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A NOTE ON PRICE CAPS WITH DEMAND UNCERTAINTY, QUANTITY PRECOMMITMENT AND DISPOSAL*

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Abstract

Since Littlechild (1983)'s report, price cap regulation has been regarded as an effective instrument to mitigate market power when precise information about cost and demand is available. Earle, Schmedders and Tatur (2007) establishes that the comparative static properties of price caps that hold when the demand is deterministic fail for a generic stochastic demand schedule. This note concerns the validity and interpretation of this result in a setting in which firms choose how much to produce ex-ante, but then upon observing the realization of demand choose how much of their output to supply, freely disposing of the output it does not supply.

Keywords: Price Cap Regulation, Capacity Investment and Withholding, Demand Uncertainty.

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Since Littlechild (1983)'s report, price cap regulation has been regarded as an effective instrument to mitigate market power: When precise information about cost and demand is available, the introduction of a binding price cap leads to an increase of the equilibrium output and surplus, and to a decrease of the market price. Moreover, under broad regularity conditions on the demand and cost functions, output and surplus decrease and the market price increases with the price cap at any price cap above marginal cost.

A more recent paper by Earle, Schmedders and Tatur (2007) – henceforth EST(2007) – shows that when demand is uncertain and firms make output decisions ex-ante, the total output is suboptimally low for price caps near marginal cost, and may increase with the price cap. Moreover, it establishes that the comparative static properties of price caps that hold when the demand is deterministic fail for a generic stochastic demand schedule. These findings lead these authors to conclude that "standard arguments supporting the imposition of price caps break down in the presence of demand uncertainty."

Lemus and Moreno (2014) – henceforth LM(2014) – studies the impact of price caps with demand uncertainty and with and without quantity precommitment and contends that EST(2007)'s sweeping conclusion is not justified. This note concerns the validity and interpretation of the claim in EST(2007)'s Theorem 6 about the generic failure of the comparative static properties of price caps in a setting in which firms choose how much to produce ex-ante, but then upon observing the realization of demand choose how much of their output to supply, freely disposing of the output it does not supply. For clarity and brevity's sake, we discuss this result for a market in which the uncertain demand is given by $P(Q,\Theta) = \max\{\Theta - Q, 0\}$, where Θ is a random variable with support on $[0,1]$, which is monopolized by a single profit maximizing firm that produces the good with constant marginal cost $c \in (0, E(P(0,\Theta))$. The imposition of a price cap $p \in \mathbb{R}_+$ implies that if the monopolist supplies q units of output and the realized demand is θ, then the market price is $\min\{P(q,\theta), p\}$.

When the monopolist must supply its entire output for each demand realization, EST(2007)'s Theorem 4 establishes the following result: *Let* (p', p'') *be a subinterval of* $[0,1]$, *and let* F *be the set of all cumulative distribution functions* F *whose support in contained on* $[0,1]$ *and such that* $E_F(\min\{P(q,\Theta),p\}) > c$ *for all* $p \in (p', p'')$ *and* $F(q^*_F(\bar{p}) + \bar{p}) < 1$ *for some* $\bar{p} \in (p', p'')$, *where* $q^*_F(\cdot)$ *is the monopolist equilibrium output. Then the set of distributions* $F \in F$ *for which* $q^*_F(\cdot)$ *is non-increasing on* (p', p'') *is nowhere dense in* $\mathcal{F}$. EST(2007) establishes this result by showing that for any continuously differentiable *c.d.f.* $\bar{F} \in \mathcal{F}$ with convex support such that

$dq^*_{\bar{F}}(\bar{p})/dp < 0$ for some $\bar{p} \in (p', p'')$ there is another *c.d.f.* $\tilde{F} \in \mathcal{F}$ arbitrary close to $\bar{F}$ such that $dq^*_{\tilde{F}}(\bar{p})/dp > 0$. The *c.d.f.* $\tilde{F}$ is obtained perturbing $\bar{F}$ on an arbitrarily small interval around $\bar{p}$ by shifting the probability on the interval to the endpoints and creating two atoms.

EST(2007)'s Theorem 6 claims that this results holds also in the setting in which the firm chooses it output ex-ante, and upon observing the realization of demand chooses how much of its output to supply, freely disposing of the output it does not supply, and argues that the same proof applies to this setting. We show that this is not the case. LM(2014) shows that the monopolist's marginal revenue for price caps p and output levels q such that $p < q < 1 - p$ is

$$MR_F(p, q) = p[1 - F(p + q)]. \tag{1}$$

(For these price cap-output pairs – region A in LM(2014) – an additional unit of output would be supplied only for high demand realization $\theta \geq p + q$.)

Consider the uniform distribution $\bar{F}$ given for all $\theta \in [0, 1]$ by $\bar{F}(\theta) = \theta$. Take $\bar{p} \in [0, 1]$ and assume that the monopolist's marginal cost satisfies $c < p\,(1 - 2p)$ for p in a small interval (p', p'') around $\bar{p}$. Then $E_{\bar{F}}(\min\{P(q, \Theta), p\}) > c$ for $p \in (p', p'')$. Moreover, for these parameter values, for each $p \in (p', p'')$ the monopolist's equilibrium output is in the interval $(p, 1-p)$, and its marginal revenue is given by equation (1). Hence the function $q^*_{\bar{F}}$ solves the equation $MR_{\bar{F}}(p, q) = c$, and is given for $p \in (p', p'')$ by $q^*_{\bar{F}}(p) = 1 - p - c/p$. Assume that

$$\frac{dq^*_{\bar{F}}(\bar{p})}{dp} = -1 + \frac{c}{\bar{p}^2} < 0.$$

We show that the method used in the proof of EST(2007)'s Theorem 4 to perturb $\bar{F}$ in order to obtain a *c.d.f.* $\tilde{F}$ such that $dq^*_{\tilde{F}}(\bar{p})/dp > 0$ does work in this setting. Specifically, $\bar{F}$ is perturbed around $\tilde{\theta} \in [0, 1]$, where $P(q^*_{\bar{F}}(\bar{p}), \tilde{\theta}) = \bar{p}$, i.e., $\tilde{\theta} = 1 - c/\bar{p}$. Denote by $\tilde{F}$ the *c.d.f.* given for $\theta \in [0, 1]\backslash[\tilde{\theta} - \varepsilon, \tilde{\theta} + \varepsilon)$ by $\tilde{F}(\theta) = \bar{F}(\theta) = \theta$, and for $\theta \in [\tilde{\theta} - \varepsilon, \tilde{\theta} + \varepsilon)$ by $\tilde{F}(\theta) = \bar{F}(\tilde{\theta} - \varepsilon) + 2\alpha\varepsilon = \tilde{\theta} - (1 - 2\alpha))\,\varepsilon$, where $\varepsilon > 0$ is arbitrarily small, and $\alpha \in (0, 1)$ is such that $q^*_{\tilde{F}}(\bar{p}) = q^*_{\bar{F}}(\bar{p}) := \bar{q}$. Let p be near $\bar{p}$ and $q \in (p, 1-p)$. Obviously, $MR_{\tilde{F}}(p, q) = MR_{\bar{F}}(p, q)$ whenever $p + q \in [0, 1]\backslash[\tilde{\theta} - \varepsilon, \tilde{\theta} + \varepsilon)$. Let $p + q \in [\tilde{\theta} - \varepsilon, \tilde{\theta} + \varepsilon)$; then

$$MR_{\tilde{F}}(p, q) = p[1 - \tilde{F}(p + q)] = p[1 - \tilde{\theta} + \varepsilon - 2\alpha\varepsilon] = p[c/\bar{p} + (1 - 2\alpha)\,\varepsilon],$$

is independent of q; and $MR_{\tilde{F}}(\bar{p}, \bar{q}) = c$ implies $\alpha = 1/2$, and therefore $MR_{\tilde{F}}(p, q) = cp/\bar{p}$. Thus, $MR_{\tilde{F}}(p, q)$ is greater (less) than c if $p > \bar{p}$ ($p < \bar{p}$). Hence $q^*_{\tilde{F}}(p) = \tilde{\theta} - p - \varepsilon$ if $p > \bar{p}$, and $q^*_{\tilde{F}}(p) = \tilde{\theta} - p + \varepsilon$ if $p < \bar{p}$, and therefore $dq^*_{\tilde{F}}(p)/dp = -1$ near $\bar{p}$.

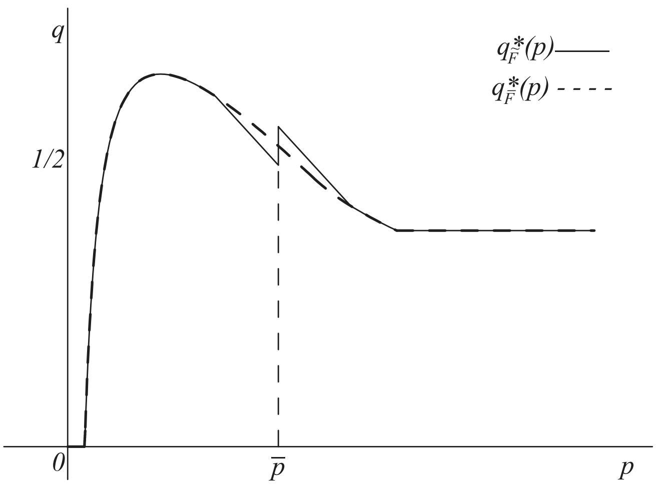

Figure 1: The Functions $q^*_{\bar{F}}$ and $q^*_{\tilde{F}}$.

Figure 1 provides the graphs of $q^*_{\bar{F}}$ and $q^*_{\tilde{F}}$ for $c = 1/32$, $\bar{p} = 2/5$ and $\varepsilon = 1/30$. Although $q^*_{\tilde{F}}$ becomes a correspondence at $\bar{p}$, the comparative statics of $q^*_{\tilde{F}}$ for p near $\bar{p}$ remain standard, i.e., output is decreasing in the price cap for p near $\bar{p}$. Therefore this perturbation fails to change the sign of the impact of the price cap on output in a neighborhood of $\bar{p}$. Thus, EST(2007)'s proof, which relies on this perturbation, does not apply when the monopolist may freely dispose of its output. In fact, this perturbation has an effect on the monopolist profit and the profit maximizing level of output akin to that of creating a flat spot on a deterministic demand.

Although it is unclear whether or not a version of EST(2007)'s genericity result holds in this setting, on the broad set of probability distributions considered the Banach-Mazurkiewicz Theorem would suggest that such results are to be expected. Nonetheless, if we restrict attention to continuous random variables and consider perturbations less drastic than those used in the proof of EST(2007)'s Theorem 4, the comparative static properties of price caps are preserved in the region of the parameter space in which the monopolist's marginal revenue near the equilibrium output is given by equation (1). In this region we may differentiate implicitly the equation $MR_F(p, q) = c$ to obtain

$$\frac{dq^*_F(p)}{dp} = \frac{[1 - F(p+q)]}{pf(p+q)} - 1,$$

where f is the *p.d.f.* of F. Hence the sign of the derivative $dq^*_F(p)/dp$ is preserved on a small neighborhood around F in, e.g., the topology of uniform convergence.

To conclude, interpreting the vulnerability of the sign of $dq^*_F(p)/dp$ to these questionably *small* perturbations as evidence of a lack of predictability of price cap regulation seems a stretch. One suspects that such perturbations would lead to analogous results in most economic settings involving uncertainty. Moreover, over the set of continuous distributions such genericity results do not hold (at least on some topologies). Finally, as shown by LM(2014) for the case of free disposal and by Grimm and Zoettl (2010) for the case of no disposal, under some regularity assumptions on the distribution of the demand the comparative static properties of price caps with demand uncertainty and quantity precommitment are similar, although more complex, to those obtained when the demand is deterministic.

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