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Give Peace a Chance: The Effect of Incomplete and Imperfect Information on Mediation*

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Abstract

In this paper we study mediation when two countries might fight a war over the ownership of a resource. Under complete information mediation is always successful, but a little bit of asymmetric information or some imperfect observability may render mediation impossible

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1 Introduction

Rationalist theories of war explain conflict as the rational choice of countries see Hirshleifer (1991), Skaperdas (2002), Jackson and Morelli (2007) and the surveys by Garfinkel and Skaperdas (2007) and Jackson and Morelli (2011). This approach has studied how trade, long term relationships or availability of resources may amplify or efface the incentives of war, see Skaperdas and Syropoulos (1996), (2001), Garfinkel and Skaperdas (2000) and Beviá and Corchón (2010). In the latter paper, it is shown that, under complete information if resource inequality is not very large and the outcome of war is neither too sensitive nor too insensitive to war expenses, a war fought in terms of a "decisive battle" -a battle that will give control of all the remaining resources to the victor- can be stopped by means of transfers from the "rich" to the "poor" country. As Clausewitz (1832) noted, war is seldom fought on these terms.[1] More often war takes place when two countries dispute a resource.[2] The war ends when one of the countries decisively defeats the other and gains control over the resource.

In this paper we address the following question: Are there self-enforcing divisions of a resource under dispute? In other words, are there divisions of the resource such that, if attained, both countries have incentives to respect?

We consider two setups. The first, which we call the Unmediated Game has two stages. In the first stage countries decide if they declare war or not. If one of the countries decides to declare war, in the second stage there is war. If both countries decide not to fight there is peace and they get zero payoffs.

In the second setup, which we call the Mediated Game, there is a mediator whose objective is to achieve peace. This mediator designs and implements a partition of the resource. Once this partition has been implemented and both countries have taken possession of their share in the resource, we have

[1] An example in which one of the sides was beaten so decisively that it dissapeared from history was the defeat of the Visigoths by the Arabs in the river Guadalete in southern Spain, 711.

[2] Modern examples include the Scramble for Africa between all major European powers in 1881-1914 and the Great Game between British and Russian empires in 1813-1907.

the same situation as in the unmediated game, i.e., countries may declare war, etc. Thus the mediator can alter the initial conditions of the game but she cannot stop further conflict. We are aware that we endow the mediator with considerable powers. But if such a mediator fails to obtain peace, mediators with less power will fail too. Thus our study may be interpreted as providing necessary conditions for mediation to be successful.

In some cases the mediator can be thought to be a real person like in the treaty of Tordesillas (1494) in which the Pope suggested a division of south America between Spain and Portugal (which lasted up to the end of Spanish and Portuguese domination in the Americas). In others the mediator is just a surrogate of the bargaining of both countries about the shares of the resource. Finally we may think of a distribution of the resource achieved for historical or geographical reasons or by a previous conflict like in the case of Cyprus.[3]

We first study complete information. In the unmediated case war is the only equilibrium (Proposition 1). The explanation is that countries can always choose war efforts in such a way that expected payoffs from war are positive. In the mediated case, we show that there is a division of the resource such that, in any equilibrium, both countries will choose peace (Proposition 2). The reason for this result is that the status quo for each country is its share of the resource and the possibility of losing it should war occur makes countries reluctant to go to war. In contrast with Beviá and Corchón (2010) this result does not depend on resource inequality or on how war expenses affect the outcome of the war.

Next we consider asymmetric information. We assume that country one has private information about how valuable the resource is. In particular, this country may have a high or a low valuation.[4] As an illustration we may think that country one has done research on the existence of a valuable resource in

[3]Mediation has been widely considered in mechanism design following the influencial work of Myerson (1982 (see also Crawford and Sobel (1982)). However, the issue there is the incentives of agents to correctly reveal the information they possess to the mediator. In our model there is no transmission of information from agents to the mediator.

[4]We show that private information about other parameters of the game are equivalent to private information about valuations.

a territory under dispute and the result of the test is only in the hands of this country only. We assume that the mediator cannot distinguish between the two valuations (types) of country one. Thus, in order to achieve peace, she has to devise a partition of the resource such that country one has incentives to remain peaceful no matter its type. We show that if this country is peaceful under high valuation it is peaceful under low valuation. Thus, the share to this country is dictated by its high valuation. We will see that this is one of the key ingredients of the failure of mediation to achieve peace under asymmetric information.

We prove that under no mediation, war is the unique Bayesian equilibrium of the asymmetric information game (Proposition 3). Under mediation, we show a necessary and sufficient condition for peace to hold as the unique Bayesian equilibrium (Proposition 4). This condition holds when

1. It is very likely that the valuation of country one is high.
2. The high and the low valuations of country one are similar.
3. The strength of country two is low.
4. The valuation of country two is low.

Points 1 and 2 are very intuitive because, when uncertainty is small we are back, essentially, to the full information scenario. Points 3 and 4 are explained because when the strength/valuation of country two is low, it will not fare well in war, so a small share of the resource would be enough to keep it peaceful.

Conversely war is likely to arise in the following cases (proposition 5)

1. Low probability that country one has a high valuation
2. Large dispersion in the valuations of country one.
3. The strength of country two is large.
4. Large valuation of country one.

Again Points 2-4 are intuitive. But Point 1 is completely counterintuitive. It says that mediation fails when we are very close to complete information! The explanation is that the share of country one is dictated by its high valuation. But when this country is weak with a high probability, war looks like a good prospect for country two, so this country needs a high share for peace. Thus, being close to complete information might be bad for peace![5]

So far we have considered the actions of players to be perfectly observable. However, as *Data Fusion* emphasizes, information about true actions is seldom reliable. The purpose of our paper is not directed to its foundations (see, e.g. Herrera, Herrera-Viedma and Verdegay, 1996) but to its consequences on the achievement of war and peace. In particular we assume that the declaration of war is imperfectly observable. This might be due to unreliability of communication channels like when Japan declared war on the US in 1941 but got the timing wrong and thus the attack on Pearl Harbor came before the war declaration. Also, if we understood war declaration not as a letter, but as taking certain actions that can only mean war, i.e., putting a fleet or tanks in attacking position, launching planes or rockets, etc. these actions could be imperfectly observed by the other country.[6]

To simplify our calculations, we assume that both countries have identical war strengths. We show that under imperfect observability, war is the only equilibrium outcome of the unmediated game (proposition 7). Under mediation, we find a necessary and sufficient condition for peace to be feasible. We show that this condition is likely to be met when

1. The observability errors are small
2. The inequality in the valuations of countries is small, too.

Point 1 is very intuitive because if errors are small, we are close to complete information. However, Point 2 is not obvious. Moreover, we show that, given any degree of imperfect observability, a sufficiently large difference in valuations makes peace impossible (Propositions 8, 9 and 10). This is because if one of the countries is stronger in valuations, its benefits under conflict are greater than

[5] Invoking the Myerson-Satterthwaite (1983) theorem to explain the failure of mediation in our case might be misleading because the assumptions needed for the theorem to hold do not necessarily hold here; see Corchón (2009) for a discussion. Also in our case, there is incomplete contracting because nothing can stop war if countries decide so. Thus, as usual, asymmetric information wreaks havoc on our model, but the reason for this is, in some cases, a subtle one.

[6] An example of this is the Nazi invation of USSR in 1941 which despite all preparations -and even warnings- came as a surprise to the soviet leaders.

under mediation, resulting in a more arduously achievable mediation. Thus, under imperfect observability, mediation failure is not as extreme as it could be under asymmetric information where mediation can fail for all values in a sequence of π tending to zero. Under imperfect observability, mediation failures can only happen when q or o tend to one *and* γ to zero (or infinite).

The rest of the paper goes as follows. Section 2 spells the model. Section 3 studies the full information case. Section 4 considers asymmetric information. Section 5 studies imperfect observability. Finally, Section 6 gathers our final comments.

2 The Model

Two countries dispute a resource which they value in V_1 and V_2 respectively. In case of war, they incur irrecuperable expenses of G_1 and G_2.

Let p_i be the probability that country $i = 1, 2$ wins the war. It could also be interpreted as the share of i in the resource after the war. p_i is determined by an asymmetric contest success function of the following form:

$$p_i = \frac{\beta_i G_i^\alpha}{\sum_{j=1}^{2} \beta_j G_j^\alpha} \tag{1}$$

$\alpha \in [0, 1]$ measures the responsiveness of the probability of winning to expenses. When $\alpha = 1$, the probability of winning is proportional to relative war efforts, and when $\alpha = 0$, the probability of winning is independent of war efforts. $\beta_i \in (0, \infty)$ is the productivity of country i in war efforts. Defining $\theta \equiv \beta_2/\beta_1$ as the relative productivity of country two in war, the contest success function can be written as:

$$p_1 = \frac{G_1^\alpha}{G_1^\alpha + \theta G_2^\alpha} \quad \text{and} \quad p_2 = \frac{\theta G_2^\alpha}{G_1^\alpha + \theta G_2^\alpha}, \tag{2}$$

where $\theta \in (0, \infty)$ is the relative productivity of country two in war efforts.

Both countries are risk-neutral. In case of war, their expected payoffs are

$$u_1 = V_1 \frac{G_1^\alpha}{G_1^\alpha + \theta G_2^\alpha} - G_1 \quad \text{and} \quad u_2 = V_2 \frac{\theta G_2^\alpha}{G_1^\alpha + \theta G_2^\alpha} - G_2, \tag{3}$$

We consider two setups. In the first one, which we call the *Unmediated Game*, countries decide in the first stage if they declare war or not. If one of the countries declares war, in the second stage, the conflict is waged and payoffs are delivered. If both countries decide not to declare war, they obtain zero payoffs.

In the second setup, which we call the *Mediated Game*, there is a mediator whose objective is to achieve peace. The mediator designs a partition of the resource and has the power to implement this partition. Once both countries have taken possession of their share in the resource, we have the same situation as in the unmediated game: countries may engage in war, but in this case, the payoffs of peace are the shares of the resource given by mediation.

3 Full Information Case

Assume that all the parameters defining the game are common knowledge between the two countries. We solve the game beginning with the second stage. Assuming that there is war, first order conditions (FOC) of expected payoff maximization for each country are:

$$\frac{\partial u_1}{\partial G_1} = V_1 \frac{\alpha\theta G_1^{\alpha-1} G_2^{\alpha}}{\left(G_1^{\alpha} + \theta G_2^{\alpha}\right)^2} = 1 = V_2 \frac{\alpha\theta G_2^{\alpha-1} G_1^{\alpha}}{\left(G_1^{\alpha} + \theta G_2^{\alpha}\right)^2} = \frac{\partial u_2}{\partial G_2} \tag{4}$$

It can be verified that these conditions are also sufficient. Thus, the efforts under war are given by (4). This implies that

$$\frac{V_2}{V_1} = \frac{G_2}{G_1}. \tag{5}$$

Plugging (5) in (4) and defining $\gamma \equiv \left(\frac{V_2}{V_1}\right)^{\alpha}$, we obtain the full information war effort, G_i^F, for $i = 1, 2$.

$$G_1^F = \frac{V_1 \alpha\theta\gamma}{\left(1+\theta\gamma\right)^2} \tag{6}$$

$$G_2^F = \frac{V_2 \alpha\theta\gamma}{\left(1+\theta\gamma\right)^2} \tag{7}$$

Substituting expressions (6) and (7) in (3), we obtain the equilibrium payoff for each country:

$$u_1^F = \frac{V_1\left[1+\theta\gamma\left(1-\alpha\right)\right]}{\left(1+\theta\gamma\right)^2} \tag{8}$$

$$u_2^F = \frac{V_2\theta\gamma\left(1+\theta\gamma-\alpha\right)}{\left(1+\theta\gamma\right)^2} \tag{9}$$

We check that payoffs are strictly positive. Thus, we have the following result:

Proposition 1 *Under complete information, war is the unique equilibrium of the unmediated game.*

Let us now study the mediated game under the assumption that the mediator knows all parameters that define the game. Suppose the mediator suggests that country one take a share $\varepsilon \in (0,1)$ in the resource. For peace to hold in equilibrium, the following condition should hold for the first country.

$$u_1^F = \frac{V_1\left[1+\theta\gamma\left(1-\alpha\right)\right]}{\left(1+\theta\gamma\right)^2} \leq \varepsilon V_1. \tag{10}$$

which amounts to

$$\frac{1+\theta\gamma\left(1-\alpha\right)}{\left(1+\theta\gamma\right)^2} \leq \varepsilon \tag{11}$$

Performing a similar calculation for the second country, we obtain

$$\varepsilon \leq \frac{1+\theta\gamma\left(1+\alpha\right)}{\left(1+\theta\gamma\right)^2}. \tag{12}$$

Then, if peace is achieved, conditions (11) and (12) imply

$$\frac{1+\theta\gamma\left(1-\alpha\right)}{\left(1+\theta\gamma\right)^2} \leq \varepsilon \leq \frac{1+\theta\gamma\left(1+\alpha\right)}{\left(1+\theta\gamma\right)^2} \tag{13}$$

Thus, peace is possible iff

$$\frac{1+\theta\gamma\left(1+\alpha\right)}{\left(1+\theta\gamma\right)^2} \geq \frac{1+\theta\gamma\left(1-\alpha\right)}{\left(1+\theta\gamma\right)^2} \tag{14}$$

which always holds while, $\alpha \in [0,1]$. We have proved the following:

Proposition 2 *Under complete information, there is a division of the resource such that peace is the unique equilibrium of the mediated game.*

4 Asymmetric Information

In this section, we take into account that some parameters of the game are privately known. This may be when valuation V_i or war productivity β_i is known by country i while country j has beliefs about the values of these variables. In order to simplify our calculations, we assume that $\alpha = 1$.

We first show that both sources of uncertainty -valuation or war productivity- are equivalent when $\alpha = 1$. In this case, the payoff functions are:

$$u_i = V_i \frac{\beta_i G_i}{\sum_{j=1}^{2} \beta_j G_j} - G_i \tag{15}$$

Defining a new decision variable $\phi_i = \beta_i G_i$, we rewrite (15) as follows:

$$u_i = V_i \frac{\phi_i}{\phi_1 + \phi_2} - \frac{\phi_i}{\beta_i} \tag{16}$$

Now multiplying both sides of the equation with β_i, we have that

$$v_i = \beta_i V_i \frac{\phi_i}{\phi_1 + \phi_2} - \phi_i \tag{17}$$

where v_i is the scaled utility of the conflict for each country and in which the productivity in conflict efforts and valuations play the same role. Thus, without loss of generality, we assume that valuations are the only source of uncertainty.

In order to focus on the simplest case, we assume that only country one has private information about $V_1 \in \{\rho V, V\}$ where $\rho \in [0, 1]$, and the probability that $V_1 = V$ is π. When country one has valuation ρV (resp. V), her war effort is $\underline{G}_1$ (resp. $\overline{G}_1$). We also define here the ratio of the valuations, γ, as the valuation of the second country in terms of the valuation of the high-type first country, i.e., $\gamma = \frac{V_2}{V}$.

As we did in the previous section, we begin by analyzing war in the unmediated case. In this case, payoffs in case of conflict for the first country are:

$$\overline{u}_1 = V \frac{\overline{G}_1}{\overline{G}_1 + \theta G_2} - \overline{G}_1 \tag{18}$$

$$\underline{u}_1 = \rho V \frac{\underline{G}_1}{\underline{G}_1 + \theta G_2} - \underline{G}_1 \tag{19}$$

Since payoff functions are concave, using the FOCs, the best reply functions of the first country according to her valuation are:[7]

$$\overline{G}_1 = \sqrt{V\theta G_2} - \theta G_2 \tag{20}$$

$$\underline{G}_1 = \sqrt{\rho V\theta G_2} - \theta G_2 \tag{21}$$

The utility of the second country is:

$$u_2 = V_2\pi \left\{ \frac{\theta G_2}{\overline{G}_1 + \theta G_2} \right\} + V_2 (1-\pi) \left\{ \frac{\theta G_2}{\underline{G}_1 + \theta G_2} \right\} - G_2 \tag{22}$$

The FOC for the second country is:

$$\pi V_2 \frac{\theta \overline{G}_1}{\left(\overline{G}_1 + \theta G_2\right)^2} + (1-\pi) V_2 \frac{\theta \underline{G}_1}{\left(\underline{G}_1 + \theta G_2\right)^2} = 1 \tag{23}$$

Note that the payoff function is concave. Substituting (21) and (20) into the equation above, we obtain the equilibrium efforts for the second country :

$$G_2^* = \frac{\rho V}{\theta} \left[\frac{1 - \pi\left(1 - \sqrt{\rho}\right)}{\frac{\rho}{\theta\gamma} + 1 - \pi (1-\rho)} \right]^2 \tag{24}$$

Substituting (24) into (21) and (20), we find the equilibrium arming by a high and low value type as follows:

$$\overline{G}_1^* = V\sqrt{\rho} \frac{\left(\frac{\rho}{\theta\gamma} + (1-\pi)\left[1 - \sqrt{\rho}\right]\right)\left(1 - \pi\left[1 - \sqrt{\rho}\right]\right)}{\left(\frac{\rho}{\theta\gamma} + 1 - \pi\left[1-\rho\right]\right)^2} \tag{25}$$

$$\underline{G}_1^* = \rho V \frac{\left(\frac{\rho}{\theta\gamma} - \pi\sqrt{\rho}\left[1 - \sqrt{\rho}\right]\right)\left(1 - \pi\left[1 - \sqrt{\rho}\right]\right)}{\left(\frac{\rho}{\theta\gamma} + 1 - \pi\left[1-\rho\right]\right)^2} \tag{26}$$

Assuming $\sqrt{\rho} \geq \pi\theta\gamma\left[1 - \sqrt{\rho}\right]$, we obtain $\underline{G}_1^* \geq 0$ as desired. Then, using the payoff functions of the conflict of each type and each country, i.e., equations (18), (19), and (22) we get the following equilibrium payoffs for each country:

For a high-valuation first country:

$$\overline{u}_1^* = V \left[\frac{\frac{\rho}{\theta\gamma} + (1-\pi)\left(1 - \sqrt{\rho}\right)}{\frac{\rho}{\theta\gamma} + 1 - \pi (1-\rho)} \right]^2 \tag{27}$$

[7] Here we disregard non-negativity constraints. Later on, we will make an assumption that will make sure that war efforts are indeed positive.

For a low-valuation first country:

$$\underline{u}_1^* = \rho V \left[\frac{\frac{\rho}{\theta\gamma} - \pi\left(1 - \sqrt{\rho}\right)}{\frac{\rho}{\theta\gamma} + 1 - \pi\left(1 - \rho\right)} \right]^2 \tag{28}$$

Finally, for the second country:

$$u_2^* = V_2 \frac{\left[1 - \pi\left(1 - \rho\right)\right]\left[1 - \pi\left(1 - \sqrt{\rho}\right)\right]^2}{\left(\frac{\rho}{\theta\gamma} + 1 - \pi\left(1 - \rho\right)\right)^2} \tag{29}$$

We check that payoffs are strictly positive. Thus, we have the following result:

Proposition 3 *Under asymmetric information war is the unique equilibrium of the unmediated game.*

We now focus on the mediated case. We assume that the mediator does not know the type of country one. Thus, her proposed share of the resource, ε, cannot depend on the type of this country. Thus, for peace to hold in equilibrium, the following conditions should be satisfied:

$$\begin{aligned} \overline{u}_1^* &\leq \varepsilon V \\ \underline{u}_1^* &\leq \varepsilon \rho V \\ u_2^* &\leq \left(1 - \varepsilon\right) V_2 \end{aligned}$$

Using equations (27), (28), and (29), we find the following conditions for peace to hold in equilibrium:

$$\left[\frac{\frac{\rho}{\theta\gamma} + \left(1 - \pi\right)\left(1 - \sqrt{\rho}\right)}{\frac{\rho V}{\theta V_2} + 1 - \pi\left(1 - \rho\right)} \right]^2 \leq \varepsilon \tag{30}$$

$$\left[\frac{\frac{\rho}{\theta\gamma} - \pi\left(1 - \sqrt{\rho}\right)}{\frac{\rho}{\theta\gamma} + 1 - \pi\left(1 - \rho\right)} \right]^2 \leq \varepsilon \tag{31}$$

$$1 - \frac{\left[1 - \pi\left(1 - \rho\right)\right]\left[1 - \pi\left(1 - \sqrt{\rho}\right)\right]^2}{\left(\frac{\rho}{\theta\gamma} + 1 - \pi\left(1 - \rho\right)\right)^2} \geq \varepsilon \tag{32}$$

Note that the left-hand side of inequality (30) is always larger than the left-hand side of inequality (31). That implies that (31) is not binding, i.e., peace holds in equilibrium for the low-valuation first country as long as it holds for the high-valuation first country. Hence, the condition for peace is reduced to

$$1-\frac{\left[1-\pi\left(1-\rho\right)\right]\left[1-\pi\left(1-\sqrt{\rho}\right)\right]^{2}}{\left(\frac{\rho}{\theta\gamma}+1-\pi\left(1-\rho\right)\right)^{2}}\geq\left[\frac{\frac{\rho}{\theta\gamma}+\left(1-\pi\right)\left(1-\sqrt{\rho}\right)}{\frac{\rho}{\theta\gamma}+1-\pi\left(1-\rho\right)}\right]^{2} \tag{33}$$

Rearranging (33), we reach the following result for peace to hold in equilibrium:

Proposition 4 *Peace is the unique equilibrium of the mediated game iff*

$$\frac{1}{\gamma}\geq\frac{\theta\left(1-\pi\right)\left(1-\sqrt{\rho}\right)^{2}\left\{\left(1-\pi\right)^{2}-\pi^{2}\rho\right\}}{2\rho\sqrt{\rho}\left(1-\pi\left(1-\sqrt{\rho}\right)\right)} \tag{34}$$

The right-hand side of (34) will be denoted as $F(\pi,\rho)$. Recall the definitions of the parameters involved in (34):

γ is the ratio between V_2 and V_1

θ is the relative strength of country two

ρV is the low value of V_1

π is the probability that V_1 is high

If the right hand side of (34) is negative, the inequality is fulfilled automatically and mediation can be successful. This occurs iff $\left(1-\pi\right)^{2}/\pi^{2}<\rho$. Thus, peace has a good chance if ρ is as close to one as possible (so both types have similar valuations) or when π is close to one so the probability that the valuation is high is overwhelming. In both cases, the uncertainty is small.

When the right-hand side of (34) is non-negative (i.e., $\left(1-\pi\right)^{2}/\pi^{2}\geq\rho$), things are more involved. Let us discuss the role of each parameter separately.

The roles of γ and θ are clear. When the strength of country two or its valuation is low, peace holds because this country could only demand a small share and this is always feasible.

To find the role of π, we partially differentiate the right-hand side of (34) with respect to π and find the following expression:

$$\frac{\left[-2+2\pi\left(1+\sqrt{\rho}\right)-\sqrt{\rho}\right]\left(\sqrt{\rho}-1\right)^{2}}{2\rho^{3/2}}. \tag{35}$$

The term $\frac{\left(\sqrt{\rho}-1\right)^2}{2\rho^{3/2}}$ is always positive. Therefore, the sign of this derivative depends on the following expression, which we define as a function, $K(\pi,\rho)$, as follows.

$$K(\pi,\rho) = -2 + 2\pi\left(1+\sqrt{\rho}\right) - \sqrt{\rho}. \tag{36}$$

It is a matter of simple calculation to see that there exists a $\widehat{\pi} \in (0,1)$ such that $K(\pi,\rho) = 0$; indeed, $\widehat{\pi} = \frac{2+\sqrt{\rho}}{2\left(1+\sqrt{\rho}\right)}$. However, recall that peace certainly holds if $(1-\pi)^2/\pi^2 < \rho$. Now let us define

$$L(\pi,\rho) = (1-\pi)^2 - \pi^2\rho$$

First of all, $L(\pi,\rho)$ is strictly decreasing in π, as $\frac{\partial L(\pi,\rho)}{\partial \pi} = -2(1-\pi) - 2\pi\rho < 0$. Solving $L(\pi,\rho) = 0$, there is a critical value for π, $\pi^c = \frac{-1+\sqrt{\rho}}{-1+\rho}$ such that $L(\pi^c,\rho) = 0$.[8] Now realize that $\pi^c < \widehat{\pi}$, $\forall \pi \in (0,1)$, implying that the peace condition is fulfilled at $\widehat{\pi}$ and beyond. Thus we conclude that a large value of π gives peace a good chance. This is very intuitive because uncertainty is very small. However, a sufficiently low value of π makes war very likely. Indeed when $\pi \simeq 0$, the condition for war is, approximately,

$$2\frac{\rho\sqrt{\rho}}{\left(1-\sqrt{\rho}\right)^2} < \gamma\theta. \tag{37}$$

The left-hand side is increasing in ρ, so in this case, war arises as a combination of a low probability of country one being of the high type, a large valuation and/or strength of country two and small a valuation for the low type. In all these cases, the share demanded by the high type looks too expensive for country two, which has good chances of winning a sizeable chunk of the prize by going to war.

It is remarkable that despite the fact that when $\pi = 0$, war cannot happen in the mediated case (see Proposition 2), war is perfectly possible when π is very close to zero. Even more, war is more likely the closer the value of π is to zero. This reveals an interesting discontinuity in the prevention of war.

[8] The other solution is $\pi = \frac{-1-\sqrt{\rho}}{-1+\rho} \notin (0,1)$.

Finally, it is easily seen that $\frac{\partial F(\pi,\rho)}{\partial \rho} < 0$ for $F(\pi,\rho) \geq 0$. Again, if ρ is close to 1, $F(\pi,\cdot)$ is close to 0 and peace has a fair chance. This is because we are close to complete information. But if ρ is low, $F(\pi,\cdot)$ could be large and make peace impossible. This is because the low type will make very little effort in war and will lose it with a high probability but its share in the resource is determined by the high type, so war looks like a good prospect for country two.

Summing up, from the discussion above, we learn the following:

Proposition 5 *Failure of mediation under incomplete information is due to:*

1. Large dispersion in the valuation of countries.

2. Large relative strength of country two.

3. Low probability that country one has a high valuation

4. Large dispersion in the valuations of country one.

The mechanism under which war occurs is that country one appears, in expected terms, as a weak opponent in war, but successful mediation demands that the share of this country is given by the characteristics of the high type.

We now study the expected cost of war, which we measure by expected war expenses. Thus, we consider only a part of the destruction brought by war; civilian casualties, capital losses and the like are disregarded. As a further simplification we assume that $\theta = \gamma = 1$. Using equations (24), (25) and (26), we obtain the expected cost of war:

$$A(\pi,\rho) = G_2^* + \pi \overline{G}_1^* + (1-\pi)\underline{G}_1^* = V\sqrt{\rho}\frac{\left[\pi\left(\sqrt{\rho}-1\right)^2(1-\pi)+\sqrt{\rho}\right]}{1+\rho+\pi(\rho-1)} \tag{38}$$

We obtain that $\partial A(.,\rho)/\partial\pi$ equals to

$$-V\frac{1+\pi^2\left(\sqrt{\rho}-1\right)^2\left(\sqrt{\rho}+1\right)+2\rho-\rho^{\frac{3}{2}}+2\pi\left(\sqrt{\rho}-\rho+\rho^{\frac{3}{2}}-1\right)}{\left[1+\pi(\rho-1)+\rho\right]^2}\left(\sqrt{\rho}-1\right)\sqrt{\rho}$$

The expression outside the numerator is positive. Hence, the sign of this derivative depends on the expression in the numerator. Now let us define

$$J(\pi,\rho) = 1+\pi^2\left(\sqrt{\rho}-1\right)^2\left(\sqrt{\rho}+1\right)+2\rho-\rho^{\frac{3}{2}}+2\pi\left(\sqrt{\rho}-\rho+\rho^{\frac{3}{2}}-1\right)$$

If $J(\pi, \rho) > 0$ for $\pi \in (0,1)$, and for any $\rho \in (0,1)$, then expected cost of war is increasing in π. Now observe that $J(0, \rho) = 1 + 2\rho - \rho^{3/2} > 0$. Since

$$\frac{\partial J(., \rho)}{\partial \pi} = 2\left(\sqrt{\rho} - 1\right)\left(1 + \rho + \pi\left(\rho - 1\right)\right) < 0$$

$J(\pi, \rho) > 0$. This result enables us to state the proposition below.

Proposition 6 *Given $\theta = 1$, $\gamma = 1$ the cost of war is strictly increasing in π.*

It can be shown that the cost of war is also increasing in ρ but the proof is cumbersome and is available upon request.

An interesting exercise would be to maximize the cost of war subject to the condition that mediation is unsuccessful. Unfortunately, such an exercise yields expressions that are intractable. We were not able to produce an example in which the cost of war exceeds $V/4$. Given that, under no mediation the cost of war may be as high as $V/2$ (for $\pi = \rho = 1$), this suggests that mediation prevents the most costly wars, but more work would be needed on this issue.

5 Imperfect Observability

In this section, we assume that the declaration of war is imperfectly observable. In order to simplify the picture, assume that $\alpha = \theta = 1$. Suppose, say, country two decides to declare war. In such a case, country one perceives a probability $q \in (0,1)$ that she faces an army G_2 strong and with probability $1 - q$ that she perceives that the other country is peaceful.[9] In the case analyzed so far, $q = 1$. Thus, expected payoffs for player one in cases in which she declares war are

$$u_1 = q(\frac{G_1}{G_1 + G_2} V_1 - G_1) + (1 - q)(V_1 - G_1), \tag{39}$$

and similarly for country two, i.e.

$$u_2 = q(\frac{G_2}{G_1 + G_2} V_2 - G_2) + (1 - q)(V_2 - G_2). \tag{40}$$

[9] A more general assumption would be that country *one* observes different values of G_2 with certain probabilities. Reasons of tractability inclined us towards the simple assumption used in the main text.

First-order conditions of payoff maximization are

$$\frac{G_2}{(G_1+G_2)^2}V_1 = \frac{1}{q} \text{ and } \frac{G_1}{(G_1+G_2)^2}V_2 = \frac{1}{q}, \tag{41}$$

so we obtain $G_2/G_1 = \gamma$ as before. Equilibrium values of war expenses are:

$$G_1 = \frac{qV_1\gamma}{(1+\gamma)^2} \text{ and } G_2 = \frac{qV_2\gamma}{(1+\gamma)^2}. \tag{42}$$

Notice that these values of war expenses are "rational expectation" values in the sense that they are the correct anticipation made by players taking into account the imperfections of the problem.

Finally, plugging the values of G_1 and G_2 obtained in (42) into (39) and (40), we obtain

$$u_1 = \frac{V_1}{(1+\gamma)^2}\left\{1+(1-q)\,\gamma(2+\gamma)\right\} \text{ and} \tag{43}$$

$$u_2 = \frac{V_2}{(1+\gamma)^2}\left\{\gamma^2+(1-q)\left[1+2\gamma\right]\right\} \tag{44}$$

When information about the moves of the other country is perfect, i.e., $q = 1$, (43) and (44) collapse into (8) and (9). When $q = 0$, countries can successfully hide their movements, so they expect to conquer the resource without any fight. Thus, $u_i = V_i$.[10] Notice that the expected payoffs from war decrease with q.

We now evaluate the benefits of peace. In the unmediated case, if both countries do not declare war, they cannot fight for the resource under dispute, so they get zero anyway. Thus, as it happened under complete information, we have that:

Proposition 7 *Under imperfect observability, war is the only equilibrium outcome of the unmediated game.*

We now turn to the mediated case. Here the mediator suggests that country one takes a share $\varepsilon \in (0,1)$ in the resource. Suppose that when, say, country

[10] Actually, this is only true at the limit, since when the army built by, say, country *two* is zero, there is no best reply for country *one*; i.e., the payoff for country *one* gets larger as long as its war expenses go to zero. But at the limit, the CSF is not defined.

two has chosen not to declare war, country one perceives that country two has declared war with probability $1 - o$ where $o \in (0, 1)$. In the previous analysis $o = 1$. Thus, expected payoffs of peace are

$$u_1 = o\varepsilon V_1 \text{ and} \quad (45)$$

$$u_2 = o(1 - \varepsilon)V_2. \quad (46)$$

Notice that under perfect observability, $o = 1$, expressions (45) and (46) are reduced to those under complete information. Thus, peace holds iff

$$1 - \frac{\gamma^2 + (1-q)(1+2\gamma)}{o(1+\gamma)^2} \geq \varepsilon \geq \frac{1 + (1-q)\gamma(2+\gamma)}{o(1+\gamma)^2}, \quad (47)$$

which, given that the bounds in (47) guarantee that $\varepsilon \in [0, 1]$, it amounts to

$$o \geq \frac{1 + \gamma^2 + (1-q)(1 + 2\gamma + \gamma(2+\gamma))}{(1+\gamma)^2} \quad (48)$$

Under full information, $q = 1$ and $o = 1$, so (48) is now

$$1 \geq \frac{(1+\gamma)^2 - 2\gamma}{(1+\gamma)^2} \quad (49)$$

and mediation is always feasible.

The following propositions yield conditions for mediation to be (or not to be) successful. Before this, let us introduce a new definition. Let

$$F(o, \gamma, q) \equiv o(1+\gamma)^2 - 1 - \gamma^2 - (1-q)(1 + 2\gamma + \gamma(2+\gamma)). \quad (50)$$

From (48), it follows that peace holds iff $F(o, \gamma, q) \geq 0$. We are now prepared to study the impact of o, γ and q on the feasibility of mediation. Since the implications of our results are similar, we will discuss them after the propositions.

Proposition 8 *If $q(1 + 4\gamma + \gamma^2) \geq (1+\gamma)^2$, $\exists \hat{o}$, such that $\forall o < \hat{o}$ war occurs and $\forall o > \hat{o}$ mediation can be successful. If $q(1 + 4\gamma + \gamma^2) < (1+\gamma)^2$, mediation cannot be successful irrespective of the value of o.*

Proof. Obviously, $F(\cdot, \gamma, q)$ is increasing in o. Now we compute

$$F(0, \gamma, q) = -\gamma^2 - (1-q)(2\gamma + \gamma(\gamma+2) + 1) - 1$$

$$F(1, \gamma, q) = q - 2\gamma + 4q\gamma - \gamma^2 + q\gamma^2 - 1$$

Thus, $F(0,\gamma,q) < 0$. Therefore, if $F(1,\gamma,q) \geq 0$, by the intermediate value theorem, there is a $\hat{o}$ such that $F(\hat{o},\gamma,q) = 0$. Given that $F(\cdot,\gamma,q)$ is increasing in o, this point is unique and the proposition follows.
Finally, if $F(1,\gamma,q) < 0$ since $F(\cdot,\gamma,q)$ is increasing in o, $F(o,\gamma,q) < 0$ $\forall o$ and mediation is impossible. ■

Proposition 9 *If $o(\gamma+1)^2 \geq \gamma^2+1$, there is a value of q, $\hat{q}$, such that for all $q < \hat{q}$ war occurs, and for all $q > \hat{q}$, mediation can be successful. If $o(\gamma+1)^2 < \gamma^2+1$, mediation cannot be successful irrespective of the value of q.*

Proof. Obviously, $F(o,\gamma,\cdot)$ is increasing in q. We now compute

$$\begin{aligned} F(o,\gamma,0) &= o - 4\gamma + 2o\gamma - 2\gamma^2 + o\gamma^2 - 2 \\ F(o,\gamma,1) &= o(\gamma+1)^2 - \gamma^2 - 1. \end{aligned}$$

Since $F(o,\gamma,0)$ is increasing in o and $F(1,\gamma,0) = -2\gamma - \gamma^2 - 1 < 0$, $F(o,\gamma,0) < 0$. Thus, if $F(o,\gamma,1) \geq 0$ by the intermediate value theorem, $\hat{q}$ exists and, since $F(o,\gamma,\cdot)$ is increasing in q, is unique.
Finally, if $F(o,\gamma,1) < 0$ since $F(o,\gamma,\cdot)$ is increasing in q, $F(o,\gamma,q) < 0$ $\forall q$ and mediation is impossible. ■

Proposition 10 *If $.5o + .75q > 1$, there is an interval $[\underline{\gamma},\bar{\gamma}]$ such that mediation can be successful iff $\gamma \in [\underline{\gamma},\bar{\gamma}]$. If $.5o + .75q < 1$, mediation can never be successful.*

Proof. We first check that $\partial^2 F(o,\gamma,q)/\partial\gamma^2 < 0$, so $F(o,\cdot,q)$ is strictly concave. Now we solve $F(o,\gamma,q) = 0$, whose largest solution is

$$\gamma = \frac{o + 2q - 2 + \sqrt{-4q + 2oq + 3q^2}}{2 - o - q} \tag{51}$$

The expression under the root is positive iff $2o + 3q > 4$. This implies $o + 2q > 2$, so the γ found in (51) is positive. Note that $\lim_{\gamma\to 0} F(o,\gamma,q) = o + q - 2 < 0$ and $\lim_{\gamma\to\infty} F(o,\gamma,q) = -\infty$, so the value of γ found in (51) is $\bar{\gamma}$. The smallest value of $F(o,\gamma,q) = 0$ serves as $\underline{\gamma}$. If this value does not exist take $\underline{\gamma} = 0$.

Finally, the last part of the proposition follows from the fact that, as we note before, $F(o,\gamma,q)$ is negative at the extremes, so if the interval where $F(o,\gamma,q) \geq 0$ does not exist, $F(o,\gamma,q) < 0$ for all γ. ■

Propositions 8-10 assert that, as expected, good observability is good for peace. But inequality in the valuations of countries has an important role, too, since the condition in proposition 8 (resp. 9) can be written as $q \geq (1+\gamma)^2/(1+4\gamma+\gamma^2)$ (resp. $o \geq (\gamma^2+1)/\left(\gamma+1\right)^2$).[11] The expression on the right hand side of this inequality has a minimum at $\gamma = 1$ and tends to 1 when $\gamma \to 0$ or $\gamma \to \infty$. Thus, high or low values of γ make mediation more difficult. This is also seen in proposition 10. Note that the minimum of $(1+\gamma)^2/(1+4\gamma+\gamma^2)$ (resp. $(\gamma^2+1)/\left(\gamma+1\right)^2$) is 2/3 (resp. 1/2). Thus, any value of q (resp o) less than 2/3 (resp. 1/2) makes war unstoppable.

Finally, we remark that an implication of Propositions 8-10 is that, given any degree of precision in the observability of o and q, a sufficiently large difference in valuations makes peace impossible.

6 Final Comments

In this paper, we study how mediation can stop war. We show that a little asymmetric information or observability problems when valuations between countries are very different may cause mediation to fail.

In order to make the model tractable, we have made a number of assumptions and leave aside some questions that we discuss now.

1. We assumed that after war, no compensation is paid by the loser. But there are several historical examples showing otherwise, i.e., the Franco-Prussian War (1870-1), World War I (1914-1918), etc. Farmer and Pecorino (1999) have shown that when the loser has to pay the expenses of the winner, total expenses might sky-rocket due to the fact that the winner pays nothing. It would be interesting to know the possibilities of mediation under this case. Given that

[11] Since $\frac{(1+\gamma)^2}{(1+4\gamma+\gamma^2)} > \frac{(\gamma^2+1)}{(\gamma+1)^2}$ the condition on q is more demanding than the condition on o.

payoffs under war are smaller than under no compensation, intuition suggests that mediation can work even better in this case than under no compensation, which is the case considered in our paper.

2. When stopping the war for both types becomes impossible, it is not clear what a peace-seeking mediator should do. For instance, she might suggest a mediation that would stop war if country one would be of the low type. This would give room for peace to hold, at least for some cases.

3. Another extension is to consider a political bias as in Jackson and Morelli (2007). In this paper, the agent running a country might receive high profits from the victory but pay only a fraction of the cost of war. In this case, it looks like if the bias is sufficiently large, mediation cannot be successful, even under complete information. This would add to our list of mediation failures.

4. Finally, mediation usually takes place in several rounds. It would be interesting to model mediation as in the model of Rubinstein (1982).

We hope that our paper sheds light on the powers and limitations of mediation and pinpoints the cases in which mediation is bound to fail. Therefore, other measures like a direct UN intervention have to be taken.

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