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Essays on the Economic Theory of Managerial Incentives

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Overview

Corporations are very common in the business world. In this kind of organizations shareholders are protected by limited liability and, furthermore, they can easily transfer their shares. As a consequence, investors might be interested in buying a corporation's shares just to diversify their portfolios, without any real interest in getting involved in management. It is therefore much easier for corporations to obtain external finance than other organizational forms, and this might well be the basic reason for their wide diffusion. For the very same reason, however, it is necessary to hire professional managers to make all the relevant decisions, and this contains the seed of their problematic governance. In fact, the separation of ownership and control produces a conflict of interest between shareholders, interested in maximizing the firm value, and managers, who can be interested in pursuing a variety of different objectives (empire building, entrenchment, shirking, etc.).

This dissertation is composed by three research papers dealing with the economics of managerial incentive provision. It is common to interpret the relationship between shareholders and managers as an agency relationship affected by both a moral hazard and adverse selection problem. Usually, managerial incentives are affected by several elements such as, for example, their compensation packages and career concerns, the internal monitoring of the board of directors, the external monitoring of the market for corporate control, etc. This dissertation suggests that it might be necessary to consider the interactions between alternative incentive mechanisms both to better understand their functioning and, at least as importantly, to help interpreting empirical observations.

The first chapter, *Paying for Observable Luck*, proposes a simple hidden action model which explains recent empirical evidence of asymmetric benchmarking in managerial compensation: managers appear to be insulated from bad luck but not from good luck. The explanation hinges on the interaction between explicit contractual incentives and implicit incentives deriving from the possibility of bankruptcy. The second chapter, *Career Concerns and Competitive Pressure*, studies how the level of competition in the product market affects the strength of managerial career concerns. Good managers are in short supply so that firms are willing to compete for them. However, the value of good managers depends on the profit differential they are able to produce on the product market. It is then shown that increased competition makes career concerns stronger if it increases such profit differential. The third chapter, *Managerial Entrenchment and the Market for CEOs*, suggests that the observed trends of increased managerial pay and increased board independence might be related. Boards captured by an entrenched managers are not active on the demand side of the managerial labor market. Therefore, increased board independence, reducing the number of captured boards, also increases competition for good managers, then rising their pay and making their career concerns stronger.

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Contents

Chapter 1

Paying for Observable Luck

In this chapter I present a simple hidden action model in which the agent has explicit contractual incentives but also implicit incentives created by the possibility of bankruptcy. An observable exogenous shock affects the agent's performance and determines the probability of liquidation. Furthermore, after signing the contract, but before choosing his action, the agent observes a private signal on the future shock. The observation of a bad signal strengthens the agent's implicit incentive and reduces the conflict of interest with the principal. If the agent had no private information, the principal could completely filter out the observable luck. However, when the agent has private information, the contract optimally adjusts explicit to implicit incentives. As a result, observable luck is not completely removed from the agent compensation schedule. The model explains recent empirical evidence of asymmetric benchmarking in managerial compensation: managers appear to be insulated from bad luck but not from good luck. The result obtains in a model that shares most of the assumptions typically made in the empirical literature. In particular, asymmetric benchmarking arises even though the managerial productivity and the exogenous shock are independent.

Keywords: Pay for Luck, Asymmetric Benchmarking, Relative Performance Evaluation.

JEL classification: D82, D86, M52.

1.1. Introduction

The relationship between shareholders of a modern public corporation and their CEOs has long been studied within the framework of agency theory. This approach stresses the trade off between insurance and incentive provision in the design of optimal contracts. Shareholders are typically well diversified and then better suited than their risk averse managers to bear the uncertainty affecting firm performance. As a consequence managerial exposure to risk finds its only rationale in the provision of incentives. Therefore, any noise that could be removed from CEOs' compensation packages, without affecting underlying incentives, should indeed be filtered out. This intuitive idea is a direct consequence of the informativeness principle (Holmstrom, 1979) and it is usually believed to imply that managerial compensation should be insulated from events that are beyond their control such as, for example, macroeconomic fluctuations. To do this one should evaluate firm performance relative to some appropriate benchmark, that reflects stochastic elements which cannot be affected by managerial activity. For example, the widely recommended use of Relative Performance Evaluation (RPE) hinges on the idea that performances within a group of peers (e.g. CEOs within the same industry) are affected by common shocks. Hence, it is argued, compensation should be increasing in own performance but decreasing in others'. Other typical recommended benchmarks include stock price indexes, input and output prices.

In spite of its intuitive appeal, this apparently straightforward implication of agency theory has found very limited support in the data.[1] In contrast, the recent works by Bannister and Newman (2003) and Garvey and Milbourn (2006) suggest the existence of a form of asymmetric benchmarking: managers appear to be insulated from bad luck but not from good luck.[2] For example, in stock option plans the strike price typically coincides with the market price at the time of award and it is not linked to general stock price indexes. In this way managers can appropriate windfalls generated by a bull market,

[1] See for example the surveys by Rosen (1992), Murphy (1999) or Prendergast (1999).

[2] When the adopted benchmark is relative performance, this phenomenon is also referred to as one sided RPE.

as it seems to have happened during the 1990s. However, when the stock price falls below the exercise price, it is often renegotiated down.[3]

The typical principal-agent model that supports the idea that managerial compensation should be decreasing in an appropriate benchmark is indeed very simple. Usually, it involves a standard hidden action model in which the firm performance is represented as the sum of three independent components: managerial productivity, an aggregate shock and a noise term. However, it is possible to modify this basic structure to produce contractual arrangements in which compensation is not decreasing in the benchmark. For example, in Himmelberg and Hubbard (2000), and Celentani and Loveira (2006), the managerial productivity is correlated with the aggregate state and, in this case, optimal benchmarking does not necessarily involve a smaller payment when the performance benchmark is high.

The lack of appropriate benchmarking in managerial compensation has however been considered by some authors as striking evidence of executive self dealing. Crystal (1991), Bertrand and Mullainathan (2001) and Bebchuk, Fried and Walker (2002) have pointed out that CEOs might have an important influence on their own pay process. According to this view, CEOs would manipulate the members of the compensation committee in order to obtain the most favorable conditions, just trying to avoid the shareholders' outrage. However, as Garvey and Milbourn (2006) have argued, it is not clear why the complete absence of any form of benchmarking should be desirable for CEOs. If compensation is linked to market movements, executives can only expect to receive the risk premium determined in the market and it is not clear whether it is high enough for the given managerial risk aversion. They also claim that asymmetric benchmarking is a more robust signal of self serving behavior since managers would prefer it to no benchmarking whatsoever.

This paper shows that asymmetric benchmarking can be a characteristic of optimal contracts adopted by completely independent and self-interested principals. The

[3] A common justification for this conduct is that with a stock price decline the plan loses its motivational value (Murphy, 1999, and Garvey and Milbourn, 2006).

main contribution is to show that, contrary to the general presumption, asymmetric benchmarking can be obtained even when managerial productivity is independent of the aggregate shock. The model stresses that, besides the contractual (explicit) incentives, managers also respond to other sources of (implicit) incentives that shareholders should take into account. In particular, I consider the indirect discipline created by bankruptcy: when a firm performs poorly and gets liquidated its manager also suffers a cost. For example, it could take some time to find a new job and, having performed poorly in the previous one, new job conditions will presumably be less attractive.[4] I also assume that, after signing the contract but before choosing his action, the agent can observe a private signal on the future state of the world (i.e. on the level of the benchmark). Bad luck, reflected in low levels of the benchmark, makes performance levels below the liquidation threshold more likely then sharpening indirect incentives. Since the agent observes the private signal before choosing an action, he can base his conduct on this information. As a consequence, the principal can provide the agent with different explicit incentives when different signals are observed. Therefore, the optimal contract should mitigate explicit incentives when the agent receives bad news because the relatively higher probability of being liquidated provides sharper indirect incentives. Because the signal observed by the manager is private, his compensation cannot be made contingent on it, but compensation depends on the firm's performance and on the realized state that are both public. Furthermore, an ex-post low level of the benchmark makes it more likely that the signal received by the agent was a bad one. Hence, to provide the agent with weaker incentives when a bad signal is observed it is sufficient to make the compensation flatter when the benchmark is low. Asymmetric benchmarking then arises from this characteristic of the optimal contract: if the benchmark is high, compensation is more sensitive to the firm outcome so that a good performance leads to a high managerial payment, however, when the benchmark is low, the wage schedule is flatter so that a poor performance is not penalized that much.

[4] Schmidt (1997) offers the first formalization within a principal-agent model of the intuitive idea that the probability of bankruptcy reduces the conflict of interest between stockholders and managers interested in keeping their job.

In the literature there are several papers that have tackled the puzzling absence of RPE in managerial contracts. In a first strand of it, pioneered by Salas Fumás (1992) and then followed, among others, by Aggarwal and Samwick (1999) and Joh (1999), the main focus is on product market interactions. It is stressed that, especially in tough competitive environments, it could be in the interest of the firms' owners to sign contracts that make managerial compensation increasing, instead of decreasing, in the market performance. In this way their commitment not to maximize profits is credible and can enforce some degree of collusion that, at the end, turns out to be better than straight competition. In a different vein, Oyer (2004) and Himmelberg and Hubbard (2000) stress the role of participation constraints. If the value of the managerial outside opportunity is positively correlated with wide industry movements, it could be necessary for the firms' owners to pay more in case of good luck in order to keep the participation constrained satisfied. Himmelberg and Hubbard (2000) also stress the role of the managerial labor market. They notice that managerial talent is relatively more productive in good states of the world so that the demand for high level executives increases during a boom period. At the same time, the supply is relatively inelastic and, then, they predict a positive relation between managerial pay and industry-wide performance, at least for the highest skilled managers.

None of the papers mentioned so far is able to explain the evidence of asymmetric benchmarking. In fact, their arguments always predict a positive relation between managerial pay and the level of the benchmark so that they can at most be useful to explain the observed mixed result on RPE. An exception is the recent work by Celentani and Loveira (2006). In a simple principal agent model they obtain that one sided RPE is optimal if the productivity of the managerial effort is sufficiently higher in the good state. Under this assumption, the observation of a good performance is more suggestive of managerial high effort in good times than in bad times and, similarly, poor performance is more suggestive of managerial effort in bad states than in good ones. The corresponding optimal contract then displays asymmetric benchmarking. In other words, the correlation between the aggregate state and managerial productivity can explain asymmetric benchmarking.[5]

In the model developed here, the relationship between firm performance, aggregate state and managerial activity is a simple linear equation of the kind commonly used in empirical studies and, even assuming independence between aggregate state and managerial productivity, asymmetric benchmarking emerges as an optimal contractual arrangement. The key element of the explanation is that for agents with a private information on the strength of their implicit incentives, benchmarking is not only used to filter out observable luck but also to adapt explicit incentives to the hidden information.

The paper is organized as follows. Section two introduces a standard principal agent model in which the agent performance is affected by an observable and uncontrollable shock (the benchmark). Furthermore, performances below a given threshold trigger the liquidation of the firm. In this framework the optimal benchmarking rule is obtained and discussed. In section three I consider the possibility for the manager of observing a signal on the future level of the benchmark. I first study optimal contracts that induce the manager to exert *unconditional effort*, i.e., the level of effort he chooses is independent of his private information. I show that, in this case, the optimal benchmarking rule displays the kind of asymmetry observed in the data. I then characterize contracts that induce the agent to exert *conditional effort*, i.e., his effort choice depends on the realization of the private signal. Section 4 contains some numerical examples and, finally, section 5 concludes.

1.2. The Baseline Analysis of an Uninformed Agent

A principal (she) has to hire a manager (he) to implement an investment project whose result x depends on a managerial action $a \in \{0, 1\}$, on an aggregate state variable

[5] They also show that the opposite assumption naturally leads to the opposite result: if the managerial effort is more productive in bad times then the managerial pay should appear to be insulated from good luck but not from bad luck.

$y \in \{B, G\}$ and on a random term ξ according to:

$$x = \gamma a + y + \xi \tag{1.1}$$

Action a can be interpreted as effort and can be high ($a = 1$) or low ($a = 0$). High effort has a utility cost of $c > 0$ for the manager while low effort involves no such cost. The term $\gamma > 0$ represents productivity of the managerial effort and is independent of the aggregate state. The noise term ξ is normally distributed with zero mean and variance σ^2. Finally, the binary variable y has a distribution $p(y)$ and it captures stochastic elements affecting the project result that are observable and verifiable. Assuming $G > 0$ and $B = 0$, the value $y = G$ can be interpreted as the favorable (Good) aggregate state while $y = B$ as the unfavorable (Bad) one. The project outcome x and the aggregate shock y are both observable and verifiable and whenever x is below a critical value $\underline{x} \leq 0$, the principal goes bankrupt. In this case the agent incurs a liquidation cost $\beta > 0$ which is expressed in utility terms. The managerial action cannot be observed by the principal and the idiosyncratic noise term ξ is not observable by anybody. All remaining parameters are commonly known. In this framework a contract is a wage schedule of the form $w : \mathbb{R} \times \{B, G\} \rightarrow [\underline{w}, \infty)$, notice in particular that the manager is wealth constrained so that feasible wage offers must be above the threshold $\underline{w}$. The timing is as follows: the principal offers a contract to the agent that can either accept or reject it. If the contract is accepted, the manager has to decide whether to exert high or low effort. After such decision, which remains hidden to the principal, the aggregate state y and the term ξ are determined and then, the project outcome x is realized according to (1.1). Both x and y are publicly observed and if the outcome is below the critical value, the principal goes bankrupt. Finally, the wage payment is carried out.[6] The principal is risk neutral and maximizes total expected profits $E(x-w)$, while the agent is risk averse with ex post utility

[6] Notice in particular that the agent receives the promised wage payment even in case of bankruptcy. It could be argued that a wage payment $w(x, y)$ should belong to $[\underline{w}, x]$ to be credible and further restrictions would be necessary in case of bankruptcy. However, including such restrictions wouldn't affect any result and, therefore, it is for the sake of simplicity that I consider general contracts of the type $w : \mathbb{R} \times \{B, G\} \rightarrow [\underline{w}, \infty)$.

$U(w,a,x) = u(w) - c(a) - \beta(x)$, where $u : [\underline{w}, \infty) \to \mathbb{R}$ is twice continuously differentiable, strictly increasing and concave, and satisfying $\lim_{w\to\infty} u'(w) = 0$ and $\lim_{w\to\underline{w}^+} u'(w) = \infty$, $c(1) = c > 0$ and $c(0) = 0$, and, finally, $\beta(x \leq \underline{x}) = \beta$ and $\beta(x > \underline{x}) = 0$. The agent also has an outside opportunity which is worth $\overline{U}$ to him. Notice that when the agent chooses action a in state y, the outcome x is normally distributed with mean $\gamma a + y$ and variance σ^2. Let $F_y(\cdot\,, a)$ be the corresponding distribution function and $f_y(\cdot\,, a)$ the associated density. The probability of bankruptcy in state y when the agent's action is a, is therefore $F_y(\underline{x}, a)$. The total probability of bankruptcy when the agent takes action a is $b(a) = \sum_y p(y) F_y(\underline{x}, a)$. It is immediate to check that $b(0) > b(1)$, meaning that high effort reduces the probability of bankruptcy. Notice that the quantity $[b(0) - b(1)]\,\beta$ is the expected reduction in turnover costs induced by $a = 1$, so that it represents the *implicit* value that effort has for the agent. Such quantity is increasing in β and, more importantly, it is also increasing in the difference $b(0) - b(1)$, i.e. the more effective high effort is in taking the company away from bankruptcy, the higher its implicit value. Therefore, as in Schmidt (1997), the possibility of bankruptcy creates an implicit incentive that mitigates the conflict of interest with the principal. The minimum cost contract inducing high effort solves:

$$\min_{w(x,z)\geq \underline{w}} \sum_y p(y) \int w(x,y) f_y(x,1)dx \tag{1.2}$$

subject to:

$$\sum_y p(y) \int u\left[w(x,y)\right] f_y(x,1)dx \geq \overline{U} + c + b(1)\beta \tag{1.3}$$

$$\sum_y p(y) \int u\left[w(x,y)\right]\left[f_y(x,1) - f_y(x,0)\right] dx + \left[b(0) - b(1)\right]\beta \geq c. \tag{1.4}$$

The incentive compatibility constraint (1.4) reflects the existence of both explicit incentives, derived from the contract, and implicit incentives, derived from bankruptcy. If (1.4) is satisfied, the agent finds it in his own interest to choose effort $a = 1$. For this to be the case the value of effort, measured on the left hand side of (1.4) must be not smaller than the cost of effort reported on the right hand side. Notice that the value of effort is the sum of two components. First, the quantity $\sum_y p(y) \int u[w(x,y)][f_y(x,1) - f_y(x,0)]\,dx$ measures the value of effort in inducing higher wage payments. This is an explicit incentive and is controlled by the principal through the contract w : the higher the pay-for-performance sensitivity, the higher the explicit value of effort. Second, the quantity $[b(0) - b(1)]\,\beta$ measures the implicit value of effort. Notice also that if $[b(0) - b(1)]\,\beta \geq c$ the agent prefers to exert effort even when he is offered a flat wage schedule that gives effort no explicit value. I rule out this possibility assuming that $\beta < \frac{c}{[b(0)-b(1)]}$ so that the agent's bankruptcy cost is not sufficient to perfectly align his interest with that of the principal. Let λ and μ be multipliers for constraints (1.3) and (1.4) respectively and define with $L_y(x) = \frac{f_y(x,1)-f_y(x,0)}{f_y(x,1)}$ the likelihood ratio corresponding to outcome x in state y. Hence, the minimum cost contract w^* inducing the high level of effort satisfies the following condition:

$$\frac{1}{u'[w^*(x,y)]} = \lambda + \mu L_y(x) \tag{1.5}$$

for all pairs (x,y) for which (1.5) has a solution $w^*(x,y) \geq \underline{w}$, otherwise $w^*(x,y) = \underline{w}$ whenever $\lambda + \mu L_y(x) < 0$. Notice that for a fixed a the density function of the outcome in the favorable state is obtained shifting to the right the corresponding density in the unfavorable state by the amount G, that is: $f_G(x+G,a) = f_B(x,a)$. This also implies that $F_G(x+G,a) = F_B(x,a)$ and $L_G(x+G) = L_B(x)$, and then also $\lambda + \mu L_G(x+G) = \lambda + \mu L_B(x)$, which in particular implies the following condition:

$$w^*(x+G,G) = w^*(x,B). \tag{1.6}$$

Hence, if we denote with $E(w^* \mid y)$ the equilibrium expected wage in state y, an immediate consequence of (1.6) is that:

$$E(w^* \mid G) = E(w^* \mid B) \tag{1.7}$$

To verify this last condition just notice that:

$$E(w^* \mid B) = \int w^*(x,B) f_B(x,1)dx = \int w^*(x+G,B) f_G(x+G,1)dx =$$

$$\int w^*(\widetilde{x}, G) f_G(\widetilde{x}, 1) d\widetilde{x} = E\left(w^* \mid G\right)$$

where the last equality in the first line follows from the change of variable $\widetilde{x} = x + G$. The expected wage payment is the same in both aggregate states so that, in expected terms, there is no reward associated to observable luck. Figure 1 gives an illustration of the optimal contract in this case.

This characteristics of the optimal wage schedule should not come as a surprise: the variable y represents in fact a benchmark against which managerial performance can be evaluated. Put it another way, the aggregate term y is beyond managerial control but, nevertheless, it affects his performance. It is then optimal to filter it out by benchmarking the managerial compensation according to the rule described in (1.6) that simply states that in good times a given outcome $x + G$ induces the same wage payment that would have been induced in the unfavorable state by the smaller outcome x. In this way the risk induced by the uncertain aggregate state is completely removed so that the risk averse agent can be hired and properly motivated at a lower cost. As commonly obtained in this class of models the resulting wage schedule is then decreasing in the benchmark.[7]

The benchmarking condition (1.6) has been obtained in a very simple model. In more sophisticated environments it isn't necessarily so. For example, in Celentani and Loveira (2006) the productivity of the managerial effort can vary across aggregate states and in this case the simple benchmarking condition (1.6) no longer holds. In particular, they show that, if effort productivity is sufficiently higher in good times than in bad times, the optimal contract displays one sided RPE.[8] In the following section I maintain the hypothesis that effort productivity is independent of the aggregate state and nevertheless

[7] Remember that the contract design problem has a statistical interpretation: the outcome x is used as a signal about the managerial action and wage payments increases with the likelihood of the manager having exerted high effort. Given the simple (linear) structure in equation 1.1, an outcome $x + G$ in favorable conditions has the same informational content about the managerial action as the outcome x in bad times.

[8] In terms of the model presented here, if managerial productivity in state y is denoted with γ_y and the variability of the noise term ξ in state y is denoted with σ_y^2, it could be shown that if $\frac{\gamma_G}{\gamma_B} > \frac{\sigma_B}{\sigma_G}$, the optimal contract displays one sided benchmarking, that is, the wage payment is increasing in y above some given performance threshold, and decreasing in y below it. However, this modified benchmarking rule would still factor out completely the effect of luck, i.e. condition (1.7), would still hold.

asymmetric benchmarking arises whenever the agent can observe a private signal on the future state of the world.

1.3. Informed Agent

In the situation described in the previous section the probability of bankruptcy doesn't play any role in shaping the benchmarking rule adopted in the optimal contract. However, the utility cost that the manager suffers in case of bankruptcy mitigates the conflict of interest with the principal. Notice also that, everything else being constant, the probability of bankruptcy is higher in bad times than in good times so that, if the agent were able to forecast the future aggregate state, the agency problem would be less severe in case bad times were anticipated to come. This idea suggests that if the agent is able to observe some signal about the future level of y, the principal could find it optimal to take advantage of the disciplining effect of bankruptcy, then, in particular, reducing the managerial exposure to risk after the observation of bad news, and increasing it after good news. As it will be shown in this section, the optimal contractual arrangement resulting in this case exhibits asymmetric benchmarking.

Consider a private signal z that is received by the agent after signing the contract, but before choosing the level of effort. The signal can either be good, $z = G$ or bad, $z = B$, and its conditional probability, given the future state of nature is given by:

$$\rho(z \,|\, y) = \begin{cases} 1-\varepsilon & if \quad z = y \\ \varepsilon & if \quad z \neq y \end{cases}$$

where $\varepsilon \in \left[0, \frac{1}{2}\right]$ measures how noisy the signal is. If $\varepsilon = 0$, the signal is perfectly informative, while $\varepsilon = \frac{1}{2}$ corresponds to a completely noisy signal.[9] Define with $\rho(z)$ the total probability of observing signal z, that is $\rho(z) = (1-\varepsilon)\, p(y = z) + \varepsilon p(y \neq z)$. The

[9] In the latter case the agent is completely uninformed and the baseline analysis in the first section applies.

posterior probability of the future aggregate state held by the manager after receiving signal z is:

$$p(y\,|\,z) = \frac{\rho(z\,|\,y)p(y)}{\rho(z)}. \tag{1.8}$$

It is a matter of simple computations to check that for any $\varepsilon \in \left[0, \frac{1}{2}\right)$

$$p(y\,|\,z = y) > p(y) > p(y\,|\,z \neq y),$$

meaning that the observation of signal $z = y$ raises the conditional probability of observing state y in the future while the observation of signal $z \neq y$ decreases it. Define the conditional probability of bankruptcy given signal z and action a as $b(a\,|\,z) = \sum_y p(y\,|\,z)F_y(\underline{x}, a)$. Notice that for both $z = B$, G, $b(0\,|\,z) > b(1\,|\,z)$, i.e., whatever the observed signal, high effort has always an implicit value because it reduces the conditional probability of bankruptcy and, therefore, the expected cost of bankruptcy. However, the following lemma shows that the implicit value of effort is higher if bad times are anticipated to come.

Lemma 1 *The implicit value of effort increases after $z = B$ and decreases after $z = G$:*

$$b(0\,|\,B) - b(1\,|\,B) > b(0) - b(1) > b(0\,|\,G) - b(1\,|\,G)$$

Proof Computing explicitly probabilities $b(a)$ and $b(a\,|\,z)$ and rearranging terms yield:

$$b(a\,|\,z) - b(1\,|\,z) = \int_{-\infty}^{\underline{x}} \{p(G\,|\,z)\,[f_G(x,0) - f_G(x,1)] + p(B\,|\,z)\,[f_B(x,0) - f_B(x,1)]\}\,dx,$$

$$b(0) - b(1) = \int_{-\infty}^{\underline{x}} \{p(G)\,[f_G(x,0) - f_G(x,1)] + p(B)\,[f_B(x,0) - f_B(x,1)]\}\,dx,$$

Notice that for $x \leq \underline{x} \leq 0$ it results that $f_B(x,0) - f_B(x,1) > f_G(x,0) - f_G(x,1) > 0$ and then the lemma immediately follows from the fact that $p(G\,|\,G) > p(G) > p(G\,|\,B)$ and

$p(B \mid B) > p(B) > p(B \mid G)$. ■

The interpretation of lemma 1 is straightforward: the observation of bad news strengthens implicit incentives because it makes high effort more valuable (i.e. more effective in reducing the conditional probability of bankruptcy). Similarly, the observation of good news makes bankruptcy a less important concern and relaxes implicit incentives. Notice that the availability of the private signal for the agent enlarges his action space. He can now condition the level of effort on his private forecast of the future aggregate state. Let (a_B, a_G) be one such possible action profile where $a_z \in \{0, 1\}$ represents effort chosen after the observation of signal z. Notice that, because the private signal is correlated with the aggregate state and the agent can condition his action on it, the *managerial productivity*, represented by the term γa in equation (1.1), can be correlated with the aggregate state y even if *productivity* of the managerial high effort, represented by the term γ, is independent of y. In particular, $(a_B, a_G) = (1, 1)$, $(0, 0)$ are unconditional effort profiles and if the agent adopts one of them, there still is independence between the aggregate state y and the managerial productivity γa. However, if the agent chooses a conditional effort profile, i.e. $(a_B, a_G) = (0, 1)$, $(1, 0)$, there would be a correlation between aggregate state and managerial product, even if the outcome is determined according to the simple equation (1.1). In the rest of this section I will mainly focus on the minimum cost contract implementing the unconditional effort profile $(a_B, a_G) = (1, 1)$. It is important to notice that, even if no correlation is produced by this contract, incentives constraints must prevent the agent from choosing conditional action profiles that would create such correlation. For this reason benchmarking is not only used to filter out observable luck as in (1.6), but also to avoid such deviations.

1.3.1. Informed Agent Exerting Unconditional Effort

The minimum cost contract implementing high effort after both signals solves:

$$\min_{w(x,z)\geq \underline{w}} \sum_y p(y) \int w(x,y) f_y(x,1)dx \tag{1.9}$$

subject to:

$$\sum_y p(y) \int u\left[w(x,y)\right] f_y(x,1)dx \geq \overline{U} + c + b(0)\beta \tag{1.10}$$

and for $z \in \{B, G\}$

$$\sum_y p(y \,|\, z) \int u\left[w(x,y)\right]\left[f_y(x,1) - f_y(x,0)\right]dx + \left[b(0 \,|\, z) - b(1 \,|\, z)\right]\beta \geq c \tag{1.11}$$

The objective function in (1.9) and the (IR) constraint (1.10) are the same as in the previous section, but now there are two incentive compatibility constraints in (1.11), one for each possible realization of the private signal. Remember from lemma 1 that $b(0 \,|\, B) - b(1 \,|\, B) > b(0 \,|\, G) - b(1 \,|\, G)$ so that the IC constraint corresponding to the observation of $z = B$ is now less demanding, while after the observation of $z = G$ sharper explicit incentives are needed to induce the high level of effort. Let λ be the multiplier for the IR constraint and $\mu(z)\rho(z)$ the multiplier for constraint IC with signal z. The optimal wage schedule w^{ε} satisfies now the following condition:

$$\frac{1}{u'\left[w^{\varepsilon}(x,y)\right]} = \lambda + \left[(1-\varepsilon)\mu(z=y) + \varepsilon\mu(z \neq y)\right] L_y(x) \tag{1.12}$$

for all pairs (x, y) for which (1.12) has a solution $w^{\varepsilon}(x,y) \geq \underline{w}$, otherwise $w^{\varepsilon}(x,y) = \underline{w}$ whenever $\lambda + \left[(1-\varepsilon)\mu(z=y) + \varepsilon\mu(z \neq y)\right] L_y(x) < 0$. The next proposition shows how the availability of the private information modifies the simple benchmarking rule (1.6).

Proposition 1 *For each $\varepsilon \in \left[0, \frac{1}{2}\right)$ the minimum cost contract $w^{\varepsilon}(x,y)$ implementing $(a_B, a_G) = (1,1)$ is unique and for both $y = B, G$ it is a continuous function of x. Furthermore, observable luck is not completely removed from managerial compensation.*

In particular, the following holds:

(1) *if* $x > \frac{\gamma}{2}$, *then* $w^{\varepsilon}(x+G,G) > w^{\varepsilon}(x,B)$;

(2) *if* $x < \frac{\gamma}{2}$, *then* $w^{\varepsilon}(x+G,G) \leq w^{\varepsilon}(x,B)$;

(3) *if* $\lambda = 0$, *then* $E\left(w^{\varepsilon} \mid G\right) > E\left(w^{\varepsilon} \mid B\right)$.

Proof Problem (1.9) - (1.11) defining the optimal wage schedule w^{ε} is not a convex program. Following a common practice, it is better to define an equivalent problem stated in terms of the utility levels $u(w(x,y))$. Let U be the range of the utility function u, and $h : U \rightarrow [\underline{w}, \infty)$ be its inverse. Define $\underline{u} = u(\underline{w})$ and notice that h is twice continuously differentiable, strictly increasing , strictly convex and such that $\lim_{u \rightarrow \underline{u}} h'(u) = 0$ and $\lim_{u \rightarrow \sup U} h'(u) = \infty$. With a slight abuse of notation let's write $u(x,y)$ to denote $u(w(x,y))$ and notice that $w(x,y) = h(u(x,y))$. Consider now the following problem:

$$\min_{u(x,z) \geq \underline{u}} \sum_{y} p(y) \int h(u(x,y)) f_y(x,1) dx \tag{1.13}$$

subject to

$$\sum_{y} p(y) \int u(x,y) f_y(x,1) dx \geq \overline{U} + c + b(1)\beta \tag{1.14}$$

and for $z \in \{B, G\}$

$$\sum_{y} p(y \mid z) \int u(x,y) \left[f_y(x,1) - f_y(x,0)\right] dx + \left[b(0 \mid z) - b(1 \mid z)\right] \beta \geq c. \tag{1.15}$$

This is now a convex program and it is equivalent to (1.9) - (1.11) in the sense that $u^{\varepsilon}(x,y)$ solves (1.13) - (1.15) if and only if $w^{\varepsilon}(x,y) = h\left[u^{\varepsilon}(x,y)\right]$ solves (1.9) - (1.11). Let λ be the multiplier for the IR constraint and $\mu(z)\rho(z)$ the multiplier for constraint IC with signal z. The optimal utility schedule u^{ε} satisfies now the following condition:

$$h'\left[u(x,y)\right] = \lambda + \left[(1-\varepsilon)\mu(z=y) + \varepsilon\mu(z \neq y)\right] L_y(x) \tag{1.16}$$

for all pairs (x,y) for which (1.16) has a solution $u(x,y) \geq \underline{u}$, otherwise $u(x,y) = \underline{u}$

whenever $\lambda+[(1-\varepsilon)\mu(z=y)+\varepsilon\mu(z\neq y)]\,L_y(x)<0$. The advantage of this formulation is that being (1.13) - (1.15) a convex program its solution is unique and multipliers $\lambda,\mu(B)$ and $\mu(G)$ are non negative.

To facilitate the subsequent exposition, define $x_B=\frac{\gamma}{2}$, and $x_G=\frac{\gamma}{2}+G$ and notice that the quantities $f_y(x_y,1)-f_y(x_y,0)$ and $L_y(x)$ have the same sign as $x-x_y$. Furthermore, simple algebra shows that for any $v\geq 0$, the following holds:

$$f_y(x_y+v,1)-f_y(x_y+v,0)=$$

$$\frac{1}{\sqrt{2\pi\sigma^2}}\left\{\exp\left[-\frac{1}{2\sigma^2}\left(v-\frac{\gamma}{2}\right)^2\right]-\exp\left[-\frac{1}{2\sigma^2}\left(v+\frac{\gamma}{2}\right)^2\right]\right\}\equiv g(v)$$

which is independent of y. Similarly it can be noticed that for $v\geq 0$:

$$f_y(x_y-v,1)-f_y(x_y-v,0)=g(-v)=-g(v)$$

i.e., the function g is odd. Using this new notation it is possible to write the IC constraint associated to signal z in the following more convenient way:

$$\sum_y p(y\,|\,z)\int_0^\infty[u(x_y+v,y)-u(x_y-v,y)]\,g(v)dv+[b(0\,|\,z)-b(1\,|\,z)]\,\beta\geq c.$$

The rest of the proof is organized in three steps.

Step 1 Let's show here that points (1) and (2) follow from $\mu(G)>\mu(B)\geq 0$ that, in turn, is established in step two. If $\varepsilon\in\left[0,\frac{1}{2}\right)$ and $\mu(G)>\mu(B)\geq 0$ it is also true that $[(1-\varepsilon)\mu(G)+\varepsilon\mu(B)]>[(1-\varepsilon)\mu(B)+\varepsilon\mu(G)]$, so that, using the first order condition (1.16), recalling that $L_y(x)$ is larger for $y=B$ and has the same sign of $(x-x_y)$, for $x>x_B=\frac{\gamma}{2}$, it results that:

$$h'\left[u(x+G,G)\right]=\lambda+[(1-\varepsilon)\mu(G)+\varepsilon\mu(B)]\,L_G(x+G)>$$

$$\lambda+[(1-\varepsilon)\mu(B)+\varepsilon\mu(G)]\,L_G(x+G)=$$

$$\lambda + [(1-\varepsilon)\mu(B) + \varepsilon\mu(G)]\, L_B(x) = h'\,[u(x,B)]\,.$$

Hence, being h an increasing and convex function, it results that $u^{\varepsilon}(x+G,G) > u^{\varepsilon}(x,B)$ which is the same as $w^{\varepsilon}(x+G,G) > w^{\varepsilon}(x,B)$. As for $x < x_B = \frac{\gamma}{2}$ let's distinguish two cases. Consider first the case in which $\lambda + [(1-\varepsilon)\mu(G) + \varepsilon\mu(B)]\, L_G(x+G) \geq 0$. Condition (1.16) still defines the optimal level of utility to be assigned to the manager for both $y = B, G$ so that, since $L_G(x+G) < 0$, we have now:

$$h'\,[u(x+G,G)] = \lambda + [(1-\varepsilon)\mu(G) + \varepsilon\mu(B)]\, L_G(x+G) <$$

$$\lambda + [(1-\varepsilon)\mu(B) + \varepsilon\mu(G)]\, L_G(x+G) =$$

$$\lambda + [(1-\varepsilon)\mu(B) + \varepsilon\mu(G)]\, L_B(x) = h'\,[u(x,B)]\,.$$

Hence, it follows that $w^{\varepsilon}(x+G,G) < w^{\varepsilon}(x,B)$. In the second complementary case in which $\lambda + [(1-\varepsilon)\mu(G) + \varepsilon\mu(B)]\, L_G(x+G) < 0$, we have that the wage offer is $w^{\varepsilon}(x+G,G) = \underline{w}$ and then for sure not larger then $w^{\varepsilon}(x,B)$. Finally, to establish continuity of the schedule $w^{\varepsilon}(x,z)$ it is sufficient to check that at the point $\omega_y \equiv L_y^{-1}\left(-\frac{\lambda}{[(1-\varepsilon)\mu(z=y)+\varepsilon\mu(z\neq y)]}\right)$ it results that:

$$\lim_{x\to\omega_y^+} u(x,y) = \lim_{x\to\omega_y^+} h'^{-1}\left(\lambda + [(1-\varepsilon)\mu(z=y) + \varepsilon\mu(z\neq y)]\, L_y(x)\right) =$$

$$\lim_{\upsilon\to 0^+} h'^{-1}(\upsilon) = \underline{u}$$

which is clearly true since, being h the inverse of u, h' converges to 0 as its argument converges to $\underline{u}$.

Step 2 Let's show that $\mu(G) > \mu(B) \geq 0$. Assume by contradiction that $\mu(B) \geq \mu(G) \geq 0$ and notice that the two IC multipliers cannot be both zero and then $\mu(B) > 0$ follows, i.e. the IC constraint associated to signal B is binding. Now, similarly to what have been found in step 1, the first order condition (1.16) implies that

$$[u^\varepsilon(x+G,G)-u^\varepsilon(x,B)]\,(x-x_B)\le 0$$

so that we can write:

$$\sum_y p(y\,|\,G)\int_0^\infty [u(x_y+v,G)-u(x_y-v,G)]\,g(v)dv =$$

$$\int_0^\infty p(G\,|\,G)\,[u(x_G+v,G)-u(x_G-v,G)]+p(B\,|\,G)\,[u(x_B+v,G)-u(x_B-v,G)]\,g(v)dv\le$$
$$\int_0^\infty p(G\,|\,B)\,[u(x_G+v,B)-u(x_G-v,B)]+p(B\,|\,B)\,[u(x_B+v,B)-u(x_B-v,B)]\,g(v)dv=$$
$$c-[b(0\,|\,B)-b(1\,|\,B)]\,\beta<c-[b(0\,|\,G)-b(1\,|\,G)]\,\beta$$

which clearly violates the incentive compatibility constraint corresponding to the signal $z=G$. Notice that the first (weak) inequality follows from the fact that being $v\ge 0$, we also have $u(x_G+v,G)\le u(x_B+v,B)$ and $u(x_G-v,G)\ge u(x_B-v,G)$ so that $u(x_G+v,G)-u(x_G-v,G)\le u(x_B+v,B)-u(x_B-v,B)$ and furthermore $P(G\,|\,G)>P(G\,|\,B)$, $P(B\,|\,G)<P(B\,|\,B)$. The second (strict) inequality is a consequence of lemma 1.

Step 3 Let's finally show point (3) in the proposition. Notice that from the first order condition (1.12), or equivalently from (1.16), if $\lambda=0$, it is possible to obtain w^ε as follows:

$$w^\varepsilon(x,y)=u'^{-1}\left([\max\{0,\ [(1-\varepsilon)\mu(z=y)+\varepsilon\mu(z\neq y)]\,L_y(x)\}]^{-1}\right),$$

where, because of the concavity of u, the function $u'^{-1}:[0,\infty]\to[\underline{w},\infty]$ is decreasing with $u'^{-1}(\infty)=\underline{w}$. Therefore, the following holds:

$$E\left(w^\varepsilon\,|\,G\right)=\int w^\varepsilon(x,G)f_G(x,1)dx=$$

$$\int u'^{-1}\left(\left[\max\left\{0,\ \left[(1-\varepsilon)\mu(G)+\varepsilon\mu(B)\right]L_G(x)\right\}\right]^{-1}\right)f_G(x,1)dx =$$

$$\int u'^{-1}\left(\left[\max\left\{0,\ \left[(1-\varepsilon)\mu(G)+\varepsilon\mu(B)\right]L_B(x-G)\right\}\right]^{-1}\right)f_B(x-G,1)dx =$$

$$\int u'^{-1}\left(\left[\max\left\{0,\ \left[(1-\varepsilon)\mu(G)+\varepsilon\mu(B)\right]L_B(x)\right\}\right]^{-1}\right)f_B(x,1)dx >$$

$$\int u'^{-1}\left(\left[\max\left\{0,\ \left[(1-\varepsilon)\mu(B)+\varepsilon\mu(G)\right]L_B(x)\right\}\right]^{-1}\right)f_B(x,1)dx =$$

$$\int w^{\varepsilon}(x,B)f_B(x,1)dx = E\left(w^{\varepsilon}\mid B\right),$$

where, in particular, the inequality follows from $\mu(G) > \mu(B)$ and the monotonicity of the function u'^{-1}. This completes the proof. ■

Proposition 1 states that the simple benchmarking rule obtained in (1.6) does not hold in the present context and, therefore, observable luck is not necessarily removed from managerial compensation. This result follows because the optimal contract inducing unconditional effort adjusts explicit incentives to implicit incentives. After the bad signal $z = B$, implicit incentives are stronger so that explicit incentives can be weaker and this is achieved by reducing the pay-for-performance sensitivity in state $y = B$. Similarly, after the good signal $z = G$, the pay-for-performance sensitivity in state $y = G$ has to increase in order to make up for the weaker discipline brought about by bankruptcy. As a result, if the net performance, i.e. performance net of the contribution of luck, is above the critical level $\frac{\gamma}{2}$, compensation in state $y = G$ is higher than the level required to eliminate the effect of luck, given in condition (1.6) in the previous section. From the other hand, net performances below the same threshold lead to compensation levels in bad times that are above what is required by the same rule (1.6. For this reason it is not possible to predict in general how expected wages rank in the two states $y = B$, G. In fact, $E(w^{\varepsilon} \mid G)$ is increased by compensation levels corresponding to net performances above $\frac{\gamma}{2}$ but it is decreased by compensations associated to performances below $\frac{\gamma}{2}$. The opposite happens to $E(w^{\varepsilon} \mid B)$. However, proposition 1 also shows that if the agent extract some rents from the contract, he also receives a higher expected wage in the good state. In fact, when the individual rationality constraint is not binding (i.e. $\lambda = 0$), net performances below the critical level $\frac{\gamma}{2}$ receive a constant wage so that only the distortion corresponding to net performances above $\frac{\gamma}{2}$ are relevant and this points unambiguously toward higher compensation in good times. Finally, notice that the condition $\lambda = 0$ is sufficient but not necessary for the result as shown by the numerical examples contained in section 4. The following result provides an additional characterization of the optimal contract that makes it explicit the use of asymmetric benchmarking.

Proposition 2 *For each $\varepsilon \in \left[0, \frac{1}{2}\right)$ the unique minimum cost contract w^ε implementing $(a_B, a_G) = (1, 1)$ pays more in the good state if the performance is good and it pays more in the bad state if the performance is bad. More precisely, there exists a performance threshold $\widehat{x}(\varepsilon) > G + \frac{\gamma}{2}$ such that:*

(1) *if $x > \widehat{x}(\varepsilon)$, then $w^\varepsilon(x, G) > w^\varepsilon(x, B)$;*

(2) *if $x < \widehat{x}(\varepsilon)$, then $w^\varepsilon(x, G) \leq w^\varepsilon(x, B)$.*

Proof For a given ε, define $\widehat{x}(\varepsilon)$ as the solution to:

$$[(1-\varepsilon)\mu(G) + \varepsilon\mu(B)]\, L_G(x) = [(1-\varepsilon)\mu(B) + \varepsilon\mu(G)]\, L_B(x)$$

that, after rearranging terms, can be written as:

$$H(x, \varepsilon) \equiv \mu(B)\,[L_G(x) - L_B(x)] + [\mu(G) - \mu(B)]\,\{L_G(x) - \varepsilon\,[L_G(x) + L_B(x)]\} = 0. \tag{1.17}$$

Notice that H is a continuous and strictly increasing function of x and furthermore $H(x_G, \varepsilon) < 0$ and $\lim_{x\to\infty} H(x, \varepsilon) = [\mu(G) - \mu(B)]\,(1 - 2\varepsilon) > 0$.[10] This implies that $\widehat{x}(\varepsilon)$ exists and it is unique and, furthermore, $\widehat{x}(\varepsilon) > x_G = G + \frac{\gamma}{2}$. Let's show now that such quantity has the properties claimed in the proposition. To this end notice that for $x > \widehat{x}(\varepsilon)$, $L_y(x) > 0$ for both $y \in \{B, G\}$ so that condition (1.16) determines the optimal

[10] To see this just check that for $x \to \infty$, both $L_y(x)$ converge to 1 and remember that $\varepsilon < \frac{1}{2}$

wage $w^{\varepsilon}(x,y)$. Furthermore, for any such x it results that:

$$[(1-\varepsilon)\mu(G)+\varepsilon\mu(B)]\,L_G(x) > [(1-\varepsilon)\mu(B)+\varepsilon\mu(G)]\,L_B(x)$$

immediately implying that $w^{\varepsilon}(x,G) < w^{\varepsilon}(x,B)$, that is point (1) in the proposition. As for point (2) just notice that it is an implication of proposition 1 for any $x < x_G$ while for $x \in (x_G, \widehat{x}(\varepsilon))$ an argument similar to the one used to establish point (1) applies. ■

This second proposition reveals that optimal contracting does not simply lead to a failure to filter out aggregate risk. It also requires that above the performance threshold $\widehat{x}(\varepsilon)$ compensation be increasing in the benchmark, i.e., for any given $x > \widehat{x}(\varepsilon)$ the manager receives a higher compensation if favorable aggregate conditions are observed while, for $x < \widehat{x}(\varepsilon)$ the compensation is higher in bad states. The proof of both results relies on the same intuitive idea that can be grasped referring to figure 2 which displays how the possibility of observing a signal on the future aggregate state distorts the wage schedule that would be optimal in the absence of the signal. If the agent receives some private information before choosing his action, the incentive he needs to choose high effort is sharper in case of good news because liquidation is perceived to be less likely and therefore is less effective as an incentive device. The way to provide the agent with sharper incentives after the observation of good news is to make the schedule $w(x,G)$ steeper, thus increasing his exposure to risk. The reason is that, after observing the signal $z = G$ it is relatively more likely that the future state will be $y = G$ and then the relevant wage schedule is likely to be $w(x,G)$. Similarly, the observation of $z = B$ weakens the incentive constraint that must be met to induce high effort, and lead to a flatter compensation schedule $w(x,B)$. It can be noticed in figure 2 that the overall result is then the form of asymmetric benchmarking described in proposition 1 and 2.

Let $EW_y^{\varepsilon} = \int w^{\varepsilon}(x,y)f_y(x,1)dx$ be the expected wage payment in state y when the available signal is affected by a noise term ε and define the ex ante expected compensation cost as $EW^{\varepsilon} = \sum_y p(y) EW_y^{\varepsilon}$. The next result shows that the availability of a more precise private signal for the agent makes it more costly to induce unconditional high effort.

Proposition 3 *If for each $\varepsilon \in \left[0, \frac{1}{2}\right)$ the principal offers in equilibrium the minimum cost contract implementing $(a_B, a_G) = (1,1)$, then the agent expected wage EW^{ε} is a decreasing function of ε.*

Proof The proof relies on the simple observation that the availability of a more informative signal shrinks the set of incentive compatible wage schedules. Define the following quantity:

$$\Delta_y = \int_0^{\infty} \left[u(x_y + v, y) - u(x_y - v, y)\right] g(v) dv.$$

With this notation the IC constraint corresponding to signal z can be written as follows:

$$p(G \mid z)\Delta_G + p(B \mid z)\Delta_B + \left[b(0 \mid z) - b(1 \mid z)\right] \beta \geq c.$$

The quantity Δ_y can be seen as an index measuring the pay-for-performance sensitivity in state y. For example, if the wage payments $w(x, y)$ above $\underline{w}$ were linear in x, the quantity Δ_y would be increasing in its slope. Using this notation it is possible to see that each IC constraint defines a hemiplane in the space (Δ_G, Δ_B). Furthermore, straightforward calculations show how increasing the signal precision, i.e. decreasing ε, shrinks the intersection of the two hemiplanes defining the region corresponding to incentive compatible contracts. ■

Endogenous independence between managerial productivity and the aggregate state also arises when the principal implements $(a_B, a_G) = (0, 0)$. In this case a constant wage equal to $\overline{U} + b_0 B$, i.e., high enough to meet the agent's participation constraint, would be optimal.[11]

[11] Notice however that if $B > \frac{c}{b_0(B) - b_1(B)}$, it is impossible to induce the agent to choose $a = 1$ if he observes the private signal $z = B$.

1.3.2. Informed Agent Exerting Conditional Effort

An endogenous correlation between managerial productivity and the aggregate state emerges when the principal implements either $(a_B,\, a_G) = (0, 1)$ or $(a_B,\, a_G) = (1, 0)$. The first conditional effort profile produces a positive correlation while the second a negative one. Notice that if the agent adopts the first profile his productivity in the good state is larger because he exerts high effort only after the observation $z = G$ which, for any $\varepsilon \in \left[0,\, \frac{1}{2}\right)$, is more likely when the future state of the world is $y = G$.[12] Similarly, if the agent adopts $(a_B,\, a_G) = (1, 0)$ and the signal is not completely noisy, the agent will choose $a = 1$ more often in state $y = B$ than in state $y = G$ then producing a negative correlation. The following proposition describes the characteristics of the optimal benchmarking rule corresponding to the minimum cost contract implementing $(a_B,\, a_G) = (0, 1)$.

Proposition 4 *Given $\varepsilon \in \left[0, \frac{1}{2}\right)$, if $\beta < \frac{c}{b(0\,|\,B)-b(1\,|\,B)}$, the unique minimum cost contract $w^{01}(x, y)$ implementing $(a_B,\, a_G) = (0, 1)$ is a continuous function of x. Furthermore, if the incentive compatibility constraint corresponding to signal $z = B$ is slack, it results that:*

(1) *if* $x > \frac{\gamma}{2}$, *then* $w^{01}(x + G, G) > w^{01}(x, B)$;

(2) *if* $x < \frac{\gamma}{2}$, *then* $w^{01}(x + G, G) \leq w^{01}(x, B)$.

Proof If the agent adopts the (conditional or unconditional) effort profile $(a_B,\, a_G)$, the outcome pdf in state y is:

$$f_y\left[x, (a_B, a_G)\right] = \left[(1-\varepsilon) f_y(x,\, a_{z=y}) + \varepsilon f_y(x,\, a_{z\neq y})\right].$$

[12] Notice that for $\varepsilon = \frac{1}{2}$, even the adoption of a conditional action profiles would not induce any correlation between the aggregate state and the managerial product. In fact, with a completely noisy signal, both $(a_B, a_G) = (0, 1)$ and $(a_B, a_G) = (1, 0)$ are equivalent to the behavioral strategy of playing either level of effort with probability one half.

Define the corresponding likelihood ratio as $L_y^{(a_B,a_G)}(x) = \frac{f_y(x,1)-f_y(x,0)}{f_y[x,(a_B,a_G)]}$. The ex ante[13] probability of bankruptcy is:

$$b_{(a_B,a_G)} = \sum_y p(y)\left[(1-\varepsilon)F_y(\underline{x}, a_{z=y}) + \varepsilon F_y(\underline{x}, a_{z\neq y})\right],$$

and the ex ante probability of exerting high effort is:[14]

$$q(a_B, a_G) = \sum_y p(y)\left[(1-\varepsilon)I(a_{z=y}=1) + \varepsilon I(a_{z\neq y}=1)\right].$$

It is now possible to state the problem characterizing the minimum cost contract implementing $(a_B,\, a_G) = (0,1)$:

$$\min_{u(x,y)\geq \underline{u}} \sum_y p(y) \int h\left[u(x,y)\right])f_y\left[x,(0,1)\right]dx \tag{1.18}$$

subject to

$$\sum_y p(y) \int u(x,y) f_y\left[x,(0,1)\right]dx \geq \overline{U} + q(0,1)c + b_{(0,1)}\beta \tag{1.19}$$

$$\sum_y p(y\,|\,B) \int u(x,y)\left[f_y(x,1) - f_y(x,0)\right]dx + \left[b(0\,|\,B) - b(1\,|\,B)\right]\beta \leq c \tag{1.20}$$

$$\sum_y p(y\,|\,G) \int u(x,y)\left[f_y(x,1) - f_y(x,0)\right]dx + \left[b(0\,|\,G) - b(1\,|\,G)\right]\beta \geq c. \tag{1.21}$$

The problem stated in terms of utility levels $u(x,y)$ is a convex program which admits a unique solution u^{01}. Let λ be the multiplier for the IR constraint (1.19) while $\mu(B)\rho(B)$ and $\mu(G)\rho(G)$ are multipliers for IC constraints (1.20) and, respectively, (1.21). The optimal utility schedule u^{01} satisfies now the following conditions:

$$h'\left[u^{01}(x,G)\right] = \lambda + \left[(1-\varepsilon)\mu(G) - \varepsilon\mu(B)\right]L_G^{(0,1)}(x) \tag{1.22}$$

[13] Ex ante here means before the observation of the private signal.

[14] In what follows the function $I(A)$ denotes the indicator function of an event A, and its value is one if A is true and zero otherwise.

$$h'\left[u^{01}(x,B)\right] = \lambda + \left[\varepsilon\mu(G) - (1-\varepsilon)\mu(B)\right] L_B^{(0,1)}(x) \tag{1.23}$$

for all pairs (x, G) and (x, B) for which (1.22) and, respectively, (1.23) have a solution above $\underline{u}$, otherwise $u(x,y) = \underline{u}$. Because $L_y^{(0,1)}(x)$ is a continuous function, it is immediate to check that u^{01} is continuous too. Furthermore, if we assume that $\mu(B) = 0$, it must be $\mu(G) > 0$ and first order conditions (1.22), (1.23) can be rewritten as follows:

$$h'\left[u^{01}(x,G)\right] = \lambda + \mu(G)\left[1 - \frac{f_G(x,0)}{f_G\left[x,(0,1)\right]}\right] \tag{1.24}$$

$$h'\left[u^{01}(x,B)\right] = \lambda + \mu(G)\left[1 - \frac{f_B(x,0)}{f_B\left[x,(0,1)\right]}\right]. \tag{1.25}$$

Notice that $f_G(x+G,0) = f_B(x,0)$ and for $x > \frac{\gamma}{2}$ also $f_G\left[x+G,(0,1)\right] < f_B\left[x,(0,1)\right]$. Furthermore, the solution is characterized by (1.24), (1.25) and it results:

$$h'\left[u^{01}(x+G,G)\right] = \lambda + \mu(G)\left[1 - \frac{f_G(x,0)}{f_G\left[x,(0,1)\right]}\right] >$$

$$\lambda + \mu(G)\left[1 - \frac{f_B(x,0)}{f_B\left[x,(0,1)\right]}\right] = h'\left[u^{01}(x,B)\right]$$

which is equivalent to $w^{01}(x+G,G) > w^{01}(x,B)$ as claimed in point (1) of the proposition. Point (2) follows from a similar argument. ■

The intuition behind this first result is that if the agent's private signal is not completely noisy, in order to induce high effort after the observation of $z = G$ but not after $z = B$, the wage schedule has to be steeper in state $y = G$ than in state $y = B$.[15] The asymmetry in the benchmarking rule described in proposition 4 is then a consequence of this characteristic. Notice that if $\beta \geq \frac{c}{b(0 \mid B) - b(1 \mid B)}$ it would not be possible to implement $(a_B, a_G) = (0,1)$ because in this case the liquidation cost β is so large that the manager would choose high effort after $z = B$ even if his wage were completely independent of the firm's outcome. Notice that points (1) and (2) in the previous proposition have been shown under the hypothesis that the incentive constraint corresponding to signal $z = B$

[15] If the signal is perfectly informative, the optimal wage schedule in state $y = B$ is constant.

is not binding. This condition for example holds when the private signal is perfectly informative. In fact, if $\varepsilon = 0$ first order conditions (1.22), (1.23) implies that $\mu(B) = 0$,[16] therefore leading to a constant wage in state B.

The next result describes the characteristics of the optimal benchmarking rule corresponding to the minimum cost contract implementing $(a_B, a_G) = (1, 0)$.

Proposition 5 *Given $\varepsilon \in \left[0, \frac{1}{2}\right)$, if $\beta < \frac{c}{b(0 \mid G) - b(1 \mid G)}$, the unique minimum cost contract $w^{10}(x, y)$ implementing $(a_B, a_G) = (1, 0)$ is a continuous function of x. Furthermore, if the incentive compatibility constraint corresponding to signal $z = G$ is slack, it results that:*

(1) *if* $x > \frac{\gamma}{2}$, *then* $w^{10}(x + G, G) < w^{10}(x, B)$;

(2) *if* $x < \frac{\gamma}{2}$, *then* $w^{10}(x + G, G) \geq w^{10}(x, B)$.

The proof closely resembles the argument given for proposition 4 and is then omitted. The intuition here is similar to the previous one. If the principal wants to induce $(a_B, a_G) = (1, 0)$ and the agent's signal brings some information on the future state, the wage schedule has to be steeper in state $y = B$ than in state $y = G$. Then, the kind of asymmetric benchmarking described in proposition 5 turns out to be optimal. Again, if $\beta \geq \frac{c}{b(0 \mid G) - b(1 \mid G)}$, it would be impossible to prevent the agent from choosing $a = 1$ after the signal $z = G$, and then also after $z = B$. However, the assumption that $\beta < \frac{c}{b(0) - b(1)}$ maintained throughout, implies that $\beta < \frac{c}{b(0 \mid G) - b(1 \mid G)}$.

Proposition 4 and 5 resemble the main results in Celentani and Loveira (2006). They found that if the agent productivity is larger in the good state, in order to induce high effort, optimal payments must be increasing in the benchmark for large outcome realizations and decreasing in the benchmark for small outcome realizations. Proposition 4 contains a similar result for the optimal contract that induces the agent to adopt the conditional effort profile $(0, 1)$ that, in turns, produces a positive correlation between the aggregate shock and the managerial product. Notice, however, that in Celentani and Loveira (2006) the assumption over the managerial productivity leads to an optimal

[16] Otherwise the wage schedule $w(x, B)$ would be decreasing in x and this cannot be optimal.

contract displaying asymmetric benchmarking, while in proposition 4 it is the contract that is designed to induce such positive correlation. In other words, in Celentani and Loveira (2006) the positive correlation creates asymmetric benchmarking while here the opposite happens: to induce positive correlation, i.e. the conditional effort profiles $(0, 1)$, asymmetric benchmarking is needed. Notice also that in order to produce the desired correlation between managerial product and aggregate shock, the contract has to satisfy a larger number of incentive constraints (there are three possible deviations instead of one). Finally notice that at the same time that the contract is creating the desired positive correlation, it is also ruling out independence (i.e. the unconditional effort profiles) and the opposite negative correlation, i.e., the conditional effort profile $(1, 0)$. Similar remarks apply to proposition 5. Celentani and Loveira (2006) show that if the agent productivity and the aggregate state are negatively correlated, optimal payments are increasing in the benchmark for small outcome realization and decreasing in the benchmark for large realizations. Proposition 5 shows that one needs a contract with similar characteristics to create this negative correlation and to rule out other possible statistical relationships between the managerial productivity and the aggregate state.

Results contained in proposition 1 and 2 goes one step further in this direction. They show that, even if the optimal contract is designed to produce independence between aggregate shock and agent's productivity, wage payments can display asymmetric benchmarking. This happens because the optimal contract has to rule out possible correlations that could emerge when the agent observes a private signal on the strength of his indirect incentives.

1.4. Numerical Examples

This section contains numerical examples highlighting most of the findings of the paper. Consider an agent with a CRRA Bernoulli utility of the form $u(w) = \frac{w^{1-r}}{1-r}$ and the following parameter values: $r = 0.7$, $\underline{w} = 0$, reservation wage $\overline{W} = 1$ (corresponding to a reservation utility $\overline{U} = 3.\overline{3}$), $p(G) = 0.6$, $\gamma = 100$, $G = 300$, $\sigma = 150$, $\underline{x} = 0$, $B = 2.5$, $c = 1.5$. If the agent does not observe a private signal on the future state of the world, bankruptcy probabilities are $b(0) = 0.21$ and $b(1) = 0.10$. The minimum cost contract inducing high effort has an expected wage $EW = 5.54$ with a standard deviation $\sigma_W = 3.06$. Inducing high effort, the principal obtains profits equal to 283.24, while inducing low effort obtains 176.13. Figure 3 shows the optimal wage schedules in both the good and bad state. Notice that, the horizontal difference between the two schedules is exactly 300.

Consider now the case in which the agent can observe a private signal on the future state of the world up to a noise term $\varepsilon = 0.2$. The conditional bankruptcy probabilities are then : $b(0 \mid G) = 0.09$, $b(1 \mid G) = 0.04$, $b(0 \mid B) = 0.37$, $b(1 \mid B) = 0.18$.

Table 1: Optimal Contract Inducing (1,1)

Expected Wage		Standard Deviation	
EW	5.67	$sigma^{w}$	3.39
EW_G	6.12	$sigma^{w_G}$	3.97
EW_B	5.03	$sigma^{w_B}$	2.12

Table 1 shows wage expectations and standard deviations in both the good and bad state of the minimum cost contract implementing $(a_B, a_G) = (1, 1)$. The principal obtains profits equal to 283.11. Notice in particular that the ex ante expected wage is larger with respect to the case in which the agent does not observe a private signal. Furthermore, when the agent has private information, his expected wage is larger in the good state than in the bad state. As in Bertrand and Mullainathan (2001), managerial pay is then positively affected by the observable shock. Notice also that the wage volatility is higher in the good state because the manager has to receive a sharper incentive in this case. Figure 4 shows the optimal wage schedule implementing high effort after both realizations of the private signal. Notice that above the threshold $\hat{x}(0.2)\approx 455$ the wage payment is larger in the good state than in the bad state. Notice also that outcomes above $\hat{x}(0.2)$ have a 40% of probability in the good state but only a 1% of probability in the bad state. Furthermore, for outcomes below the threshold $\hat{x}(0.2)$, the wage payment is larger in the bad state. Therefore, the figure describes a situation in which the optimal wage displays asymmetric benchmarking.

Table 2: Optimal Contract Inducing (0,1)

Expected Wage		Standard Deviation	
EW	4.64	$sigma^w$	3.61
EW_G	5.26	$sigma^{w_G}$	4.37
EW_B	3.71	$sigma^{w_B}$	1.56

Table 2 shows wage expectations and standard deviations corresponding to the minimum cost contract implementing $(a_B, a_G) = (0, 1)$. With this contract profits are equal to 252.26. Notice that the ex ante expected wage is smaller with respect to previous cases because the agent is induced to provide effort only after the observation of signal $z = G$. For the same reason the expected outcome is smaller too and, given other parameter values, profits decrease. Notice that the expected wage payment is both larger and more volatile in state $y = G$. This is because the aggregate state G is relatively more likely after the observation of signal $z = G$, which is the signal that triggers high effort. Figure 5 shows the optimal wage schedule implementing the conditional effort profile $(0, 1)$. In this example, the incentive constraint corresponding to the observation of signal $z = B$ is not binding. Therefore, we observe the situation described in proposition 4. In particular, notice that for large outcome realizations (success), good luck is not completely removed, i.e. $w(x, G) > w(x - \delta, B)$. In other examples it is possible to obtain a wage structure that induces larger payments in the good state for outcomes above a certain threshold.

Table 3: Optimal Contract Inducing (1,0)

Expected Wage		Standard Deviation	
EW	3.96	$sigma^w$	2.19
EW_G	3.72	$sigma^{w_G}$	1.38
EW_B	4.32	$sigma^{w_B}$	2.94

Table 3 shows wage expectations and standard deviations generated by the minimum cost contract implementing $(a_B, a_G) = (1, 0)$. Corresponding profits are equal to 232.61. The expected wage is even smaller than for the contract implementing $(0, 1)$. This is because high effort here is induced after the observation of signal $z = B$ that sharpens indirect incentives. Notice that wage expectation and volatility are both larger in state B. Figure 6 shows the optimal wage schedule implementing the conditional effort profile $(1, 0)$. The incentive constraint corresponding to signal G is not binding so that proposition 5 applies. Notice in particular that payments are increasing in the benchmark for small outcome realizations while they are decreasing for large outcome realizations.

Finally, notice that the contract inducing the unconditional effort profile $(1, 1)$ maximizes profits in this example and, therefore, it would be adopted by the principal.

1.5. Conclusion

The model presented in this paper describes a mechanism that explains how an optimal contractual arrangement between a principal and an agent could display asymmetric benchmarking even if managerial productivity and aggregate shocks are uncorrelated. There are two key elements behind this result. First, the manager has implicit incentives deriving from the possibility of bankruptcy and, second, after signing the contract but before choosing his action, he observes a private signal on the future state of the world. The signal affects managerial indirect incentives: the observation of bad news increases the conditional probability of bankruptcy in case of misbehavior and therefore reduces the conflict of interest with the principal. The availability of the private signal also allows the agent to adopt conditional or unconditional effort profiles. Therefore it introduces the possibility of observing some correlation between managerial product and aggregate state, even if the exogenous productivity of managerial high effort is constant across states. The optimal benchmarking rule is used not only to filter out the observable shock but also to adjust the contractual explicit incentives to the variable implicit incentives. As a result, even when the managerial productivity and the observable shocks are uncorrelated, the optimal contract can display the kind of asymmetric benchmarking observed in the data.

The focus of the paper is on managerial incentives but, because the model adopted is very simple, it could readily be applied in different contexts where RPE considerations are important. For example, the recognition that apparently suboptimal practices, like asymmetric benchmarking, can indeed correspond to the most desirable arrangement may be important for the analysis of yardstick competition (Shleifer 1985, Sobel 1999). Regulated firms can in fact be induced to efficiently reduce their costs by setting up incentive schemes that relies on relative performances. The findings in this paper are then of some interest to asses the most desirable form of such incentives structures.

In this model, optimal contracts are usually non linear in the outcome. An interesting possibility would be to restrict to contracts with a base salary and a call option on the firm's stock. As in Aseff and Santos (2005) one could obtain the optimal contract within this class and evaluate how it performs relative to the optimal non linear contract. This modification of the model would also allow to study how the strike price of the optimal option plan is affected by the observable shock.

As a final remark notice that results in this paper rely on the interplay between explicit and implicit incentives. One could obtain similar results considering other sources of indirect incentives as long as they are sharper in bad states of the world. For example, negative aggregate shocks reducing the value of the firm, could increase the probability of a takeover or could trigger a closer monitoring activity by large stakeholders. In both cases managers would have stronger implicit incentives in bad states and optimal contracts should not overlook their effects.

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Figure 1: The reference benchmarking rule

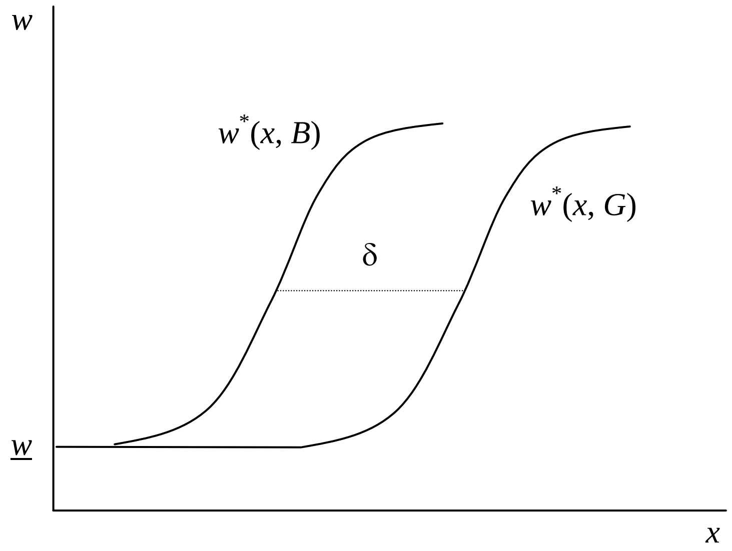


Figure 2: Asymmetric benchmarking

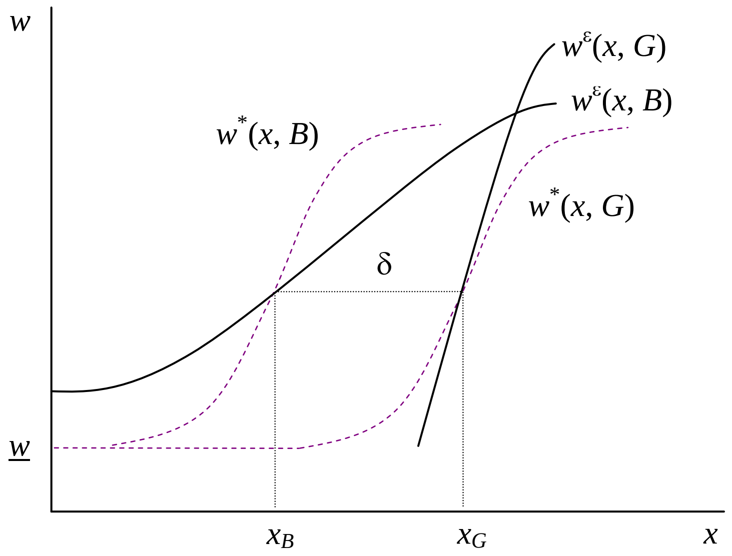


Figure 3: Minimum cost contract implementing high effort with no signal

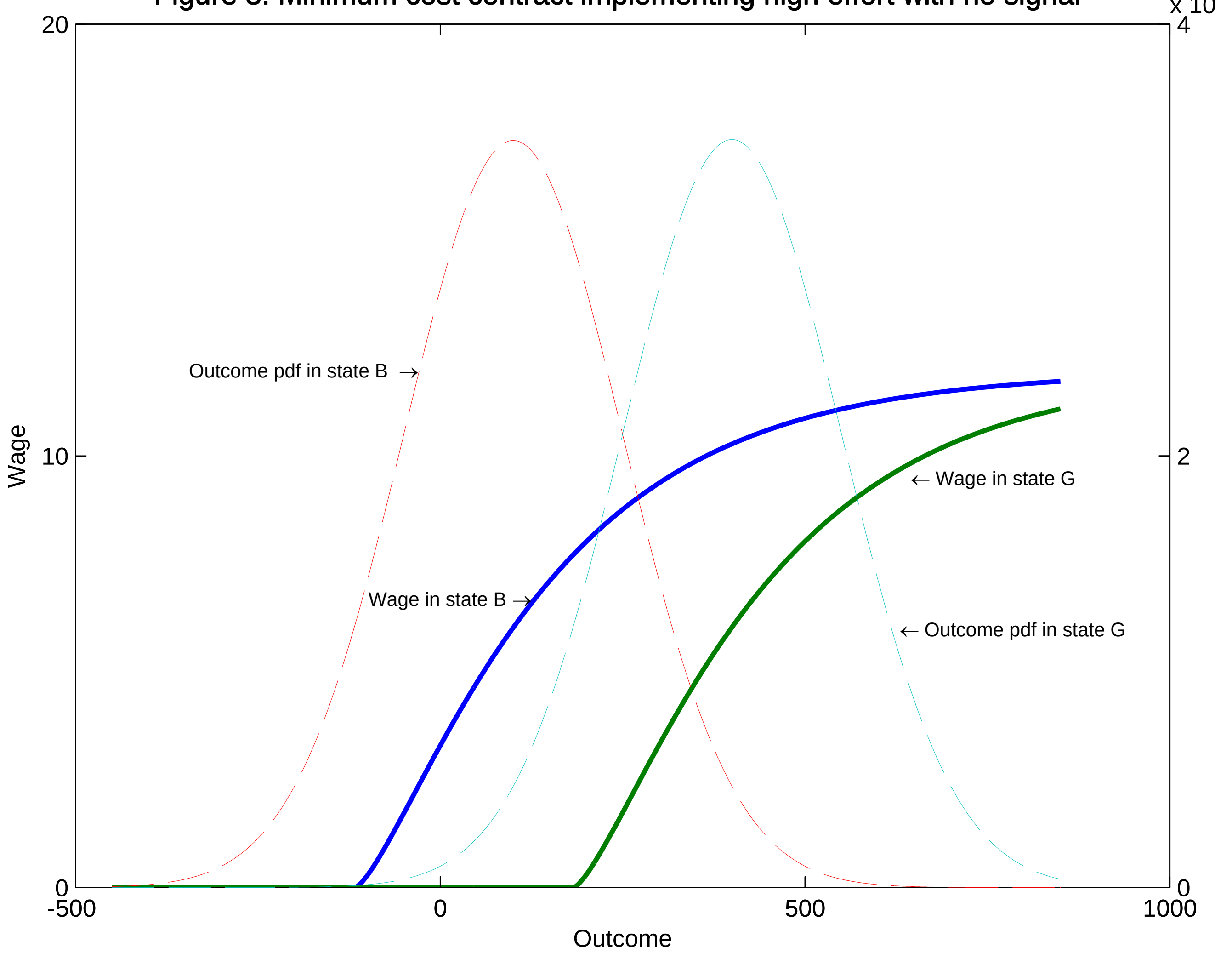
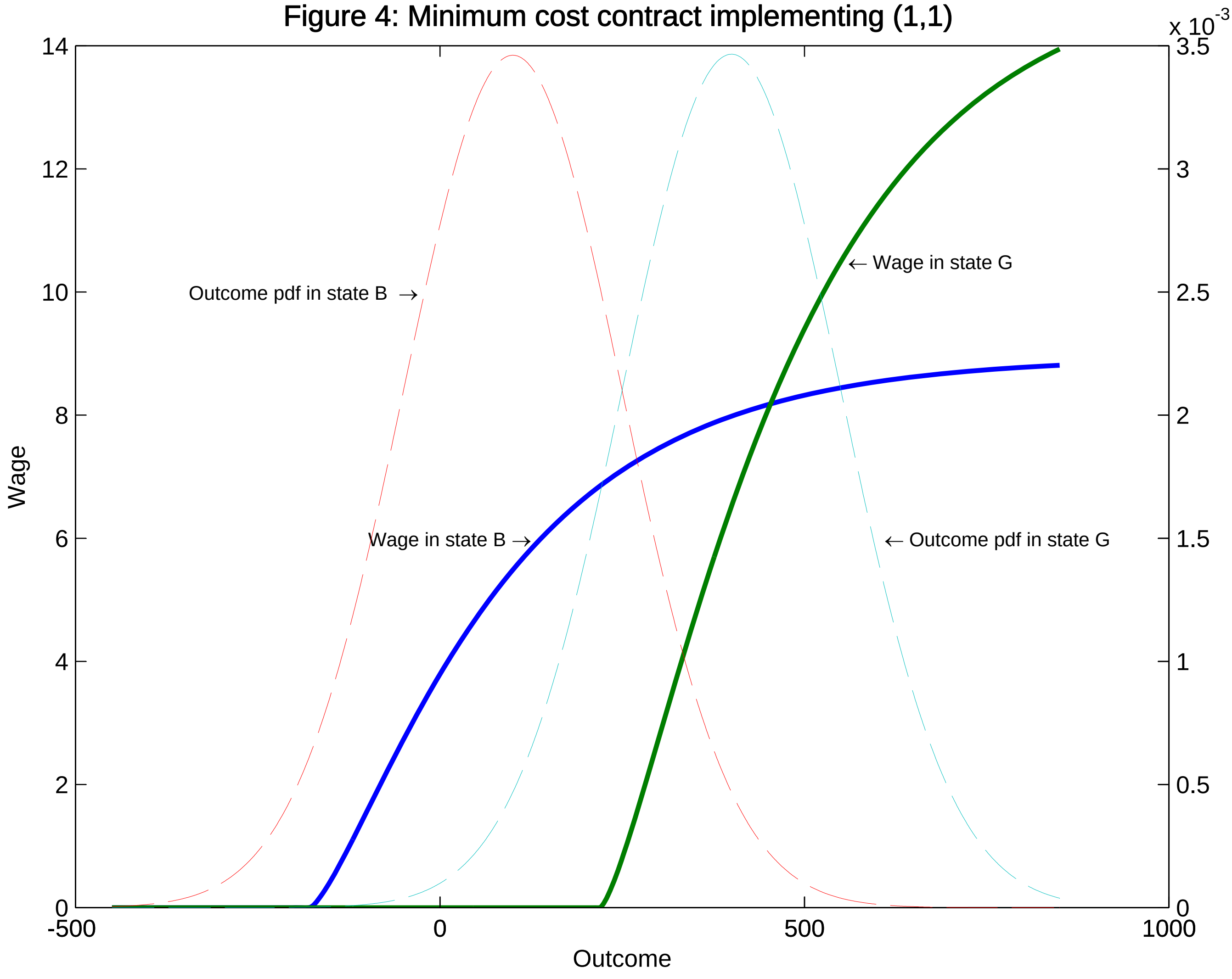


Figure 4: Minimum cost contract implementing (1,1)



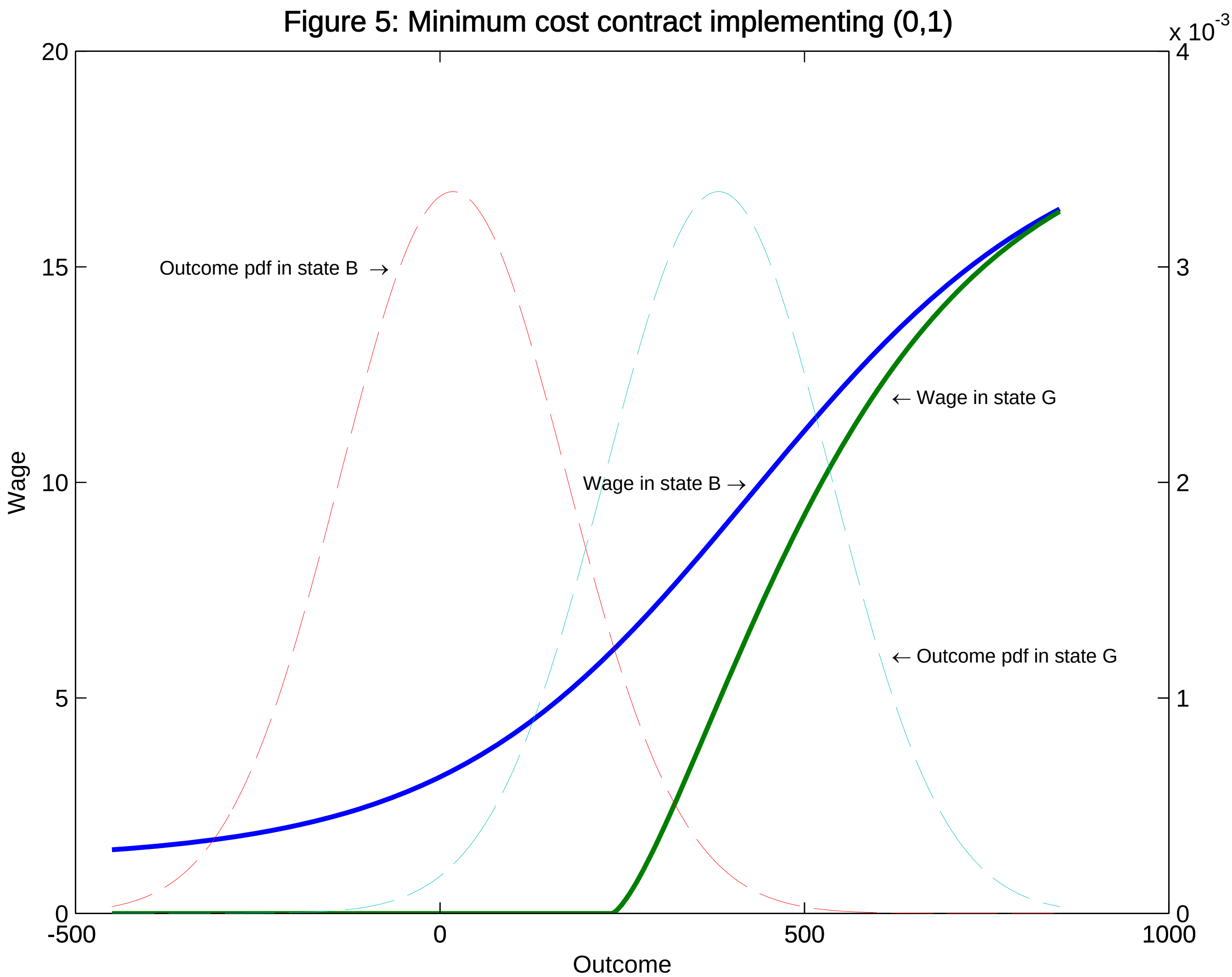
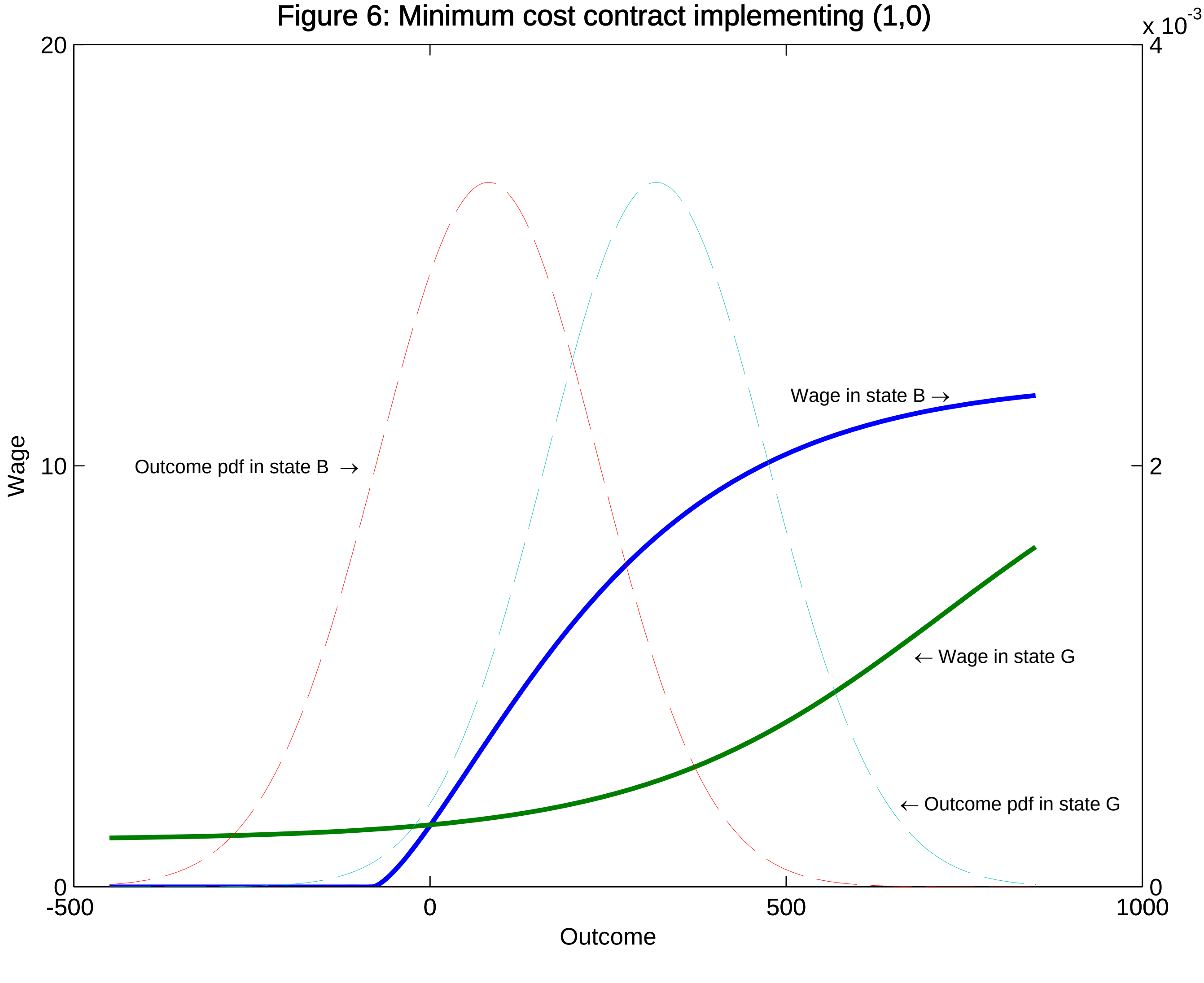


Figure 5: Minimum cost contract implementing (0,1)
x 10^-3
Wage
Outcome
Outcome pdf in state B →
← Wage in state G
Wage in state B →
← Outcome pdf in state G
20
15
10
5
0
4
3
2
1
0
-500
0
500
1000

Figure 6: Minimum cost contract implementing (1,0)



Chapter 2

Career Concerns and Competitive Pressure

In a duopoly model I study the effects of increased competitive pressure in the product market on managerial career concerns. In an early stage of their careers, managers have unknown ability and, with no further information, they look identical to firms. Later on, managers' observed performances, allow firms to learn about their ability and to rank them accordingly. Good managers (i.e. managers with good past performances) are both valuable and in short supply, so that firms are forced to compete for their hiring. In ideal conditions, good managers are able to capture their entire value, that coincides with the expected profits they produce in excess with respect to less talented managers. Competition in the product market affects such profit differential and, therefore, it also determines the strength of managerial career concerns. However, the effect of increased competition is ambiguous: it raises the reputational concerns to the extent that it makes to hire a good manager more valuable.

Keywords: career concerns, product market competition.

JEL classification: D43, D83, G30

2.1. Introduction

The idea that firms should be run in the owner's interest is usually accepted so that in modern public corporations, where property and management are commonly separated, a problem arises of providing managers with the right incentives to implement the shareholders' value. Most of the corporate governance literature addresses exactly this agency problem and describes a number of possible solutions to it. In their comprehensive survey, Becht et al. (2003) identify five mechanisms currently used to discipline managers: the presence of a large shareholder, the market for corporate control, the board of directors, executive compensation packages and, finally, the managers' loyalty duty coupled with an effective shareholder legal protection.[1] Even if in the last twenty years a large body of both empirical and theoretical analysis has emerged, the real functioning and effectiveness of these governance mechanisms are not well understood yet.

At least since Smith (1776), competition in the product market has been considered as a further source of managerial discipline. The basic idea goes as follows: in firms that operate under a strong competitive pressure, any lack of efficiency reduces profits and seriously threaten the survival possibility in the market. Therefore, managers concerned with the very conservation of their job, work as hard as they can to ensure profit maximization. As intuitive as it may appear at first, a closer consideration of this idea rises at least two questions. First: what does a "more competitive market" exactly mean? Second: which mechanisms create a link between the degree of product market competition and managerial behavior? A part from its intuitive incentive effect due to bankruptcy, competition may also interact with other sources of managerial incentives. For example, Hart (1983), Scharfstein (1988), Hermalin (1993) and Schmidt (1997), among others, analyze how the characteristics of the product market affect the optimal compensation package to be offered to managers, and competition has been assumed to either affects

[1] Another very well known survey of corporate governance is by Shleifer and Vishny (1997). Here the authors suggest that the essential elements of a successful governance system are some form of concentrated ownership and legal protection of investors. There is a number of other general treatment of the issue. For example Tirole (2001) tries to analyze the role of the so called stakeholder society while Zingales (1998) frames the corporate governance problem in an incomplete contract approach.

the information structure behind the optimal contract (Hart 1983, Scharfstein 1988), or firms profitability and the "incentive to give incentives" (Hermalin 1993, Schmidt 1997).

The contribution of this paper is to analyze how product market competition, through its effect on firms profitability, affects managerial career concerns.[2] The basic idea is that, at least for managers at the top of the firm's ladder, like the CEO or the CFO for example, career concerns mainly depends on the external labor market, that is, on alternative job opportunities in other firms. From the other hand, firms' willingness to pay for managerial talent depends on its profitability which, in turns, is determined by the characteristics of the product market. In this way, the level of product market competition affects managerial behavior through its effect on the strength of career concerns.

The model has two periods and in each period two firms compete in the product market. Firms have to hire managers before each round of product market competition, and they can only commit to pay a fixed wage. Hence, no explicit contractual incentives are allowed. Managerial ability and effort determine the firm efficiency and competitive strength on the product market and, therefore, its profits. In the first period managerial skills are symmetrically unknown to everybody, and effort is not observable by firms. All the managers have the same, commonly known, priors over their ability so that they are homogeneous from the firms' point of view. Because of this homogeneity, young managers have a very weak bargaining position in the labor market, here represented as a sequential game where firms offer a wage to each manager and then managers choose one of them (if any). Such labor market structure allows firms to hire young managers at the reservation wage. In period two, however, the observation of past performances allows firms to make some inferences about managerial skills. The manager who performed the best is now more valuable, and obtains a wage premium on the labor market equal to the extra profits that he is able to produce with respect to the other manager. This extra profit is referred

[2] Fama (1980) first proposed the idea that career concerns can be an important source of incentives in large corporation. In his original view, the agency problem created by the separation of ownership and control is completely resolved by the managerial incentives to build a good reputation. Later on, even if Holmstrom (1999) recognized that career concerns are in general suboptimal, a large body of literature as emerged, and managers are usually believed to derive powerful incentives from their careers. (For example, see Murphy, 1999, or Becht et al., 2003).

to as the value of efficiency in this paper, and it plays a fundamental role in determining the strength of career concerns. In fact, a central result of the analysis is that a change in the product market that rises the marginal value of efficiency, also increases managerial effort. The basic insight for this result is that the manager with the best past performance is more valuable, and firms are willing to pay more to hire him. In the ideal labor market described above, firms compete a là Bertrand for the best manager, so that the wage premium that they are forced to pay coincides with the amount of profits that he is expected to produce in excess to the other manager. Therefore, it is exactly such profit differential that determines how strong career concerns are, and it may either increase or decrease in a more competitive product market.

There are several papers in the literature that study the incentive effects of product market competition. However, they usually assume that managers sign a formal contract that makes their compensation contingent on some measure of the firm efficiency (cost reductions, accounting profits, etc.). Because optimal compensation packages depend on the competitiveness of the environment, the degree of competition indirectly affects managerial incentives. In the model by Hart (1983) the principal observes a cost index which depends on both an industry wide shock and the managerial effort. A more competitive product market then allows the principal to make better inferences about the agent's contribution. With a very special assumption on the agent preferences, Hart shows that agents work harder in more competitive markets. Scharfstein (1988) however shows that this conclusion is not robust to alternative specifications on the managerial utility function.

In a different vein, Hermalin (1993) and Schmidt (1997) assume that increased competition reduces firms profitability. In both cases smaller profits have an ambiguous effect on optimal contracts and managerial effort levels. A mechanism that induce ambiguity in both models is what Schmidt calls the value-of-a-cost-reduction effect and Hermalin calls the change-in-the-relative-value-of-actions effect. To grasp the general idea consider a situation in which the efficiency of a firm can be either high or low and let π_H and π_L be the corresponding profits in the two cases. Of course $\pi_H > \pi_L$. More competition decreases both π_H and π_L, but what is really relevant is how competition affects the difference $\pi_H - \pi_L$. Intuitively, $\pi_H - \pi_L$ measures the loss associated with a lack of efficiency, and the higher it is, the larger the equilibrium effort induced by optimal contracts. Because competition can either increase or decrease $\pi_H - \pi_L$, its incentive effect is ambiguous.[3] As discussed above, in this paper I show that a similar (ambiguous) effect also holds when managers have career concerns as their main source of incentives.

A different approach is taken by Willig (1987). Still retaining the usual principal-agent framework, he identifies increased competition in the product market with a smaller and more elastic (residual) demand function. From his analysis emerges that a smaller demand tends to reduce efficiency, while increased elasticity raises it. Again, the overall effect is ambiguous.

A common characteristic of the literature discussed so far, is that the strategic interaction among firms operating in an imperfectly competitive product market is ignored and the market structure is then assumed to be exogenous. An exception is Raith (2003): he explicitly analyzes a market game among firms run by managers rewarded in accordance to the cost reduction they induce. He shows that more substitutable products or a larger market size induce managers to provide more effort while a reduction in the entry cost reduces managerial effort. To some extent his results are still ambiguous: smaller entry cost or larger substitutability could both be regarded as increased competition but they have opposite effects on the equilibrium effort.[4]

The effect of competition on career concerns has received very little attention in the literature. An important exception is Vickers (1995). He suggests that the most basic characteristic of competition is the very existence of competitors, and, at least, it allows firms to evaluate their performance relative to that of other firms. In the case of career

[3] Schmidt also identifies a bankruptcy effect that unambiguously rises managerial effort: in a more competitive market the probability of bankruptcy is higher, and managers tend to work harder to keep their job. Contrary to Schmidt (1997), Hermalin (1993) allows for managerial risk aversion in his model, and, as a consequence, he finds that increased competition also has an income effect and a risk adjustment effect, both with an ambiguous sign.

[4] A remarkable property of his model is that changes in any parameter value result in a positive correlation between the pay-for-performance sensitivity and the profit volatility.

concerns, the performance of a competitor represents a further source of information that can be used to learn something about the unknown managerial ability. Of course, this form of relative performance evaluation is relevant only if there exists some correlation among agents' abilities in the market, and its effect depends on the sign of such correlation. In particular, if managerial abilities are positively correlated, then the observation of a good industry result is the signal that each manager's ability is indeed high, so that, in this case, future managerial wages are increasing in the market performance. Managers would therefore like to free ride on other firms' good performances, and incentives are in fact reduced in this case. The opposite happens if managerial abilities are negatively correlated.[5] However, in this paper I assume that unobservable managerial abilities are independently distributed so that I completely abstract from the learning effects suggested by Vickers.

The remainder of the paper is organized as follows. Section two introduces the basic model and characterizes the equilibrium effort. It is then argued that the main determinant of career concerns is the value of efficiency. Therefore, the impact of a change in the market environment on indirect incentives passes through its effect on the value of an efficient manager. In section three I consider some examples of explicit market games and, using the results previously obtained, I am able to evaluate how parameter values affect incentives. To conclude, section four contains some final remarks.

[5] Vickers also shows that the overall effect on the ex-ante incentives to provide effort depends crucially on the correlation of the measurement errors affecting individual performances: if there is a large positive correlation, incentives to provide effort are increased, the intuition being that if this correlation is strong the precision in observing managerial ability is higher and then any given level of effort has a higher impact on the learning process.

2.2. Career Concerns within a Duopoly

2.2.1. The Basic Model

There are two periods $t = 1, 2$ and in each period two firms compete in the product market. Each firm is made by a principal (the owner) and an agent (the manager) who has to be hired at the beginning of each period t with a constant salary w_t.[6] Contingent payments are not allowed and long term binding contracts cannot be signed. There are two managers to be hired whose innate ability, or skill, is symmetrically unknown at the beginning of period one. To make things simpler I assume that each manager has a reservation salary $\overline{w}$ which is independent of his age and past experience. The competitive strength of a firm is summarized by an efficiency parameter x whose value is affected by the managerial skill and activity. More precisely firm hiring manager i in period t has in that period an efficiency parameter:

$$x_{i,t} = \eta_i + e_{i,t} + \varepsilon_{i,t} \tag{2.1}$$

where η_i is manager i's innate ability (or skill or talent), $e_{i,t} \in [0, \overline{e}]$ is his effort in period t, and $\varepsilon_{i,t}$ is an idiosyncratic random component. The manager's ability and effort are then substitutes in rising such an efficiency parameter and then the firm strength in the product market. Such x-value can be thought of as some measure of what Leibenstein (1966) called X-efficiency, as opposed to allocative efficiency. The X-efficiency of a firm is typically determined by those cost reducing activities (plant restructuring, waste reductions, work methods and so on) that are directly under the managerial control. In period one $\eta_i \sim N(0, \sigma_\eta^2)$ while $\varepsilon_{i,t} \sim N(0, \sigma_\varepsilon^2)$ for any manager and any period. All random variables are independent. In what follows I identify a generic manager with the superscript i, while superscript j denotes the other one, finally superscript n denotes a generic firm.

[6] In the following the firm owner will be referred to as a female and the manager as a male.

The timing of events in period one is as follows: 1) Managerial abilities are determined independently according to a $N(0, \sigma_\eta^2)$, and no one observes them; 2) Firms bid to hire a manager; 3) Both managers decide how much effort to exert, and efficiency parameters are determined and publicly observed; 4) Firms compete in the product market. In period two events from 2) to 4) take place anew. Agents are assumed to be risk neutral and their utility is simply $w_1 + w_2 - g(e_1) - g(e_2)$, where $g(e)$ is the cost of exerting effort e. The function g is twice continuously differentiable strictly increasing and strictly convex, furthermore $g(0) = g'(0) = 0$ and $\lim_{e \to \bar{e}^-} g'(e) = \infty$. Firms maximize total expected profits.

In order to fully describe the extensive form game to be analyzed, it is necessary to specify how the bidding phase in point 2 is realized. I assume that both firms simultaneously submit a wage offer to each manager. Then, in period 1 an equal probability lottery decides which of the two managers has to make the choice between the offers he faces, if any, while the other manager will not be able to accept the offer received by the firm that closed its vacancy. In period two, if both managers worked in the first period, the manager who previously performed the best has the advantage of being the first to make a decision. Notice that observing exactly the same managerial efficiency in the first period is a zero probability event, and it induces a subgame in which managers are still identical, in such a case the rules of the first period are still applied. If a manager is not assumed by any firm in the first period, he will exit the industry so that he will not be on the labor market in the second period. His lifetime utility in this case is then $2\overline{w}$.

I consider these particular bidding rules to capture two relevant characteristics of the managerial labor market. First, in the market for young and inexperienced managers, firms have the strongest contractual position: the point here is that young managers are very close substitute to one another, for example because their past careers are not very informative about their talent as CEO in that particular industry, so that they compete very closely and firms can finally extract almost all the surplus generated by the relationship (in fact all the surplus in the model). Second, a senior manager with a good past performance is a "scarce good" in the managerial labor market and then he has a stronger bargaining position allowing him to obtain part of the surplus. The model captures this feature with the rule that assigns to the good manager the priority in choosing between the firm offers.

For the purposes of this paper it is better not to consider an explicit market game. I will rather describe the firms interaction in the product market by means of a (reduced form) profit function. In particular if $x_t = (x_{1,t}, x_{2,t})$ are the realized efficiency parameters in period t, for firm 1 and 2 respectively, product market competition yields to the firm hiring manager i at time t the amount of profits:

$$\pi_{i,t} = \pi(\phi, x_{i,t}, x_{j,t}), \tag{2.2}$$

where the function $\pi : \Phi \times \mathbb{R}^2 \to \mathbb{R}$ is bounded both above and below, it is increasing in $x_{i,t}$ and decreasing in $x_{j,t}$ (monotonicity properties being strict in at least one argument) and, furthermore, it is twice continuously differentiable. The parameter ϕ belongs to some open interval $\Phi \subset \mathbb{R}$, and will be used to index the degree of competition in the product market with the interpretation that a higher ϕ corresponds to a more competitive environment.

Notice that with the assumptions made on the function π, a higher efficiency parameter corresponds to higher profits for the firm who realizes it, and to lower profits for its competitor. Then a larger efficiency parameter corresponds to a stronger firm in the market. Also notice that firms are in a substantial symmetric position in the market: only realized efficiency parameters determine profits and not their particular distribution among firms. This formalization, then, is not descriptive of those circumstances in which some firms have other sources of market power, as for example it would be the case if one of the firms were a Stackelberg leader, or had an information advantage over the demand structure, etc.[7] In principle, a firm could be run without a manager, that would be the

[7] However, the formulation could easily allow for changing market conditions between period one and two: it would be enough to have different profit functions in the two periods. To add this possibility wouldn't change anything in the analysis so I prefer to stay with the notation introduced in the text.

case if some manager prefers the outside option, but I assume that the profits would then be so low that any strategy involving wage offers below the managers' reservation value are weakly dominated and are suboptimal whenever the probability of hiring nobody is positive. More precisely, I assume that if a firm doesn't hire a manager its profits are $\underline{\pi}(\phi) < \inf_{x,y} \pi(\phi, x, y) - \overline{w}$, while a firm managed with efficiency x and facing a competitor with no manager obtains profits equal to $\overline{\pi}(\phi, x) = \sup_y \pi(\phi, x, y)$.

How the degree of product market competition (here indexed with the parameter ϕ) affects the amount of profits that can be earned is not very clear in general terms. For example, Boone (2002, 2004) analyzes several examples of oligopoly markets where the strength of competition is naturally identified with the value of some parameter.[8] He finds that as competition increases, the amount of profits earned by the least efficient firm decreases, but he also finds that the ratio between the profits of any firm and those of a less efficient one increases. With identical firms this result simply means that increased competition decreases profits for every firm. However, when firms with different efficiencies coexist in the market, the result suggests the traditional "selection effect" of competition already described for example in Vickers (1985).[9] Since the main focus of this paper is on the relationship between the strength of career concerns and product market competition, I'll rather consider the following two alternative conditions that, as it will be shown thereafter, play a crucial role.

Condition 1 (IVE) *For each (x, y) the difference $\pi_2(\phi, x, y) - \pi_3(\phi, y, x)$ is strictly increasing in ϕ.*

Condition 2 (DVE) *For each (x, y) the difference $\pi_2(\phi, x, y) - \pi_3(\phi, y, x)$ is strictly decreasing in ϕ.*

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[8] He considers three different sources of increased competition: a larger number of firms, more aggressive market interactions and more efficient competitors.

[9] Boone also finds that some quantities commonly used to empirically asses the degree of competition in an industry (e.g. the Herfindhal index, price cost margins, etc.) are not monotonic in the level of competition as measured by the relevant parameter. He then proposes a new empirical measure based on profit ratios.

To interpret these VE (marginal Value of Efficiency) conditions, let's consider the difference $\pi(\phi, x, y) - \pi(\phi, y, x)$. It represents the profits differential that a firm of efficiency x can produce when it competes with a firm of efficiency y. Such differential has of course the sign of $x - y$ and depends on the product market degree of competition. The derivative $\pi_2(\phi, x, y) - \pi_3(\phi, y, x)$ represents then the marginal value of efficiency x, and condition IVE (Increasing VE) requires it to be increasing in the value of competition, while condition DVE (Decreasing VE) requires it to be decreasing in ϕ. According to IVE, then, to be more efficient is more important in a more competitive environment.[10] The opposite is true according to DVE. None of these conditions is to be intended an exact characterization of how the strength of competition in the product market affects firm profitability. The idea of imperfect competition is indeed a vague one. It refers to the existence of some kind of rivalry among firms that strategically interact in the product market, but the exact nature of such rivalry, as well as its intensity and consequences, have to be better specified in any particular context. Broadly speaking, competition has both an exogenous and an endogenous component: there are characteristics in the product market such as entry fees, size, substitutability among different brands of the same product, transparency, the eventual threat of a potential entrant etc, that naturally affect the strength of the firm competition within an industry. These elements are, to a large extent, exogenous and in this model I exactly refer to these kind of determinants of the market competitiveness. However, the number of firms in any particular industry as well as their respective market shares are important determinants of the degree of competition and, of course, they are endogenous. I will not attempt to consider this other aspect of competition in this paper. In principle, the exogenous characteristics of a markets determines its endogenous structure so that changes in the first can affect the second and the overall effect will be the sum of the two. Hence, it is incomplete to analyze only the effects of exogenous elements but this is a first step toward a better

[10] Note that for each (x, y) the difference $|\pi(\phi, x, y) - \pi(\phi, y, x)|$ measures the value of the most efficient manager and condition IVE implies that such quantity is strictly increasing in ϕ. This latter condition closely resembles the selection effect of increased competition found by Boone in his examples, but it is here expressed in terms of differences in profits rather than profits ratios.

understanding of the whole process. Of course, changes in the exogenous structure that do not affect the number of firms find here a complete treatment.

2.2.2. Equilibrium Analysis

The concept of equilibrium that will be used is the Perfect Bayesian Equilibrium, which will be simply referred to as the equilibrium. A pure strategy for a firm specifies in each period a wage offer for each manager on he labor market as a function of the observed history of the game.[11] A pure strategy for a manager has to specify in each period which offer to accept (if any) and the level of effort to exert in case he is hired by a firm as a function of the past observed history of the game. I will only consider pure strategy equilibria. Players also have beliefs about managerial talents. In period one everybody shares the same priors described above. The (possible) observation of the first period efficiency parameters then allows to update such beliefs in period two. The process of belief revision taking place after he observation of first period efficiency parameters depends on the amount of effort that i is expected to exert in period one, say $\widehat{e}_{i,1}$. Given (2.1) and given such expectation, the observation of $x_{i,1}$ is equivalent to the observation of:

$$z_{i,1} = \eta_i + \varepsilon_{i,1} = x_{i,1} - \widehat{e}_{i,1}.$$

A simple process of normal learning then then takes place and the updated beliefs about manager i's ability is $\eta_i / x_{i,1} \sim N(\tau z_{i,1}, \tau\sigma_\varepsilon^2)$, where $\tau = \frac{\sigma_\eta^2}{\sigma_\eta^2+\sigma_\varepsilon^2}$ is the signal to noise ratio. Note that manager i could choose in principle $e_{i,1} \neq \widehat{e}_{i,1}$ so distorting the market learning process about his talent. In such a case, from manager i's standpoint, it would result that $\eta_i / x_{i,1} \sim N(\tau(z_{i,1} + e_{i,1} - \widehat{e}_{i,1}), \tau\sigma_\varepsilon^2)$ which first order stochastic dominates the previous one as long as $e_{i,1} > \widehat{e}_{i,1}$. However, in equilibrium firms have rational expectation in the sense that they correctly anticipate the level of effort chosen by the managers.

[11] Hence, if a manager doesn't work in period one he won't be on the labor market in period two so that, after any such history, firms cannot make any wage offer to such manager.

Given $\widehat{e}_{i,1}$ and $\widehat{e}_{j,1}$, if both managers are hired in period one, the firm hiring manager i has first period expected profits, gross of wage payments, given by:

$$\Pi_{i,1} = E_{(\varepsilon_1,\eta)}\left[\pi(\phi, \eta_i + \widehat{e}_{i,1} + \varepsilon_{i,1}, \eta_j + \widehat{e}_{j,1} + \varepsilon_{j,1})\right],$$

while if only manager i is hired in period one his principal first period expected gross profits are:

$$\overline{\Pi}_{i,1} = E_{(\varepsilon_1,\eta)}\left[\bar{\pi}(\phi, \eta_i + \widehat{e}_{i,1} + \varepsilon_{i,1})\right],$$

where the expectation above are evaluated at the beginning of period one and take into account the prior distribution of the random variable $\eta = (\eta_i, \eta_j)$.

It is immediate to recognize that in period two a hired manager has no incentive to provide a positive level of effort, that is $e_{i,2} = e_{j,2} = 0$. Hence, if both managers are rehired in period two, the firm hiring manager i has expected second period gross profits given by:

$$\Pi_{i,2} = E_{(\varepsilon_2,\eta)}\left[\pi(\phi, \eta_i + \varepsilon_{i,2}, \eta_j + \varepsilon_{j,2}) \,|\, x_1\right].$$

From the other hand, if only manager i is rehired in period two, his principal has expected second period profits, gross of wage payments, given by:

$$\bar{\Pi}_{i,2} = E_{(\varepsilon_2,\eta)}\left[\bar{\pi}(\phi, \eta_i + \varepsilon_{i,2}) \,|\, x_{i,1}\right],$$

where these second period expectations are evaluated at the beginning of period two, using the distribution of η resulting after the observation of x_1.

In period two, after the observation of first period efficiencies, managers are no longer homogeneous in terms of their talent, even if they are both expected to exert no effort. Furthermore, firms compete à la Bertrand to hire the most skilled of them, so that they will end up paying out to the good manager a wage premium that completely exhausts the profit differential he is able to produce in the product market. This intuition is shown in the following two lemmas.

Lemma 1 *In any equilibrium firms earn the same amount of expected net profits in each subgame starting at the beginning of period two.*

Proof There are three different types of subgames starting at the beginning of period two. First, in subgames following histories where no manager was hired in the first period both firms obtain $\underline{\pi}(\phi)$ with probability one. Second, in subgames following histories where exactly one manager was hired in the first period, say manager i, at least one principal won't be able to hire a manager in period two and her profits will be $\underline{\pi}(\phi)$. Let $\bar{\Pi}_{i,2} - w_{i,2}$ be the expected profits of the other principal when she hires the manager with a salary $w_{i,2} \geq \bar{w}$(note that in equilibrium it is not possible that the manager turns out to be unemployed in period two). If $w_{i,2} > \bar{\Pi}_{i,2} - \underline{\pi}(\phi)$ the principal hiring the manager would prefer not to hire him and if $w_{i,2} < \bar{\Pi}_{i,2} - \underline{\pi}(\phi)$ the principal hiring no manager could make a wage offer $w_{i,2} + \varepsilon$ that for $\varepsilon \in \left(0, \bar{\Pi}_{i,2} - \underline{\pi}(\phi)\right)$ attracts the manager and allow the deviating firm to increase its profits. Hence, it must be $w_{i,2} = \bar{\Pi}_{i,2} - \underline{\pi}(\phi)$. Finally, in subgames following histories where both managers were hired in the first period, they are rehired in the following period in any equilibrium. Hence, let $w_{i,2}$ and $w_{j,2}$ be their salaries in period two, it must be shown that:

$$\Pi_{i,2} - w_{i,2} = \Pi_{j,2} - w_{j,2}.$$

Assume by contradiction that $\Pi_{i,2} - w_{i,2} > \Pi_{j,2} - w_{j,2}$, i.e. it is more profitable to hire manager i. There must be at least one principal hiring manager j with positive probability and she could attract manager i with probability one by offering him a slighter higher wage, say $w_{i,2} + \varepsilon$ (with $\varepsilon > 0$) and withdrawing at the same time the wage offered to manager j, such a deviation is convenient for each $\varepsilon < \frac{1}{2}\left[(\Pi_{i,2} - w_{i,2}) - (\Pi_{j,2} - w_{j,2})\right]$. A similar contradiction arises from the alternative assumption $\Pi_{i,2} - w_{i,2} < \Pi_{j,2} - w_{j,2}$ and this completes the proof. ■

Lemma 2 *In any equilibrium both managers are hired in both periods. Furthermore, the wage earned by manager i in period one is independent of $e_{i,1}$ while the wage he earns* *in period two is given by:*[12]

$$w_{i,2} = \bar{w} + (\Pi_{i,2} - \Pi_{j,2})\, I\,(x_{i,2} \geq x_{j,2})\,.$$

Proof Let's show first that both managers are employed in period one. If, by contradiction, manager i doesn't work in the first period, his lifetime utility is $2\bar{w}$ and at least one firm is earning $\underline{\pi}(\phi)$ in the same period. Lemma 1 then implies that such a principal will not be able to earn more than $\underline{\pi}(\phi)$ in period two either, so that to offer $\bar{w} + \varepsilon$ to manager i in period one is a profitable deviation for each $\varepsilon \in (0, \Pi_{i,1} - \underline{\pi}(\phi))$, because she attracts manager $i$with such an offer and obtain strictly larger expected net profits. Hence, both managers are hired in the first period and this immediately implies that they are both hired also in period two. To obtain the expression of $w_{i,2}$ consider first the situation in which $x_{i,1} < x_{j,1}$. Let's show that in this case $w_{i,2} = \bar{w}$. Assume by contradiction that $w_{i,2} > \bar{w}$ (note that it cannot be $w_{i,2} < \bar{w}$ in any equilibrium) and consider the following alternative bid for the principal hiring manager i: the offer made to manager i is slightly reduced while the other is kept constant. With such a strategy she will still end up hiring manager i but at a smaller wage and then she has an incentive to deviate. Similarly, if $x_{i,1} > x_{j,1}$ then $w_{j,2} = \bar{w}$. Note also that if $x_{i,1} = x_{j,1}$, then $\Pi_{i,2} = \Pi_{j,2}$, so that it must be $w_{i,2} = w_{j,2} = \bar{w}$ since there's no need for a firm to offer more in order to hire one of the two perfectly identical managers. Together with lemma 1 and the fact the both managers always work in a firm, this last result immediately implies the expression given for $w_{i,2}$. To complete the proof it remains to show that $w_{i,1}$ doesn't depend on $e_{i,1}$ but this is trivially true since firms cannot observe effort levels (however the wage offers in the first period do depend on $\widehat{e}_{i,1}$ and $\widehat{e}_{j,1}$). ■

Lemma 2 makes it clear that there are two reasons for a manager to build a career through the exertion of some positive level of effort in the first period. First of all, a manager can earn more than the reservation wage only if he performs better than the

[12] Thereafter I'll use the notation $I(E)$to denote the indicator function of an event E, that is, $I(E) = 1$ if E is true and $I(E) = 0$ otherwise.

other one and, second, the wage premium that the best manager obtains, increases with his perceived ability.

Also notice that the lemma suggests the existence of what could be called an "implicit, lagged relative performance evaluation": the wage earned by a manager in period two depends on his relative performance in period one, and this is not an explicit contractual agreement but simply a consequence of the firm equilibrium behavior in the model.[13]

The fact that in this model the best manager completely appropriates the extra profits he is able to produce may seem quite extreme. In a more realistic model firms would retain part of such profits but, what is really at stake here is how the competitiveness of the firms' product market shapes the managerial incentives to build a career. The previous lemma, then, simply suggests that such incentives depend on how profitable to hire a good manager is, which in turn depends on the characteristics of the product market. It seems reasonable to expect that in markets where firms profitability is not strongly linked to their X-efficiency, the incentives for managers to build a career are probably not very high. To make this point more explicit, consider the following quantities:

$$
\begin{aligned}
z_{i,1} &= \eta_i + \varepsilon_{i,1} \sim N\left(0, \sigma_\eta^2 + \sigma_\varepsilon^2\right); \\
z_{i,2} &= \eta_i + \varepsilon_{i,2} \sim N\left(\tau\left(z_{i,1} + e_{i,1} - \widehat{e}_{i,1}\right), (1+\tau)\sigma_\varepsilon^2\right); \\
z_{j,1} &= \eta_j + \varepsilon_{j,1} \sim N\left(0, \sigma_\eta^2 + \sigma_\varepsilon^2\right); \\
z_{j,2} &= \eta_j + \varepsilon_{j,2} \sim N\left(\tau z_{j,1}, (1+\tau)\sigma_\varepsilon^2\right).
\end{aligned}
$$

Their interpretation is as follows: $z_{i,1}$ and $z_{j,1}$ are simply the sum of the unknown talent and the noise term in the first period for manager i and, respectively, manager j. Notice in particular that the distributions of such quantities are those commonly held at the beginning of period one, and they are independent of the managerial choice of effort. From the other hand the quantity $z_{i,2}$ represents manager i's ability plus the second period

[13] When managers are given contingent compensation contracts, the use of relative performance evaluation allows to reduce the managerial exposure to risk, and, therefore, the cost of incentive provision. However, in this model managers receive a fixed wage payment in each period, and the use of relative performance evaluation emerges for a completely different reason.

noise, and its distribution is conditioned on the first period information (here summarized by $z_{i,1}$) in the hypothesis that manager i exerts effort $e_{i,1}$ while he is expected to exert effort $\widehat{e}_{i,1}$. The quantity $z_{j,2}$ has a similar meaning for manager j but its distribution is computed assuming that his effort choice is correctly anticipated (and is equal to $\widehat{e}_{j,1}$). Hence, increasing the effort he provides, manager i can bias firms' learning process making the distribution over his ability in period two better, in the sense of first order stochastic dominance. Of course, in equilibrium the manager will not fool the market.

In terms of the notation just defined, manager i is paid above his reservation wage in period two if and only if $z_{i,1} > z_{j,1} + \widehat{e}_{i,1} - e_{i,1}$, furthermore, the wage premium that she obtains in such an event can be written as follows:

$$wp\left(\phi, z_{i,1} + e_{i,1} - \widehat{e}_{i,1}, z_{j,1}\right) = E_{(z_{i,2}, z_{j,2})}\left[\pi\left(\phi, z_{i,2}, z_{j,2}\right) - \pi\left(\phi, z_{j,2}, z_{i,2}\right)\right].$$

Notice that this quantity is twice continuously differentiable. At the beginning of period one manager i's expected wage in period two can then be written as follows:

$$w\left(\phi, e_{i,1} - \widehat{e}_{i,1}\right) = \bar{w} + E_{(z_{i,1}, z_{j,1})}\left[wp\left(\phi, z_{i,1} + e_{i,1} - \widehat{e}_{i,1}, z_{j,1}\right) I\left(z_{i,1} + e_{i,1} - \widehat{e}_{i,1} > z_{j,1}\right)\right] =$$
$$\bar{w} + \textstyle\int_{-\infty}^{\infty}\int_{v+\widehat{e}_{i,1}-e_{i,1}}^{\infty} wp\left(\phi, u + e_{i,1} - \widehat{e}_{i,1}, v\right) dF_{z_{i,1}}(u) dF_{z_{j,1}}(v).$$

The above quantity, which is continuously differentiable and strictly increasing in $e_{i,1}$, can be used to characterize the equilibrium effort exerted in the first period. This is done in the following proposition.

Proposition 1 *In any equilibrium the two managers choose in the first period the same level of effort $e_1(\phi)$ which is uniquely identified by the condition:*

$$\int_{-\infty}^{\infty}\int_{v}^{\infty} wp_2(\phi, u, v) dF_{z_{i,1}}(u) dF_{z_{j,1}}(v) = g'\left(e_1(\phi)\right). \qquad (2.3)$$

Proof As a preliminary step, let's show that for each $(\phi, u, v) \in \Phi \times \mathbb{R}^2$, the quantity wp_2 is indeed strictly positive. Consider a given $(\phi, u, v) \in \Phi \times \mathbb{R}^2$, and define the random variables $x \sim N\left(\tau u, (1+\tau)\,\sigma_{\varepsilon}^{2}\right)$ and $y \sim N\left(\tau v, (1+\tau)\,\sigma_{\varepsilon}^{2}\right)$. It is therefore possible to write:

$$wp\left(\phi, u, v\right) = E_{(x,y)}\left[\pi\left(\phi, x, y\right) - \pi\left(\phi, y, x\right)\right],$$

where the difference $\pi\left(\phi, x, y\right) - \pi\left(\phi, y, x\right)$ is strictly increasing in x. Since a larger value for u induces a strictly dominant distribution on x (in the sense of first order stochastic dominance), it result that $wp_2\left(\phi, u, v\right) > 0$. This property of wp_2 is important: it ensures that the left hand side of (2.4), measuring the marginal value of effort for manager i, is indeed strictly positive, so that (2.4) has in fact a unique solution. Let's now show that condition (2.4) effectively identifies the optimal effort level for both managers in period one. To do so, notice that the level of effort exerted by manager i in period one can only affect his expected wage in period two, hence, in equilibrium, his choice solves the following problem:

$$\max_{e_{i,1} \in [0,\bar{e}]} w\left(\phi, e_{i,1} - \widehat{e}_{i,1}\right) - g\left(e_{i,1}\right).$$

The solution $e^{*}\left(\widehat{e}_{i1}\right)$ exists and maps $[0, \bar{e}]$ into itself. Furthermore, such solution must be interior and, therefore, it is identified by the first order condition $w'\left(\phi, e^{*}\left(\widehat{e}_{i1}\right) - \widehat{e}_{i,1}\right) = g'\left(e^{*}\left(\widehat{e}_{i1}\right)\right)$. Furthermore, the equilibrium effort chosen by manager i must be correctly anticipated by the firm, i. e. it must be a fixed point of the function $e^{*}\left(e\right)$. When the objective function in the above maximization problem is not quasi concave, such fixed point, call it $e_{i,1}(\phi)$, may not exist, and it would therefore not be possible to obtain an equilibrium in pure strategy. However, when such fixed point does exist, it is the unique solution to the equation $w'\left(\phi, 0\right) = g'\left(e_{i,1}(\phi)\right)$ which, taking into account that $wp(\phi, v, v) = 0$ for each ϕ and v, can be written as follows:

$$\int_{-\infty}^{\infty} \int_{v}^{\infty} wp_2(\phi, u, v) dF_{z_{i,1}}(u) dF_{z_{j,1}}(v) = g'\left(e_{i,1}(\phi)\right). \tag{2.4}$$

Notice also that the equilibrium effort level chosen by manager j is similarly characterized by:

$$\int_{-\infty}^{\infty}\int_{v}^{\infty} wp_2(\phi,u,v)dF_{z_{j,1}}(u)dF_{z_{i,1}}(v) = g'\left(e_{j,1}(\phi)\right). \tag{2.5}$$

Since the random variables $z_{i,1}$ and $z_{j,1}$ are identically distributed, conditions (2.4) and (2.5) both coincides with condition (2.3). This shows the statement in the proposition. ■

Notice that, in equilibrium, firms correctly anticipate the managerial effort in the first period, so that, in equilibrium, it does not affect the expected wage that a manager will earn in period two. In fact, such quantity can be computed as follows:

$$w^* = w(\phi,0) = \int_{-\infty}^{\infty}\int_{v}^{\infty} wp(\phi,u,v)dF_{z_{j,1}}(u)dF_{z_{i,1}}(v). \tag{2.6}$$

The above proposition characterizes the effort level that managers choose in equilibrium in the first period. Such effort is a proxy for X-efficiency within the industry, and it depends on the level of competition in the product market. Therefore, implicitly differentiating expression (2.3) with respect to ϕ it is possible to evaluate how competition affects managerial incentives to build a career:

$$e_1'(\phi) = \frac{\int_{-\infty}^{\infty}\int_{v}^{\infty} wp_{12}(\phi,u,v)dF_{z_{i,1}}(u)dF_{z_{j,1}}(v)}{g''\left(e_1'(\phi)\right)}. \tag{2.7}$$

The following proposition establishes some comparative statics results.

Proposition 2 *If condition IVE is satisfied, then $e_1'(\phi) > 0$, while if condition DVE is satisfied, then $e_1'(\phi) < 0$.*

Proof The proposition immediately follows from the property that, for each $(\phi,u,v) \in \Phi\times\mathbb{R}^2$, if condition IVE is satisfied, then $wp_{12}(\phi,u,v) > 0$, while, if condition DVE is satisfied, then $wp_{12}(\phi,u,v) < 0$. To show this property, consider a given $(\phi,u,v) \in \Phi\times\mathbb{R}^2$, and define the random variables $x \sim N\left(\tau u,(1+\tau)\sigma_{\varepsilon}^2\right)$ and $y \sim N\left(\tau v,(1+\tau)\sigma_{\varepsilon}^2\right)$.

Therefore, it is possible to write:

$$wp_1(\phi, u, v) = E_{(x,y)}\left[\pi_1(\phi, x, y) - \pi_1(\phi, y, x)\right].$$

The difference $\pi_1(\phi, x, y) - \pi_1(\phi, y, x)$ is strictly increasing in x if condition IVE is satisfied and it is strictly decreasing in x if condition DVE is satisfied. Since a larger value for u induces a strictly dominant distribution on x (in the sense of first order stochastic dominance), the sign of wp_{12} is as claimed above. ■

Notice that VE conditions are sufficient but by no means necessary for the result in the above proposition. In particular situations, weaker versions of them could suffice. For example, consider the following weak version of the VE conditions:

Condition 3 **(IVE-W)** *For each (x, y) with $x > y$ the difference $\pi_2(\phi, x, y) - \pi_3(\phi, y, x)$ is strictly increasing in ϕ.*

Condition 4 **(DVE-W)** *For each (x, y) with $x > y$ the difference $\pi_2(\phi, x, y) - \pi_3(\phi, y, x)$ is strictly decreasing in ϕ.*

In other terms conditions IVE-W requires that the derivative $\pi_1(\phi, x, y) - \pi_2(\phi, y, x)$ be increasing in x in the hemiplane $x > y$ only. Similarly, condition DVE-W requires that $\pi_1(\phi, x, y) - \pi_2(\phi, y, x)$ be decreasing in x only for $x > y$. The following proposition 3 states that under the weaker version of the VE conditions, the same result as in proposition 1 holds, provided that in period two the residual uncertainty on the managerial talent is small enough.

Proposition 3 *If condition IVE-W is satisfied, then it exists $\sigma^+ > 0$ such that $\sigma_\epsilon^2 < \sigma^+ \Rightarrow e_1'(\phi) > 0$. Similarly, if condition DVE-W is satisfied, then it exists $\sigma^- > 0$such that $\sigma_\epsilon^2 < \sigma^- \Rightarrow e_1'(\phi) < 0$.*

Proof I only show the first statement in the proposition, the second one is similar. Let's proceed in two steps.

Step 1 I first show that, if $\sigma_\epsilon^2 = 0$ and condition IVE-W is satisfied then it results $e_1'(\phi) > 0$. Given an expectation $\widehat{e}_1$ on the first period effort, the observation of x_{i1}

perfectly reveals the efficiency of manager i which is $\eta_i = x_{i1} - \widehat{e}_1$. By choosing a different level of effort, say $e_{i,1}$, manager i could induce the market to believe him of talent $\eta_i + e_{i,1} - \widehat{e}_1$ and then the expected second period wage for manager i can be written as follows:

$$w(e_{i,1} - \widehat{e}_1) = \overline{w} +$$
$$\int_{-\infty}^{\infty} \int_{v+\widehat{e}_1 - e_{i,1}}^{\infty} \pi\left(\phi, u + e_{i,1} - \widehat{e}_1, v\right) - \pi\left(\phi, v, u + e_{i,1} - \widehat{e}_1\right) dF_{z_{i,1}}(u) dF_{z_{j,1}}(v).$$

Thus, the first period equilibrium effort $e_1(\phi)$ is characterized by:

$$\int_{-\infty}^{\infty} \int_{v}^{\infty} \left[\pi(\phi, u, v) - \pi(\phi, v, u)\right] dF_{z_{i,1}}(u) dF_{z_{j,1}}(v) = g'\left(e_1(\phi)\right),$$

from which, implicitly differentiating, one obtains:

$$e_1'\left(\phi\right) = \frac{\int_{-\infty}^{\infty} \int_{v}^{\infty} \left[\pi_{12}(\phi, u, v) - \pi_{13}(\phi, v, u)\right] dF_{z_{i,1}}(u) dF_{z_{j,1}}(v)}{g''\left(e_1'\left(\phi\right)\right)}. \tag{2.8}$$

so that the claim in step 1 immediately follows from condition IVE-W.

Step 2 To complete the proof, consider the numerator of the right side of 2.7 and note that it is a continuous function of the parameter σ_ε^2, converging to the numerator of the right side of 2.8 as $\sigma_\varepsilon^2 \to 0^+$. ■

According to both propositions 2 and 3, what is really relevant for managerial career concerns is the marginal value of efficiency. A change in the market conditions that increases the marginal value of an efficient manager also increases the incentives to build a good reputation. This is so in this model because in the second period labor market, the good manager fully appropriates of the value of his (possibly) larger efficiency, measured by the difference in profits that he is able to produce. This result closely resembles proposition 4 in Schmidt (1997). A major difference consists in the source of managerial incentives: in this paper they indirectly arises from career concerns, while Schmidt uses explicit contingent contract in his model.

More competitive product markets are usually thought of as inducing smaller profits (e.g. smaller price-cost margins) to the firms. However, contrary to the model of Schmidt (1997) and Hermalin (1992), the amount of profits doesn't play any role in the present context. This wouldn't be so if managers were not assumed to be risk neutral. With more general managerial preferences, an income effect and a risk adjustment effect similar to those described by Hermalin (1992) would arise both with ambiguous sign. An explicit possibility of bankruptcy, with an associated turnover cost for the failing manager, would also create a scope for for the amount of profits to the extent that, as it seems reasonable, smaller profits rises the probability of bankruptcy. As in Schmidt (1997), an increased probability of bankruptcy would naturally rises the managerial incentives in the first period.

In this model, the bargaining power that managers with a good reputation acquires on the labor market has a key role: they are interested in building a career only to the extent that they can capture the value of such reputation. Any element that negatively affects their bargaining power, as for example the existence of switching costs for a manager who decides to change firm, for example in the form of lost specific human capital, would then reduce their incentives. Note also that the results in this paper especially hold for managers at the top level in the firm hierarchy. In fact, at lower levels career concerns are mainly driven by the internal labor market, that is, by the possibility of getting better employment conditions within the same firm. To evaluate how competitive pressure in the product market affects the internal labor market of a firm and, therefore, the incentives throughout the firm structure, would surely be an interesting issue to address, but it would probably require a completely different model.

As a final remark, note that it is not clear in this setting whether the equilibrium level of effort in the first period is too high or too low with respect to the efficient level. Of course, the second period level is too low for sure but this depends on the fact that there isn't any future after period two and then, there isn't any scope for building a career. Efficiency here has to be defined with respect to profits, namely we could say that the efficient (symmetric) level of effort in period t is e_t^{FB} defined as follows:

$$e_t^{FB} \in \arg\max_{0 \leq e \leq \bar{e}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \pi(\phi, u, v) + \pi(\phi, v, u)\, dF_{x_{i,t}(e)}(u) dF_{x_{j,t}(e)}(v) - 2g(e). \quad (2.9)$$

The distribution of period t efficiencies $x_{i,t}(e)$ and $x_{j,t}(e)$ take into account all past information and assume that managers provide the level of effort e. The first order necessary condition characterizing this first best effort level is then:

$$\frac{\partial}{\partial e} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \pi(\phi, u, v) + \pi(\phi, v, u)\, dF_{x_{i,t}(e^{FB})}(u) dF_{x_{j,t}(e^{FB})}(v) = 2g'(e^{FB}). \quad (2.10)$$

Comparing condition (2.10), defining the first best level of effort, with condition (2.3) defining the equilibrium first period effort, it is not clear at all whether young managers are overworking or shirking too much. Furthermore, in this model the discount factor is for simplicity assumed to be one, namely that the future is as important as the present in determining the lifetime utility of agents. More generally, however, a small enough discount factor could induce young managers to provide an amount of effort below the efficient level. But in a model with a finite horizon like this, it could also be the case that "the future" is more important that "the present", so that the discount factor could be larger than one. A large enough discount factor could then induce managers to overwork.

2.3. Examples

In this section I discuss some examples of explicit market games. Propositions 2 and 3 directly allow to evaluate the impact of specific market parameters on managerial incentives within the industry. In the first two examples I consider firms producing a homogeneous good and competing à la Cournot with a linear demand, in the first place, and then with an isoelastic demand. The third example is very common in IO and it represents the easiest way of modeling price competition among firms producing differentiated products. A common feature of all the examples is that a larger market size corresponds to stronger incentives. A more complete analysis would allow for an endogenous market structure. This possibility is partially pursued in the first example where the number of firms and their market shares is in fact endogenous, even if only two of them can potentially employ a manager. As a final example I consider a switch from Cournot to Bertrand competition as a way of representing an increased competitive level between two firms.

2.3.1. Cournot Competition with Linear Demand

There's a continuum of mass n of entrepreneurial firms (EF), i.e. firms run directly by their owner, and two managerial firms (MF), i.e. firms run by a manager. They compete choosing the quantity to sell on the product market. The inverse demand function is given by $p(Q) = A - Q$, where $Q = q_i + q_j + \int_0^n q(h)dh$ is the aggregate production, being $q(h)$ the "production intensity" of a generic EF $h \in [0, n]$, and (q_i, q_j) the quantities produced by the two MF. The parameter $A > 0$ measures the size of the market. Each EF has a constant marginal cost equal to $c > 0$, while a MF has a marginal cost equal to $c = \kappa(x)$, where x denotes its manager efficiency and κ is a differentiable, positive and decreasing function, bounded above by c. Assuming that the parameters always allow for an interior solution (i.e. $A > 3c$), the profit function for a MF managed with efficiency x and competing against n EF and a MF of efficiency y is:

$$\pi(x, y) = \left[\frac{A + nc + \kappa(y) - (n+2)\kappa(x)}{n+3}\right]^2.$$

The function π is clearly bounded, strictly increasing in x and strictly decreasing in y. The parameter ϕ could be here identified with n, A or c. Furthermore it results that:

$$\pi(x, y) - \pi(y, x) = \frac{n^2 + 4n + 3}{(n+3)^2}\left[\kappa^2(x) - \kappa^2(y)\right] + \frac{2A - 2nc}{(n+3)}\left[\kappa(x) - \kappa(y)\right]$$

and then it is possible to obtain:

$$\frac{\partial}{\partial A}\left[\pi(x,y)-\pi(y,x)\right]=\frac{2[\kappa(y)-\kappa(x)]}{n+3}$$
$$\frac{\partial}{\partial c}\left[\pi(x,y)-\pi(y,x)\right]=\frac{2n[\kappa(y)-\kappa(x)]}{n+3}$$
$$\frac{\partial}{\partial n}\left[\pi(x,y)-\pi(y,x)\right]=\frac{2}{(n+3)^2}\left[\kappa^2(y)-\kappa^2(x)\right]-\frac{2(A-3c)}{(n+3)^2}\left[\kappa(y)-\kappa(x)\right].$$

Note that the derivatives of the difference $\pi(x,y)-\pi(y,x)$ with respect to A and with respect to c are both strictly increasing functions of x so that condition IVE is satisfied in those cases. Hence, a larger market or less efficient entrepreneurial competitors induce larger incentives to build a career. It is also possible to compute:

$$\frac{\partial^2}{\partial n \partial x}\left[\pi(x,y)-\pi(y,x)\right]=\frac{2}{(n+3)^2}\kappa'(x)\left[2\kappa(x)+A-3c\right]$$

which is for sure a negative quantity whenever $A>3c$, then, since this is always the case in any interior solution, the more EF are in the market, the smaller the implicit incentives created by career concerns.[14]

To make the market structure endogenous consider the existence of a fixed entry cost $F>0$.[15] Notice that since the MF are always more efficient than any EF and the profit function is increasing in own efficiency, if an EF is in the market then the two MF are also in the market. Restrict attention to a range of parameters that allow the entrance of at least one EF. Let $n^*=n(A,c,F)$ be the number of EF which optimally decide to enter as a function of the other parameters; of course n^* is increasing in A, and decreasing in both c and F. It is now possible to evaluate the impact of A, c or F on the managerial indirect incentives in case of endogenous market structure. A decrease in the entry cost increases the number of EF so that indirectly reduces the managerial incentives. Usually, markets protected by smaller entry costs are considered as more competitive, but this example

[14] This result is similar to what found by Martin (1993). He considers a model of Cournot competition among firm run by a manager, and in which incentives are provided through explicit contingent contracts. He finds that the optimal effort induced in equilibrium decreases when the number of firms increases.

[15] This entry cost has to be paid at the beginning of period one and allows to remain in the market for two periods.

shows that they also tend to be less efficient.[16] The impact of changes in other parameters is now ambiguous: for example, a larger market size would in principle increase incentives but the entrance of more EF in the market tends to outweigh this effect, and the final result cannot be predicted.

2.3.2. Cournot Competition with Isoelastic Demand

Consider a Cournot duopoly in the market for a homogeneous good whose inverse demand function is $p(Q) = \left(\frac{A}{Q}\right)^{\left(\frac{1}{\varepsilon}\right)}$, where, $Q = q_i + q_j$ is the total quantity produced by firms i and j, and $\varepsilon > 1$ is the constant elasticity of the demand function. Firm i marginal cost is constant and is given by $\kappa(x)$ where x denotes its manager efficiency and κ is a differentiable, positive, bounded and decreasing function. The profit function for a firm of efficiency x competing with a firm of efficiency y is then the following:

$$\pi(x, y) = \frac{A(2\varepsilon - 1)^{\varepsilon - 1}\left[(1-\varepsilon)\kappa(x) + \varepsilon\kappa(y)\right]^2}{\varepsilon^{\varepsilon}\left[\kappa(x) + \kappa(y)\right]^{\varepsilon+1}};$$

where parameters are assumed to be such that no corner solutions arises, i.e. it is assumed that $\varepsilon < \frac{\sup \kappa}{\sup \kappa - \inf \kappa}$.

Note that the function π is continuous, differentiable, strictly increasing in x and strictly decreasing in y. The parameter ϕ can here be identified with either A or ε. It is immediate to compute:

$$\pi(x, y) - \pi(y, x) = A\left[\frac{(2\varepsilon - 1)}{\varepsilon\left[\kappa(x) + \kappa(y)\right]}\right]^{\varepsilon}\left[\kappa(y) - \kappa(x)\right],$$

[16] Raith (2004) also finds that a smaller entry cost reduces efficiency and for the same reason: the presence of a larger number of firms tend to reduce incentives.

which can be shown to be a strictly increasing function of x and a strictly decreasing function of y. It is then possible to compute:

$$\frac{\partial}{\partial A}\left[\pi(x,y)-\pi(y,x)\right]=\left[\frac{(2\varepsilon-1)}{\varepsilon[\kappa(x)+\kappa(y)]}\right]^{\varepsilon}\left[\kappa(y)-\kappa(x)\right],$$
$$\frac{\partial}{\partial\varepsilon}\left[\pi(x,y)-\pi(y,x)\right]=A\left[\kappa(y)-\kappa(x)\right]\left[\frac{(2\varepsilon-1)}{\varepsilon[\kappa(x)+\kappa(y)]}\right]^{\varepsilon}\left[\ln\frac{2\varepsilon-1}{\varepsilon[\kappa(x)+\kappa(y)]}+\frac{1}{2\varepsilon-1}\right].$$

The former of such derivatives is a strictly increasing function of x so that condition IVE is satisfied, meaning that a larger market size induces managers to exert more effort. From the other hand, the latter derivative is not monotone so that the impact of a more elastic demand on the managerial incentives cannot be predicted unambiguously. As in Willig (1987) a smaller market size tends to reduce managerial incentives, but an increased demand elasticity has not a well defined effect here. The point is that for firm i equilibrium requires:

$$\frac{p-c_i}{p}=\frac{1}{\varepsilon}\frac{q_i}{Q};$$

so that a larger value of the elasticity parameter has not a well defined effect on profits (and on profits differentials). In fact, the most efficient firm will be able to get a larger market share if the demand is more elastic, but this doesn't ensure that the overall profits increases. However, it is possible to note that if $\sup\kappa<1$ the quantity $\frac{\partial}{\partial\varepsilon}\left[\pi(x,y)-\pi(y,x)\right]$ is the product of three positive and strictly increasing functions of x in the hemiplane $x>y$ so that, in this region, it is a strictly increasing function of x. Condition IVE-W is satisfied and it is possible to conclude that, if the residual uncertainty is small enough, an increased demand elasticity improves managerial incentives. The intuition is that in such a case there cannot be a big difference between an efficient and an inefficient firm (recall that κ is a positive function), so that the effect of a changing market share is not very important and it is dominated by the "direct" effect on profits, which is the only one showing up in the model by Willig.

2.3.3. Differentiated Products

In the spirit of Hotelling (1929), consider a linear city of length 1 where consumers are uniformly distributed with density A. Two firms are located at the opposite ends of the city, and they sell the same good. The first firm location is at $s = 0$ and the other firm is located at $s = 1$. Consumers' demand can be either one or zero, and $v > 0$ denotes their common valuation for the good. To move from their location to one of the firm, consumers incur a transportation cost of t for unit of length. Firm hiring manager of efficiency x has constant marginal cost $\kappa(x)$, where the function κ has the same properties as in the previous example. To fix notation, assume that manager i is hired by the firm located at zero while the other firm is hiring manager j. Competition is in prices and, if (p_i, p_j) are the prices charged by the two firms, to buy one unit of the good the consumer located at s faces a total cost of $p_i + ts$ or of $p_j + t(1 - s)$ depending on whether he goes to firm i or to firm j. The consumer located at $s(p_i, p_j) = \frac{1}{2} + \frac{p_j - p_i}{2t}$ is indifferent between the two firms, so that those located at his left prefer to buy from firm i, and those located to his right prefer to buy from firm j. The parameter t is traditionally interpreted as a measure of the product substitutability: the smaller its value the closer substitute the two products are. Competition among firms producing closer substitutes is usually considered to be tougher so that it is quite natural to interpret a decrease in t as an increase in competition. As above, it is also interesting to evaluate the impact of the parameter A on managerial incentives. Assuming that v is large enough to ensure that, in equilibrium, each consumer is willing to buy one unit of the good, the profit function of a firm having efficiency x and competing with a firm with efficiency y is:

$$\pi(x,y) = \begin{cases} A\left[\kappa(y) - \kappa(x) - t\right] & if \quad \kappa(x) < \kappa(y) - 3t \\ \frac{A}{18t}\left[3t + \kappa(y) - \kappa(x)\right]^2 & if \quad |\kappa(x) - \kappa(y)| \leq 3t \\ 0 & if \quad \kappa(x) > \kappa(y) + 3t. \end{cases}$$

Notice that only when $|\kappa(x) - \kappa(y)| \leq 3t$ both firms are producing a positive amount of goods while in the other cases the most efficient firm only is supplying the whole market. If κ is decreasing then the profit function satisfies all the properties stated in section 2 but it doesn't have partial derivatives, and then it is not differentiable, in the region $\{(x,y) : |\kappa(x) - \kappa(y)| = 3t\}$. However, this region has measure zero in the real plane so that all the results in the previous section still go through with the exception of proposition 3 that requires continuity for the partial derivatives of π. It is possible to obtain:

$$\frac{\partial}{\partial x}\left[\pi(x,y) - \pi(y,x)\right] = -A\kappa'(x)\left[\frac{2}{3} + \frac{1}{3}I\left(|\kappa(x) - \kappa(y)| > 3t\right)\right].$$

The above quantity is strictly increasing in A and then a larger market size increases the managerial incentives to build a career. However, it doesn't depend on t if the parameter configuration only allows for an interior solution, i.e. $\sup\kappa - \inf\kappa \leq 3t$. This corresponds to a situation in which firm efficiency doesn't make a big difference (possible cost reductions are small compared to transportation costs). However, when corner solutions are possible, i.e. $\sup\kappa - \inf\kappa > 3t$, the above derivative is a decreasing function of t for any (x,y). Hence, provided that firm efficiency is important enough, an increased product substitutability makes managerial career concerns sharper.[17]

2.3.4. A Switch from Cournot to Bertrand Competition

An increase in competitive pressure is sometimes represented by a switch from Cournot to Bertrand competition. In order to analyze how such a change affects the managerial career concerns let's consider a market with linear demand $P(Q) = A - Q$, in which two firms compete in prices with probability q, and in quantities with probability $(1-q)$. As above, each firm has constant marginal cost equal to $\kappa(x)$ where x is its managerial efficiency and κ is a differentiable, positive and decreasing function bounded above by some value $c > 0$. Of course, this is a fictitious market, but it exactly reproduces the Bertrand game when $q = 1$ and the Cournot game when $q = 0$. Assume that parameter

[17] This result seems to be robust to alternative specification of the transportation cost, as long as it is the same for each consumer. For example, using a convex transportation cost as $c(x) = tx^2$ or a concave one as $c(x) = t(2x - x^2)$ exactly yields the same results.

values are such that both firms produce a positive quantity in the Cournot competition (i.e. $A > 2\sup\kappa - \inf\kappa$), and that in the Bertrand game the less efficient firm's marginal cost is always below its competitor monopoly price (i.e. $A > 3\sup\kappa$, note that this condition implies the previous one). With these restrictions the profit function in case of Cournot and Bertrand competition are respectively:

$$\pi^C(x,y) = \left[\frac{A+\kappa(y)-2\kappa(x)}{3}\right]^2$$

$$\pi^B(x,y) = \begin{cases} [A-\kappa(y)]\,[\kappa(y)-\kappa(x)] & if \quad x \geq y \\ 0 & if \quad x < y. \end{cases}$$

Hence the overall profit function is:

$$\pi(x,y) = q\pi^B(x,y) + (1-q)\pi^C(x,y).$$

It is then possible to compute:[18]

$$\frac{\partial^2}{\partial x \partial q}\left[\pi(x,y) - \pi(y,x)\right] = \begin{cases} -\kappa'(x)\left[\frac{A+2\kappa(x)-3\kappa(y)}{3}\right] & if \quad x > y \\ -\kappa'(x)\left[\frac{A+3\kappa(x)-4\kappa(y)}{3}\right] & if \quad x < y. \end{cases}$$

The last quantity is strictly positive as long as $A > 3\sup\kappa$, which is always the case for the previous profit function to make sense. Therefore, the IVE condition is satisfied, and an increase in the probability q induces managers to exert a larger effort in the first period. In particular, this means that in a Bertrand game career concerns are sharper then in a Cournot game, provided that the market size is large enough.

[18] As in the previous example, the function π is not differentiable everywhere. In this case π doesn't have partial derivatives along the line $x = y$ but such region has measure zero in the real plane and all the previous results, but proposition 3, still hold.

2.4. Concluding Remarks

This paper studies the effects of product market competition on managerial career concerns. Differently from Vickers (1995), the analysis abstracts from the existence of any correlation among managerial abilities, and, therefore, relative performances are not useful in the firms' learning process. However, competition among managers in the labor market reintroduces relative performance evaluation in two ways. First, managers are ranked according to past performances and the best receives a wage premium in the future. Second, the future wage premium is equal to the profits that the best manager is expected to produce in excess with respect to the competitor, and, in turns, this expected profit differential depends itself on past relative performances (the larger the gap in managerial performances observed in the past, the more likely that such a gap will be large also in the future). The profit differential produced by a more efficient manager was called the value of efficiency, and it was shown that changes in market parameters that reduce such quantity also reduce ex-ante incentives to provide effort. I also provided several examples showing that increased competition does not always produce stronger career concerns.

The effect of product market competition on managerial career concerns emerges from the interaction between two markets: the firm product market and the managerial labor market. As such, it would be better treated in a general equilibrium framework but, unfortunately, there isn't any satisfactory way of treating imperfect competition in general equilibrium models. The analysis in this paper, which is framed in a partial equilibrium context, then suffers of several limitations. For example, top executive skills are usually of a general nature and are worth more or less the same in many different markets. It is in fact not infrequent that the CEO of a firm in a given industry moves to some firm in another industry. In the partial equilibrium framework used in this paper, this would mean that the exogenous outside option for any given manager depends in fact on his performance in the industry, but the kind of dependence would certainly be better treated as a general equilibrium phenomenon. The model should also allow for more then two managers and, possibly, for overlapping generations of managers, but these extensions shouldn't add too much to the results obtained above.

At a more general level, much remains to be understood on the interaction between competition (in product or factor markets) and corporate governance mechanisms, but my impression is that a reasonable treatment of strategic interactions in general equilibrium is needed to properly address these and related issues.

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Chapter 3

Managerial Entrenchment and the Market for CEOs

In this chapter I present a simple model of entrenchment in which CEOs' have private benefits of control and their incentives derive from turnover in the labor market. Managers have unknown ability which affects their past performances. The ability of successful managers is then perceived to be high, and they obtain higher compensation while retaining private benefits of control. Failing managers, on the contrary, reveal to be less skilled and are fired, then losing private benefits. Entrenchment allows failing managers to keep their job with some probability, and has two effects on the managerial labor market. First, it prevents captured companies from seeking better managers, then reducing the market value of a successful CEO. This *demand effect* weakens career concerns. Second, it decreases the number of successful managers, then increasing their market value. This *supply effect* strengthens career concerns. The model predicts that if the probability of firing an entrenched manager is small, the demand effect dominates, so that a reduction in the ex-ante probability of entrenchment increases managerial compensation.

Keywords: Managerial Entrenchment, Career Concerns.

JEL classification: D83, D86, G34.

3.1. Introduction

Corporations are very common forms of business organization. Despite their diffusion, the separation of ownership and control, typical in this kind of companies, entails a basic agency problem between shareholders and management. Moreover, especially when stock ownership is dispersed among many investors, agency problems are exacerbated by the absence of a well identified principal. To control management is surely valuable, but it has the nature of a public good: any shareholder benefits from anybody else being looking after managers, but, trying to free ride on each other's controlling effort, shareholders may well end up leaving managers with substantial discretion. Typically, small shareholders consider their investment in the company as a way of diversifying their portfolio, and they are not interested in getting involved in management.[1]

To overcome this problem shareholders appoint a board of directors that is supposed to act on their behalf in controlling management. Board powers are usually defined very broadly in the corporate charter, but the most important decisions in which boards play a significant role are those concerning the selection, monitoring and replacement of the CEO (see for example Mace 1971, Vancil 1987, and, more recently, Hermalin and Weisbach 2003). However, the CEO may have in practice a significant influence in selecting directors, and may also serve as the chairman of the board. It is therefore not surprising that CEOs might try to entrench themselves in their positions, filling the board with people willing to rubber stamp any managerial decision and, at least as importantly, unwilling to replace the CEO when it would be in the shareholders' interest to do it. Entrenched managers are therefore hard to replace. Typically, removing them from their position requires a hostile takeover. However, takeovers can be expensive for the raider, who might be forced to pay a substantial premium on the firm's market value. Furthermore, managers can be protected by antitakeover provisions in the corporate charter, or by some form of antitakeover legislation. Not surprisingly, hostile takeovers are relatively

[1] Of course, they can sell out their shares if they are not satisfied with the company's performance.

rare events.[2] As a result, a board that is captured by the CEO is ineffective in protecting the shareholders' interest.[3] There are however alternative sources of discipline that can make up for the absence of an effective board. Among these, managerial career concerns are surely of great importance. In fact, both entrenched and non-entrenched managers are concerned with the value that their reputation has on the market for managerial services. Fama (1980) first proposed the idea that career concerns can completely resolve the agency problems involved in large corporations. Holmstrom (1999) criticized this extreme conclusion on the ground that the value of a good reputation only accrues at some point in the future, so that, if agents are impatient, career concerns at most provides suboptimal incentives. In some circumstances career concerns could even exacerbate an incentive problem, as for example for fund managers (Holmstrom and Ricart i Costa 1986). A large body of literature as emerged since the original work of Fama, and, at least for CEOs, it is generally accepted that career concerns are an important disciplining device (Murphy 1999, Becht et. al. 2003).

This paper studies the relationship between entrenchment, managerial pay, and managerial career concerns. The basic insight of the analysis is that good managers are in short supply, so that firms compete to hire them and are willing to pay more than the reservation wage. Managers have therefore an incentive to build a good reputation, and this incentive is stronger when the equilibrium pay on the labor market is higher. Entrenchment affects the characteristics of the managerial labor market and, as a consequence, it also affects managerial pay and career concerns. In a nutshell, one of the main consequences of entrenchment is that the board of directors is not willing to replace the CEO, but this means that a captured firm will not be active on the demand side of the managerial labor market. Therefore, entrenchment dampens competition on the demand side of the market for CEOs. Because of this *demand effect*, the equilibrium wage

[2] See Becht, Bolton and Roell (2003) for a general discussion of takeovers, and Jahera and Pugh (1991) for an analysis of the antitakeover legislation in Daleware.

[3] However, Almazan and Suarez (2003) argue that, in order to save on the overall managerial compensation cost, some degree of managerial entrenchment could be optimal for shareholders. In a similar vein, Adams and Ferreira (2007) argue that, taking into account the dual role of directors as both monitors and advisors of the CEO, some degree of board friendliness can be optimal.

tends to decrease and, as a consequence, managerial career concerns tends to be weaker. However, entrenchment also has a *supply effect* that runs in the opposite direction. In fact, entrenchment reduces the availability of good managers, then rising their market value and making their career concerns stronger. Which of the two effects dominates cannot be predicted in general. However, I show that if the probability of replacement of an entrenched manager is small, for example because of a strong antitakeover legislation, then the demand effect is the most important. In this case, reducing the number of entrenched manager (i.e. of captured firms) increases the competition on the demand side of the market for CEOs and, as a result, the managerial pay increases, and makes career concerns stronger.

In the last twenty years, at least in the US, we have indeed observed a constant increase in the level of managerial compensation (see for example Murphy 1999) and there is also evidence that, over the same period, the probability of managerial entrenchment has decreased (Huson et. al 2001). The trend in managerial compensation is uncontroversial and it has sometimes be interpreted as evidence of increased self dealing of CEOs (see for example Bebchuk and Fried 2003). As for managerial entrenchment, what has been observed is a trend toward more independent directors sitting on the board, and the adoption of more incentive compensation for directors. Overall, boards appear to have increased their independence from the CEO, and it is likely that the proportion of captured board has been decreasing over time. The findings in this paper allow to interpret the increase in managerial pay as a result of increased board independence: if the demand effect of entrenchment is dominant a smaller proportion of captured board in the economy should translate into increased competition for CEO hiring, and therefore into higher wages. As mentioned above, the demand effect of entrenchment is surely dominant when the probability of firing an entrenched manager is very low. In fact, during the 1980s there has been a widespread adoption of antitakeover legislation in the US, and it is likely that, entrenched managers have become more difficult to replace with a hostile takeover.

This paper is related to the literature on boards which is extensively reviewed by Hermalin and Weisbach (2003). The empirical studies are predominant, and among the most common findings are that: 1) The board composition do not affect corporate performance while the board size is negatively correlated with it; 2) Both board composition and size appear to be significantly correlated with such firm decisions as CEO replacement, executive compensation, adoption of poison pills and acquisitions; 3) The board structure evolves over time according to the evolution of the CEO bargaining power relative to existing directors. Theoretical studies of boards are relatively more scarce. An outstanding exception is the paper by Hermalin and Weisbach (1998). They stress that board willingness to monitor the CEO is larger for more independent boards, but they find that a good performing CEO strengthens his bargaining position within the board, and is able to (endogenously) reduce its independence. In a related paper, Hermalin (2005) elaborate the theoretical framework of his previous paper with Weisbach, and he is able to tight together some observed trends in corporate governance. In particular, he also proposes an argument that allows to interpret increased managerial pay as a result of increased board independence. According to Hermalin, more independent board are more willing to replace the CEO, and therefore he has to be payed a premium to accept a job with an expected shorter tenure. Notice that, this argument is different from the one proposed in this paper, and an important distinction is that Hermalin's idea is particularly important for changes in board independence within a firm, being unchanged the level of independence in other firms. In fact, in such a situation the firm that gets stronger knows that the outside opportunities of potential CEOs are unchanged, and that is exactly why it has to offer a higher compensation to attract a manager into a job with a faster turnover. The argument that I propose in this paper relies instead on a market effect, and it applies to changes in the probability of entrenchment that affect all the firms in the same way (e.g. changes in the legislation on boards).

The remainder of the paper is organized as follows. Section two describes the model and highlights the learning process taking place after the observation of first period performances. Section three describes the market for CEOs and characterizes its equilibrium. In turn, section four studies the ex-ante incentive effect of job opportunities available in the labor market, and establishes the link between managerial entrenchment and the market for CEOs. Section five concludes.

3.2. The Model

3.2.1. Setup

There are two periods $t = 1, 2$. At the beginning of period 1 there is a continuum of firms and each firm is run by an incumbent, young manager (she) receiving a fixed wage payment normalized to zero and enjoying private benefits of control $B > 0$. Without loss of generality, the mass of firms is assumed to be one. Firm $n \in [0, 1]$ has an investment opportunity that can result in a success, in which case it yields $V(n) > 0$, or a failure, thus yielding zero. Let G be the cumulative distribution of investment returns in case of success, that is, $G(v)$ is the fraction of firms whose project returns in case of success are at most v. I assume that G has a continuous density g and that its support is the interval $\left[0, \overline{V}\right]$. Managers differ in their ability of successfully bringing about the investment project. Such ability, or skill, or talent, is summarized by a real number $\theta \in \left[0, \frac{1}{\eta}\right]$, with $\eta > 1$, whose cumulative distribution in the population is F with continuous density f. The specific ability of any particular manager is not known to anybody, including the manager herself. Let $\bar{\theta}$ denotes its mean and σ^2 its variance in the population.[4] The manager working in firm n, henceforth manager n, can get entrenched in the first period. I model entrenchment as a binary variable: either manager n captures the firm's board she works for, or she doesn't. Entrenched managers are harder to remove from their

[4] In what follows, the capital letter Θ will be used to refer to the random variable "managerial skill" while θ will refer to its specific values.

position. The entrenchment probability depends on elements that could be collectively referred to as *corporate governance quality.* Such elements could for example include the number of independent directors sitting on the board, the existence of independent remuneration and auditing committees, the participation and activism of institutional investors (mutual funds, pension funds, insurance companies, etc.). It is clear that the probability of entrenchment within a firm is to some extent idiosyncratic and depends on its particular history and characteristics. However, there are also elements that are common to everyone in the economy and that affect how likely is for a CEO to gain control of the board. Among them, legislation plays surely a big role: it may for example require specific characteristics of the board for a company to be incorporated or listed. The analysis in this paper is concerned with those aspects of corporate governance which depend on the existing legislation so that cross section variability of governance quality will be completely disregarded. Let's define with $\gamma \in (0, 1]$ the probability of entrenchment within any firm in the economy. A smaller γ corresponds to a tighter corporate governance legislation, making it harder for a manager to capture the board. In what follows, it will also be useful to refer to the quantity $\lambda = \frac{1-\gamma}{\gamma}$ directly measuring the quality of governance and ranging from zero (entrenchment always occurs) to ∞ (entrenchment never occurs).[5] If a manager does not succeed in capturing the board, she can be removed at the end of the period while an entrenched manager *secures* his position (a precise description of the consequences of entrenchment follows). Whether a manager captures the board or not is publicly observed in the economy. Each manager n, whether she got entrenched or not, implements the firm investment project and the probability of success $p(n)$ depends on

[5] The entrenchment probability could also depend on some form of managerial pressure to capture the board. For example, an incumbent manager could choose an action $a \in [0, 1]$ and the resulting probability of entrenchment could be γa. Action a would represent the managerial attempt to decrease board independence within the restrictions imposed by the legislation. For example, even if a minimum number of independent directors must be appointed but the condition of independence is not well defined, a manager willing to make the board more friendly could try to impose directors that are formally in a condition of independence but that, for other reasons, are close to her. However, in the model considered here, entrenched managers obtain a larger utility than non-entrenched ones (see in particular proposition 2). Therefore, as long as no costs are associated with any degree of entrenchment pressure, each manager would choose $a = 1$. This is precisely the scenario analyzed in the text. Notice however that entrenchment actions could be costly (for example, a CEO could try to bribe some important independent director) but, as most of the results presented in the paper are independent of this assumption, I will not consider this possibility.

her ability $\theta(n)$ and effort $e(n) \in [1, \eta]$ according to:[6]

$$p(n) = \theta(n)e(n). \tag{3.1}$$

Managerial effort e cannot be observed and has a utility cost $c(e) > 0$ satisfying $c'(e) > 0$, $c''(e) > 0$, $c'(1) = 0$, $c'(\eta) = \infty$. At the end of period one, firm n receives the realized returns of the investment project while manager n receives $B - c(e)$. To summarize, the timing of events in period one is the following: 1) The ability of each manager is determined according to F; 2) With probability γ each manager gets entrenched, and everyone observes the realization of this event; 3) Each manager chooses an effort level; 4) Each firm investment project produces its result which is publicly observed; 5) Agents receive first period payoffs.

In period two each firm n has a new investment opportunity that, again, can result in a success, thus yielding the same amount $V(n)$ as in period one, or in a failure, thus yielding zero. However, before implementing their projects, firms have the opportunity to evaluate and, possibly, dismiss their incumbent manager. In particular, unsuccessful managers who didn't capture their board are fired and they are not able to find a new job as chief executives. Therefore, they leave the scene, thus losing their private benefits of control, and receive their reservation wage equal to zero in some alternative occupation. From the other hand, unsuccessful managers who did entrench themselves in the first period are fired only with some probability $\alpha \in [0, 1]$. If they are fired, again they lose control benefits and obtain zero elsewhere, while, if they keep their position, they also keep their private benefits of control, even if their wage is equal to zero in this case. The main mechanism that allows a company to dismiss an entrenched manager is a hostile takeover, so that α can be interpreted as the probability that a successful raid occurs. The value of α reflects several elements that can reduce the profitability of a hostile takeover. The free rider problem described by Grossman and Hart (1980)

[6] Notice that a manager with the highest ability exerting the maximum level of effort succeeds with certainty.

can force a rival to pay a substantial premium on the firm's share pre-bid price. This erodes the profit margin for the bidder and can undermine its incentives to undertake the operation.[7] Takeover legislation or anti takeover provisions in the corporate charter may also make it costly to conduct a successful raid, and, furthermore, the price to be paid for a successful takeover may increase if some competing bidder becomes interested in the same target. Of course, the free rider problem, anti takeover provisions, and bidders competition could be more or less important for different firms so that, at least in principle, the probability of firing an entrenchment manager could vary across firms. Again, this cross section variability is disregarded and α is assumed to measure the strength of antitakeover legislation. Therefore, a smaller α corresponds to a stronger legislative protection against hostile bids.

Successful (senior) managers, whether they were entrenched or not, enter a perfectly competitive labor market where they are reallocated among independent firms. A firm is independent in three cases: first, when it was not captured by its manager in period one, second, when even if it was captured, its investment project was a success and, third, when even if it was captured and failed, its manager were fired anyway.[8] Put it another way, non-independent firms, that do not participate the labor market, are firms that failed in period one and did not fire their managers. The labor market is perfectly competitive so that both managers and firms are price (i.e. wage) takers. At the given market wage firms can demand at most one senior manager and, to the other side, senior managers decide whether to offer their services or not. Senior managers willing to supply their work at the

[7] Imagine that the value of a firm's share under current management is v, while it could be $v' > v$ under alternative management. Let $p < v'$ be the price that a raider offers to buy any of the firm's share, and consider the holder of a single share who does not believe to be pivotal in the success of the raid. If he thinks that the raid will be successful, then he has an incentive not to tender his share (in this way he gets v' instead of $p < v'$). In other words, he has an incentive to free ride on other shareholders' tender decision, and enjoy the full capital gain $v' - v$. However, if each shareholder decides not to tender his share, the takeover cannot be successful. Hence, to have some chances to succeed, the raider must pay at least a price $p = v'$ but, in this way, he's paying to shareholders the entire capital gain, and the incentives to buy the firm are completely destroyed.

[8] In the second case, the board of directors has no reason to fire the manager so that the fact that she is entrenched is immaterial. Notice that, firing decisions are embedded in the structure of the model instead of being explicitly analyzed. However, their explicit consideration wouldn't alter any result: it would clearly be optimal to fire a failing manager whenever possible. It is therefore for the sake of simplicity that I use the "reduced-form" model described in the text.

given market wage are allocated among firms willing to hire one of them, giving priority to more productive firms, i.e. firms whose project returns are higher in case of success. If a firm does not hire a senior manager on the market it will hire a young one with unknown ability distributed according to F. Young managers receive their reservation wage that, as in period one, is normalized to zero. Furthermore, if a senior manager does not find a job in a firm, she leaves the market, loses private benefits of control and receives in some alternative occupation her reservation wage equal to zero. After having been hired by a firm, neither senior nor young managers exert any productive effort so that the probability of success of their investment project equals their unknown ability.[9] At the end of period two, each firm receives the realized returns of the investment project, managers who found a job enjoy private benefits of control B and, among them, senior manager receive the market wage. Finally, all remaining managers receive a wage equal to zero. To summarize, the timing of events in period two is the following: 1) Unsuccessful managers are fired with probability one if they are not entrenched and with probability α if they are entrenched; 2) Independent firms and senior successful managers enter the labor market; 3) Managers do not exert any productive effort; 4) Each firm investment project produces its result which is publicly observed; 5) Agents receive second period payoffs.

There is no discounting so that agents maximize total expected payoffs. Notice that firms are heterogeneous in terms of investment returns in case of success and, at least in principle, this heterogeneity could reflect underlying differences in either the firm size or the firm productivity. However, private benefits of control extracted by managers in bigger firms are likely to be higher, while they are constant and equal to B in the model. It therefore appears more natural to think of firms as being roughly the same size (measured for example by the employment level) but having available projects of different quality.[10]

[9] This is so because there are no returns on effort in period two: compensation within the period is fixed and there is not a future to justify further concerns to build, or keep, a good reputation.

[10] Notice that managers also have *private benefits of shirking* that can be extracted by exerting a low level of effort. In fact, the cost of effort $c(e)$ could be thought of as the amount of such private benefits that a manager of ability θ has to give up to induce a probability of success equal to θe.

In this setup a pure strategy for manager n specifies productive efforts $[e(n), e_E(n)]$ to be exerted in case of no entrenchment and entrenchment respectively, and also a rule to decide whether to supply her labor services in the second period labor market, in case she succeeded in period one, as a function of the market wage. A pure strategy for firm n specifies a rule to decide whether to demand or not a successful manager at the given market wage, in case it participates as an independent firm to the second period labor market.

3.2.2. Learning the Managerial Ability

At the beginning of period one, every agent believes that the ability of manager n is distributed according to F. However, the observation of first period investment results allow to update such beliefs. In particular, if manager n is expected to exert efforts $\widehat{e}(n)$, the posterior cumulative distribution $\widehat{F}$ of her ability in case of success is:

$$\widehat{F}(\theta) = \frac{\int_{\underline{\theta}}^{\theta} \widehat{e}(n) u f(u) du}{\int_{\underline{\theta}}^{\frac{1}{2}} \widehat{e}(n) u f(u) du} = \frac{E(\Theta \mid \Theta \leq \theta)}{E(\Theta)} F(\theta) \tag{3.2}$$

which dominates the unconditional distribution F (in the sense of first order stochastic dominance) and, furthermore, is independent of $\widehat{e}(n)$. It is also independent of n so that each successful manager's ability is believed to be distributed according to $\widehat{F}$. Let $\widehat{f}(\theta) = \theta f(\theta)/\bar{\theta}$ be the corresponding density and $\widehat{\theta}$ the corresponding mean. Because of first order stochastic dominance, it clearly results $\widehat{\theta} > \bar{\theta}$. In fact, it is a matter of simple algebra to obtain:

$$\widehat{\theta} = \bar{\theta} + \frac{\sigma^2}{\bar{\theta}}. \tag{3.3}$$

Notice that the higher the volatility of managerial ability, the larger the improvement on managerial expected skill in case of success. If the uncertainty about θ is big, i.e. σ^2 is large, success is interpreted as a more reliable signal of managerial high skill. Conversely, if almost all the probability mass is concentrated around $\bar{\theta}$ in period one, i.e. σ^2 is small, even in case of success managerial skills will not be considered to be significantly above the average.

The learning process described above takes into account that entrenchment is observable. However, as it is clear from (3.2) the quantity $\widehat{e}(n)$ does not affect posterior beliefs and, therefore, unobservability of the entrenchment history would not change anything in the present context. Of course, this is a consequence of the particular firm technology described in (3.1). With more general technologies the assumption of entrenchment unobservability would be necessary to prevent the entrenchment history from affecting the learning process. However, even maintaining such assumption, more general technologies would make the learning process depending on anticipated effort levels $[\widehat{e}(n), \widehat{e}_E(n)]$. In particular, the higher $\widehat{e}(n)$ or $\widehat{e}_E(n)$, the worst the posterior distribution of the ability of a successful manager. Intuitively, the more effort managers are expected to put into their job, the less informative their possible success is about their true skills. This possibility would not necessarily break the relevance of symmetric learning processes (i.e. learning processes that do not depend on n) as, in equilibrium, anticipated effort levels equal the actual managerial effort choices that can be symmetric (i.e. the same for every manager). However, a consequence would be that the larger the equilibrium effort, the smaller the value of a successful manager, the smaller their wage in the second period labor market. However, in section 3 I will discuss another reason that introduce a similar relationship between equilibrium effort and second period market wage: higher equilibrium effort levels enlarge the set of successful managers in period two, and strengthen competition among them. As a result, they earn smaller wages in the labor market. Hence, more general technologies do not introduce any new effect in the model. It is therefore for the sake of simplicity that I use the technology described in (3.1): it makes the learning process in case of success quite straightforward, and allows a very simple characterization of equilibrium behavior without affecting qualitative results.

Let's also consider the learning process that takes place after the observation of a failure in period one. Let again $\check{e}(n)$ be the expectation on the effort level exerted by manager n. If firm n fails the posterior distribution of her ability is:

$$\check{F}_n(\theta) = \frac{\int_{\underline{\theta}}^{\theta} [1 - u\check{e}(n)]\, f(u)du}{\int_{\underline{\theta}}^{\frac{1}{2}} [1 - u\check{e}(n)]\, f(u)du} = \frac{1 - \check{e}(n)E(\Theta \mid \Theta \leq \theta)}{1 - \check{e}(n)E(\Theta)} F(\theta).$$

Such distribution is dominated by F for every expectation on manager n exerted effort $\check{e}(n)$. Notice also that the corresponding density is $\check{f}_n(\theta) = \frac{1-\theta\check{e}(n)}{1-\bar{\theta}\check{e}(n)} f(\theta)$, and manager n expected skill is:

$$\check{\theta}_n = \bar{\theta} - \frac{\check{e}(n)\sigma^2}{1 - \check{e}(n)\bar{\theta}}.$$

Therefore, a young manager is expected to be better than a manager who failed in period one, for any possible expectation $\check{e}(n)$ on her effort level. As a consequence, the firm that was hiring a manager who failed has an incentive to fire her, and no other firm in the economy has an incentive to offer her a new job as chief executive. To give a further interpretation of α, assume for a moment that entrenched managers cannot be fired. If firm n was captured by its manager and then failed in period one, its market value at the beginning of period two is $\check{\theta}_n V(n)$, while it would be worth at least $\bar{\theta}V(n) > \check{\theta}_n V(n)$ if its manager were fired. It could then be profitable for a rival to buy firm n's share, take control of the company and then fire the incumbent manager to look for a replacement in the labor market.[11] Such a takeover would clearly be hostile because the incumbent manager has an incentive to resist and retain her position to keep enjoying private benefits of control. As a consequence, all failing firms not announcing their willingness to replace their manager becomes potential targets for a hostile tender offer bid. Therefore, the probability α that an entrenched and failing manager is fired is a simple way of modeling the existence of potential raiders: if manager n is entrenched and fails, with probability α she suffers a hostile takeover and is fired, while firm n participates to the managerial labor market. If firm n is not taken over, its entrenched manager cannot be removed and will keep her position within the firm.

[11] In fact, if a raider buys 100% of the target's shares at $\check{\theta}_n V(n)$, removes the board and fires the incumbent manager, the market value of the company increases at least to $\bar{\theta}V(n)$. Therefore, the raider obtains a capital gain of at least $\left(\bar{\theta} - \check{\theta}_n\right) V(n) = \frac{\check{e}(n)\sigma^2}{1-\check{e}(n)\bar{\theta}} V(n) > 0$.

3.3. Labor Market Equilibrium

The outside wage for a senior manager on the second period labor market is zero, but she would lose private benefits of control in an alternative job. As a consequence, senior managers optimally supply their services at any market wage above $-B$. Similarly, firm n optimally demands a senior manager if the expected value she can generate, net of wage payment, is not smaller than the expected value generated by a young manager. That is, if w is the market wage, firm n optimally demands a senior manager whenever:

$$\widehat{\theta}V(n) - w \geq \bar{\theta}V(n),$$

which is equivalent to:

$$w \leq \frac{\sigma^2}{\bar{\theta}}V(n). \tag{3.4}$$

At the beginning of period two, what is relevant for the subsequent functioning of the labor market is the set of potential buyers and the set of potential sellers. Potential buyers are all independent firms, and potential sellers are all successful managers. Which subsets of firms and managers are relevant in the labor market then crucially depends on managerial behavior in the first period. In what follows I only consider the case in which managers behave symmetrically in period one, that is, each of them chooses the same levels of productive effort $(e,\, e_E)$. I assume that a strong law of large number holds in this model for a continuum of random variables.[12] In this case, in period two the following will happen with probability one: the set of independent firms will have a measure of $(1-\gamma)+\gamma\bar{\theta}e_E+\gamma(1-\bar{\theta}e_E)\alpha$, which can also be written as $1-\gamma(1-\alpha)(1-\theta e_E)$. With probability one, the distribution of project returns among independent firms is therefore

[12] Judd (1985) shows that there are some difficulties associated with such a law of large numbers when it involves a continuum of random variables, but it is not in contradiction with basic mathematics. In particular, he shows that it is always possible to construct a probability measure in which a continuum of independent random draws is representable, and such that a strong version of the law of large numbers in fact holds. However, it is also possible to construct alternative probability measures that are as meaningful, and for which it fails. Hence, while it is perfectly admissible to assume that the probability measure one is working with is compatible with a strong law of large numbers, this approach doesn't allow to relate the continuum model to some limit of large but finite models. See Al-Najjar (2004) for an attempt in this latter direction.

$\left[1-\gamma(1-\alpha)(1-\bar{\theta}e_E)\right] G$, and the measure of successful managers is $(1-\gamma)\bar{\theta}e+\gamma\bar{\theta}e_E$, which can also be written as $\bar{\theta}e-\gamma\bar{\theta}(e-e_E)$. An equilibrium wage w in the labor market is then implicitly defined by:

$$\left[1-\gamma(1-\alpha)(1-\bar{\theta}e_E)\right]\left[1-G\left(\frac{\bar{\theta}}{\sigma^2}w\right)\right]=\bar{\theta}e-\gamma\bar{\theta}(e-e_E). \tag{3.5}$$

To interpret condition (3.5), remember that, according to (3.4), at a given wage w, all firms whose project returns is at least $\frac{\bar{\theta}}{\sigma^2}w$, optimally demand a senior manager. Hence at the wage w the measure of firms demanding a senior manager is exactly $\left[1-\gamma(1-\alpha)(1-\bar{\theta}e_E)\right]\left[1-G\left(\frac{\bar{\theta}}{\sigma^2}w\right)\right]$, so that (3.5) can be interpreted as a market clearing condition: at w the measure of firms that demands a senior manager is equal to the measure of senior manager, all of whom are supplying their services at any non-negative market wage.[13] Denoting with H the inverse function of G and recalling that $\lambda=\frac{1-\gamma}{\gamma}$, it is possible to solve (3.5) for the equilibrium wage w as a function of e and e_E:[14]

$$w(e,e_E)=\frac{\sigma^2}{\bar{\theta}}H\left[\frac{\lambda\left(1-\bar{\theta}e\right)+\alpha\left(1-\bar{\theta}e_E\right)}{\lambda+\bar{\theta}e_E+\alpha\left(1-\bar{\theta}e_E\right)}\right]. \tag{3.6}$$

To interpret condition (3.6), remember that successful managers are allocated to independent firms, giving priority to more productive ones. Furthermore, the market clearing

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[13] More generally, Let $\Lambda\subset[0,1]$ be the set of independent firms, and let $S\subset\Lambda$ be the subset of such firms that were successful in period one. Consider sets with the following characteristics: S and $\Lambda(v)=\{n\in\Lambda: V(n)\leq v\}$ are measurable for each v. Denoting with $\mu(A)$ the measure of a set of real numbers, it is then possible to define the cumulative distribution of project returns in case of success for firms in Λ which is given by:

$$G_\Lambda(v)=\mu\left[\Lambda(v)\right].$$

An equilibrium wage w in the labor market is then defined by:

$$\mu(\Lambda)-G_\Lambda\left[\frac{\bar{\theta}}{\sigma^2}w\right]=\mu(S).$$

Notice that, if the set S of successful managers or one of the sets $\Lambda(v)$ is not measurable, then such definition of equilibrium wage cannot be used. However, as long as both effort profiles $[e(n),e_E(n)]$ are measurable functions with means given by $\bar{e}_E$ and, respectively, $\bar{e}$, then with probability one $\mu(S)=(1-\gamma)\bar{\theta}\bar{e}+\gamma\bar{\theta}\bar{e}_E$ and $\mu\left[\Lambda(v)\right]=\left[1-\gamma(1-\alpha)(1-\bar{\theta}\bar{e}_E)\right]G(v)$. Notice also that if a manager deviates unilaterally from the above profiles, the measure of the relevant sets of buyers and sellers in the second period will not be affected. Of course, a deviation would change the probability that the firm run by the manager and the manager herself will belong to such sets.

[14] More precisely, $H:[0,1]\rightarrow\left[\underline{V},\overline{V}\right]$ is the inverse of the restriction of G to the interval $\left[\underline{V},\overline{V}\right]$ where G is strictly increasing.

wage is such that the least profitable firm hiring a senior manager breaks even. Hence, it is exactly this marginal firm that "commands" the equilibrium wage, that is given exactly by its willingness to pay. A larger set of senior managers reduces the profitability of the breaking even firm and then the equilibrium wage in the market. Similarly, a larger set of potential buyers increases the profitability of the marginal firm that will then command a higher equilibrium wage. Notice that condition (3.6) defines the equilibrium wage for given values of e and e_E but, in turn, equilibrium effort levels depend also on the market wage that a successful CEO can earn on the labor market. The overall equilibrium has then the nature of a fixed point and is analyzed in the following section. Some remarks are useful at this point. First, notice that the equilibrium wage is a decreasing function of both e, e_E and $\bar{\theta}$. This is so because the larger e, e_E or $\overline{\theta}$ in period one, the bigger the set of successful manager in period two, the stronger the competition that every senior manager receives by the group of peers. Second, the equilibrium wage is increasing in the first period skill volatility, the reason being that the larger σ^2 the bigger the expected ability of a senior manager (see expression 3.3), the larger the wage that firms are willing to pay for them. Third, notice that the equilibrium wage is increasing in α. In fact, a larger probability of firing an entrenched an failing manager increases the measure of independent firms and then boosts the equilibrium market wage. Finally, the equilibrium wage also depends on the characteristics of the distribution of project returns in case of success. In particular, shifting probability mass toward higher value of investment returns would increase the equilibrium wage. To conclude this section notice that from condition (3.5) it is clear that a reduction in the ex-ante probability of entrenchment γ (i.e. an increase in λ) has two different effects on the equilibrium wage. First, a reduction in γ has a *demand effect*: it enlarges the set of independent firms that demand a senior CEO, and, through this channel, it tends to increase the equilibrium wage. However, a smaller γ also has a *supply effect*: it affects the size of the set of successful managers. In particular, if $e > e_E$, the measure of successful managers increases in the economy when γ decreases and, therefore, the supply effect tends to reduce the equilibrium wage. Which of the two effects is prevalent cannot be predicted in general terms. Notice that in condition (3.6) the equilibrium wage is also expressed as a function of λ and it is simple to see that the sign of the partial derivative $\frac{\partial w}{\partial \lambda}$ is equal to the sign of :

$$(1-\bar{\theta}e)\bar{\theta}e_E - \alpha(1-\bar{\theta}e_E)\bar{\theta}e.$$

Assuming that $e > e_E$, as in fact will be established in the next section, the sign of this last expression depends on α and on the difference $e - e_E$. However, for $\alpha = 0$, and by continuity also for α positive small enough, the equilibrium wage is unambiguously an increasing function of λ. The intuition is that with $\alpha = 0$ entrenchment has the strongest impact on the competition for good managers. In fact, none of the captured firms will be active on the managerial labor market in this case and, therefore, a reduction in the entrenchment probability, i.e. an increases in λ, strongly increases demand side competition. As a consequence, at least in this case the equilibrium wage increases with λ. This property holds also when the values of e and e_E are endogenized, which is the purpose of the next section.

3.4. Entrenchment: Private vs Market Benefits

Let's consider the problem of choosing optimal levels of effort $[e(n),\, e_E(n)]$ for a manager n. Assuming that any other manager is choosing $(e,\, e_E)$, manager n chooses $[e(n),\, e_E(n)]$ to solve the following problem:

$$\max_{[e(n),e_E(n)]\in[1,\eta]^2}(1-\gamma)\left\{\bar{\theta}e(n)\left[B+w(e,e_E)\right]-c\left[e(n)\right]\right\}+$$

$$+\gamma\left\{\bar{\theta}e_E(n)\left[B+w(e,e_E)\right]+\left[1-\bar{\theta}e_E(n)\right](1-\alpha)B-c\left[e_E(n)\right]\right\}. \tag{3.7}$$

Notice that problem (3.7) has a unique interior solution with first order, necessary and sufficient, conditions given by:

$$\bar{\theta}w(e, e_E) + \bar{\theta}B = c'\left[e(n)\right];$$

$$\bar{\theta}w(e, e_E) + \alpha\bar{\theta}B = c'\left[e_E(n)\right].$$

Right hand sides in the above expressions, represent marginal cost of effort for non-entrenched and, respectively, entrenched managers, while, left hand sides represent marginal values. Therefore, it is clear that the marginal value of effort is higher for non-entrenched managers. In fact, effort increases both the probability of gaining a higher wage and the probability of keeping private benefits of control. Entrenched managers are less concerned with the possibility of being fired and, therefore, job conservation is a less effective incentive for them. In a symmetric equilibrium it must be that $e(n) = e$, and $e_E(n) = e_E$ so that, taking into account the expression for the market wage given in (3.6), optimal, common, effort levels (e^*, e_E^*) satisfy:

$$\sigma^2 H\left[\frac{\lambda\left(1-\bar{\theta}e^*\right)+\alpha\left(1-\bar{\theta}e_E^*\right)}{\lambda+\bar{\theta}e_E^*+\alpha\left(1-\bar{\theta}e_E^*\right)}\right]+\bar{\theta}B = c'\left(e^*\right); \tag{3.8}$$

$$\sigma^2 H\left[\frac{\lambda\left(1-\bar{\theta}e^*\right)+\alpha\left(1-\bar{\theta}e_E^*\right)}{\lambda+\bar{\theta}e_E^*+\alpha\left(1-\bar{\theta}e_E^*\right)}\right]+\alpha\bar{\theta}B = c'\left(e_E^*\right). \tag{3.9}$$

The following proposition characterizes some properties of effort levels exerted in the symmetric equilibrium.

Proposition 1 *There exists a unique couple of optimal effort levels $(e^*,\, e_E^*)$, and, if $\alpha < 1$, then $e^* > e_E^*$. Furthermore, both e^* and e_E^* are increasing in σ^2 and e_E^* is also increasing in α.*

Proof Existence follows from a straightforward fixed point argument. Define the function $\mathcal{F} : [1,\eta]^2 \to [1,\eta]^2$ as:

$$\mathcal{F}(e, e_E) = \left(c'^{-1}\left[\sigma^2 H\left[\frac{\lambda(1-\bar{\theta}e)+\alpha(1-\bar{\theta}e_E)}{\lambda+\bar{\theta}e_E+\alpha(1-\bar{\theta}e_E)}\right]+\bar{\theta}B\right], c'^{-1}\left[\sigma^2 H\left[\frac{\lambda(1-\bar{\theta}e)+\alpha(1-\bar{\theta}e_E)}{\lambda+\bar{\theta}e_E+\alpha(1-\bar{\theta}e_E)}\right]+\alpha\bar{\theta}B\right]\right).$$

A symmetric equilibrium is then a fixed point of $\mathcal{F}$, but $\mathcal{F}$ is a continuous function from the compact set $[1,\eta]^2$ into itself and, therefore, it clearly admits at least one fixed point. Existence is then established. As for uniqueness, notice that from (3.8) and (3.9), any solution (e^*, e_E^*) must satisfy:

$$c'(e^*) - c'(e_E^*) = (1-\alpha)\bar{\theta}B. \tag{3.10}$$

Assume by contradiction the existence of two different solutions $(e^*, e_E^*) \neq (e^{**}, e_E^{**})$. Condition (3.10) then implies that $e^* \neq e^{**}$ and $e_E^* \neq e_E^{**}$. Without loss of generality, consider $e^* > e^{**}$. From condition (3.9) it follows that $e_E^* < e_E^{**}$, so that (e^*, e_E^*) and (e^{**}, e_E^{**}) cannot both satisfy (3.10) which is necessary for a solution. Uniqueness, hence, follows. The property $e^* > e_E^*$ for $\alpha < 1$ follows directly from (3.10) and the strict convexity of c. As for comparative statics results, consider $\alpha' > \alpha$ and let (e', e'_E); (e, e_E) be corresponding optimal effort levels. Assume by contradiction that $e'_E \leq e_E$. From (3.9) it then follows that $e' > e$ which is incompatible with (3.10). Hence, $e'_E > e_E$. Other comparative statics results are established similarly. ■

According to proposition 1, non-entrenched managers exert more effort than those who secured their position within the firm. This result is very intuitive: non-entrenched managers exert effort for two reasons: first, to reduce the probability of being fired and, second, to increase the probability of building a good reputation. For entrenched managers the first incentive is less important so that their effort has a smaller marginal value and, as a consequence, they exert less effort in equilibrium. As for comparative statics results, notice that a larger skill volatility σ^2 increases the value of a good reputation, thus making career concerns stronger for both entrenched and non-entrenched managers. The effect of a change in $\bar{\theta}$ on equilibrium effort levels is ambiguous. Expressions (3.8) and (3.9) are quite clear on this point. The marginal value of effort is the sum of two components. First, the marginal benefits on the labor market, measured by $\sigma^2 H(.)$, which is decreasing in $\bar{\theta}$. Second, the marginal benefits of retained control, measured by $\bar{\theta}B$ for non-entrenched managers, and by $\alpha\bar{\theta}B$ for entrenched managers, which, on the contrary, is increasing in $\bar{\theta}$. A variation in $\bar{\theta}$ affects then these two components in opposite directions and the overall effect cannot be predicted. Notice however that in the extreme case $\alpha = 0$, the equilibrium effort in case of entrenchment is unambiguously decreasing in $\bar{\theta}$. In fact, in this case, entrenched managers keep their job with certainty so that effort has no effect on the probability of retaining private benefits of control. As for comparative statics with respect to α, notice in condition (3.9) that this parameter affects directly the marginal value of effort for entrenched managers. In particular, a larger α makes effort more productive in keeping private benefits, then inducing a higher level in equilibrium. This is quite intuitive: for entrenched managers the larger the probability of a takeover in case of failure (i.e. the larger α), the stronger their incentive to be successful.[15] From the other hand, non-entrenched managers are affected by variations in α only through its effect on the equilibrium market wage. However, this effect is ambiguous. A larger α increases the measure of independent firms, but, through its effect on e_E^*, it also increases the measure of successful managers. The first effect tends to increase the equilibrium wage, but the second one goes in the opposite direction, and the total effect is unpredictable. However, if the equilibrium wage increases (decreases) because of a variation in α, the equilibrium effort provided by non-entrenched managers also increases (decreases).

To get the analysis one step further, let $w^* = w(e^*, e_E^*)$ be the equilibrium wage for senior managers and define expected utilities U_E^* and U^* for an entrenched and, respectively, a non-entrenched manager, that is:

$$U_E^* = \bar{\theta}e_E^*\,(B + w^*) + (1 - \bar{\theta}e^*)(1 - \alpha)B - c(e_E^*);$$

$$U^* = \bar{\theta}e^*\,(B + w^*) - c(e^*).$$

[15] This effect resembles the disciplining role of takeovers analyzed by Scharfstein(1988), and it is indeed due to a very similar mechanism: in both cases the incentive effect of takeover stems from the existence of a raider that can replace the current board when this is ineffective. However, in the original work by Scharfstein, the source of board incapacity is not managerial entrenchment, but the existence of post contractual asymmetric information. The idea is that, after signing an incentive contract the CEO may receive some new information that creates new opportunities of self dealing, not properly addressed in the original contract.

It is then possible to define the value of entrenchment as follows:

$$U_E^* - U^* = (1 - \bar{\theta} e^*)(1-\alpha)B + [c(e^*) - c(e_E^*)] - \bar{\theta}\,(B + w^*)\,(e^* - e_E^*)\,. \tag{3.11}$$

Expression (3.11) makes it clear that an entrenched manager is partially shielded from the possibility of losing private benefits of control, and extracts more private benefits in the form of a smaller effort. From the other hand, reduced effort also reduces the expected labor market benefits that accrue to successful managers. However, this cost, measured by the term $\bar{\theta}\,(B + w^*)\,(e^* - e_E^*)$ has a smaller impact than the benefits associated with a smaller effort, measured by $c(e^*) - c(e_E^*)$. Notice, in fact, that from (3.9), it results that $\bar{\theta}\,(B + w^*) = c'(e_E^*)$. Therefore, the value of entrenchment can be rewritten as:

$$U_E^* - U^* = (1 - \bar{\theta} e^*)(1-\alpha)B + [c(e^*) - c(e_E^*)] - c'(e_E^*)\,(e^* - e_E^*)\,.$$

As long as $\alpha < 1$, the last expression is strictly positive because the term $c'(e_E^*)\,(e^* - e_E^*)\,,$ measuring expected labor market loss from entrenchment, is a linear approximation of the increment $c(e^*) - c(e_E^*)$, measuring private benefits from entrenchment. Furthermore, the convexity of c ensures that the difference $[c(e^*) - c(e_E^*)] - c'(e_E^*)\,(e^* - e_E^*)$ is indeed strictly positive. A further intuition for this result is that effort level e^* is a possible choice also for entrenched managers. However, they prefer e_E^* which, therefore, has to make them better off. Also notice that if $\alpha = 0$, condition (3.10) implies that $e^* = e_E^*$ so that, in this case, entrenchment has no value, i.e. $U_E^* = U^*$. This shows the following proposition.

Proposition 2 *If entrenchment reduces the probability of firing in case of failure, i.e.* $\alpha < 1$, *then the value of entrenchment is strictly positive:* $U_E^* > U^*$.

So far I haven't analyzed the effect of changes in the probability of entrenchment γ on the characteristics of the equilibrium, but this is the purpose of the remainder of this section. Remember that the main interpretation of a reduction in γ is a change in the legislation that increases the quality of corporate governance, like for example the introduction of the Sarbanes-Oxley act in 2002, the adoption of more stringent listing requirements, etc. Notice that the parameter γ affects equilibrium effort indirectly through its effect on the equilibrium wage in the labor market. Unfortunately, as discussed at the end of the previous section, it is impossible to derive any general comparative statics result with respect to γ, or, equivalently, with respect to λ. The problem is that, in general, a reduction in the probability of entrenchment, i.e. an increase in λ, has two conflicting effects on the managerial labor market. More firms are willing to contract a senior CEO, but more senior CEOs are available for contracting. Whether the demand or the supply effect prevails is therefore not predictable in general terms. However, the following proposition allows to obtain precise comparative statics result in the special case $\alpha = 0$. While certainly special, this is however an important case. In fact, throughout the 1980s many states in the US have been adopting very strong antitakeover legislation whose effects can be represented in this model exactly with a very small value of α.

Proposition 3 *If the probability α of firing and entrenched manager in case of failure is zero, then the equilibrium wage w^*, the equilibrium effort levels (e^*, e_E^*), and the expected utility of both entrenched and non-entrenched managers $(U^*,\, U_E^*)$ are all increasing in the quality of governance, as measured by λ.*

Proof Assume $\alpha = 0$. Let's show first that, in this case, equilibrium effort levels are increasing in λ. consider $\lambda' > \lambda$ and let $(e', e_E')\,;(e, e_E)$ be corresponding optimal effort levels. Assume by contradiction that $e' \leq e$. From (3.9) and taking into account that $\alpha = 0$, it follows that $e_E' > e_E$ which is incompatible with (3.10). Hence, $e' > e$. Assuming $e_E' \leq e_E$ immediately produces a similar contradiction so that $e_E' > e_E$ follows. Let's show now that w^* increases in λ. Consider again $\lambda' > \lambda$ and let $(e', e_E')\,;(e, e_E)$ be corresponding optimal effort levels. We know that $e' > e$ and $e_E' > e_E$. Assume by contradiction that $w(e', e_E') \leq w(e, e_E)$. Taking into account the expression of the equilibrium wage (3.6), the first order condition (3.9), and the condition $\alpha = 0$, it is possible to write:

$$c'(e'_E) = \sigma^2 H \left[\frac{\lambda(1-\bar{\theta}e')}{\lambda + \bar{\theta}e'_E} \right] \leq \sigma^2 H \left[\frac{\lambda(1-\bar{\theta}e)}{\lambda + \bar{\theta}e_E} \right] = c'(e_E),$$

then, $e'_E \leq e_E$, which is a contradiction. The quantities U^* and U^*_E are increasing in λ because of their definition and the envelope theorem. ■

Notice that the arguments used in the proof remain valid for α positive but small enough, because all the quantities involved, i.e e^*, e^*_E, w^*, U^*, and U^*_E, are continuous functions of α. Therefore, such comparative statics results still hold if the probability of firing an entrenched manager is small. According to proposition 3, the model predicts that if entrenched managers are hard to remove, for example because of strong takeover legislation, a reduction in the probability of entrenchment increases managerial payment, at least for managers with a good reputation. The intuition is very simple: in this case the demand effect dominates. The prevailing effect of a better governance is then to diminish the number of entrenched managers and, therefore, to increase the number of firms competing for good managers. The equilibrium wage then increases as a consequence of the increased level of competition. Hermalin (2005) establishes a similar relationship between corporate governance quality (interpreted as board independence in his article) and managerial pay. The argument he proposes is the following: more independent boards are more willing to replace their CEO in case of poor performance, and, as a result, CEOs have a shorter tenure. Therefore, to offset the cost associated with a faster turnover, managerial compensation must increase. However, it should be noticed that this argument applies naturally to a situation in which board independence increases in one firm but not in the others. In fact, a manager has to be compensated for the expectation of a shorter tenure only if there are other firms with weaker governance in which tenures can be reasonably be expected to be longer. In other words, if a firm strengthens its governance, while all the others do not, it is reasonable that a manager will require a wage premium to accept the job in the firm that is getting stronger. Without such wage premium, the manager could prefer the outside option of working in a weaker firm. This logic, however, does not apply to time series variation in the quality of governance when this variations affect all the firm in roughly the same way (for example, because of a change in the existing legislation). In fact, if working conditions become tougher (e.g. tenures get shorter) in any firm, a manager's outside option changes exactly in the same way as her actual working conditions. In such a situation it is not clear why managers should receive a higher wage.[16]

Proposition 3 offers a different argument that exactly works when Hermalin's seems to be less convincing. Furthermore, the proposition clarifies that the upward pressure exercised on managerial payments by better governance, also increases the utility of both entrenched an non-entrenched managers. Notice, however, that from an-ex ante point of view, stronger boards means that managers can entrench themselves, then enjoying the entrenchment value $U_E^* - U^*$, less easily. A trade off then emerges ex-ante: if the probability of entrenchment decreases, expected labor market benefits increases, but expected private benefits derived from entrenchment decreases.

To conclude this section let's analyze briefly the comparative statics effects of λ in the opposite special case $\alpha = 1$. As it is clear from (3.10), in this case it must be that $e^* = e_E^*$, and then expression (3.11) also implies that $U^* = U_E^*$. In a sense, entrenchment is immaterial and has no effect whatsoever. This is reasonable because the condition $\alpha = 1$ exactly means that failing managers are fired with probability one, independently of whether they are entrenched or not. We should also expect the probability of entrenchment, or equivalently λ, to have no effect on the equilibrium. This is indeed the case, and, in fact, equilibrium conditions (3.8) and (3.9), with $\alpha = 1$ and taking into account that $e^* = e_E^*$, are both equivalent to:

$$\sigma^2 H \left(1 - \bar{\theta} e^*\right) + \bar{\theta} B = c'\left(e^*\right), \tag{3.12}$$

[16] The argument proposed by Hermalin may also fail if the employment contract leaves managers with more than their reservation utility. Managers can extract rents through optimal contracts for several reasons: limited liability, an established good reputation, etc. In this case, even if the improvement in the quality of governance does not affect the value of the managerial outside option, it is not obvious why it should increase managerial compensation.

so that equilibrium effort levels are independent of λ. The same is true for the equilibrium wage which is given by:

$$w^* = \frac{\sigma^2}{\bar{\theta}} H \left(1 - \bar{\theta} e^*\right). \tag{3.13}$$

A similar result obtains when $\alpha < 1$ but $\gamma = 0$. In this case, entrenched managers do secure their position to some extent but entrenchment has zero probability ex-ante, and it is irrelevant in determining the expected equilibrium wage in the labor market. In fact, it is immediate to check that, in this case, condition (3.12) defines optimal effort for non-entrenched managers, and condition (3.13) defines the equilibrium wage. However, equilibrium effort for entrenched managers is now defined by:

$$\sigma^2 H \left(1 - \bar{\theta} e^*\right) + \alpha \bar{\theta} B = c' \left(e_E^*\right).$$

The value of α does have an effect on the equilibrium effort exerted by entrenched managers, but entrenchment occurs with probability zero so that α does not affect the equilibrium wage and the equilibrium effort exerted by non-entrenched managers.

3.5. Conclusion

This paper analyzed the interaction between managerial entrenchment, compensation and career concerns. Contrary to most of the existent literature on boards, the analysis is at a market level, and it allows to identify effects that are not immediately evident at the firm level. In particular, entrenchment was shown to affect the market for CEOs and, therefore, the strength of managerial career concerns. Less entrenchment increases the competition for CEOs on the demand side, but it also increase the competition among CEOs on the supply side, and the total effect is not predictable in general terms. However, when the probability of dismissal of an entrenched manager is low, for example because of a strong legal protection against hostile takeover, the demand side effect dominates, and less entrenchment unambiguously increases the equilibrium managerial payment, thus having a beneficial effect of career concerns.

To focus on these market effects, the analysis treats in a very simple way the consequences of entrenchment: it just makes it harder to remove an incumbent CEO. Of course, such effects would also be present in a more realistic treatment in which for example, as in Hermalin and Weisbach (1998), friendly boards were less willing to monitor their CEO.

The theoretical findings suggest that the steady increase in managerial compensation observed in the last 20 years can be explained, at least partially, with a corresponding steady improvement in board independence. According to this point of view, the increased level of managerial pay is due to increased competition for good CEOs, and it should also have produced stronger career concerns for young generations of managers.

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