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Acyclicity and Singleton Cores in Matching Markets*

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Abstract

This paper analyzes the role of acyclicity in singleton cores. We show that the absence of simultaneous cycles is a sufficient condition for the existence of singleton cores. Furthermore, acyclicity in the preferences of either side of the market is a minimal condition that guarantees the existence of singleton cores. If firms or workers preferences are acyclical, unique stable matching is obtained through a procedure that resembles a serial dictatorship. Thus, acyclicity generalizes the notion of common preferences. It follows that if the firms or workers preferences are acyclical, unique stable matching is strongly efficient for the other side of the market.

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1 Introduction

This paper studies the relationship between acyclicity and singleton cores of many-to-one matching markets. A cycle in the preferences of firms arises when there is an alternating list of firms and workers "in a circle" such that every firm in the cycle prefers the worker on its clockwise side to the worker on its counterclockwise side but finds both acceptable. The preferences of one side of the market are acyclical if they have no cycles of any length. We prove that the absence of simultaneous cycles implies that the core is a singleton. Furthermore, acyclicity in the preferences of either side of the market is a minimal condition that guarantees that the stable set is a singleton.

We show that the notion of acyclical preferences is a generalization of the notion of common preferences. Indeed, if the preferences of one side of the market are acyclical, unique stable matching can be obtained through a "corrected" serial dictatorship. It follows that if firm preferences (workers) are acyclical and publicly known, it is a dominant strategy for workers (firms) to reveal their true preferences. Finally, we prove that the absence of simultaneous cycles implies that unique stable matching is strongly efficient for both sides of the market.

The existence of singleton cores is relevant to the matching market literature. Sönmez 1996) studies strategy-proofness in the context of college admissions problems and shows that there exits an allocation rule that is Pareto efficient, individually rational and strategy-proof if and only if each college has an unlimited number of slots or, in other words, if the core is single valued. Sönmez (1999) generalizes this result to more general matching problems. Alternative conditions for singleton core have been presented in the literature. Eeckhout (2000) identifies a sufficient condition for singleton cores in the context of marriage problem. This condition requires that no male or female prefers a mate of the opposite six with the same rank order below his or her own order. Clark (2006) too introduces a sufficient condition for singleton cores called no crossing condition. Our condition is independent from the conditions in Eeckhout (2000) and Clark (2006).[1] Ehler and Massó (2007) explore the relationship between singleton cores and the existence of equilibrium in centralized matching markets with incomplete information. They show that truth-telling is an ordinal Bayesian Nash equilibrium of the revelation game, which is induced by a common belief and stable mechanism if and only if all profiles that are in support of the common belief have singleton cores.

Ergin (2002) introduces an alternative notion of acyclicity and shows that worker-optimal stable matching is efficient if and only if the preferences of the firms are acyclical .[2] Haeringer and Klijn (2009) show that Ergin acyclicity is a necessary and sufficient condition for Nash implementation of the stable correspondence. In the context of housing markets, Kesten (2006) showed that for some fixed priority profiles the deferred acceptance rule and the top trading cycle rule are equivalent if and only if the preference profile is acyclic (see also Kesten 2010).

Finally, Romero-Medina and Triossi (2011) prove that when the hospital-optimal stable rule is employed, acyclicity is the minimal condition that guarantees the stability of NE of the capacity manipulation games.

The structure of the paper is as follows. Section 2 presents the model and in Section 3 and its relation to serial dictatorship and efficiency. Section 4 concludes.

[1] Examples are available upon request.

[2] Ergin's acyclicity is weaker than the concept of acyclicity used in this paper but it is independent of the notion of "absence of simultaneous cycles". Furthermore, Ergin's acyclicity does not guarantee that the stable set is a singleton.

2 The Model

In the hiring problem examined in this paper, there are a number of workers, each of whom is seeking a position at one of many firms. Let $F = \{f_1, ..., f_k\}$ be the set of firms, let $W = \{w_1, ..., w_t\}$ be the set of workers, let $P_F = (P_{f_1}, ..., P_{f_k})$ be a list of the firms' preferences on workers and let $P_W = (P_{w_1}, ..., P_{w_t})$ be a list of workers' preferences on firms. The triple (F, W, P), where $P = (P_F, P_W)$ is called a **Matching Market**. For every $f \in F$, P_f, there is a strict order defined on 2^W, the set of all subsets of W.[3] Let $W' \subset W$ be a set of workers. The preferred group of workers for firm f among the ones belonging to W' is called the **choice set from** W'. It is denoted by $Ch_f(W', P_f)$ or by $Ch_f(W')$, when there is no potential for ambiguity. Formally, $Ch_f(W', P_f) = \arg\max_{P_f} \{W'' : W'' \subset W'\}$. If $\varnothing P_f W'$, firm f prefers not to employ any worker rather than jointly employ the workers in W' and W' is called **unacceptable to** f. Otherwise, W' is **acceptable to** f. $A(f)$ denotes the set of workers who are individually acceptable to f. The maximum numbers of workers firm f is willing to hire is f's **capacity** and is denoted by q_f, or formally $q_f = \max\{|W'| : Ch_f(W', P_f) \neq \varnothing\}$.[4] For every $w \in W$, P_w there is a strict order defined on $F \cup \{w\}$. Any firm f such that wP_wf is called **unacceptable to** w. Otherwise, f is **acceptable to** w. $A(w)$ denotes the set of firms that are acceptable to w. For every agent $x \in F \cup W$, R_x denotes x's weak preference relation.

A matching allocates workers to firms. A **matching** on (F, W) is a function $\mu : F \cup W \to 2^W \cup F$ such that for every $(f, w) \in F \times W$: (i) $\mu(f) \in 2^W$, (ii) $\mu(w) \in F \cup \{w\}$ and (iii) $\mu(w) = f \Leftrightarrow w \in \mu(f)$. We denote by $\mathcal{M}$ the set of matchings on (F, W). A matching μ

[3]For all $w, w' \in S$ $\; wP_fw'$, $wP_f\varnothing$ and $\varnothing P_fw$ denote $\{w\} P_f \{w'\}$, $\{w\} P_f \varnothing$ and $\varnothing P_f \{w\}$, respectively.
[4]$|W'|$ stands for the cardinality of the set W'

is **individually rational** if (i) $Ch_f(\mu(f)) = \mu(f)$ for all $f \in F$ and (ii) $\mu(w) R_w w$ for all $w \in W$. A matching μ is **blocked by the pair** $(f, w) \in F \times W$ if (i) $f P_w \mu(w)$ and (ii) $w \in Ch_f(\mu(f) \cup \{w\})$. A matching μ is **stable in** (F, W, P) if it is individually rational and no pair blocks it. Otherwise, μ is **unstable**. $\Gamma(F, W, P)$ denotes the **stable set**, which is the set of matchings that are stable in market (F, W, P).

The stable set may be empty. This is why the literature has focused on preference restrictions according to which workers are not seen as complements. A firm f has responsive preferences if, for any two assignments that differ by one worker only, the firm prefers the assignment associated with the preferred worker. Formally, P_f are **responsive** if for all $W' \subset W$ such that $\sharp W' \leq q_f - 1$ and for all w, $w' \in W$: (i) $W' \cup \{w\} P_f W' \cup \{w'\} \Leftrightarrow w P_f w'$ and (ii) $W' \cup \{w\} P_f W' \Leftrightarrow w \in A(f)$. Under this restriction, the deferred acceptance algorithm (Gale and Shapley 1962) produces either the firm-optimal or the worker-optimal stable matching, which is denoted by μ^F and μ^W, respectively, depending on whether the firms or the workers make the offers (see Roth and Sotomayor 1990). Throughout the paper we assume that the preferences of every firm are responsive.

When there is no risk of ambiguity, we use P_F and P_W to denote the following binary relations within the set of matchings. For every μ,ν matchings, let $\mu P_F \nu$ if and only if $\mu(f) R_f \nu(f)$ for all $f \in F$ and $\mu(f) P_f \nu(f)$ for at least one f. Let $\mu P_W \nu$ if and only if $\mu(w) R_w \nu(w)$ for all $w \in W$ and $\mu(w) P_w \nu(w)$ for at least one w.

3 Acyclicity and singleton cores

A cycle in the preferences of the firms is a list of firms and workers "in a circle" in which every listed firm prefers the worker on its clockwise side to the worker on its counterclockwise side and finds both acceptable. Formally:

Definition 1 *A* ***cycle (of length*** $T+1$***) in the preferences of the firms*** *is given by* $f_0, f_1, ..., f_T$ *such that* $f_t \neq f_{t+1}$ *for* $t = 0, ..., T$ *and distinct* $w_0, w_1, ..., w_T$ *such that*

1. $w_T P_{f_T} w_{T-1} .... w_1 P_{f_1} w_0 P_{f_0} w_T$,

2. *for every* t*,* $w_t \in A(f_t) \cap A(f_{t-1})$.[5]

Assume that a cycle exists. If every w_t is initially assigned to f_{t+1}, every firm is willing to exchange its assigned worker with its successor. The notion of a cycle in the preference of the workers is specular.

A simultaneous cycle arises when there is a list of firms and workers that are simultaneously a cycle for the preferences of the firm and the preferences of the worker. Formally:

Definition 2 *A* ***simultaneous cycle*** *of length* $T+1$ *is a set of firms* $f_0, f_1, ..., f_T$ *and workers* $w_0, w_1, ..., w_T$ *forming a cycle both in the preferences of the firms and of the workers.*

If there are not simultaneous cycles, the stable set is a singleton.

Proposition 1 *Assume that no simultaneous cycle exists. If this is the case, the stable set of* (F, W, P) *is a singleton.*

[5] From now on indices are considered modulo $T+1$.

Proof. To prove that the stable set of (F,W,P) is a singleton when no simultaneous cycle exists, we show that if the stable set is not a singleton then there exists a simultaneous cycle. Assume that the stable set is not a singleton. Then there are two stable matching μ, ν such that $\mu P_F \nu$ and $\nu P_W \mu$. Set $W' = \{w : \nu(w) P_w \mu(w)\} \neq \emptyset$. Let $f_0 \in \mu(W')$, then $\mu(f_0) P_m \nu(f_0)$. Let $w_0 \in \mu(f_0) \setminus \nu(f_0)$, $w_0 \in M' = \mu(W')$. For all $n \geq 1$ set $f_{n+1} = \nu(w_{n+1})$ if $w_n \neq w_t$ for every $t < n$ and set $f_{n+1} = f_n$ otherwise. Observe that $f_0 \neq f_1$. Let $w_n = max_{P_{f_{n-1}}} \mu(f_{n-1}) \setminus (\nu(f_{n-1}) \cup \{w_1, ..., w_{n-1}\})$ if $\mu(f_{n-1}) \nsubseteq \nu(f_{n-1}) \cup \{w, ..., w_{n-1}\}$ and set $w_{n+1} = w_n$ otherwise. The sequence is stationary because W is finite and it stops at some $\bar{n} > 1$ such that $f_{\overline{n}} = f_{\overline{n}+1}$. Let l be such that $f_l = f_{\overline{n}}$. Set $j_n = w_{n+l}$ and $r_n = f_{n+l}$ for every $n \leq \overline{n} - l$. The sequence comprises different workers and two consecutive distinct firms. This sequence satisfies $\mu(j_n) = r_n = \nu(j_{n+1})$ for $n \leq \overline{n} - l$, and $\nu(j_l) = r_0$. It follows that (i) $j_0 P_{r_0} j_k P_{r_{k-1}} j_{k-1} ...... j_2 P_{r_2} j_1 P_{r_1} j_0$ and (ii) $r_0 P_{j_k} r_k P_{j_k} r_{k-1} ... P_{j_0} r_0$. Thus, $j_0, ..., j_l, r_0, ..., r_l$ is a simultaneous cycle. ■

From Proposition 1, it follows that the acyclicity in the preferences of either side of the market guarantees that the stable set is a singleton. From this perspective, the acyclicity condition is a minimal condition that guarantees that the stable set is a singleton.

Proposition 2 *Assume that there is a cycle in P_F (in P_W). If this is the case, there exists a profile of preferences for the workers, P_W (for the firms, P_F), such that the stable set of (F, W, P_F, P_W) contains at least two matchings.*

Proof. Assume that there a is cycle in P_F. Let $f_0, ..., f_T$ and $w_0, ..., w_T$ be as they are in Definition 2. Let $F' = F \setminus \{f_0, ..., f_T\}$ and let $W' = W \setminus \{w_0, ..., w_T\}$. Let $P_{W'}$ be any vector of the preferences for the workers in W' such that $A(w) \subset F'$ for all $w' \in W'$. Let $\bar{\mu}$ be any stable matching of $(F', W', P_{F'}, P_{W'})$. Let $P_{w_i} : f_i, f_{i+1}$ for $w = 0, ..., T-1$ and

$P_{w_T} : f_T, f_0$. Let $P_w : w$ if $w \notin \{w_0, ..., w_T\}$. Let $P_W = (P_{F'}, P_{w_0}, ..., P_{w_T})$. Define the matchings μ and ν as follows: $\mu(w_i) = f_i$ and $\nu(w_i) = f_{i+1}$ for $i = 0, ..., T-1$, $v(w_T) = f_0$. Let $\mu(w) = \nu(w) = \bar{\mu}(w)$ if $w \in W'$. Both μ and ν are stable in (F, W, P_F, P_W), so the stable set of (F, W, P_F, P_W) contains at least two stable matchings, μ and ν.

The proof of the claim when there is a cycle in P_W is identical and thus omitted. ∎

3.1 Acyclicity and serial dictatorship.

Next, we attempt to determine the restrictiveness of the acyclicity assumption. To this end, we first consider the case in which every worker (firm) is acceptable to all firms (workers). In this case, the preferences are acyclical if and only if they are the same for every firm (worker).

Proposition 3 *Assume that $wP_f\emptyset$ (fP_ww) for every $w \in W$ and for every $f \in F$. The preferences of the firms (workers) are acyclical if and only if the firms (respectively workers) have the same preferences on individual workers (respectively firms).*

Proof. Assume that the preferences of the firms are acyclical and that $wP_f\emptyset$ for every $w \in W$ and for every $f \in F$. Then there is no cycle of length two. This implies that all firms have the same preferences because all workers are acceptable to every firm Next, we prove that if the firms have the same preferences on individual workers, then there is no cycle in the preference of the firms. The proof is obtained by contradiction. Assume that $w_0 P_{f_0} w_T P_{f_T} w_{T-1} .... w_1 P_{f_1} w_0$ for some T and some $w_0, ..., w_T$, $f_0, ..., f_T$. Because $P_{f_0} = P_{f_T}$, we have $w_0 P_{f_0} w_1$ and $w_1 P_{f_0} w_0$, which yields a contradiction.

The proof of the claim when the preferences of the workers are acyclical and fP_ww is identical and is thus omitted.

■

As we can see in the following example, the result does not hold when some workers are not acceptable to every firm.

Example 1 *Let* $F = \{f_1, f_2, f_3\}, W = \{w_1, w_2, w_3\}$. *Let* $P_{f_1} : w_1w_2,\ P_{f_2} : w_2w_3,\ P_{f_3} : w_3w_1$. *Let* $P_{w_1} : f_3f_1,\ P_{w_2} : f_1f_2,\ P_{w_3} : f_2f_3$

There is a simultaneous cycle of length three $w_1P_{f_1}w_2P_{f_2}w_3P_{f_3}w_1,\ f_3P_{w_1}f_1P_{w_2}f_2P_{w_3}f_3$, *but there is no cycle of length two. Let* $q_{f_1} = q_{f_2} = q_{f_3} = 1$. *The market* (F, W, P) *has two stable matchings* μ *and* ν *defined by* $\mu(w_1) = f_1,\ \mu(w_2) = f_2,\ \mu(w_3) = f_3$ *and* $\nu(w_1) = f_3$, $\nu(w_2) = f_1,\ \nu(w_3) = f_2$.

It is well known that under common preferences, the stable set can be generated by a serial dictatorship. This result still holds true when the preferences are acyclical. If the preferences of the firms are acyclical, there is an underlying order on the set of the workers such that the unique stable matching is generated by a "corrected" serial dictatorship: the first worker chooses from among the firms at which she is acceptable, worker t chooses from among the firms at which she is acceptable and which have at least one position available, and so forth.

Proposition 4 *If the preferences of the firms are acyclical, there is an ordering of the workers* $w_{i_1}, ..., w_{i_n}$ *such that the stable matching* μ *is given by:*

$$\mu(w_{i_1}) = \max_{P_{w_{i_1}}} \{f \in F : w_{i_t}P_f\emptyset\}$$

$$\mu\left(w_{i_{t+1}}\right) = \max_{P_{w_{i_{t+1}}}} \left\{f \in F : w_{i_{t+1}}P_f\emptyset\right\} \setminus \{f \in F : |\{w_{i_s} : s \leq t, \mu(w_{i_s}) = f\}| = q_f\}$$

for all t, $1 \leq t \leq n-1$.

Proof. Assume that the preferences of the firms are acyclical. Let $w_{i_1} \in W$ such that there are no $w \in W$ and $f \in F$ such that $wP_f w_{i_1} P_{\hat{f}} \emptyset$. Such a w_{i_1} exists because P_F is acyclical. For $0 \leq t \leq n-1$, let $w_{i_{t+1}} \in W$ such that there are no $w \in W \setminus \{w_{i_1}, ..., w_{i_t}\}$ and $f \in F$ such that $wP_f w_{i_{t+1}} P_f \emptyset$. Such a $w_{i_{t+1}}$ exists because P_F is acyclical. To complete the proof it suffices to show that the matching μ defined in the claim is stable. The proof is obtained by contradiction. Let i_t be such that (w_{i_t}, f) blocks the μ. Set $F_t = \{f \in F : w_{i_t} P_f \emptyset\} \setminus \{f \in F : |\{w_{i_s} : s \leq t, \mu(w_{i_s}) = f\}| = q_f\}$. We have $\mu(w_{i_t}) = \max_{P_{w_{i_t}}} F_t$. First, assume that $|\mu(f)| < q_f$. Then, $f \in F_t$, yields a contradiction. Second, consider the case in which $|\mu(f)| = q_f$. Because (w_{i_t}, f) blocks μ, $w_{i_t} P_f w$ for some $w \in \mu(f)$. From the definition of the sequence $i_1, ..., i_n$, it follows that $w = w_{i_s}$ for some $s > t$. Thus, $f \in F_t$, which yields a contradiction. ■.

An analogous result holds when the preferences of the workers are acyclical.

Proposition 5 *If the preferences of the workers are acyclical, there is an ordering of the firms* $f_{i_1}, ..., f_{i_m}$ *such that the stable matching* μ *is given by:*

$$\mu(f_{i_1}) = Ch\left(P_{f_{i_1}}, \{w \in W : f_{i_t} P_w w\}\right)$$

$$\mu\left(f_{i_{t+1}}\right) = Ch\left(P_{f_{i_{t+1}}}, \left\{w \in W : f_{i_{t+1}} P_w w\right\} \setminus \{w \in W : w \in \mu(f_{i_s}) \text{ for some } s \leq t\}\right),$$

for all t, $1 \leq t \leq m-1$.

The proof of Lemma 5 is similar to the proof of Lemma 4 and is thus omitted.

Specifically, if the preferences of the workers are acyclical and publicly known, it is a dominant strategy for the firms to reveal their true preferences and capacities if using any selection from the stable set is used (see Romero-Medina and Triossi, 2011).

Corollary 1 *Let $\mu\left(P_F, P_W\right) \in \Gamma\left(F, W, P_F, P_W\right)$ for all P_F, P_W.*

1. *Assume that P_F is acyclical. Then, for every $w \in W$ $\mu\left(P_F, P_W\right) R_w \mu\left(P_F, P_w', P_{-w}\right)$ or all P_w'.*

2. *Assume that P_W is acyclical. Then, for every $f \in F$ $\mu\left(P_F, P_W\right) R_f \mu\left(P_f', P_{-f}, P_W\right)$ for all P_f'.*

Additionally, the existence of singleton cores in the many to one matching problems avoids simple manipulation strategies on the colleges' side as the use of truncation strategies.

3.2 Simultaneous Cycles and Efficiency.

For the marriage model, Roth (1982) shows that there is no individually rational matching that all agents of one side on the market strictly prefer to their corresponding optimal stable matching. Roth (1985) extends this result to the college admissions problem, which shows that this weak Pareto optimality property holds for the "one side" of the market if the preferences of the firms are responsive.[6] Roth (1985) also shows that the property is, in general, false for the "many side" of the market.

From Proposition 4 (5), it follows that if the preferences of the firms (workers) are acyclical, then unique stable matching is strongly efficient for the workers (firms). The same result holds when the preferences do not have simultaneous cycles.

[6] Martínez et al. (2004) shows that this property does not holds if the preferences are substitutable but not responsive.

Proposition 6 *Assume that there are not simultaneous cycles and let μ be the unique stable matching. There are no individually rational matchings ν,ρ such that $\nu P_W \mu$ or $\rho P_F \mu$.*

Proof. We prove that if there exists a matching ν such that $\nu P_W \mu$, then there exists a simultaneous cycle. Define two sequences of workers and firms $\{w_t\}_{0 \leq t \leq T-1}$ and $\{f_t\}_{1 \leq t \leq T}$ as follows. Let $w_1 \in W$ such that $\nu(w_1) \neq \mu(w_1)$. For all $t \geq 1$ set $f_t = \nu(w_t)$. For all $t \geq 1$, let $w_t \in \mu(f_t) \setminus (\mu(f_t) \cup \{w_1, ..., w_{t-1}\})$. The sequence stops when it reaches some T such that $\mu(f_T) \subset (\nu(f_T) \cup \{w_1, ..., w_{T-1}\})$. Such a T exists because W and F are finite. Set $K = max\{t < T \mid f_t = f_T\}$. There is no loss of generality in assuming $K = 1$. The firms $f_1, ..., f_{T-1}$ are distinct and $T \geq 3$ because $f_t \neq f_{t+1}$ for every $t = 1, ..., T-2$. The workers $w_1, ..., w_{T-1}$ are distinct by construction. For $0 \leq t \leq T-2$, $\nu(w_t) = f_t = \mu(w_{t+1})$ and $f_T = \mu(w_1)$. We have $f_1 P_{w_1} f_2 P_{w_2} f_3 ... f_{T-1} P_{w_{T-1}} f_T = f_1$. Let $t \geq 1$. We have $w_t \notin \mu(f_t)$, $w_{t+1} \in \mu(f_t)$ and $\nu(w_t) P_{f_t} \mu(w_t)$. Thus $w_{t+1} P_{f_t} w_t$, otherwise (f_t, w_t) would block μ. It follows that $w_1, ..., w_T, f_1, ..., f_T$ constitute a simultaneous cycle.

The proof of the other part of the claim is identical and is thus omitted.

■

4 Conclusions

The contribution of this paper has been to show the roles that the acyclicity of the preferences and, in particular, the absence of simultaneous cycles have in the design of well behaved and efficient mechanisms. We have shown that by imposing acyclical priorities or by verifying that acyclicity is satisfied by the agents' preferences, we can guarantee the existence of singleton cores.

Singleton cores play an important role in the practical design of markets. As an example, in the NMRP, Roth and Peranson (1999) found that the core in the NMRP for a set of reported preferences was small but not a singleton. This result was both surprising and extremely relevant for deciding the proper version of the deferred acceptance algorithm to be implemented in the redesign of the NMRP. In addition, the existence of singleton cores in the many-to-one matching problems avoids simple manipulation strategies on the colleges' side by using truncation strategies (Ma, 2002, 2010, see also Sotomayor, 2011). In this paper, we have shown that the absence of simultaneous cycles implies that the core is a singleton. Additionally, acyclicity in the preferences of either side of the market guarantees that the core is a singleton. This finding is particularly relevant because in problems in which the agents are endowed with priorities acyclicity can be implemented in the design.

Acyclicity can also contribute to avoiding strategic behavior and facilitating the provision of efficient allocations. In this sense, we find that if the preferences of the firms are acyclical and publicly known, it is a dominant strategy for workers to reveal their true preferences. Therefore, the strategic interaction among the both sides of the market is simplified.

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