



## IMPACT OF THE OPERATIONS MANAGER'S DUAL ROLE ON INVENTORY POLICY \*

JOSÉ ALFARO <sup>1</sup> JOSEP A. TRIBÓ <sup>2</sup>

### *Abstract*

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In modern corporations, the Operations Manager's role in defining of firm's strategy is becoming more important. In this paper we describe how firms can use this tendency for Operations Managers to make strategic decisions as a mechanism to prevent inventory mismanagement. These managers have incentives to speculate with inventory cost reductions, thereby avoiding sharp reductions in a single period, because it would hinder further reductions in the future. Remarkably, firms may prevent such behavior by stimulating the Operations Managers' strategic orientation, without losing sight of inventory-efficient management.

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**Keywords:** Operations Manager behavior, Compensation model, Inventory policy.

<sup>1</sup> Business Department. University of Navarra. Campus Universitario 31080 Pamplona. Spain.  
E-mail: [jalfaro@unav.es](mailto:jalfaro@unav.es)

<sup>2</sup>Business Department, Carlos III, C/ Madrid 126 (Getafe), 28903. E-mail: [joatribo@emp.uc3m.es](mailto:joatribo@emp.uc3m.es)

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## 1/ INTRODUCTION

Operations decisions are playing an increasingly important role in a firm's overall strategy to achieve competitive advantage. This is not surprising, as Wild (1979) emphasized a long time ago, "Operations managers' decisions influence the entire organization, the jobs within it, and the manner in which the whole is managed". White and Wharton (1990) surveys 900 US manufacturing firms and gives a score of 5.39 for production on a scale from 0 to 7, thus considering it as a significant element in a corporate strategy. These features lead Armistead and Mapes (1992) to conclude that operations managers' roles are likely to be more focused on corporate targets than on performance measures for manufacturing. A consequence follows: these managers should move their focus from production planning, scheduling and control (Gerwin, 1993), to coordination tasks (*i.e* connecting production and marketing, Nie and Young, 1997; or to design and coordinate networks of knowledge, Mak and Ramaprasad, 2003).

However, this process of gaining more "managerial skills" (Oakland and Sohal, 1989) is far from a reality. As D'Netto and Sohal (1999) cites, "the UK's operations managers were seen as mechanics with dirty finger nails, rather than gentlemen", and their orientation is mainly cost focused rather than strategic oriented (Hum and Leow, 1992). Interestingly, this is in sharp contrast to the potential capabilities of these managers. D'Netto and Sohal (1999) shows, making use of a sample of Australian firms, that current Operations Managers are academically well-qualified in order to undertake staff supervision functions. This leads these authors to argue that Operations Managers in the future should have a greater role to play in setting the strategic direction of the company and defining priorities.

Along this line, our main objective in this paper is to investigate whether Operations Managers' involvement in activities not directly related to operational management, like the definition of a firms' strategy, may affect detrimentally their time and effort devoted to managing inventory in a context where these managers have incentives to behave opportunistically. As a consequence, inventory policy may not be optimum. We intend to prove that this is not true.

We approach this problem by developing a simple model, where Operations Managers are compensated when they achieve two complementary goals. First, when there is a reduction in inventory costs. And second, when they dedicate efforts to other activities not directly related to operations. These efforts are devoted, for example, to designing a knowledge supply network. This is a network that integrates manufacturing, distribution, engineering, technology deployment, marketing and customer services. OMs are especially able to coordinate these different sources of knowledge as the operational routines needed are quite familiar to them. This reinforces a firm's strategy by ensuring efficient delivery of end products and services to markets (Mak and Ramaprasad, 2003). Also, these efforts can be spent in analysing outside information like demand perspectives for a firm's products and competitors' policies. This analysis will facilitate the coordination task between agents within the firm like workers (Haltiwanger and Maccini, 1994) or between different divisions like marketing and production (Nie and Young, 1997), in order to define a well-grounded strategy<sup>1</sup>. Considering only the first part of the compensation scheme, inventory cost reductions in the short-term cannot be drastic, because this would hinder the achievement of relevant reductions in inventory costs in the future (opportunistic behavior). However, the compensation for managerial efforts not devoted to operational activities would justify drastic reductions in inventory costs. This is because the latter earnings may offset the expected future losses that an initial drastic inventory cost reduction brings about. To achieve such an

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<sup>1</sup> Throughout the paper, we are going to refer to these efforts as managerial efforts that are considered not directly related to inventory management.

outcome an Operations Manager should combine intensive efforts to reduce inventory costs with low managerial efforts in the first period. This allows a greater possibility for future compensation linked to improvements in these latter efforts that can offset the reduction in future compensation related to inventory costs reduction. Thus, through this mechanism, the firm's owner can use monitoring of Operations Managers behavior in implementing these non-inventory related efforts (reduction in agency costs) as a way to influence inventory cost reductions.

Paradoxically, by incorporating compensation from increases in these managerial efforts, we find that inventory costs are reduced, and a steady-state inventory level is reached, in a shorter period of time than by compensating only on inventory cost reductions.

We can extract several conclusions from our paper. Firstly, firms can try to stimulate the managerial role of their Operations Managers without giving up inventory-efficient management. The correct design of the compensation package can make both objectives compatible. Interestingly, our point is that this scheme also works when these alternative tasks have even no relationship with inventory management and when Operations Managers have large incentives to behave opportunistically. Secondly, long-serving Operations Managers with additional non-inventory responsibilities may fix inventories at their optimal level in a shorter period of time than recently-appointed managers without these responsibilities. Once that inventory level has been reached, there is no further inventory variability. Consequently, this variability in the medium-term is smoother in those firms with long-serving Operations Manager in comparison with those firms with newly appointed managers (Alfaro and Tribó, 2003). Thirdly, firms with expertise in monitoring managers should handle inventories more efficiently. Lastly, if we consider the management of manpower levels as an Operations Manager's responsibility, we can conclude that Operations Managers may manipulate workforce levels as a complementary mechanism to achieve lower inventory variability than by only managing inventories. This is in accordance with other studies (Haltiwanger and Maccini, 1994).

This paper is divided into six sections. In the second section we build up the model, which is solved in the third section. We discuss the main theoretical findings in the fourth section. Section five inspects possible extensions of the model. The paper concludes with some final remarks.

## 2/ THE MODEL

This is a two-period model, with an Operations Manager (OM henceforth) of a representative firm deciding inventory policy as well as implementing some non-verifiable managerial efforts. Firm's owner monitors these efforts to compensate the OM contingent on them. The model is based on the following assumptions:

### Assumptions

1/ The firm faces demand  $D_t = \bar{D} + e_t$ , where  $e \in [\underline{e}, \bar{e}]$   $E\{e\} = 0$  and  $|e| \leq \bar{D}$  (to avoid a negative demand, that is  $\bar{D}$  is large enough). These  $e$  are deviations from the mean and are known at the end of each period. They are independent and uniformly distributed with a zero mean value. These deviations allow us to abstract from issues related to the demand structure and its impact on firms' inventory policy. Similar demand consideration can be found in Khan (1991), although this paper does not specify the distribution of demand shocks<sup>2</sup>.

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<sup>2</sup> The adoption of a non-uniform, but symmetric distribution would have generated qualitatively the same type of results. The main difference is that the comparative static analysis would have been more dependent on the initial conditions over total inventory costs.

2/ Total Inventory Cost ( $TC$ ) per period is defined as the sum of the costs associated with filling customers' orders, the cost of carrying inventories and the stockout costs<sup>3</sup>. An expression of this function<sup>4</sup> is:

$$TC = c_f \frac{D}{Q} + c_h q \frac{S^2}{2Q} + c_r q \frac{(Q-S)^2}{2Q} \quad (1)$$

Where:  $c_f$  = Filling cost per order;  $D$  = Total demand per period;  $c_h$  = Unitary inventory holding cost;  $c_r$  = Unitary inventory stockout cost;  $S$  = Inventory level;  $Q$  = Lot size;  $q$  = Planning period (if we take it as 1 we accommodate planning period with demand one).

In each period, a firm faces a demand that arrives at a continuous rate and, at the end of the period, total demand comes out to be  $D_t$ . Besides, we assume, in order to simplify, that demand is attended with constant lots with a  $Q$  size. We can think of the existence of some technological constraints to justify this assumption<sup>5</sup>.

3/ An OM is risk neutral and has a two-period temporal horizon<sup>6</sup>. The compensation is defined as follows (we implicitly assume a zero discount rate):

$$w_t = \mathbf{a} + \mathbf{b}(TC_{t-1} - TC_t + k(e_t - e_{t-1})) \quad (2)$$

By notation,  $w_t$  is the period-t wage that it is composed of two parts: a fixed part,  $\mathbf{a}$ , which is high enough<sup>7</sup>, and a variable part. The variable part has two terms. First term ( $TC_{t-1} - TC_t$ ) measures the decrease in the total inventory costs between period t-1,  $TC_{t-1}$ , and period t,  $TC_t$ . The second term,  $e_t - e_{t-1}$  measures the increase in the observed managerial effort (see next assumption) between period t-1,  $e_{t-1}$ , and period t,  $e_t$ <sup>8</sup>. With this kind of scheme, the firm's owner gives OMs incentives to reduce inventory costs as well as to increase managerial effort,  $e$ <sup>9</sup>.

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<sup>3</sup> Inventory carrying cost  $c_h q \frac{S^2}{2Q}$  is the average cost of holding inventories between different lots of production considering a uniform demand. Stockout cost  $c_r q \frac{(Q-S)^2}{2Q}$  is the cost of not attending a uniform demand (negative inventory) between different lots of production.

<sup>4</sup> We have decided to work with (1) because, although the demand includes a stochastic part, the random noise is uniformly distributed, and, on average, total cost function can be characterized by this function once we substitute the demand for its mean value.

<sup>5</sup> To consider an endogenous lots size does not bring about relevant new insights.

<sup>6</sup> If we had added an additional period, the main difference would have been that the first period solution would have been more complex and, in general the reduction in inventory costs would have been less drastic. In general, the larger the number of periods in the model, the smoother the inventory policy.

<sup>7</sup> This  $\mathbf{a}$  should be high enough to avoid situations of negative wages if the OM has not achieved an inventory cost reduction and has not increased managerial effort.

<sup>8</sup> Note that this managerial effort is not observable and can only be inferred from a signal that can be measured in cardinal terms after the implementation of some monitoring (-see next assumption). This signal is the output generated which is a standard assumption in agency theory (Milgrom and Roberts, 1992). Moreover, we have assumed, for simplicity, that this output is proportional (with a factor  $K$ ) to the effort  $e$ . The larger the effort  $e$ , the more skilled is the manager in defining a successful strategy generating tangible results. Note also that parameter  $k$  controls the relative weight for the OM's compensation for managerial effort in comparison with inventory cost reduction, which will allow us to conduct some comparative static analysis in later sections.

<sup>9</sup> An alternative incentive scheme to compensate overall effort,  $e$ , in each period instead of the differences in these efforts, introduces an asymmetry with regard to the proposed scheme that compensates the reduction in inventory costs. In any case, this alternative incentive scheme would produce more pronounced positive effects of managerial efforts on inventory cost reduction.

4/ Owners only observe managerial effort  $e$  if they implement a monitoring intensity,  $M$ <sup>10</sup>. In particular, we follow what is established as standard in the agency theory literature (e.g. Diamond, 1991); monitoring intensity defines the probability of detecting effort  $e$  inferred from the output that this effort generates<sup>11</sup>. Finally, for the sake of simplicity, we assume that this monitoring intensity is exogenously given with the same value in both periods<sup>12</sup>.

5/ The OM can implement managerial effort,  $e$ , with a cost given by the function  $C[e]$  that satisfies  $C'(e) > 0$ ,  $C'(e=0) = 0$  and  $C''(e) > 0$ . Moreover, we separate the cost related to these efforts from those efforts devoted to operational activities. Hence, we do not force *ex-ante* the interaction between operational and managerial efforts by assuming complementarities/substitutibilities in the effort cost function. Moreover, the efforts devoted to the definition of an inventory policy are observable (see footnote 10) and can be fixed *ex-ante* in a contract. This justifies their exclusion from the maximization program solved by an OM.

6/ An OM can be fired at the end of the first period, if he or she has not created net value. In this case, to simplify, firing costs are assumed to be zero. Thus, the *ex-ante* probability of an OM's continuation,  $p_c$ , equals the probability of observing that an OM has created value. This can be achieved by reducing  $TC$ , or by increasing managerial effort,  $e$ . Therefore,  $p_c$  is defined as:

$$p_c = M \text{prob}(TC_0 - TC_1 + k(e_1 - e_0) > 0) + (1 - M) \text{prob}(TC_0 - TC_1 > 0)$$

This is the sum of two terms. First, the  $(M)$  probability of knowing effort,  $e$ , times the probability of an increase in the observed value generated by the OM (including  $e$ ). Second, the probability  $(1-M)$  of not knowing effort,  $e$ , times the probability of an increase in the value generated by the OM due to the reduction in inventory costs.

The OM knows the probability,  $p_c$ , which means that the monitoring intensity implemented by the owner is known. We justify such an assumption because the OM can infer  $M$  from the pressures exerted by the principal in order to determine managerial effort  $e$ .

### **Time-line of the Model**

0/ The OM simultaneously determines first-period inventory level,  $S_1$ , as well as first-period effort  $e_1$ . To do so, he or she takes into consideration his or her expectation of future demand realizations as well as the owner's monitoring intensity  $M$ .

1/ First-period demand realization  $D_1 = \bar{D} + e_1$  is known at the end of that period. OM receives the wage and continues in the firm if enough value is generated. If not, the OM is fired and a new one is hired.

2/ The OM defines second-period inventory policy as well as second-period effort. To do so *he or she* takes into consideration first-period total inventory costs,  $TC_1$ , first-period effort,  $e_1$ , as well as his or her expectation of second-period demand realizations.

3/ Last-period demand is realized, and the OM receives his or her payments.

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<sup>10</sup> The lack of observability for this effort conforms to the idea that they are not operational and that they are embedded in the definition of the overall company strategy. Given the intangible nature of these efforts, which is in sharp contrast with the mechanical nature of the operational ones, it is quite natural to assume that managerial "strategic" efforts are not observable without monitoring whereas operational ones are.

<sup>11</sup> Some authors like Repullo and Suarez (2004) consider that the probability of knowing effort,  $e$ , depends not only on monitoring intensity  $M$ , but also on effort  $e$ . We abstract from these issues given that our focus is not on the interaction between monitoring efforts and strategic efforts but on the effect of the latter on inventory management.

<sup>12</sup> By allowing  $M$  to be endogenous would have introduced technical complexities without giving new insights into the connection between effort,  $e$ , and inventory cost reduction.

### 3/ SOLVING THE MODEL

We solve the model backwards. Thus, we first characterize second-period OM decisions. Then, we move to the first period.

#### Second-period Problem

At  $t=2$ , an OM determines second-period inventory level,  $S_2$ , as well as second-period managerial effort,  $e_2$ . The maximization problem to solve is the following:

$$\begin{aligned} \text{Max}_{\{S_2, e_2\}} E_2[w_2] &\equiv E_2[\mathbf{a} + \mathbf{b}M(TC_1 - TC_2 + k(e_2 - e_1)) + \mathbf{b}(1-M)(TC_1 - TC_2) - C[e_2]] = \\ &= E_2[\mathbf{a} + \mathbf{b}(TC_1 - TC_2) + \mathbf{b}Mk(e_2 - e_1) - C[e_2]] \end{aligned} \quad (3)$$

$$\text{With } TC_i = c_f \frac{D_i}{Q} + c_h \mathbf{q} \frac{S_i^2}{2Q} + c_r \mathbf{q} \frac{(Q - S_i)^2}{2Q} \quad \text{and } i = \{1, 2\}$$

Where,  $e_2$ , can only be observed with a probability which is given by owner's monitoring intensity ( $M$ ).

First Order Conditions (FOC) of the previous problem leads to<sup>13</sup>:

$$\partial_{S_2} E_2\{w_2\} = p_c (\mathbf{q}c_h \frac{S_2}{Q} - \mathbf{q}c_r \frac{Q - S_2}{Q}) = 0 \Rightarrow S_2 = \left(\frac{c_r}{c_h + c_r}\right)Q \equiv \hat{S} \quad (4)$$

$$\partial_{e_2} E_2\{w_2\} = \mathbf{b}Mk - C'[e_2^*] = 0 \Rightarrow \mathbf{b}Mk = C'[e_2^*] \quad (4')$$

We can see that increases in  $M$  and/or  $k$  lead to increases in second-period effort  $e_2$  due to  $C'' > 0$ . Note also, that effort  $e_2$  depends on  $\mathbf{b}M$  factor. Either an increase in the incentive intensity,  $\mathbf{b}$ , and/or in the monitoring intensity,  $M$ , can be used as substitute mechanisms to stimulate second-period OM efforts,  $e_2$ . In words of Chang and Lai (1999) "there is a trade-off between the wage (carrot) incentive and the supervision (stick) incentive".

#### First-period Problem

At  $t=0$ , OM determines first-period inventory,  $S_1$ , and first-period effort,  $e_1$ , taking into consideration the optimal solution found in the second period. The problem the OM solves is the following ( $E_0$  stands for period-zero expectations):

$$\text{Max}_{\{S_1, e_1\}} E_0[\mathbf{a} + \mathbf{b}(TC_0 - TC_1 + Mk(e_1 - e_0)) + p_c (\mathbf{a} + \mathbf{b}(TC_1 - TC_2^* + Mk(e_2^* - e_1)))] - C[e_1]$$

$$\text{S.t. } S_2^* = \left(\frac{c_r}{c_h + c_r}\right)Q \equiv \hat{S}, \quad TC_2^* = TC[S_2^*] \quad \text{and } \mathbf{b}Mk = C'[e_2^*]$$

By re-arranging this expression and neglecting constant terms, we can transform the previous maximization problem into minimization of the following objective function ( $O_1$ ):

$$\text{Min}_{\{S_1, e_1\}} E_0\{O_1\} = \mathbf{b}(E_0 TC_1 - Mke_1) + p_c \mathbf{b}(E_0 TC_2^* - E_0 TC_1 + Mk(e_1 - e_2^*) - \frac{\mathbf{a}}{\mathbf{b}}) + C[e_1] \quad (5)$$

$$\text{Where }^{14} p_c = M \frac{Q}{c_f \Delta \mathbf{e}} \left[ TC_0 - E_0 TC_1 + k(e_1 - e_0) - \frac{c_f}{Q} \mathbf{e} \right] + (1-M) \frac{Q}{c_f \Delta \mathbf{e}} \left[ TC_0 - E_0 TC_1 - \frac{c_f}{Q} \mathbf{e} \right]$$

$$\text{Thus, } p_c = \frac{Q}{c_f \Delta \mathbf{e}} \left[ TC_0 - E_0 TC_1 + Mk(e_1 - e_0) - \frac{c_f}{Q} \mathbf{e} \right]$$

<sup>13</sup> Second order conditions are satisfied straightforwardly.

<sup>14</sup> The expression of  $p_c$  is obtained using the assumption made that error term  $\mathbf{e}$  follows a uniform distribution.

First order conditions lead to (see point 1 in the Appendix):

$$\partial_{e_1} E_0 \{O_1\} = -MbkT + C'(e_1) \quad (6)$$

$$\partial_{S_1} E_0 \{O_1\} = bT \frac{\partial E_0 TC_1}{\partial S_1} \quad (7)$$

$$T \equiv \frac{Q}{c_f \Delta \mathbf{e}} \left( \frac{c_f \bar{\mathbf{e}}}{Q} + \Delta_2 - \Delta_1 + \frac{\mathbf{a}}{\mathbf{b}} + Mk(e_2 + e_0 - 2e_1) \right) \text{ where } \Delta_2 \equiv (E_0 TC_1 - E_0 TC_2) \text{ and } \Delta_1 \equiv (TC_0 - E_0 TC_1) \quad (8)$$

The solution of this problem leads to an equilibrium that depends on the value of:

$$\hat{T} \equiv T(e_1 = 0, e_2 = e_2^*, S_1 = S_2 = \hat{S}) = \frac{Q}{c_f \Delta \mathbf{e}} \left( \frac{c_f (\bar{D} + \bar{\mathbf{e}})}{Q} + \mathbf{q} \frac{c_r c_h Q}{2(c_r + c_h)_1} + \frac{\mathbf{a}}{\mathbf{b}} - TC_0 + Mk(e_2^* + e_0) \right) \quad (9)$$

With  $\hat{S}$  and  $e_2^*$  given in (4) and (4') respectively.

### Proposition 1

The optimal inventory policy is defined in terms of T given in (8):

a) If  $\hat{T} \equiv T(e_1 = 0, e_2 = e_2^*, S_1 = S_2 = \hat{S}) < 0$

$$e_1^* = 0 \quad Mbk = C'(e_2^*) \quad S_1^* = \bar{S} \quad S_2^* = \hat{S} \text{ where } T(e_1^*, e_2^*, S_2^*, S_1 = \bar{S}) \equiv 0$$

b) If  $\hat{T} > 0$ , in that case the equilibrium is given by:

$$MbkT^* = C'(e_1^*) \quad Mbk = C'(e_2^*) \quad S_1^* = S_2^* = \hat{S} \text{ with } T^* \equiv T(e_1^*, e_2^*, S_1 = S_2 = \hat{S})$$

### Proof

See point 2 in the Appendix.

## 4/ DISCUSSION

Proposition 1 shows that equilibrium inventory level,  $\hat{S}$ , as well as its associated  $TC$ , is achieved in a single period when the OM implements some managerial efforts in the first period. On the contrary, when he or she exercises no effort in the initial period, the steady-state equilibrium is not achieved until the second period. Obviously, in the former situation, in comparison with the latter, there is no inventory variability between period one and period two. Also, from this equilibrium, we obtain that once the steady-state inventory level,  $\hat{S}$ , is achieved, there is a drastic increase of managerial efforts from  $e_1 = 0$  to  $e_1 = e_1^*$ .

Remarkably, whenever  $TC_0$  is high, important reductions in  $TC$  are required and, the compensation scheme must favor managerial efforts,  $e$ , in order to make the  $\hat{T} > 0 \Leftrightarrow e_1 > 0$  outcome more likely. A natural way to achieve this is by raising  $k$  (the relative weight of the managerial effort in the compensation scheme). Specifically, expression (8) shows that when there is an important reduction in first-period total inventory cost ( $\Delta_1 = TC_0 - TC_1$  high), there is also a significant decrease in the  $T$  value, which, in turn, also decreases -by (6) and  $C'' > 0$ - first-period effort,  $e_1$ . The logic of this result is that a remarkable reduction of  $TC_1$  generates limited second-period OM gains because of reductions in  $TC$  ( $TC_1 - TC_2$ ) are more difficult to achieve when  $TC_1$  is low. A way to offset these diminished revenues is by reducing first-period effort  $e_1$ , because this opens the possibility of substantial second-period OM gains linked to managerial effort ( $k(e_2 - e_1)$  may be high when  $e_1$  is low). Thus, increasing OM effort compensation with a high  $k$ , favors the possibility of steep inventory cost reduction in the first period. This is stated in the following Proposition.

## Proposition 2

*OM compensation for managerial efforts allows for significant reductions in inventory-related costs so as to achieve the optimal long-term inventory level in a short period of time. This generates, in future periods, a reduced variation in a firm's inventory level.*

### Proof

Follows directly from Proposition 1.

This result may justify a firm's policy of promoting the managerial role of OMs. In that case OMs try to reduce  $TC$  to achieve the optimal level in a single period and after that, they focus on increasing managerial efforts. This describes a pattern of OMs' behavior that should be tested empirically.

Interestingly, these efforts, as we stated in the introduction, may be completely unrelated to inventory management. In fact, we are assuming this to be true, otherwise, the compensation scheme perceived by OMs would rely exclusively on reductions in an "amplified" total inventory cost function, as we discuss in the next section. Note also that by requiring some monitoring for verification, we assume implicitly that these managerial efforts are of a different nature to those implemented in the definition of inventory management (these latter efforts do not require any monitoring).

An example of these managerial efforts, although to some degree related with inventory management, is labor hiring. Some authors show that in case of high demand shocks variability, an OM with both responsibilities may decide to use worker turnover to smooth inventory variability (Haltiwanger and Maccini, 1994). In our model, high demand variability (high  $\bar{\epsilon}$ ) makes the  $\hat{T} > 0$  outcome in (9) more probable. This leads to the equilibrium with non-null managerial effort (*i.e.* labor hiring effort) and smoother inventory policy.

Proposition 1 also allows describing those scenarios with high inventory variability in the medium term as we define it. This is the difference between second-period inventory level and the first-period one. Basically, this analysis relies on the inspection of the expression of  $\hat{T}$  when  $\hat{T} < 0$ ,<sup>15</sup> which can be rearranged (see point 3 in the appendix) as:

$$\hat{T} = \frac{2Q}{c_f \Delta \bar{\epsilon}} (E_0(TC_1[\hat{S}]) - E_0(TC_1[\bar{S}])) \quad (10)$$

Thus, a more negative  $\hat{T}$  means an increase in the difference between  $\bar{S}$  (first-period inventories) and  $\hat{S}$  (second-period inventories). This will lead to an increase in inventory variability<sup>16</sup>.

Also, expression  $\hat{T}$  in (9) and (10) allows making a comparative static analysis with regard to structural parameters like  $\frac{a}{b}$ ,  $TC_0$ ,  $M$ ,  $k$ . This defines the following Proposition.

## Proposition 3

*When the fixed portion of the OM compensation is larger than the variable part ( $\frac{a}{b}$  high); and/or the owner's monitoring intensity is large enough ( $M$  large); and/or effort incentive scheme is high enough ( $k$  high) and/or the initial inventory costs are not very high ( $TC_0$  low), three consequences follow. Firstly, there is a sharp reduction in inventory  $TC$  and the steady-state level is achieved in a single period. Secondly, inventory policy is quite stable in the medium term, and lastly, OM implements managerial efforts during the initial period.*

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<sup>15</sup> The case of  $\hat{T} \geq 0$  leads to null inventory variability.

<sup>16</sup> To try to smooth this variability is relevant because at a macroeconomic level up to 87% of GDP variance during short-term recessions is linked with drastic inventory reductions (Blinder and Maccini, 1991).



## Proof

Directly by inspecting  $\hat{T}$  and making use of the result of Proposition 1.

This proposition describes different mechanisms that can lead to a medium-term inventory variability that is low when an owner tries to reduce inventory costs. Firstly, the design of compensation scheme that combines a high fixed part with large payments for OM managerial effort (high  $k$ ). Second, an intensive monitoring (high  $M$ ) of OM managerial efforts. This leads to some testable predictions like a lower inventory variability in those firms owned by banks (recognized as specialists in monitoring), in comparison with those owned by non-banks (Tribó, 2006).

This model also predicts that whenever initial  $TC$  is high and, eventually, a new OM is hired to deal with this situation, an OM implements no managerial efforts and focuses mainly on reducing inventory  $TC$ . On the other hand, when a long-tenure OM is in charge, generally, he/she is also involved in non inventory-related activities. These OMs have developed the required skills to carry out other responsibilities not directly related to inventories. In that case, our model shows that by receiving good incentives, these OMs are able to: reduce inventory  $TC$  to a greater extent; ensure a convergence to the steady-state inventory level in a shorter period of time; and define a smoother inventory policy. This result is consistent with some empirical studies that compare inventory variability between firms with long-tenure OMs and those with recently-appointed ones (Alfaro and Tribó, 2003).

As a final comment, we can integrate in our analysis factors related to firm's market structure as well as the characteristics of the goods that a firm produces. As a first approximation, we can consider that a high (low) value of  $c_r$  may represent mainly competitive (monopolistic) markets, while a high (low) value of  $c_h$  may be linked to perishable (perennial) goods. Simple inspection of  $\hat{T}$  in (9) reveals that  $\frac{\partial \hat{T}}{\partial c_r} > 0$  and  $\frac{\partial \hat{T}}{\partial c_h} > 0$ .

Thus, from this point of view, the more competitive the markets and/or the more perishable the goods traded, the more likely are those features linked to the  $\hat{T} > 0$  equilibrium: drastic  $TC$  reduction in the short-term and low inventory variability in the medium term. However, this result relies also on the value of  $TC_0$ , which depends not only on the initial conditions of the firm but also on the market structure and the type of goods.

## 5/ EXTENSIONS

This model can be extended in two ways in order to reinforce our conclusions.

One possibility is to consider that OM has to achieve an objective total cost ( $TC^o$ ). From that level, OM is compensated (penalized) for any further reductions (increases). To be more realistic, we can assume that the principal (entrepreneur) does not know exactly what the optimal level to be accomplished by the OM should be (asymmetric information). In this case, a solution for the principal is to update this optimal level, taking as reference the reductions in  $TC$  that OM has achieved in the previous period. Formally:  $TC_{i+1}^o = TC_i^o - \mathbf{d}(TC_i^o - TC_i)$ , where  $0 \leq \mathbf{d} \leq 1$ <sup>17</sup>. In this case,  $\mathbf{d} = 1$  means that the reduction in the total cost achieved by the OM is a complete indication of what the objective  $TC$  function should be in the next period (there is a full information updating). On the contrary,  $\mathbf{d} = 0$  means that the objective  $TC$  is the same for both periods. Interestingly, when there is information updating ( $\mathbf{d} \rightarrow 1$ ), the OM compensation scheme coincides with that of the model. Note that  $TC_2^o \rightarrow TC_1$  as  $\mathbf{d} \rightarrow 1$ . In this case, OM compensation is:

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<sup>17</sup> By penalizing appropriately increases in the  $TC$  from the objective level, we prevent OM from behaving opportunistically by setting a high  $TC$  in the previous period in order to induce the entrepreneur to fix an achievable objective  $TC$  function in the present one.

$$w_i = \mathbf{a} + \mathbf{b}(TC_{i-1}^o - \mathbf{d}(TC_{i-1}^o - TC_{i-1}) - TC_i) + Mk(e_i - e_{i-1}) \rightarrow \mathbf{a} + \mathbf{b}(TC_{i-1} - TC_i) + Mk(e_i - e_{i-1})$$

More specifically, there exists a threshold value  $0 < \hat{\mathbf{d}} < 1$  such that for  $\mathbf{d} > \hat{\mathbf{d}}$  the equilibrium in inventory and effort levels is of the type given in Proposition 1 and all the analysis made throughout the paper applies. However, when  $\mathbf{d} < \hat{\mathbf{d}}$ , only the equilibrium with low inventory variability and significant  $TC$  reduction exists. Interestingly, in this latter situation strong first-period  $TC$  reductions do not preclude OM from implementing significant first-period effort,  $e_1$ , as we have found in our model. Conversely, large first-period OM efforts do not hinder first-period  $TC$  reductions that characterize the so-called good-equilibrium. In this case, the beneficial effects of managerial efforts are even greater than in our model as there is less need to give up substantial first-period managerial efforts to achieve a strong first-period inventory  $TC$  reduction. In particular, the marginal loss in the compensation for inventory cost reduction is proportional to  $((1+\mathbf{d})e_1)$ . When  $\mathbf{d}$  is low, implementing high managerial efforts has a limited detrimental effect on the OM's compensation for reductions achieved in inventory costs.

Another extension is to consider that managerial efforts are exclusively linked to inventory management. For example, in the JIT approach the OM implements efforts to continuously improve the parameters of the existing cost function. In this case, there are different modeling possibilities. On the one hand, we can consider that these managerial efforts reduce  $TC$  function in an additive way, that is,

$TC_i = c_f \frac{D_i}{Q} + c_h \mathbf{q} \frac{S_i^2}{2Q} + c_r \mathbf{q} \frac{(Q - S_i)^2}{2Q} - Ke_i$ . In this case, OM compensation based on reducing the overall  $TC$  from an objective  $TC$  level ( $\mathbf{d}=1$ ) transforms to an equivalent function to expression (2) of the model:

$w_i = \mathbf{a} + \mathbf{b}(TC_{i-1} - ke_{i-1} - (TC_i - ke_i)) = \mathbf{a} + \mathbf{b}(TC_{i-1} - TC_i + k(e_i - e_{i-1}))$ . Hence, the analysis made throughout the paper applies.

On the other hand, there is an alternative that is to introduce these managerial efforts in a multiplicative way; for example, by reducing  $c_f, c_h, c_r$  by a factor  $(1-e)$ . In this case, the manager will implement low managerial efforts in the first period in order to make the objective  $TC$  function in the second period achievable. Then, in this latter period the OM will implement such efforts to ensure steep reduction in the  $TC$  and cash them in accordingly. Hence, this dynamic is quite similar to that described in our model, but in this case these managerial efforts are less beneficial for stimulating  $TC$  reductions.

## 6/ CONCLUSIONS

In this paper we show that an Operations Manager (OM) who devotes some efforts to define a firm's strategy may outperform an OM exclusively devoted to inventory management because the latter has more incentive to behave opportunistically. This is achieved by implementing a compensation scheme that rewards inventory cost reductions as well as the increase in other value-generating managerial efforts. With such a scheme, an OM has incentives to reduce significantly inventory costs in the short-term although this lowers the possibility to cut inventory costs in the future. Thus, an OM will only have incentives to make such reductions if these future losses are offset with other gains. These may be achieved by rewarding managerial efforts adequately.

According to the proposed incentive scheme, we conclude the following OMs behavior. In the first period, OMs devote more efforts to reduce inventory costs than to their managerial responsibilities. However, when the steady-state inventory level is reached, the managerial component of the OM efforts linked to strategic tasks is more important. Thus, paradoxically, by combining both types of efforts, there is an intense inventory cost reduction

in the short-term that drives the firm's inventory level to its steady-state value. This may not be true when an OM only manages firm's inventory. In this case he or she behaves strategically and avoids sharp reductions in inventory costs in the short-term because this would hinder future reductions in these costs.

Our analysis also highlights different features. Firstly, in an efficiently design contract framework, when we compare firms with long-serving OMs with those recently appointed, a smoother medium-term inventory variation is expected in the former in comparison with the latter. Secondly, our model has some relationship with those models that compare centralized versus non-centralized inventory decision-making. An OM with managerial and operations responsibilities forms part of a highly centralized decision-making framework. In that case our result of lower inventory variability value in comparison with less centralized frameworks coincides with Matsuura and Tsubone (1993). Thirdly, those firms with an OM involved in an increasing amount of additional responsibilities need not be concerned about their lack of focus. The appropriate design of the compensation package based, paradoxically, on relevant payments for those non-inventory related activities could overcome all these problems. This result is consistent with some common features of behavioral theory: In a context with information asymmetries, due to uncertainty, investment variability may be reduced once we have designed an efficient compensation scheme that weights variables that are sufficiently informative in capturing managerial behavior (non-inventory related efforts). This is a clear message that can be extracted from this paper: it is possible to shape the "natural" tendency of OMs to act as strategic managers, in a way that does not damage a firm's inventory policy and even prevent them from behaving opportunistically on inventory management.

Our model has some limitations that open new avenues for future research. This is a two-period static model that does not allow us to analyze the dynamics of the OM role within the firm. A dynamic model would open up the possibility to study issues of OM related to career concerns. We expect that in such a model, the OM would internalize their promotion possibilities which, in turn, would provide further incentives to exert managerial efforts. Hence, we expect the results of our model to be reinforced when dynamic effects are incorporated. Questions of information asymmetries are also ruled out for the sake of simplicity. However, although simple, our model conclusions are robust to different modeling alternatives. Also a set of empirical predictions concerning firms' inventory variability emerges from our theoretical results. We expect lower inventory variability in those firms with: long-serving OMs (who carry out these managerial efforts); large fix-part managerial compensation; and efficiently monitoring (*i.e.* with banks as shareholders). Also, from our model we find that OM compensation is based mainly on inventory cost reduction in the short-term and on managerial efforts in the medium-term. Finally, once we incorporate some additional questions of market microstructure, we get that in monopolistic markets with low unitary holding cost inventory variability is especially high. Empirically test these theoretical outcomes will be the subject of future research.

## REFERENCES

- Alfaro, J., and Tribó, J. 2003. Operations manager turnover and inventory fluctuations. *International Journal of Production Economics*, 81-82: 51-58.
- Armistead, C.G., and Mapes, J. 1992. The changing role of supply chain operations managers. *Logistics Information Management*, 5 (2): 10-17.
- Blinder, A.S., and Maccini, L.J. 1991. Taking stock: A critical assessment of recent research on inventories. *Journal of Economic Perspectives*, 5 (1): 73-96.
- Chang, J., and Lai, C. 1999. Carrots or sticks? A social custom viewpoint on worker effort. *European Journal of Political Economics*, 28 (2): 297-310.
- Diamond, D. W. 2001. Monitoring and Reputation: The Choice between Bank Loans and Directly Placed Debt. *The Journal of Political Economy*, 99 (4): 689-721.
- D'Netto, B., and Sohal, A. S. 1999. Changes in the production manager's job: past, present and future trends. *International Journal of Operations and Production Management*, 19 (2): 157-181.
- Gerwin, D. 1993. Manufacturing flexibility: A strategic perspective. *Management Science*, 39 (4): 395-411.
- Haltiwanger, J.C., and Maccini L.J. 1994. Inventories and multi-period labor contracts: Implications for business cycle analysis. *Inventory Cycle and Monetary Policy*: 175-205. New York: Springer-Verlag.
- Hum, S. H., and Leow, L. H. 1992. The perception of the strategic role of manufacturing amongst operations managers: An empirical study based on a newly industrialized economy. *International Journal of Operations and Production Management*, 12 (11): 15-23.
- Khan, J., 1991. *Why is production more volatile than sales? Theory and evidence on the stockout-avoidance motive for inventory-holding*, Working Paper, E-91-7. Hoover Institution.
- Mak, K. T., and Ramaprasad, A. 2003. Knowledge supply network. *The Journal of the Operational Research Society*, 54 (2): 175-183.
- Matsuura, H., and Tsubone, H. 1993. A comparison of centralized and decentralized control rules in push-type production ordering systems. *European Journal of Operational Research*, 70 (2): 199-211.
- Milgrom, P. and Roberts, J. 1992. Economics, Organization and Management, Prentice Hall (ed.)
- Nie, W., and Young, S. T. 1997. A study of operations and marketing goal consensus in the banking industry. *International Journal of Operations & Production Management* 17 (8): 806-820.
- Oakland, J.S., and Sohal, A. 1989. The education, training and career of production managers in British industry. *International Journal of Operations and Production Management*, 9 (8): 63-90.
- Repullo, R., and Suarez, J. 2004. Venture Capital Finance: A Security Design Approach. *Review of Finance*, 8 (1): 75-108.
- Tribó, J. A. 2006. Ownership Structure and Inventory Policy. *International Journal of Production Economics*, in press.
- Wild, R. 1979. Organization Design, Technology and the Influence of Operations Management. *Personnel Review*, 8 (3): 5-15.
- White, E. M., Wharton, T. J., 1990. Current Manufacturing Approaches: Opinion and Practice. *Production and Inventory Management Journal* 31 (4) 38-44.

## APPENDIX

$$1/ \partial_{e_1} E_0 \{O_1\} = -\mathbf{b}(1-p_c)Mk + \mathbf{b} \frac{\partial p_c}{\partial e_1} (E_0 TC_2 - E_0 TC_1 + Mk(e_1 - e_2) - \frac{\mathbf{a}}{\mathbf{b}}) + C'[e_1] \quad (\text{A1.1})$$

$$\partial_{S_1} E_0 \{O_1\} = \mathbf{b}(1-p_c) \left( \frac{\partial E_0 TC_1}{\partial S_1} \right) + \mathbf{b} \frac{\partial p_c}{\partial S_1} (E_0 TC_2 - E_0 TC_1 + Mk(e_1 - e_2) - \frac{\mathbf{a}}{\mathbf{b}}) \quad (\text{A1.2})$$

$$\text{With } p_c = \frac{Q}{c_f \Delta \mathbf{e}} \left[ TC_0 - E_0 TC_1 + Mk(e_1 - e_0) - \frac{c_f}{Q} \bar{\mathbf{e}} \right] \Rightarrow \frac{\partial p_c}{\partial e_1} = \frac{MkQ}{c_f \Delta \mathbf{e}} \quad \text{and} \quad \frac{\partial p_c}{\partial S_1} = -\frac{Q}{c_f \Delta \mathbf{e}} \frac{\partial E_0 TC_1}{\partial S_1}$$

Arranging the previous expressions, we get in:

$$\partial_{e_1} \{O_1\} = -\mathbf{Mbk} \left\{ \frac{Q}{c_f \Delta \mathbf{e}} \left( \frac{c_f \bar{\mathbf{e}}}{Q} + \Delta_2 - \Delta_1 + \frac{\mathbf{a}}{\mathbf{b}} + Mk(e_2 - 2e_1 + e_0) \right) \right\} + C'(e_1) = -\mathbf{Mbk}T + C'(e_1) \quad (\text{A1.3})$$

$$\partial_{S_1} E_0 \{O_1\} = \mathbf{b} \frac{Q}{c_f \Delta \mathbf{e}} \left( \frac{c_f \bar{\mathbf{e}}}{Q} + \Delta_2 - \Delta_1 + \frac{\mathbf{a}}{\mathbf{b}} + Mk(e_2 - 2e_1 + e_0) \right) \frac{\partial E_0 TC_1}{\partial S_1} = \mathbf{b}T \frac{\partial E_0 TC_1}{\partial S_1} \quad (\text{A1.4})$$

$$\text{With } T \equiv \frac{Q}{c_f \Delta \mathbf{e}} \left( \frac{c_f \bar{\mathbf{e}}}{Q} + \Delta_2 - \Delta_1 + \frac{\mathbf{a}}{\mathbf{b}} + Mk(e_2 + e_0 - 2e_1) \right) \quad \Delta_2 \equiv (E_0 TC_1 - E_0 TC_2) \quad \text{and} \quad \Delta_1 \equiv (TC_0 - E_0 TC_1)$$

2/ By inspecting (A1.3) and (A1.4), we can distinguish two situations:

a) If  $\hat{T} \equiv T(e_1 = 0, e_2 = e_2^*, S_1 = S_2 = \hat{S}) > 0$  with  $\hat{S} \equiv \left( \frac{c_r}{c_h + c_r} \right) Q$  from  $\frac{\partial E_0 TC_1}{\partial \hat{S}} = 0$  and  $\mathbf{Mbk} = C'(e_2)$

In this case, the optimal inventory and effort values are  $S_1^* = \hat{S}_1$   $\mathbf{Mbk}T^* = C'(e_1^*)$ . Where:

$$T^* \equiv T(e_1^*, e_2^*, S_1 = S_2 = \hat{S}) = \frac{Q}{c_f \Delta \mathbf{e}} \left( \frac{c_f (\bar{D} + \bar{\mathbf{e}})}{Q} + \mathbf{q} \frac{c_r c_h Q}{2(c_r + c_h)_1} + \frac{\mathbf{a}}{\mathbf{b}} - TC_0 + Mk(e_2^* + e_0 - 2e_1^*) \right) \quad \text{with } \mathbf{bMk} = C'(e_2^*)$$

Note that  $C'[e_1 = 0] = 0$  and  $C'[e_1 > 0] > 0$  joint with  $\frac{\partial T}{\partial e_1} < 0$  ensures an interior solution for effort  $e_1$  (see A1.3).

Second order condition of (A1.4) for  $S = \hat{S}$ , ensure that:

$$\partial_{S_1}^2 \{O_1\} \Big|_{\hat{S}_1} = \mathbf{b} \frac{\partial T}{\partial S_1} E_0 TC_1 + \mathbf{b}T \frac{\partial^2 E_0 TC_1}{\partial S_1^2} \Big|_{\hat{S}_1} = \mathbf{b}T \frac{\partial^2 E_0 TC_1}{\partial S_1^2} \Big|_{\hat{S}_1} > 0 \quad \text{as } T(\hat{S}_1) > 0 \quad \text{and} \quad \frac{\partial^2 E_0 TC_1}{\partial S_1^2} > 0$$

Thus, this is a minimum. Moreover, as  $\hat{S}$  is a minimum for  $T(S_1)$ , then the condition  $\hat{T} > 0$  ensures that  $T(S_1) > 0$  for any  $S_1$ . This feature neglects  $T=0$  as a minimum.

Finally, note that the Hessian is also positively definite as

$$\partial_{S_1}^2 \{O_1\} \Big|_{e^* \hat{S}_1} > 0; \quad \partial_{e_1}^2 \{O_1\} \Big|_{e^* \hat{S}_1} > 0 \quad \text{and} \quad \partial_{e_1 S_1}^2 \{O_1\} \Big|_{e^* \hat{S}_1} = -\mathbf{Mbk} \frac{\partial T}{\partial S_1} \Big|_{e^* \hat{S}_1} = -\frac{2\mathbf{Mbk}Q}{c_f \Delta \mathbf{e}} \frac{\partial E_0 (TC_1)}{\partial S_1} \Big|_{e^* \hat{S}_1} = 0$$

b) If  $\hat{T} \equiv T(e_1 = 0, e_2 = e_2^*, S_1 = S_2 = \hat{S}) < 0$ . In this case,  $C'(e_1 = 0) = 0$  with  $C'(e_1 > 0) > 0$  in (A1.3) ensures that  $\partial_{e_1} \{O_1(e_1 = 0)\} > 0 \Rightarrow e_1^* = 0$  (we are minimizing). Moreover, (A1.4) shows

$$\text{that } \hat{S}_1 \text{ cannot be optimal, as } \partial_{S_1}^2 \{O_1\} \Big|_{\hat{S}_1} = \mathbf{b} \frac{\partial T}{\partial S_1} \frac{\partial E_0 TC_1}{\partial S_1} + \mathbf{b}T \frac{\partial^2 E_0 TC_1}{\partial S_1^2} \Big|_{\hat{S}_1} = \mathbf{b}T \frac{\partial^2 E_0 TC_1}{\partial S_1^2} < 0,$$

which is the condition of a maximum. Thus, the solution of  $\partial_{S_1}\{TC\} = \mathbf{b}T \frac{\partial E_0 TC_1}{\partial S_1} = 0$  cannot be

$$\frac{\partial E_0 TC_1}{\partial \hat{S}} = 0 \text{ but } S_1^* = \bar{S} \text{ that satisfies } T(e_1 = 0, e_2^*, S_1 = \bar{S}, S_2 = \hat{S}) = 0: ^{18}$$

$$T(e_1 = 0, e_2^*, S_1 = \bar{S}, S_2 = \hat{S}) = \frac{Q}{c_f \Delta \mathbf{e}} \left( \frac{c_f(\bar{D} + \bar{\mathbf{e}})}{Q} + 2c_h \mathbf{q} \frac{\bar{S}_1^2}{2Q} + 2c_r \mathbf{q} \frac{(Q - \bar{S}_1)^2}{2Q} - \mathbf{q} \frac{c_r c_h Q}{2(c_r + c_h)_1} + \frac{\mathbf{a}}{\mathbf{b}} - TC_0 + Mk(e_2^* + e_0) \right)$$

Thus, by choosing the root of the previous polynomial with  $\bar{S}_1 > \hat{S}_1$ , we can ensure that

$$\partial^2_{S_1} \{O_1\} \Big|_{\bar{S}_1} = \mathbf{b} \frac{\partial T}{\partial S_1} E_0 TC_1 + \mathbf{b} T \frac{\partial^2 E_0 TC_1}{\partial S_1^2} \Big|_{\bar{S}_1} = \mathbf{b} \frac{\partial T}{\partial S_1} E_0 TC_1 \Big|_{\bar{S}_1} > 0$$

$$\text{For } S_1 = \bar{S}_1, \partial_{e_1} \{O_1\} \Big|_{(e_1=0, \bar{S}_1)} = C'(e_1 = 0) = 0 \text{ with } \partial^2_{e_1} \{O_1\} \Big|_{(e_1=0, \bar{S}_1)} = -Mbk \frac{\partial T}{\partial e_1} + C'(e_1) > 0.$$

Moreover,  $e_1^* = 0$  and  $S_1 = \bar{S}_1$  is a minimum as the Hessian (H) is positively definite: <sup>19</sup>

$$H = (\partial^2_{S_1} \{O_1\})(\partial^2_{e_1} \{O_1\}) - (\partial^2_{e_1 S_1} \{O_1\})^2 \Big|_{e_1=0, \bar{S}_1} = \left( \frac{2MbkQ}{c_f \Delta \mathbf{e}} \right)^2 \frac{\partial E_0(TC_1)}{\partial S_1} \left[ E_0(TC_1) - \frac{\partial E_0(TC_1)}{\partial S_1} \right] \Big|_{e_1=0, \bar{S}_1} > 0 \text{ as } \bar{D}$$

is large enough (Assumption 1). Note also that

$$H \Big|_{e_1=0, \bar{S}_1} = \left( \frac{2MbkQ}{c_f \Delta \mathbf{e}} \right)^2 \frac{\partial E_0(TC_1)}{\partial S_1} \left[ E_0(TC_1) - \frac{\partial E_0(TC_1)}{\partial S_1} \right] \Big|_{e_1=0, \bar{S}_1} > 0 \text{ where } E_0(TC_1) \text{ is minimum for}$$

$$S_1 = \hat{S}_1$$

$$3/ T(e_1 = 0, e_2^*, S_1 = \bar{S}, S_2 = \hat{S}) = \frac{Q}{c_f \Delta \mathbf{e}} \left( \frac{c_f(\bar{D} + \bar{\mathbf{e}})}{Q} + 2c_h \mathbf{q} \frac{\bar{S}_1^2}{2Q} + 2c_r \mathbf{q} \frac{(Q - \bar{S}_1)^2}{2Q} - \mathbf{q} \frac{c_r c_h Q}{2(c_r + c_h)_1} + \frac{\mathbf{a}}{\mathbf{b}} - TC_0 + Mk(e_2^* + e_0) \right) = 0$$

$$\Rightarrow \frac{c_f(\bar{D} + \bar{\mathbf{e}})}{Q} + \mathbf{q} \frac{c_r c_h Q}{2(c_r + c_h)_1} - \frac{\mathbf{a}}{\mathbf{b}} + TC_0 - Mk(e_2^* + e_0) = 2 \frac{c_f \bar{D}}{Q} + 2c_h \mathbf{q} \frac{\bar{S}_1^2}{2Q} + 2c_r \mathbf{q} \frac{(Q - \bar{S}_1)^2}{2Q} \equiv 2E_0(TC_1[\hat{S}])$$

If we use the definition of  $\hat{T}$  in (9) and the fact that  $E_0(TC_1[\hat{S}]) = \frac{c_f \bar{D}}{Q} + \mathbf{q} \frac{c_r c_h Q}{2(c_r + c_h)_1}$ , we obtain:

$$\hat{T} = \frac{Q}{c_f \Delta \mathbf{e}} \left( 2 \frac{c_f(\bar{D})}{Q} + 2\mathbf{q} \frac{c_r c_h Q}{2(c_r + c_h)_1} - \left( \frac{c_f(\bar{D} + \bar{\mathbf{e}})}{Q} + \mathbf{q} \frac{c_r c_h Q}{2(c_r + c_h)_1} - \frac{\mathbf{a}}{\mathbf{b}} + TC_0 - Mk(e_2^* + e_0) \right) \right) \Rightarrow$$

$$\Rightarrow \hat{T} = \frac{2Q}{c_f \Delta \mathbf{e}} (E_0(TC_1[\hat{S}]) - E_0(TC_1[\bar{S}]))$$

<sup>18</sup> This is a second-order polynomial with a real solution when  $\hat{T} < 0$ . This is the case because  $\bar{T}(\bar{S}_1 = \hat{S}_1) = \hat{T}$  and  $\bar{T}$  increases with  $\bar{S}_1$  for  $\bar{S}_1 > \hat{S}_1$ . Thus it will exist a  $\bar{S}_1 > \hat{S}_1$  such that  $\bar{T} = 0$

<sup>19</sup> See assumption 5 for the cost function C.