Akademia Górniczo-Hutnicza

Faculty of Mechanical Engineering and Robotics

Cracks detection in Aluminium plates by ultrasounds using Lamb waves.

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For me, this work means the continuation of my great plan of life and many experiences that I have lived, which I have grown rapidly. And on the other hand, it is for me the beginning of a new stage as a professional. During its implementation have learned to appreciate much more my loved ones, friends and all those who have been very supportive to reach the end of this stage and who undoubtedly dedicate this symbol of effort so important to me.

To my family who have instilled in me values of humility, perseverance and struggle. My parents taught me that in adversity just have to stay strong and keep going harder. Thanks to them I have managed to become that I am. To my brother, who has always been my role model to follow, and He is the best model that anyone can have.

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1. INTRODUCTION AND AIM OF THE PROJECT

By the emerging application of ultrasound in different industrial fields, it is necessary to develop inspection procedures to detect the presence of discontinuities that affect the properties of these materials. The development of this project is expects a study using the technique of ultrasonic nondestructive testing that allows detect failures in plates of Aluminium through the utilization of Lamb waves.

It is necessary a preliminary study about ultrasound, types of waves existing and a more concrete study of the waves used in this project, which are Lamb waves, getting to know something more about transducers and how they act in according to their placement on the plate. And it is also essential a study of data representation and how to analyze them in function of the type of presentation.

The two first experiments are performed in an isotropic plate of 1mm thickness and $50 \times 60cm^2$ of size.

The first experiment is to obtain the dispersion curves for the mode in which the wave is excited (A_0), through pitch-catch method.

In the second experiment it is based in detection of defects through pulse-echo method and also consists of the study and analysis of the rest of the wave.

The aim of the third and the last experiment is the creation of a program in Matlab to be able to detect the location of the defect based on the properties of the ellipses, this experiment also takes place in an Aluminium isotropic plate as in the two previous experiments, but in this case the plate is bigger: $1 \times 1m^2$.

2. TECHNIQUES AND CHARACTERISTICS OF ULTRASOUND

2.1 NONDESTRUCTIVE TESTING:

This section has been written with the help of references [1] and [2].

The field of nondestructive testing (NDT) is a broad and interdisciplinary field, which plays a critical role to guarantee that the components and systems perform their function of a reliable way.

These tests are carried out so as not to affect the future usefulness of the material. These materials are examined without being damaged, without interfering with the final use. The nondestructive test provides an excellent balance between the control of quality and profitability.

The current system of nondestructive testing consists of various subsystems as shown as follow:

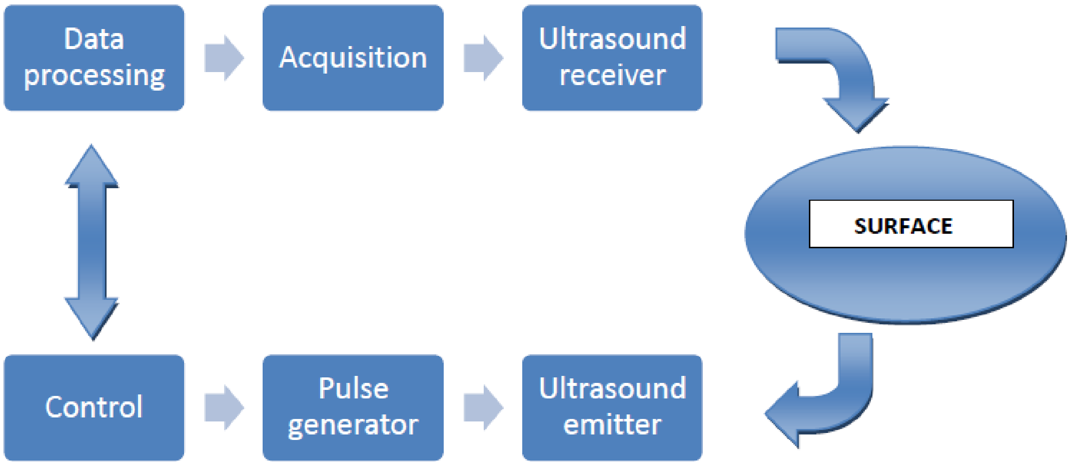

Image 2.1: Block diagram of the current system of nondestructive testing.

There are several types of nondestructive testing. One of the most representative, and the one used in this project is the technique of ultrasound testing.

2.2 ULTRASOUND:

This section has been written with the references [3] and [4].

The ultrasonic sound waves are identical in nature to sound waves differing from these that its frequency range is above the audible zone and, therefore they are also mechanical waves which need to be transmitted a material medium (solids, liquids and gases) to their propagation. These waves have led to the emergence of numerous technical and scientific applications, the most significant being the non-destructive control of the quality from the structural materials.

The association of use of high frequency acoustic waves with sensitive electronic devices for control, broadcasting and reception, along with the directional characteristics of these waves, has improved the sensitivity of detection of very small defects.

Ultrasonic signal is composed of a series of mechanical vibrations transmitted in the material through waves as the same nature as the sound, but with a higher frequency to 20.000 cycles/s (Hz).

Inside the emission spectrum of acoustic waves (vibrations), infrasound is not audible low frequency below 10 Hz. The audible waves by human ear are between 10Hz and 16000Hz and ultrasonic are neither audible by the human ear, because they are above of 16000Hz.

The physical principle from this method of inspection is the constant speed transmission of ultrasonic waves through the material and capturing the echo produced when there is a change in acoustic impedance “Z” (resistance from the material to oppose the passage of a ultrasonic wave).

Ultrasonic inspection is one of the nondestructive methods most commonly used and metal-mechanical area and it is mainly used for thickness measurement and detection of surface discontinuities, subsurface and internal.

ADVANTAGES OF ULTRASOUND:

The main advantages of ultrasound inspection with respect to other non-destructive testing for the inspection of materials are:

- Greater penetration power ranging from millimeters up to 6m in some cases
- High sensitivity which allows the determination of elastic constants
- Detection of small cracks
- Greater accuracy in determining the position of internal cracks
- Ability to estimate the size and shape of the cracks
- Need only a surface to study the material
- Possibility of electronic transaction that enables instant information and process control automation

2.3 SENSITIVITY AND RESOLUTION:

The two concepts more important used in ultrasound are sensitivity and resolution: sensitivity is the ability to locate small discontinuities and resolution is the ability of the system to locate discontinuities that are close together within the material or located near the part surface. Both terms increase when the frequency increases. [5] and [6].

2.4 DISCONTINUITY, DEFECT AND CRITICAL DISCONTINUITY:

DISCONTINUITY:

Is the lack of homogeneity or interruption in the normal physical structure of a material, also be a deficiency in the physical configuration of a piece, part or component.

DEFECT:

Is any discontinuity or indication of a discontinuity whose shape or shape has exceeded the acceptable limits established by the code, standard or applicable specification.

CRITICAL DISCONTINUITY:

Critical discontinuity is the largest discontinuity than can be accepted or the smallest that can be rejected.

The wavelength of the ultrasound is a very important role in the crack detection probability. A shorter wavelength causes an increase in the frequency, which improves the detection of smaller discontinuities because the discontinuity must be larger than one-half the wavelength to be able to detect. This concept is shown in the next image. As it can be seen in the figure, with a relatively large wavelength, it can straight past defects or obstacles. However, with a wavelength smaller size of the defect is intercepted by the wave.

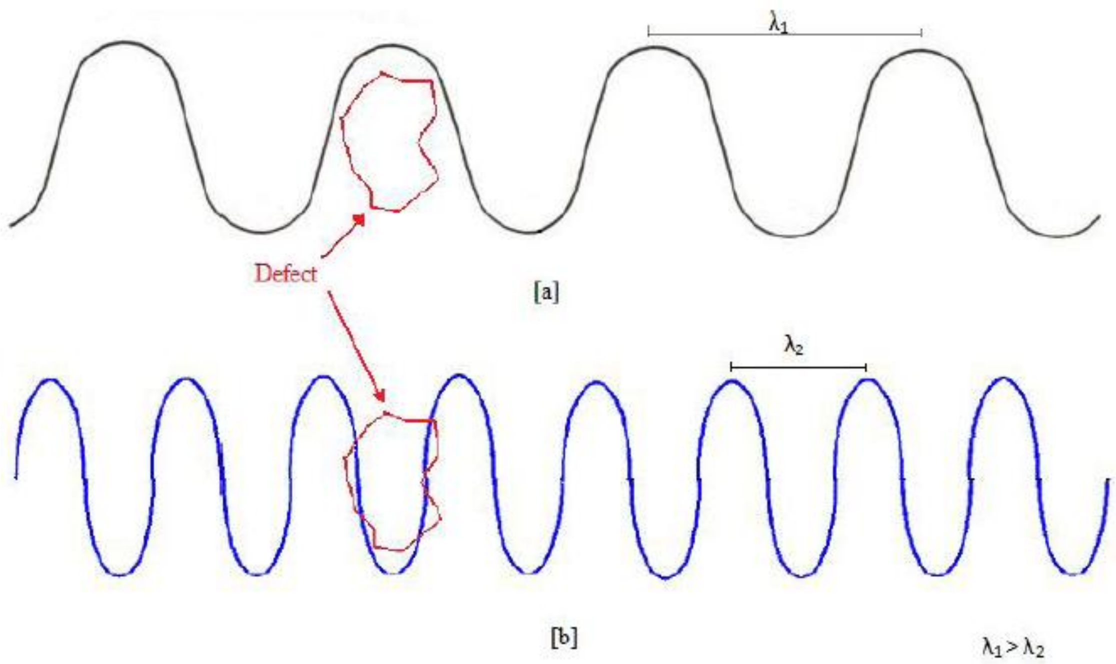

Image 2.2: Relation between wavelength and the detection of obstacles.[6]

2.5 WAVES:

For the writing of this section have been used a mixing of these references: [7], [8], [9], [10], [11] and [12].

When a wave travels through a medium, the constituent particles of this change their equilibrium or rest. After any type of wave through a medium, the particles return to their position of balance.

The particles are altered in direction parallel to the direction the wave propagates

CHARACTERISTICS OF THE WAVES:

- **Period, frequency and wavelength:**

The vibrations travel as a wave, similar to how the light does. However, unlike light waves, which can travel in the vacuum, ultrasound requires an elastic medium as a liquid or solid. The basic parameters of a continuous wave are wavelength (λ) and the period (T) of a complete cycle. These parameters can be seen in the figure below.

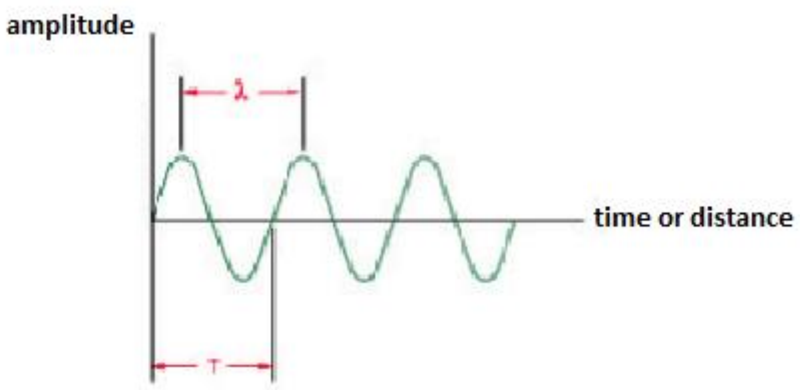

Image 2.3: Parameters of a wave.

The number of complete cycles in a second is the frequency (f) and it is measured in Hertz (Hz). The time required to complete a full cycle is the period (T), which is measured in seconds. The relationship between the frequency and length of a continuous wave is:

$$f = \frac{1}{T}$$

- **The velocity of ultrasounds and wavelength:**

The relationship between velocity and wavelength is the next:

$$\lambda = \frac{v}{f}$$

Where:

λ is the wavelength

v is the velocity of the wave

f is the frequency of the wave

TYPES OF WAVES:

The particles of a medium in which an ultrasonic wave propagates undergo diverse displacements, leading to different types of waves:

Each type of wave is characterized by its direction, velocity and transmitted energy (associated with the direction of oscillation of the particles with respect to direction of wave propagation). The different velocities of these waves are related and depend on the intrinsic parameters of the material, as its elastic modulus (E), its Poison coefficient, its shear dynamic modulus or Coulomb modulus (G) and its density.

- **Longitudinal waves**

Longitudinal waves are characterized by the movement of the particles, which are parallel to the direction of propagation of ultrasound, creating areas of compression and expansion separated half wavelength in the material which is spread. The sound is propagated in this way, is caused by elastic link between particles, where each of them to vibrate, push the adjacent particles, these in turn transmit the energy to neighboring particles and so on.

They can be transmitted through solids, liquids and gases; their speed of travel is the highest compared to the other modes.

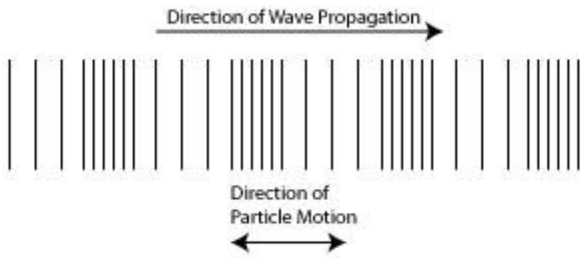

Image 2.4: longitudinal wave. [10]

- **Transverse waves**

The direction of oscillation of the particles is perpendicular to the direction of propagation, so that these waves travel through the material it is necessary that the particles exhibit sufficient attraction between them to enable it to spread the wave. Gases and liquids are practically unable to transmit shear waves, because their molecules are offer very little resistance to the transversal slide, and so there is not exist elastic ties that bind its zero position. This context it matches the property of having a stiffness modulus equal to zero, except for highly viscous liquids which have a “complex modulus of stiffness”, and therefore can spread though strongly damped transverse waves, in other words, the transverse waves can propagate only in solids.

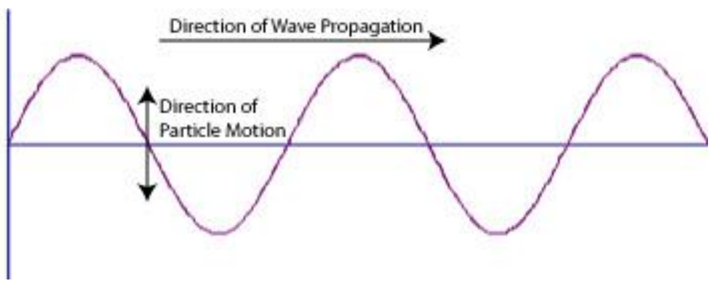

Image 2.5: Transverse wave. [10]

- **Rayleigh**

Rayleigh wave or surface waves are ultrasonic waves spread only on the surface of the material following the profile of the object. Provided there are no abrupt changes in it, the depth of penetration of a Rayleigh wave is equal to its wavelength.

The oscillations of the particles are elliptical, and so this wave has a less velocity than the previous two. The superficial character of this wave makes that the travel time of a certain distance depend on the irregularities of the surface.

Rayleigh wave can be transmitted only on a solid surface.

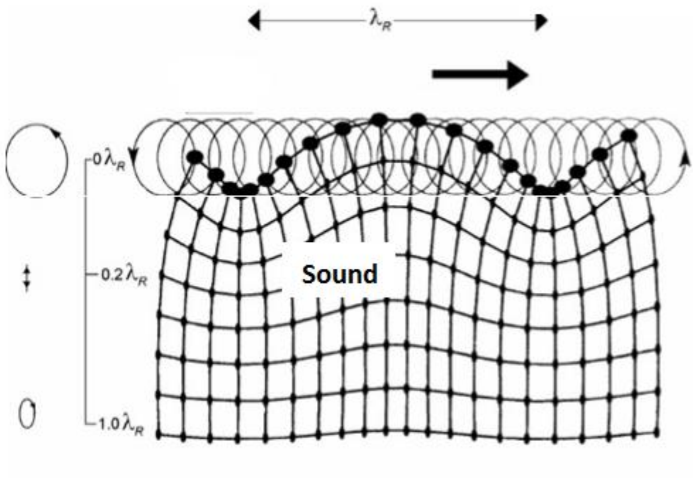

Image 2.6: Rayleigh wave [11]

- **Lamb**

Lamb waves or plate waves occur when the solid has a thickness much smaller than its width or length (so the model of a semi-infinite solid is no longer valid.) The solid can be assimilated to a plate in which the pure surface wave no exists, except if its wavelength is much smaller than the thickness of the plate.

When the thickness is the same order of magnitude as the wavelength, Lamb waves occur, which have components of the oscillation of particles perpendicular to the surface.

WAVES UTILIZED IN THIS PROJECT:

The waves used in this project are Lamb waves. The Lamb waves refer to elastic waves propagating in a solid plate with free boundaries. The displacements of the Lamb waves occur both in the direction of the wave propagation and perpendicularly to the plane of the plate.

Lamb waves in plates are produced in two basic shapes which in practice are mixed:

- Asymmetric waves or waves of flexion: the particles of the central or neutral axis oscillate by transverse way.

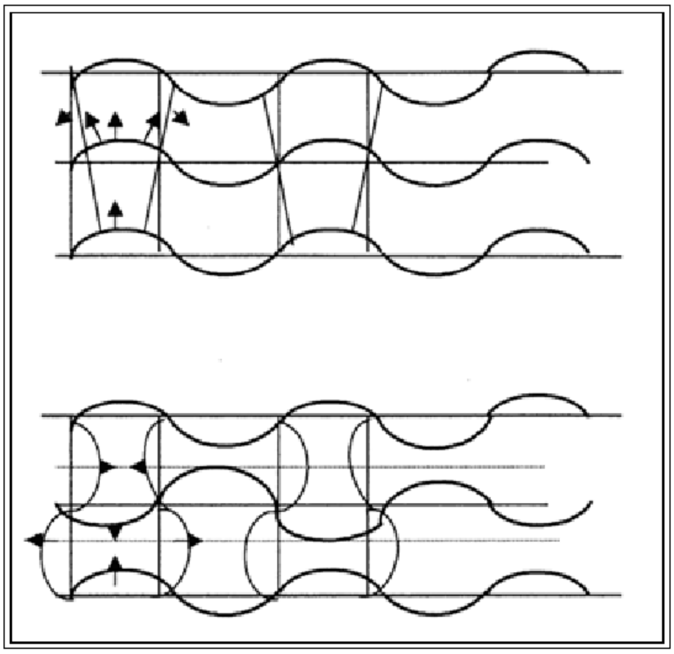

Image 2.7: Asymmetric wave. [12]

- Symmetric waves or waves of expansion. The particles of the central zone oscillate longitudinally.

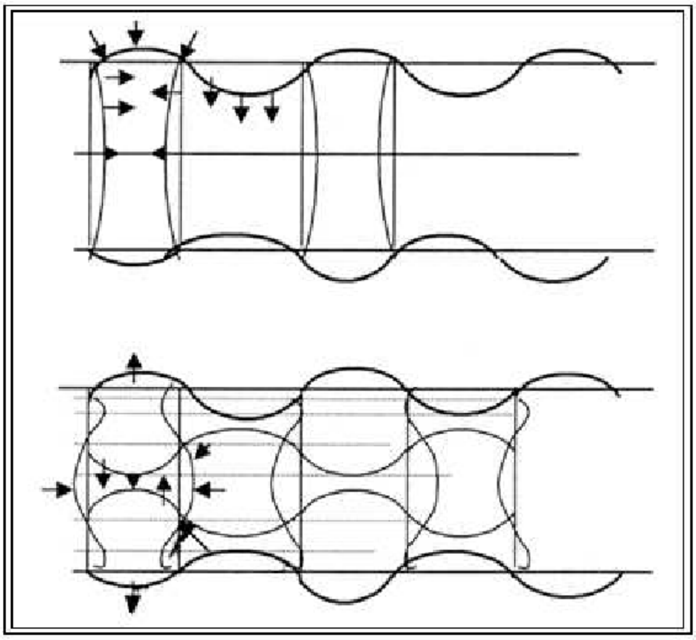

Image 2.8: Symmetric wave. [12]

2.6 TRANSDUCER:

For written this section, the references were necessary: [13], [14] and [15].

A transducer is a device capable of converting one type of input power in another different output. An ultrasonic transducer converts electrical energy into mechanical energy in the form of sound and vice versa.

TYPES OF TRANSDUCERS:

- **Piezoelectric transducers:**

Consist of a piezoelectric crystal that contracts with electrical impulses applied to its surface. It is important to note that does not use any type of magnetism, and it contain magnets neither.

It has a high performance, sensitivity, or efficiency, but as the surface of radiation is very small, they only are used for high frequency reproduction. They are also utilized in sonar or ultrasound scan where the frequencies used are above our audible range.

Piezoelectric transducers are cheap to manufacture and they support high powers, since it is very difficult to destroy the crystal itself.

- **Piezoelectric ceramics transducers**

The piezoceramic materials are more widely used materials for applications such as actuators or sensors. The crystal structure is face-centred cubic (isotropic) before polarization, and after polarization of these materials exhibit tetragonal symmetry Curie temperature, bellow this temperature the ferroelectric domains become an unguided random arrangement. Above this temperature, ceramic materials lose their piezoelectric properties.

- **Piezocomposite transducers (PZT)**

An alternative to piezoelectric ceramics are piezocomposites, materials made of two or more phases, one of which is piezoelectric. A piezocomposite is a multiphase material consisting of a piezoelectric ceramic and piezoelectric passive polymer so that it reduces the specific acoustic impedance of the new material and that produces the piezoelectric effect. Physical and mechanical properties of the composite depends on the proportion and characteristics of the phases and how they are interconnected, so it is relatively easy to vary the properties of the composite depending on your specific application.

TRANSDUCER UTILIZED IN THIS PROJECT:

The transducers utilized in this project are PZT. Transducers are supplied by the manufacturer Noliac [16].

Noliac CMAP's are typically used when it is required a low displacement at high force. Noliac CMAP's are coffined monolithic piezoelectric low voltage actuators that convert an electrical input to a very high-resolution linear movement at high force. Noliac CMAP's are manufactured with ceramic layer thickness down to 20μm as standard in order to allow very low operating voltage for full performance of the piezo material. The concrete name of transducer utilized is CMAP 12.

The product facts are in the Appendix A.

2.7 METHODS:

PITCH-CATCH METHOD

The measurement done by pitch-catch through integrated transducers are based on changes detection that happen in the Lamb waves as they propagate through a damaged region of the structure. This method utilized pairs of transducer one acting as transmitter and the other as receiver. These transducers are permanently stuck on the surface.

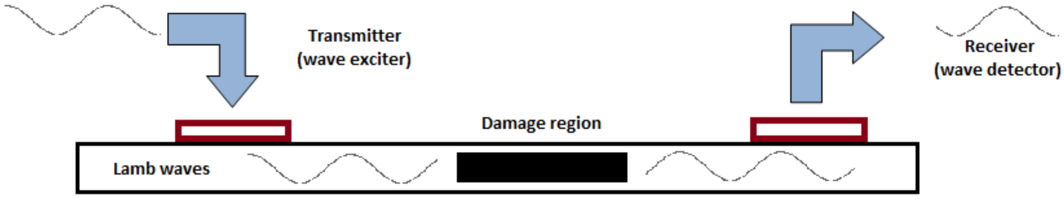

Image 2.9: Pitch-catch method

PULSE-ECHO METHOD

The pulse-echo measurements through integrated transducers follow the general principles of the pitch-catch inspection by Lamb waves, but in this case, a piezoelectric transducer coupled to the structure acts as the same time like a transmitter and receiver of the guided Lamb waves.

The sent wave by the transducer is partially reflected in the crack and the echo is captured by the same transducer, which acts like a receiver. To guarantee the correct detection of the defect is important to use a little dispersive Lamb mode.

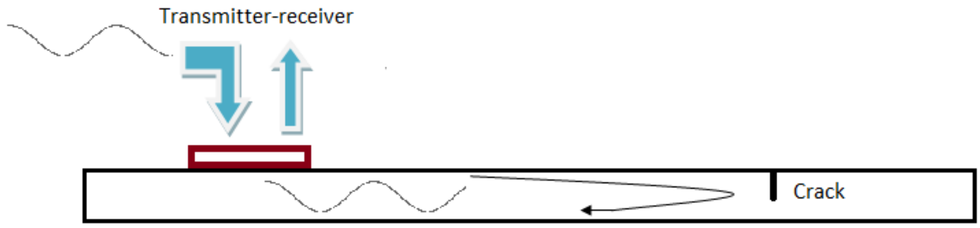

Image 2.10: Pulse-echo method

2.8 THE PRESENTATION OF DATA:

The ultrasound data can be collected and displayed in different formats. The three most common are known as A-scan, B-scan and C-scan. The reference of this section is [4].

Each mode of presentation provides a different way to view and evaluate the region of material to be inspected. Modern computer systems of ultrasonic scan can show the date in each of these three forms of presentation at the same time.

A-SCAN PRESENTATION:

The A-scan display shows the quantity of ultrasonic energy received as function of time. This energy received is plotted along the vertical axis and the horizontal axis represents the time of flight.

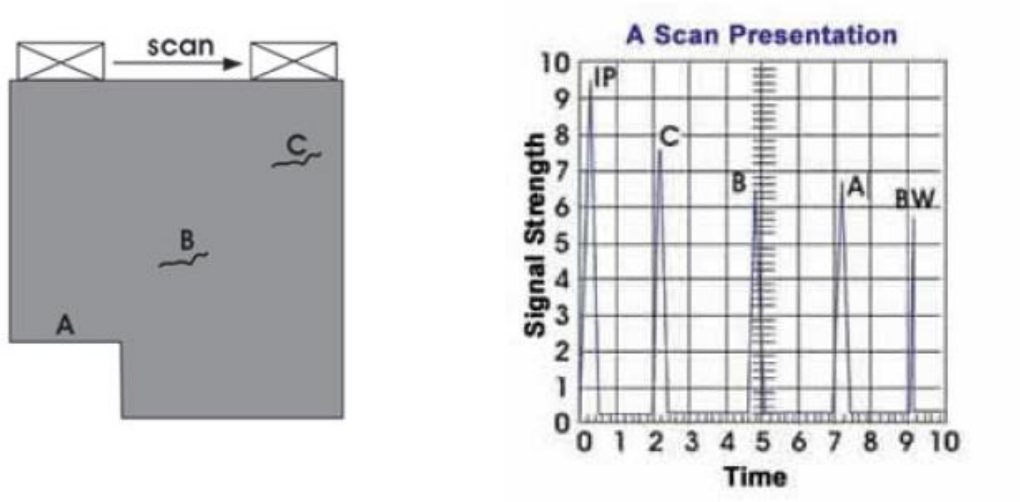

Image 2.11: A-scan presentation.[4].

B-SCAN PRESENTATION:

B-scan representations are a cross-sectional view of the sample. In the B-scan, the travel time of sound energy is displayed along the vertical axis and the linear position of the transducer is shown along the horizontal axis.

With the B-scan, the depth of reflection and its approximate linear dimensions in the scan direction can be determinate.

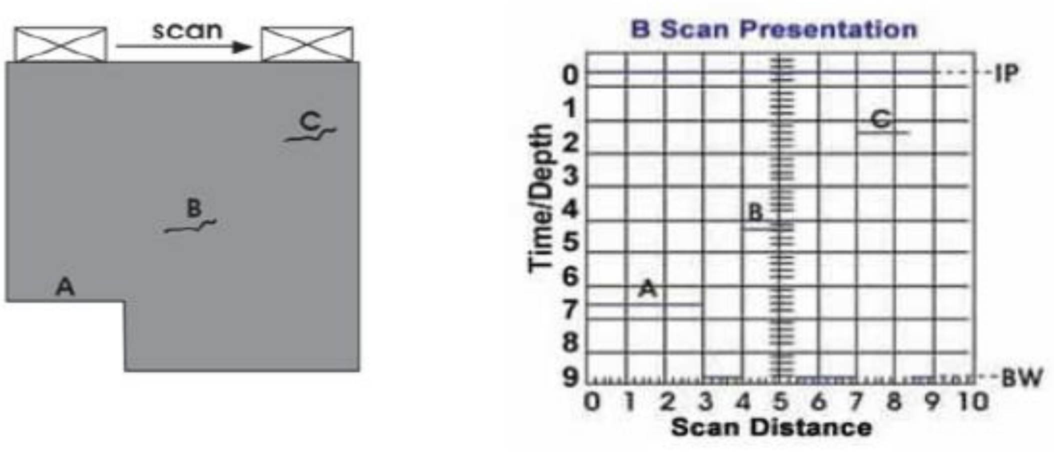

Image 2.12: B-scan representation.[4]

C-SCAN PRESENTATION:

C-scan presentation provides a view of the location type and size defect of the sample to inspect. The image plane is parallel to the scanning direction of the transducer.

C-scan presentation provides an image of features that reflect and disperse the sound inside and the surfaces of the sample.

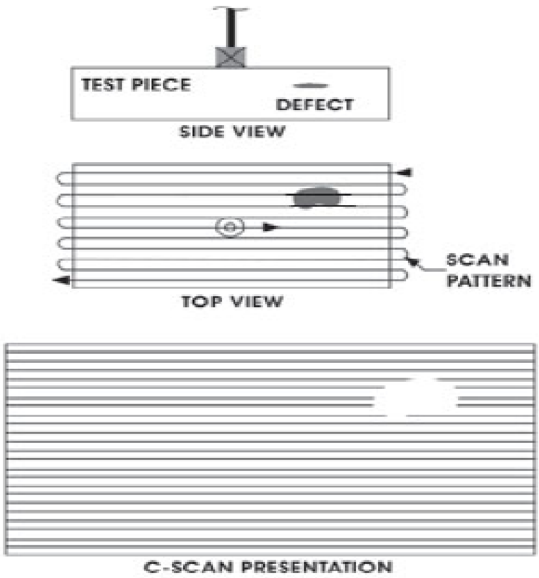

Image 2.13: C-scan representation.[4]

3. SOFTWARE

The program utilized was Matlab. The name MATLAB is an acronym of "MATrix LABoratoy". It is a computing environment and development of fully integrated applications directed to carry out projects where there are involved in higher mathematical calculations and graphic display of the same. MATLAB integrates numerical analysis, matrix calculation, signal processing and graphical display in a complete environment where problems and solutions are expressed in the same way that they traditionally would be written, without making use of traditional programming.

Image 3.1: Program used (Matlab).[17].

This program extracts all the information and displays the resulting waveform, allowing us to carry out the necessary studies. Below it is showing an image of this program.

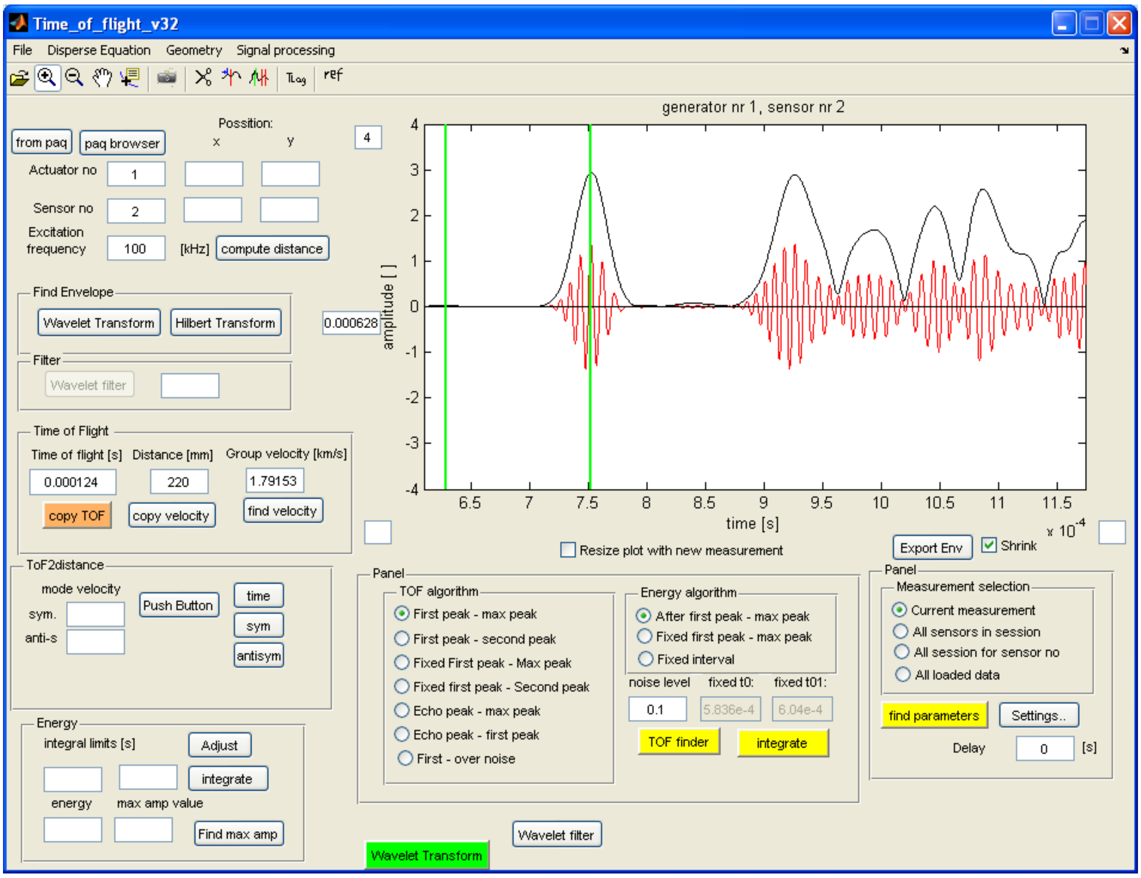

Image 3.2: Screen of the program.

4. EXPERIMENTS

In the experiments had been used epoxy resin to hold the transducers in fixed position.

4.1 THE FIRST EXPERIMENT

The first experiment took place in a plate of Aluminium with 1 mm of thickness.

OBJECTIVE:

The first experiment consist of calculate the velocities for the different excitation frequencies (from 100 kHz to 500 kHz, step 50kHz) and obtain the dispersion curves for Lamb modes in an isotropic plate through pitch-catch setup.

THEORETICAL INTRODUCTION

For this theoretical introduction was necessary the next references: [7], [18], [4], [14], [19] and [20].

It is necessary defining some important properties of Lamb waves for acoustic image conformation. The phase velocity is the fundamental feature that it needs to know. The group velocity can be obtained from the phase with the relationship:

$$c_g = c - \lambda * \frac{\partial c}{\partial \lambda}$$

Where:

c_g is the group velocity

λ is the wavelength of the Lamb wave

c is the phase velocity

The Lamb wave transmitted through the material of the particles moving in two different ways. If the movement of the section of the plate is perpendicular to the symmetrical about the same, are called symmetric Lamb waves (S0, S1, S2…). If the movement is antisymmetric, the waves are called antisymmetric Lamb waves (A0, A1, A2…). The symmetric and antisymmetric modes of Lamb have different phase and group velocities as well as the distribution of particle displacement and strains along the thickness of the plate.

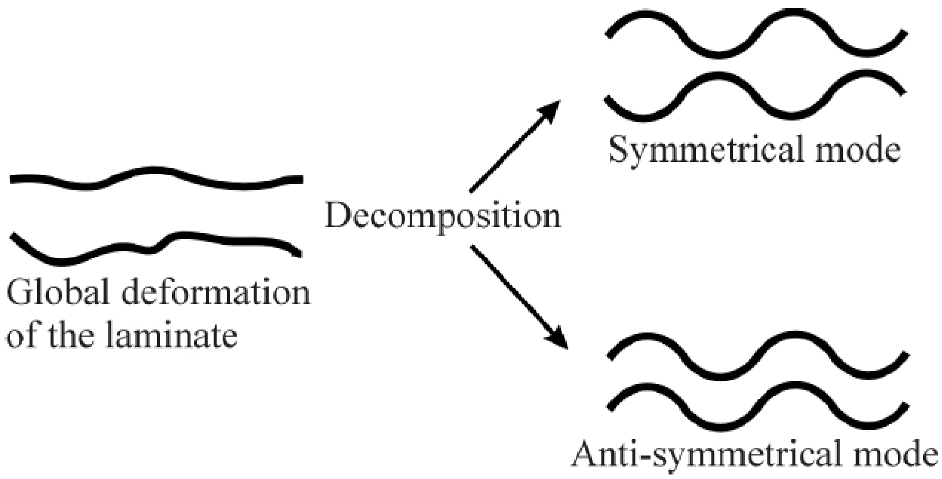

Image 4.1: Descomposition in symmetric and antisymmetric modes of a Lamb wave. [14]

It is known the relation between wavelength and velocity($\lambda = \frac{v}{f}$), but in the case of Lamb waves exists a small difference, because this type of waves is dispersive which means that wavelength and velocity is a function of frequency. That is the reason why the equation changes a bit, becoming in:

$$\lambda(frequency) = \frac{v(frequency)}{f}$$

Lamb waves are dispersive, also means that their phase velocities and group change with frequency. Dispersion curves of phase and group can represent in function of frequency due to multiple modes can be exist for a given frequency. It is obtained a family of dispersion curves.

These dispersion curves for Lamb modes in an isotropic plate structure are developed in the appendix B.

DATA COLLECTION

The two transducers in pitch-catch setup are separate 220mm. the actuator is excited with the different frequencies.

Parameters used in the program are the following:

- ❖ Number of periods in package ➔ 3
- ❖ Output gain ➔ 1/2
- ❖ Input gain ➔ 1/80
- ❖ Window modulation ➔ hanning

With these parameters is extracted the Time Of Flight (TOF) information through two possible ways: Hilbert transform or Wavelet transform. The data shown in the graphics are represented by A-scan. The TOF is the measurement between the two green lines, the first green line is the time zero second and the second line is the catch of the signal. Once obtained the TOF, it introduces the distance between the two transducers and the same program shows the results of velocity thanks to the formula described above.

The experiment starts with 100 kHz of frequency:

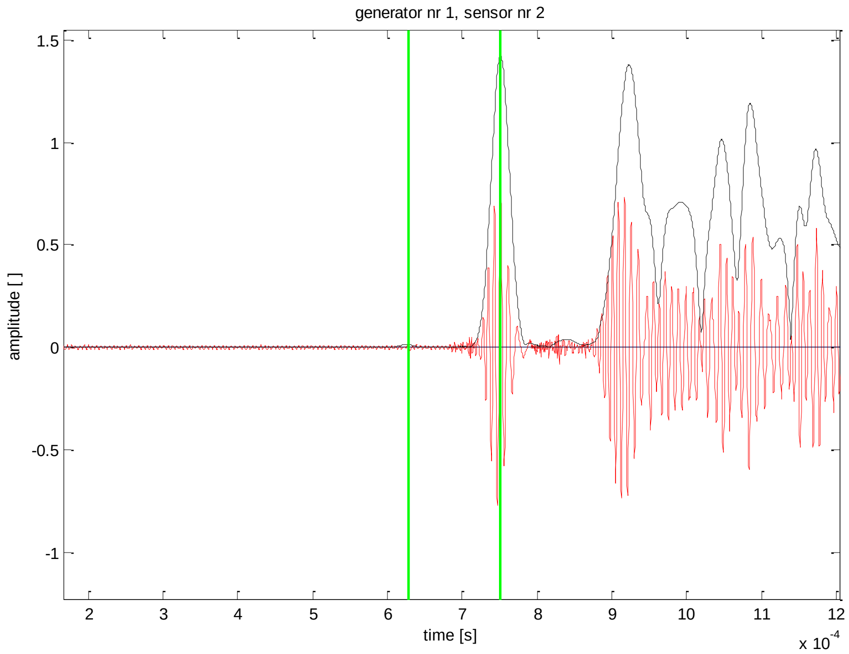

Image 4.2: Graphic for 100kHz

The first wave shown in the graphic is the signal received by the catch. Not can be confused with the signals reflected from the edge of the plate because the two transducers are placed in the center of the plate, far away from the edge. This is common for the rest of measurements in whole first experiment.

The Time Of Flight obtained in this case is 0.0001228 s and the velocity is 1.79153 km/s.

Increasing the frequency in 50 kHz ➔ **150 kHz:**

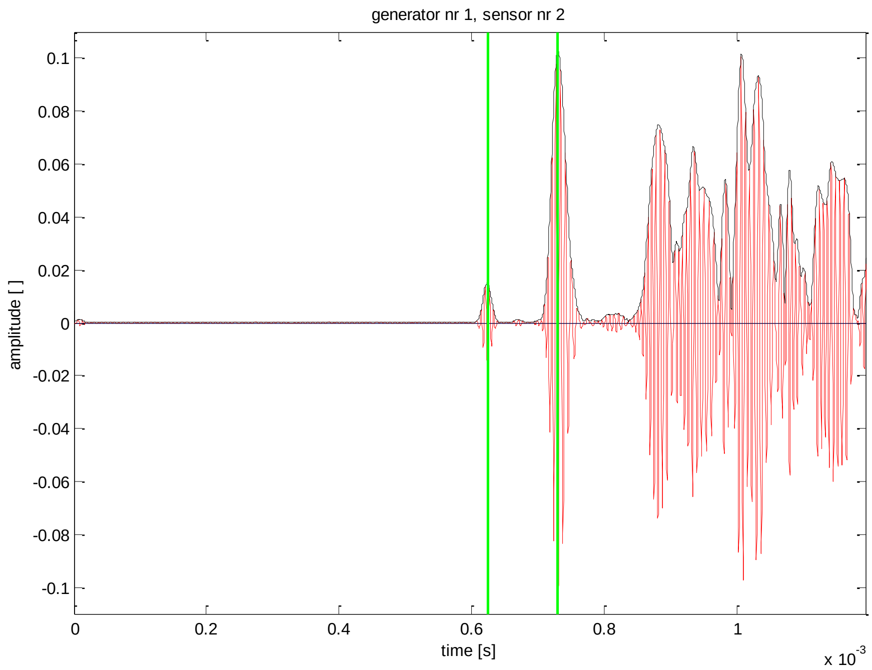

Image 4.3: Graphic por 150kHz

In this frequency yielded a Time Of Flight of 0.0001064 s and a velocity of 2.06767 km/s.

For a frequency of **200 kHz:**

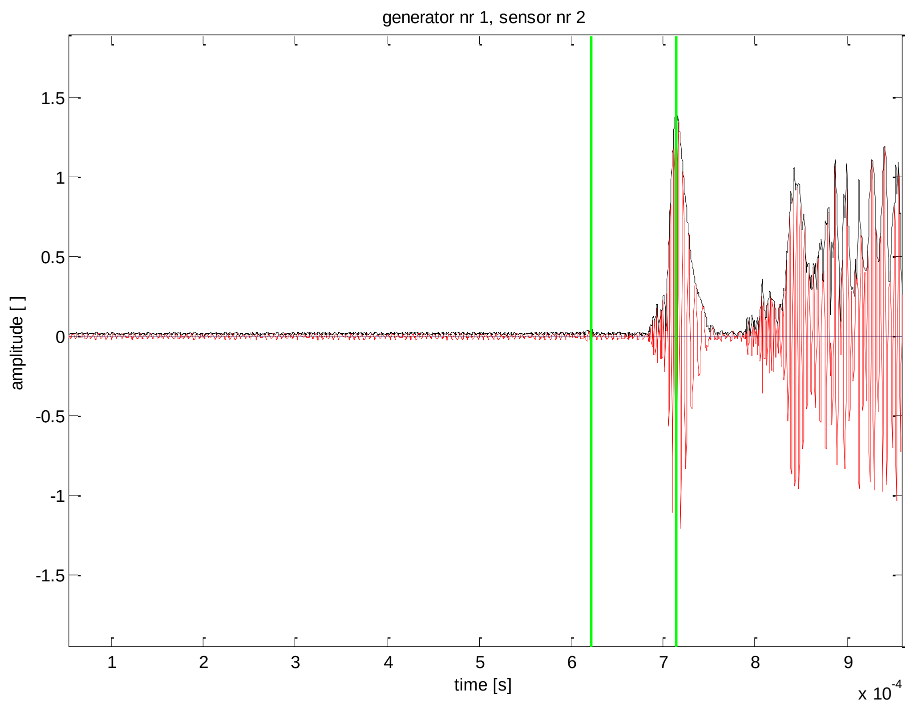

Image 4.4: Graphic for 200kHz

The results obtained in this case are that the Time Of Flight is 9.56e-5 s and the velocity is 2.3126 km/s.

For a frequency of **250 kHz** :

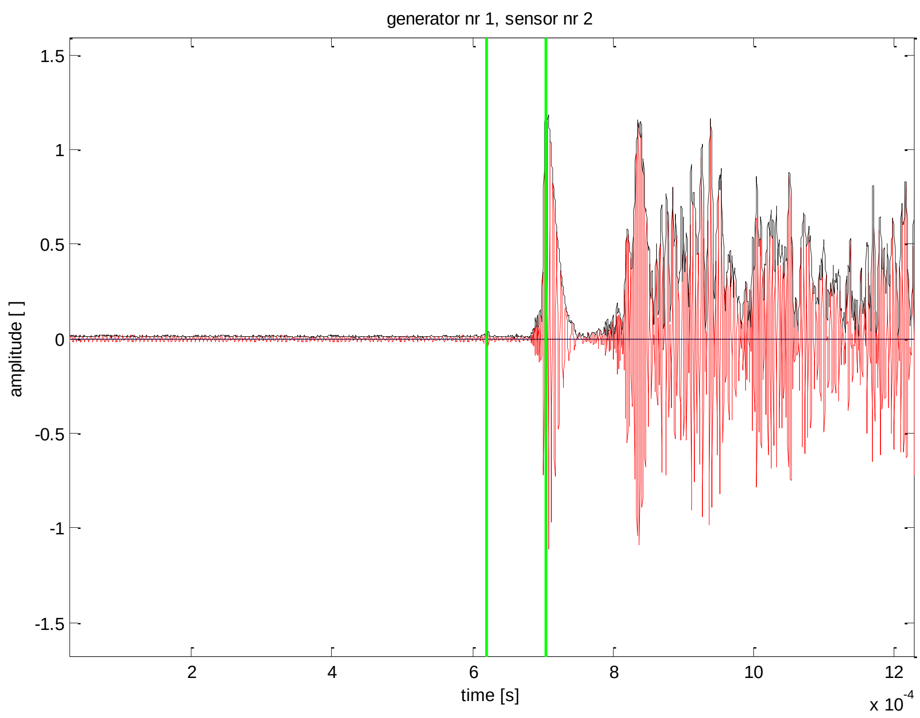

Image 4.5. Graphic for 250kHz

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The Time of Flight is 8.96e-5 and the velocity is 2.45536 km/s.

For a frequency of **300 kHz:**

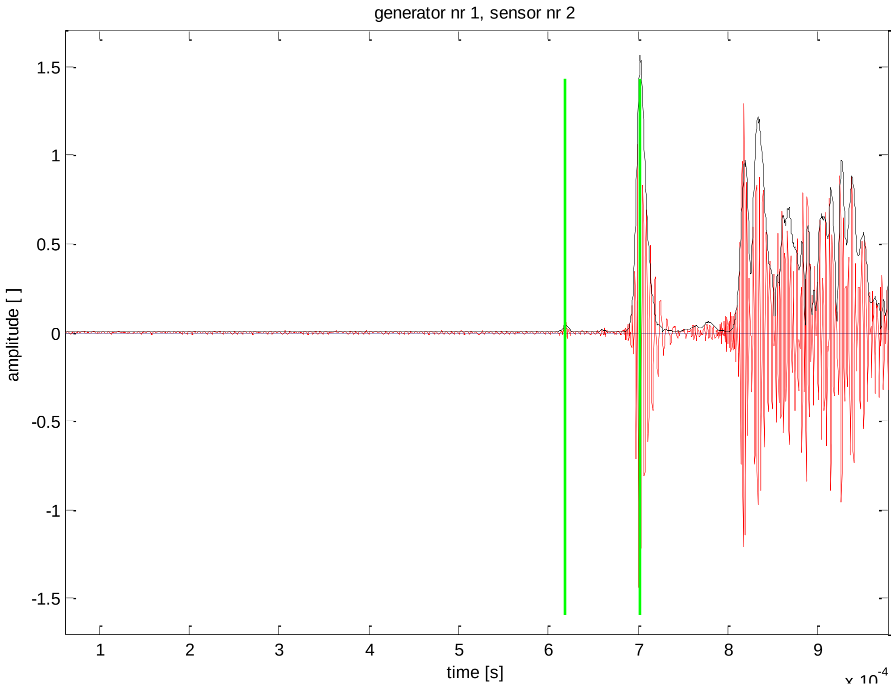


Image 4.6: Graphic for 300kHz

The results obtained for 300 kHz of frequency are the following:

- The Time Of Flight is 8.36e-5 s
- The velocity is 2.63158 km/s.

For a frequency of **350 kHz.**

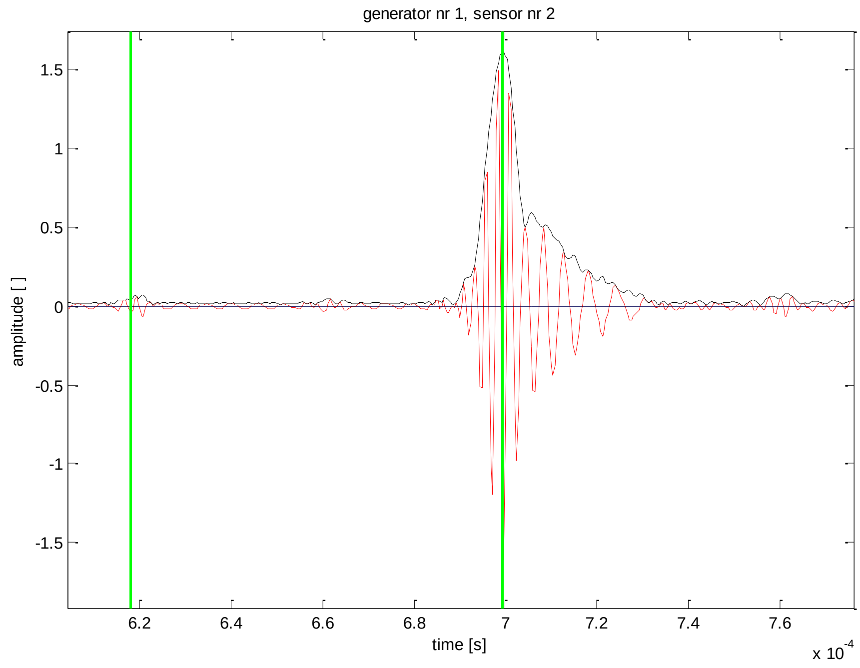


Image 4.7: Graphic for 350kHz

It is obtained a Time Of Flight of 8.2e-5s and a velocity of 2.68293 km/s.

For a frequency of **400 kHz** it is obtained the next graphic:

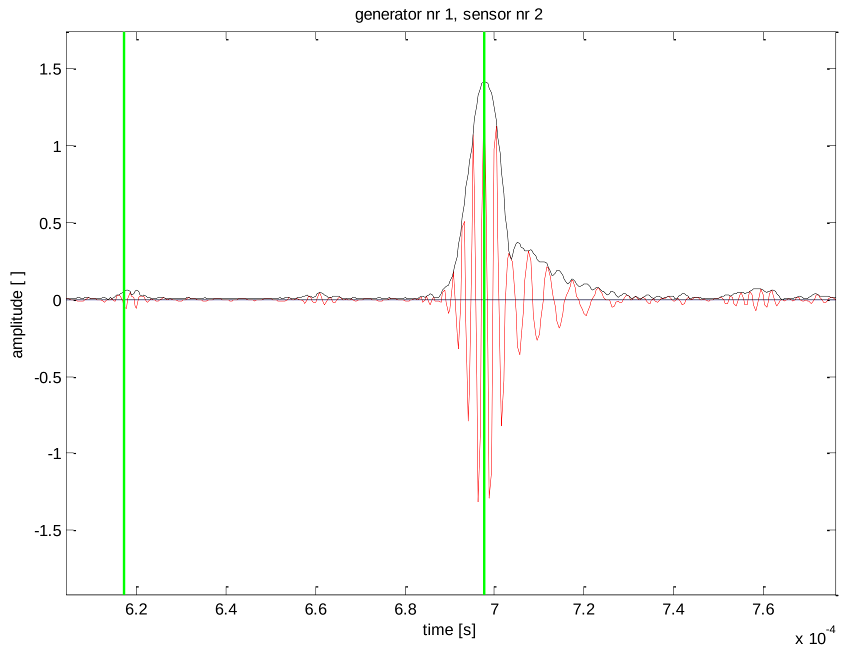


Image 4.8: Graphic for 400kHz

And the numerical results are:

Time Of Flight = 8.8e-5s

Velocity = 2.68293 km/s

For a frequency of **450 kHz:**

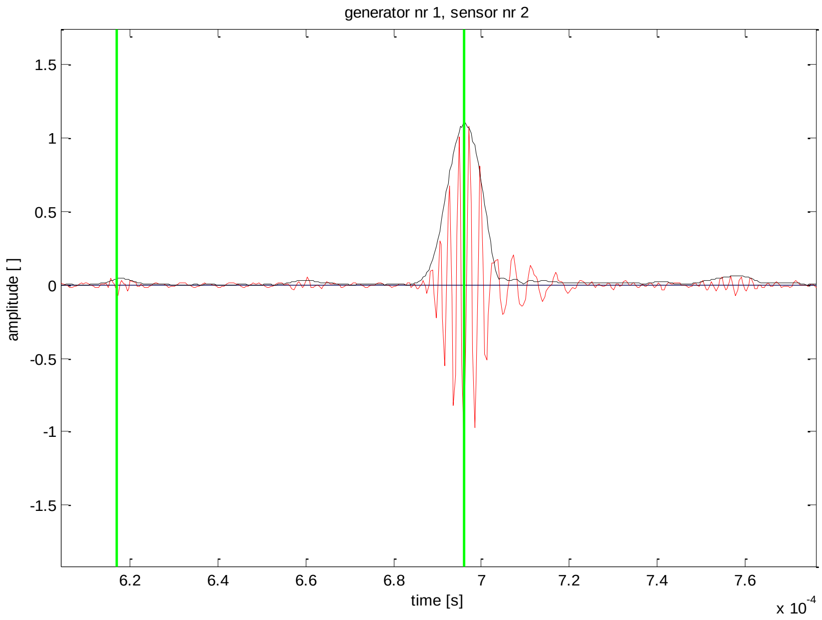


Image 4.9: Graphic for 450kHz

The Time Of Flight obtained in this case is 7.92e-5s and the velocity is 2.7778 km/s.

Finally, it obtains the result for a frequency of **500 kHz**:

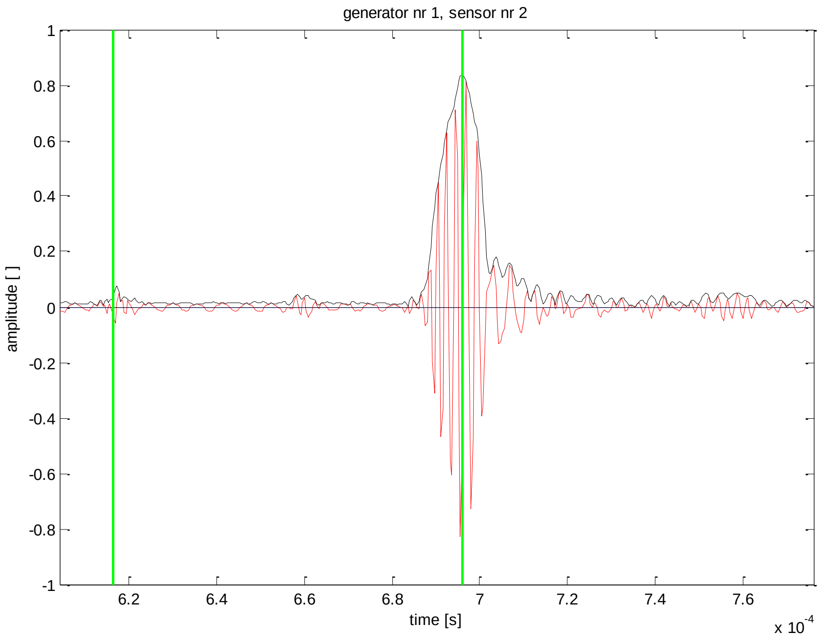


Image 4.10:Graphic for 500kHz

In this last case the Time Of Flight is 7.88e-5 s and the velocity is 2.79188 km/s.

RESULTS:

Collecting all data, create a table in excel with the results and show in a plot the relation velocity versus excitation frequency.

Excitation frequency (kHz)	Distance (mm)	TOF(s)	Velocity (km/s)
100	220	0.0001228	1.79153
150	220	0.0001064	2.06767
200	220	9.56e-005	2.30126
250	220	8.96e-005	2.45536
300	220	8.36e-005	2.63158
350	220	8.2e-005	2.68293
400	220	8.08e-005	2.72277
450	220	7.92e-005	2.77778
500	220	7.88e-005	2.79188

Chart 4-1: Results for the first experiment

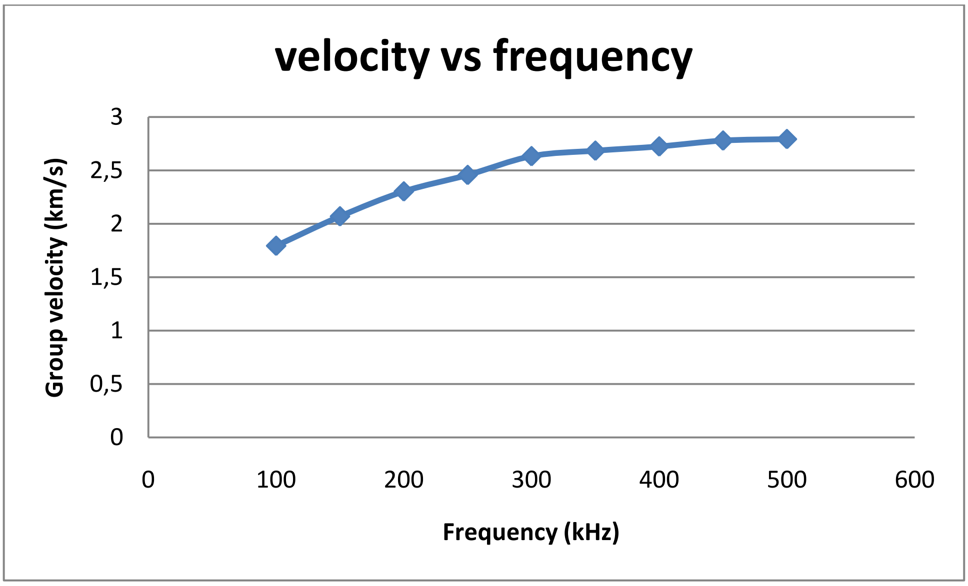


Image 4.11: Dispersion curves obtained (GROUP velocity)

Matlab program shows this graphical used to obtain the TOF.

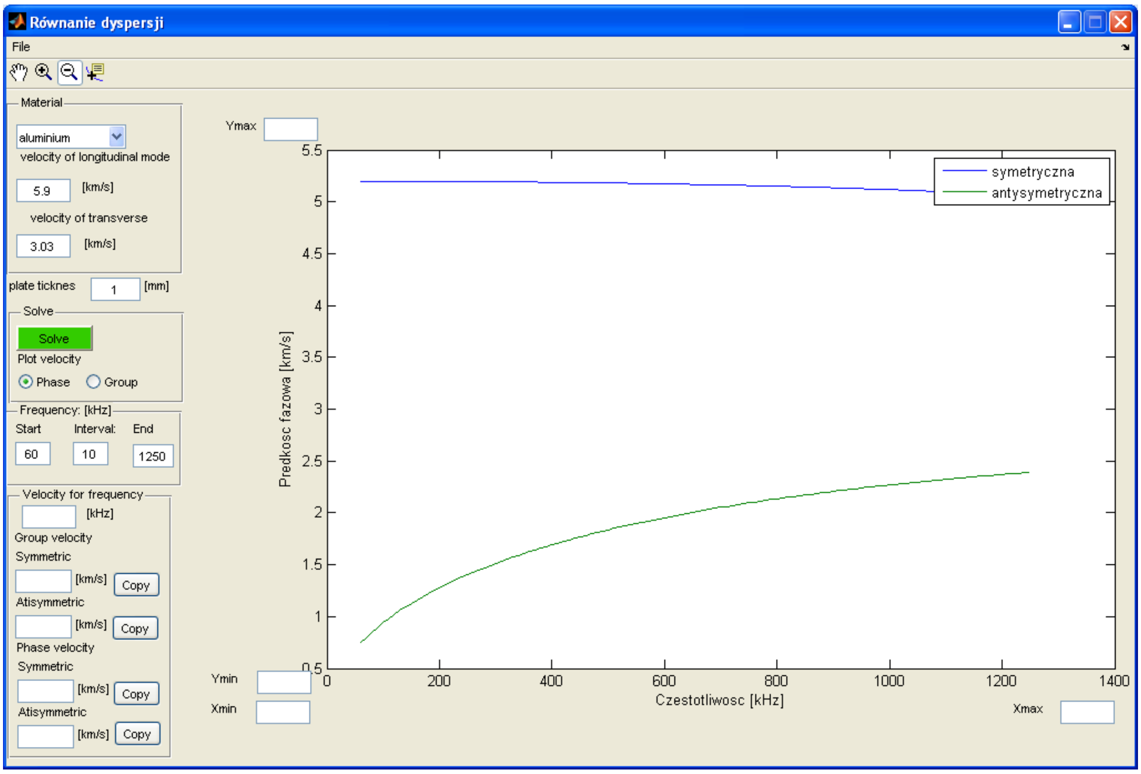


Image 4.12: Dispersion curves obtained (PHASE velocity)

CONCLUSION

In the graph of dispersion curves it is obtained by excel for the group velocity (Figure 4.11), it can be seen clearly how the dispersion curve drawn corresponds to the antisymmetric mode (A_0).

In the graph of dispersion curves it is obtained by Matlab for the phase velocity (Figure 4.12), it is possible to see how the dispersion curve drawn corresponds to the antisymmetric mode (A_0) for low phase of velocity and the symmetric mode (S_0) for high phase or velocity.

S_0 mode exhibits reasonable sensitivity to defects anywhere in the thickness, while A_0 is more sensitive to surface cracks or corrosion, but the latter may not be suitable for long distance propagation because of its high attenuation ratio.

Also, it can be observed as due to the existence of noise appears small amplitude waves, which at low frequencies are practically invisible, whereas as we increase the frequency becomes more significant. These waves due to noise can be seen in the figure below:

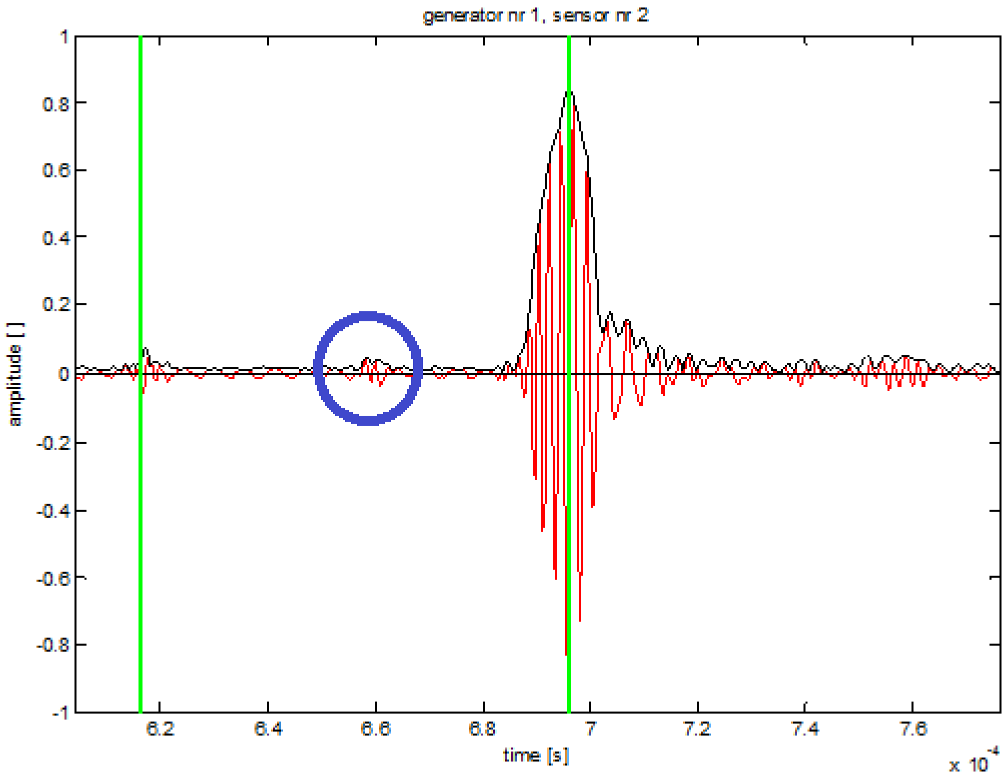


Image 4.13: Existence of noise

4.2 THE SECOND EXPERIMENT

OBJECTIVE:

The second experiment consists of detection of failures in a plate of Aluminum of 1mm of thickness through pulse-echo setup.

Bellow shows a sketch of the experiment:

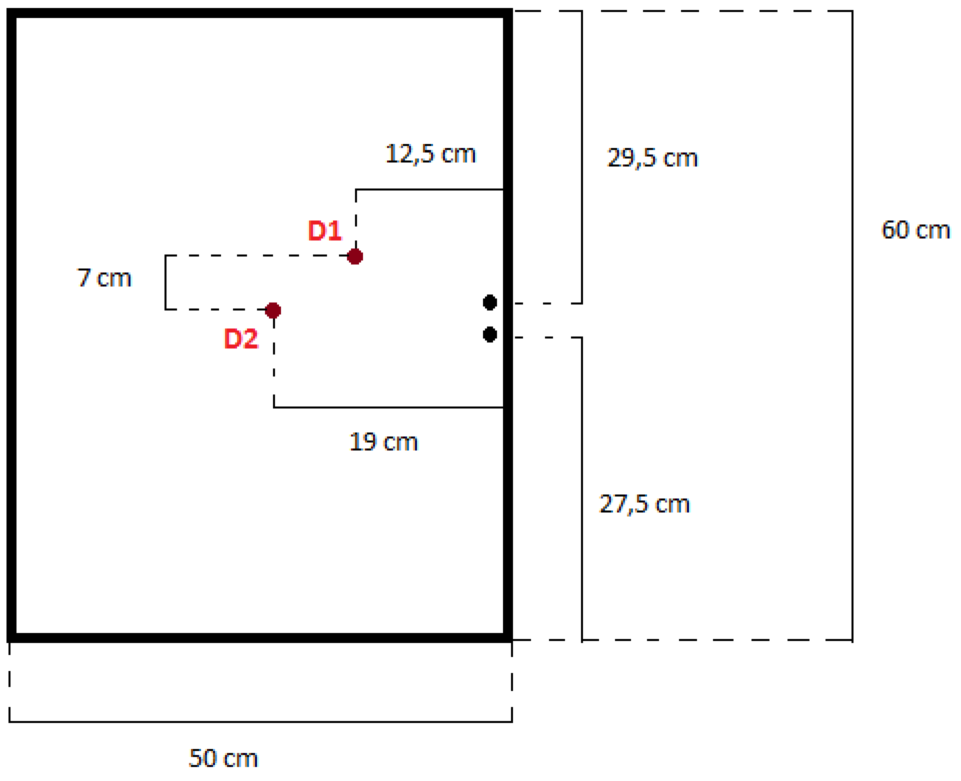


Image 4.14:Sketch of the second experiment

The red points are the defects with sufficient size to be detected, because as previously mentioned, the size of the defect must be at least half the wavelength. Because it was placed another smaller defect before, but as the size was not sufficient to be detected, results were not obtained.

THEORETICAL INTRODUCTION

The sound emanated from an ultrasound transducer is not from a single point, it is originated at many points along the surface of the piezoelectric element. This implies a sound field with many waves interacting or interfering with each other

An important point to highlight is that when two waves interact, they superimpose and the amplitude of the sound pressure or particle displacement at any point of interaction is the sum of the amplitudes of the two individual waves. Is different if the waves are in phase or if they are out of phase. If the waves are in phase (the valleys and peaks of one are exactly aligned with those of the other) they combine to double the displacement of either wave acting alone. When they are out of phase they combine to cancel each other out. When the two waves are not completely in phase or out of phase, the resulting wave is the sum of the wave amplitudes for all points along the wave. [4]

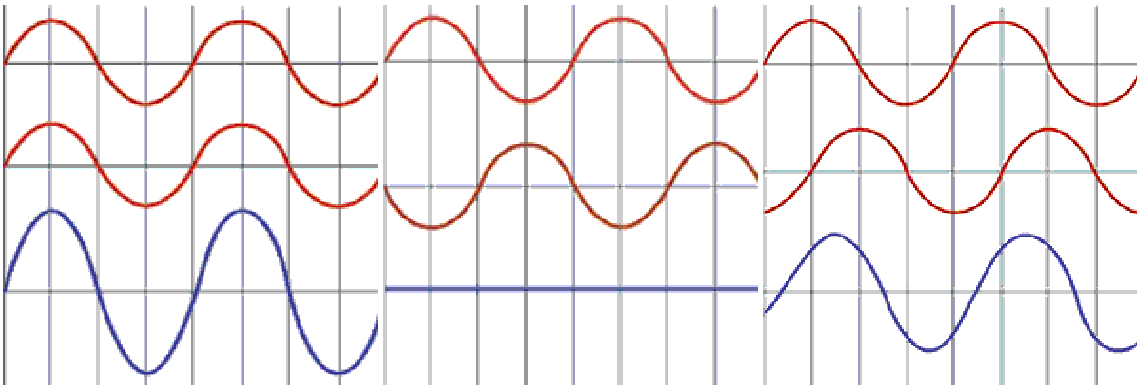


Image 4.15: The result of two interacting waves. [4]

DATA COLLECTION

Knowing the velocities for different frequencies obtained in the first experiment, the results from the graphs were interpreted by the following way: inserting the known velocity for a certain frequency, the program gives the distances that are the potential failures.

Parameters used in the program are the following:

- Number of periods in package ➔ 3
- Output gain ➔ 1/8
- Input gain ➔ 1/80
- Window modulation ➔ hanning

Frequency of 100 kHz was chosen but only was detect the reflection about the boundaries but nothing else. Increasing the frequency, the wavelength decreases, which smaller defects will be detect, since most detectable crack size have to be half the wavelength. For all this the frequency was increased, choosing 300 kHz. Introducing in the program the corresponding velocity to this frequency, which is 2.63158km/s, it was obtained the next image:

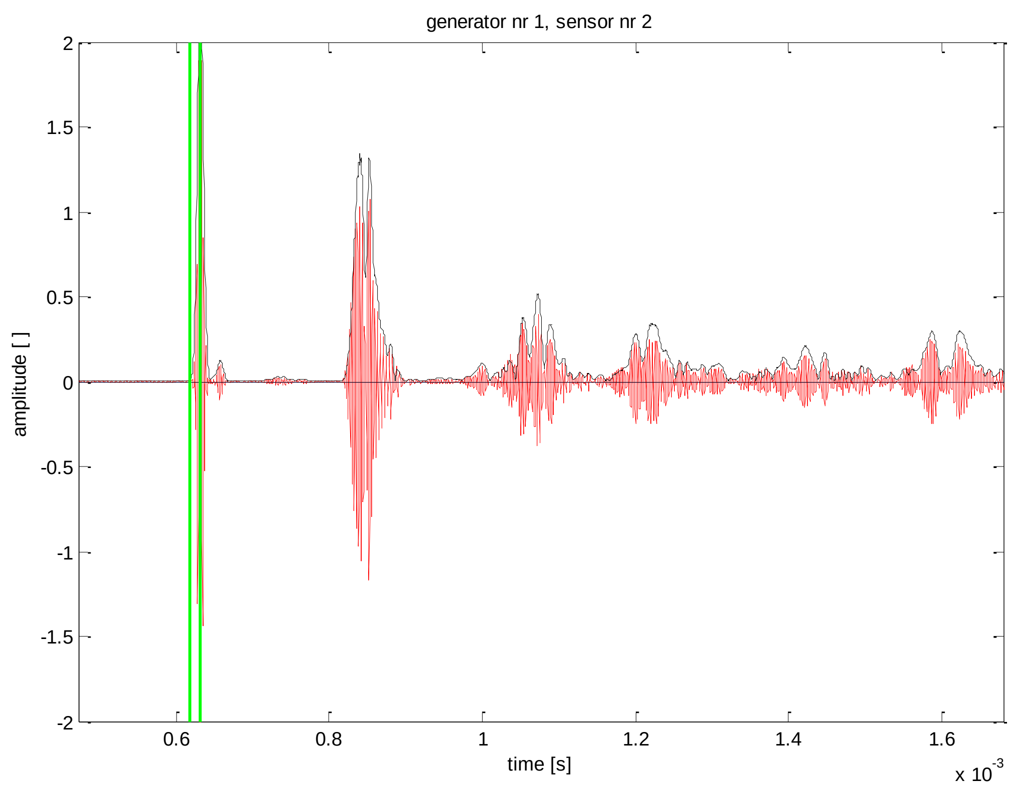


Image 4.16:Graphic for the second experiment

RESULTS

The two fist green lines are very close, which means that the two transducers are placed very near. The first wave is due to the first transducer emits the signal and, on the way the second finds the other transducer and reflected the wave.

Behind this first wave, two large amplitude waves can be seen. The meaning of these two waves is as follows:

- For the first wave, the program shows that the possible failure is located at a distance of 560.736mm, this distance is what it takes to receive the second transducer signal first, so 560.736mm is the total distance (return path); if we divide this distance by two, we will get the real length which is the possible defect. A distance of 280.368mm is obtained by the division, which means it is a reflection of the end of the plate, because, as it is possible to see in the sketch of the plate (figure 4.14) one end is located at 27.5cm.
- The program shows that the second wave is at a distance of 582.808mm (return path), which divided by two is a distance of 291.404mm. This distance is approximately the distance shown by the sketch (29.5cm). Then this distance is the reflection of the second end of the plate.

Between the first two waves of great amplitude it finds something hardly visible. Zooming the area exists between the echo of another transducer and the first reflection from the end of the plate, it was found the next:

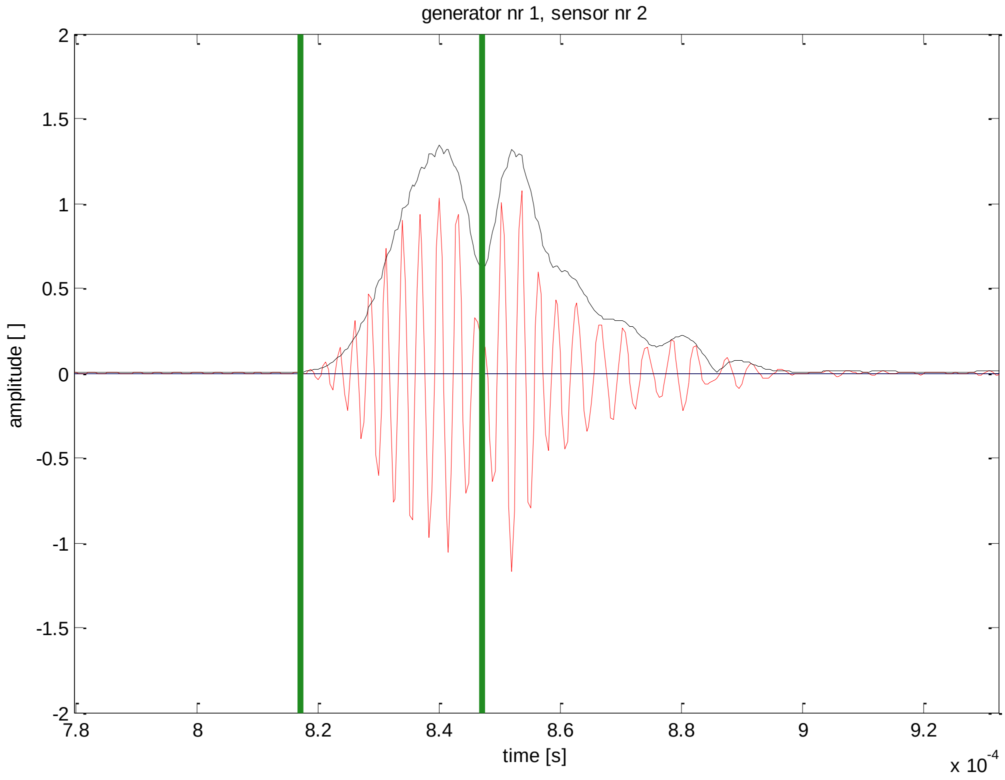
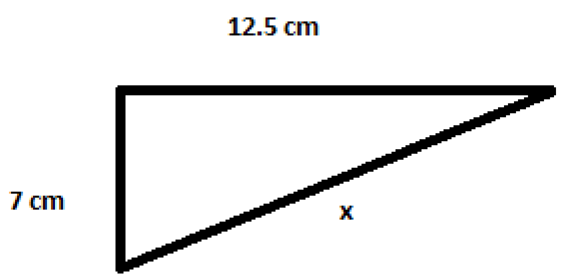


Image 4.17:Damage number 1

The shape of this wave is the result of superposition of the two waves. In accord to previously discussed, in the section called Theoretical introduction.

Moving the two green lines to exactly know where the first maximum amplitude, it is obtained that the maximum is located at a distance of 300.872mm. Dividing this number by two it gets the position of the first defect. So the first defect found is placed at a distance of 150.436mm. Then the distance was calculated according to the image:



$$x = \sqrt{12.5^2 + 7^2} = 14.33cm$$

The distance of the second defect is shown below:

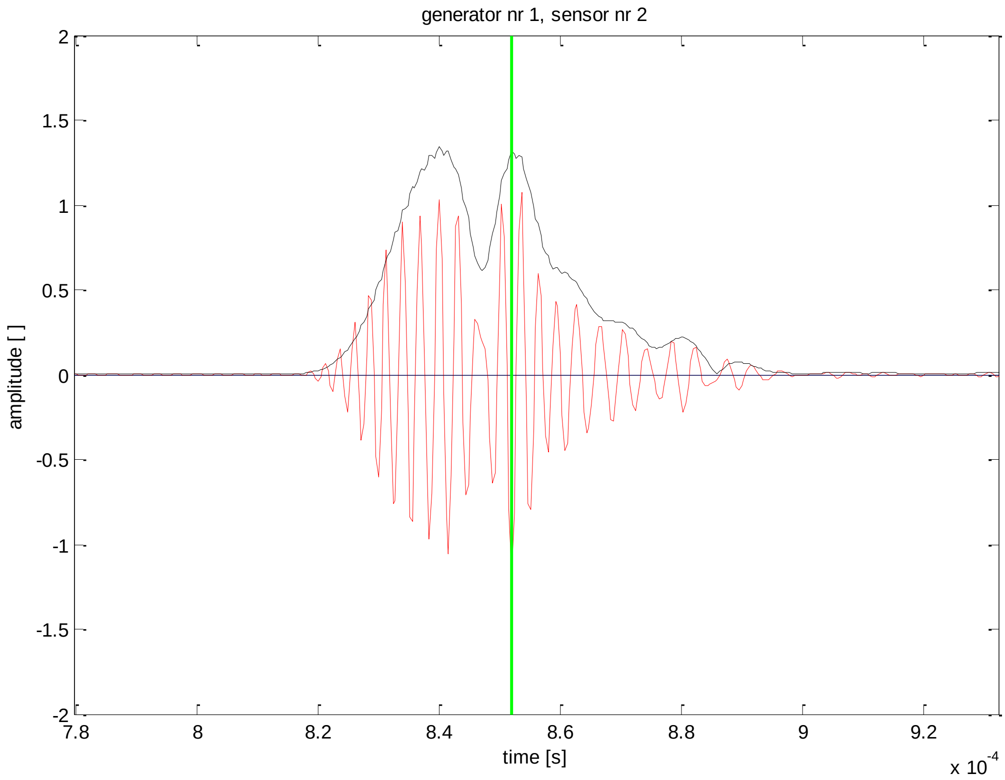


Image 4.18:Damage number 2

Moving the green line as the same way, it was found that a distance of 390.15 is placed the second maximum amplitude of the wave. Dividing by two; the distance of the second defect is 195.075mm.

Observing this distance in the image of the plate, it can be seen that the second defect is placed at 19 cm from the first transducer.

The results obtained with this experiment are summed up following chart:

Points	Frequency (kHz)	Velocity (Km/s)	Distance obtained by the program (mm)	Experimental distance (mm)	Real distance(mm)	Edge or damage
1	300	2.63158	560.736	280.368	275	Edge
2	300	2.63158	582.808	291.404	295	Edge
3	300	2.63158	300.872	150.436	143.3	D1
4	300	2.63158	390.15	195.075	190	D2

Chart 4-2:Results for the second experiment

CONCLUSION

Ultrasound through Lamb waves is a good way to detect defects. A relatively low frequency is preferable where only two fundamental modes, namely the zeroth antisymmetrical mode (A_0) and the zeroth symmetrical mode (S_0), are available. It has also been found that the dominant components of the S_0 mode vibrate mainly in-plane, whereas they are out-of-plane for the A_0 mode, which means that the A_0 mode has a significant vertical shear and lateral displacement. The A_0 mode is more sensitive to small damage because of its shorter wavelength than that of the S_0 mode at the same frequency, but it shows more severe dispersion at low frequencies.

The different between the results of the experimental and the distances from the image of the plate are first of all due to calculating distances in the image of the plate, it was measured the total distance from the first transducer (transmitter); to be exact it should only be taken into account the one way road from the first transducer, and the return way from the second transducer (receiver).

Another possible reason for these differences is that the distances measured on the plate were made very clumsy with a ruler. Probably if the measurements had been made more accurate, the results also would be more precise.

4.3 THE THIRD EXPERIMENT

OBJECTIVE:

The main objective of this experiment consist of detect the crack situation through a new method does not used in the experiment before.

To start the excited wave is produced by the first transducer, which acts as a transmitter, while the other three transducers act like receptors, capturing the signal. It creates three different ellipses, to each of these three receivers. It aims to find the intersection point of these three ellipses, because this is the point where the defect is located. The second part of the experiment was performed in the same way as the first, but in this case is the transducer number two which acts as emitter, and the transducers number 1, 3 and 4 are which act as receivers, and so on is achieved by varying the number of transducer acts as transmitter.

To realize this experiment it was necessary the creation of a program in Matlab, which plots the three ellipses in the same image and find the intersection point of these ellipses. The program needs several inputs arguments which are the velocity of the wave, the time of flight of each receiver, the location of each transducer and the number of steps to do the ellipse point to point. And the output is a plot with these three drawn ellipses and the intersection point. The Matlab program are in the Appendix C.

In this case the plate of Aluminum is 1m*1m with a thickness of 2mm, the emitter transducer is excited with a frequency of 100kHz and the mode of excitation is A_0.

Bellow it is shown a sketch for the placement of the transducers on the plate and the defect to be detect.

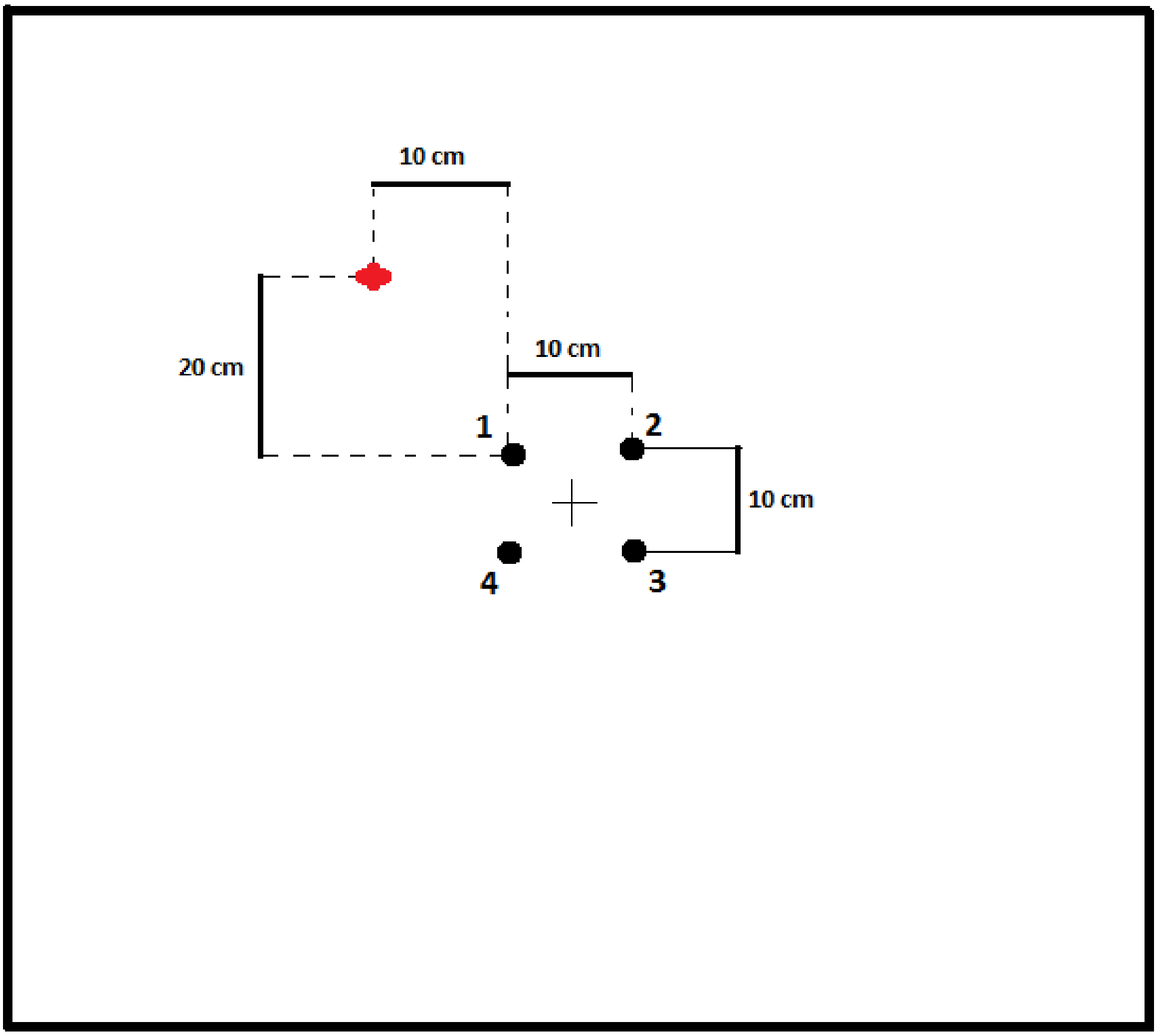


Image 4.19: Sketch of the third experiment.

The red mark is the crack to found, while the four black points are the position of the actuators or sensor, depend on which act like as a sensor in each measurement. These black points are situated symmetrically respect the center of the plate. In the sketch the center is represented with a cross. These transducers are 10cm apart each one.

THEORETICAL INTRODUCTION

ELLIPSE:

For the parameters and the main properties of ellipses are necessary the references: [21], [22] and [23].

Firstly it is necessary to remain some characteristics and properties about ellipses to understand this experiment.

The main parameters of an ellipse are the next:

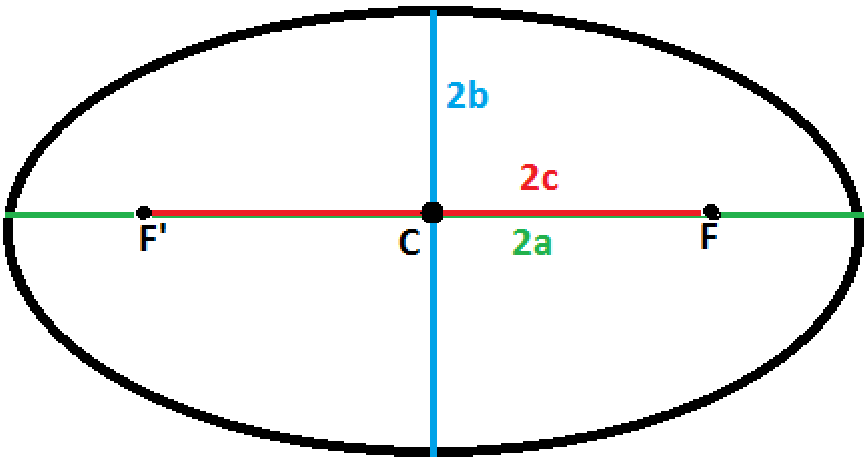


Image 4.20: Ellipse parameters

The major axis (2a): is the longest length in an ellipse that passes through the foci. It is equal in length that to the constant like it is explained after. This axis is represented by a green line in the figure above.

The minor axis (2b): is the length which is perpendicular to the major axis. It is the shortest length which passes through the center. This axis is represented by a blue line in the figure above.

The vertices: are the end points of the major axis.

The center (C): is the midpoint of the major axis.

The foci (F' and F): are located on the major axis, one on each side of the center. If a and b are the semi-major and semi-minor axes respectively, then the length from the focus to the center is c:

$$a^2 = c^2 + b^2$$

The eccentricity (e): is the ratio between the semi length focal and semi-major axis:. $\mathrm{e} = {}^{\mathrm{c}}/_{\mathrm{a}}$.

All eccentricity of ellipses are between 0 and 1. An eccentricity of 0 means that the ellipse is a circle and a long, thin ellipse has an eccentricity that approaches 1.

The angle of the ellipse: not all the ellipses are with the major axis parallel to the x-axis, because many times this axis is rotated a certain angle. The angle of the ellipse is called to which forms the x-axis with the major axis of the ellipse, being positive in the direction of the clockwise, as the following way:

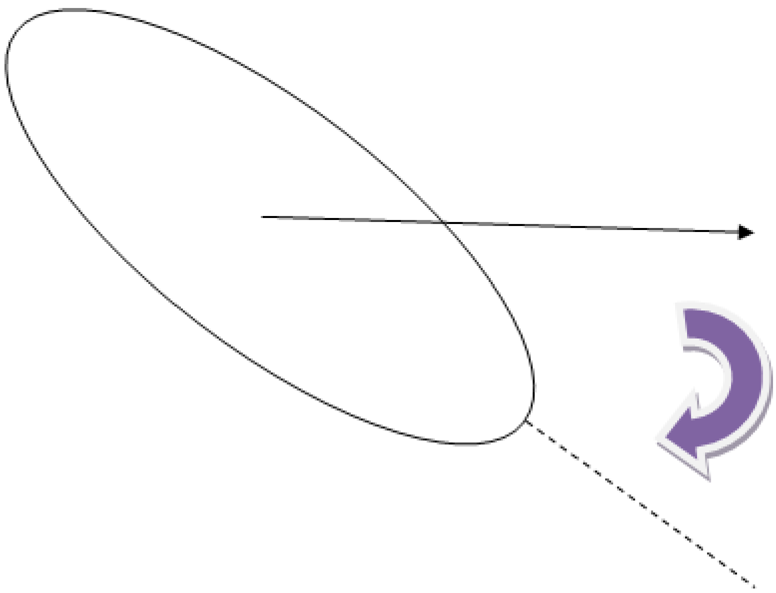


Image 4.21: Angle of the ellipse.

The principal property of the ellipses and the base of this experiment is: every point of the ellipse satisfies that the sum of the distances from two fixed points (foci) is constant. Whit this it can be written:

$$2a = d1 + d2$$

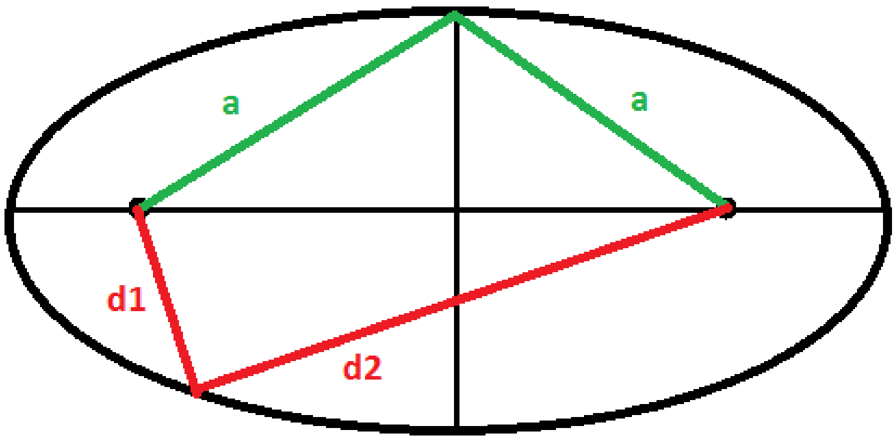


Image 4.22: Main property of the ellipses.

The standard equation for a ellipse is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

But it is valid only when the centre of the Cartesian axis is the same than the centre of the ellipse, if they are not the same, the formula changes as the following way:

$$\frac{(x - x_c)^2}{a^2} + \frac{(y - y_c)^2}{b^2} = 1$$

Where:

x_c is the distance x of the centre of the ellipse respect the origin

y_c is the distance y of the centre of the ellipse respect the origin

EXPERIMENT IN WHICH THIS IS BASED:

In this section it shows a brief summary of the experiment done, the experiment in which is based and the differences between both.

Taken the experiment realized by Ye Lu, Lin Ye and Zhongquing Su [24] as a starting point and called "*Crack identification in aluminium plates using Lamb wave signals of a PZT sensor network".* This consists on the next:

$$\frac{\sqrt{(x-x_i)^2+(y-y_i)^2}}{V_{Si}}+\frac{\sqrt{x^2+y^2}}{V_{S0}}-\frac{\sqrt{x_i^2+y_i^2}}{V_{S0}}=T_{i-lag}$$

Where:

(x,y) is the position of the crack.

(x_i,y_i) is the position of the transducer number i.

V_{Si} is the velocity of the excitation mode for each transducer S_i.

$V_{S0}$is the velocity of the excitation mode S_0.

T_{i-lag} is the Time Of Flight between the incipient S_0 mode and the crack-induced wave components.

And the figure which represented this formula is the following:

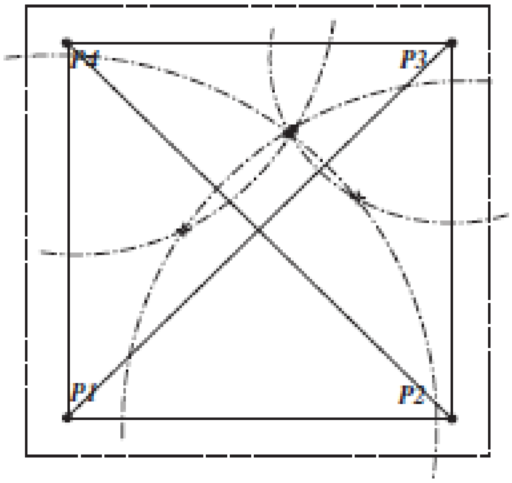


Image 4.23: Shapes used in the experiment of Ye Lu.[24]

But in the object of study about this Master Thesis, the formula changes because the excitation mode is the same, and only antisymmetric mode (A_0), then de velocity is constant, and also the time which appear in that formula changes because in the case of this work the time of flight starts in zero, this is the time starts to count in the moment than the wave is emitted by the first transducer. The changed formula for this experiment has become in the next:

$$\frac{\sqrt{(x-x_i)^2+(y-y_i)^2}}{V_{A0}}+\frac{\sqrt{x^2+y^2}}{V_{A0}}-\frac{\sqrt{x_i^2+y_i^2}}{V_{A0}}=TOF$$

Where:

$V_{A0}$is the velocity of the excitation mode A_0.

$TOF = T_{i-lag} + T_3$ where T_3 is the time between the moment that the wave is excited and the first peak of amplitude.

$$\sqrt{(x-x_i)^2+(y-y_i)^2}+\sqrt{x^2+y^2}-\sqrt{x_i^2+y_i^2}=TOF * V_{A0}$$

With this formula and the main property of ellipses can been written:

$$d_1 + d_2 = TOF * V_{A0}$$

Where:

d_1 is the distance from the focus to one point of the ellipse (the crack).

d_2 is the distance from the another focus to one point of the ellipse (the crack).

$TOF * V_{A0}$ is a constant

Thank to this, it achieves one of the most important property of the ellipses.

The difference between the experiment of Ye Lu, and the experiment which is part of this project is the shape of the figures utilized. In the experiment of Ye Lu the shape developed are circles because the transducer which acts like emitter is the same that the transducer which receives the wave. But in the case of the experiment of this Master Thesis the transducers which act like emitter and receptor are different, and not only different, they are also separated a necessary distance between both for be able to describe ellipses instead of circles, because if receptor and emitter were very close the image obtained would be a circle.

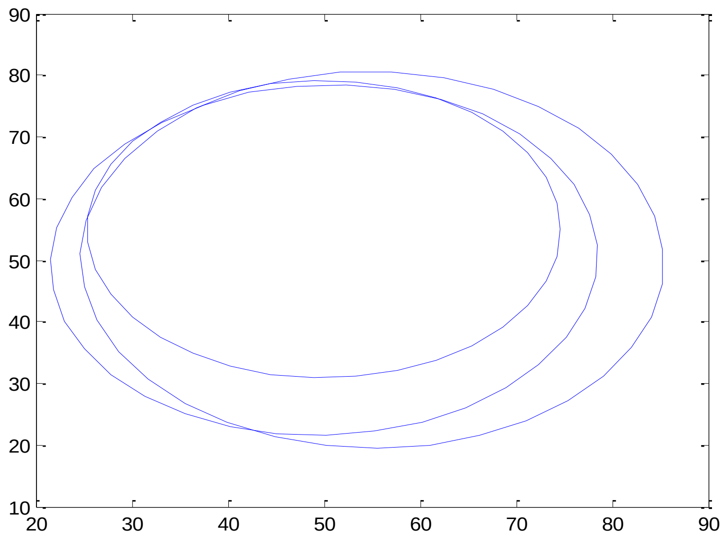


Image 4.24: Shapes used in the experiment done.

DATA COLLECTION

This experiment is divided in two parts: the first part of the experiment was performed in the same way as previous experiments in order to obtain the necessary parameters to introduce in the Matlab program, while the second part consist of using the program created in Matlab, and thus identify the location of the defect.

THE FIRST PART OF THE EXPERIMENT:

The four transducers are separate 10mm among themselves, and symmetrically located respect to the centre. The actuator is always excited with the same frequency, being of 100kHz, the mode of excitation is A_0 for this experiment. The black point number one (transducer number 1) works like an actuator, the other three act as sensors.

Parameters used in the program in this experiment are the following:

- Number of periods in package ➔ 3
- Output gain ➔ 1/8
- Input gain ➔ 1/80
- Window modulation ➔ hanning

The first thing to do in this experiment is to calculate the wave velocity, for this it is necessary realize the same that in previous experiments. First calculate the Time of flight of the wave and knowing the distance between the first two transducers (10cm) it will be calculate the velocity of the wave. The program shows that the flight of time is 4.68e-005 s and introducing 100mm in the distance, it is obtained that the velocity of the wave is 2.13675 km/s., as it is shown:

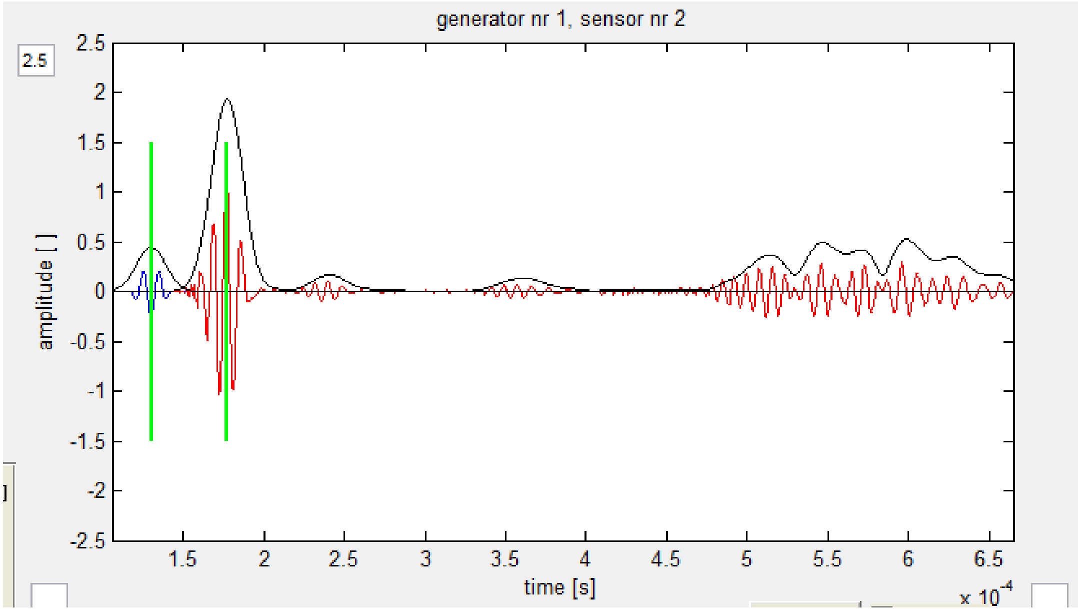


Image 4.25: Calculate the velocity of the wave

With this velocity it is possible to obtain the dispersion curves:

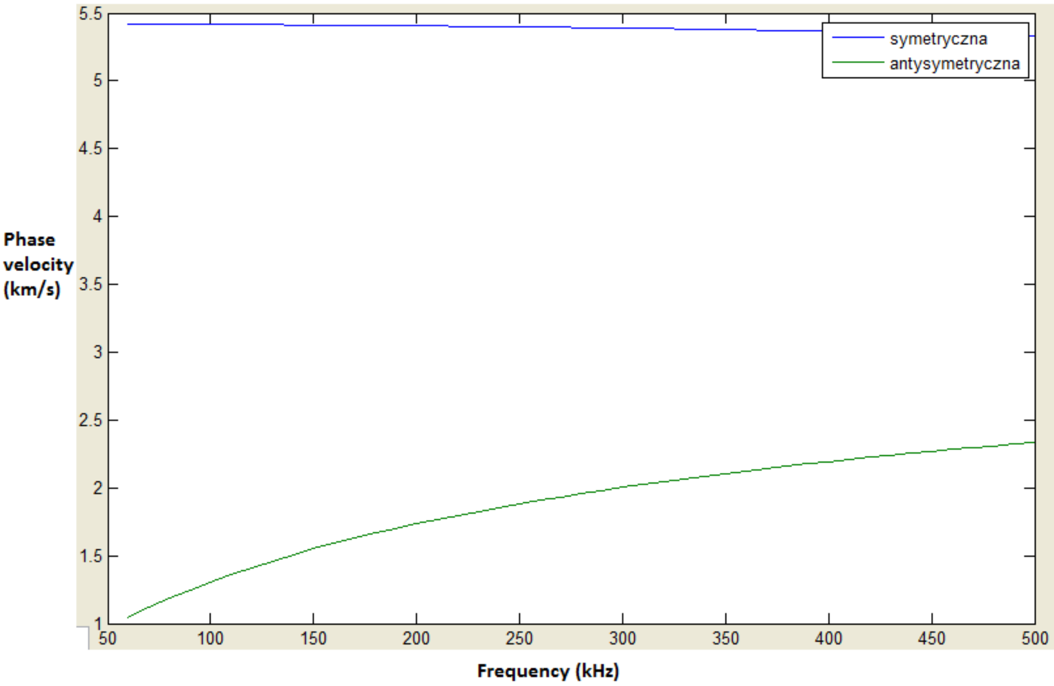


Image 4.26.Dispersion curves (PHASE velocity).

As it is known, the velocity is characteristic for each material and to the specifications made. At low frequencies this is the graphic for the phase velocity for an Aluminium plate of 2mm of thickness. The green line represents the A0 mode while the blue line represents S0 mode. It will be ideally that only appears A0 mode but this is practically impossible, but thanks to the transducers chosen this mode appears at least possible.

Moving the green cursors to analyze the next peak of amplitude:

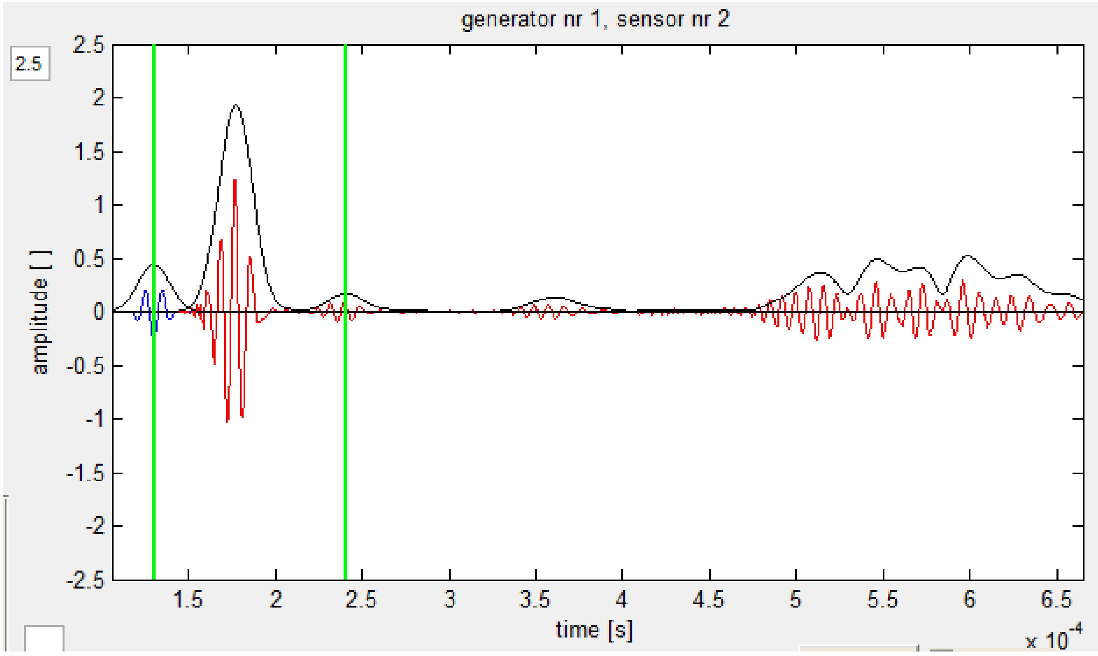


Image 4.27: Reflection of another transducer. (generator=1, sensor=2)

In this graphic it is obtained a time of flight of 0.0001104s. With this time and introducing the constant velocity, it is obtained that this peak is located at a distance of 235.897mm. It is possible that this point could be the reflection of another transducer. To analyze if this is correct, the real distance is calculated as follow:

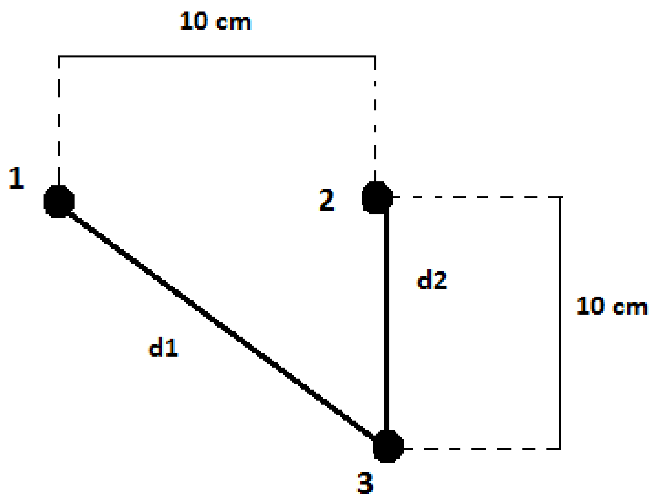


Image 4.28: Real distance from de reflection of the transducer number 3. (generator=1, sensor=2)

$$d1 = \sqrt{10^2 + 10^2} = 14.1421cm$$

$$d2 = 10cm$$

The total distance is:

$$d1 + d2 = 14.1421 + 10 = 24.14cm$$

It is checked that this distance is the reflection of the transducer number 3.

Investigating the next peak of amplitude as the same way:

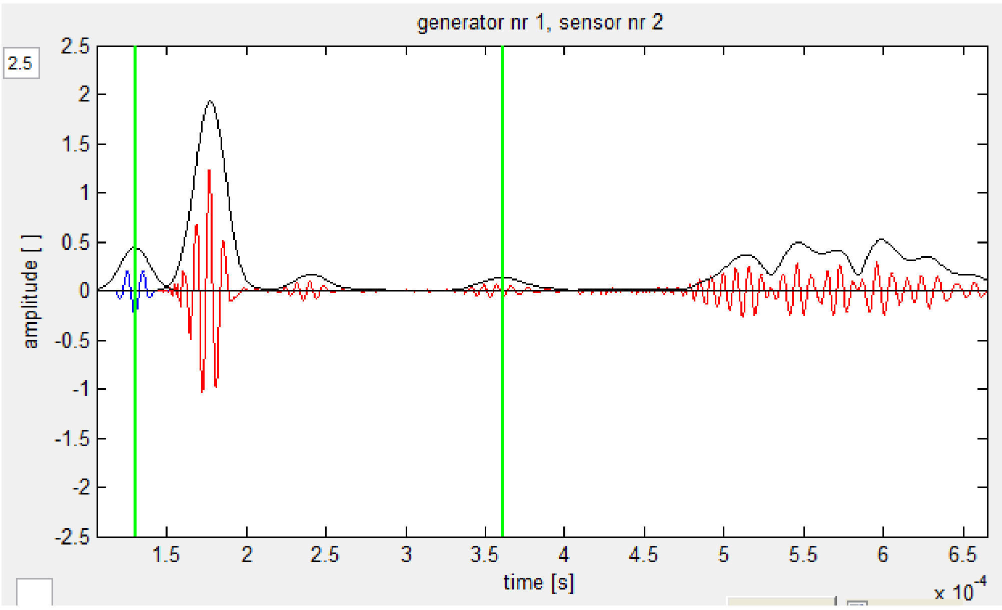


Image 4.29: location of the defect. (generator=1, sensor=2)

It is found that the distance is 493.162mm for a Time of Flight of 0.0002308s.

Checking if this distance is the real distance to de defect:

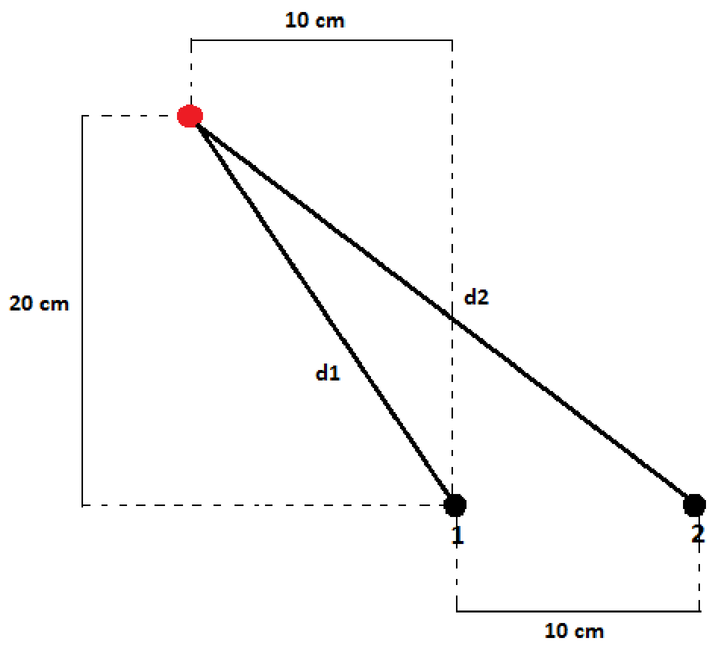


Image 4.30: Real distance of the defect (generator=1, sensor=2).

$$d1 = \sqrt{20^2 + 10^2} = 22.36068cm$$

$$d2 = \sqrt{20^2 + 20^2} = 28.28427cm$$

The total distance is:

$$d1 + d2 = 22.36068 + 28.28427 = 50.654cm$$

This confirms that this is the point where the defect is placed.

Now it is realized the same but in this case the transducer number 3 acting as a receiver of the wave:

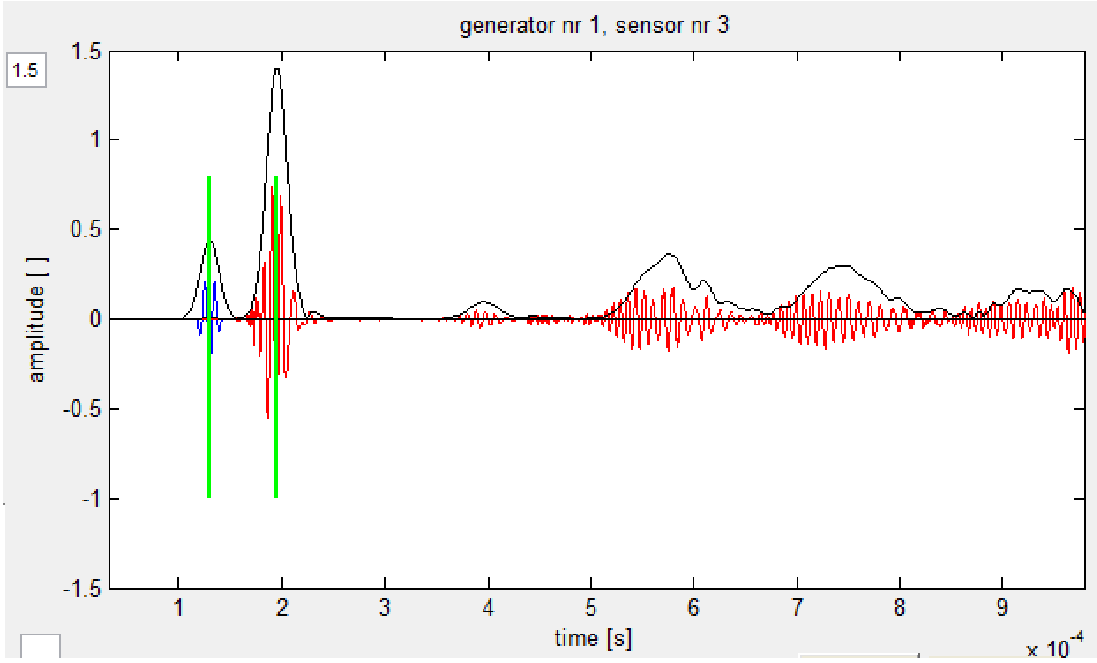


Image 4.31: Distance between transducer number 1 and 3.

Obtaining 139.316mm of distance corresponding to the distance between the transducer which it is working (1 and 3). The time of flight is 6.52e-005s.

Moving the cursors to discuss the next peak of the wave:

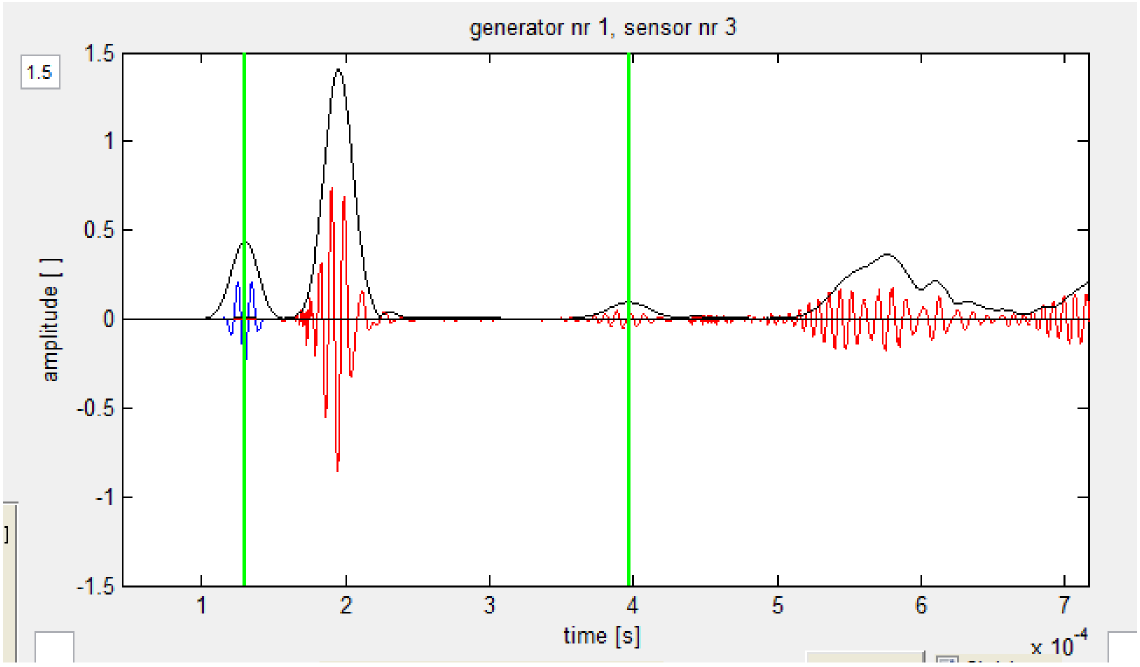


Image 4.32: Location of the defect. (generator=1, sensor=3).

It is obtained 0.0002668s as this time of flight and with the same velocity, the program shows a distance of 570.085mm that probably correspond to the defect.

It is calculated the real distance that the defect is placed to see if it really corresponds to the distance obtained.

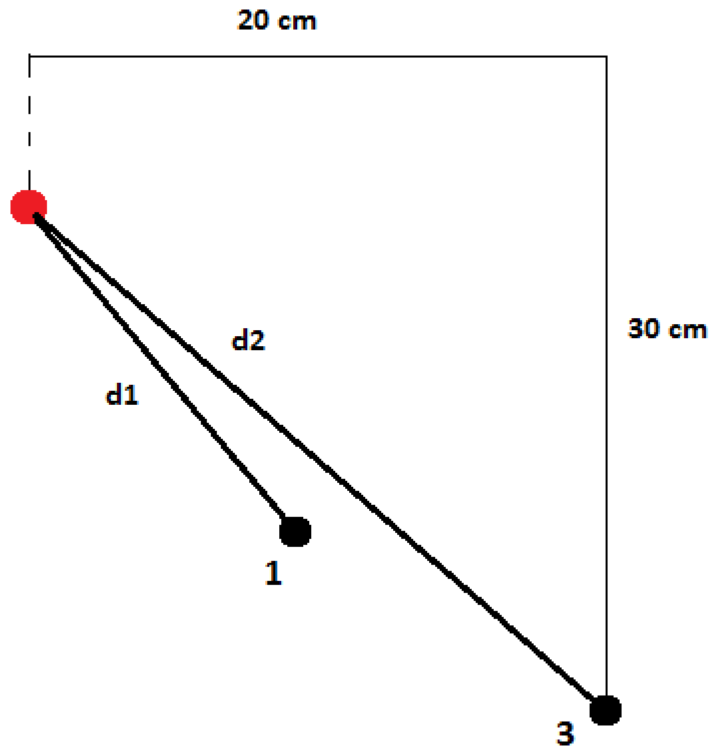


Image 4.33: Real distance of the defect (generator=1, sensor=3).

$$d1 = 22.36068cm$$

$$d2 = \sqrt{20^2 + 30^2} = 36.0555cm$$

The total distance is:

$$d1 + d2 = 22.36068 + 36.0555 = 58.41cm$$

It can been observed that this time of flight and this distance actually corresponds to the defect detection.

Again changing the transducer that is going to act as receiver, in this case involves the transducer number 4:

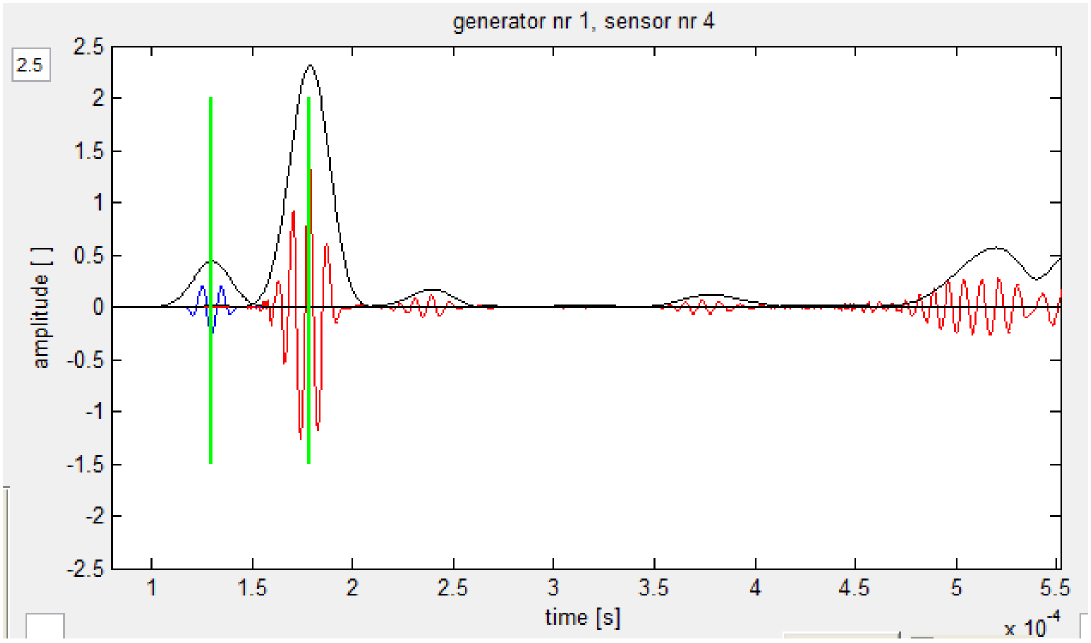


Image 4.34: Distance between the transducers number 1 and 4.

Obtaining a time of flight of 4.88e-005s and a distance of 104.273mm (actually the two transducers are separated a distance of 10cm).

Moving the green lines to analyze the next amplitude wave it is obtained:

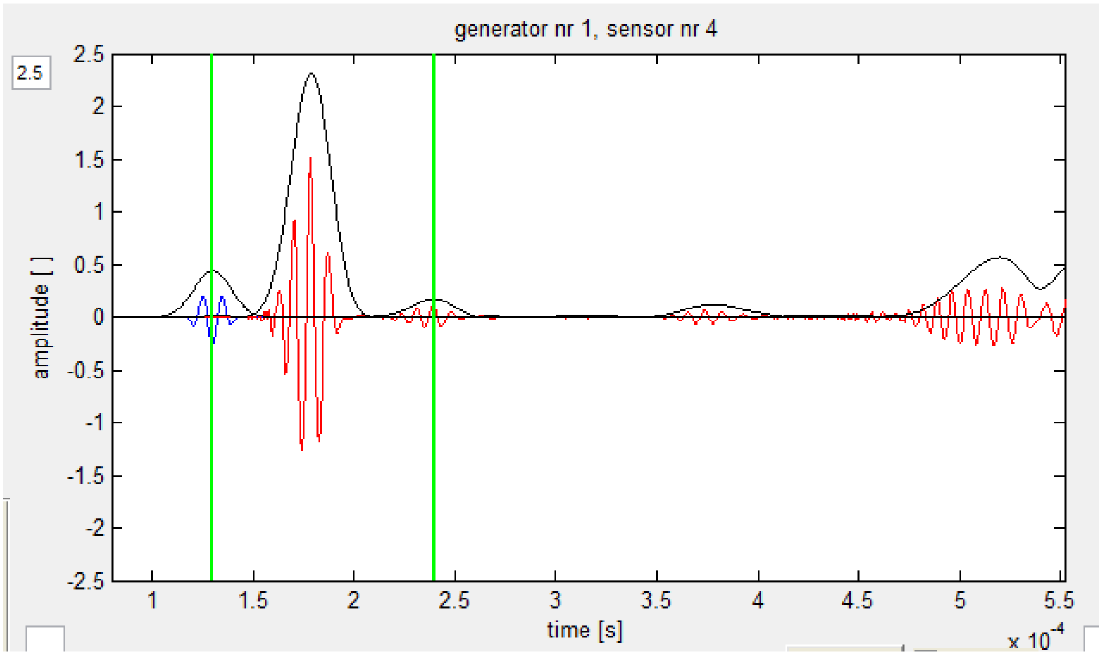


Image 4.35: Reflection from another transducer (generator=1, sensor=4)

The time of flight is 0.0001096s and the distance is 234.188, for reflection from the transducer number 3. This distance should have been 24.14cm, as previously calculated.

Investigating the next wave amplitude it is obtain:

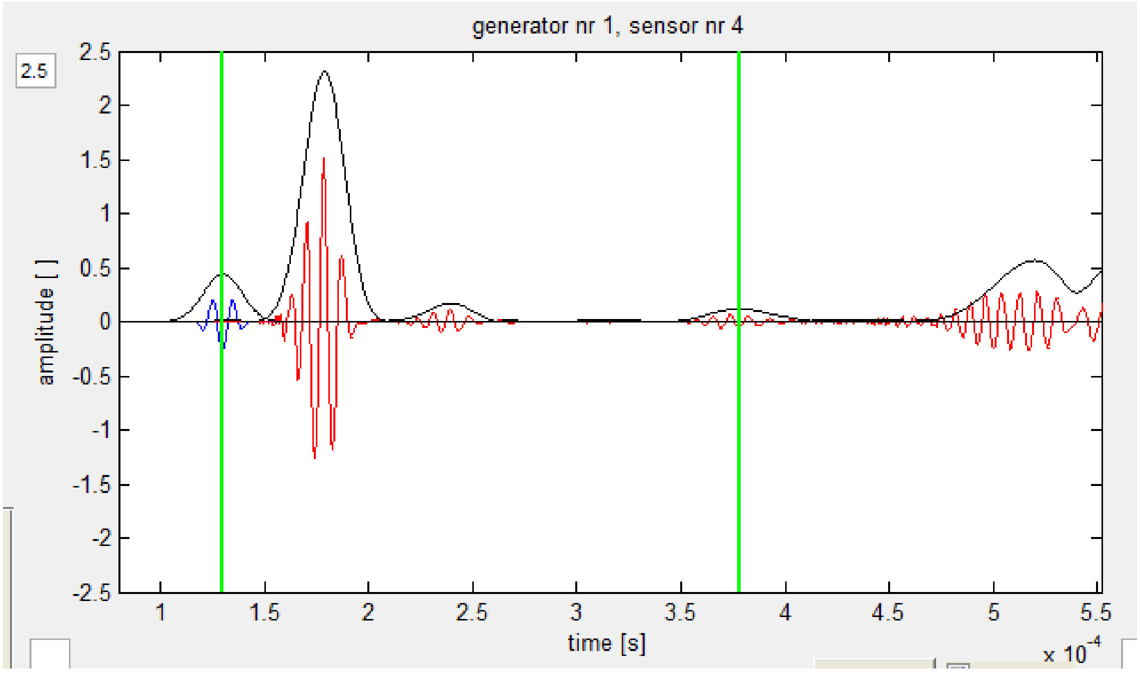


Image 4.36: Location of the defect. (generator=1, sensor=4)

It is obtained a time of flight of 0.000248s and a distance of 529.914mm.

Verifying that this is really the default TOF:

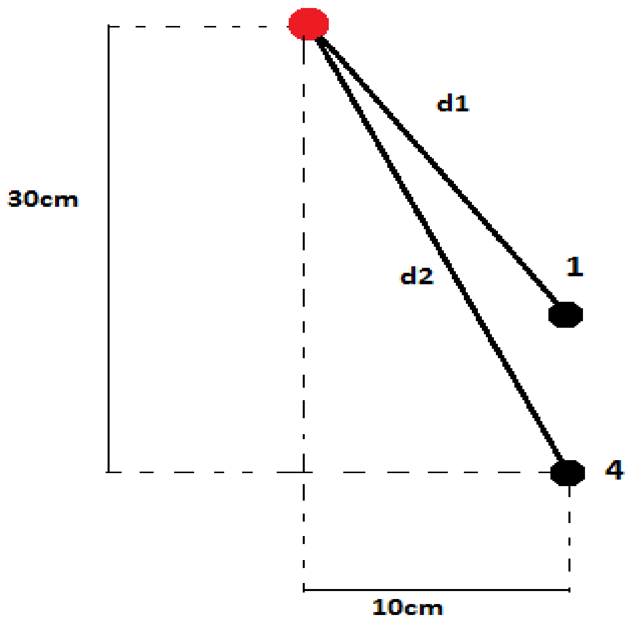


Image 4.37. Real distance of the defect (generator=1, sensor=4).

$$d1 = 22.36068cm$$ ➔ Previously calculated

$$d2 = \sqrt{30^2 + 10^2} = 31.62277cm$$

Then, the total distance is: 53.62277cm

THE SECOND PART OF THE EXPERIMENT:

The inputs required by the program created by Matlab are v, TOF1, TOF2, TOF3, steps, xt1, yt1, xt2, yt2, xt3, yt3, xt4, yt4, where:

v➔ is the velocity of the Lamb wave. In this concrete case: 213675cm/s.

TOF1➔ is the Time of Flight for the excited wave in the first transducer and received in the second transducer. In this case it is 0.0002308s.

TOF2,➔ is the Time of Flight for the excited wave in the first transducer and received in the third transducer. In this case it is 0.0002668s.

TOF3 ➔ is the Time of Flight for the excited wave in the first transducer and received in the fourth transducer. In this case it is 0.000248s.

Steps➔ is the number of steps to draw the ellipses. The steps used in this program are 360000.

xt1➔ x coordinate of the transducer number 1. In this case is 45cm.

yt1 ➔ y coordinate of the transducer number 1. In this case is 55cm.

xt2 ➔ x coordinate of the transducer number 2. In this case is 55cm.

yt2 ➔ y coordinate of the transducer number 2. In this case is 55cm.

xt3 ➔ x coordinate of the transducer number 3. In this case is 55cm.

yt3 ➔ y coordinate of the transducer number 3. In this case is 45cm.

xt4 ➔ x coordinate of the transducer number 4. In this case is 45cm.

yt4➔ y coordinate of the transducer number 4. In this case is 45cm.

With these inputs, the first that the program does is to calculate the characteristic parameters for each of the ellipses, so in this way it calculates the major axis, the minor axis, the focal length, the location of the center of the ellipse the angle of the ellipse. Once all these parameters are calculated for each ellipse, then the program will calculate the complete ellipses. Assuming the number 1 is the transducer that acts as emitter of the wave and the rest of transducers act as receivers (2, 3, 4), the program calculates each of the three ellipses formed by the pairs of transducers 1 and 2, 1 and 3, and 1 and 4. It can also change the transducer which acts like as emitter, for which only would be necessary to reorder the parameters introduced into the program, calling number 1 which in this case acts as a emitter and 2, 3 and 4 to act as receivers. After calculate the three ellipses, the screen shows a drawing of the three overlapped ellipses.

RESULTS

When the black point number two (transducer number 1) works like an actuator, the other three act as sensors

Generator-sensor	Velocity (km/s)	Distance (mm)	TOF (s)	DEFECT?
1-2	2.13675	100	4.68e-005	No
1-2	2.13675	235.897	0.0001104	No
1-2	2.13675	493.162	0.0002308	DEFECT
1-3	2.13675	139.316	6.52e-005	No
1-3	2.13675	570.085	0.0002668	DEFECT
1-4	2.13675	104.273	4.88e-005	No
1-4	2.13675	234.188	0.0001096	No
1-4	2.13675	529.914	0.000248	DEFECT

Chart 4-3: Resume for the third experiment when the transducer number 1 acts like an emitter.

Introducing the Time Of Flight to the different defects found, the program created in Matlab shows the next:

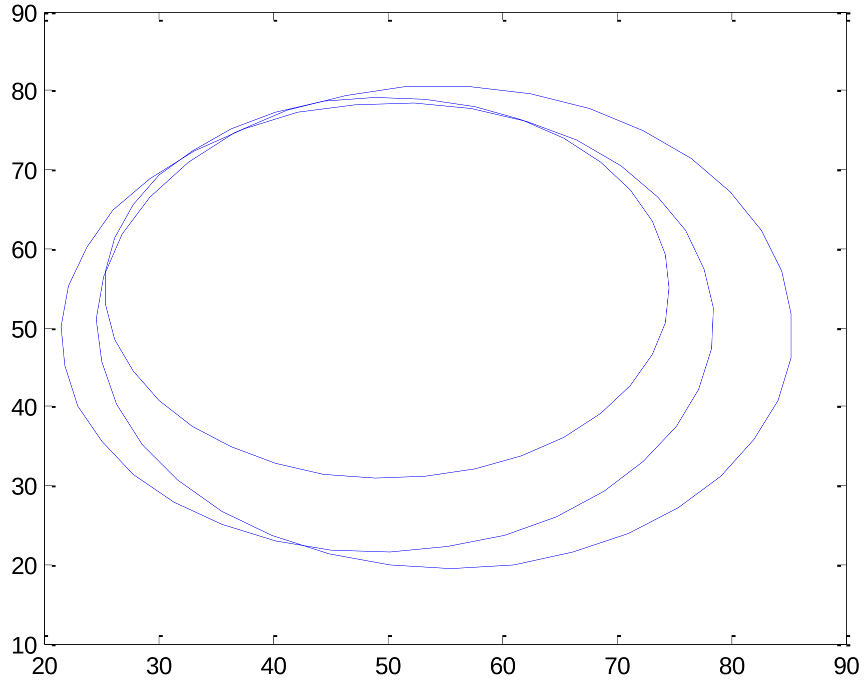


Image 4.38: output of Matlab

And zooming the intersection point between the three ellipses:

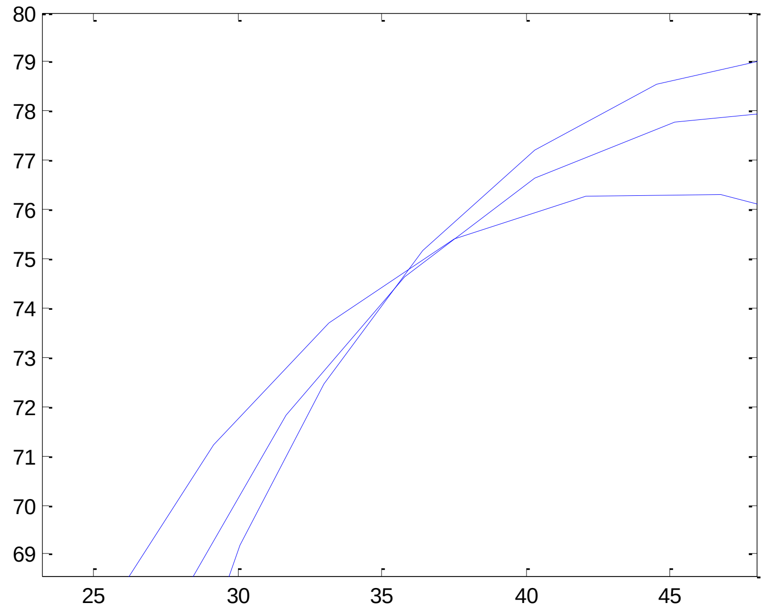


Image 4.39: zoom of the critical point.

The intersection point of these three ellipses is around (35.25; 74.75). This point found is the placement of the defect.

When the black point number two (transducer number 2) works like an actuator, the other three act as sensors

Generator-sensor	**Velocity (km/s)**	**Distance (mm)**	**TOF (s)**	**DEFECT?**
2-1	2.13675	100	4.68e-005	No
2-1	2.13675	235.897	0.0001104	No
2-1	2.13675	493.162	0.0002308	DEFECT
2-3	2.13675	101.709	4.76e-005	no
2-3	2.13675	243.59	0.000114	no
2-3	2.13675	613.675	0.0002872	DEFECT
2-4	2.13675	141.88	6.64e-005	no
2-4	2.13675	578.632	0.0002708	DEFECT

Chart 4-4: Resume for the third experiment when the transducer number 2 acts like an emitter.

In this case and in the rest the image obtained by Matlab and the results are the same than the previously case.

When the black point number three (transducer number 3) works like an actuator, the other three act as sensors.

Generator-sensor	Velocity (km/s)	Distance (mm)	TOF (s)	DEFECT?
3-1	2.13675	138.461	6.48e-005	No
3-1	2.13675	569.23	0.0002664	DEFECT
3-2	2.13675	101.709	4.76e-005	No
3-2	2.13675	243.59	0.000114	No
3-2	2.13675	613.675	0.0002872	DEFECT
3-4	2.13675	104.273	4.88e-005	no
3-4	2.13675	234.188	0.0001096	No
3-4	2.13675	660.683	0.0003092	DEFECT

Chart 4-5: Resume for the third experiment when the transducer number 3 acts like an emitter.

When the black point number four (transducer number 4) works like an actuator, the other three act as sensors.

Generator-sensor	Velocity (km/s)	Distance (mm)	TOF (s)	DEFECT?
4-1	2.13675	104.273	4.88e-005	no
4-1	2.13675	233.333	0.0001092	no
4-1	2.13675	529.914	0.000248	DEFECT
4-2	2.13675	141.88	6.64e-005	no
4-2	2.13675	580.341	0.0002716	DEFECT
4-3	2.13675	104.273	4.88e-005	no
4-3	2.13675	232.478	0.0001088	no
4-3	2.13675	659.828	0.0003088	DEFECT

Chart 4-6: Resume for the third experiment when the transducer number 4 acts like an emitter.

CONCLUSION

Zooming more the intersection point, it can been found the next:

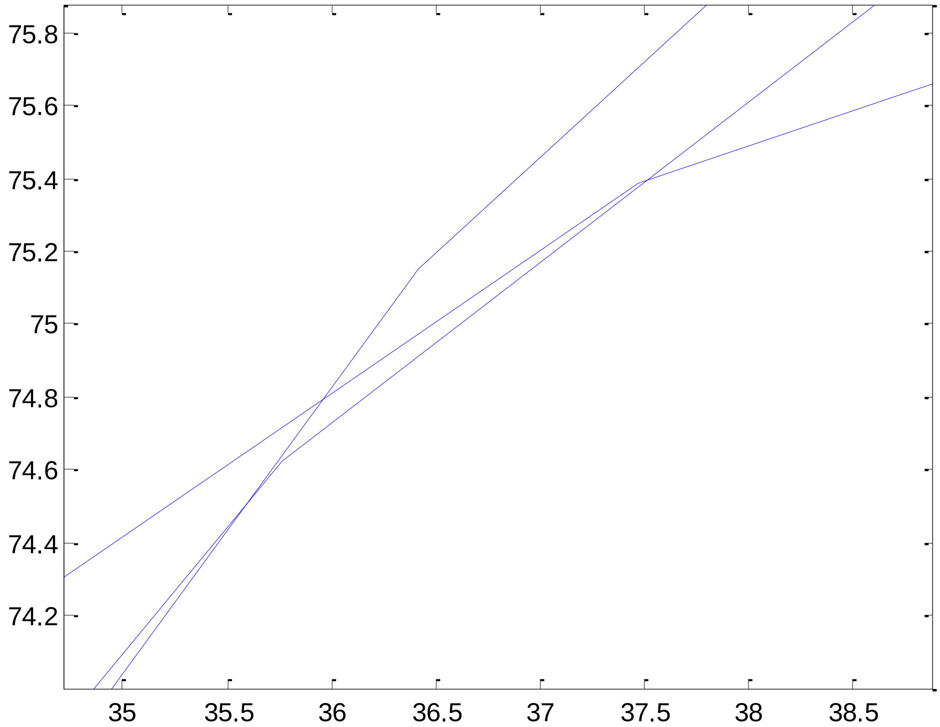


Image 4.40: Second zoom to the intersection point.

It is impossible to found exactly the placement of the defect but the program created shown very good results. As it can been seen in the image the intersection point is found between 35.5 and 37.5cm for the x axis and for the y axis it is between 74.5 and 75.4cm. In a range of two centimeters the defect is found in a plate of one meter. And for plates bigger the results will be the same.

Knowing that the defect is located in a space of at the most $2 * 2cm^2$, it is much easier to find the defect, instead of having to inspect a whole plate of $100 * 100cm^2$.

5. SUMMARY

Aluminium plates are found in practically all industrial applications and this is the reason of the great interest in the study of maintenance of these plates. These are often required to have a significant life span, requiring a frequent maintenance and inspection of critical areas. This, among many other applications, has triggered the development of testing methods that can assess the state of a structure or material without affecting its functionality so called non-destructive testing (NTD).

A more innovative solution is the technique of defects detection by ultrasounds trough Lamb waves. This technique allows continuous monitoring of the condition of the structure and allows also the integration of detection systems in critical areas of the structure that require constant maintenance. For the generation of these waves have been used as transducers piezocomposites because of its low cost, low weight and low volume; these are features that make extraordinarily useful when it comes to introduce into the structure to be analyzed.

One of the main goal of this Master Thesis consist of learning, study and analysis of Lamb waves, and the characteristics that they carry, such as obtaining the dispersion curves for the introduced mode of excitation. In addition to putting into practice the study of these waves to detect failures in the plate.

Since for a given frequency can be multiple modes of propagation, it was proceeded to select and identify through a simple method the Lamb modes, focusing solely on the propagation of antisymmetric mode A_0. It is practically impossible to isolate one mode, this is the reason to choose these PZT transducers, wich get minimize symmetric mode S_0.

The present project satisfies with another of the fundamental purpose, creating a program in Matlab based on the properties of the ellipses which finds the placement of the defect. Although the results are not exact, the program gives reliable results as well as precise. Future experiments can improve this program to provide more accurately in the location of the defect, because this created program is precise but not exact, since the defect is located in a range of $\pm 1cm$. However for large plates that range is fairly accurate since it would save the inspection of the whole plate.

This program serves as a preventive and maintenance of structures in the detection of cracks, thanks to this, it can make an archive of mechanical behavior done periodically, but the most important of these inspections, is that it provides a large decrease in cost.

Finally, it is significant to mention that the presence of noise has made difficult the analysis of the obtained wave, therefore in the future might try to filter this noise and delete all or partially to facilitate the analysis of the resulting wave.

A. APPENDIX

	VALUE	TOLERANCE
Length/outer diameter	2mm	+/-0.10mm
Width/inner diameter	2mm	+/-0.10mm
Height	2mm	+/-0.05mm
Operating voltage	120V	
Free stroke	2.8μm	+/-15%
Blocking force	160N	+/-20%
Capacitance	36Nf	+/-15%
Stiffness	57N/ μm	+/-20%
Maximum operating temperature	150ºC	
Curie temperature	350ºC	
Material	NCE57	
Unloaded resonance frequency	>500kHz	
External electrodes	Screen-printed Ag	
wires	Optional	

Chart A-1: Characteristic of the transducer utilized. [16].

B. APPENDIX

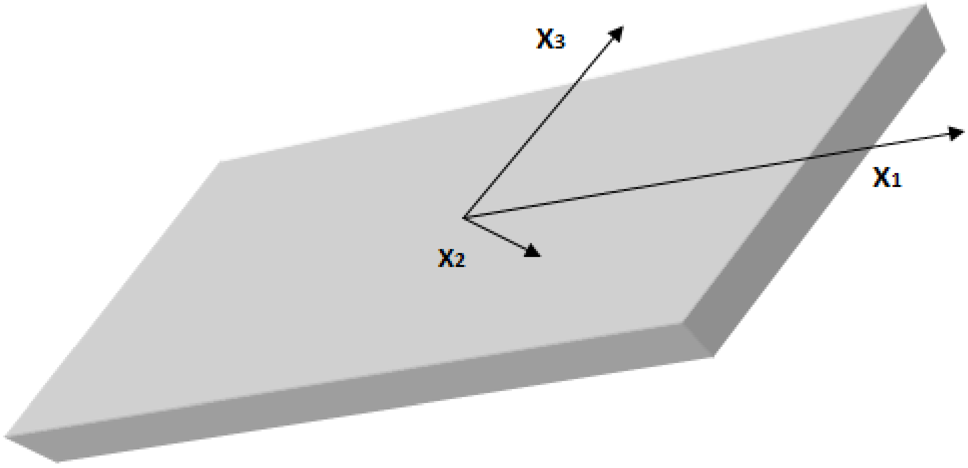


Image B.1: Infinite isotropic plate

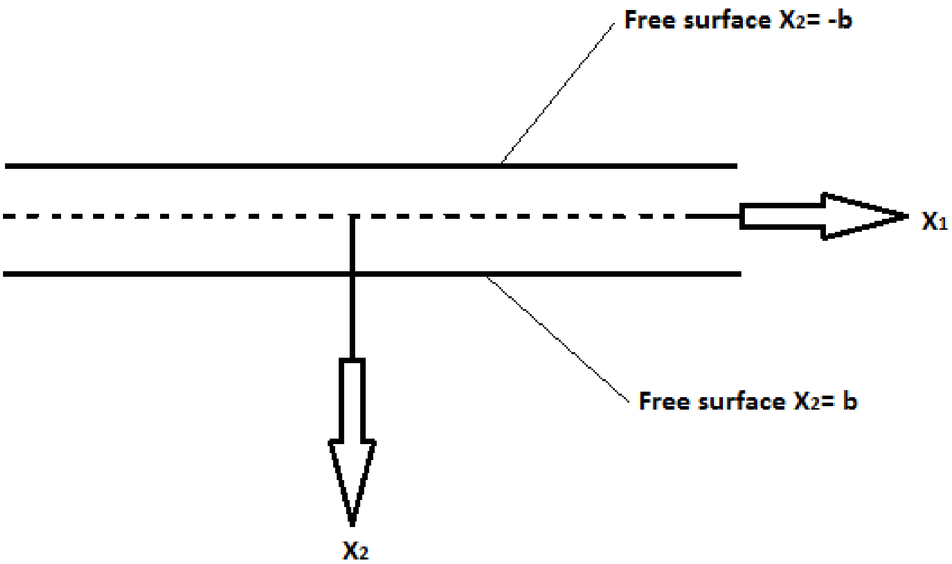


Image B.2: Cross-sectional view

Supposing guided waves propagation along the x1 axis and the plate is two-dimensional (2D), we can disregard the variations along de x3 axis, so we can ignore the displacements along this axis.

The equation of the movement is the next:

$$(\lambda + \mu)\nabla\nabla * \mathrm{u} + \mu\nabla^2\mathrm{u} = \rho\ddot{\mathrm{u}}$$

Where:

u is the displacement vector

λ is a Lamé constant of the material

μ is another Lamé constant of the material

ρ is the density of the material

By the decomposition of Helmholtz,

$$\mathrm{u} = \nabla\phi + \nabla \times \mathrm{H} \quad \text{and } \nabla\mathrm{H} = 0$$

And the movement equations in terms of the Helmholtz components are:

$$\nabla^2\phi = \frac{1}{c_1^2}\emptyset \;, \quad \nabla^2\mathrm{H}_3 = \frac{1}{c_2^2}\mathrm{H}_3 \text{ and } \mathrm{H}_1 = \mathrm{H}_2 = 0$$

Where:

$c_1 = \sqrt{(\lambda + 2\mu)/\rho}$ ➔ the bulk longitudinal wave velocity

$c_2 = \sqrt{\lambda/\rho}$ ➔ the bulk shear wave velocity

The solutions are:

$$\phi = \mathrm{f}(\mathrm{x}_2)e^{i(\xi x_1 - \omega t)} \quad \text{and} \quad \mathrm{H}_3 = \mathrm{h}_3(\mathrm{x}_2)e^{i(\xi x_1 - \omega t)}$$

Thus, we have the next differential equations:

$$\frac{d^2 f}{dx_2^2} + \alpha^2 f = 0 \quad \text{and} \quad \frac{d^2 h_3}{dx_2^2} + \beta^2 h_3 = 0$$

Where:

$$\alpha^2 = \frac{\omega^2}{c_1^2} = -\xi^2 \quad \text{and} \quad \beta^2 = \frac{\omega^2}{c_2^2} = -\xi^2$$

These differential equations solved are:

$$f(x_2) = A\sin\alpha x_2 + B\cos\alpha x_2$$

$$h_3(x_2) = C\sin\beta x_2 + D\cos\beta x_2$$

Where:

A,B,C and D are constants.

The conditions of traction-free are imposed because $x_2 = \pm b$:

$\sigma_{22} = \sigma_{21} = 0$ at $x_2 = \pm b$

Writing the tractions in terms of the Helmholtz:

$$\sigma_{22} = (\lambda + 2\mu)\nabla^2\phi - 2\mu\left(\frac{\partial^2\phi}{\partial x_1^2} + \frac{\partial^2 H_3}{\partial x_1\,\partial x_2}\right)$$

$$\sigma_{21} = \mu\left(2\frac{\partial^2\phi}{\partial x_1\,\partial x_2} + \frac{\partial^2 H_3}{\partial {x_2}^2} - \frac{\partial^2 H_3}{\partial {x_1}^2}\right)$$

Replacing:

$$\begin{bmatrix} -(\xi^2 - \beta^2)\cos\alpha b & 2i\xi\beta\cos\beta b \\ 2i\xi\alpha\sin\alpha b & (\xi^2 - \beta^2)\sin\beta b \end{bmatrix} \begin{bmatrix} B \\ C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -(\xi^2 - \beta^2)\sin\alpha b & -2i\xi\beta\sin\beta b \\ 2i\xi\alpha\cos\alpha b & (\xi^2 - \beta^2)\cos\beta b \end{bmatrix} \begin{bmatrix} A \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

To resolve these, it is necessary the Rayleigh-Lamb equations, which are:

$$\frac{\tan\beta b}{\tan\alpha b} = \left[\frac{-4\alpha\beta\xi^2}{(\xi^2 - \beta^2)^2}\right]^{\pm 1}$$

The positive exponent is to the symmetric Lamb modes (S_0), while the negative exponent corresponds to the antisymmetric Lamb modes ((A_0). The Rayleigh-Lamb equations relate the excitation angular frequency with the phase velocity of the guided waves in the plate. These Rayleigh-Lamb equations support the phase-velocity dispersion curve.

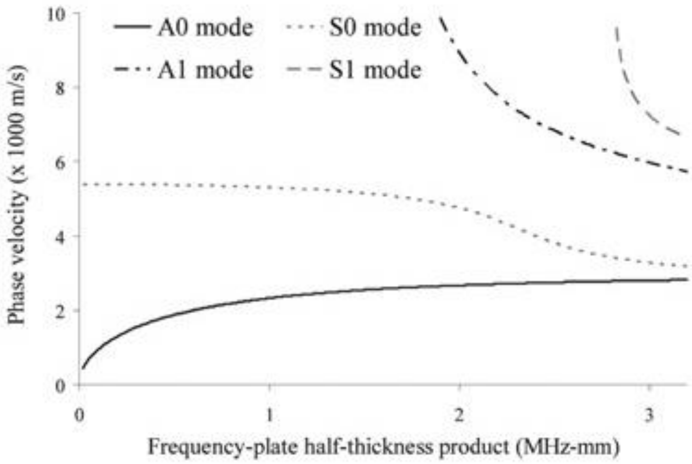


Image B.3: Dispersion curves for Lamb modes in an isotropic aluminum plate structure (PHASE velocity).[20]

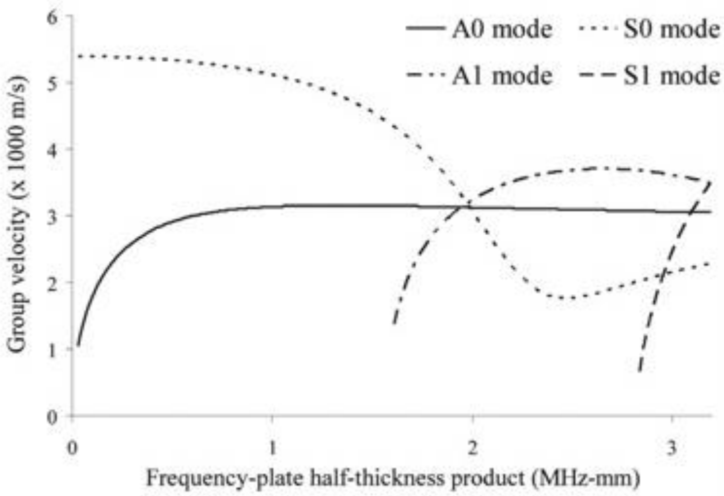


Image B.4: Dispersion curves for Lamb modes in an isotropic aluminum plate structure (GROUP velocity).[20]

C. APPENDIX

The Matlab program realized to draw the three ellipses in the same plot, and to find the intersection point of these ellipses:

function [X1 Y1 X2 Y2 X3 Y3] = crackdetection(v, TOF1, TOF2, TOF3, steps, xt1,yt1, xt2, yt2, xt3, yt3, xt4, yt4)

```
a1=(v*TOF1)/2;

x1=(xt1-xt2)/2+xt2;

y1=(yt1-yt2)/2+yt2;

difx1=xt1-xt2;

dify1=yt1-yt2;

if xt1==xt2;

   d1=abs(dify1);

else if yt1==yt2

   d1=abs(difx1);

else

   d1=sqrt((abs(difx1)).^2+(abs(dify1)).^2);

   end

end


c1=d1/2;

b1=sqrt(a1.^2-c1.^2);
```

```
if dify1==0

   angl1=0;

else if difx1==0

      angl1=90;

   else

angl1=atan((dify1)/(difx1));

   end

end

angle1=-angl1*180/pi;


a2=(v*TOF2)/2;

x2=(xt1-xt3)/2+xt3;

y2=(yt1-yt3)/2+yt3;

difx2=xt1-xt3;

dify2=yt1-yt3;

if xt1==xt3

   d2=abs(dify2);

else if yt1==yt3

   d2=abs(difx2);

else

d2=sqrt((abs(difx2)).^2+(abs(dify2)).^2);

   end

end
```

```
c2=d2/2;

b2=sqrt(a2.^2-c2.^2);

if dify2==0

   angl2=0;

else if difx2==0

      angl2=90;

   else

angl2=atan((dify2)/(difx2));

   end

end

angle2=-angl2*180/pi;


a3=(v*TOF3)/2;

x3=(xt1-xt4)/2+xt4;

y3=(yt1-yt4)/2+yt4;

difx3=xt1-xt4;

dify3=yt1-yt4;

if xt1==xt4;

   d3=abs(dify3);

else if yt1==yt4

   d3=abs(difx3);

else

d3=sqrt((abs(difx3)).^2+(abs(dify3)).^2);
```

```
    end
end
c3=d3/2;
b3=sqrt(a3.^2-c3.^2);
if dify3==0
    angl3=0;
else if difx3==0
        angl3=90;
    else
angl3=atan((dify3)/(difx3));
    end
end
angle3=-angl3*180/pi;

 error (nargchk(10, 20, nargin))
    if nargin<20, steps=36;end

    beta1 = -angle1 * (pi / 180);
    sinbeta1 = sin(beta1);
    cosbeta1 = cos(beta1);

    alpha1 = linspace(0, 360, steps)' .* (pi / 180);
    sinalpha1 = sin(alpha1);
```

```
cosalpha1 = cos(alpha1);


X1 = x1 + (a1 * cosalpha1 * cosbeta1 - b1 * sinalpha1 * sinbeta1);

Y1 = y1 + (a1 * cosalpha1 * sinbeta1 + b1 * sinalpha1 * cosbeta1);

plot (X1,Y1)

hold on




beta2 = -angle2 * (pi / 180);

sinbeta2 = sin(beta2);

cosbeta2 = cos(beta2);


alpha2 = linspace(0, 360, steps)' .* (pi / 180);

sinalpha2 = sin(alpha2);

cosalpha2 = cos(alpha2);


X2 = x2 + (a2 * cosalpha2 * cosbeta2 - b2 * sinalpha2 * sinbeta2);

Y2 = y2 + (a2 * cosalpha2 * sinbeta2 + b2 * sinalpha2 * cosbeta2);

plot (X2,Y2)

hold on
```

```
beta3 = -angle3 * (pi / 180);

sinbeta3 = sin(beta3);

cosbeta3 = cos(beta3);


alpha3 = linspace(0, 360, steps)' .* (pi / 180);

sinalpha3 = sin(alpha3);

cosalpha3 = cos(alpha3);


X3 = x3 + (a3 * cosalpha3 * cosbeta3 - b3 * sinalpha3 * sinbeta3);

Y3 = y3 + (a3 * cosalpha3 * sinbeta3 + b3 * sinalpha3 * cosbeta3);

plot (X3,Y3)


hold off

if nargout==1,

   X1 = [X1 Y1];

   X2 = [X2 Y2];

   X3 = [X3 Y3]; end


if X1==X2==X3


   if Y1==Y2==Y3

      display('The intersection point is:')
```

```
            x= X1

            y= X2

        end



    else

        display ('There is no point of intersection')

    end




end
```

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