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ESSAYS ON CARTEL BEHAVIOR AND DETERRENCE POLICY

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Resumen en Castellano

Esta tesis consta de tres artículos sobre el impacto de distintas políticas para la disuasión de delitos corporativos en el comportamiento estratégico de los agentes económicos. Combina temas de Organización Industrial, Derecho y Economía, con fundamentos teóricos de microeconomía. El primer artículo analiza el impacto de la política de defensa de la competencia en la disuasión de la colusión y en la eficiencia productiva de los cárteles que sobreviven, en contextos en donde ocultar la colusión es costoso. En este artículo se considera una política de defensa de la competencia basada en multas, inspecciones y programas de clemencia.[1] El segundo artículo analiza el impacto de la divulgación de información sobre próximas inspecciones en la sostenibilidad de los cárteles. El tercer artículo analiza la efectividad de recompensar la denuncia de irregularidades en la disuasión de delitos empresariales, así como su impacto en el contrato óptimo entre el dueño de la empresa y el empleado denunciante.

En el primer capítulo, titulado *¿Qué pasa si las multas por colusión no son lo suficientemente altas? Implicaciones en la disuasión y la eficiencia productiva*, se analiza el impacto de la política de defensa de la competencia en las decisiones de las empresas relacionadas a la colusión y a la producción. En el modelo, las empresas del cártel dedican esfuerzo costoso a actividades relacionadas con la eficiencia productiva y la ocultación del cártel: lo primero reduce los costes marginales de producción y lo segundo reduce la probabilidad de detección. El esfuerzo es costoso, y por ello limitado, por lo que las empresas del cártel deben decidir respecto de su distribución entre eficiencia productiva y ocultación. En este contexto, un primer análisis se refiere a cómo la política de defensa de la competencia distorsiona el interés de las empresas en la eficiencia productiva con respecto a la ocultación. Se demuestra que cuando las multas son bajas y/o

[1] Los programas de clemencia ofrecen una reducción de multas a la empresa del cártel que brinde pruebas fehacientes del mismo a la autoridad de defensa de la competencia y que coopera con ella a lo largo de la fase de enjuiciamiento. La efectividad de estos programas reside en incrementar la tentación al desvío.

la probabilidad de inspección es baja, las empresas encuentran rentable destinar la totalidad de sus esfuerzos a la eficiencia productiva. Sin embargo, a medida que las multas o las inspecciones suben, las empresas sustituyen esfuerzo desde la eficiencia productiva a la ocultación. Esta reasignación de esfuerzos hace que la colusión sea sostenible en industrias en las que no lo es de otra manera y crea ineficiencias no consideradas en modelos tradicionales de colusión.

A la luz de estos resultados, un segundo análisis se refiere al impacto de un cambio de política en la disuasión del delito y en el bienestar. Se demuestra que un aumento de la multa puede tener dos efectos opuestos sobre el bienestar, mientras que lo puede mejorar a través de un menor número de cárteles, también lo puede reducir a través de una mayor ineficiencia productiva de aquellos cárteles que sobreviven a la política. En particular, para niveles intermedios de la multa, un aumento de ésta puede conllevar a un aumento de bienestar derivado de un menor número de cárteles que no compensa la pérdida de bienestar derivada de una mayor ineficiencia productiva de aquellos que sobreviven. A raíz de este resultado, se recomienda la fijación de multas muy elevadas tal que ningún cártel sobreviva. Sin embargo, en la práctica esto no siempre es creíble o posible de implementar. En este contexto, se demuestra que la disuasión no es monótona ni en el nivel de multas ni en el de inspecciones considerados en forma individual: un aumento en cualquiera de estos instrumentos puede mejorar la sostenibilidad de la colusión en lugar de su disuasión, si el otro instrumento no se ajusta en consecuencia. Por lo tanto, el mensaje principal en cuanto a la política de defensa de la competencia es que ésta debe ser cuidadosamente diseñada, combinando cuidadosamente ambos instrumentos, multas e inspecciones.

Finalmente, el análisis recomienda el uso de programas de clemencia. Dado que estos programas exigen plena colaboración por parte de la empresa informante, una solicitud de clemencia implica que esta empresa no destina esfuerzo a la ocultación de evidencia, lo que a su vez implica mayor eficiencia productiva durante la fase de persecución y enjuiciamiento.

En el segundo capítulo, titulado *Un inspector llama: Sobre la optimalidad de inspecciones anunciadas*, se amplía el modelo anterior con la introducción de una nueva política: ahora, la autoridad de defensa de la competencia puede revelar fehacientemente información sobre la probabilidad de inspección en el período vigente antes de que las empresas tomen decisiones estratégicas sobre precio, producción y actividades asociadas a la colusión. Se considera que esta política se lleva a cabo a través del envio de señales a aquellas empresas con alta probabilidad de inspección en el período en curso.

La posibilidad de tomar decisiones sobre la base de una probabilidad de inspección más precisa afecta los incentivos a la colusión en dos formas opuestas. Por un lado, aumenta la rentabilidad de la colusión: información precisa respecto de la probabilidad de inspección en curso permite a las empresas minimizar pérdidas de beneficios por (*i*) dedicar esfuerzo costoso a la ocultación en períodos en donde no se recibe una alerta de inspección, y (*ii*) asignar demasiado esfuerzo a la eficiencia productiva en períodos en donde sí se recibe tal alerta. Ambos efectos conllevan a mayores incentivos a la colusión. Sin embargo, por otro lado, la firma que se desvía se beneficia igualmente de esta información, lo que aumenta los incentivos al desvío y, por tanto, dificulta el sostenimiento de la colusión.

En este contexto, la pregunta principal es si la introducción de un programa de alerta puede mejorar la disuasión (es decir, si el último efecto descrito anteriormente puede prevalecer sobre el primero). Se demuestra que este caso ocurre cuando la multa y la probabilidad de inspección son altas. Para tales valores de los parámetros, el programa apenas distorsiona el comportamiento de la empresa que sigue el acuerdo de colusión, pero, sin embargo, distorsiona ampliamente el de la empresa que se desvía. Con altos costes esperados, el programa no implica mucho para la empresa que sigue el acuerdo de colusión: ésta asigna todo su esfuerzo a la ocultación (*a*) siempre, si no hay un programa de alerta, y (*b*) casi siempre, si lo hay. Sin embargo, el programa implica mucho para la empresa que se desvía. Dado que los períodos de no inspección son pocos, información precisa acerca de cuando se está en uno de ellos es de gran relevancia: los beneficios del desvío se maximizan si la empresa se desvía en uno de estos períodos porque los costes de detección son mínimos (nulos).[2] Por lo tanto, en este contexto, la introducción del programa de alertas conlleva una mejora en la disuasión de la colusión.

En el tercer capítulo, titulado *Premiando la denuncia de irregularidades en un modelo de principal-agente*, se analiza el impacto de recompensar a empleados que denuncian delitos corporativos en la disuasión del delito y en la eficiencia productiva interna de las empresas. En el modelo, hay un principal que posee una empresa con dos empleados. Ambos empleados son contratados para dedicar esfuerzo costoso a la producción, pero uno de ellos también puede dedicar, secretamente, esfuerzo a cometer un delito corporativo (*el delincuente*), y el otro a reunir pruebas de tal acto (*el agente*). El crimen no sólo produce beneficios privados para el delincuente, sino también una externalidad para el principal.

En este contexto, se demuestra que es posible disminuir la existencia de delitos corporativos a partir de la introducción de programas que recompensan a empleados que, siendo ajenos al delito, exponen públicamente evidencia del mismo: las recompensas inducen a estos empleados a reunir pruebas del delito, lo que aumenta la probabilidad de detección del mismo; frente a una mayor probabilidad de detección, los beneficios de delinquir disminuyen y con ello los deseos del delicuente de comenter tal acto. Sin embargo, también se observa que esta política de recompensas tiene efectos secundarios sobre la eficiencia productiva de las empresas: si las externalidades netas que el principal percibe por el delito son elevadas, éste retribuirá en exceso el esfuerzo destinado a la producción, en un intento de sesgar la asignación de esfuerzo del agente en detri-

[2] El supuesto clave en este resultado es que la evidencia de la colusión dura un sólo período, el actual, de tal manera que la empresa que se desvía no puede ser castigada por haber prácticado la colusión en períodos anteriores. Este supuesto es habitual en modelos de colusión.

mento de la recopilación de evidencia del delito. Lo contrario acontece cuando las externalidades netas que el principal percibe por el delito son muy bajas (altamente negativas). Se destacan dos casos extremos: uno en el que el principal contrata al agente sólo por sus actividades relacionadas con la recopilación de pruebas del delito, y otra en el que, para minimizar la probabilidad de detección del delito, el principal no contrata al agente. Este último implica una pérdida total de bienestar derivada de los programas de recompensas.

Por último, se demuestra que el principal puede estar enteresado en introducir un programa de recompensas de accionar interno a la empresa, independientemente de la existencia de un programa de recompensas externo a la misma. Se observa tal interés cuando el delito corporativo en cuestión es altamente perjudicial para el principal y, en el caso de existir un programa de recompensas externos, cuando además la recompensa que ofrece el programa externo no es lo suficientemente alta como para disuadir el delito tanto como desea el principal. Además, se demuestra que el uso de un programa interno de recompensa máximiza la disuasión del delito. Este resultado sugiere la implementación de programas públicos de recompensa solamente para la persecución de delitos que favorecen los intereses de los propietarios de las empresas (fraudes impositivos, colusión y delitos ambientales, entre otros).

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Chapter 1

Introduction

This thesis consists of three papers on the impact of deterrence policies for corporate crimes in the strategic behavior of economic agents. It combines topics from Industrial Organization and Law and Economics, with theoretical grounds from Microeconomics. The first chapter analyzes the impact of the antitrust policy on deterring collusion and the productive efficiency of surviving cartels when concealing collusion is costly. This chapter considers an antitrust policy based on fines, inspections and leniency programs.[1] The second chapter analyzes the impact of disclosing information on the likelihood of inspections on the sustainability of cartels. The third chapter analyzes the effectiveness of rewarding whistle-blowing to deter occasional corporate crimes, as well as its impact on the optimal contract between the firm owner and the whistle-blower.

In the first chapter, entitled *What if Fines on Collusion are not high enough? Implications on Deterrence and Productive Efficiency when Concealment is Costly*, I analyze the impact of the antitrust policy on the firm's decisions over crime and production. In the model, cartel firms devote costly effort to activities

[1] Leniency programs reduce sanctions against the cartel firm that reports evidence of the cartel to the antitrust authority and cooperates with it along the prosecution phase. The effectiveness of these programs to improve deterrence lies in enhancing the temptation to deviate.

related to productive efficiency and concealment, as the former reduces marginal costs from production and the latter reduces the probability of detection. Effort is costly, and so that limited; thus cartel firms have to decide on how to allocate it among productive efficiency and concealment. In this context, a first analysis refers to how the antitrust policy distorts firms' interest in productive efficiency with respect to that in concealment. I show that when fines are low and/or the probability of inspection is low, firms find it profitable to allocate all effort to productive efficiency. However, as fines or inspections go up, firms substitute effort from productive efficiency to concealment. This reallocation of effort makes collusion sustainable in industries where it wouldn't be otherwise and create inefficiencies not considered in standard models of collusion.

In the light of these results, a second analysis refers to the impact of a policy change in deterrence and welfare. I show that a fine increase can have two opposite effects on welfare, while it can improve welfare through fewer cartels, it can also reduce it through more inefficient surviving ones. Particularly, for intermediate fine levels, a fine increase can imply a welfare gain from fewer cartels that does not compensate the welfare loss from more inefficient surviving ones. This result favors setting very high fines such that no cartel survives. However, in practice this is not always credible or possible to implement. In this context, I show that deterrence is non-monotonic neither in the level of fines nor in that of inspections individually considered: an increase in any of them may enhance collusion sustainability rather than deterrence if the other instrument is not changed accordingly. Therefore, the main policy recommendation is that the antitrust policy has to be carefully designed, such that combining both instruments, fines and inspections, conveniently.

Finally, the analysis favors the use of leniency programs. Since leniency programs demand full collaboration from the reporting firm, a leniency application implies no effort on concealment from that firm, and thus an efficiency gain under deviation.

In the second chapter, entitled *An Inspector Calls: On the Optimality of Warning Cartel Firms before an Inspection*, I extend the previous model by introducing a new policy in the game: the Antitrust Authority can credibly disclose information on the likelihood of a current inspection before firms take strategic decisions on price, production and cartel activities. I consider that the Antitrust Authority performs this policy by sending warnings to firms with a high probability of a current inspection.

The possibility to take decisions on the basis of a more accurate probability of inspection affects cartel firms incentives to collude in two opposite ways. On the one hand, it raises collusion profitability: with accurate information on the likelihood of a current inspection, firms can minimize profit losses from (*i*) devoting costly effort to concealment anytime that they do not receive a warning, and from (*ii*) an unprofitable effort allocation towards productive efficiency each time that they receive a warning. In this case, incentives to collude go up and collusion is facilitated. However, on the other hand, the deviants also benefit from this information; this enhances incentives to deviate and makes collusion harder to sustain.

In this context, the main question is whether the introduction of a warning program can improve deterrence (i.e., whether the latter effect described above can prevail over the former). I show that disclosing information on the likelihood of a current inspection improves deterrence when both the fine and the probability of inspection are high. For such parameter values the program does not distort much the behavior of a firm that follows the collusive agreement, but it does so for a deviant. Indeed, faced with high expected costs, the program does not imply much for the former: this allocates all its effort to concealment (*a*) always, if there is not a warning program, and (*b*) almost always, if there is. However, for a deviant the program implies a lot. Since periods of no inspection are few, accurate information about when one is in one of these is of huge relevance: in these periods deviation not only implies the standard higher gains from sales, but also minimum (no) detection costs.[2] Hence, in this context, whereas firm's benefits derived from the program are little under collusion, there are huge under deviation; and so the program implies an improvement in deterrence.

In the third chapter, entitled *Rewarding Whistle-Blowers: Implications on Deterrence and on Principal-Agent Contracts*, I analyze the impact of rewarding whistle-blowers on the deterrence of corporate crimes and the internal efficiency of firms. In the model, there is a principal that owns a firm with two employees. Both employees are hired to devote costly effort to production, but one of them can secretly devote effort to commit corporate crime (*the offender*) and the other to gather evidence of such an act (*the agent*). The crime not only yields private gains to the offender employee, but also an externality to the firm owner.

In this context, I demonstrate that whistle-blower programs that offer a reward to non-offender employees for the public exposure of corporate crimes can improve deterrence: rewards induce non-offender employees to gather crime evidence, which raises the probability of crime detection; faced with a higher probability of detection (and thus with lower net gains from crime), the offender's willingness to commit crime go down. However, I also show that this reward policy has side effects on the productive efficiency of firms: if the principal's net externalities from crime are very high, this will overpay effort devoted to production in an attempt to bias the agent's effort allocation away from crime detection. The opposite will hold when the principal's net externalities from crime are very low (highly negative). Two extreme cases stand out: one in which the principal hires the agent only for his activities related to gathering crime evidence, and other in which, to minimize crime detection, he does not hire the agent. The latter one implies that rewards create a total loss of welfare.

Finally, I demonstrate that it may be in the principal's interest to create

[2] A key assumption in this result is that evidence from collusion lasts for one period, such that a deviant can not be punished from colluding in past periods. This is a usual assumption in models of collusion.

a reward-program private to the firm, regardless of the existence of a (public) whistle-blower program. A program of this type arises when crime is highly detrimental to the principal and, if there exists a whistle-blower program, when also the public reward is not high enough to deter crime as much as the principal would like. Moreover, I demonstrate that the use of a private reward-program implies maximum deterrence. This result suggests the implementation of public whistle-blower programs to prosecute crimes that work in favor of the interest of firm owners (e.g., tax-frauds, collusion, environmental crimes, etc.).

Chapter 2

What if cartel fines are not high enough? Implications on deterrence and productive efficiency

2.1 Introduction

To succeed, cartels concentrate on two targets: profit maximization and concealment. To achieve these, cartel firms devote resources to productive efficiency and to concealment. When resources are limited, firms face the challenge to allocate them optimally, sacrificing productive efficiency in favor of concealment, or *vice versa*. In this decision, the antitrust policy has a key role, as it affects the expected detection costs, and through it, the relative importance of the targets. In this context, very large fines achieve full deterrence and, therefore, also productive efficiency. But, what if fines are not high enough? How do low and moderate fines distort cartel firms' effort allocation? What are the implications on deterrence and on firms' productive efficiency? And on social welfare? This paper sheds light to these questions.

I develop a model in which cartel firms devote effort to productive activities and to concealment: effort devoted to production reduces marginal costs and effort devoted to concealment reduces the probability of detection. Effort is costly and limited, thus firms have to decide on how to allocate it among productive efficiency and concealment. The intuition goes as follows: cartel survival depends on the success of each of its member firms, not only as firms that collude in a cartelized market, but also as firms that individually operate in complex markets. Thus, in a cartel, senior executives have to be cautious on how to allocate their time, effort and attention among the own productive efficiency and the cartel organization, in order to guarantee a balanced success on both.[1] For simplicity purposes, among the activities related to the cartel organization, I focus on concealment activities. These include the attendance to secret meetings all over the world and the conduct of a joint sales agency, among other activities.[2] For further simplification, I reduce the three dimensions of care (effort, time and attention) to one: effort.

In this setup, cartel firms' effort allocation depends on fines and inspections. When fines are low and/or the probability of inspection is low, firms find it profitable to allocate all effort to productive efficiency. However, as fines or inspections go up, firms substitute effort from productive efficiency to concealment. This reallocation of effort makes collusion sustainable in industries where

[1] Aware of how time and effort-consuming are cartel activities (not only concealment), cartel members create complex hierarchical structures that set the role of each member in the cartel, as well as the rules to follow in case of eventual problems. In this way, the cartel is intended to be conducted as efficiently as a legal organization. For evidence on the hierarchical operativeness of cartels, see Baker & Faulkner (1993), Griffin (2000), Levenstein & Suslow (2006) and Harrington (2006).

[2] Using data from 19 discovered cartels, Levenstein & Suslow (2006) show that cartels that used joint sales agencies were among the more successful cartels in terms of their long-lastingness and fewer coordination problems. They find evidence on the use of a joint sales agency to conceal cartel practices in the following cartels: bromine (1885-1895), cement (1922-1962), diamonds (1870s-1970s), ocean shipping (1870-1924), oil (1871-1874), potash (1877-1897), and European steel (1926-1939).

it wouldn't be otherwise and create inefficiencies not considered in standard models of collusion. In the light of these results, a fine increase can have two opposite effects on welfare, while it can improve welfare through fewer cartels, it can also reduce it through more inefficient surviving ones. Particularly, for intermediate fine levels, a fine increase implies a welfare gain from fewer cartels that does not compensate the welfare loss from more inefficient surviving ones. This analysis suggests a carefully design for the antitrust policy, as deterrence is not monotonic in the level of the fine. Indeed, a fine increase may enhance collusion sustainability and a welfare loss rather than deterrence if inspections are not set accordingly.

In the analysis I also consider the effectiveness of leniency programs. These programs reduce sanctions against the cartel firm that reports evidence of the cartel to the antitrust authority (AA) and cooperates with it along the prosecution phase.[3] The effectiveness of these programs to improve deterrence lies in enhancing the temptation to deviate. In terms of my model, the prospect of an amnesty enhances deviation incentives more than in models without effort on concealment, as the firm that deviates saves effort costs associated to concealment (a deviant that applies for leniency has no incentives to devote costly effort to concealment).

The paper continues as follows. In Section 2, I provide a brief description of the related literature. In Section 3, I set up the model. In section 4, I solve it without effort on concealment (benchmark case), and in Section 5, I solve it with effort on concealment. In Section 6, I discuss the implications of a fine increase on deterrence and on firms' productive efficiency. In Section 7, I analyze the welfare implications of using leniency programs. I conclude in Section 8.

[3] Spagnolo (2008) provides an extensive review of literature on leniency in collusion.

2.2 Related Literature

This paper is closely related to studies on collusion that analyze productive inefficiencies created by antitrust policies. Aubert, Kovacic & Rey (2006) show that whistle-blowing programs improve the deterrence effect of high fines, but that, however, may induce (*i*) cartel firms to bribe informed employees and hold their under-performance to avoid possible crime reports, and (*ii*) non-cartelized firms to deter good cooperation between them when this can not be distinguished from the type of communication involved in price-fixing agreements. Therefore, although these programs can improve deterrence, they can also reduce the productive efficiency of surviving cartels and of non-cartelized firms.

Within a principal-agent model, Aubert (2009) achieves this result for individual leniency programs. Under the assumption that competition requires less managerial effort than collusion, and this, in turn, less than deviation, a manager that privately chooses market conduct and productivity-enhancing effort may opt for an anti-competitive conduct to save costly effort. With the same logic, a manager that colludes is highly tempted to deviate from the collusive agreement. Thus, to avoid cartelization or, under collusion, to prevent deviation, shareholders provide the manager with weak incentives to exert effort. In this context, individual leniency raises the costs of inducing collusion; but also makes it more likely the payment of informational rents and the request of inefficient effort levels when it is desired to induce competition. Therefore, while individual leniency contributes to deterrence, it also tempts competition-prone shareholders to induce collusion rather than competition. Regardless of the market conduct, productive efficiency is not achieved.

Similar to Aubert (2009), I also get into the firm's 'black-box' to analyze how the antitrust policy distorts the decision problem of those who decide on the behavior of the firm. However, the mechanism in this paper is different to that in Aubert. While Aubert focuses the analysis on how the antitrust policy can distort the agency problem of a principal and its subordinate, I focus the analysis on how the antitrust policy can distort firms' interest in productive efficiency with respect to that in concealment.

A key element in my framework is the possibility of destroying evidence of collusion. Aubert *et al.* (2006) suggest that firms keep evidence of the cartel if they fear that rivals will apply for leniency. Jellal & Souam (2004) point to firms' interest in keeping evidence taking into consideration that concealment is costly and negatively related to the inspector's performance. The higher the cost of effort devoted to concealment or the lower the inspector's effort devoted to discovering evidence, the more the evidence that firms prefer to keep. Following Jellal *et al.* (2004), I consider costly concealment as the driving force behind the keeping of evidence of the cartel. However, I assume that concealment can create productive inefficiencies by making use of effort previously devoted to production. This trade-off explains why firms keep cartel evidence in the absence of leniency programs or under-performance of the inspectors.

Other key element in my framework is the endogeneity of the probability of detection. Jellal *et al.* (2004) consider the probability of detection endogenous to the firms' and the inspector's efforts devoted to hide and discover collusion, respectively. Harrington (2004 and 2005) considers the probability of detection endogenous to current and previous periods' prices, since he assumes that anomalous price movement make customers and the AA suspicious that a cartel is operating. Harrington & Chen (2005) extends these works to leniency programs. Similar to the probability of detection, the probability of paying penalties is endogenous to the cartel firms' perception regarding the severity of the antitrust policy, Harrington & Chang (2009), and on the AA's resources devoted to prosecute and convict discovered cartels, Harrington (2011).

This paper is in line with those that consider the probability of detection (penalty) endogeneous to the firm's, not the AA's, behavior. The novelty of my work lies in the productive inefficiencies associated to concealment and, ultimately, to the antitrust policy. The reason is that allocating effort to concealment implies effort to be devoted away from productive activities, being effort devoted to concealment increasing in the severity of the antitrust policy.

This paper is also related to the literature on the impact of leniency programs in antitrust enforcement. Two main results stand out in this literature. First, high amnesties, and particularly total amnesty, improve deterrence by making self-reporting attractive and, therefore, inducing cartel members to defect and report, Motta & Polo (2003), Aubert *et al.* (2006), Chen & Rey (2007), Harrington (2008), among others. Second, low and intermediate amnesties may have a perverse effect on deterrence: when self-reporting becomes attractive, the threat of self-reporting to punish an agent that did not behave as agreed upon by the cartel may also become credible, and can be used by smart wrongdoers to enforce cartels that would not be sustainable in the absence of this threat, Spagnolo (2000), Buccirossi & Spagnolo (2001 and 2006). Regarding leniency, my paper has a very specific objective: whether a generally accepted leniency program distorts cartel firms' effort allocation, and if so, what implication does it have on firms' productive efficiency. To the best of my knowledge, this question has not been explored in the literature before.[4]

Finally, this paper also addresses the issue of antitrust policies with perverse effects, i.e., antitrust policies that contribute to cartel sustainability rather than to deterrence. Spagnolo (2000) and Buccirossi & Spagnolo (2001 and 2006) emphasis the perverse effect of leniency programs in deterrence. Harrington (2004, 2005) shows how a fine increase can (negatively) affect profits from deviation more than the net value of future profits from collusion, facilitating collusion (i.e., relaxing the incentive compatibility constraint of the cartel). Similarly, I show perverse effects from an antitrust policy that distorts profits from deviation more than those from collusion, facilitating collusion.

[4]For a *generally accepted leniency program* I consider a program that offers amnesty to the first informant firm for its full collaboration in the detection of the cartel.

2.3 The Model

Consider an economy with a continuum of industries. In each industry, there are two firms producing perfect substitutes and there is an inelastic demand for two units with reservation price v. I assume $v \sim U[\underline{v}, \overline{v}]$. Firms maximize profits over an infinite time horizon with constant discount parameter δ. To this end, they compete or collude on prices.

To produce, firms have a fixed marginal cost β, which can be privately reduced for the current period through *effort devoted to productive efficiency* $a_i \geq 0$, $i = 1, 2$. Then, the firm's marginal cost is $c_i = \beta - a_i$.

The market demand goes to the lowest priced firm or, in case of a price tie, to the firm with the lowest production cost. Under a price tie and equal production costs, firms equally split demand.

Collusion requires communication, which constitutes hard evidence for cartel detection. Evidence lasts for one period and can be discovered by the AA during an inspection. However, firms can privately destroy some of evidence through costly effort and, consequently, reduce the likelihood of finding evidence in an inspection.

To model this, I set the probability of finding cartel evidence in an inspection to firm i: e^{-z_i}, $i = 0, 1$, where $z_i \geq 0$ is firm i's *effort devoted to concealment*. The higher is this effort, the lower is the probability of finding cartel evidence in an inspection to a firm.

Effort is costly. I set the firm's effort disutility function as $\frac{(a_i+z_i)^2}{2}$. This specification for the disutility of effort follows Holmstrom & Milgrom (1991) and is common in multitask analyses. It is consistent with the view that efforts are technological substitutes and that disutility depends on total effort (not on the firm's effort allocation).[5]

[5]For the effort allocation to be also relevant, one can introduce a weighting parameter

To fight cartels, the AA has two instruments, fines and inspections. Both instruments are specific to firms, which implies: (i) in a single period, the AA can inspect either firm i, or firm j, or both firms, and (ii) under detection, each firm pays a fine F.

I assume that the probability of an inspection to a firm, denoted by $\rho \in [0,1]$, is exogenously given. Hence, the cartel probability of detection is:

$$h\left(z_1, z_2 \mid \rho\right) = \rho \left(e^{-z_1} + e^{-z_2} \right) - \rho^2 e^{-z_1} e^{-z_2}$$

There is cartel detection if the AA finds evidence after inspecting one, or both, of the firms.

Note that each firm's effort devoted to concealment creates a positive externality to rivals by reducing the cartel probability of detection. The lower is z_i, the higher is the externality that firm i perceives from and additional unit of z_j ($\frac{\partial^2 h}{\partial z_i \partial z_j} < 0$).

The timing of the game is as follows. At stage 0, firms choose whether to collude or compete. If one firm chooses to compete, competition takes place and the game ends. If, instead, there is an agreement on collusion, at stage 1, firms decide whether to follow the collusive agreement or to deviate. Under deviation, the deviant either slightly reduces its price, or increases its effort devoted to productive efficiency, or both. In this way, it gets all demand.

At stage 2, effort, production and price decisions are executed and the rival's price is observed. Also, inspections take place. At stage 3, firms get their payoffs from sales. Under cartel detection, firms pay a fine F and the game starts again from stage 0. If the cartel is not detected, but one firm has deviated, a punishment phase takes place. Finally, if none of the firms have deviated and the cartel is not detected, the game repeats itself from stage 1.

In this setup, firms make simultaneous pricing and effort decisions in every

$\mu \in \Re_0^+$ such that $\frac{(a_i+\mu z_i)^2}{2}$. The assumption of $\mu = 1$ affects the degree of substitution between efforts, but in no case restricts the results of the paper.

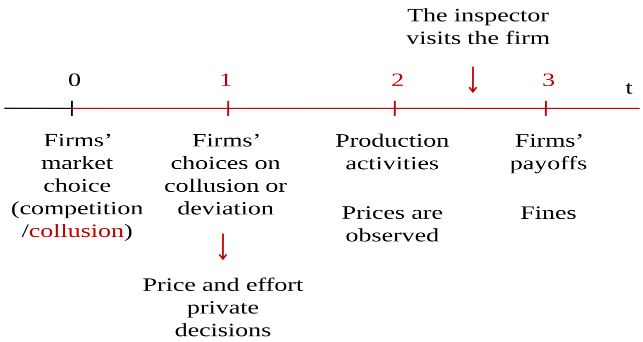

Figure 2.1: Time-structure of the model

period t. With an infinite horizon, firm i, $i = 1, 2$, chooses prices $p_{it} \in [0, v]$ and efforts $a_i, z_1 \in [0, 1]$ (effort z_i is relevant only under deviation), in every t, $t = 1, 2, ..., \infty$.

Under collusion, price choices at date t depend on the history of previous sales, so that p_{it} depends on $H_{it} = (q_{i1} \ ; \ q_{i2} \ ; \ . \ . \ . \ ; \ q_{i,t-1} \ )$, $i = 1, 2$. The rational behind this rule goes as follows: under collusion firms charge the same price and split the demand in halves, $q_i = 1$, $i = 1, 2$; thus, for a firm, no sales implies that the rival deviated (in price, in effort or in both). Therefore, the collusive strategy for firm i is to initially price at the collusive price p^c in period 1 and to continue pricing according to:

$$p_{it} = p^c \qquad if: \ q_i^\tau = 1 \quad \forall \tau \in \{1, ..., t-1\}, \ j = \{1, 2\}$$

as long no firm has deviated from this path. If a firm has deviated, there is a reversion to the single-period Nash equilibrium strategy of pricing, since Nash reversion can assure zero profits for the deviant.

In the one-shot game, firms choose price and effort devoted to productive efficiency to maximize current profits:

$$\Pi_i = [p_i - (\beta - a_i)] \, q_i - \frac{a_i^2}{2}$$

Proposition 1 *There exists a one-shot game Nash equilibrium in which one* *firm obtains zero profits.*

In the one-shot game there is a pure strategy equilibrium in weakly dominated strategies that yields zero profits for both firms. Also, there are undominated mixed-strategy equilibria that yield zero profits for one firm and positive profits for the other. Since at the static Nash equilibrium there is at least one firm that obtains zero profits, Nash reversion in which the deviant obtains zero profits constitutes an optimal penal code.

2.4 Collusion without Effort on Concealment

Without effort on concealment, the probability of finding cartel evidence in an inspection to a firm is 1. Therefore, the cartel probability of detection is exogenously determined as a function of ρ: $h^B = 2\rho - \rho^2$.

The firm's problem is to chose price and effort to maximize:

$$\Pi_i = [p_i - (\beta - a_i)]\, q_i - \frac{a_i^2}{2} - F\rho\,(2-\rho)$$

The first term is the firm's payoff from production and the second and third ones its costs associated to effort and to detection, respectively.

Solving for effort: $a_i = q_i$, $i = 1, 2$.

Regarding price, under collusion firms charge the same price and split the demand in halves: $p_i = p^c$ and $q_i^c = 1$, $i = 1, 2$. Thus, in each period, firms make one unit of effort ($a_i^c = 1$) and obtain profits:

$$\Pi_i^c = p^c - \beta + \frac{1}{2} - F\rho\,(2-\rho)$$

If a firm decides to deviate, it either slightly reduces its price, or increases its effort devoted to productive efficiency (to reduce marginal costs), or both. In this way, it gets all demand. A price reduction does not have side effects on firm's efficiency, however the increase of effort on productive efficiency does it. Thus, to maximize profits, a deviant always reduce its price slightly and chooses the effort level a^d that maximizes the current value of profits from deviation. Assuming firm i deviates:[6]

$$a_i^d = \arg\max \left\{ 2 \; [\; p^c - (\beta - a_i) \;] - \frac{a_i^2}{2} - F\rho\,(2-\rho) \right\} = 2$$

Under deviation, the firm behaves as an efficient monopolist: it devotes two units of effort to produce the two units of the good that the market demands. Profits from deviation are:

$$\Pi_i^d = 2\,(p^c - \beta) + 2 - F\rho\,(2-\rho)$$

in the current period, and zero thereafter.

2.4.1 Cartel's Sustainability

Collusion is sustainable as long as firms have no incentives to deviate, i.e., when the current gains from deviation (G) are no greater than the present value of net future profits from collusion.

$$(ICC) \qquad G = \Pi^d - \Pi^c \;\leq\; \frac{\delta}{1-\delta}\,\Pi^c \tag{2.1}$$

In this model:

$$p^c - \beta + \frac{3}{2} \;\leq\; \frac{\delta}{1-\delta}\left[\, p^c - \beta + \frac{1}{2} - F\rho\,(2-\rho) \,\right]$$

For $\delta > \frac{1}{2}$, a price increase relaxes ICC, which implies that firms always charge the reservation price under collusion, $p^c = v$. Prices lower than v, make collusion harder to sustain, and prices higher than v would imply no sales. So, collusion is sustainable if and only if it is sustainable at price v. Along the paper I assume $\delta > \frac{1}{2}$.[7]

[6] Since the optimal penal code yields zero profits for the deviant forever after deviation, the current value of total profits from deviation equates current profits from deviation: $\pi_i^d + \delta\, 0 + \delta^2\, 0 + \delta^3\, 0 + \ldots = \pi_i^d$.

[7] Otherwise, collusion is not profitable. $\delta > \frac{1}{2}$ is the standard level of patient assumed in models of collusion.

Solving for v in ICC:

$$v \;\geq\; v_1 = \beta + \; \frac{\frac{3}{2} - 2\delta + \delta\rho\,(2-\rho)\,F}{(2\delta - 1)}$$

Proposition 2 *(Without effort on concealment) There exists* $v_1 \in [\underline{v}, \overline{v}]$ *such that collusion is sustainable in all industries with high enough reservation price,* $v \geq v_1$*.* v_1 *is increasing in* F *and* ρ*.*

From the AA's point of view, v_1 states the effectiveness of the antitrust policy to deter cartels: an increase in fine and/or in the likelihood of an inspection raises the threshold parameter v_1, making collusion harder to sustain.

2.5 Collusion with Effort on Concealment

Allowing for effort on concealment, the firm's problem is to choose price and effort levels that maximize:

$$\Pi_i = [p_i - (\beta - a_i)]\, q_i - \frac{(a_i + z_i)^2}{2} - F\;\rho \left[\left(e^{-z_i} + e^{-z_j}\right) - \rho e^{-z_i - z_j}\right]$$

Solving for efforts, the interior solution is:

$$a_i + z_i = q_i \tag{2.2}$$

$$q_i = F\rho e^{-z_i}\left(1 - \rho e^{-z_j}\right) \tag{2.3}$$

Equations (2.2) and (2.3) characterize firm's optimal behavior under both collusion and deviation. Equation (2.2) states that, for the same level of production, an increase in effort devoted to productive efficiency must be compensated with an equal reduction in effort devoted to concealment. Equation (2.3) states that z_i's marginal benefits to i's profits (LHS) must equate its marginal costs (RHS).

Rewriting (2.3):

$$R_i(z_j) = -\ln\left[\frac{q_i}{F\rho\left(1-\rho e^{-z_j}\right)}\right] \tag{2.4}$$

Equation (2.4) is an effort reaction curve. It represents each firm's effort devoted to concealment in terms of the rival's effort devoted to concealment.[8] Particularly, the higher is the rival's effort devoted to concealment, the higher is the own effort devoted to this activity too. To see this, assume that j increases its effort on concealment. Immediately, the cartel probability of detection decreases distorting i's equilibrium condition: now, i's marginal utility from effort devoted to concealment is lower than its marginal cost. To restore equilibrium, i increases its effort devoted to concealment, too.[9]

Regarding antitrust parameters, $R_i(z_j)$ is upward sloping in F and in ρ: the more severe is the antitrust policy, the more incentivized is the firm to conceal evidence, and thus the higher is the firm's effort devoted to this activity. Regarding firm's market share, $R_i(z_j)$ is downward sloping in q_i: the higher is the level of production, the lower is the firms's willingness to devote effort to concealment, as higher market shares makes concealment relatively less important with respect to productive efficiency.

Under collusion, firms charge the same price and split demand in halves. Setting $q_i^c = 1$ in equations (2.2) and (2.4), reaction curves $R_1(z_2)$ and $R_2(z_1)$ have a unique intersection point, that is on 45° line. Therefore, there exists a

[8] Using equation (2.2), one can rewrite equation (2.4) in terms of efforts devoted to productive efficiency.

[9] Analytically, for the same level of production ($dq_i = 0$), an increase in z_j ($dz_j > 0$) implies:

$$dq_i = -F\rho e^{-z_i}\left[\left(1-\rho e^{-z_j}\right)dz_i + \rho\ d\left(e^{-z_j}\right)\right] = 0$$

where $d\left(e^{-z_j}\right) < 0$. Solving for dz_i:

$$dz_i = -\frac{\rho\ d\left(e^{-z_j}\right)}{1-\rho e^{-z_j}} > 0$$

unique interior solution:

$$z_i^c = 1 - a_i^c = -\ln\left[\frac{F - \sqrt{F^2 - 4F}}{2F\rho}\right] \tag{2.5}$$

To assure a solution in the set of rational numbers, I assume $F > \underline{F} = 4$. This assumption is purely numerical and does not restrict the results of the paper.

Lemma 1 *Under collusion, $a_i^c + z_i^c = 1$, and there exist F_0 and F_1, where $F_0 < F_1$, such that: for $F < F_0$, all effort is allocated to productive efficiency ($a_i^c = 1$), and for $F > F_1$, all effort is allocated to concealment ($z_i^c = 1$). For $F \in (F_0, F_1)$, effort is allocated partially to each activity as determined by (2.5), thus $a_i^c, z_i^c \in (0,1)$.*

For $F < F_0$, productive efficiency is the relatively more important activity, thus firms allocate all effort to it. However, as fines go up, the relative importance of concealment increases, such that for $F \in (F_0, F_1)$ firms find it profitable to allocate effort among both productive efficiency and concealment: the higher the fine and/or the probability of inspection, the more biased the firms' effort allocation towards concealment. Finally, for $F > F_1$, concealment is the relatively more important activity, and thus firms allocate all effort to it.

The critical fine value F_0 is downward sloping in ρ: the higher is this probability, the higher is the relative importance of concealment with respect to productive efficiency, and therefore the lower is the critical fine value at which firms find it profitable to devote effort to concealment.[10]

If firm i decides to deviate, it slightly reduces its price to get all demand ($q_i = 2$), and redetermines effort allocation considering that its rival follows the

[10] Actually, both of the critical fine values, F_0 and F_1, are downward sloping in ρ:

$$F_0 = \begin{cases} \frac{1}{\rho(1-\rho)} & if\ \rho \leq \frac{1}{2} \\ 4 & if\ \rho > \frac{1}{2} \end{cases} \qquad F_1 = \frac{1}{\rho e^{-1}\left(1 - \rho e^{-1}\right)}$$

collusive agreement ($q_j = 0$).[11] Setting $q_i = 2$ in equations (2.2) and (2.4), Lemma 2 follows immediately:

Lemma 2 *(Assume firm i deviates) Under deviation $a_i^d + z_i^d = 2$, and there exist F_0^d and F_1^d, where $F_0 < F_0^d < F_1 < F_1^d$, such that: for $F < F_0^d$, firm i allocates all effort to productive efficiency ($a_i^d = 2$), and for $F > F_1^d$, to concealment ($z_i^d = 2$). For $F \in \left(F_0^d, F_1^d\right)$, it allocates effort partially to each activity as determined by $R_i(z_j^c \mid q_i = 2)$, thus $a_i^d, z_i^d \in (0, 2)$.*

Critical fine values F_0^d and F_1^d are downward sloping in ρ.[12]

From Lemmas 1 and 2 we observe that firms allocate effort similarly under collusion and deviation: for low values of the fine, all effort is allocated to productive efficiency; but, as fines go up effort is reallocated from productive efficiency to concealment.

However, the critical fine value at which the firm finds it profitable to devote effort to concealment is higher under deviation, $F_0 < F_0^d$. The reason is that under deviation, more units of the good are produced. Therefore, for $F \in \left(F_0, F_0^d\right)$ while the firm that follows the collusive agreement finds it profitable to devote effort to concealment, the firm that deviates does not. Similarly, for $F \in \left(F_1, F_1^d\right)$, the firm that follows the collusive agreement devotes all the effort to concealment, whereas the deviating firm does not. In this case, further reductions in the cartel probability of detection depend exclusively on the deviant. (Figure 2.2).

Two final comments are in order. First, for $\hat{F} = \frac{2}{e^{-1}\rho(1-e^{-1}\rho)} \in \left(F_1, F_1^d\right)$, efforts devoted to concealment under collusion and deviation are equal, $z_i^d =$

[11] As discussed in the benchmark case, firm i can deviate with a slight reduction in its price, an increase in its effort on productive efficiency, or both. In this way, it gets all demand. However, while a price reduction does not alter i's productive efficiency, an increase of effort on productive efficiency does it. Thus, to maximize profits from deviation, the firm always reduces its price. Whether it also increases its effort devoted to productive efficiency depends on the antitrust parameters ρ and F (Lemma 2).

[12] $F_0^d = \frac{4}{\rho(2-\rho)}$ and $F_1^d = \frac{4}{\rho e^{-2}\left(2-\rho e^{-2}\right)}$.

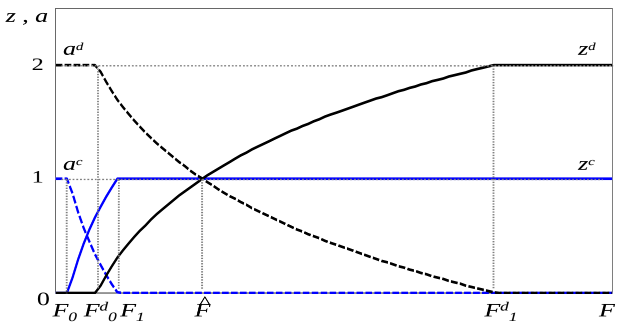

Figure 2.2: Firm's effort allocation under collusion (a^c, z^c) and under deviation (a^d, z^d) in terms of F, for $\rho < 1/2$. Effort devoted to productive efficiency in dashed lines and effort devoted to concealment in solid lines.

$z_j^c = 1$. Hence, the corresponding probabilities of cartel detection are equal as well, $h^d = h^c$. For $F < \hat{F}$, there is more effort devoted to concealment under collusion, and for $F < \hat{F}$, under deviation. Thus:

Corollary 1 *There exists $\hat{F} \in \left(F_1, F_1^d\right)$ such that: $z_i^d > z_j^c$ if and only if $F > \hat{F}$. Hence, for $F > \hat{F}$, the cartel probability of detection following a deviation is lower as compared to when no deviation has taken place. Otherwise, the opposite holds.*

The second comment refers to firm's productive efficiency under collusion and under deviation. Firm's relative productive efficiency following a deviation is higher as compared to when no deviation has taken place. To see this, define the ratio of effort devoted to productive efficiency over effort devoted to concealment: $r^c = \frac{a_i^c}{z_i^c}$, under collusion, and $r^d = \frac{a_i^d}{z_i^d}$, under deviation. These ratios lie in $\Re_0^+$ and are decreasing and convex in F. Then, the higher the fine, the more biased the firm's effort allocation towards concealment. But, they are not equal: $r^d \geq r^c$, as under deviation more units of the good are produced and, thus, each unit of effort devoted to productive efficiency is more valued then that under collusion. Thus, the deviant's relative productive efficiency is higher than (or equal to) that of the firm that follows the collusive agreement. (Figure

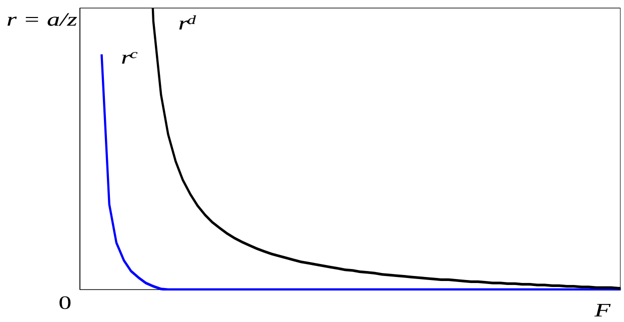

Figure 2.3: Firm's ratio of effort devoted to productive efficiency over effort devoted to concealment, in terms of F. r^c and r^d denote the ratio under collusion and deviation, respectively.

2.3)

2.5.1 Cartel's Sustainability

Effort on concealment does not affect the previous result that states that collusion is sustainable if and only if it is sustainable at the reservation price, $p^c = v$.[13] However, it affects the result that an increase in F or ρ always improves deterrence. To see this, recall ICC:

$$G \leq \frac{\delta}{1-\delta}\Pi^c$$

For the benchmark case (without effort on concealment), an increase in F or ρ reduces firm's profits through higher expected detection costs. This profit loss is independent of whether the firm colludes or deviates, as the cartel probability of detection is exogenous to the firm's effort allocation. Therefore, whereas a more severe antitrust policy reduces the RHS of ICC, it does not affect the LHS. As a direct consequence, the more severe the antitrust policy, the fewer the number of cartels.

[13]Since the collusive price does not depend on firms' effort allocation, whether firms devote effort on concealment (and how much effort they devote to it) does not distort the previous result that a price increase relaxes ICC.

Allowing for effort on concealment, an increase in F or ρ affects firm's profits in two ways: directly through higher expected detection costs, and indirectly through a distortion in the effort allocation. The magnitude of these two effects depends on whether the firm colludes or deviates (Lemmas 1 and 2). Thus, in this context, both the profits from collusion and the gains from deviation depend on F and ρ. Whether a more severe antitrust policy improves deterrence depends on how it distorts the gains from deviation (in sign and magnitude) in comparison to how it distorts the expected profits from collusion.

In what follows, I analyze in detail the endogenous nature of the gains from deviation with respect to fines. On the basis of this analysis, the global implications of a fine increase on deterrence follow immediately.

Endogenous gains from deviation: assume firm i deviates. A fine increase distorts i's gains from deviation as follows:

$$\frac{\partial G}{\partial F} = \underbrace{\left(h^c - h^d\right)}_{Direct\ Effect} + \underbrace{\left(2\ \frac{\partial a_i^d}{\partial F} - F\ \frac{\partial h^d}{\partial F}\right) - \left(\frac{\partial a_j^c}{\partial F} - F\ \frac{\partial h^c}{\partial F}\right)}_{Indirect\ Effect} \tag{2.6}$$

Equivalently:

$$\frac{\partial G}{\partial F} = \underbrace{\left(h^c - h^d\right)}_{Direct\ Effect} + \underbrace{F\ \frac{\partial z_j^c}{\partial F}\left(\frac{\partial h^c}{\partial z_j^c} - \frac{\partial h^d}{\partial z_j^c}\right)}_{Indirect\ Effect} \tag{2.7}$$

The *direct effect* measures the effect of a fine increase on G from different probabilities of cartel detection under collusion and deviation. For $F < F_0$, this effect is zero: when fines are low, all effort is devoted to productive efficiency under both collusion and deviation; thus $h^c = h^d = h^B = 2\rho - \rho^2$. For $F \in \left(F_0, \hat{F}\right)$, this effect is negative because there is more effort devoted to concealment under collusion and, consequently, the cartel probability of detection is lower then, $h^c < h^d$. However this argument is reversed for $F > \hat{F}$, and the *direct effect* is positive, $h^c > h^d$.

The *indirect effect* measures the effect of a fine increase on G from different reallocations of effort under collusion and under deviation. This is clearly stated in (2.6): following a fine increase, firms may find it convenient to reallocate effort from productive efficiency to concealment; this effort reallocation depends on whether the firm colludes or deviates (Lemmas 1 and 2).

Taking into account forthcoming discussions in this paper, I find it more appropriate to analyze the *indirect effect* as stated in equation (2.7). The key element behind this formulation is that each firm determines its own, but not the rival's, effort allocation. This implies that, following a fine increase, each firm reallocates effort from productive efficiency to concealment so as to equate the (own) profit losses from a lower productive efficiency to the (own) profit gains from a lower probability of detection, given a rival that follows the collusive agreement. This effort reallocation has zero algebraical counterpart in profits (thus does not appear in (2.7)). Yet, firms' profits alter as the rival's reallocation of effort creates externalities. This is what (2.7) demonstrates.[14] Note that under both collusion and deviation, the 'rival firm' follows the collusive agreement, thus the *indirect effect* highly depends on $\frac{\partial z_j^c}{\partial F}$.

For $F < F_0$, the firm that follows the collusive agreement finds fines too low to worry about. Thus, the *indirect effect* is zero. For $F > F_1$ the *indirect effect* is zero too, but for a different reason: for $F > F_1$, fines are so high that the firm

[14]Technically, since: (*i*) $\frac{\partial a_i}{\partial F} = -\frac{\partial z_i}{\partial F}$, and (*ii*) $\frac{\partial h}{\partial F} = \frac{\partial h}{\partial z_i}\,\frac{\partial z_i}{\partial F} + \frac{\partial h}{\partial z_j}\,\frac{\partial z_j}{\partial F}$, the *indirect effect* associated to deviation in (2.6) is:

$$2\,\frac{\partial a_i^d}{\partial F} - F\,\frac{\partial h^d}{\partial F} = -\frac{\partial z_i^d}{\partial F}\underbrace{\left(2 + F\,\frac{\partial h^d}{\partial z_i^d}\right)}_{0} - F\,\frac{\partial h^d}{\partial z_j^c}\,\frac{\partial z_j^c}{\partial F} = -F\,\frac{\partial h^d}{\partial z_j^c}\,\frac{\partial z_j^c}{\partial F}$$

In the RHS, the first term is zero, as in brackets there is the equilibrium condition (2.3). The second term is the change in profits that the deviant obtains from a change in the rival's effort allocation.

One can obtain the *indirect effect* associated to collusion analogously. In this way, the *indirect effect* can be written as:

$$F\,\frac{\partial z_j^c}{\partial F}\,\left(\frac{\partial h^c}{\partial z_j^c} - \frac{\partial h^d}{\partial z_j^c}\right)$$

which is what equation (2.7) states.

has already allocated all its effort to concealment. What if $F \in (F_0, F_1)$? For intermediate fine values, a fine increase induces the firm to reallocate effort from productive efficiency to concealment, $\frac{\partial z_j^c}{\partial F} > 0$. As a whole, the *indirect effect* is negative, as $\frac{\partial h}{\partial z_i}$ is downward sloping in the total effort devoted to concealment, which is higher under collusion $\left(\frac{\partial h^d}{\partial z_i^d} > \frac{\partial h^c}{\partial z_i^c}\right)$,

Lemma 3 *With effort on concealment:*

(i) *for $F < F_0$, the gains from deviation are independent of F, and are equal to those for the benchmark case, and*

(ii) *for $F > F_0$, the gains from deviation are U-shaped in F, with a minimum at $\hat{F}$.*

One final comment related to the negative slope of G with respect to F: for $F \in \left(F_0, F_0^d\right)$, a fine increase induces both firms to reallocate effort under collusion, whereas under deviation only one firm is induced (the one that follows the collusive agreement). Therefore, the negative effect of a fine increase on profits is less mitigated under deviation. In other words, a fine increase reduces more profits from deviation. This effect gets stronger as ρ increases, i.e., the higher the ρ, the higher the expected detection costs perceived by the deviant.

Corollary 2 *For $F \in \left(F_0, F_0^d\right)$, the higher the ρ, the higher the reduction in G that follows from a fine increase.*

Solving for v in ICC, and given Lemma 3 and Corollary 2:

Proposition 3 *(With effort on concealment) There exist $v_2 \in [\underline{v}, \overline{v}]$, $\hat{\rho} \in [0, 1]$ and $\tilde{F} \in \left(F_0, \hat{F}\right)$, such that collusion is sustainable in all industries with $v > v_2$, and:*

(i) *for $\rho < \hat{\rho}$, v_2 is upward sloping in F, thus a fine increase improves deterrence.*

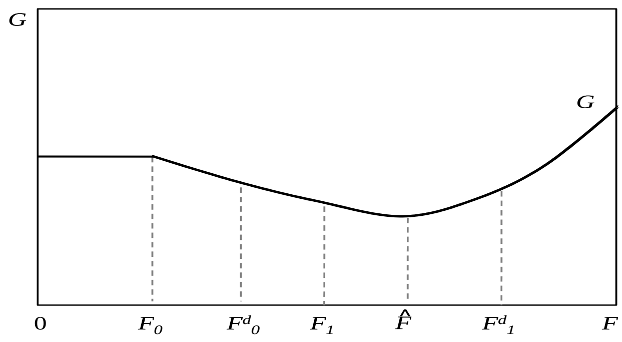

Figure 2.4: With effort on concealment, for $F > F_0$, the gains from deviation are U-shaped in F, with a minimum at $\hat{F}$.

(ii) *for $\rho > \hat{\rho}$, v_2 inherits the U-shaped form of G with respect to F: for $F \notin \left(F_0, \tilde{F}\right)$, v_2 is upward sloping in F, and a fine increase improves deterrence. Otherwise, v_2 is downward sloping in F, and a fine increase facilitates collusion.*

Briefly, points (i) and (ii) in Proposition 3 summarize the following: for $F < F_0$, a fine increase reduces the present value of net future profits from collusion, leaving the gains from deviation unaffected. Therefore, a fine increase improves deterrence unambiguously. For $F \in \left(F_0, \hat{F}\right)$, a fine increase reduces the present value of net future profits from collusion and the gains from deviation. For $\rho > \hat{\rho}$ the reduction in the gains from deviation is high, and in particular, higher than observed for the present value of net future profits from collusion. Thus, a fine increase facilitates collusion. The opposite holds for $\rho < \hat{\rho}$, where the reduction in the gains from deviation is little relatively to that in the present value of net future profits from collusion. Thus, a fine increase improves deterrence. Finally, for $F > \hat{F}$, a fine increase reduces the present value of net future profits from collusion and increases the gains from deviation. Both effects improve deterrence.

Let me stress the perverse effects that Proposition 3 states for intermediate

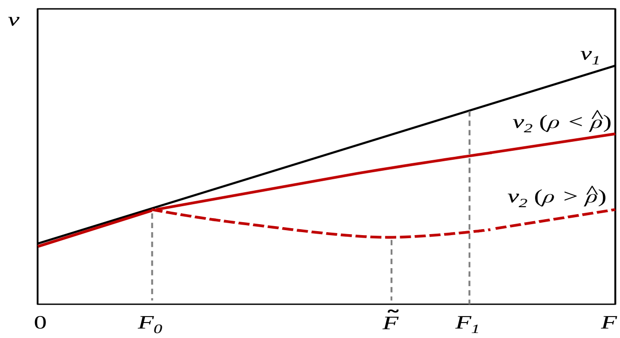

Figure 2.5: With effort on concealment, collusion is sustainable in all industries with $v > v_2$.

values of the fine: when the probability of inspection is high ($\rho > \hat{\rho}$), the threshold price v_2 inherits the U-shaped form of G with respect to F. In this case, the deviant is severely affected by a fine increase; so severely that collusion is facilitated.

Finally, it is important to mention that the threshold price v_2 lies below that for the benchmark case, $v_2 \leq v_1$. By a revealed preference argument, if it were not the case, firms would not have chosen to devote effort to concealment in the first place.

Corollary 3 *For $F = F_0$, $v_2 = v_1$, and for $F > F_0$, $v_2 < v_1$.*

2.6 Social Welfare

In this economy demand is perfectly inelastic, thus welfare depends exclusively on whether production is efficient. In other words, collusion creates an efficiency loss if and only if the good is inefficiently produced as compared to when competition takes place. Under competition only one firm serves demand, devoting as much effort to productive efficiency as output produced (the rival does not produce, neither devotes effort to production). Thus, production is efficient if production is efficiently allocated among firms (i.e., if only one firm serves demand)[15] and if each firm is technologically efficient (i.e., if each firm devotes to productive efficiency as much effort as output privately produced). Under collusion, the former condition never holds, as both firms produce. Whether the latter holds depends on the antitrust policy: when fines are low, firms are technologically efficient, but as fines go up, their productive efficiency goes down.

In this setup, a fine increase can have two welfare effects. On the one side, it can increase welfare through fewer cartels. But, on the other side, it can reduce welfare through more inefficient surviving ones.

Let W^* and W^c denote the social welfare in an industry under competition and under collusion, respectively. Industries are uniformly distributed in $[\underline{v}\ ,\ \overline{v}]$, thus total welfare in this economy is:

$$W = \int_{\underline{v}}^{v_2} W^* \ \frac{v}{(\overline{v}-\underline{v})} \ dv + \int_{v_2}^{\overline{v}} W^c \ \frac{v}{(\overline{v}-\underline{v})} \ dv$$

Within industries, social welfare is the sum of the consumer surplus (CS) and the producer surplus ($\Pi = \Pi_1 + \Pi_2$). Under collusion: $W^c = \Pi^c + R$, as the consumer surplus is equal to the expected revenues from fines ($CS^c = R$). Under competition: $W^* = CS^*$, as firms' profits are zero.[16] Therefore:

$$W = \int_{\underline{v}}^{v_2} CS^* \ \frac{v}{(\overline{v}-\underline{v})} \ dv + \int_{v_2}^{\overline{v}} (\Pi^c + R) \ \frac{v}{(\overline{v}-\underline{v})} \ dv$$

[15]Considering total profits, the net contribution of total effort devoted to productive efficiency is higher when only one firm serves demand. To see this, assume $F < F_0$. For low values of the fine, total effort devoted to productive efficiency is 2 under both competition and collusion, and there is no effort on concealment under collusion. However, while under competition only one firm serves demand, under collusion demand is split in halves. In this context, under competition, the contribution of effort to social welfare is 4 ($a_i = q_i = 2$ and $a_j = q_j = 0$, $i \neq j \Rightarrow a_i q_i + a_j q_j = 2 \times 2 = 4$). Under collusion, instead, the contribution of effort to social welfare is 2 ($a_i^c = q_i^c = 1$, $i = 1, 2 \Rightarrow 2(a_i^c q_i^c) = 2$). Regarding effort costs, under competition these are 2 $\left(\frac{a_i^2}{2} + \frac{a_j^2}{2} = \frac{4}{2} + \frac{0}{2} = 2\right)$, and under collusion 1 $\left(2\frac{(a_i^c)^2}{2} = 2 \times \frac{1}{2} = 1\right)$. Consequently, the net contribution of total effort devoted to productive efficiency under competition $(2 = 4 - 2)$ is higher than that under deviation $(1 = 2 - 1)$.

For higher levels of the fine, there is less effort devoted to productive efficiency under collusion, and, thus, the inefficiencies associated to production under collusion are higher.

[16]In the analysis, I consider competitive profits from the one-shot Nash equilibrium in pure strategies, which yields zero profits to each firm. Considering the equilibria in mixed-strategies would imply positive profits for one firm and, thus, industry profits higher than zero. For social welfare purposes, the distribution of profits between firms in an industry is irrelevant. For details on the one-shot Nash equilibria, please see the Appendix.

Taking partial derivative of W with respect to F:[17]

$$\frac{\partial W}{\partial F} = \frac{1}{(\overline{v} - \underline{v})} \left[\; \left(\, CS^{*}\left(v_2\right) - \Pi^{c}\left(v_2\right) - R \, \right) \, \frac{\partial v_2}{\partial F} \; - \; 2 \; \left(\overline{v} - v_2\right) \; \frac{\partial z_i^c}{\partial F} \; \right] \tag{2.8}$$

Inside the brackets, the first term denotes the welfare gains/losses from a change in the number of competitive industries. The sign of this term depends on whether the fine increase improves deterrence or not (i.e., $\frac{\partial v_2}{\partial F} > 0$). Indeed, the term $CS^{*}\left(v_2\right) - \Pi^{c}\left(v_2\right) - R$ is strictly positive, reflecting the inefficiencies on production from an inefficient allocation of production under collusion (i.e., inefficiencies from two firms producing, instead of one). Thus, if there are fewer cartels after a fine increase, there is a welfare gain. If, instead, there are more cartels after a fine increase, there is a welfare loss.

The second term in brackets $\left(2 \; \left(\overline{v} - v_2\right) \; \frac{\partial z_i^c}{\partial F}\right)$, represents the welfare losses from less efficient cartels. Since higher fines can induce firms to reallocate effort from productive efficiency to concealment, this term is non-negative.

Note that for $F \notin (F_0, F_1)$, the second term is zero, as a fine increase does not distort the effort allocation under collusion (Lemma 1). Thus, a fine increase improves total welfare if and only if it improves deterrence (i.e., iff the first term in (2.8) is positive). This result is standard in models of collusion. However, for $F \in (F_0, F_1)$, the second term is negative, as higher fines induce colluding firms to increase effort on concealment (Lemma 1). In this case, the final effect of a fine increase on total welfare depends on the antitrust parameters F and ρ. This result is a novelty in models of collusion.

Using our previous results described in Proposition 3 in equation (2.8), Proposition 4 follows immediately:

Proposition 4 *There exists* $\check{\rho} \in [0, \hat{\rho}]$*, such that*

(i) *for* $\rho < \check{\rho}$*,* W *is upward sloping in* F*, thus a fine increase improves total*

[17]When taking the derivative, keep in mind that $\frac{\partial(\Pi^c + R)}{\partial F} = -2 \, \frac{\partial z_i^c}{\partial F}$, and $\frac{\partial CS^*}{\partial F} = 0$.

welfare.

(ii) *for $\rho > \check{\rho}$, W inherits the U-shaped form of v_2 with respect to F: for $F \notin (F_0, F_1)$, W is upward sloping in F, and a fine increase improves total welfare. Otherwise, W is downward sloping in F, and a fine increase reduces total welfare.*

Proposition 4 reinforces the perverse effects of intermediate fine levels: when fines are not high enough, a fine increase may be eventually detrimental for social welfare despite its effectiveness in deterring cartels. The latter is the case in which the welfare gains from fewer cartels are not high enough to compensate society for the welfare losses associated to more inefficient surviving cartels. This result strongly favors setting very large fines such that no cartel survives.

Note that this result is in line with standard literature on collusion, which favors the use of very high fines to achieve deterrence: high fines achieve deterrence at a lower cost than many inspections. However, this paper suggests something else: fines and inspections are not exchangeable instruments anymore. Indeed, increasing one of these instruments may have negative consequences on the other instrument's impact on deterrence, firm's productive efficiency and, ultimately, social welfare. Thus, the general recommendation is that the antitrust policy should be carefully designed, pushing crime detection too much with a single instrument may be detrimental for deterrence and social welfare.

2.7 Leniency Programs

Consider a leniency program that offers a fine amnesty to the first cartel firm to come forward with hard evidence of the cartel. Denoting the amnesty parameter by $\theta \in [0, 1]$, the fine amnesty is $(1 - \theta) F$.

Leniency applications are public, i.e., leniency reports are observed by rivals. This implies that the cartel breaks after a leniency application, and that, therefore, there are no leniency applications under collusion. Indeed, a leniency application is a betrayal to the collusive agreement, and the knowledge of a rival's leniency application leads to cartel breakdown, regardless of whether the application finally ends in a sentence for collusion.

Thus, leniency applications only take place under deviation. For a deviant, the introduction of a leniency program implies two strategies to choose from: (a) deviation with report, and (b) deviation without report.

Without effort on concealment (benchmark case), this decision is simple: a deviant applies for leniency if and only if the fine payed after reporting is lower than the expected fine to be paid without it. In other words, there is a leniency application if and only if the amnesty parameter θ is lower than the cartel probability of detection: $\theta < \hat{\theta}^B = \rho\,(2-\rho)$.

With effort on concealment, a deviant that applies for leniency has no incentives to devote effort on concealment, as it will pay θF regardless of its effort allocation. Thus, assuming firm i deviates, alternative (a) implies maximum productive efficiency and deviation with report, which yields profits:[18]

$$\Pi_i^l = 2\,(v-\beta) + 2 - \theta F$$

And alternative (b) implies an effort allocation as stated in Lemma 2 without reporting. In this case, profits from deviation are:

$$\Pi_i^d = 2\,(v-\beta) + 2\;a_i^d - 2 - Fh^d$$

[18]Applying for leniency, the problem of a deviant is:

$$\max_{a_i^l} : \;\Pi_i = 2\,[v-(\beta-a_i)] - \frac{a_i^2}{2} - \theta F$$

where $\frac{\partial \Pi_i}{\partial a_i} = a_i - 2$. Thus: $\left(a_i^l, z_i^l\right) = (2,0)$, and $\Pi_i^l = 2\,(v-\beta) + 2 - \theta F$.

There is a leniency application if $\Pi_i^l > \Pi_i^d$. Equivalently:[19]

$$\theta \ < \ \hat{\theta} \ = \ h^d \ + \ \frac{2\left(2 - a_i^d\right)}{F} \in \left[0 \ , \ \hat{\theta}^B\right]$$

Intuitively, if the deviant devotes effort to concealment, it is because such an effort allocation allows it to achieve the highest expected profits. Hence, to induce the deviant to collect cartel evidence and apply for leniency, the AA should offer a fine amnesty that more than compensates the firm's profit losses associated to a different effort allocation.

Proposition 5 summarizes:

Proposition 5 *There exist* $\hat{\theta}^B, \ \hat{\theta} \in (0,1)$*, where* $\hat{\theta} < \hat{\theta}^B$*, such that a leniency program improves deterrence if and only if it sets an amnesty parameter:*

(i) *Without effort on concealment:* $\theta < \hat{\theta}^B$*.*

(ii) *With effort on concealment:* $\theta < \hat{\theta}$*.*

Two comments to conclude. First, deterrence is maximized at $\theta = 0$, regardless of whether we allow for effort on concealment. Thus, the analysis strongly favors full amnesties. Second, with effort on concealment, a successful leniency program implies a welfare gain beyond deterrence, as reporting implies full productive efficiency for the firm that deviates. This 'efficiency' gain from leniency programs is a novelty in models of leniency in games of collusion.

2.8 Conclusion

In this paper I develop a model in which cartel firms devote effort to productive efficiency and to concealment: the former reduces marginal costs from production and the latter reduces the probability of detection. Effort is costly and

[19]With a little bit of algebra, one can easily prove that $\hat{\theta}$ is downward sloping in F and $\hat{\theta} \in \left(\ \rho\left(e^{-2} + e^{-1} - \rho e^{-3}\right) \ , \ \rho\left(2 - \rho\right) \ \right)$.

limited, thus firms have to decide on how to allocate it among productive efficiency and concealment.

When fines are low, productive efficiency is relatively more important than concealment, thus firms allocate all effort to productive efficiency. But, as fines go up (or if inspections become more likely), the relative importance of concealment goes up, and firms find it profitable to reallocate effort from productive efficiency to concealment. In this context, a fine increase can have two opposite effects on welfare, while it can improve welfare through fewer cartels, it can also reduce it through more inefficient surviving ones.

Two results stand out. First, firm's possibility to reduce the likelihood of cartel detection makes collusion sustainable in industries where it wouldn't be otherwise. This result is intuitive: concealment is costly, implying that if firms devote effort to it, it must be because it facilitates collusion.

The second result states perverse effects from the antitrust policy: a fine increase can reduce social welfare, by inducing surviving cartels to be highly inefficient, or by facilitating collusion, or both. For the second effect, the trigger element is that the effort allocation under deviation is biased towards productive efficiency as compared to that under collusion (as in the former case there are produced more units of the good). For low/intermediate fine values, this implies that the cartel probability of detection is higher under deviation. In this context, a fine increase may relatively affect the deviant so negatively that eventually it induces cartel sustainability rather than deviation.

On the basis of these results, the analysis favors setting very high fines such that no cartel survives. However, in practice this is not always credible or possible to implement. In this context, the main message from the paper is that the antitrust policy has to be carefully designed, such that combining both instruments, fines and inspections, conveniently: since deterrence is non-monotonic in the level of any of these instruments individually considered, pushing crime detection too much with a single instrument can lead to undesirable outcomes.

This result leads to a number of interesting observations, some of which may be lines for future work. For instance, what if fines are endogenous to some measure of the crime damage (e.g., to the price mark-up achieved under collusion)? This new element in the model may lead to imperfect collusion, which would distort the relative importance of productive efficiency with respect to concealment. In this context, it becomes crucial the analysis of the implications of endogenous fines on the non-monotonicity observed between deterrence and fines, firms' productive efficiency, and welfare. Other interesting line for future work is related to the modeling assumption on inspections. In this model, inspections are firm specific, but what if inspections are industry-specific? Industry-specific inspections implies that each firm can not reduce the probability of detection by itself. In this context, how does the critical fine value at which firms find it convenient to substitute effort from production to concealment change? We should expect this critical value to be greater, as neither firm will devote effort to concealment without being sure that its rival has strong incentives to do so as well. These types of questions lead one to think about the importance of establishing the optimal detection policy under different frameworks; a clear challenge for future work on the subject.

Finally, in Section 7 I show that leniency programs can improve welfare beyond a deterrence improvement. Since leniency programs demand full collaboration from the reporting firm, a leniency application implies no effort on concealment. Thus, by inducing reporting, leniency programs improve deterrence and assure full productive efficiency from the deviant. This result is restricted to the case where deviation takes place, but, nevertheless, is novel in the literature.

Appendix

♦ **Proposition 1: Equilibrium in pure strategies**

Let's first prove that there is no NE with $a_i = a_j$.

Assume $p_i < p_j$. Since i has the lowest price, it serves all demand. But, this implies that one of the firms is not optimizing. Indeed, firm i serving demand and both firms optimizing implies: $a_i = q_i = 2$ and $a_j = q_j = 0$, which contradicts the initial statement.

Assume $p_i = p_j = p$, then firms split demand in halves, $q_i = q_j = 1$. Optimization implies $a_i = a_j = 1$, and profits $\Pi_i = p - (\beta - 1) - \frac{1}{2}$, $i = 1, 2$. Assume firm i slightly reduces its price: it gets all demand, $q_i = 2$, and makes effort $a_i = 2$. In this context, i's profits are $\Pi_i = [p_i - \epsilon - (\beta - 2)]\, 2 - 2$, $\epsilon > 0$, greater than before for low ϵ. As there is a profitable deviation to the candidate outcome, this can not be a NE.

Hence, if there exists an equilibrium, it must be at $a_i \neq a_j$.

Let's prove that there is no NE with $a_i \neq a_j$ and $p_i \neq p_j$.

Assume $p_i < p_j$, then firm i serves all demand, $q_i = 2$ and $q_j = 0$. The optimality condition implies $a_i = 2$ and $a_j = 0$. Notice that firm i can increase profits with a slight increase in its price. In fact, i's most profitable deviation is to charge $p_i = p_j$. But, then, firm j would find it profitable to reduce its price below p_i. This process repeats itself anytime $p_i \neq p_j$. The outcome $p_i \neq p_j$ with $a_i \neq a_j$ is not stable and, therefore, can not be a NE.

Let's prove that there is no NE with $a_i \neq a_j$ and $p_i = p_j \neq p^* = \beta - 1$.

Assume $p_i = p_j > p^* = \beta - 1$ and $a_i < a_j$, then firm i serves all demand, $q_i = 2$ and $q_j = 0$. The optimality condition implies $a_i = 2$ and $a_j = 0$. Since i's profits are positive for $p_i > p^* = \beta - 1$, nothing prevents j to reduce its price and get all demand. But this is a contradiction with the initial statement of equal prices.

Assume $p_i = p_j < p^* = \beta - 1$ and $a_i > a_j$. Firm i obtains negative profits for $p < p^* = \beta - 1$, so it won't charge a price below p^*. But, this contradicts the initial statement.

Finally, let's prove that $a_i \neq a_j$ with $p_i = p_j = p^* = \beta - 1$ is a NE.

Assume $a_i > a_j$, then firm i serves all demand and obtains profits $\Pi_i = (p^* - \beta + 2)\, 2\ {-2} = 0$. Since j does not produce, neither makes effort, it obtains zero profits too. As both firms are maximizing profits: $a_i = q_i = 2$ and $a_j = q_j = 0$. If i reduces its price, it obtains negative profits. If, instead, i increases its price, j charges p^* and serves all demand. In this case, we are back to the initial statement with one firm serving demand and both firms making zero profits. As there is no profitable deviation from the candidate outcome, this is a NE.

Mixed-strategy Equilibria

Each firm's payoff is given by:

$$\Pi_i = (p_i - c_i)\, q_i - \frac{a_i^2}{2}$$

Let $\underline{p}_i$ and $\overline{p}_i$ denote the infimum and supremum, respectively, of the support of firm i's strategy.

Assume $a_i > a_j$, then $c_i < c_j = c$.

First, note that $\underline{p}_i = \underline{p}_j \geq c$. This follows from the facts that $p_i \geq c_i$, and that profits are strictly increasing in the firm's price whenever it is the lowest.

Then observe that firm i obtains zero profits if $\overline{p}_i > \overline{p}_j$. The same is true if $\underline{p}_i = \underline{p}_j < \overline{p}_i = \overline{p}_j = \overline{p}$ and either no one plays $\overline{p}$ with positive probability or if some firm does (there is at most one), it is firm j. It follows that at least one firm earns zero profits in any mixed-strategy equilibrium. As $c_i < c$, this is not firm i, which can always guarantee positive profits by pricing below c; so $\overline{p}_i \leq \overline{p}_j$. Further more, $\overline{p}_i = c$, since otherwise firm j could obtain positive profits by undercutting.

Consequently, if $a_i > a_j$, such that $c_i < c_j = c$, there exist mixed-strategy equilibria in which firm i charges $p_i = c$ with probability 1 and firm j mixes price over the range $[c, p')$ for any $p' \in (c, v]$, according to some strategy $F_j(p) = Pr\,(p_j \leq p)$ that satisfies $F_j(p) \geq \frac{p-c}{p-\beta+a_i}$, so as to deter firm i from raising its price. Given firm j's strategy, firm i's profits from deviating and charging a price $p > c = \beta$ is $[1 - F_j(p)]\,(p - \beta + a_i)\ 2 - \frac{a_i^2}{2} \leq (c - \beta + a_i)\ 2 - \frac{a_i^2}{2}$.

Given above strategies, firms' optimal effort levels are $a_i = 2$ and $a_j = 0$, being profits $\Pi_i = 2$ and $\Pi_j = 0$.

Note that while outputs and costs of the set of mixed-strategy equilibria are identical to those of the pure-strategy equilibrium, profits are not. Note further that while the pure-strategy equilibrium involves firm j playing a weakly-dominated strategy, in any mixed-strategy equilibrium firm j plays an undominated strategy almost surely.

Now, assume $a_i = a_j = a$, then $c_i = c_j = \beta - a$, and $\min\{p_i, p_j\} = \beta - a$, since otherwise either firm could obtain positive profits by undercutting. It follows that there does not exist a mixed-strategy equilibrium in this case.

⧫ **Proposition 2:** In main text.

⧫ **Lemma 1:** Recalling equations (2.2) and (2.3), firm i's optimal behavior, $i = 1, 2$ is given by:

$$a_i + z_i = q_i$$

$$q_i = F\rho e^{-z_i}\left(1 - \rho e^{-z_j}\right)$$

Under collusion, firms split the market in halves, $q_i^c = q_j^c = 1$. Thus the former equation is: $a_i^c + z_i^c = 1$. This is the first statement in Lemma 1.

For the second statement in Lemma 1 note that the latter equation can be reduced to:

$$F\rho e^{-z_i^c} = 1 + F\rho^2 e^{-z_i^c} e^{-z_j^c} \tag{2.9}$$

for firm i, and:

$$F\rho e^{-z_j^c} = 1 + F\rho^2 e^{-z_i^c} e^{-z_j^c} \tag{2.10}$$

for firm j.

The RHSs of equations (2.9) and (2.10) are equal, so that the LHSs are equal too, which implies: $z_i^c = z_j^c$. Particularly:

$$z_i^c = z_j^c = -\ln\left[\frac{F - \sqrt{F^2 - 4F}}{2F\rho}\right]$$

With a little bit of algebra, the reader can proof that $z_i^c \in (0,1)$ for $F \in (F_0, F_1)$, where:

$$F_0 = \begin{cases} \frac{1}{\rho(1-\rho)} & if\ \rho \leq \frac{1}{2} \\ 4 & if\ \rho > \frac{1}{2} \end{cases} \qquad\qquad F_1 = \frac{1}{\rho e^{-1}\left(1 - \rho e^{-1}\right)}$$

♦ **Lemma 2:** Assume i deviates: i has two units of effort to allocate among production and concealment (condition 2.2).

Setting $q_i^d = 2$ and $z_j = z_j^c$ in equation (2.3): $F\rho e^{-z_i} = 2 + F\rho^2 e^{-z_i} e^{-z_j^c}$. Solving for z_i, there is a unique solution at:

$$z_i^d = -\ln\left(\frac{2}{F\rho\left(1 - \rho^2 e^{-z_j^c}\right)}\right)$$

With a little bit of algebra, the reader can proof that $z_i^d \in (0,2)$ for $F \in \left(F_0^d, F_1^d\right)$, where: $F_0^d = \frac{4}{\rho(2-\rho)}$ and $F_1^d = \frac{4}{\rho e^{-2}(2-\rho e^{-2})}$.

♦ **Lemma 3:** Holds from considering Lemmas 1 and 2 and Corollary 1 in equation (2.7). See main text.

♦ **Proposition 3:** By definition, v_2 follows from setting $p^c = v$ in ICC and solving for v.

$$v > v_2 = \begin{cases} \beta + \frac{\delta F\rho(2-\rho)+\frac{1}{2}(3-4\delta)}{2\delta-1} & if: F < F_0 \\ \beta + \frac{5-4\delta+\delta\left(F-\sqrt{F^2-4F}\right)-2\ln(A^c)-\rho(1-\delta)\left(F+\sqrt{F^2-4F}\right)}{2(2\delta-1)} & if: F_0 < F < F_0^d \\ \beta + \frac{1+\delta\left(F-\sqrt{F^2-4F}\right)-2\ln(A^c)+4\ln\left(A^d\right)(1-\delta)}{2(2\delta-1)} & if: F_0^d < F < F_1 \\ \beta + \frac{\frac{1}{2}+F\rho e^{-1}\left(1+\delta-\rho e^{-1}\right)+2(1-\delta)\ln\left(\hat{A}^d\right)}{2\delta-1} & if: F_1 < F < F_1^d \\ \beta + \frac{\left(-\frac{3}{2}\right)+F\rho e^{-1}\left(1-e^{-1}\right)(1+\delta)-F\rho^2 e^{-2}\left(1+e^{-1}\delta\right)}{2\delta-1} & if: F > F_1^d \end{cases}$$

where $A^d = \frac{4}{\rho\left(F+\sqrt{F^2-4F}\right)}$ and $\hat{A}^d = \frac{2}{F\rho(1-\rho e^{-1})}$.

The partial derivative of v_2 with respect to F is negative; except when $F \in \left(F_0, \tilde{F}\right)$, $\tilde{F} < \hat{F}$, and $\rho > \hat{\rho}$, case in which v_2 inherits the U-shaped form of G with respect to F (See Lemma 3).

♦ **Proposition 4:** Holds from considering the results described in Lemma 1 and Proposition 3 in equation (2.8). See main text.

♦ **Proposition 5:** In main text.

Chapter 3

An inspector calls: On the optimality of not-by-surprise inspections

3.1 Introduction

Cartel firms' willingness to undertake illegal activities depends on the profit gains from crime and the associated detection costs. Regarding the latter, any information on the likelihood of inspection is crucial to firms. In this paper, I show that the Antitrust Authority (AA) can make use of firms' interest on the likelihood of inspections to improve deterrence: by disclosing information on the likelihood of a current inspection, the AA can distort cartel firms' behavior over time, which can destabilize the current collusive agreement. I address two questions in this paper: how does the disclosure of accurate information on the likelihood of a current inspection distort cartel firms' strategic decisions? Can this policy induce an improvement in deterrence?

I develop a model in which cartel firms devote effort to productive activities and to concealment: effort devoted to production reduces marginal costs and effort devoted to concealment reduces the probability of detection. Effort is costly and limited, thus firms have to decide on how to allocate it among productive efficiency and concealment. The intuition goes as follows: cartel survival depends on the success of each of its member firms, not only as firms that collude in a cartelized market, but also as firms that individually operate in complex markets. Thus, in a cartel, senior executives have to be cautious on how to allocate their time, effort and attention among the own productive efficiency and the cartel organization, in order to guarantee a balanced success on both.[1] For simplicity purposes, among the activities related to the cartel organization, I focus on concealment activities. These include the attendance to secret meetings all over the world and the conduct of a joint sales agency, among other activities.[2] For further simplification, I reduce the three dimensions of care (effort, time and attention) to one: effort.

Regarding the antitrust policy, the AA can credibly disclose information on the likelihood of current inspection before firms decide on how to allocate effort. The AA performs this policy by sending warnings to firms with a high probability of current inspection. The possibility to take decisions on the basis of a more accurate probability of inspection raises cartels' profitability: firms can minimize profit losses from devoting costly effort to concealment each time that they do not receive a warning, and from not doing so each time they receive a warning. However, since deviants also benefit from this, incentives to deviate

[1]Taking into consideration how time and effort-consuming are cartel activities (not only concealment), cartel members create complex hierarchical structures that set the role of each member in the cartel, as well as the rules to follow in case of eventual problems. In this way, the cartel is intended to be conducted as efficiently as a legal organization. For evidence on the hierarchical operativeness of cartels, see Baker & Faulkner (1993), Griffin (2000), Levenstein & Suslow (2006) and Harrington (2006).

[2]Using data from 19 discovered cartels, Levenstein & Suslow (2006) show that cartels that used joint sales agencies were among the more successful cartels in terms of their long-lastingness and fewer coordination problems. They find evidence on the use of a joint sales agency to conceal cartel practices in the following cartels: bromine (1885-1895), cement (1922-1962), diamonds (1870s-1970s), ocean shipping (1870-1924), oil (1871-1874), potash (1877-1897), and European steel (1926-1939).

are also enhanced, making collusion harder to sustain.

In this context, I show that a warning program of this type improves deterrence when both the fine and the probability of inspection are high. The key issue behind this result is that for such parameter values the program does not distort much the behavior of a firm that follows the collusive agreement, but it does so for a deviant. Indeed, faced with high expected costs, the program does not imply much for the former: this allocates all its effort to concealment (a) always, if there is not a warning program, and (b) almost always, if there is. However, for a deviant the program implies a lot. Since periods of no inspection are few, accurate information about when one is in one of these is of huge relevance: in these periods deviation not only implies the standard higher gains from sales, but also minimum (no) detection costs.[3] Hence, in this context, whereas firm's benefits derived from the program are little under collusion, there are huge under deviation; and so the program implies an improvement in deterrence.

The driving force behind this result is that warnings create fluctuations in the probability of inspection. These fluctuations distort cartel firms' incentives to deviate over time, destabilizing the collusive agreement. In this sense, this paper is related to Harrington (2008), which considers i.i.d probability of cartel condemnation once an investigation is launched. Allowing for leniency applications, this framework explains why at some initial point in time it is sustainable for firms to collude and not apply for leniency, but at some later time it becomes optimal to apply.[4] Harrington (2011) extends the analysis by considering i.i.d. signals on this probability that are private to firms (signals that can be sent by the AA). These signals magnify firms' concerns about a leniency ap-

[3] A key assumption in this result is that evidence from collusion lasts for one period, such that a deviant can not be punished from colluding in past periods. This is a usual assumption in models of collusion.

[4] Leniency programs reduce sanctions against the first cartel firm that reports information of the cartel to the Antitrust Authority and cooperates with it along the prosecution phase. The effectiveness of these programs to improve deterrence lies in their capability to increase the temptation to deviate. Spagnolo (2008) provides an extensive review of literature on leniency in collusion.

plication even when the likelihood of condemnation is low. Even though both papers consider fluctuations in an antitrust parameter, my setup is closer to the latter, in the sense that the fluctuations can derive from the execution of an antitrust policy. However, the driving force for deterrence is different: in Harrington (2011), the driving force for deterrence is firms' rivalry in obtaining the amnesty in a post-cartel environment, which is magnified with private information on the likelihood of condemnation. In my paper, instead, the driving force for deterrence is the distortion that i.i.d. signals on the likelihood of inspection create on firms' incentives to deviate in a pre-cartel environment.

This paper is also related to papers on collusion with demand fluctuations over time. Rotemberg & Saloner (1986) analyze optimal collusive pricing under observable demand shocks which are i.i.d. over time. They find that collusion is more difficult to sustain when demand is high, as the gains from deviation are increasing in the size of the market to be served by the deviant. In this context, pricing countercyclically realigns incentives to collude over time, facilitating collusion. Haltiwanger & Harrington (1991) extend Rotemberg & Saloner (1986) to a business cycle analysis, where the demand is subject to (deterministic) cyclical fluctuations.[5] In this setup, it is no longer clear whether booms (i.e., when demand is raising) are the toughest time for firms to collude: during boom, the temptation to deviate is high, however immediate future profits from collusion (which are heavily weighted) also are. On the contrary, Haltiwanger *et al.* (1991) show that recessions are the toughest time for firms to collude. Considering two points on the cycle with equal demand, such that demand is increasing in one of them and decreasing in the other, losses from cheating are greater at the point at which demand is raising, as immediate profits from collusion are expected to be higher. During booms, the expectation of immediate higher future gains from collusion acts as a deterrent to cheat, and turns recessions into the toughest time for firms to collude. Numerical simulations suggest

[5] Although in Haltiwanger *et al.* (1991) demand movements are specified to be deterministic, their results are robust to allowing for (nonobservable) i.i.d. demand shocks. In that case, there are firms' expectations on future demand that move cyclically.

a tendency for firms to price countercyclically during recessions.

This paper differs from the previous ones in that I consider parameter fluctuations derived from an antitrust policy. The decision on whether to disclose information on the likelihood of inspection, as well as on how much information to disclose and how to do it (e.g., the timing to follow in the disclosure, whether the disclosure is public or private, etc) is an attribute of the AA. Another difference is that, in my paper, firms do not find it optimal to price differently over time to reduce the impact of parameter fluctuations. Instead, they find it optimal to reallocate effort from production to concealment. This realigns incentives to deviate over time better than price movements.

The paper continues as follows. In Section 2, I set up the model. In Section 3, I solve it for the case in which the AA does not disclose information on inspections (benchmark case), and, in Section 4, I solve it for the case in which it does. Particularly, I consider that in each period the AA warns firms in industries with a high probability of current inspection before they take decisions on price and effort. In Section 5, I perform a welfare analysis considering the effects of this policy on deterrence and on the productive efficiency of surviving cartels. In Section 6, I introduce some variations to the basic framework. I conclude in Section 7.

3.2 The Model

Consider an economy with a continuum of industries. In each industry, there is an inelastic demand for two units with reservation price v, and two firms producing perfect substitutes. Firms maximize profits over an infinite time horizon with time-invariant discount parameter $\delta \in [0, 1]$ and, to this end, they compete or collude on prices. Within industries, firms have the same δ. However, across industries the discount factor might differ. In particular, I assume that the discount factor across industries follows a uniform distribution function $U[0,1]$. Thus, industries are identified through the degree of patience of their firms.[6]

To produce, firms have a fixed marginal cost β, which can be privately reduced for the current period through *effort devoted to productive efficiency* $a_i \geq 0$, $i = 1, 2$. Then, the firm's marginal cost is $c_i = \beta - a_i$.

The market demand goes to the lowest priced firm or, in case of a price tie, to the firm with the lowest production cost. Under a price tie and equal production costs, firms equally split demand.

Collusion requires communication, which constitutes hard evidence for cartel detection. Evidence lasts for one period and can be discovered by the AA during an inspection. However, firms can privately destroy some of evidence through costly effort and, consequently, reduce the likelihood of finding evidence in an inspection.

To model this, I assume $z_i \in \{0, 1\}$, $i = 1, 2$, such that the probability of finding cartel evidence in an inspection to firm i, denoted φ_i, takes two values: if firm i does not devote effort to concealment ($z_i = 0$), this probability is 1; otherwise, if $z_i = 1$, this probability is reduced to $\underline{\varphi}_i < 1$.[7]

Effort is costly. I set the firm's effort disutility function as $\frac{(a_i+z_i)^2}{2}$. This specification for the disutility of effort follows Holmstrom & Milgrom (1991) and is common in multitask analyses. It is consistent with the view that efforts are technological substitutes and that disutility depends on total effort (not on the firm's effort allocation).[8]

[6]Different industries is not an essential component of the model; however it facilitates the interpretation of the welfare analysis in Section 5. Also, industries can differ in any parameter of the model; w.l.o.g. I chose δ.

[7]A general version of this model can be found at Avramovich (chapter 1 in thesis dissertation, 2012), where $z_i \geq 0$ and the probability of finding cartel evidence in an inspection to firm i is $\varphi_i = e^{-z_i}$, $i = 1, 2$. The restriction on z_i to be zero or one responds to the interest of simplification and in no case restricts the results of this work.

[8]For the effort allocation to be also relevant, one can introduce a weighting parameter $\mu \in \Re_0^+$ such that $\frac{(a_i+\mu z_i)^2}{2}$. The assumption of $\mu = 1$ affects the degree of substitution between efforts, but in no case restricts the results of the paper.

To fight cartels, the AA has three instruments: fines, inspections and warnings. Both warnings and inspections, are industry-specific policies: either both firms in the industry are inspected (warned), or none.[9]

Warnings are signals on the likelihood of current inspection sent to firms prior to inspections. Particularly, these inform each firm on whether its probability of current inspection is high or low. These signals are i.i.d. across time and are sent once and at the same time for all firms. Throughout the paper I refer to firms that receive a signal on a high probability of current inspection as warned firms. Similarly, I refer to firms that receive a signal on a low probability of current inspection as not-warned firms.[10]

To model this, I define the events $W = \mathit{warning}$ and $I = \mathit{inspection}$, $W, I \in \{0, 1\}$. $W = 1$ states that the firm is warned, and $W = 0$ states that the firm is not warned. With the same logic, $I = 1$ states that the firm is inspected, and $I = 0$ states no inspection.

The probabilities associated to these events are: $Pr(W = 1) = \eta$, the probability of being warned, and $Pr(\ I \mid W = 1) = \overline{\rho}$ and $Pr(\ I \mid W = 0) = \underline{\rho} < \overline{\rho}$, the probabilities of inspection given a warning and no-warning, respectively. $\eta, \underline{\rho}, \overline{\rho} \in [0, 1]$.

Therefore, the expected probability of inspection to a firm is:

$$\rho \ = \ \eta \, \overline{\rho} \ + \ (1 - \eta) \ \underline{\rho} \tag{3.1}$$

and the associated cartel probability of detection is:

$$h\left(z_i, z_j \mid \rho\right) \ = \ \rho \ \left(\ \varphi_i + \varphi_j - \varphi_i \varphi_j \ \right) \tag{3.2}$$

There is cartel detection if there is an inspection and the inspector finds cartel

[9] I relax this assumption in Section 6, where both warnings and inspections, are firm-specific policies.

[10] One can think on warned firms as firms that receive a letter from the AA informing of a high probability of current inspection for them, and on not-warned firms as firms that do not receive this letter. Hence, after the signaling time, firms that receive the letter (warned firms) know that their probability of current inspection is high. Similarly, firms that do not receive the letter (not-warned firms) know that their probability of current inspection is low.

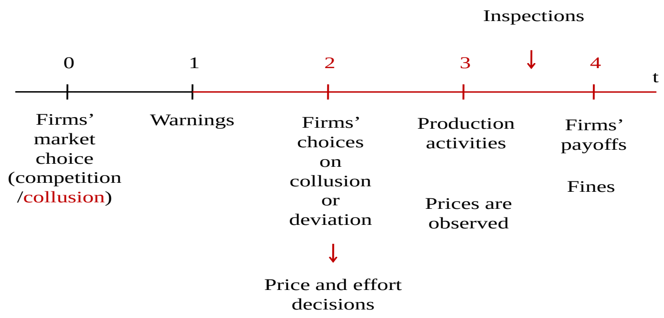

Figure 3.1: Time-structure of the model

evidence after inspecting any of the firms. Note that none of the firms can reduce the probability of cartel detection by itself. Indeed, assume there is an inspection and $z_i = 1$, but $z_j = 0$, then: $h(1,0 \mid \rho) = \rho \left(\underline{\varphi}_i + 1 - \underline{\varphi}_i \right) = \rho$. Firm i's effort devoted to concealment is useless to reduce the probability of detection if firm j does not devote effort to concealment too.

The timing of the game is as follows. At stage 0, firms choose whether to collude or compete. If one firm chooses to compete, competition takes place and the game ends. At stage 1, the AA makes warnings.

If there was an agreement on collusion, at stage 2 firms decide whether to follow the collusive agreement or to deviate. Under deviation, the deviant either slightly reduces its price, or increases its effort devoted to productive efficiency, or both. In this way, it gets all demand.

At stage 3, effort production and price decisions are executed and the rival's price is observed. Also, inspections take place. At stage 4, firms obtain their payoffs from sales. Under cartel detection, firms pay a fine F and the game starts again from stage 0. If the cartel is not detected, but one firm has deviated, a punishment phase takes place. Finally, if none of the firms have deviated and the cartel is not detected, the game repeats itself from stage 1.

In this setup, firms make simultaneous pricing and effort decisions in every period t. With an infinite horizon, firm i, $i = 1, 2$, chooses prices $p_{it} \in [0, v]$ and efforts a_i, z_i (effort z_i is relevant only under deviation), in every t, $t = 1, 2, ..., \infty$.

Under collusion, price choices at date t depend on the current warning (W_t) and on the history of previous sales; so that p_{it}, depends on $H_{it} = (\ q_{i1}(W_1)\ ;\ q_{i2}(W_2)\ ;\ .\ .\ .\ ;\ q_{i,t-1}(W_{t-1})\ ;\ W_t\ )$. The rational behind this rule goes as follows: under collusion firms charge the same price and split the demand in halves, $q_i = 1$, $i = 1, 2$; thus, for a firm, no sales implies that the rival deviated (in price, in effort or in both). Therefore, the collusive strategy for firm i is to initially price at the collusive price $p^c_{W_1}$ in period 1 and to continue pricing according to:

$$p_{it} = p^c_{W_t} \qquad if:\ q_i^{\tau} = 1 \ \ \forall \tau \in \{1, ..., t-1\}\ ,\ W_t \in \{0, 1\}\ ,\ j = 1, 2$$

as long no firm has deviated from this path. If a firm has deviated, there is a reversion to the single-period Nash equilibrium strategy of pricing, since Nash reversion can assure zero profits for the deviant.

In the one-shot game firms choose price and effort to maximize current profits:

$$\Pi_i = [p_i - (\beta - a_i)]\, q_i - \frac{a_i^2}{2}$$

Proposition 6 *There exists a one-shot game Nash equilibrium in which one firm obtains zero profits.*

In the one-shot game there is a pure strategy equilibrium in weakly dominated strategies that yields zero profits for both firms. Also, there are undominated mixed-strategy equilibria that yield zero profits for one firm and positive profits for the other. Since at the static Nash equilibrium there is at least one firm that obtains zero profits, Nash reversion in which the deviant obtains zero profits constitutes an optimal penal code.

3.3 Collusion without Warnings

As a benchmark case assume there is not a warning program. Consequently, no information on inspections is disclosed to firms.[11] Throughout the paper, I frequently reference this case with the shortcut 'without warnings', and the case in which there is a warning program with the shortcut 'with warnings'.

The firm's problem is to choose price and effort levels that maximize:

$$\Pi_i = [p_i - (\beta - a_i)]\, q_i - \frac{(a_i + z_i)^2}{2} - F\ \rho\,(\varphi_i + \varphi_j - \varphi_i\varphi_j)$$

The first term is the firm's payoff from production and the second and third ones its costs associated to effort and to detection, respectively.

Solving for effort a_i:

$$a_i + z_i = q_i \tag{3.3}$$

This equation characterizes firms' optimal behavior under both collusion and deviation. For the same level of production, an increase in effort devoted to productive efficiency must be compensated with an equal reduction in effort devoted to concealment.

Regarding price, under collusion firms charge the same price and split the demand in halves: $p_i = p^c$ and $q_i^c = 1$, $i = 1, 2$. Setting $q^c = 1$ into equation (3.3) and recalling that $z_i \in \{0, 1\}$, two effort allocations satisfy this condition: one that maximizes productive efficiency, and other that maximizes concealment. Firm's optimal behavior is given by the one that maximizes profits:

Lemma 4 *Under collusion $a_i^c + z_i^c = 1$, and there exists F_E such that: for $F < F_E$, all effort is allocated to productive efficiency ($a_i^c = 1$), and for $F > F_E$, all effort is allocated to concealment ($z_i^c = 1$).*

[11] In this setup, the event W is not defined, so neither are the probabilities associated to it, η, $\overline{\rho}$ and $\underline{\rho}$.

For $F < F_E$,[12] the marginal contribution of an additional unit of effort devoted to productive efficiency is higher than the marginal contribution of an additional unit of effort devoted to concealment. Hence, firms find it optimal to allocate all effort to productive efficiency. Beyond F_E this condition is reversed and, to restore the equilibrium, firms substitute effort from productive efficiency to concealment.[13]

The critical fine value F_E is negatively related to the probability of inspection ρ: the higher is this probability, the more important is concealment with respect to productive efficiency, and therefore the lower is the critical fine value at which firms find it profitable to devote effort to concealment.

If a firm decides to deviate, it either slightly reduces its price, or increases its effort on productive efficiency (to reduce marginal costs), or both. In this way, it gets all demand. A price reduction does not have side effects on firm's efficiency, however the increase of effort on productive efficiency does it. Thus, to maximize profits, a deviant always reduce its price slightly and redetermines effort allocation considering that its rival follows the collusive agreement.

Lemma 5 *(Assume firm i deviates) Under deviation* $a_i^d + z_i^d = 2$*. For* $F < 2F_E$*, firm i allocates all effort to productive efficiency* ($a_i^d = 2$)*, and for* $F > 2F_E$*, it allocates one unit of effort to each activity* ($a_i^d = z_i^d = 1$).

From Lemmas 1 and 2 we observe that firms allocate effort similarly under collusion and deviation: for low values of the fine, all effort is allocated to productive efficiency; but, as the fine goes up effort is reallocated from productive efficiency to concealment.

However, the critical fine value at which the firm finds it profitable to devote

[12] $F_E = \frac{1}{\rho[1-\underline{\varphi}(2-\underline{\varphi})]}$.

[13] For $F > F_E$ there are two equilibria, one in which firms maximize productive efficiency, and other in which they maximize concealment. Since the latter yields higher profits, it is the one that takes place, and therefore the one considered throughout the paper. See the Appendix for details.

effort to concealment is higher under deviation. This is so since the deviant produces more units of the good and, therefore, finds it productive efficiency relatively more important than the firm that follows the collusive agreement. Also note that for high fine values the deviant devotes one unit of effort to each activity, concealment and productive efficiency; instead, the firm that follows the collusive agreement allocates its single unit of effort to concealment. Hence, the former is always relatively more efficient than the latter.[14]

Finally, but not less important, I want to stress that for $F \in (F_E, 2F_E)$ there is effort devoted to concealment under competition but not under deviation. This implies: (i) by deviating, a firm obtains high sale revenues due to more goods sold and a higher productive efficiency, and (ii) the cartel probability of detection under collusion is lower than that under deviation, which yields gains from deviation downward sloping in F. Both results are a novelty in models of collusion.

To understand why the gains from deviation can be downward sloping in F, consider a fine increase. If the firms want to reduce the negative impact of this policy in profits, they have to devote effort to concealment – both of them –, such that reducing the cartel probability of detection. However, for $F \in (F_E, 2F_E)$ only the firm that follows the collusive agreement finds it profitable to do it, thus the cartel probability of detection is reduced only under collusion. Therefore, a fine increase affects more profits from deviation than from collusion and, consequently, the gains from deviation go down. (Figure 3.2)

3.3.1 Cartel's Sustainability

Collusion is sustainable as long as firms have no incentives to deviate, i.e., when the gains from deviation are no greater than the current value of net future

[14]Considering the ratios $r^d = a^d/z^d$ and $r^c = a^c/z^c$, it holds that $r^d \geq r^c$ for all values of F and ρ.

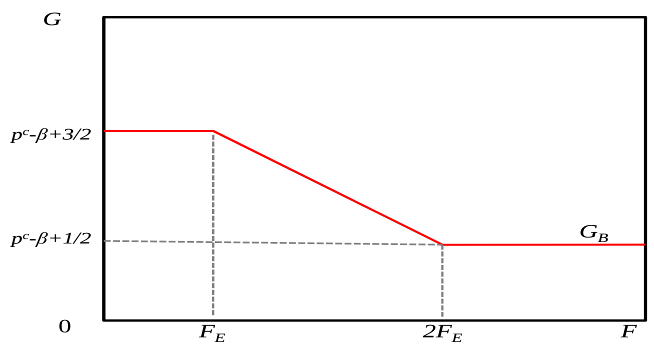

Figure 3.2: Gains from deviation for the benchmark case G_B in terms of F. For $F < F_E$ and $F > 2F_E$, G_B is not related to F, as the cartel's probabilities of detection under collusion and deviation are equal. For $F \in (F_E, 2F_E)$, G_B is downward sloping in F, as the cartel probability of detection is higher under deviation and, therefore, a fine increase damages more profits from deviation.

profits from collusion:

$$(ICC) \qquad G = \Pi^d - \Pi^c \leq \frac{\delta}{1-\delta}\,\Pi^c \tag{3.4}$$

In this model, for $F < F_E$, $(a_i^c, z_i^c) = (1,0)$ and $\left(a_i^d, z_i^d\right) = (2,0)$. Thus, ICC is:

$$p^c - \beta + \frac{3}{2} \leq \frac{\delta}{1-\delta}\left[p^c - \beta + \frac{1}{2} - F\rho\right]$$

For $F \in (F_E, 2F_E)$, $(a_i^c, z_i^c) = (0,1)$ and $\left(a_i^d, z_i^d\right) = (2,0)$. Thus, ICC is:

$$p^c - \beta + \frac{5}{2} - F\rho\left[1 - \underline{\varphi}\left(2 - \underline{\varphi}\right)\right] \leq \frac{\delta}{1-\delta}\left[p^c - \beta - \frac{1}{2} - F\rho\underline{\varphi}\left(2 - \underline{\varphi}\right)\right]$$

Finally, for $F > 2F_E$, $(a_i^c, z_i^c) = (0,1)$ and $\left(a_i^d, z_i^d\right) = (1,1)$. Thus, ICC is:

$$p^c - \beta + \frac{1}{2} \leq \frac{\delta}{1-\delta}\left[p^c - \beta - \frac{1}{2} - F\rho\underline{\varphi}\left(2 - \underline{\varphi}\right)\right]$$

For $\delta > 1/2$, a price increase relaxes ICC condition, which implies that firms always charge the reservation price under collusion, $p^c = v$. Prices lower than v, make collusion more difficult, and prices higher than v would imply no

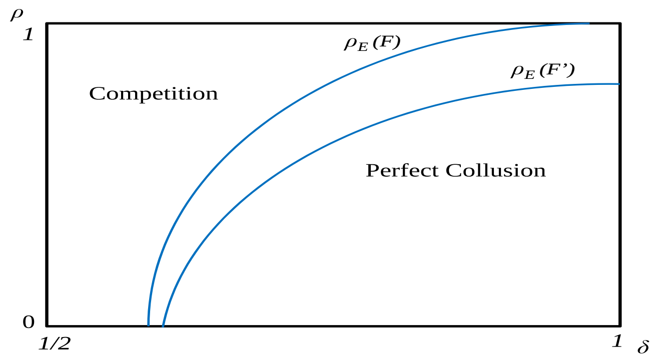

Figure 3.3: Critical probability value ρ_E for F and F', $F < F'$. Perfect collusion is sustainable if and only if $\rho < \rho_E$. Otherwise, competition takes place.

sales. So, collusion is sustainable if and only if perfect collusion is sustainable. Along the paper I assume $\delta > 1/2$, as otherwise collusion is not profitable.[15]

Proposition 7 *Without warnings, there exists ρ_E such that perfect collusion is sustainable if and only if $\rho \leq \rho_E$. Otherwise, competition takes place.*

Figure 3.3 illustrates Lemma 3 in terms of ρ and δ: the higher is the probability of inspection ρ, the greater is the patience required to collude. The critical probability value ρ_E is negatively related to F: the higher is the fine, the lesser is the amount of industries that find collusion sustainable for each value of ρ (ρ_E moves to the right). On the contrary, ρ_E is positively related to v: the higher is the reservation price, the higher is the amount of industries that find collusion sustainable for each value of ρ (ρ_E moves to the left).

3.4 Warnings

Now, assume that there is a warning program. Through warnings, the AA discloses valuable information for firms: at some initial point in time firms decide

[15]Profitability is a necessary condition for collusion. In this model, profitability implies $\delta > 1/2$, which is a standard level of patient in models of collusion.

whether to form a cartel on the basis of an expected probability of inspection $\rho = \eta\,\overline{\rho} + (1-\eta)\,\underline{\rho}$, but knowing that in each later period they will take price and effort decisions on the basis of a more accurate probability $\overline{\rho}$ or $\underline{\rho}$.

In the interest of simplicity, and w.l.o.g., I assume $\underline{\rho} = 0$ and $\overline{\rho} = 1$. This assumption implies certainty about the occurrence of a current inspection, rather than a more accurate prediction of its likelihood of occurrence.

The firm's problem is to choose price and effort levels that maximize:

$$\Pi_i = [\ p_i - (\beta - a_i)\ ]\ q_i \ - \frac{(a_i + z_i)^2}{2} \ - \ W\ F\ \ (\varphi_i + \varphi_j - \varphi_i\varphi_j)$$

The first term is the firm's payoff from production and the second and third ones are firm's costs associated to effort and to detection, respectively. The latter is relevant only if the firm is warned of current inspection, i.e., when $W = 1$.

Solving for a_i: $a_i + z_i = q_i$, $i = 1, 2$. Under collusion this is $a_i^c + z_i^c = 1$, $i = 1, 2$, as firms split demand in halves.

Straightforward, for $W = 0$ a firm devotes all effort to productive efficiency, as it is not going to be inspected. For $W = 1$, instead, a firm allocates either all effort to productive efficiency or to concealment depending on the value of the fine.

Lemma 6 *Under collusion $a_i^c + z_i^c = 1$, and*

(i) *for $W = 0$, all effort is allocated to productive efficiency,*

(ii) *for $W = 1$, there exists $F_N = F_E(\rho = 1)$ such that: for $F < F_N$, all effort is allocated to productive efficiency ($a_i^c = 1$), and for $F > F_N$, all effort is allocated to concealment ($z_i^c = 1$).*

Note that when fines are low, firms allocate all their effort to productive efficiency, regardless of warnings. This is so since for low fine values firms' profit losses associated to detection costs are little as compared to those associated to low sale revenues from an effort allocation away from productive efficiency.

If firm i decides to deviate, it slightly reduces its price to get all demand ($q_i = 2$), and redetermines effort allocation considering that its rival follows the collusive agreement.[16]

Lemma 7 *(Assume firm i deviates) Under deviation* $a_i^d + z_i^d = 2$, *and*

(i) *for* $W = 0$, *the firm allocates all effort to productive efficiency,*

(ii) *for* $W = 1$: *for* $F < 2F_N$, *the firm allocates all effort to productive efficiency* ($a_i^d = 2$), *and for* $F > 2F_N$, *the firm allocates one unit of effort to each activity* ($a_i^d = z_i^d = 1$).

3.4.1 Cartel's Sustainability

As firms behave differently with respect to whether they are warned of an ongoing inspection or not, ICC differs on whether the current W is 0 (ICC_0) or 1 (ICC_1):

$$(ICC_W) \qquad G_W \leq \frac{\delta}{1-\delta} \; [\; (1-\rho) \; \Pi^c(\; p_0^c \mid W=0) \; + \; \rho \, \Pi^c(\; p_1^c \mid W=1) \;]$$

where: $G_W = \Pi^d \left(\; p_W^d \mid W \right) - \Pi^c \left(\; p_W^c \mid W \right)$, for $W \in \{0,1\}$

The RHS of ICC_W is independent of the current value of W, as it states information regarding future periods: in future periods, with probability ρ firms are warned and charge p_1^c, and with probability $1-\rho$ they are not warned and charge p_0^c.[17]

[16]As discussed for the benchmark case, firm i can deviate with a slight reduction in its price, an increase in its effort on productive efficiency, or both. In this way, it gets all demand. However, whereas a price reduction does not alter i's productive efficiency, an increase of effort on productive efficiency does it. Thus, to maximize profits from deviation, the firm always reduces its price. Whether it also increases its effort devoted to productive efficiency depends on the antitrust parameters ρ and F (Lemma 4).

[17]The key assumption behind this issue is that W is i.i.d. across time.

The LHS of ICC_W, instead, depends on the current value of W. If in the current period $W = 0$, firms allocate all effort to productive efficiency under both collusion and deviation. This implies equal probabilities of detection and, therefore, G_0 independent of F.

Instead, if in the current period $W = 1$, firms allocate effort depending on the fine value. For $F < F_N$ and $F > 2F_N$, they allocate effort to concealment equally, under both collusion and deviation. This implies equal probabilities of detection and, therefore, G_1 independent of F. However, for $F \in (F_N, 2F_N)$, all effort is allocated to concealment under collusion and to productive efficiency under deviation. In this case, the probability of detection is higher under deviation and, therefore, a fine increase reduces profits from deviation more than those from collusion. Consequently, G_1 is downward sloping in F.

Figure 3.4 shows G_0 and G_1 in terms of F when firms charge the same price in all periods ($p_0^c = p_1^c = p^c$). Note that for equal prices $G_1 \leq G_0$.[18] In this context, ICC_0 is more restrictive than ICC_1. This result is of high relevance for the rest of the paper.

Lemma 8 *If firms charge the same price in all periods, then* $G_1 \leq G_0$ *and* ICC_0 *is more restrictive than* ICC_1*.*

Optimal Price under Collusion

Under collusion, firms charge the highest possible price compatible with cartel sustainability. For the benchmark case, this price is v, as a price increase relaxes ICC. However, with warnings, whether an increase in the current price relaxes ICC_W depends on W and ρ:

Lemma 9 *There exist* ρ_0 *and* ρ_1 *such that:*

[18]This is so since the gains from deviation are upward sloping in the current price.

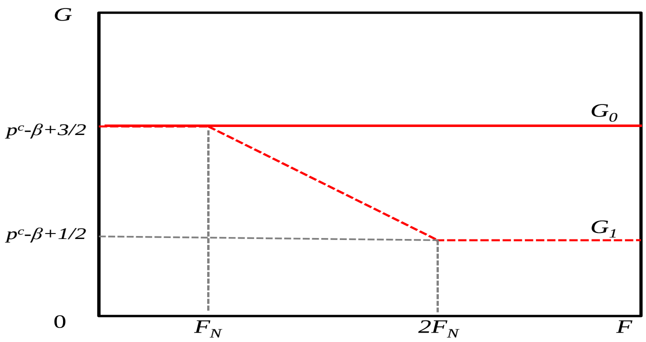

Figure 3.4: Gains from deviation in terms of F for $p_0^c = p_1^c = p^c$.

(i) *Assume $W = 0$, then for $\rho > \rho_0$, an increase in p_0^c restricts ICC_0, making collusion more difficult to sustain. On the contrary, for $\rho < \rho_0$, an increase in p_0^c relaxes ICC_0, facilitating collusion.*

(ii) *Assume $W = 1$, then for $\rho > \rho_1$, an increase in p_1^c relaxes ICC_1, facilitating collusion. On the contrary, for $\rho < \rho_1$, an increase in p_1^c restricts ICC_1, making collusion more difficult to sustain.*

For a price reduction, the opposite holds.

Assume $W = 0$: firms are not warned in the current period and charge p_0^c. Furthermore, assume that ρ is high ($\rho > \rho_0$): the probability of receiving a warning in any future period is high, and so p_1^c is the price generally charged in the future. In this context, an increase in p_0^c rises the continuation value of profits (CVP) little, and in particular less of what it rises G_0, making collusion more difficult to sustain. Analogously, a reduction in p_0^c facilitates collusion. Now, assume that ρ is low ($\rho < \rho_0$): the probability of receiving a warning in any future period is low, and so p_0^c is the price generally charged in the future. In this context, an increase in p_0^c rises CVP greatly, and in particular more of what it rises G_0; therefore, collusion is facilitated. Analogously, a reduction in p_0^c makes collusion more difficult to sustain. The same arguments hold in understanding the case $W = 1$.[19]

Figure 3.5 shows ρ_0 and ρ_1 in terms of δ.[20] ρ_0 and ρ_1 cross each other at $(\delta, \rho) = \left(\frac{1}{2}, \frac{2}{3}\right)$, setting three areas of interest. Area I, for $\rho < \rho_0$, states pairs (δ, ρ) for which if perfect collusion is not sustainable, neither is other type of collusion.[21] Area II, for $\rho \geq \max\{\rho_0, \rho_1\}$, states pairs (δ, ρ) for which if perfect collusion is not sustainable, imperfect collusion at prices $(p_0^c, p_1^c) = (p', v)$ with $p' < v$, may still be sustainable. This comes from the fact that a reduction in p_0 relaxes ICC_0, facilitating collusion. Finally, Area III, for $\rho \in (\rho_0, \rho_1)$, states pairs (δ, ρ) for which if perfect collusion is not sustainable, imperfect collusion at prices (p', v) or (p'', p''') such that $p'' < p' < v$ and $p'' < p''' < v$, may still be sustainable. Imperfect collusion at prices (p'', p''') comes from the fact that ICC_W is relaxed with price reductions.

Note that imperfect collusion is direct consequence of Lemma 5: if firms always charge the same price, collusion is sustainable if and only if ICC_0 holds. But, by reducing price in those periods in which they are not warned on a current inspection, the gains from deviation G_0 go down, facilitating collusion. However such a price reduction also reduces CVP, which makes collusion more difficult to sustain. Lemma 6 states critical values for ρ that assure that this pricing strategy (e.g., imperfect collusion at prices (p', v)) truly facilitates collusion.

Lemma 10 *With warnings, there exist $\rho_{PC} \geq \rho_{IC} \geq \rho_{IC2}$, such that:*

[19]Assume $W = 1$: firms are warned in the current period and charge p_1^c. When $\rho > \rho_1$, the probability of receiving a warning in any future period is high, and so p_1^c is the price generally charged in the future. Consequently, an increase in p_1^c rises CVP greatly, and in particular more of what it rises G_0, facilitating collusion. The opposite holds for a reduction in p_1^c. On the contrary, when $\rho < \rho_1$, the probability of receiving a warning in any future period is low, and so p_0^c is the price generally charged in the future. In this context, a reduction in p_1^c reduces CVP little, and in particular less of what it reduces G_0, facilitating collusion. Analogously, an increase in p_1^c makes collusion more difficult to sustain.

[20]Critical probability levels ρ_0 and ρ_1 are: $\rho_0 = \frac{2\delta - 1}{\delta}$ and $\rho_1 = \frac{1-\delta}{\delta}$.

[21]Consider $\rho \in (\rho_0, \rho_1)$: a price increase relaxes ICC_W. Thus, if perfect collusion is not sustainable, neither is other type of collusion (under perfect collusion, firms already charge the highest possible price v). Consider $\rho \leq \min\{\rho_0, \rho_1\}$: a price increase relaxes ICC_0, but restricts ICC_1. Assume perfect collusion, then (*i*) $(p_0^c, p_1^c) = (v, v)$, and (*ii*) ICC_0 is most restrictive than ICC_1. If perfect collusion is not sustainable, it is so (particularly) because ICC_0 does not hold. Can one change p_0^c to make this condition to hold? No, since under perfect collusion it is already charged the highest possible price v. Therefore, if perfect collusion is not sustainable, neither is other type of collusion.

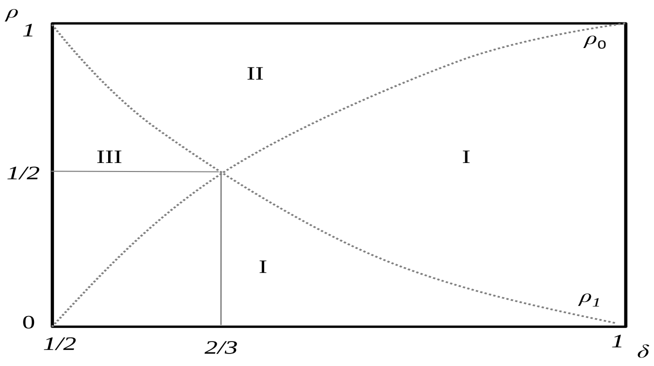

Figure 3.5: Critical parameter values ρ_0 and ρ_1 define three critical areas of interest for collusion. Area I: if perfect collusion is not sustainable, neither is other type of collusion. Areas II and III: if perfect collusion is not sustainable, imperfect collusion may still be sustainable.

(i) *Perfect collusion is sustainable if and only if* $\rho \leq \rho_{PC}$*,*

(ii) *Imperfect collusion at prices* (p', v) *is sustainable if and only if* $\rho \leq \rho_{IC}$*,*

(iii) *Imperfect collusion at prices* (p'', p''') *is sustainable if and only if* $\rho = \rho_{IC2}$*.*

Corollary 4 *If imperfect collusion is sustainable, perfect collusion also is.*

Lemma 7 states sustainability conditions for perfect and imperfect collusion. These conditions are not mutually exclusive, i.e., for some parameter values firms can sustain more than one type of collusion. Among two o more types of collusion, firms choose the one that maximizes profits: perfect collusion is preferred to collusion at prices (p', v), which, in turns, is preferred to collusion at prices (p'', p'''). This is so since firms maximize profits and these are increasing in price.

Following this profit choice, a direct result from Corollary 1 is that imperfect collusion never takes place.

Proposition 8 *With warnings, firms play perfect collusion if and only if* $\rho < \rho_{PC}$*. Otherwise, competition takes place.*

The critical probability value ρ_{PC} is upward sloping in δ and v. The more patient are the firms or the higher is the reservation price v, the higher is the probability of inspection at which collusion is sustainable. On the contrary, ρ_{PC} is downward sloping in F: the higher is the fine, the lower is the probability of inspection at which collusion is sustainable.

3.5 Social Welfare

Preceding Sections characterize the sustainability of cartels in terms of a threshold value for the probability of inspection ρ, such that one can argue that (perfect) collusion is sustainable if and only if the probability of inspection is lesser than a certain threshold. However, that threshold differs on whether the AA makes use of a warning program or not: whereas without the program (benchmark case), that threshold is ρ_E, with the program, it is ρ_{PC}. In the light of these results, does the warning program improve deterrence? And social welfare? This Section sheds light to these questions.

The use of a warning program has two opposite effects on the sustainability of cartels. On the one side, the program raises the profitability of collusion: accurate information on the likelihood of current inspections prevents *(i)* not-warned firms from devoting costly effort to concealment, and (*ii*) warned firms from an unprofitable effort allocation towards productive efficiency. Thus, the expected profits from collusion in the presence of the program are higher than those from the benchmark case.

However, on the other side, the warning program also raises the gains from deviation, as the firm that deviates is also beneficed from the accurate information on inspections. Technically: in the presence of the program, the relevant gains from deviation are G_0, which are equal to those for the benchmark case (G_B) for low fine values, but higher than these for high fine values.[22] Therefore,

[22]Please, see the Appendix for details.

there can be also a deterrent effect from warnings.

Figure 3.6 illustrates these two opposite effects, and Proposition 4 states the condition under which one or the other dominates (i.e., the condition under which ρ_E is lesser/greater than ρ_{PC}), as well as the welfare implications of both cases.

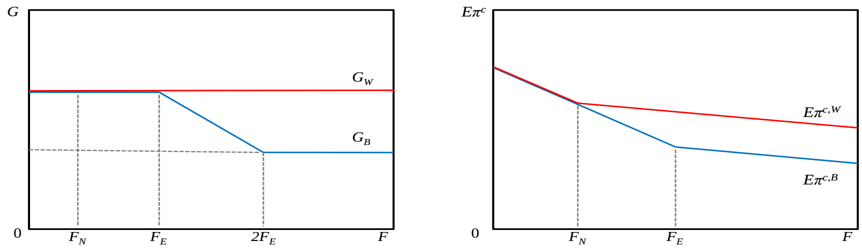

Figure 3.6: Left: gains from deviation with and without a warning program, G_W and G_B respectively. Right: expected profits from collusion with and without a warning program, $E\pi^{c,W}$ and $E\pi^{c,B}$ respectively.

Proposition 9 *There exists ρ^* such that:*

(i) *For $F < F_N$, a warning program has no effect on deterrence, neither on the productive efficiency of cartels,*

(ii) *For $F \in (F_N, F_E)$, a warning program facilitates collusion and creates productive inefficiencies on cartels,*

(iii) *For $F > F_E$, a warning program facilitates collusion for $\rho < \rho^*$, and makes it more difficult otherwise. In no case the program affects the productive efficiency of cartels.*

For $F < F_N$, fines are too little to worry about, and thus firms maximize productive efficiency regardless of the existence of the warning program. In this case, $\rho_E = \rho_{PC}$, the program has no effect on deterrence, neither on the productive efficiency of cartels.

For $F \in (F_N, F_E)$, $F < F_E$ the gains from deviation with and without the warning program are equal (Figure 3.6, left), thus there is no deterrence effect from it. However, there is a cartel's sustainability effect: in the presence of the program, firms reduce their expected detection costs by substituting effort from productive efficiency to concealment whenever they receive a warning on an inspection. Undoubtedly, the program facilitates collusion. In addition, the program creates productive inefficiencies, as firms are efficient in its absence, but only in some periods in its presence.

Finally, for $F > F_E$, the warning program facilitates collusion by preventing firms from devoting costly effort to concealment each time they do not receive a warning. However, the program also makes collusion harder to sustain by increasing the gains from deviation. In this context, when ρ is high ($\rho > \rho^*$), the latter effect prevails, as the profit gains derived from the program are little: when the probability of receiving a warning in any future period is high, firms behave as in the benchmark case most of the time, and the program's favorable effect on the cartel's sustainability is little as compared to its adverse effect on it. However, when the probability of receiving a warning in any future period is low ($\rho < \rho^*$), firms' profit gains derived from the program are huge. In this case, the warning program facilitates collusion.

This analysis suggests that when the fine and the probability of inspection are high, the AA should make use of a warning program as part of the antitrust policy. But, how high the probability of inspection should be?, i.e., which is the critical level of inspection ρ^*? It depends on the firm's patience coefficient δ: the higher is the value of δ, the higher is the critical value ρ^*, and hence the greater must be the probability of inspection in order to recommend the use of a warning program. Considering δ equal to 0.6 and 0.7, the critical value ρ^* is 0.33 and 0.57, respectively.

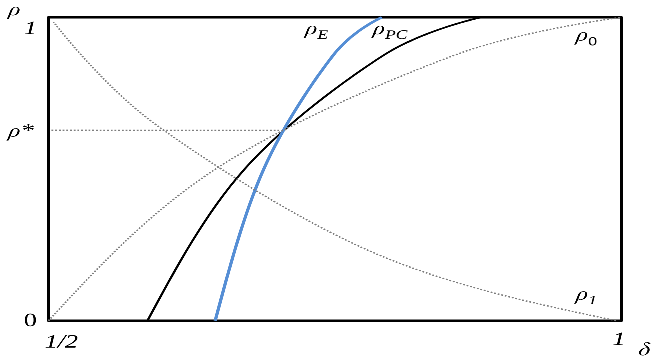

Figure 3.7: ρ_E and ρ_{PC} in terms of F, for $F > 2F_E$. For $\rho < \min\{\rho_E, \rho_{PC}\}$ firms collude regardless of the existence of a warning program. For $\rho \in (\rho_E, \rho_{PC})$ firms collude when there is a warning program and compete otherwise. For $\rho \in (\rho_{PC}, \rho_E)$, firms compete when there is a warning program and collude otherwise. For $\rho > \max\{\rho_E, \rho_{PC}\}$, firms compete.

3.6 Extensions and Variations

Preceding Sections analyzed the impact of disclosing information on the likelihood of current inspection on cartel's deterrence and on the productive efficiency of surviving cartels. The analysis assumed industry-specific inspections and warnings, and a collusive agreement that requires communication in all periods. This Section relaxes each of these two assumptions in turn.

□ **Firm-specific inspections and warnings.** I first extend the analysis by allowing inspections and warnings to be firm-specific policies instead of industry-specific policies.

Firm-specific inspections allows for single-firm inspections. In this context, each cartel firm can reduce the cartel probability of detection by devoting effort to concealment. Following notation of previous Sections, the cartel probability of detection is:[23]

$$h\,(z_i, z_j \mid \rho) \;=\; \rho\;\left(\,\varphi_i + \varphi_j - \rho\,\varphi_i\,\varphi_j\,\right)$$

There is cartel detection if there is an inspection in at least one firm and the inspector finds cartel evidence during it.

In terms of my model, the cartel probability of detection only affects firm's marginal benefits from effort devoted to concealment. Thus, different specifications for this probability (one calculated on the basis of industry-specific inspections and other on the basis of firm-specific inspections) affects the relative importance of concealment with respect to productive efficiency, but not firms' willingness to devote effort to concealment when fines are high:

Lemma 11 *When fines are low, firms allocate all effort to productive efficiency; however when fines go up, they find it profitable to reallocate effort from productive efficiency to concealment. This behavior holds under both collusion and deviation.*

From Lemma 8 it follows that Lemmas 1 and 2 are robust to inspections specific to the firm or industry.[24]

Firm-specific warnings implies a collusive game in which firms may allocate effort differently in each period. E.g., assuming that only firm i receives a warning, firm j does not devote effort to concealment, as it is not inspected; however firm i may find it profitable to do so, as this can reduce the probability of detection. Therefore, given a collusive agreement, in each period each firm takes strategic decisions depending on whether it receives a warning and on an expected probability of warnings to rivals.[25]

[23]Note that under firm-specific inspections, the probability of inspection to a firm is ρ, and the probability of inspection to both firms in the industry is ρ^2. Instead, under industry-specific inspections, these two probabilities are equal, ρ.

[24]Technically, whether inspections are firm-specific or industry-specific only matters in identifying the critical fine level at which firms find it profitable to substitute effort from productive efficiency to concealment. Please, see the Appendix for a detailed algebraical analysis.

[25]Note that even though that a posteriori (after warnings) firms' effort allocations may differ,

Thus, there is only one single difference between a setup in which warnings are specific to the industry to other in which they are specific to firms: in the latter each firm takes decisions knowing that its rival may devote more/less effort to concealment than itself. This asymmetry distorts the marginal benefit that each firm obtains from devoting effort to concealment, but not its willingness to devote effort to concealment when fines are high.

Holding the assumption of fully informative warnings, Lemmas 9 and 10 state firm's effort allocation under collusion and deviation.

Lemma 12 *Under collusion:*

(i) *For $W_i = 0$, $i = 1, 2$: firm i finds it profitable to allocate all effort to productive efficiency.*

(ii) *For $W_i = 1$, $i = 1, 2$: firm i finds it profitable to allocate all effort to productive efficiency for low values of the fine, and to substitute effort from productive efficiency to concealment otherwise*

Lemma 13 *(Assuming firm i deviates) Under deviation:*

(i) *For $W_i = 0$: firm i finds it profitable to allocate all effort to productive efficiency.*

(ii) *For $W_i = 1$: firm i finds it profitable to allocate all effort to productive efficiency for low values of the fine, and to substitute effort from productive efficiency to concealment otherwise*

From Lemmas 9 and 10 it follows that Lemmas 3 and 4 are robust to inspections specific to the firm or industry.

a priori, at the time of deciding the cartel formation, firms' (expected) effort allocations are equal. Indeed, at stage 0 cartel firms decide whether to form a cartel on the basis of an expected joint probability of receiving a warning, which is equal to both firms.

Finally, holding Lemmas 1-4, the discussion in Section 5 regarding the implications of a warning program on deterrence and on social welfare holds too.

Proposition 10 *Results exposed in Proposition 4 are robust to warnings specific to the firm or industry.*

□ **Communication conditional on warnings.** For the next variation assume that firms establish a collusive agreement in which communication is conditional on warnings: firms communicate if and only if they are not warned.

Since communication is crucial in the price-fixing practice, this variation implies that firms compete each time they receive a warning and collude otherwise. In this way, firms avoid detection costs; however, they sacrifice high profits in exchange (firms obtain competitive profits each time they are warned, which are lower than those from collusion).

This variant in the game does not distort firms' behavior under competition, neither under collusion in the absence of the warning program. Thus, Lemmas 1 and 2 from the main text still hold. Instead, Lemmas 3 and 4 does not hold anymore, as in the presence of the warning program firms collude only when $W = 0$. Lemmas 10 and 11 substitute Lemmas 3 and 4, respectively:

Lemma 14 *Given a collusive agreement with communication conditional on warnings: for $W = 1$, firms compete, and for $W = 0$, firms collude. Under collusion $(a_i^c, z_i^c) = (1, 0)$.*

Lemma 15 *Firm i deviates with an effort allocation $\left(a_i^d, z_i^d\right) = (2, 0)$ when $W = 0$.*

Corollary 5 *In the presence of a warning program, firms are fully efficient on production.*

In this context, whereas a fine increase restricts ICC in the absence of the warning program (benchmark case), it does not affect this in its presence (ICC_W does not dependent on F). Indeed, without the program the cartel probability of detection is $h(z_i, z_j \mid \rho) > 0$; instead, with the program, this probability is zero, as collusion takes place only when firms are not inspected. Proposition 6 follows immediately:

Proposition 11 *There exists F^* such that a warning program improves deterrence if and only if $F < F^*$. Otherwise, the program facilitates collusion.*

For $F < F^*$, firms' profit gains from avoiding detection costs are little and, in particular, lower than their profit losses from less sales revenues. Thus, the warning program improves deterrence for low values of the fine. The opposite holds for $F > F^*$, and the program facilitates collusion.

Note that Propositions 4 and 6 state opposite results regarding the deterrent effect of the warning program conditional to fines. In the basic model (with communication in all periods), high fines are a necessary condition for the program to improve deterrence (Proposition 4). However, with communication conditional on warnings, low fines are.[26] Intuitively, the reason goes as follows: in the basic model, firms sacrifice high sale revenues in favor of concealment only for high values of the fine. Instead, with communication conditional on warnings, firms do that for all values of the fine. Hence, when fines are low, firms sacrifice relatively too much in the later setup, as they sacrifice high sale revenues in exchange of avoiding little detection costs. The opposite holds when fines are high.[27]

Two final comments are in order. First, in the interest of simplicity the analysis above assumed that the collusive agreements with communication in

[26] Actually, with communication conditional on warnings, low fines are a necessary (and sufficient) condition for the program to improve deterrence.

[27] When fines are high, firms sacrifice relatively too little when communication is conditional on warnings: firms achieve zero detection costs when communication is conditional on warnings, but they can only aspire to a reduction of these in the basic model.

all periods (basic model) and with communication conditional on warnings were mutually exclusive. However, nothing prevents these agreements from coexist. If this were the case, at stage 0 firms would decide whether to form a cartel and, if so, which collusive agreement to follow. This decision over the collusive agreement allows firms to reduce the negative effect of the warning program: firms would choose following the collusive agreement of the basic model when fines are low, and the one with communication conditional on warnings when these are high. In this case, the program may not improve deterrence never.

My second comment is related to the assumption of evidence lasting for one period. Relaxing this assumption, such that evidence lasts for several periods, does not invalidate the results from the basic model, however, it invalidates those from the alternative setup developed in this subsection. With evidence lasting for several periods, a collusive agreement in which firms communicate and collude only when they do not receive a warning does not present any advantage for them: firms would sacrifice high net sale revenues for nothing, the evidence would be still there. Note that this second comment reduces the relevance of the former one.

3.7 Conclusion

This paper is a first step in understanding the impact of disclosing private information on inspections on the sustainability of collusion. In particular, I analyze whether the disclosure of information on the likelihood of current inspection can improve deterrence, as well as possible side-effects of this policy on the productive efficiency of surviving cartels.

In my model cartel firms devote costly effort to productive activities and to concealment, as the former reduces marginal costs from production and the latter reduces the probability of detection. Since effort is costly, is limited; thus firms have to decide on how to allocate it among productive efficiency and concealment. When fines are low, productive efficiency is relatively more important than concealment, thus firms allocate all effort to productive efficiency. Instead, when fines are high, concealment is the relatively most important activity, and firms allocate all effort to it.

In this context, a fine increase can have two opposite effects on welfare, while it can improve welfare through fewer cartels, it can also reduce it through more inefficient surviving ones. This analysis favors large fines such that no cartel survives, however, in practice this policy is not always credible or possible to apply.

In this context, I show that the disclosure of private information on the likelihood of current inspections can improve social welfare. When the AA introduces a warning program, firms are prevented from (*i*) devoting costly effort to concealment anytime that they do not receive a warning, and (*ii*) an unprofitable effort allocation towards productive efficiency each time that they receive a warning. Both effects raise profits from collusion, facilitating collusion. However, since deviants also benefit from these effects, incentives to deviate are also enhanced, making collusion harder to sustain. I show that when both fines and inspections are high, the latter effect prevails and the program improves deterrence. Also, since for high fine values cartels are highly inefficient, fewer cartels implies a welfare gain beyond higher deterrence.

Appendix

⧫ Proposition 1: Equilibrium in pure strategies

Let's first prove that there is no Nash Equilibrium (NE) with $a_i = a_j$.

Assume $p_i < p_j$. Since i has the lowest price, it serves all demand. But, this implies that one of the firms is not optimizing. Indeed, firm i serving demand and both firms optimizing implies: $a_i = q_i = 2$ and $a_j = q_j = 0$, which contradicts the initial statement.

Assume $p_i = p_j = p$, then firms split demand in halves, $q_i = q_j = 1$. Optimization implies $a_i = a_j = 1$, and profits $\Pi_i = p - (\beta - 1) - \frac{1}{2}$, $i = 1, 2$. Assume firm i slightly reduces its price: it gets all demand, $q_i = 2$, and makes effort $a_i = 2$. In this context, i's profits are $\Pi_i = [p_i - \epsilon - (\beta - 2)]\, 2 - 2$, $\epsilon > 0$, greater than before for low ϵ. As there is a profitable deviation to the candidate outcome, this can not be a NE.

Hence, if there exists an equilibrium, it must be at $a_i \neq a_j$.

Let's prove that there is no NE with $a_i \neq a_j$ and $p_i \neq p_j$.

Assume $p_i < p_j$, then firm i serves all demand, $q_i = 2$ and $q_j = 0$. The optimality condition implies $a_i = 2$ and $a_j = 0$. Notice that firm i can increase profits with a slight increase in its price. In fact, i's most profitable deviation is to charge $p_i = p_j$. But, then, firm j would find it profitable to reduce its price below p_i. This process repeats itself anytime $p_i \neq p_j$. The outcome $p_i \neq p_j$ with $a_i \neq a_j$ is not stable and, therefore, can not be a NE.

Let's prove that there is no NE with $a_i \neq a_j$ and $p_i = p_j \neq p^* = \beta - 1$.

Assume $p_i = p_j > p^* = \beta - 1$ and $a_i < a_j$, then firm i serves all demand, $q_i = 2$ and $q_j = 0$. The optimality condition implies $a_i = 2$ and $a_j = 0$. Since i's profits are positive for $p_i > p^* = \beta - 1$, nothing prevents j to reduce its price and get all demand. But this is a contradiction with the initial statement of equal prices.

Assume $p_i = p_j < p^* = \beta - 1$ and $a_i > a_j$. Firm i obtains negative profits for $p < p^* = \beta - 1$, so it won't charge a price below p^*. But, this contradicts the initial statement.

Finally, let's prove that $a_i \neq a_j$ with $p_i = p_j = p^* = \beta - 1$ is a NE.

Assume $a_i > a_j$, then firm i serves all demand and obtains profits $\Pi_i = (p^* - \beta + 2)\, 2 - 2 = 0$. Since j does not produce, neither makes effort, it obtains zero profits too. As both firms are maximizing profits: $a_i = q_i = 2$ and $a_j = q_j = 0$. If i reduces its price, it obtains negative profits. If, instead, i increases its price, j charges p^* and serves all demand. In this case, we are back to the initial statement with one firm serving demand and both firms making zero profits. As there is no profitable deviation from the candidate outcome, this is a NE.

Mixed-strategy Equilibria

Each firm's payoff is given by:

$$\Pi_i = (p_i - c_i)\, q_i - \frac{a_i^2}{2}$$

Let $\underline{p}_i$ and $\overline{p}_i$ denote the infimum and supremum, respectively, of the support of firm i's strategy.

Assume $a_i > a_j$, then $c_i < c_j = c$.

First, note that $\underline{p}_i = \underline{p}_j \geq c$. This follows from the facts that $p_i \geq c_i$, and that profits are strictly increasing in the firm's price whenever it is the lowest.

Then observe that firm i obtains zero profits if $\overline{p}_i > \overline{p}_j$. The same is true if $\underline{p}_i = \underline{p}_j < \overline{p}_i = \overline{p}_j = \overline{p}$ and either no one plays $\overline{p}$ with positive probability or if some firm does (there is at most one), it is firm j. It follows that at least one firm earns zero profits in any mixed-strategy equilibrium. As $c_i < c$, this is not firm i, which can always guarantee positive profits by pricing below c; so $\overline{p}_i \leq \overline{p}_j$. Further more, $\overline{p}_i = c$, since otherwise firm j could obtain positive profits by undercutting.

Consequently, if $a_i > a_j$, such that $c_i < c_j = c$, there exist mixed-strategy equilibria in which firm i charges a price $p_i = c$ with probability 1 and firm j mixes price over the range $[c, p')$ for any $p' \in (c, v]$, according to some strategy $F_j(p) = Pr\,(p_j \leq p)$ that satisfies $F_j(p) \geq \frac{p-c}{p-\beta+a_i}$, so as to deter firm i from raising its price. Given firm j's strategy, firm i's profits from deviating and charging a price $p > c = \beta$ is $[1 - F_j(p)]\,(p - \beta + a_i)\ 2 - \frac{a_i^2}{2} \leq (c - \beta + a_i)\ 2 - \frac{a_i^2}{2}$.

Given above strategies, firms' optimal effort levels are $a_i = 2$ and $a_j = 0$, being profits $\Pi_i = 2$ and $\Pi_j = 0$.

Note that while outputs and costs of the set of mixed-strategy equilibria are identical to those of the pure-strategy equilibrium, profits are not. Note further that while the pure-strategy equilibrium involves firm j playing a weakly-dominated strategy, in any mixed-strategy equilibrium firm j plays an undominated strategy almost surely.

Now, assume $a_i = a_j = a$, then $c_i = c_j = \beta - a$, and $\min\{p_i, p_j\} = \beta - a$, since otherwise either firm could obtain positive profits by undercutting. It follows that there does not exist a mixed-strategy equilibrium in this case.

♦ **Lemma 1:** (Collusion) Recalling equation 3.3, firm's optimal behavior is given by: $a_i + z_i = q_i$, for $i = 1, 2$. Since $q_i^c = 1$ and $z_i \in \{0, 1\}$, there are two alternative effort allocations: $(a_i, z_i) = \{(1,0), (0,1)\}$.

(i) Assume i sets $(a_i, z_i) = (1, 0)$. If j sets $(a_j, z_j) = (1, 0)$, its profits are $\pi^c(1, 0) = p^c - \beta + 1/2 - \rho F$. If j deviates and sets $(a_j, z_j) = (0, 1)$, j reduces its net sale revenues (as its marginal cost would be high) in exchange of no reduction in its expected detection costs, as both firms should devote effort to concealment to reduce the cartel probability of detection. Thus, setting $(a_i, z_i) = (1, 0)$, $i = 1, 2$, is a Nash equilibrium.

(*ii*) Assume now that i sets $(a_i, z_i) = (0, 1)$. If j sets $(a_j, z_j) = (0, 1)$, it contributes to reduce the cartel probability of detection, and obtains profits $\pi^c(0, 1) = p^c - \beta - 1/2 - \rho\underline{\varphi}\left(2 - \underline{\varphi}\right) F$. If j deviates and sets $(a_j, z_j) = (1, 0)$, its net sale revenues are higher, but so are its expected detection costs (as the cartel probability of detection goes up). j's profits in this case are: $\pi^c(1, 0) = p^c - \beta + 1/2 - \rho F$. There exists $F_E \in \Re^+$ such that: for $F < F_E$, j maximizes profits with an effort allocation $(a_j, z_j) = (1, 0)$, and for $F > F_E$, j maximizes profits with an effort allocation $(a_j, z_j) = (0, 1)$. Thus, setting $(a_i, z_i) = (0, 1)$ for $F > F_E$, $i = 1, 2$, is a Nash equilibrium.

For $F > F_E$ there are two equilibria, one in which firms maximize productive efficiency, and other in which they maximize concealment. Since the latter yields higher profits, this is the one that takes place: firms set $(a_i, z_i) = (1, 0)$ for $F < F_E$, and $(a_i, z_i) = (0, 1)$ otherwise, $i = 1, 2$.

♦ **Lemma 2:** Assume i deviates: i has two units of effort to allocate among productive efficiency and concealment (condition 3.3).

For $F < F_E$, j sets $(a_j, z_j) = (1, 0)$, and i's best response is to set $(a_i, z_i) = (2, 0)$: any deviation from this effort allocation implies lower net sale revenues in exchange of no reduction in its expected detection costs (as both firms should devote effort to concealment to reduce the cartel probability of detection).

For $F > F_E$ firm j sets $(a_j, z_j) = (0, 1)$. If i sets $(a_i, z_i) = (2, 0)$, its profits are $\pi^d(2, 0) = 2(p^c - \beta) + 2 - \rho F$. If i sets $(a_i, z_i) = (1, 1)$, its profits are $\pi^d(1, 1) = 2(p^c - \beta) - \rho\underline{\varphi}\left(2 - \underline{\varphi}\right) F$. Comparing profits: i maximizes profits by setting $(a_i, z_i) = (2, 0)$ for $F \in (F_E, 2F_E)$, and $(a_i, z_i) = (1, 1)$, otherwise.

♦ **Proposition 2:** Follows from setting $p^c = v$ in ICC and solving for ρ.

♦ **Lemma 3:** Firm's optimal behavior is given by: $a_i + z_i = q_i$, where $q_i = 1$. Assume $W = 0$. Since no inspection takes place, $z_i = 0$ and, therefore, $a_i = 1$, $i = 1, 2$. Now, assume $W = 1$. This case corresponds to the benchmark case but for $\rho = 1$. Following Lemma 1, firms set $(a_i, z_i) = (1, 0)$ for $F < F_N = F_E(\rho = 1)$ and $(a_i, z_i) = (0, 1)$ otherwise, $i = 1, 2$.

♦ **Lemma 4:** Same proof as for Lemma 3, but for $\rho = 1$.

♦ **Lemma 5:** G_0 and G_1 are the gains from deviation for $W = 0$ and $W = 1$, respectively:

$$G_0 = p_0^c - \beta + \frac{3}{2}$$

Firms maximize productive efficiency under collusion and under deviation. Thus, expected detection costs under both collusion and deviation, are equal and, therefore, G_0 is independent of F.

$$G_1 = \begin{cases} p_1^c - \beta + \frac{3}{2} & if: \ F < F_N \\ p_1^c - \beta + \frac{5}{2} - \left[1 - \underline{\varphi}\left(2 - \underline{\varphi}\right) F\right] & if: \ F \in (F_N, 2F_N) \\ p_1^c - \beta + \frac{1}{2} & if: \ F > 2F_N \end{cases}$$

For $F < F_N$, firms maximize productive efficiency under collusion and under deviation. Thus, G_1 is independent of F. For $F \in (F_N, 2F_N)$, productive efficiency is maximized under deviation, but concealment under collusion. Expected detection costs are higher under deviation and, therefore, G_1 is download sloping in F. For $F > 2F_N$, it is devoted one unit of effort to concealment under both collusion and deviation; thus, G_1 is independent of F.

If firms charge $p_0^c = p_1^c$, $G_0 \geq G_1$, ICC_0 is more restrictive than ICC_1, as the continuation value of profits does not depend on the current value of W.

♦ **Lemma 6:** Holds from partial derivatives of ICC_0 and ICC_1 with respect to current prices, p_0^c and p_1^c respectively.

$$\rho < \rho_0 = \frac{2\delta - 1}{\delta} \qquad\qquad \rho > \rho_1 = \frac{1-\delta}{\delta}$$

♦ **Lemma 7: Perfect collusion:** perfect collusion is sustainable if and only if ICC_0 holds. Setting $p_0^c = p_1^c = v$ in ICC_0 and solving for ρ: perfect collusion is sustainable if and only if:

$$\rho < \rho_{PC} = \begin{cases} \frac{(2\delta-1)(v-\beta)+2\delta-3/2}{\delta F} & if: \ F < F_N \\ \frac{(2\delta-1)(v-\beta)+2\delta-3/2}{\delta\left[1+\underline{\varphi}\left(2-\underline{\varphi}\right)F\right]} & if: \ F > F_N \end{cases}$$

Imperfect collusion:

(i) For $\rho > \rho_0$, a reduction in p_0^c facilitates collusion, as it relaxes the most restrictive incentive compatibility constraint (ICC_0). Define p' as the highest price compatible with $ICC_0(p', v)$, so that it is the price at which this constraint holds with equality. Substituting p_1^c by v in ICC_0, equating this condition to zero and solving for p_0^c:

$$p_0^c = p'(v) = p' = \begin{cases} \frac{\beta - \frac{3}{2} + \delta(\rho v - 2\beta + 2 - \rho F)}{1-2\delta+\delta\rho} & if: \ F < F_N \\ \frac{\beta - \frac{3}{2} + \delta\left(\rho v - 2\beta + 2 - \rho - \rho\underline{\varphi}\left(2-\underline{\varphi}\right)F\right)}{1-2\delta+\delta\rho} & if: \ F > F_N \end{cases}$$

Substituting p_0^c and p_1^c by $p'(v)$ and v, respectively, in ICC_1, and solving for ρ: imperfect collusion at prices $(p_0^c, p_1^c) = (p', v)$ is sustainable if and only if:

$$\rho < \rho_{IC} = \begin{cases} \frac{(2\delta-1)(v-\beta)+2\delta-3/2}{\delta F} & if: \ F < F_N \\ \frac{(2\delta-1)(v-\beta)+4\delta-5/2-(2\delta-1)F\left(1-\underline{\varphi}\left(2-\underline{\varphi}\right)\right)}{2\delta+\delta F\left[2\underline{\varphi}\left(2-\underline{\varphi}\right)-1\right]} & if: \ F \in (F_N, 2F_N) \\ \frac{(2\delta-1)(v-\beta)-1/2}{\delta F\underline{\varphi}\left(2-\underline{\varphi}\right)} & if: \ F > 2F_N \end{cases}$$

IMPORTANT: the condition above is relevant only when imperfect collusion at prices $(p_0^c, p_1^c) = (p', v)$ is profitable, which requires:

1. $\rho > \rho_l = \max\left\{ \rho_0 \ , \ v - \beta + \frac{3}{2} - F \right\}$, for $F < F_N$.

2. $\rho < \rho_h = \min\left\{ \rho_0 \ , \ v - \beta + \frac{3}{2} - F \right\}$, for $F > F_N$.

Comparing ρ_{IC} and ρ_{PC}:

For $F < F_N$, $\rho_{PC} = \rho_{IC}$.

For $F > F_N$, $\rho_{PC} > \rho_{IC}$ for $\rho < \rho_0$, and $\rho_{PC} < \rho_{IC}$ otherwise. This is so since: (i) ρ_{PC} and ρ_{IC} are upward sloping in δ, with $\frac{\partial \rho_{PC}}{\partial \delta} < \frac{\partial \rho_{IC}}{\partial \delta}$, ($ii$) $\rho_{PC}(\delta^0_{PC}) = 0$ and $\rho_{IC}(\delta^0_{IC}) = 0$ for $\delta^0_{PC} < \delta^0_{IC}$, and ($iii$) $\rho_{PC} = \rho_{IC} = \rho_0$. Note that for sustainability purposes, it is only relevant that $\rho_{PC} > \rho_{IC}$ for $\rho < \rho_0$.

(ii) For $\rho \in (\rho_1, \rho_0)$, reductions in p^c_0 and p^c_1 facilitate collusion. Define (p'', p''') as the highest vector price at which ICC_W holds with equality such that $p'' < p''' < v$, i.e., is the vector price at which $G_0 = G_1$. To obtain p'' and p''', substitute p^c_0 by p'' and p^c_1 by p''' in ICC_W. Then, from ICC_0 holding with equality, solve for $p''(p''')$:

$$p''(p''') = p' = \begin{cases} p''' & if: \ F < F_N \\ p''' + 1 - F\left[1 - \underline{\varphi}\left(2 - \underline{\varphi}\right)\right] & if: \ F \in (F_N, 2F_N) \\ p''' - 1 & if: \ F > 2F_N \end{cases}$$

Plugging $p''(p''')$ into ICC_1, equating this constraint to zero and solving for ρ: imperfect collusion at prices $(p^c_0, p^c_1) = (p'', p''')$ is sustainable if and only if:

$$\rho = \rho_{IC2} = \begin{cases} \frac{(2\delta-1)\left(p'''-\beta\right)+2\delta-3/2}{\delta F} & if: \ F < F_N \\ \frac{(2\delta-1)\left(p'''-\beta\right)+4\delta-5/2-(2\delta-1)F\left(1-\underline{\varphi}\left(2-\underline{\varphi}\right)\right)}{2\delta+\delta F\left[2\underline{\varphi}\left(2-\underline{\varphi}\right)-1\right]} & if: \ F \in (F_N, 2F_N) \\ \frac{(2\delta-1)\left(p'''-\beta\right)-1/2}{\delta F\underline{\varphi}\left(2-\underline{\varphi}\right)} & if: \ F > 2F_N \end{cases}$$

for $p''' < v$.

Straightforward, for $p''' = v$, $p' = p''$; therefore, $\rho_{IC2} = \rho_{IC}$ for $p''' = v$, and $\rho_{IC2} < \rho_{IC}$ for $p''' < v$.

♦ **Proposition 3:** Holds from Lemma 6 (and Corollary 1) and firms' profit-maximization behavior.

♦ **Proposition 4:** Cartel's sustainability condition without warnings:

$$\rho < \rho_E = \begin{cases} \frac{(2\delta-1)(v-\beta)+2\delta-3/2}{\delta F} & if: \ F < F_E \\ \frac{(2\delta-1)(v-\beta)+2\delta-5/2}{F\left[\delta\ \left(1-\underline{\varphi}\left(2-\underline{\varphi}\right)\right)\right]} & if: \ F \in (F_E, 2F_E) \\ \frac{(2\delta-1)(v-\beta)-1/2}{\delta F\underline{\varphi}\left(2-\underline{\varphi}\right)} & if: \ F > 2F_E \end{cases}$$

Cartel's sustainability condition with warnings (perfect collusion):

$$\rho < \rho_{PC} = \begin{cases} \frac{(2\delta-1)(v-\beta)+2\delta-3/2}{\delta F} & if: \ F < F_N \\ \frac{(2\delta-1)(v-\beta)+2\delta-3/2}{\delta\left[1+\underline{\varphi}\left(2-\underline{\varphi}\right)F\right]} & if: \ F > F_N \end{cases}$$

For $\rho < 1/2$, $F_N < 2F_N < F_E < 2F_E$, and for $\rho > 1/2$, $F_N < F_E < 2F_N < 2F_E$.

For $F < F_N$, $\rho_E = \rho_{PC}$.

For $F \in (F_N, F_E)$, simple algebra shows that $\rho_E < \rho_{PC}$ for $F > F_N$, which is true by initial condition.

For $F > F_E$, it is useful to notice that: (i) ρ_E and ρ_{PC} are upward sloping in δ, with $\frac{\partial \rho_E}{\partial \delta} > \frac{\partial \rho_{PC}}{\partial \delta}$, and (ii) $\rho_E(\delta_E^0) = 0$ and $\rho_{PC}(\delta_{PC}^0) = 0$ for $\delta_{PC}^0 < \delta_E^0$.

ρ_{PC} and ρ_E intersect at $\rho^* \in (0,1)$ for $v > v^*$. For $v < v^*$, $\rho^* > 1$; w.l.o.g. I consider $\rho^* = 1$ in this case. Thus, for $\rho < \rho^*$, $\rho_E < \rho_{PC}$, and for $\rho > \rho^*$, $\rho_E > \rho_{PC}$.

♦ **Firm-specific inspections and warnings (Proofs for Lemmas 8-10 and Proposition 5).**

Solution for the benchmark case (without warnings)

The firm's problem is to choose price and effort levels that maximize:

$$\Pi_i = [p_i - (\beta - a_i)]\, q_i - \frac{(a_i + z_i)^2}{2} - \rho\left(\varphi_i + \varphi_j - \rho\varphi_i\varphi_j\right)\ F$$

Solving, firm's optimal behavior is given by: $a_i + z_i = q_i$, with $z_i \in \{0,1\}$, $i = 1,2$.

Collusion: Under collusion, $q_i^c = 1$, thus there are two alternative effort allocations: $(a_i, z_i) = \{(1,0), (0,1)\}$.

(i) Assume i sets $(a_i, z_i) = (1,0)$. If j sets $(a_j, z_j) = (1,0)$, its profits are $\pi^c(1,0) = p^c - \beta + 1/2 - \rho(2-\rho)F$. If j sets $(a_j, z_j) = (0,1)$, its profits are $\pi^c(0,1) = p^c - \beta - 1/2 - \rho\left[1 + \underline{\varphi}(1-\rho)\right]F$. There exists $F_H \in \Re^+$ such that: for $F < F_H$, j maximizes profits with an effort allocation $(a_j, z_j) = (1,0)$, and for $F > F_H$, j maximizes profits with an effort allocation $(a_j, z_j) = (0,1)$. Thus, setting $(a_i, z_i) = (0,1)$ for $F < F_H$, $i = 1,2$, is a Nash equilibrium.

(ii) Assume now that i sets $(a_i, z_i) = (0,1)$. If j sets $(a_j, z_j) = (0,1)$, its profits are $\pi^c(0,1) = p^c - \beta - 1/2 - \rho\underline{\varphi}\left(2 - \rho\underline{\varphi}\right)F$. If j sets $(a_j, z_j) = (1,0)$, its profits are $\pi^c(1,0) = p^c - \beta + 1/2 - \rho\left[1 + (1-\rho)\underline{\varphi}\right]F$. There exists $F_T \in \Re^+$ such that: for $F < F_T$, j maximizes profits with an effort allocation $(a_j, z_j) = (1,0)$, and for $F > F_T$, j maximizes profits with an effort allocation $(a_j, z_j) = (0,1)$. Thus, setting $(a_i, z_i) = (0,1)$ for $F > F_T$, $i = 1,2$, is a Nash equilibrium.

Since $F_T = \frac{1}{\rho\left(1-\underline{\varphi}\right)\left(1-\rho\underline{\varphi}\right)} < \frac{1}{\rho\left(1-\underline{\varphi}\right)(1-\rho)} = F_H$, for $F \in (F_T, F_H)$ there are two equilibria: one in which firms maximize productive efficiency, and other in which they maximize concealment. With a little bit of algebra one can proof that the latter yields higher profits. Thus, firms set $(a_i, z_i) = (1,0)$ for $F < F_T$, and $(a_i, z_i) = (0,1)$ otherwise, $i = 1,2$.

Deviation: Assume i deviates: $q_i^d = 2$.

For $F < F_T$, j sets $(a_j, z_j) = (1,0)$. If i sets $(a_i, z_i) = (2,0)$, its profits are $\pi^d(2,0) = 2(p^c - \beta) + 2 - \rho(2-\rho)F$. If i sets $(a_i, z_i) = (1,1)$, its profits are $\pi^d(1,1) = 2(p^c - \beta) - \rho\left[1 + \underline{\varphi}(1-\rho)\right]F$. Comparing profits, i maximizes profits by setting $(a_i, z_i) = (2,0)$.

For $F > F_T$ firm j sets $(a_j, z_j) = (0,1)$. If i sets $(a_i, z_i) = (2,0)$, its profits are $\pi^d(2,0) = 2(p^c - \beta) + 2 - \rho\left[1 + \underline{\varphi}(1-\rho)\right]F$. If i sets $(a_i, z_i) = (1,1)$, its profits are $\pi^d(1,1) = 2(p^c - \beta) - \rho\underline{\varphi}\left(2 - \rho\underline{\varphi}\right)F$. Comparing profits: i maximizes profits by setting $(a_i, z_i) = (2,0)$ for $F \in (F_T, 2F_T)$, and $(a_i, z_i) = (1,1)$, otherwise.

Incentive compatibility constraint under perfect collusion:

For $F < F_T$, $(a_i^c, z_i^c) = (1,0)$ and $\left(a_i^d, z_i^d\right) = (2,0)$. Thus, ICC is:

$$v - \beta + \frac{3}{2} \ \leq \ \frac{\delta}{1-\delta} \ \left[\ v - \beta + \frac{1}{2} - \rho \ (2-\rho) \ F \ \right]$$

For $F \in (F_T, 2F_T)$, $(a_i^c, z_i^c) = (0,1)$ and $\left(a_i^d, z_i^d\right) = (2,0)$. Thus, ICC is:

$$v - \beta + \frac{5}{2} - \rho\left[1 - \underline{\varphi}\left(1 - \rho\underline{\varphi}\right)\right] \ F \ \leq \ \frac{\delta}{1-\delta} \ \left[\ v - \beta - \frac{1}{2} - \rho\underline{\varphi}\left(2 - \rho\underline{\varphi}\right) \ F \ \right]$$

Finally, for $F > 2F_T$, $(a_i^c, z_i^c) = (0,1)$ and $\left(a_i^d, z_i^d\right) = (1,1)$. Thus, ICC is:

$$v - \beta + \frac{1}{2} \ \leq \ \frac{\delta}{1-\delta} \ \left[\ v - \beta - \frac{1}{2} - \rho\underline{\varphi}\left(2 - \rho\underline{\varphi}\right) \ F \ \right]$$

Introducing warnings

Firm i's problem is to choose price and effort levels that maximize:

$$\Pi_i = [\ p_i - (\beta - a_i)\ ] \ q_i \ - \ \frac{(a_i + z_i)^2}{2} \ - \ [W_i \ \varphi_i \ (1 - \rho\varphi_j) \ + \ \rho\varphi_j] \ F$$

Solving, firm i's optimal behavior is given by: $a_i + z_i = q_i$, with $z_i \in \{0,1\}$, $i = 1,2$.

Collusion: Under collusion, $q_i^c = 1$, thus there are two alternative effort allocations: $(a_i, z_i) = \{(1, 0), (0, 1)\}$.

Assume $W_i = 0$. Since i is not warned, $z_i = 0$. Thus, $(a_i, z_i) = (1, 0)$ and i's expected profits are:

$$\pi^c_{i,W_i=0}(1, 0 \mid W_j) = p^c - \beta + 1/2 - \rho\varphi_j F$$

Now, assume $W_i = 1$. With probability ρ, firm j is warned too ($W_j = 1$), and thus i behaves as in the setup with industry-specific policies when $W = 1$; i.e., firm i behaves as Lemma 3 states for $W = 1$.

With probability $1 - \rho$, firm j is not warned ($W_j = 0$). In this case, any reduction in the cartel probability of detection depends on i's effort devoted to concealment. If i sets $(a_i, z_i) = (1, 0)$, its profits are:

$$\pi^c_{i,W_i=1}(1, 0 \mid W_j = 0) = p^c - \beta + 1/2 - F$$

and if i sets $(a_i, z_i) = (0, 1)$, its profits are

$$\pi^c_{i,W_i=1}(0, 1 \mid W_j = 0) = p^c - \beta - 1/2 - \underline{\varphi}F$$

Comparing profits: i maximizes profits by setting $(a_i, z_i) = (1, 0)$ for $F < F_K = \frac{1}{1-\underline{\varphi}}$, and $(a_i, z_i) = (0, 1)$, otherwise.

Deviation: Assume i deviates: $q_i^d = 2$.

Assume $W_i = 0$. Since the cartel probability of detection does not depend on i's effort allocation, i sets $\left(a_i^d, z_i^d\right) = (2, 0)$.

Now, assume $W_i = 1$. With probability ρ, $W_j = 1$. In this case, i behaves as in the setup with industry-specific policies when $W = 1$; i.e., firm i behaves as Lemma 4 states for $W = 1$.

With probability $1 - \rho$, $W_j = 0$. In this case, any reduction in the cartel probability of detection depends on i's effort devoted to concealment. If i sets

$(a_i, z_i) = (2, 0)$, its profits are:

$$\pi^d_{i,W_i=1}(2, 0 \mid W_j = 0) = 2(p^c - \beta) + 2 - F$$

and if i sets $(a_i, z_i) = (1, 1)$, its profits are

$$\pi^d_{i,W_i=1}(1, 1 \mid W_j = 0) = 2(p^c - \beta) - \underline{\varphi} F$$

Comparing profits: i maximizes profits by setting $(a_i, z_i) = (2, 0)$ for $F < 2F_K$, and $(a_i, z_i) = (1, 1)$, otherwise.

Incentive compatibility constraint under perfect collusion: With a little bit of algebra, ICC is:

For $F < F_K$:

$$v - \beta + \frac{3}{2} \leq \frac{\delta}{1-\delta}\left[v - \beta + \frac{1}{2} - \rho\ (2-\rho)\ F\right]$$

For $F \in (F_K, F_N)$:

$$v - \beta + \frac{3}{2} \leq \frac{\delta}{1-\delta}\left[v - \beta + \frac{1}{2} - \rho(1-\rho) - \rho\ (2-\rho)\ \left[\rho + (1-\rho)\underline{\varphi}\right] F\right]$$

Finally, for $F > F_N$:

$$v - \beta + \frac{3}{2} \leq \frac{\delta}{1-\delta}\left[v - \beta + \frac{1}{2} - \rho - \rho\underline{\varphi}\left(2 - \rho\underline{\varphi}\right)\ F\right]$$

Effects of warnings on deterrence and firms' productive efficiency:

(i) For $F < F_K$, the warning program has no effect on deterrence, neither on cartel's productive efficiency.

(ii) For $F \in (F_K, F_N)$, the warning program facilitates collusion and creates productive inefficiencies on cartels.

(iii) For $F \in (F_N, 2F_K)$, the warning program facilitates collusion and creates productive inefficiencies on cartels if $F_T > 2F_K$ or if $F_T < 2F_K$ and $F < F_T$.

(iv) For $F \in (2F_K, 2F_N)$, the warning program facilitates collusion and creates productive inefficiencies on cartels if $F_T > 2F_N$ or if $F_T \in (2F_K, 2F_N)$ and $F < F_T$.

(v) For $F > 2F_N$, the warning program facilitates collusion if (a) $F_T > 2F_N$ and $F < F_T$, or (b) $F_T < 2F_N$ and $F < 2F_T$. In case (a) the program also creates productive inefficiencies on cartels. Finally, it can holds (c) $F_T < 2F_N$ and $F > 2F_T$. In this case, there exists ρ^* such that: for $\rho < \rho^*$, the program facilitates collusion, but for $\rho > \rho^*$, the program improves deterrence.

♦ **Lemma 11:** Recalling equation 3.3, firm's optimal behavior is given by: $a_i + z_i = q_i$, for $i = 1, 2$. Under collusion (i.e., for $W = 0$): (i) $q_i^c = 1$, and (ii) there are not detection costs. Therefore, $(a_i^c, z_i^c) = (1, 0)$.

♦ **Lemma 12:** Assume i deviates: i has two units of effort to allocate among productive efficiency and concealment (condition 3). Since deviation can occur only in periods in which firms are not warned, detection costs are zero. Therefore, $\left(a_i^d, z_i^d\right) = (2, 0)$.

♦ **Proposition 6:** Cartel's sustainability condition without warnings:

$$\rho < \rho_E = \begin{cases} \frac{(2\delta-1)(v-\beta)+2\delta-3/2}{\delta F} & if: \ F < F_E \\ \frac{(2\delta-1)(v-\beta)+2\delta-5/2}{F\left[\delta\ \left(1-\underline{\varphi}\left(2-\underline{\varphi}\right)\right)\right]} & if: \ F \in (F_E, 2F_E) \\ \frac{(2\delta-1)(v-\beta)-1/2}{\delta F \underline{\varphi}\left(2-\underline{\varphi}\right)} & if: \ F > 2F_E \end{cases}$$

Cartel's sustainability condition with warnings (perfect collusion):

$$\rho < \rho_{PC} = \frac{(2\delta-1)(v-\beta)+2\delta-3/2}{\delta\left(v-\beta+1/2-\pi_i^*\right)}$$

where π_i^* is firm i's profits under competition: $\pi_i^* = 0$ if firms play in weakly-dominated strategies, and $E\left[\pi_i^*\right] = 1$ if firms play in undominated strategies.

Comparing ρ_{IC} and ρ_{PC}: there exists $F^* \in \Re^+$ such that for $F < F^*$, $\rho_E > \rho_{PC}$, and for $F > F^*$, $\rho_E < \rho_{PC}$.

Chapter 4

Rewarding whistle-blowing in a principal-agent model

4.1 Introduction

Rewards in whistle-blowing legislation imply a serious challenge to the economic theory of enforcement. While they can improve deterrence on corporate crime, they can also create a non desired "hunt bounty" environment inside firms that distorts employees' attention from production towards activities related to gathering crime evidence. In this paper I develop a model that captures the implications of rewarding whistle-blowers on deterrence and on optimal contracts among non-offenders.

A *whistle-blower* is an individual who provides credible information related to some corporate misconduct to the pertinent authority.[1] The act of whistle-blowing is not meant to cause harm to the organization. Rather, it is meant to

[1]The term whistle-blowing originated from the practice of English policemen who blew their whistle when they observed the happening of some crime. The blowing of whistle alerted other law enforcement officers and the general public that a crime was being committed.

facilitate the public exposure of acts that occur within firms in detriment of the interest of the firm itself or social welfare.

To encourage whistle-blowing on corporate crimes, the USA legislation offers rewards to whistle-blowing. The False Claim Act (1986 and subsequent reforms), the IRS Whistle-blower Reward Program (2006), and the recent Dodd-Frank Act (2010), are the three main pieces of legislation governing rewards to whistle-blowing. These pieces of legislation differ in the crime of concern and the extent of applicability, but they have a common objective: to incentivize employees endowed with information to report corporate crimes.[2]

By considering rewards as a mechanism that creates a decision problem for employees on how to allocate effort among productive activities and activities related to gathering crime evidence, I develop a model that analyzes the implications of rewarding whistle-blowers on deterrence and on optimal contracts among a principal and an agent that are not crime offenders.

In the model, there is a principal who owns a firm with two employees. One employee can get personal gains from committing corporate crime, but to do so, he has to devote costly effort. The other employee can gather crime evidence and report it to the authority in exchange of a reward, but to do so, he has to take effort from productive activities to locate it into activities related to gathering crime evidence, as his effort is constrained.[3]

[2] The False Claims Act (FCA) fights fraud against the USA federal government. The FCA makes it a crime for any person or organization to submit a record or claim for payment for services, property or other items to the government, knowing that the information is not true. Rewards oscillate between 15 and 30 percent of funds reimbursed to the government as a result of the investigation.

The Dodd-Frank Wall Street Reform and Consumer Protection Act (Dodd-Frank Act) rewards private reports on securities violations, including violations of the Foreign Corrupt Practices Act (FCPA), that result in monetary sanctions greater than 1 million. The reward oscillates between 10 and 30 percent of the total recovery (i.e., the additional tax, penalty and other collected amounts).

The IRS Whistle-blower Reward Program rewards private reports on international financial crimes, including tax-fraud, money laundering, and the flow of narcotics and terrorist funding.

Detailed information on these pieces of legislation is available at:
http://www.justice.gov/
http://www.gpo.gov/fdsys/pkg/PLAW-111publ203/pdf/PLAW-111publ203.pdf
http://www.irs.gov/compliance/article/0,,id=180171,00.html

[3] One can think of a manager as an offender, who works as many hours as required/desired; and a secretary as the other employee (the potential whistle-blower), who has a fixed amount

Effort devoted to gathering crime evidence increases the probability of crime detection, but it also reduces the probability of crime existence (as the gains from crime are decreasing in the probability of detection) and, consequently, the probability of obtaining the reward too. Hence, the non-offender employee's effort allocation depends on: (1) reward levels, (2) his effort disutility, (3) the opportunity cost of gathering crime evidence in terms of the salary loss from less effort devoted to production, and (4) how his effort devoted to gathering crime evidence affects the probability of crime existence and that of crime detection.

There are two other important elements in the model. First, the principal observes effort devoted to production, but not so effort devoted to commit crime or to gathering crime evidence. As a result, he can not design a contract over effort devoted to commit crime or to gathering crime evidence. Second, crime creates an externality to the principal, positive or negative, which makes him interested in the existence of crime or in its deterrence.

I demonstrate that while rewards can improve deterrence, they can also distort the optimal contract between the principal and the agent. With rewards, the principal overpays effort devoted to production when he gets high positive externalities from crime, as he wants to bias the agent's effort allocation away from crime detection. Similarly, the principal underpays effort devoted to production when he gets high negative externalities from crime. There are two extreme cases, one in which the principal hires the agent only for his activities related to gathering crime evidence, and other in which, to reduce the probability of crime detection, he does not hire the agent. The latter one implies that rewards create a total loss of welfare.

By extending the analysis to private reward programs (i.e., inside-firm programs that reward the private exposure of the corporate crime), I demonstrate how these can substitute whistle-blower programs with a beneficial effect for the

of hours per working day. Assuming that both are fully efficient in their work, working hours and effort match perfectly. Hence, while the effort is limited for the secretary, it is not so for the manager (the offender).

principal and deterrence.

4.2 Related Literature

The literature on whistle-blowing can be classified into two main strands: one that conceives whistle-blowers as individuals with altruistic concerns,[4] and another that conceives whistle-blowers as self-interested individuals, who decide on whether to whistle the blow depending on an expected reward.

In this paper, I follow the latter conception. Equally important, I assume that whistle-blowers and firm owners do not take part in the crime.[5] Together, these assumptions define a working framework appropriate for the analysis of many corporate crimes (e.g, collusion at managerial level, the non compliance with safety regulations, environmental crimes), but, to the best of my knowledge, still not explored in the literature. Nevertheless, some of my results are in line with those already found in the literature regarding rewards to whistle-blowing.

Within a setup in which offender principals bribe whistle-blowers and hold their under-performance to avoid possible crime reports, Friebel & Guriev (2011) demonstrate that rewards to whistle-blowing may have undesired welfare effects. When rewards are not high enough to prevent crime, they increase losses from under-performance and bribes, as the opportunity cost of silence is increasing in rewards. Only reward levels that make bribes unprofitable improve deterrence, since crime concealment depends directly on bribes. Aubert, Kovacic & Rey (2006) analyse the same problem in a collusive game. But, in this context, even low reward levels improve deterrence, since the decision on whether to form a cartel can be highly sensible to the lower profits from collusion associated to bribes and employees' under-performance.

[4]Miceli and Near (1988), Dworkin and Near (1997), Miceli, Near, Rehg & Van Scotter (2001) and (2008), Miceli (2004), among others.

[5]This assumption implies that (*i*) whistle-blowers are not regretful offenders, neither accept a bribe for their silence, and (*ii*) firm owners are not crime offenders, neither encourage crime actively.

My results are close to these in the sense that with rewards the agent's payment from work may be higher than that without rewards, as the principal may want to distort the agent's attention away from crime detection. But in my setup, this is not associated to a contract outside the law or under-performance, as the higher payment attempts to focus the agent's attention in productive activities and, in none of the cases, allows for work inefficiencies. I also find that high rewards can improve deterrence, but mostly if the principal is interested on it, which is exactly the opposite setup of Friebel *et.al* (2011) and Aubert *et.al* (2006). On the contrary, as in my setup the principal can dispense with the whistle-blower, the combination of high rewards and a principal against crime deterrence implies no contract between the principal and the agent, and thus a total loss of welfare.

Within the literature of rewards to whistle-blowing stands out the literature of leniency programs, that analyzes the effects of offering amnesties to crime offenders in exchange of their collaboration in the detection of crime. This literature can be classified in two main strands: one that focuses on individual wrongdoers that commit occasional crimes, and another that focuses on corporate wrongdoers that systematically commit the same crime, particularly collusion.

Regarding leniency and occasional crimes, Kaplow & Shavell (1994), Malik (1993) and Innes (1999) stand out. Assuming high fines under crime detection, Kaplow & Shavell (1994) discuss the beneficial effects of leniency programs for deterrence in environmental crimes, to which Malik (1993) adds their beneficial effects associated to lower auditing costs in regulation. Inees (1999) extends these works by studying prospective ex-post benefits from remediation, as the clean-up activity is a central component of environmental law enforcement. Iness demonstrates how clean-up benefits and fine amnesties after self-reporting can be equivalent policies to deter environmental crimes.

The literature on leniency in collusion is extensive.[6] Motta and Polo (2003) is the first paper explicitly addressing the effects of leniency programs on collusion in a dynamic analytical framework. They demonstrate that these programs make enforcement more effective, but may also induce collusion since amnesties decrease the expected cost of the misbehavior. In the optimal policy, the former effect dominates and leniency programs improve deterrence. Their main contribution in this paper lies in that leniency improves deterrence even in the case where the leniency application is made after an investigation has started. Harrington (2008) extends Motta and Polo (2003) with the additional novel feature of reports along the equilibrium path: in cartels under investigation, firms may rush non-cooperatively to report information under a sufficiently generous leniency program (effect named Race to the Courthouse). Equilibrium reports during prosecution take place when the realization of the probability of a successful conviction is high. Spagnolo (2000) and Buccirossi and Spagnolo (2001, 2006) highlight the possibility that smart wrongdoers can use leniency as a threat against deviation, which allows them to sustain cartels that would not be sustainable in the absence of leniency programs. Hence, moderate forms of leniency may have the counterproductive side effect of facilitating illegal transactions.

Despite the fact that I consider positive rewards instead of amnesties, my paper differs from these mainly in that I disaggregate in two the figures of the crime offender and the whistle-blower, none of them the firm owner. This, together with a firm owner that may be interested or not in crime, allows me to analyze private incentives to execute and deter crime beyond the firm's costs associated to detection (e.g., fines, clean-up activities, etc.), as well as how these incentives are affected by the behavior of the other members of the firm. In this context, rewards to whistle-blowing entail a wide range of effects on deterrence, where both deterrence effects mentioned above are possible: a deterrence im-

[6]For an extensive analysis of the literature of leniency in collusion, please refer to Spagnolo (2008).

provement, mostly when the principal is interested in that; and a deterrence loss, otherwise. As final comment, I want to add that my paper is pioneer in getting into the firm's 'black-box' to seek the efficiency effects that rewarding non-offender whistle-blowers has in optimal contracts.

The rest of the paper proceeds as follows, I introduce the model in Section 3. In Section 4, I solve it for the case in which there are no rewards (benchmark case), and in Section 5, I solve the case with rewards. In Section 6, I consider the implications of rewards on deterrence and on the principal's utility. In Section 7, I extend the analysis for the case in which the principal can introduce a private reward program regardless of the existence of a whistle-blower program. I conclude in Section 8.

4.3 The Model

A firm operates in a competitive market. In the firm, there is a principal, the owner of the firm, and two employees. The principal designs the contracts that govern the relationship between the firm and each employee, and the employees execute productive activities following these contracts. In particular, contracts are defined on the basis of employees' efforts devoted to production, which are fully observable.

One employee can devote effort to commit corporate crime, which yields him private gains g. The crime creates hard evidence that can be found through an inspection. The other employee, instead, can not commit corporate crime, neither get any direct payoff from it. But, given crime, he can find evidence of it if he devotes effort to look for it.

Along this paper, I refer the former employee as 'the offender' and the latter employee as 'the agent'.

Effort is costly. Effort devoted to production is observable, but not effort devoted to commit crime or to the gathering of crime evidence.

To fight crime, the anti-crime authority has two instruments:

- Inspections: given crime, it is detected with probability $\rho \in [0, 1]$.
- Whistle-blower program: offers a reward R to employees who present evidence of corporate crime, excluding offenders. It rewards only if the inspector fails in detecting crime.[7]

If crime evidence is found, the firm and the offender pay corporate and individual fines F and f, respectively.

Finally, crime entails a direct payoff G to the principal, which can be positive or negative, $G \in \Re$.

In this framework, I assume that the principal can identify which employee can commit corporate crime, i.e. he can identify which job positions give an employee the attributes required to commit crime. But, he can not directly observe whether there is crime in the firm, neither gather evidence of it. I also assume that it is profitable for the principal to hire the offender employee, despite the possibility of a corporate crime.

Regarding the agent and the offender's activities, I assume that the offender's activities related to production and crime are not related. However, the agent's activities related to production and the gathering of crime evidence are technological substitutes.

Hence, in this model, the principal decides whether to hire the agent, and if so designs a contract for him.[8] The offender decides whether to commit crime, and if so how much effort devote to it. Finally, the agent decides whether to

[7]These assumptions are in line with the USA legislation of rewards to whistle-blowing.

[8]The agent's effort devoted to productive activities is the only item of interest for the purpose of a contract, since the principal (*i*) always hires the offender, and (*ii*) does not have an instrument to distort the offender's willingness to commit crime (as the offender's effort devoted to commit crime is independent of his effort devoted to productive activities).

gather crime evidence, and if so how to allocate effort among production and the gathering of crime evidence.

I define $z, e_1, e_2 \in [0,1]$, effort devoted to commit crime, to productive activities and to gathering crime evidence, respectively.

At this point it is important to define two probabilities: The probability of crime existence P_c, and the probability that the agent finds crime evidence given that there exists crime P_e. I assume $P_c = z$, so that the higher the offender's effort devoted to commit crime, the higher the probability of crime existence. Regarding P_e, I assume $P_e = e_2$, so that, given crime, the higher the agent's effort devoted to gathering crime evidence, the higher the probability that he finds it.

Finally, I assume that the agent can not falsify evidence of crime.

The timing of the game is as follows: At date 0 the principal decides whether to hire an agent from a competitive market for agents. If so, he decides on the contract to offer him.

For the contract, I consider a linear payment scheme for productive effort e_1: $w(\alpha, \beta) = \alpha + \beta e_1$, where $\alpha \in \Re$ and $\beta \geq 0$.

In the absence of a contract, the agent has zero utility, the principal hires only the offender and the game ends. If instead there is a contract, at date 1 the agent and the offender simultaneously decide on effort levels. At date 2, production, crime activities and activities related to gathering crime evidence are executed. At this time, an inspector visits the firm. The inspection ends with a report supporting or rejecting crime. If the agent has found evidence, he also submits a report with crime evidence to the anti-crime authority. At date 3, the principal gets a payoff $Y(e_1) = ye_1$ (with $y > 0$) from production and pays $w(\alpha, \beta)$ to the agent. Besides, given crime, the principal and the offender get payoffs G and g, respectively. If the inspector or the agent report crime

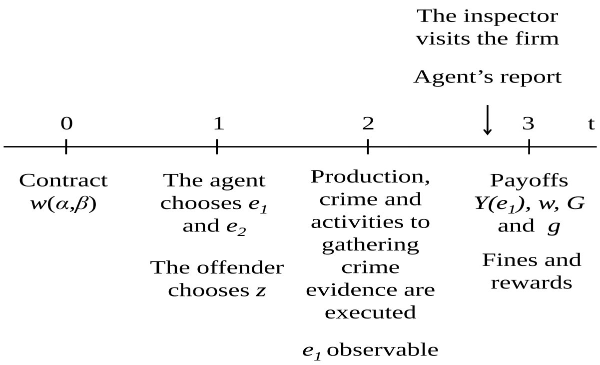

Figure 4.1: Time-structure of the model

evidence, the firm and the offender pay F and f, respectively. The agent gets the reward if he is the only one to report crime evidence.

In this setup, the agent's utility function is:

$$U(e_1, e_2) = (\alpha + \beta e_1) - \frac{(e_1 + e_2)^2}{2} + z\ (1 - \rho)\ e_2\ R$$

The first term in the RHS is the agent's utility from effort devoted to production; the second term, his disutility from effort; and the third term, his utility from effort devoted to gathering crime evidence. This last term is composed of the probability of crime existence z, times the probability of the agent being the only one to report crime evidence, $(1 - \rho)\, e_2$, times the reward R. I assume $R \leq F$.[9]

The offender's utility function is:

$$O(z) = z\ \{\ g - [\ \rho + (1 - \rho)\, e_2]\ \ f\ \}\ -\ \frac{z^2}{2}$$

The first term in the RHS is the offender's expected utility from crime. Outside brackets, his effort devoted to commit crime. In brackets, his net gains from crime, that depend on the agent's effort devoted to gathering crime evidence. The second term is the offender's disutility from effort.

[9]The upper-bound $R = F$ is consistent with a reward program funded by fines, in line with the USA whistle-blower legislation.

Finally, the principal's utility function is:

$$V(\alpha, \beta) = ye_1 - (\alpha + \beta e_1) + z \ \{ \ G - [\rho + (1 - \rho) \, e_2] \ \ F \ \}$$

The first two terms in the RHS are the principal's net utility from the agent's productive activities, and the last term his externalities from crime.

4.4 The Case of No Rewards to Whistle-Blowing

As a benchmark case, assume no rewards to whistle-blowing. Without rewards, the agent has no incentives to devote costly effort to the gathering of crime evidence ($e_2 = 0$). Hence, given crime, the probability of crime detection is ρ.

Solving by backward induction, at date 1 the agent and the offender choose the levels of effort e_2 and z, respectively, that maximize their utilities:

$$\max_{e_1} \ U(e_1) = \alpha + \beta e_1 - \frac{e_1^2}{2}$$

$$\max_{z} \ O(z) = z \ \ (\ g - \ \ \rho \ f \ ) \ \ - \ \ \frac{z^2}{2}$$

For expository reasons, throughout the paper I set $g = f = 1$ [10], so that the offender's problem simplifies to:

$$\max_{z} \ O(z) = z \, (1 - \rho) - \frac{z^2}{2} \tag{4.1}$$

Lemma 16 *Without rewards to whistle-blowing, effort devoted to crime is $z^B = 1 - \rho$, and effort devoted to production is $e_1^B = \min \{\beta, 1\}$.*

Effort levels z^B and e_1^B equate the agent and the offender's marginal utility and marginal disutility, respectively. The supra-index B denotes the *Benchmarck case*.

[10] This simplification does not restrict the results of the paper.

At date 0, the principal chooses a contract $w(\alpha, \beta)$ that maximizes his utility and is incentive compatible and acceptable to the agent:

$$\max_{\alpha,\beta} \; V(\alpha,\beta) = ye_1 - (\alpha + \beta e_1)$$

$$s.t. \quad e_1 \in arg \max_{e_1'} \left\{ \alpha + \beta e_1' - \frac{e_1'^2}{2} \right\} \qquad (IC)$$

$$\alpha + \beta e_1 - \frac{e_1^2}{2} \geq 0 \qquad (PC)$$

The (IC) and the (PC) are the agent's incentive compatibility and participation constraints, respectively.

Proposition 12 *Without rewards to whistle-blowing, the optimal contract* $w\left(\alpha^B, \beta^B\right)$ *is given by a marginal payment on productive effort given as follows:*

$$\beta^B = \begin{cases} y & if: \; y \leq 1 \\ & \\ 1 & if: \; y > 1 \end{cases}$$

and α^B *such that the principal gets all the agent's surplus.*

Corollary 6 *Without rewards to whistle-blowing, the agent exerts effort* $e_1^B = \min\{y, 1\}$ *at the optimal contract.*

Without rewards to whistle-blowing, the principal offers an efficient contract that pays effort depending on the marginal productivity from work and in which the agent makes the effort level that equates his marginal utility and marginal disutility from effort. This contract is independent of z.

4.5 Rewarding Whistle-Blowing

Rewards to whistle-blowing induce the agent to devote costly effort to the gathering of crime evidence. Given crime, this effort raises the probability of detection and, therefore, reduces the offender's willingness to commit crime. The latter, in return, reduces the agent's incentives to gather crime evidence, since the probability of crime existence is downward sloping in effort devoted to commit crime. It follows that, with rewards, efforts devoted to commit crime and to gathering crime evidence depend on each other.

For the principal, rewards to whistle-blowing imply a shift in utility from changes in his net externalities from crime and in the optimal contract. The former change arises from (now) endogenous probabilities of crime existence and condemnation. The latter change arises from the property of technical substitutability in the agent's effort: with rewards, the agent may wish to substitute effort devoted to productive activities by effort devoted to activities related to gathering crime evidence.

In what follows I solve the optimal contract between the principal and the agent in two steps. First, I solve the agent's and the offender's simultaneous effort decisions. Second, I solve the principal-agent problem given those effort decisions (i.e., I solve the optimal contract for e_1).

4.5.1 The Agent's and Offender's Simultaneous Effort Choices

At date 1, the agent choses the levels of efforts e_1 and e_2 that maximize his utility taking z as given:

$$\max_{e_1,e_2} \; U(e_1,e_2) = \alpha + \beta e_1 - \frac{(e_1+e_2)^2}{2} + (1-\rho)\; z\; e_2\; R \tag{4.2}$$

And the offender chooses the level of z that maximizes his utility taking e_2 as given:

$$\max_{z} \; O(z) = z\left\{1 - \left[\rho + (1-\rho)\, e_2\right]\right\} - \frac{z^2}{2} \tag{4.3}$$

Lemma 17 *With rewards to whistle-blowing, there exist* $\beta_0 \in [0,1)$*,* $\beta_1 = \min\{\beta_{1a}, \beta_{1b}\} \in [0,2)$*, with* $\beta_0 < \beta_1$*, and* $\hat{R} = \frac{1}{(1-\rho)^2}$*, such that:*

(i) *For* $\beta < \beta_0 : (e_1, e_2) = (0\ ,\ \beta_0)$ *and* $z = (1-\rho)(1-\beta_0)$.

(ii) *For* $\beta \in [\beta_0, \beta_1]$*:* $e_1, e_2, z \in [0,1]$*, with* e_1 *and* z *upward sloping in* β *and* e_2 *downward sloping in* β*:*

$$e_2 = \beta - e_1 = 1 - \frac{\beta}{(1-\rho)^2 R} \tag{4.4}$$

$$z = (1-\rho)(1-e_2) = \frac{\beta}{(1-\rho) R} \tag{4.5}$$

(iii) *For* $\beta > \beta_1$ *and*

- $R < \hat{R}$: $(e_1, e_2) = (\min\{\beta, 1\}\ ,\ 0)$ *and* $z = (1-\rho)$.
- $R > \hat{R}$: $(e_1, e_2) = (1\ ,\ \beta_{1b} - 1)$ *and* $z = (1-\rho)(2-\beta_{1b})$.

Boundaries β_0 and β_1 are upward sloping in R.[11]

For all values of the parameters β and R, the offender's effort devoted to commit crime is given by the probability of not being discovered, neither by the inspector $(1-\rho)$, nor by the agent $(1-e_2)$. Thus, the higher the agent's effort devoted to gathering crime evidence, the lower the offender's effort devoted to crime.

What about the agent's effort allocation? For $\beta < \beta_0$, the e_2's marginal contribution to the agent's utility is higher than the e_1's marginal contribution, as the agent is paid (relatively) little for each unit of effort devoted to production. Hence, he only devotes effort to gathering crime evidence. How much effort? It depends on R: the higher the level of R, the higher the e_2.

For $\beta \in [\beta_0, \beta_1]$ the agent allocates effort among production and the gathering of crime evidence. Equation (4.4) states the agent's effort allocation in

[11] In particular:

$$\beta_0 = \frac{(1-\rho)^2 R}{1+(1-\rho)^2 R} < 1 \qquad \beta_{1b} = 2\beta_0 \qquad \beta_{1a} = (1-\rho)^2 R$$

Whether β_{1a} higher (lower) than β_{1b} depends on the policy instruments R and ρ: $\beta_{1a} < \beta_{1b} \iff R < \hat{R} = \frac{1}{(1-\rho)^2}$.

terms of the e_1's and the e_2's marginal contributions to the agent's utility. Each level of effort is positively related to its own marginal contribution and negatively related to that of the other one. In terms of β, this implies that the agent substitutes e_2 for e_1 as β increases (i.e., the higher the value of β, the more biased is the agent's effort allocation towards productive activities), and vice versa. Note that the agent's total effort is positively related to β ($e_1 + e_2 = \beta$), so that total effort increases with β.

Equation (4.5) states the offender's effort devoted to commit crime in terms of e_2, first, and in terms of β and R, second. The higher is the agent's relative payment from gathering crime evidence (i.e., the lower the ratio β/R), the more biased is the agent's effort allocation towards this activity, and so the lower is the offender's effort devoted to crime.

Finally, for $\beta > \beta_1$, the agent maximizes effort devoted to production, as he is paid a high amount for it. What about effort devoted to gathering crime evidence? The agent devotes effort to this activity if and only if the reward is high enough, $R > \hat{R}$. Hence, for high reward values, the agent devotes one unit of effort to productive activities and effort to gathering crime evidence depending on R: the higher the reward, the higher the agent's effort devoted to gathering crime evidence. Instead, for low reward values, the agent only devotes effort to productive activities (at most one unit).[12]

Note that for $\beta > \beta_1$ the offender's effort devoted to commit crime is maximum, as the agent minimizes effort devoted to gathering crime evidence.

Figures 4.2 and 4.3 show the agent's effort allocation for different values of β for $y \leq 1$ and the case values $R < \hat{R}$ and $R > \hat{R}$, respectively.

[12]For explanatory purposes, and w.l.o.g., I assume $y \leq 1$ in all explanations throughout this section, although the general results hold for all values of y.

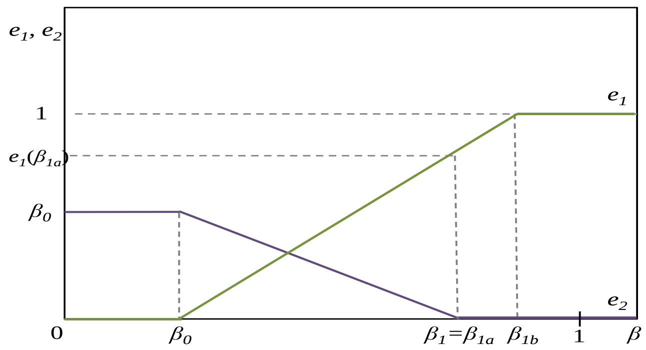

Figure 4.2: The agent's effort allocation in terms of β for $y \leq 1$ and low values of the reward ($R < \hat{R}$).

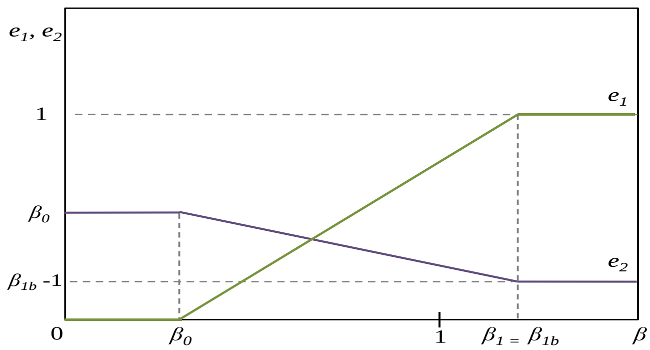

Figure 4.3: The agent's effort allocation in terms of β for $y \leq 1$ and high values of the reward ($R > \hat{R}$).

4.5.2 The Principal-Agent Problem

At date 0, the principal chooses a contract $w(\alpha, \beta)$ that maximizes his utility and is incentive compatible to the agent and acceptable to both:

$$\max_{\alpha,\beta} \; V(\alpha, \beta) = ye_1 - (\alpha + \beta e_1) + z \; \{ \; G - [\rho + (1-\rho)\, e_2] \;\; F \; \}$$

$$s.t: \quad (e_1, e_2) \in arg \max_{e_1^{'}, e_2^{'}} \left\{ \alpha + \beta e_1^{'} - \frac{\left(e_1^{'} + e_2^{'}\right)^2}{2} + (1-\rho)\, z e_2^{'} R \right\} \qquad (ICa)$$

$$z = (1-\rho)\,(1-e_2) \qquad (ICo)$$

$$\alpha + \beta e_1 - \frac{(e_1 + e_2)^2}{2} + (1-\rho)\, z e_2 R \geq 0 \qquad (PCa)$$

$$ye_1 - (\alpha + \beta e_1) + z \; \{ \; G - [\rho + (1-\rho)\, e_2] \;\; F \; \} \geq (1-\rho)\,(\; G - \rho F \;) \qquad (PCp)$$

The (ICa) and the (ICo) are the agent and the offender's incentive compatibility constraints, respectively; and the (PCa) and the (PCp) are the agent and the principal's participation constraints, respectively.

Regarding the (PCp), the principal may be better off by not hiring the agent. In his decision, he compares the payoffs he gets from not hiring the agent (RHS) and from hiring him (LHS). The principal hires the agent if, in doing that, he gets the highest payoff.

In what follows I solve the principal-agent problem for the optimal effort choices obtained in Section 5.1. To do it, I solve first the optimal contract for a 'semi-constrained' problem without the (PCp). I denote this problem the Semi-Constrained Principal-Agent (SCPA) problem. Second, I check whether the (PCp) holds at the optimal contract of the SCPA problem.

Solving the SCPA Problem

Plugging equations (4.4), (4.5) and the (PCa) binding into the principal's objective function and solving for β, the interior solution for β is given by:

$$\beta^* = \varphi \left\{ y + k \left[(1-\rho)(G-F) + (1-\rho)^2 R \right] \right\} \tag{4.6}$$

The coefficient k measures the principal's capability to distort the agent's effort allocation through β, $k = -\frac{\partial e_2}{\partial \beta} / \frac{\partial e_1}{\partial \beta} = \frac{1}{1+(1-\rho)^2 R}$. The coefficient φ is a multiplier with $\frac{\partial \varphi}{\partial R} < 0$ and $\lim_{R\to\infty} \varphi = 1$.[13]

Equation (4.6) states the agent's marginal payment for e_1 in terms of both effort's (e_1 and e_2) marginal contributions to the principal's utility. In braces, the first term is the e_1's marginal contribution to production (y). The higher the e_1's marginal contribution to production the more interested is the principal in e_1, and so the higher the β he is willing to pay for it.

The second term in braces is the e_2's marginal contribution to the principal's utility given G, F and R. This term can be positive or negative. Inside brackets: The higher is the payoff G, the higher is the principal's benefit from crime, and so the higher the β he is willing to pay to reduce e_2. The higher is the corporate fine F, the lower is the principal's benefit from crime, and so the lower the β he is willing to pay to increase e_2. Finally, the higher is the reward R, the higher is the level of e_2 the agent is willing to make, and so also the higher are the probability of detection and the β the principal has to pay to reduce e_2. Outside brackets, the coefficient k measures the principal's capability to distort the agent's effort allocation through β: the higher is the reward, the lower is the principal's capability to govern over the agent's effort allocation ($\frac{\partial k}{\partial R} < 0$). Note that the effect of rewards is ambiguous: while a reward increase induces the principal to increase β to discourage e_2, it also induces him to reduce β, as for high reward levels he can not govern over the agent's effort allocation.

[13]Particularly: $\varphi = \frac{R\left[1+(1-\rho)^2 R\right]}{(1-\rho)^2 R^2 + 2(R-F)}$, positive for $R > R_h = \frac{\sqrt{1+2F(1-\rho)^2}-1}{(1-\rho)^2}$.

Proposition 2 characterizes β^* in terms of G, and Corollary 2 states the comparative statics of β^* with respect to R.

Proposition 13 *(Interior solution) There exists $\underline{G}$, such that effort devoted to production is underpaid with respect to the benchmark case ($\beta^* < y$) if and only if $G < \underline{G}$.*

Corollary 7 ***(i)*** *There exists $\overline{G}$, with $\overline{G} > \underline{G}$, such that a reward increase rises β^* if and only if $G < \overline{G}$.*

(ii) *β^* approaches y as R goes to infinity.*

For the intuition behind these results, consider a reward increase: if $G < \underline{G}$, crime is so 'bad news' to the principal that he underpays e_1 to bias the agent's effort allocation towards activities related to gathering crime evidence. But, as R goes up, the agent gets incentives from outside the firm to do so and, to restore incentives, the principal increases β.

If, instead, $G > \underline{G}$, crime is so 'good news' to the principal that he overpays e_1 to bias the agent's effort allocation away from crime detection. As R goes up, the agent gets incentives from outside the firm to gather crime evidence so, to restore incentives, the principal increases β. However, successive increases in R make this strategy increasingly costly to the principal and so, for a high enough value of R, he finds himself better off by resigning to it and focusing only on the incentives on e_1. In terms of Corollary 2, for $G > \overline{G}$ we only observe the negative relationship between β and R. This is due to the fact that for $G > \overline{G}$, β is too high and it is not profitable for the principal to increase it further.

The second point of Corollary 2 is straightforward now: the higher the value of R, the lower the principal's capability (and possibly interest as well) to govern over the agent's effort allocation through β. So, regardless of the value of G, β^* approaches its value for the benchmark case (y) for high values of R.

In the light of these results, it will be useful to define β's interior solution in terms of R:

Corollary 8 *There exist* R_0*,* R_1*, with* $0 \leq R_0 \leq R_1$*, such that:* $R \in [R_0, R_1]$ $\Leftrightarrow \beta^* \in [\beta_0, \beta_1]$.

CORNER SOLUTIONS: For $R \notin [R_0, R_1]$, there is no optimal contract profitable to both principal and agent, with β^* and $e_1, e_2 \in (0, 1)$. Nevertheless, both parties can still find it profitable to celebrate a contract:

- For $R > R_1$ (i.e., $\beta < \beta_0$), the principal hires the agent only to gather crime evidence. Therefore, he sets $\beta = 0$.[14]
- For $R < R_0$ (i.e., $\beta > \beta_1$), the principal hires the agent for his productive activities, regardless of his possible activities related to gathering crime evidence.[15] For low reward levels ($R < \hat{R}$), the agent does not devote effort to gathering crime evidence, thus the principal will try to achieve productive efficiency: $\beta = e_1 = \min\{y, 1\}$. However, this will be only possible if $y > \beta_1$. Thus, in this corner solution, the principal sets: $\beta = \min\{\max\{\beta_{1a}, y\}, 1\}$ (remember that for $R < \hat{R}$, $\beta_1 = \beta_{1a}$).

  Instead, for high reward levels ($R > \hat{R}$), the agent devotes effort to gathering crime evidence and maximizes effort devoted to work ($e_1 = 1$). An increase of β beyond β_1 has no impact on e_1, as e_1 has already taken its maximum value, thus the principal sets $\beta = \beta_1 = \beta_{1b}$ (remember that for $R > \hat{R}$, $\beta_1 = \beta_{1b}$).

Summing up, the contract that solves the SCPA problem is the 'candidate contract' for the optimal contract. For medium/low reward values, this contract allows activities related to gathering crime evidence to co-exist with productive

[14]For $\beta < \beta_0$, the corner solution is $(e_1, e_2) = (0\ , \beta_0)$. As e_2 does not depend on the particular value that β takes in the interval $[0, \beta_0)$, w.l.o.g. β can be set equal to zero.

[15]For $R < R_0$ effort e_2 is minimized. Particularly, when $R < \hat{R}$, $e_2 = 0$.

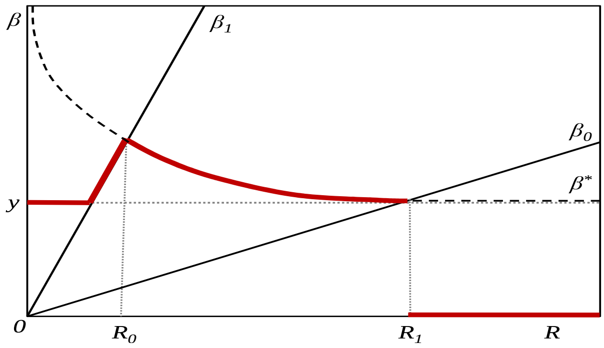

Figure 4.4: Candidate marginal payment to productive effort for $y \leq 1$ and $G > \overline{G}$ (highlighted). Boundaries β_0 and β_1 in solid lines, and the interior solution β^* in dashed line. Productive effort is efficiently paid for low reward values and overpaid for medium/high reward values. For sufficiently high reward values ($R > R_1$), $\beta = 0$.

activities ($e_1 > 0$, $e_2 \geq 0$). Instead, for high reward values, this contract allows only for activities related to gathering crime evidence ($e_1 = 0$ and $e_2 > 0$). Full productive efficiency can be achieved for reward values close to zero, otherwise productive effort is overpaid or underpaid with respect to the benchmark case (Figures 4.4 and 4.5).

The Principal's Participation Constraint and the Optimal Contract

Under what conditions is the candidate contract an optimal contract? Plugging the candidate contract $w(\alpha, \beta)$ of the SCPA model and the optimal effort levels, obtained in Section 5.1, in the (PCp), we obtain the following:

Proposition 14 *With rewards to whistle-blowing, there exist $\hat{R}$, G_0, G_1, $G_a < G_b$, and $\hat{y}$, such that the principal hires the agent if and only if:*

(i) $R \in [R_0, R_1]$ *and* $y > \hat{y}$, *or if* $y < \hat{y}$ *but* $G \notin (G_a, G_b)$*;*

(ii) $R > R_1$ *and* $G < G_0$*; and*

(iii) $R < \min\left\{R_0, \hat{R}\right\}$, *or if* $R \in (\hat{R}, R_0)$ *and* $G < G_1$.

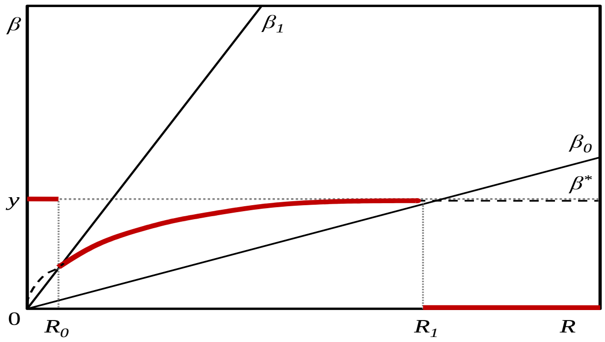

Figure 4.5: Candidate marginal payment to productive effort for $y \leq 1$ and $G < \overline{G}$ (highlighted). Boundaries β_0 and β_1 in solid lines, and the interior solution β^* in dashed line. Productive effort is efficiently paid for low reward values and underpaid for medium/high reward values. For sufficiently high reward values ($R > R_1$), $\beta = 0$.

Proposition 3 says that there is a contract in three cases. First, when the agent is not interested in gathering crime evidence. This is the benchmark case, that arises for very low reward values $\left(R < \min\left\{R_0, \hat{R}\right\}\right)$.

Second, when the agent's productive activities are profitable enough to compensate the principal's losses from the activities related to gathering crime evidence. This case arises for intermediate reward values and a high enough e_1's marginal productivity ($R \in [R_0, R_1]$ and $y > \hat{y}$, or if $y < \hat{y}$ and $G > G_b$).[16]

Third, when the agent's activities related to gathering crime evidence are profitable to the principal. This case completes Proposition 3 and arises when crime creates negative externalities to the principal, or if positive, too little.[17]

[16]For $R \in [R_0, R_1]$, the principal knows that if he hires the agent, this will allocate effort among production and the gathering of crime evidence. For high values of y, hiring the agent is profitable to the principal regardless of e_2. For low values of y, instead, this is so if and only if $G \notin (G_a, G_b)$. A low payoff from crime ($G < G_a$), makes deterrence desirable to the principal, hence he hires the agent regardless of the detrimental effect that e_2 has on production. A high payoff from crime ($G > G_b$), instead, makes deterrence not desirable to the principal, but such a high payoff allows him to set β high enough to reduce e_2 in such a way that hiring the agent is still profitable.

[17]This case contemplates three sub-cases: for $R > R_1$, the principal hires the agent to make activities related to gathering crime evidence. Such a contract takes place if and only if the principal's payoff from crime is low enough ($G < G_0$).

For $R \in [R_0, R_1]$ and $y < \hat{y}$, the agent allocates effort among production and the gathering of crime evidence. Such a contract takes place if and only if the principal's payoff from crime is low enough ($G < G_a$).

Proposition 4 defines the principal-agent contract in the presence of rewards.

Proposition 15 *With rewards to whistle-blowing, the principal-agent contract $w(\alpha, \beta)$ is given by a marginal payment on productive effort given as follows:*

(i) *For $R \in [R_0, R_1]$: β is defined as in equation (4.6).*

(ii) *For $R > R_1$: $\beta = 0$.*

(iii) *For $R < R_0$ and*

- *$R < \hat{R}$: $\beta = \min\left\{\max\left\{\beta_{1a} = (1-\rho)^2 R, y\right\}, 1\right\}$, or if*
- *$R > \hat{R}$: $\beta = \beta_{1b} = \frac{2(1-\rho)^2 R}{1+(1-\rho)^2 R}$*

and α such that the principal gets all the agent's surplus.

Introducing rewards to whistle-blowing affects the optimal contract between the principal and the agent in two ways: the decision on whether to celebrate the contract (Proposition 3), and if so, the values α and β that define it (Proposition 4).

Without rewards, there is always a contract, defined over the agent's marginal productivity from effort devoted to productive activities. With rewards, instead, the existence of contract depends on the level of the reward, on the principal's externalities from crime and, only in third place, on the agent's marginal productivity from effort devoted to productive activities. Together, high rewards and high externalities from crime, create an environment in which hiring the agent is too costly for the principal. A low marginal productivity from effort devoted to production aggravates the situation. Therefore, we should expect no contract between the principal and the agent in these cases.

For $R < R_0$, the agent maximizes effort devoted to production and devotes effort to gathering crime evidence depending on the value of R. If $R < \hat{R}$, his effort devoted to gathering crime evidence is zero, as the reward is too low (benchmark case). But, if $R > \hat{R}$, his effort devoted to gathering crime evidence is positive. In this case, there is a contract if and only if the principal's payoff from crime is low enough ($G < G_1$).

In the cases in which there is still a contract, productive efficiency can be achieved for reward values close to zero. Otherwise, productive effort is overpaid or underpaid with respect to the benchmark case.

4.6 Implications of Rewarding Whistle-Blowers

4.6.1 Deterrence

Through rewards to whistle-blowing, the anti-crime authority distorts the probability of crime existence: without rewards, this probability is $P_c(e_2 = 0) = (1 - \rho)$, with rewards, it is $P_c(e_2) = (1 - \rho)(1 - e_2)$. Corollary 4 follows immediately:

Corollary 9 *The introduction of rewards to whistle-blowing improves deterrence if and only if, after rewards, there is a contract and $e_2 > 0$.*

What if, given $e_2 > 0$, there is a reward increase?

Lemma 18 *(Assume $e_2 > 0$) A reward increase assures an improvement in deterrence for:*

(i) *low (high) reward values, and*

(ii) *intermediate reward values and high values of G.*

For low and high values of R, the agent chooses effort devoted to gathering crime evidence based solely on the value of R. In particular, the higher the reward, the higher his effort devoted to this activity. Therefore, unless e_2 is already 1 (its maximum value), a reward increase improves deterrence.

Instead, for intermediate reward values, the agent considers both R and β, to choose how much effort to devote to gathering crime evidence. In this case, a reward increase can have two counter-effects on e_2: whereas it directly encourages e_2, it may also discourage e_2 (indirectly) through its impact on β. To see this, consider the case in which crime is not desired by the principal (i.e., G is low). In this case, a reward increase is 'good news' to him: as e_2 is encouraged by the public authority, he can restore productive efficiency with a higher β. But, in return, the higher β induces the agent to reduce e_2. Hence, the reward increase improves deterrence if and only if the increase in e_2 due to the higher R more than compensates its subsequent reduction due to the higher β. Thus, for $\frac{\partial \beta}{\partial R} > 0$ (which holds for low values of G), the deterrence effect of a reward increase is ambiguous.

However, for $\frac{\partial \beta}{\partial R} < 0$ (which holds for high values of G), it is assured a gain in deterrence after a reward increase, as e_2 goes up due to the higher R and the lower β.

As a final comment, note that $\frac{\partial \beta}{\partial R} < 0$ implies that (*i*) e_1 is overpaid, as the principal gets high externalities from crime, and (*ii*) the principal can not use β to bias the agent's effort allocation away from crime detection, as he can not increase β more. In this scenario, too much incentives towards the gathering of crime evidence (i.e., successive reward increases), can put the existence of the contract at risk, as the principal may find himself better off by not hiring the agent and keeping crime in the firm. Therefore, the anti-crime authority should be cautious on the level of the reward, such that encouraging whistle-blowing without risking the existence of the contract.

4.6.2 The Principal's Utility with Rewards

The introduction of rewards to whistle-blowing is profitable for the principal if his utility with rewards is higher than that without rewards; i.e., if the following condition holds:

$$V\left(\alpha(R), \beta(R)\right) \;\geq\; \frac{y^2}{2} \;+\; (1-\rho)\left(G-\rho F\right) \tag{4.7}$$

The LHS is the principal's utility in the presence of rewards and the RHS his utility for the benchmark case. Note that condition (4.7) is similar to (PCp) described in Section 5.2, but for the additional term $\frac{y^2}{2}$ in the RHS. This term states the principal's net utility from hiring the agent in the absence of rewards.[18] This similarity suggests that the introduction of rewards implies a gain in utility for the principal if and only if those conditions for which (PCp) holds are strong enough to ensure compliance in excess over $\frac{y^2}{2}$.

Lemma 19 *The introduction of rewards implies a gain in utility for the principal if and only if rewards do not discourage the principal from hiring the agent and crime is highly detrimental for the principal (i.e., G is low enough).*

When rewards discourage the principal from hiring the agent, the principal obtains a net payoff from crime $(1-\rho)(G-\rho F)$ with and without rewards. However, without rewards he also obtains a positive payoff from the agent's work $\left(y^2/2\right)$. Undoubtedly, the principal is better off without rewards.

When rewards do not discourage the principal from hiring the agent, his net payoffs from crime and from the agent's work differ on whether there are rewards or not. In this case, the introduction of rewards implies a gain in utility for the principal if these allow him to reduce an undesired (and unprofitable) crime for him; i.e., if G is low enough.

At this point, it is worth to mention two cases in which the principal is indifferent regarding the introduction of rewards.[19] First, when rewards are very low. In this case, the agent is not interested in gathering crime evidence and the principal can offer an efficient contract almost always. Provided the

[18]Without rewards, the principal always hires the agent. From Proposition 1 and Corollary 1, the principal's utility for the benchmark case is: $U = \frac{y^2}{2} + (1-\rho)(G-\rho F)$ if $y \leq 1$, and $U = y - \frac{1}{2} + (1-\rho)(G-\rho F)$ if $y > 1$. Following the analysis in previous sections, and w.l.o.g., I assume $y \leq 1$ in condition (4.7) and in the explanations that follow it , although the general results from this section hold for all values of y.

[19]These cases correspond to scenarios for which (PCp) holds with inequality and condition (4.7) holds with equality.

efficient contract, the principal is indifferent to the introduction of rewards.[20] Second, when the principal's net gains from crime are high enough to bias the agent's effort allocation away from crime detection. In this case, the principal can set α and β such that inducing the agent an effort allocation close to that for the benchmark case ($e_2 \to 0$ and $e_1 \to e_1^B$). Provided this contract (although not efficient as productive work is overpaid), the principal is indifferent to the introduction of rewards.[21]

A second issue to discuss refers to how a reward increase affects the principal's utility. Assuming that it is profitable for the principal to hire the agent, what if rewards go up?

Lemma 20 *(Assume contract) A reward increase makes the principal's utility to go up if the agent's activities related to gathering crime evidence are profitable to the principal.*

For low reward values ($R < R_0$), the principal hires the agent for his productive activities, regardless of e_2, and the agent devotes effort to gathering crime evidence depending on R: the higher the reward, the higher this effort level. In this scenario, a reward increase implies a gain in utility for the principal if and only if he is interested in crime detection.

For high reward values ($R > R_1$), the principal hires the agent for his activities related to gathering crime evidence. In this case, a reward increase always implies a gain in utility for the principal, as e_2 is upward sloping in R.

For $R \in [R_0, R_1]$, the agent devotes effort to production and to the gathering of crime evidence, and the principal underpays/overpays productive effort.

[20]For $R < \min\left\{R_0, \hat{R}\right\}$, the optimal contract is given by $\beta = \min\{\max\{\beta_{1a}, y\}, 1\}$ and α such that the principal gets all the agent's surplus. At this contract, $(e_1, e_2) = (\beta, 0)$. The principal is indifferent regarding the introduction of rewards if these do not prevent him to offer the agent the efficient contract: $\beta = e_1 = y$ if $y \leq 1$, or $\beta = e_1 = 1$ if $y > 1$.

[21]High values of G allow the principal to overpay work such that inducing $e_2 \to 0$ and $e_1 \to e_1^B$. Since α is set such that the principal get's all the agents surplus, the principal retrieves any overpayment and his utility from hiring the agent is that for the benchmark case.

In this scenario, a reward increase has two effects on the principal's utility. First, a utility gain through a higher productive efficiency, as β approaches y with successive increases in R (Corollary 2). Second, a utility change due to a higher/lower net payoff from crime: an increase in R distorts the agent's effort allocation and, through it, the probability of crime existence. When crime is undesired by the principal (G is low), higher values of R imply less crime and the principal's net payoff from crime goes up. In this case, an increase in R raises the principal's utility undoubtedly. However, when crime is desired by the principal (G is high), the effect of higher values of R in the principal's net payoff from crime is, a priori, unknown (it depends on the parameters of the model); and therefore so is the final effect on the principal's utility.

4.7 Private Reward Programs

Private Reward Programs (PRPs) work as Whistle-Blower Programs (WBPs): both reward whistle-blowers to encourage the exposure of corporate crime. However, PRPs avoid the detection costs associated to the public exposure of the wrongdoing, as the whistle-blow is private to the firm. Instead, PRPs have to afford the payment of rewards.

To introduce PRPs in the model, I assume that these and WBPs are equally efficient in deterring crime. This assumption implies that the offender's behavior with respect to crime does not depend on the formal system used by the agent to report the evidence, but, as I have assumed so far, on how much effort the agent devotes to gathering crime evidence.

In this context, the principal chooses whether to create a PRP that rewards $r > 0$ to the agent for the private exposure of the corporate crime and offer him a contract $w\left(\alpha(r), \beta(r)\right)$; or, instead, to offer the agent a contract $(\alpha(R), \beta(R))$, without PRP ($r = 0$).

The agent and the offender's effort choices: With PRPs, the agent and the offender's effort choices are as described for WBPs (Section 5.1), as PRPs and WBPs are equally efficient in deterring crime. One simply has to consider r instead of R.

The principal-agent problem: The principal's problem consists on whether to introduce a PRP and, if so, on defining the value of r.

Assuming the existence of a WBP, this problem is described as follows:

$$\max_{\alpha,\beta,r} V(\alpha,\beta,r) = ye_1 - (\alpha+\beta e_1) + z\ \left(G - \rho F \right) + z\ (1-\rho)\, e_2\ r$$

$$s.t: \quad (e_1, e_2) \in arg\max_{e_1^{'},e_2^{'}} \left\{ \alpha + \beta e_1^{'} - \frac{\left(e_1^{'}+e_2^{'}\right)^2}{2} + (1-\rho)\, z e_2^{'} r \right\} \qquad (ICa)$$

$$z = (1-\rho)\,(1-e_2) \qquad (ICo)$$

$$\alpha + \beta e_1 - \frac{(e_1+e_2)^2}{2} + z\ (1-\rho)\ \ e_2\ r \geq 0 \qquad (PCa)$$

$$ye_1 - (\alpha+\beta e_1) + z\ \left(G - \rho F \right) + z\ (1-\rho)\, e_2\ r$$

$$\geq \max\left\{\ V\left(\alpha(R), \beta(R)\right)\ \ , (1-\rho)\,(G-\rho F)\ \right\} \qquad (PCp)$$

The principal's outside option is his highest possible payoff for $r = 0$.

In the case in which there is no WBP the problem is described alike, but for the RHS of (PCp) which is given by the principal's net payoff for the benchmark case:[22] $\frac{y^2}{2}\ +\ (1-\rho)\,(G-\rho F)$.

Solving, the optimal contract goes as follows:

[22]Following the analysis in previous sections, and w.l.o.g., I assume $y \leq 1$ in (PCp) for the benchmark case, although the general results from this section hold for all values of y.

Proposition 16 *The principal creates a PRP that offers $r > 0$ to the agent for the private report of crime evidence if and only if:*

(i) *(Assuming no WBP) crime is highly detrimental for him (G is low enough).*

(ii) *(Assuming WBP) crime is highly detrimental for him (G is low enough) and public rewards are not high enough (R is low).*

This contract is given by $\beta = 0$ and $\alpha(r) < 0$ such that the principal gets all the agent's surplus.

Corollary 10 *With PRPs,* $(e_1, e_2) = \left(0 \ , \ \frac{(1-\rho)^2 r}{1+(1-\rho)^2 r} \right)$ *and* $z = (1-\rho)(1-e_2)$.

Proposition 5 states that low externalities from crime can incentivize the principal to introduce a PRP in the absence of a WBP, and in its presence if the public reward is not high enough. These cases correspond to those in which the principal hires the agent only for his activities related to gathering crime evidence.

Regarding the size of the reward, note that the principal can set r as high as he desires, as he retrieves all reward payments through α anyways. This analysis suggests very high values of r, as the probability of crime existence (and so also the expected detection costs) is downward sloping in r.[23]

Finally, note that the introduction of a PRP implies that deterrence and the principal's utility are maximized. The latter result is intuitive: the introduction of a PRP is voluntary, thus if the principal introduces it, it must be because this implies a gain in utility for him. For the second result consider a WBP that offers R. Furthermore, assume that the parameters of the model are such that the principal hires the agent to gathering crime evidence. Hence, e_2 and deterrence are maximum: $e_2 = \frac{(1-\rho)^2 R}{1+(1-\rho)^2 R}$ and $z = \frac{(1-\rho)}{1+(1-\rho)^2 R}$, the higher is R,

[23]With PRP, the principal's utility at the optimal contract is upward sloping in r. Please, see the Appendix for details.

the higher is e_2 and the lower is z. In this context, if the principal finds it e_2 to low, he can introduce a PRP that offers $r > R$ to induce the agent to exert more effort. Therefore, for any given R, the introduction of a PRP implies an increase in e_2 and, consequently, higher deterrence.

4.8 Conclusion

Whistle-blowers play an important role in fighting fraud by companies and the government, including fraud against the government. To encourage whistle-blowing, the USA legislation offers rewards to whistle-blowing on corporate crimes. What are the implications of rewarding whistle-blowing on the contract between a firm's owner and a non-offender employee? And on deterrence? Are rewards desirable for firm owners? If so, under which conditions?

This paper gives an answer to these questions by developing a model that assumes: (*i*) rewards create a decision problem to a non-offender employee on how to allocate effort among productive activities and activities related to the gathering of crime evidence, (*ii*) effort devoted to gathering crime evidence affects the probability of crime existence which, in return, affects the expected payoff from rewards, (*iii*) effort devoted to production is observable (so that production can be contracted upon it), but efforts devoted to gathering crime evidence and committing crime are not, (*iv*) the principal is not the crime offender, but gets externalities (positive or negative) from crime, and (*v*) the principal can not design a law-enforceable contract with the offender to encourage/prevent crime.

Regarding the implications of rewarding whistle-blowing on contracts, two results stand out. First, effort devoted to production is overpaid (underpaid) if the principal wants to bias the agent's effort allocation towards (away from) productive activities. The principal wants to bias the agent's effort allocation towards productive activities when he gets high positive externalities from crime.

To this end, he overpays productive activities, as this discourages the agent from gathering crime evidence. Alternatively, the principal wants to bias the agent's effort allocation towards crime detection when he gets low externalities from crime (negative or positive but low). To this end, he underpays productive activities. As an extreme case, the principal hires the agent only for his activities related to crime detection.

Second, rewards may imply no contract between the principal and the agent. Consider the case in which the principal overpays productive activities to bias the agent's effort allocation away from crime detection: for high reward values, the required overpayment can be unprofitable for the principal, especially if the agent's marginal productivity from work is low. As a consequence, the principal may find himself better off by not hiring the agent, even if in the absence of rewards he would have done it.

In the light of these results, the introduction of rewards improves deterrence if and only if rewards do not discourage the principal from hiring the agent and $e_2 > 0$.

What is the effect of a reward increase in deterrence? And in the principal's utility? Regarding deterrence, there is an improvement in deterrence following a reward increase if crime detection is desired by the principal (i.e., if the principal gets high negative externalities from crime), or if the principal's externalities from crime are high enough to sustain hiring the agent in a scenario in which the agent's effort allocation is biased towards crime detection. With respect to the effect on the principal's utility, there is a gain in utility following a reward increase if crime detection is desired by the principal.

Finally, I also demonstrate that for low enough crime externalities, it is in the principal's interest to create a reward-program private to the firm, regardless of the existence of a whistle-blower program. A program of this type arises when crime is highly detrimental to the principal and, if there exists a whistle-blower program, when also the public reward is not high enough to deter crime as much as the principal would like. Moreover, I demonstrate that the use of a private reward-program implies maximum deterrence.

Summing up, whistle-blower programs can improve deterrence, but at the expense of productive efficiency. The loss in efficiency is lower when the principal gets low (negative) externalities from crime, as it is associated only to less effort devoted to productive activities – and not also to the possibility of no-contract. Nevertheless, in this case, the principal may also have private incentives to create a private reward program, with higher benefits for him and for crime deterrence. In the light of this result, this paper favors the use of whistle-blower programs to deter crimes that work in favor of the interest of the principal (e.g., tax-frauds, collusion, environmental crimes, etc.) and when the agent's work is highly important to the firm (in terms of the model, when the agent's marginal productivity from work (y), is high), such that the principal finds it too costly to dispense with the agent's activities (e.g., accountants, middle or low-level managers, executive secretaries, etc.)

Appendix

Lemma 1: From the partial derivative of $O(z)$ with respect to z: $z^B = 1-\rho$. Since $O(z)$ is concave in z, $z^B = arg \max \{O(z)\}$.

From the partial derivative of $U(e_1)$ with respect to e_1: $e_1^B = \min\{\beta, 1\}$, for $\beta \geq 0$ and $e_1 \in [0,1]$. Since $U(e_1)$ is concave in z, $e_1^B = arg \max \{U(e_1)\}$.

Proposition 1: Follows from (i) agent's optimal effort allocation e_1^B (Lemma 1), and (ii) α such that the principal gets all the agent's surplus.

Lemma 2: The FOC from the offender's problem is:

$$\frac{\partial O}{\partial z} = 1 - [\rho + (1-\rho)\, e_2] - z \tag{4.8}$$

The FOCs from the agent's problem are:

$$\frac{\partial U}{\partial e_1} = -e_1 - e_2 + \beta \tag{4.9}$$

$$\frac{\partial U}{\partial e_2} = -e_1 - e_2 + (1-\rho)\, zR \tag{4.10}$$

From (4.8), the best response $e_2(z)$ of the offender is: $e_2(z) = 1 - \frac{z}{1-\rho}$, and from (4.9) and (4.10), the best response $e_2(z)$ of the agent is:

$$e_2(z) = \begin{cases} \min\{1, \max\{(1-\rho)\, zR - 1, 0\}\} & if:\ z < \frac{\beta}{(1-\rho)R} \\ [\min\{1, \max\{(1-\rho)\, zR - 1, 0\}\}\ ,\ \min\{(1-\rho)\, zR, 1\}] & if:\ z = \frac{\beta}{(1-\rho)R} \\ \min\{(1-\rho)\, zR, 1\} & if:\ z > \frac{\beta}{(1-\rho)R} \end{cases}$$

Solving, the equilibrium is:

$$(e_2, z) = \begin{cases} (\beta_0\ ,\ (1-\rho)\,(1-\beta_0)) & if:\ \beta < \beta_0 \\ \left(1 - \frac{\beta}{(1-\rho)^2 R}\ ,\ \frac{\beta}{(1-\rho)R}\right) & if:\ \beta \in (\beta_0, \beta_1) \\ (\underline{e}_2\ ,\ \overline{z}) & if:\ \beta > \beta_1 \end{cases}$$

where $\beta_0 = \frac{(1-\rho)^2 R}{1+(1-\rho)^2 R}$, $\beta_1 = \min\left\{\beta_{1a} = (1-\rho)^2 R \ , \ \beta_{1b} = 2\beta_0\right\}$, and:

$$(\underline{e}_2, \overline{z}) = \begin{cases} (0 \ , \ 1-\rho) & if: \ \beta_1 = \beta_{1a} \\ \\ (\beta_{1b} - 1 \ , \ (1-\rho)(2-\beta_{1b})) & if: \ \beta_1 = \beta_{1b} \end{cases}$$

Finally, at the equilibrium, e_1 is:

$$e_1 = \begin{cases} 0 & if: \ \beta < \beta_0 \\ \\ \beta + \frac{\beta}{(1-\rho)^2 R} - 1 & if: \ \beta \in (\beta_0, \beta_1) \\ \\ \overline{e}_1 & if: \ \beta > \beta_1 \end{cases}$$

where: $\overline{e}_1 = \min\{\beta, 1\}$ if $\beta_1 = \beta_{1a}$, and $\overline{e}_1 = 1$ if $\beta_1 = \beta_{1b}$.

Proposition 2: β^* is linearly related to G and y. Thus, there exists a unique $\underline{G}$ such that $\beta^* > y$ if and only if $G > \underline{G}$.

Proposition 3: Rewrite the (PCp) as:

$$X = ye_1 - (\alpha + \beta e_1) + z \ \{ \ G - [\rho + (1-\rho) e_2] \ F \ \} - (1-\rho)(\ G - \rho F \ ) \geq 0 \quad (4.11)$$

The (PCp) holds if and only if $X \geq 0$.

(i) Assume $R \in [R_0, R_1]$. Substituting the interior solution of the SCPA's contract in (4.11), simple algebra shows that X is a polynomial of degree 2 in G, with positive coefficient in the quadratic term, and $X_{min} \geq 0$ if and only if $y \geq \hat{y} = \frac{(1-\rho)^2 R}{2}$. For $y < \hat{y}$, the roots of the polynomial are G_a and G_b, with $G_a < G_b$, such that $X \geq 0$ if and only if $G \notin [G_a, G_b]$.

(ii) Assume $R \notin [R_0, R_1]$. Efforts at the corner solution are independent of G. For $R > R_1$ and $R \in (\hat{R}, R_0)$, X is linear and downward sloping in G. Thus, there exists a unique value of G such that $X > 0$ if and only if G is below this value. Denote these critical values by G_0 and G_1 for $R > R_1$ and $R \in (\hat{R}, R_0)$, respectively.

For $R < \min\left\{R_0, \hat{R}\right\}$, $X \geq 0$ for all values of G.

Proposition 4: Follows from Lemma 2, Corollary 3 and Proposition 3.

Lemma 3: There is a deterrence effect from a reward increase if and only if $\frac{\partial e_2}{\partial R} > 0$.

Assume $R \notin [R_0, R_1]$: $e_2 \in \{0, \beta_0, \beta_{1b} - 1\}$, where $0 < \beta_0 < \beta_{1b} - 1$ and $\frac{\partial \beta_0}{\partial R}$, $\frac{\partial \beta_{1b}}{\partial R} \geq 0$. Thus, $\frac{\partial e_2}{\partial R} \geq 0$.

Assume $R \in [R_0, R_1]$: $e_2 = 1 - \frac{\beta}{(1-\rho)^2 R}$, with β upward or downward sloping in R.

Taking the partial derivative of e_2 with respect to R:

$$\frac{\partial e_2^*}{\partial R} = \frac{1}{\left[(1-\rho)^2 R\right]^2}\left[\beta\,(1-\rho)^2 - \frac{\partial \beta}{\partial R}\frac{1}{(1-\rho)^2 R}\right]$$

The first term in brackets is the direct effect of a higher reward in e_2, which is always positive: A reward increase encourages e_2. The second term in brackets is the indirect effect, which can be positive, negative or zero, depending on how the reward increase distorts β.

For high values of G, $\frac{\partial \beta}{\partial R} < 0$, thus $\frac{\partial e_2}{\partial R} > 0$. For low values of G, instead, $\frac{\partial \beta}{\partial R} > 0$, thus the effect of a reward increase on e_2 depends on the parameters of the model.

Lemma 4: Rewrite condition (4.7) in the text as:

$$W = \underbrace{ye_1 - (\alpha + \beta e_1) + z\ \{\ G - [\rho + (1-\rho)\,e_2]\ \ F\ \} - (1-\rho)\,(\ G - \rho F\ )}_{X} - \frac{y^2}{2} \geq 0 \quad (4.12)$$

The introduction of rewards improves the principal's utility if and only if $W \geq 0$. Following notation in Proposition 3: $W = X - \frac{y^2}{2}$, W and X have the same functional form with respect to G. In addition, W and X have the same coefficients with respect to G, but for the independent term, which is lower for

W. Then:

(i) Assume $R \in [R_0, R_1]$. Substituting the interior solution of the SCPA's contract in (4.12), W is a polynomial of degree 2 in G with positive coefficient in the quadratic term and $W_{min} < X_{min}$. In addition, simple algebra shows that $W_{min} < 0$, with roots G_{Wa} and G_{Wb}, $G_{Wa} < G_{Wb}$. Note that $G_{Wa} < G_a < G_b < G_{Wb}$, as the polynomial W equates the polynomial X, but for the fact that the former has a lower independent term. Hence: *given contract, the introduction of rewards improves the principal's utility for* $G \notin [G_{Wa}, G_{Wb}]$*, and reduces the principal's utility for* $G \in [G_a, G_{Wa}] \bigcap [G_b, G_{Wb}]$.

(ii) Assume $R \notin [R_0, R_1]$. In the corner solutions, efforts are independent of G. For $R > R_1$ and $R \in (\hat{R}, R_0)$, W is linear and downward sloping in G. Thus, there exists a unique value of G such that $W > 0$ if and only if G is below this value. Denote these critical values by G_{W0} and G_{W1} for $R > R_1$ and $R \in (\hat{R}, R_0)$, respectively.

For $R < \min\left\{R_0, \hat{R}\right\}$, $W \leq 0$ for all values of G.

Lemma 5: There is a gain in utility from a reward increase if and only if $\frac{\partial V}{\partial R} \geq 0$.

For $R > R_1$, the principal hires the agent only for his activities related to gathering crime evidence ($\frac{\partial V}{\partial e_2} > 0$). The agent devotes effort to gathering crime evidence depending on the reward value: the higher the reward, the higher the effort level ($\frac{\partial e_2}{\partial R} \geq 0$). Straightforward, $\frac{\partial V}{\partial R} \geq 0$.

For $R < R_0$, the principal hires the agent for his productive activities, regardless of e_2. The agent devotes effort to gathering crime evidence depending on the reward value: the higher the reward, the higher the effort level ($\frac{\partial e_2}{\partial R} \geq 0$). Two possibilities arise:

- For $R < \hat{R}$: $e_2 = 0$ and $e_1 = \beta_{1a} > y$, with $\frac{\partial e_1}{\partial R} \geq 0$. $\frac{\partial V}{\partial R} = \frac{\partial e_1}{\partial R}(y - \beta_{1a}) \leq 0$. A reward increase does not distort e_2, but discourages e_1 with respect to its efficient level. There is a loss in utility following a reward increase.

- For $R > \hat{R}$: $e_1 = 1$ and $e_2 = \beta_{1b} - 1$, with $\frac{\partial e_2}{\partial R} \geq 0$. In this case, there exists $\tilde{G} < G_1$ such that: $\frac{\partial V}{\partial R} \geq 0 \Leftrightarrow G < \tilde{G}$.

For $R \in [R_0, R_1]$, the agent devotes effort to production and to the gathering of crime evidence. A reward increase has two effects in the principal's utility: a distortion in utility due to β approaching y, and a distortion in utility due to that an increase/reduction in e_2 changes his net payoffs from crime.

$$\frac{\partial V^*}{\partial R} = \underbrace{\frac{\partial \beta^*}{\partial R}(y - \beta^*)}_{Efficiency\ Effect} - \underbrace{\frac{\partial e_2^*}{\partial R}\left[y + \frac{(y - \beta^*)}{k}\right]}_{Crime\ Net\ Payoff\ Effect}$$

The *Efficiency Effect* is positive but for $G \in \left\{\underline{G}, \overline{G}\right\}$ (where it is negative), and the sign of the *Crime Net Payoff Effect* is assured to be positive for a high enough crime externality ($G > \check{G}$). Hence, $\frac{\partial V}{\partial R} \geq 0$ if $G > \max\left\{\overline{G}, \check{G}\right\}$. Otherwise, the ultimate effect of an increase in R in the principal's utility depends on the parameters of the model.

Proposition 5: The principal introduces a PRP if and only if the payoff he obtains with it is higher than the one he obtains without it.

With PRP, the SCPA can be set as:

$$\max_{\beta, r} V(\beta, r) = ye_1 - \frac{(e_1 + e_2)^2}{2} + (1 - \rho)(1 - e_2)(G - \rho F)$$

with FOCs:

$$\frac{\partial V}{\partial \beta} = y\frac{\partial e_1}{\partial \beta} - (e_1 + e_2)\left(\frac{\partial e_1}{\partial \beta} + \frac{\partial e_2}{\partial \beta}\right) - (1 - \rho)(G - \rho F)\frac{\partial e_2}{\partial \beta} \tag{4.13}$$

$$\frac{\partial V}{\partial r} = \frac{\partial e_2}{\partial r}\left[-y - (1 - \rho)(G - \rho F)\right] \tag{4.14}$$

Condition (4.14) is positive for $G \leq G_r = \rho F - \frac{y}{(1-\rho)}$. This implies that: ($i$) the principal's utility is increasing in r for $G \leq G_r$, and (ii) given a contract with PRP, the higher the r the higher the principal's utility.

For the rest of the proof I assume $G < G_r$, such that the principal finds it profitable to introduce a PRP.

From above comments and (4.13), the principal maximizes profits in a candidate contract with PRP with $\beta^* = 0$ and $r \to \infty$. Following our assumption in the main text, α is set such that the principal gets all the agents surplus.

For this candidate contract, $e_1^* = 0$ and $e_2^* = \frac{(1-\rho)^2 r}{1+(1-\rho)^2 r}$. The principal's payoff is:

$$V_{PRP}^* = \left(1 - \rho - \frac{(1-\rho)^3 r}{1 + (1-\rho)^2 r}\right)(G - \rho F) - \frac{(1-\rho)^2 r}{1 + (1-\rho)^2 r}$$

Principal's participation constraint:

(i) Assume there is no WBP: the principal's outside option is $V_0 = (1-\rho)(G - \rho F) + \frac{y^2}{2}$. Since V_{PRP}^* and V_0 are linear in G, there exists G_{crit} such that the principal introduces the PRP if and only if G is low enough, i.e., $G < \min\{G_r, G_{crit}\}$.

(ii) Assume there exists a WBP: the principal's outside option is $\max\{V_{WBP}, V_{00}\}$, where $V_{WBP} = V(\alpha(R), \beta(R))$, and $V_{00} = (1-\rho)(G - \rho F)$. Since V_{00} is linear in G, for $V_{00} > V_{WBP}$ the proof follows as in (i). Instead, when $V_{WBP} > V_{00}$ three cases are possible.

(a) Assume $R > R_1$. At the optimal contract with WBP: $e_1^* = 0$ and $e_2^* = \frac{(1-\rho)^2 R}{1+(1-\rho)^2 R}$. In this context, V_{WBP} is linear in G and the proof follows as in (i).

(b) Assume $R < R_0$. At the optimal contract with WBP, e_1^* is maximum and e_2^* is minimum:

- For $R_0 < \hat{R}$: $(e_1, e_2) = (\min\{\max\{\beta_{1a}, y\}, 1\}\ ,\ 0)$.

- For $R_0 > \hat{R}$: $(e_1, e_2) = (1\ ,\ \beta_{1b} - 1)$.

In this context, V_{WBP} is linear in G and the proof follows as in (i).

$c)$ Assume $R \in (R_0, R_1)$. Note that if V_{WBP} is the outside option, it is because $V_{WBP} > V_{00}$. In this context, assume $V_{00} > V^*_{PRP}$. Using arguments in (i), this inequality holds when G is high. Thus, for high values of G, $V_{WBP} > V^*_{PRP}$, and the principal does not introduce a PRP. On the contrary, for low enough values of G, $V^*_{PRP} > V_{00}$, and thus the principal may find it profitable to introduce a PRP. In this context, assume that the principal does not introduce a PRP: he obtains a positive payoff from the agent's activities related to production and some payoff from his activities related to gathering crime evidence. However, if he introduces a PRP, he only obtains payoff from the latter (i.e., the principal sacrifices his payoff associated to production in favor of maximum deterrence). By a revealed preference argument, if the principal introduces a PRP it must be because his utility increases with higher deterrence. In other words, if the principal introduces a PRP it must be because his externalities from crime G are so low (negative), that maximum deterrence increases his utility.

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