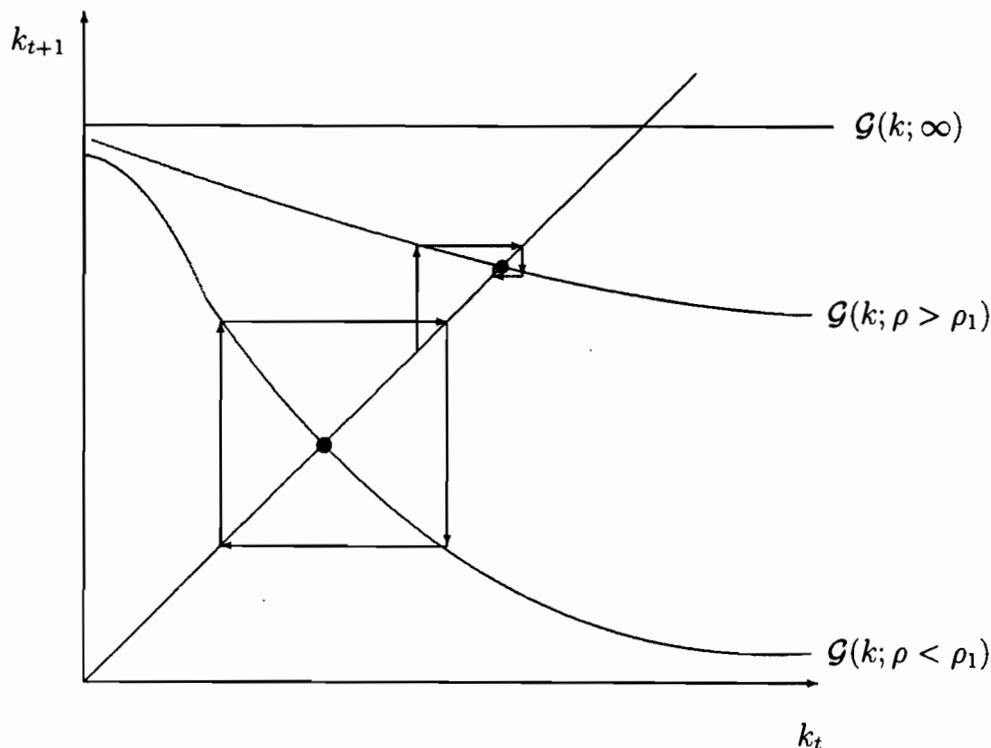
3 Conclusions

In an OLG economy, we show that technological irreversibilities and segmented labor markets, combined with idiosyncratic uncertainty, generate unemployment and underutilization of capacities. Because it takes one generation to reallocate resources among sectors, idiosyncratic shocks produce missallocation and inefficiency, which takes the form of underemployment of production factors. This framework provides an alternative approach to the search model of Pissarides (1990) to analyse structural unemployment problems. As Cooper and John (1988) pointed out, inefficiency in the search model can be interpreted as a coordination problem due to trading externalities in uncertain conditions. Inefficiency in our framework could also be interpreted under the light of the coordination failure literature. Irreversibilities engender strategic complementarities between the skill decision of the households and firms' decision in the presence of idiosyncratic uncertainty. Note also that, contrary to Pissarides' framework in which wages are fixed by a bargaining between firms and workers, our labor market is Walrasian, and wages differ across sectors.

The interest of our approach is to link heterogeneity with the presence of uncertainty

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and irreversibility, and to make possible the analysis of the effect of idiosynchratic uncertainty on growth. To illustrate this, we provide two examples in which the variance of the idiosyncratic shock, which also measures heterogeneity and mismatch in the economy, plays a crucial role. In the first example, the mismatch parameter presents a "saddle-node bifurcation" and an increase in uncertainty may put the economy in an "inescapable poverty trap." In the second example, the mismatch parameter presents a "flip bifurcation" and an increase in uncertainty may generate endogenous cycles.

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